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From vertical to horizontal structures : New optimization challenges in electricity markets

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" Tout ce que nous pouvons faire et que nous ferons, sans nous lasser, est d'en savoir de plus en plus, et d'y comprendre de moins en moins. Tout s'explique. Tout reste obscur... ! "

Jean d'Ormesson

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Résumé

La chaine d'approvisionnement énergétique a fortement évolué aux cours des 20 dernières années. La libéralisation des marchés de l'électricité et les nouvelles technologies ont fortement influencé la manière d'envisager la production et la transmission d'électricité. Les modèles mathématiques classiques utilisés dans les problèmes lié à l'énergie ont besoin d'être revus pour intégrer les contraintes pratiques modernes.

Un problème classique pour un Compagnie Génératrice (CG) est le problème de Unit Commitment (UC) qui consiste à établir un plan de production pour une demande en électricité connue. Lorsque ce problème fut considéré, le prix de l'électricité et la demande étaient relativement simple à estimer comme une seule CG nationale avait le monopole du marché. Ce problème a été étudié de manière extensive en utilisant de la Programmation Mathématique (PM). Aujourd'hui, le prix de l'électricité est relativement volatile à cause de l'introduction de marchés dérégulés et la demande du marché est répartie entre plusieurs CGs en compétition sur divers marchés. Une CG ne peut se limiter à considérer un problème de UC seul pour envisager sa production. Il y a un besoin d'intégrer les incertitudes liées au marché de l'électricité et aux quantités à produire aux modèles utilisés pour qu'une CG puisse établir un plan de production rentable.

La technologie a aussi permis d'envisager de nouveaux concept tel que les Micro-Grilles (MGs). Une MG est composée d'un ensemble de consommateurs reliés à travers un réseau de transmission, possédant des générateurs d'électricité et optimisant sa consommation interne. Ce concept est possible grâce à l'utilisation croissante d'énergies renouvelables locales ainsi que d'appareils interconnectés. Cependant, étant donné que les énergies renouvelables ont un faible rendement, sont intermittentes et que les appareils de stockage d'énergie sont encore peu efficaces, les MGs ne peuvent pas envisager d'être pleinement autonome en électricité. Il y a donc une nécessité d'avoir un fournisseur d'électricité externe pour avoir suffisamment d'électricité disponible à tout moment. Une CG jouant le rôle de fournisseur auprès d'une MG fait face énormément d'incertitude concernant la demande à cause de la gestion interne de la MG sur laquelle elle n'a pas de contrôle.

Dans cette thèse, des problèmes d'optimisation intégrant de nouvelles contraintes modernes liés à l'approvisionnement énergétique sont étudiés via la PM. Plusieurs problèmes considèrant des interactions entre plusieurs acteurs sont modélisés via des formulations bi-niveaux. Nous illustrons comment les difficultés liées aux contraintes modernes peuvent être exploitées pour obtenir des propriétés permettant de reformuler les problèmes étudiés en formulation linéaire en nombre entiers. Des heuristiques performantes sont obtenues à partir des formulations exactes dont certaines sont applicables à des problèmes plus généraux. Une analyse extensive de la performance des méthodes de résolution ainsi que de l'influence des contraintes modernes sont présentées dans diverses expériences numériques.

Abstract

The electricity supply chain has seen a strong evolution of its environnement over the past years. Liberalization of electricity markets and new technologies are having a strong influence on how to organize electricity production and transmission. Previous computational methods used in electricity related problems need to be updated in order to follow the evolution of real life constraints.

One classical problem for a generation company (GC) is the Unit Commitment problem (UC) which consists in establishing an electricity production plan over a given time horizon to satisfy a demand in electricity. When first considered, the price of electricity and demands were relatively easy to estimate as national GCs had a monopoly over the market. This problem has been widely studied and solved using Mathematical Programming (MP) methods. Today, the price of electricity can be relatively volatile due to the introduction of deregulated electricity markets and the demand of the market is split among several independent GCs competing on several different markets. When estimating profit, a GC cannot therefore consider solving only a UC problem. There is a need to integrate the uncertainty on the price of electricity and the quantities to produce when a GC must take decisions in order to establish a production plan.

Technology has also led to new conceptual organization in the electricity supply chain through Micro-Grids (MGs). A MG is composed of a group of power consumers which have their own power generation units and optimizes its internal electricity consumption. This concept is possible due to the increasing use of renewable energy sources and the increasing penetration of interconnected devices used in daily life. Still, because renewable energy sources are intermittent and storage devices are still not sufficiently efficient, MGs cannot consider being autonomous regarding electricity production. Therefore, MGs must have external power suppliers to ensure sufficient electricity supply at all time. A GC trading electricity with a MG faces a lot of uncertainty regarding its demand because of the internal management of the MG. This situation asks again for new computational methods considering the interaction between different actