



Research article

Effects of facet joint degeneration on stress alterations in cervical spine C5–C6: A finite element analysis

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Abstract: It has been demonstrated that articular facet degeneration can cause local strain alterations and induce neck pain. This study aims to quantify the biomechanical effects of normal and degenerated C5–C6 articular facets, and evaluate the correlation of mechanical strain between healthy and degenerated spine. A 3-dimensional finite element (FE) model of the C5–C6 cervical spine was developed [Model 0 (M0)]. The asymmetric models of C5–C6 bilateral articular facet joint were established separately to mimic articular facet joint degeneration. The capsule ligament stiffness of C5–C6 unilateral facet joint was altered with minimum and maximum threshold to simulate capsule ligaments' lesion and calcification [Model 1 (M1) and Model 2 (M2), respectively]. Besides, the cervical C5–C6 unilateral articular facet joint direction was changed by 5° and 10° forward to imitate the moderate joint hyperplasia and severe osteophyte (Model 3 and Model 4 respectively). M1 increased the rotation range of ipsilateral side (left), while M2 reduced, and both had limited effect on the contralateral side (right). The angle increased in Model 3 (M3) (61°) and Model 4 (M4) (55°) comparing to M0 during the axial rotation, and the angle of M4 was larger. M3 and M4 increased the nucleus pulposus pressure with and without controlled angular displacement during axial rotation. The pressure of nucleus pulposus increased during M1 rotating to the abnormal side but decreased when rotating to the other side, but the results of M2 were opposite. The capsule ligament stiffness made an impact on segmental mobility and vertebral spatial position, and the sagittal angle of articular facet joint exerted an influence on disc pressure distribution.

Keywords: spine stress and strain; spine degeneration; articular facet joint; finite element analysis; osteoporosis; regeneration; mechanical stress alteration

1. Introduction

Degeneration of spinal articular facet joint and related structures leading to spatial position deviations and abnormal functional activities can cause neck pains and related symptoms, which are surrounded by abundant micro-nerves and vascular distributions [1]. The most common risk factors for spinal articular facets and related structures' degeneration include trauma, chronic lesion, the posture of working and living, etc. In the clinic, it is generally believed that this status of the spinal articular facet can accelerate the degeneration of the structures and induce corresponding symptoms. However, to our best knowledge, the reports referred to quantitative research were currently rare.

The degeneration of cervical spinal articular facet joint comprises the changes of the material properties of the joint capsule ligament and the morphological variations of the articular facet joint. It is well known that spine ligaments not only control the motion of the segments, but also maintain spine stability. As major cervical ligaments, capsular ligaments (CL) are one of the fundamental supportive structures in the region [2]. In anatomy, the articular facet joint and CL are paired, while joint bears part of the head weight, and CL stabilizes the activities of the corresponding joint.

In the study of ligament biomechanics, researchers generally attempted to introduce the ligament resection sequentially for quantification of the mechanical effects of a certain type in the cadaveric specimens [3,4]. But, these studies only stimulated the status of complete ligaments rupture, which was incompatible with our research purpose. Previous studies demonstrated that individual differences in mechanical and material properties are ubiquitous, subjected to a variety of factors including age, gender, and cumulative micro-trauma, etc. [5–7]. For instance, aging would reduce the elastic modulus of ligaments, and some specific ligaments of males have a higher modulus than female, naturally [6,8]. In addition, certain sub-failure trauma did not completely break the ligaments, but repetitive loading would significantly reduce the rigidity of the ligaments [9]. Therefore, the relationship between the mechanical sensitivity of these CL and stiffness variation is critical to understand the biomechanical effects of CL degeneration.

Pal et al. [10] reports the direction of the cervical facet joint systematically with specimens, and the standard deviation of the middle and lower cervical spine facet joints related with the vertebral level was range from 6.16° to 9.43° . Meanwhile, the angle difference was found not only among individuals, but also between the bilateral sides of the same cervical spine. An epidemical report assessed 200 cervical spondylosis patients with axial CT scan imaging and demonstrated that aging and facet tropism concerning the sagittal plane were associated with the facet degeneration [11]. Therefore, depth understanding how physiological or pathologic changes of anatomical structural respond to physiological activities will guide us clinical practices reasonably. It is necessary to investigate the mechanism underlying physiological or pathologic changes of anatomical structural respond to physiological activities, which can provide useful information in clinical practices.

Previous literatures showed that the C5–C6 segment was more inclined to be injured, and was one of the most degenerative segments reasonably [12,13]. Although a few authors reported some meaningful data through *in vivo* and *in vitro* studies at present [4,6], biomechanical effects cannot be quantified and accurately analyzed through conventional methods. For an instant, the tissues' property

materials and spine functional activities of the formalin soaked cadaver specimen was distinctive with the live human body. Besides, there is a complex additive effect of the intact cervical spine physiological function, which could hardly figure out the microstructure changes referring to the geometrical structure and properties separately. Also, the *in vivo* studies mostly relied on radiological imaging measurement, which was difficult to evaluate the spinal three-dimensional movement.

FE analysis has been widely used by the researchers in biomechanical experiments and has illustrated great flexibility for customizing of various parameters for specific purposes [14,15]. The cervical disc degeneration led to intervertebral space stenosis, increased the facet joint pressure, and caused joint capsule and other soft tissue injury [16]. Moreover, the articular facet injury accelerated disc deterioration [17].

The hypothesis of the study is that unilateral ligament stiffness and sagittal angle changes in articular facet joints could break the balance of stress distribution under specific activities. Therefore, this study was aimed to quantitatively analyze the biomechanical changes and to comprehend the inherent law related to the potential damages and mechanical properties of C5–C6 segment's unilateral articular capsule ligament and corresponding joint sagittal angle under physiological activities.

2. Materials and methods

2.1. Model development

As shown in Figure 1, an anatomically accurate 3-dimensional FE model of the C5–C6 segment had been previously developed based on a 30-year old healthy male subject in a supine position [18]. The subject did not have any history of neck pain and related disorders, and any spine degeneration, malformation, variation, trauma and any other factors that could affect the experiment. The C5–C6 segment was the most frequently affected by disc degenerative disease [19]. Computed tomography (CT) images of a 0.625 mm slice was scanned. The CT images of C5–C6 segment were processed with commercial software Mimics 10.1 (Materialise Inc., Leuven, Belgium), Geomagic Studio 11 (Geomagic, Inc., Research Triangle Park, NC, USA), Hypermesh 11.0 (Altair Engineering, Inc., Executive Park, CA, USA) sequentially. The components of the FE model included two vertebrae, one intervertebral disc and major ligaments, and the vertebra bone composed of cortical, cancellous bones and endplates, which were modeled with hexahedron elements with isotropic, linear mechanical properties. Each articular facet was shell elements and the initial gap between the joints was 0.4 mm.

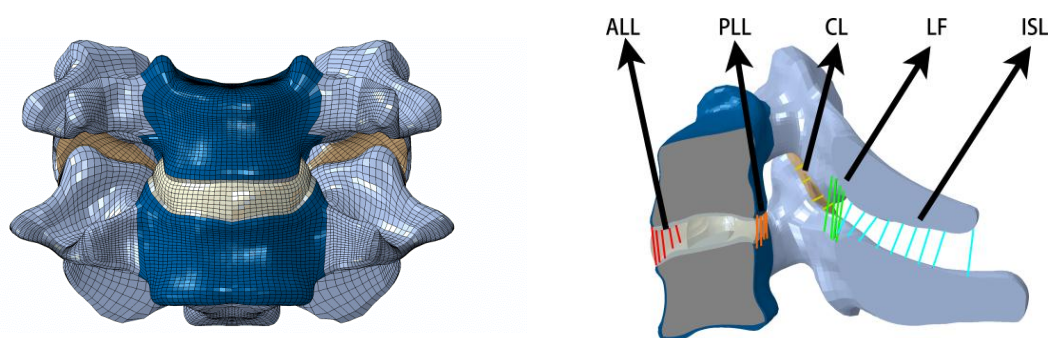


Figure 1. The cervical FE model of C5–C6 cervical spine (including vertebra, disc, and ligament).

Table 1. Material properties and element types in the FE analysis.

Component	Element type	Young's modulus (MPa)	Poisson's ratio	Reference
Cortical bone	Hexahedral (C3D8)	10000	0.3	[21]
Cancellous bone	Hexahedral (C3D8)	450	0.25	[21]
Facet cartilage	Shell (S4R)	10	0.4	[22]
Ligaments	Connectors	Nonlinear	-	[23]
Annulus fiber	Rebar	110	0.3	[24]
Annulus ground	Hexahedral (C3D8)	Nonlinear	-	[20]
Nucleus	Fluid cavity	Incompressible	-	[25]
Endplate	Hexahedral (C3D8)	1200	0.29	[26]

The inter-facets were designed to frictionless surface to surface contact. Intervertebral disc consisted of an incompressible nucleus pulposus, annulus ground substance and embedded fibers, and nucleus pulposus was created as a fluid cavity which constituted 25% of the disc volume and had a bulk modulus of 2000 MPa. The annulus fibers were modeled as rebar elements and the matrix volume of fibers concerning the surrounding ground substance was varying from 23% in the outer layer to 5% in the inner, while the orientation of fibers ranged from $\pm 25^\circ$ in the outer layers to $\pm 45^\circ$ in the inner. Annulus ground substance was modelled with hexahedral elements with nonlinear hyperelastic mechanical properties [20]. Five major ligaments, anterior longitudinal, posterior longitudinal, interspinous, capsular, and flaval ligaments, were attached to nonlinear tension-only truss element. Detailed material properties assigned to the model obtained from literature were listed in Table 1.

2.2. Model validation

We applied 1 Nm pure moment on the three planes to produce flexion, extension, lateral bending and axial rotation, and followed a 73.6 N load near the center of C5 and C6 for head physical gravity simulation. The predicted ranges of motion (ROM) of the C5–C6 segment model under same loading conditions with published experimental data were used for validation [27].

2.3. FE analysis

The basic load-displacement curve of capsular ligament was obtained from the literature by using average value of each parameter [23], the parameters controlling the load-displacement curve included the displacement, force at the transition point and stiffness of the linear region. The minimum stiffness of capsular ligament was created by raising the former one parameter and reducing the latter two parameters by one standard deviation. The maximum stiffness of capsular ligament was created

using the opposite approach, but the toe region was adjusted to a linear pattern so that the slope of the toe region was no greater than the slope of linear region. Therefore, each ligament would have three load-displacement curves which served as average, minimum and maximum stiffness.

To evaluate the biomechanical effects of capsular ligament in various stiffness on C5–C6 segment, we changed the material properties of unilateral capsular ligament (left) by the minimum or maximum stiffness separately for ligament degeneration simulation as shown in Figure 2, while the remaining ligament (right) maintained the average value, which were Model 1 (minimum stiffness capsular ligament) and Model 2 (maximum stiffness capsular ligament). Variations in ligament stiffness could be created by modifying these values obtained from the literature [18,27].

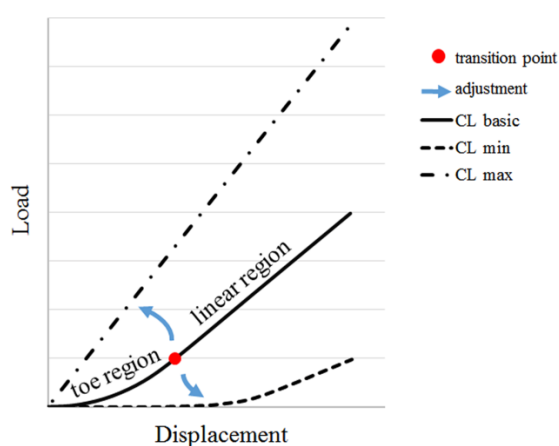


Figure 2. Adjusted three levels of capsular ligament stiffness (illustrated with displacement and loading curve, linear portion starts after pre-load).

The average cervical articular facet tropism was $56.36^\circ \pm 8.82^\circ$ [10], and the facet angle of our basic model was about 66° with the horizontal plate of upper endplate. Referred to the similar study of lumbar spine for mechanics analysis of asymmetric model [21], we altered the angles of left articular facets pair forward 5° and 10° separately to imitate the unilateral facet joint degeneration, which were Model 3 (M3) (61°) and Model 4 (M4) (55°), respectively Figure 3.

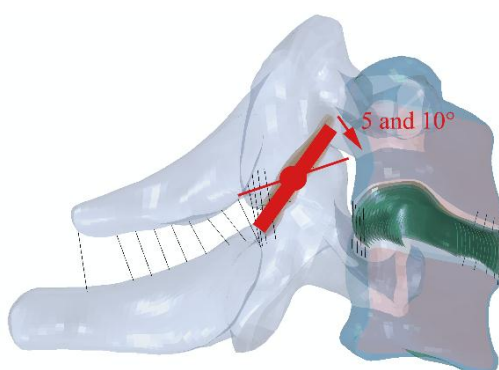


Figure 3. Adjusted method of cervical articular facet tropism of C5–C6 segment.

Above all, five FE models were compared in this study. The loadings and boundary conditions of basic model M0, and adaption models of M1, M2, M3 and M4 were maintained to calculate the segmental kinematics and disc stress distribution.

3. Results

3.1. Changes of C5–C6 segmental movement

As shown in Figure 4, the angular displacement of C5 during flexion, extension, lateral flexion, and rotation movement of M1 increased under physiological loading. However, the ranges of six degrees of M2 decreased. For the changes of C5 articular facet joint sagittal angle, we simulated the cervical flexion activities under the 1 Nm moment, and the resulted angles of M0, M3, M4 were 5.01 °, 5.05 °, and 4.84 °, separately. And during the extension, the angles of M0, M3, and M4 were 5.73 °, 5.72 °, and 5.43 °, separately. During the axial rotation, we found that the angle increased in M3 and M4 comparing to M0, furthermore, the angle of M4 was larger. However, the angle decreased in M3 and M4 during the lateral bending, and M4 had the larger magnitude of the reduction.

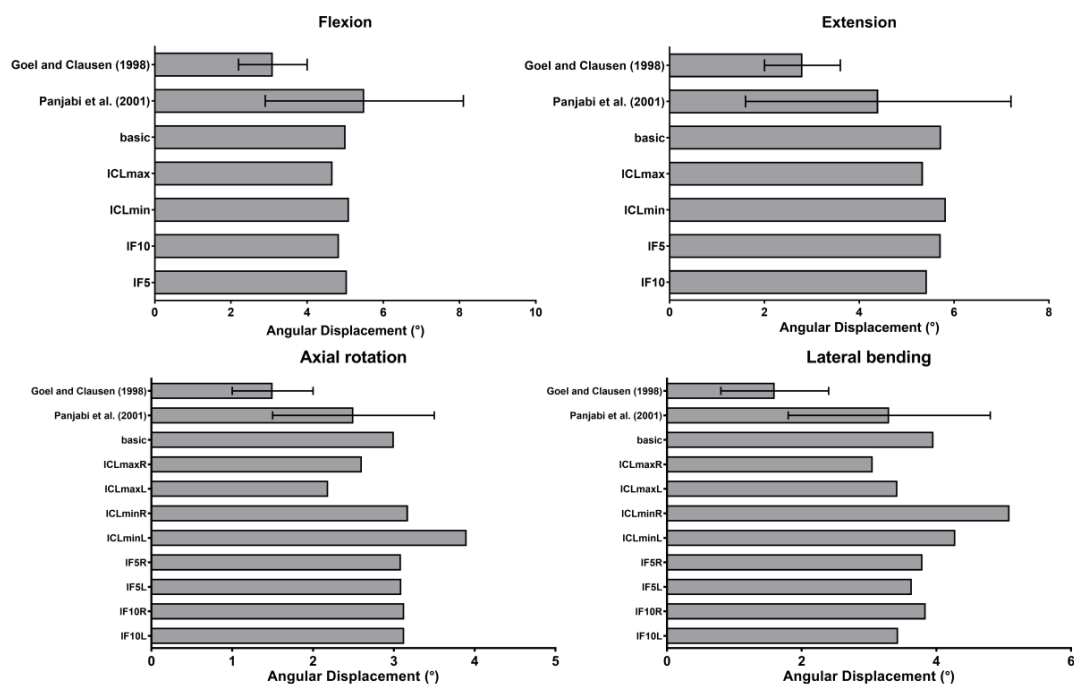


Figure 4. Angular displacement of C5 with different stiffness ligaments under the physiological activities (ICLmin = M1, ICLmax = M2, IF5 = M3, IF10 = M4).

3.2. Displacement of C5 segmental spatial position

Figure 5 showed the displacement of all asymmetric models (i.e., M1, M2, M3 and M4) in the coronal plane during flexion, extension and neutral position. In detail, the spinous process of C5 of

M1 turned to the left during flexion, while the vertebral body of C5 of M2 turned to the left. During extension, the spinous process of C5 of M2 turned to the left. M4 turned to the left under the head weight in neutral position. The displacements were remarked by red circles.

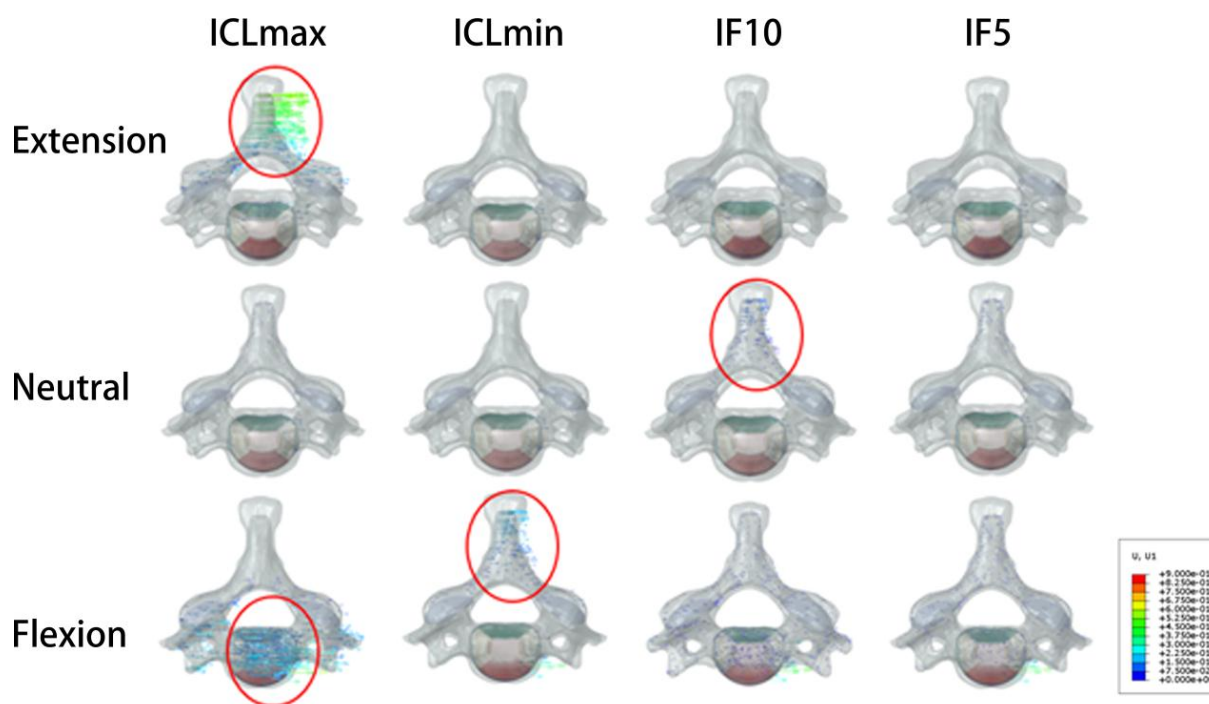


Figure 5. Displacement of C5 spatial position under the physiological activities (ICLmin = M1, ICLmax = M2, IF5 = M3, IF10 = M4).

3.3. Abnormal intervertebral disc stress distribution

As showed in Figure 6, stress distribution of C5–C6 intervertebral disc during flexion and extension, abnormal were demonstrated in the all asymmetric models (i.e., M1, M2, M3 and M4), however, insignificantly.

During axial rotation of the cervical spine, it could cause a comparatively large effect on stress concentrations on nucleus pulposus. Under the right rotation, the pressure in the nucleus of M2 was higher than M0, but M1 was lower than M0. And, M2 was lower than M0 under the left rotation. Besides, the amplitude of M4's ascending curve was lower than M0 during right rotation, but it was higher during left rotation. And, under the controlled axial rotation angle, the curve trends of M1 and M2 were consistent with moment loading condition, but the M2 was magnified as showed with the red lines.

During left lateral bending, the pressure in the nucleus of M1 was higher than M0, but M2 was lower than M0, and the amplitude of M4's ascending curve was lower than M0. However, M2 was higher than M0, and the amplitude of M4's ascending curve was higher than M0 during right lateral bending. Additionally, M3 and M4 were both higher than M0 under the same angular displacement of lateral bending.

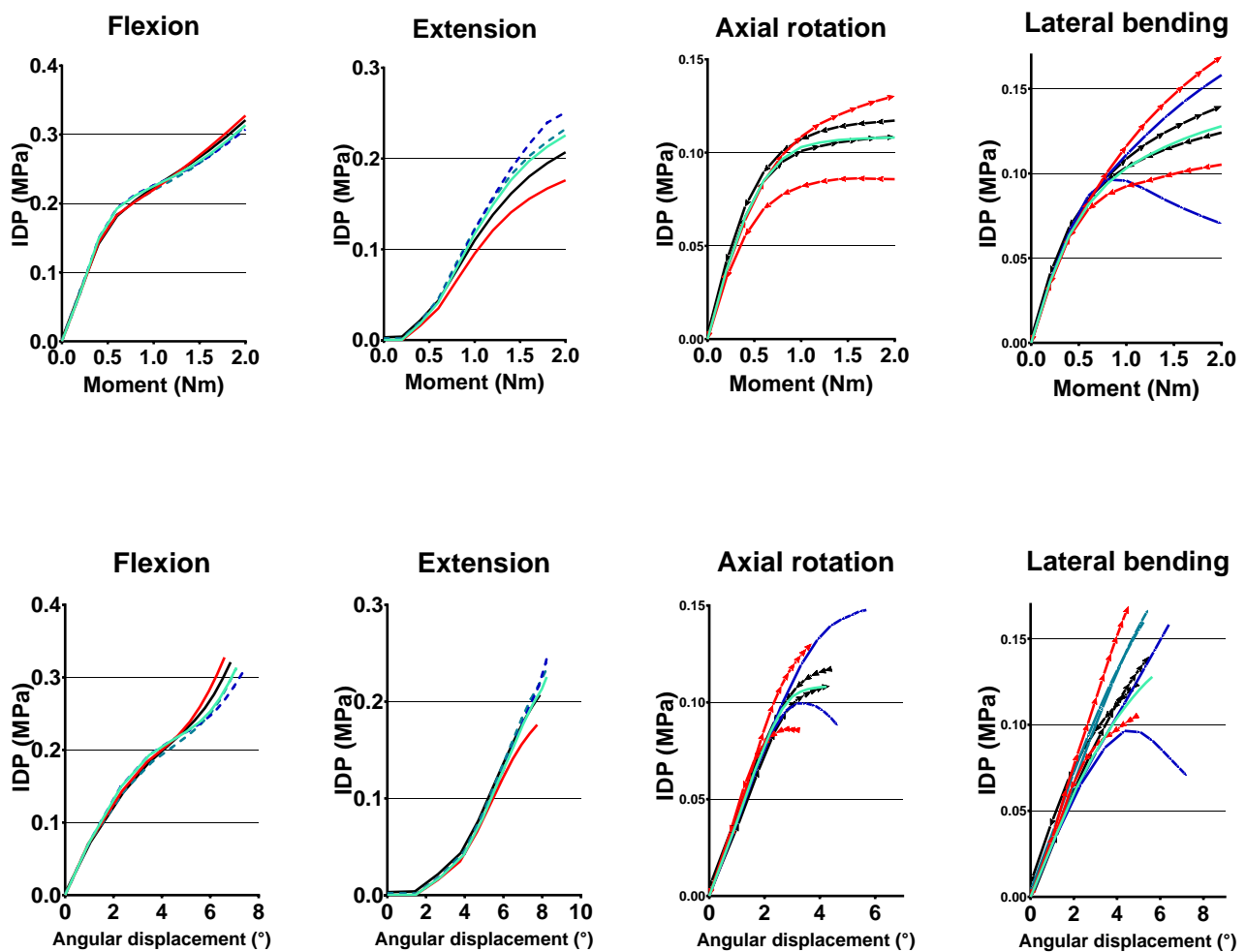


Figure 6. Average stress on C5–C6 intervertebral disc under the physiological activities (ICLmin = M1, ICLmax = M2, IF5 = M3, IF10 = M4).

4. Discussion

The human facet joint, a synovial joint located on the posterior-lateral spine, is highly susceptible to degenerative changes and plays a significant role in morbidities [28]. Previous studies reported that the range of activity of the lower cervical spine decreased with aging [29], which was attributed to the reduction of spine disc height and formation of osteophyte owing to the aging [2,23,30]. This study showed that we controlled the angular displacement of C5 articular joint within a certain range (about 2° in segmental rotational angle), the C5–C6 disc distribution of all asymmetric models (M1, M2, M3 and M4) had no significant difference with the symmetrical model (M0) during the rotation, lateral flexion activities. The results indicated that avoidance of excessive activity benefited the asymmetric degeneration cervical spine disc.

Degeneration of human spine emerges with aging naturally, while would be accelerated with repeated sub-injury, trauma and inclined the mechanical strength of soft tissues, i.e., ligaments [7,31,32]. The results showed that variation of joint capsule ligament rigidity (M1, M2) had a limited restriction

during flexion and extension but more obvious changes during rotation, lateral flexion. We found that M1 increased the rotation range of ipsilateral side (left), while M2 reduced, and both had limited effect on the contralateral side (right). During extension, the C5 spinous processes of M2 appeared to the ipsilateral side (left) while the C5 vertebra deviation to the same side during flexion, which was caused by the limitation of high stiffness of capsule ligament. In contrast, the low stiffness capsule ligament of M1 increased the ipsilateral axial rotation, and therefore the C5 spinous processes turned to ipsilateral side during flexion. In summary, the displacement of M2 was more distinct than M1 during activities, it implied that ligament lesion lead to objective segment instability at the initial stage, but with the ligament calcification changed, it would produce a kind of protective stability through limiting the neck activity range.

For the degeneration of the articular joint facet angel, M3 and M4 increased the nucleus pulposus pressure with and without controlled angular displacement during axial rotation. During lateral flexion, M3 and M4 increased the nucleus pulposus pressure significantly, which was greater than turning to the contralateral side. The result was consistent with a radiology imaging study [12], which showed that the larger angle of cervical facet joint declined the anti-shear ability and led to more vulnerable of the disc annulus fiber in the same side. Interestingly, when we controlled angular displacement, the disc stress distribution of M3 and M4 was similar and both higher than M0. This study suggested when the unilateral articular facet joint was rotated forward and closer to the horizontal plane; the range of axial rotation would be increased slightly, which was consistent with previous report [33]. Additionally, the disc pressure increasing was relating to the segment activity.

There were some limitations in this study. Firstly, *in vivo* and *in vitro* experiments are needed to reliably validate and supplement in future studies. Even though we adjusted the capsule ligament stiffness according to the load-displacement curve, but the interactions between the tissue's stiffness, geometrical morphology and other material properties were omitted. Secondly, we did not address the analysis of cervical lordosis, neither of other articular facet joint sagittal angle corresponded to different spine segment and skeletal muscles, which affected the spine kinematics [11,34]. Therefore, these findings with individual characteristics should be explained cautiously to other spine segments. The further studies should focus on these variable factors for understanding thoroughly of the spine biomechanics with more optimized experimental design and powerful computer analysis capabilities.

5. Conclusion

The capsule ligament stiffness made an impact on segmental mobility and vertebral spatial position, and the sagittal angle of articular facet joint exerted an influence on disc pressure distribution. The findings demonstrated a vicious cycle of the cervical spine degeneration, which confirmed that it is a protective calcification for stability alternated with progressive degeneration for instability.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

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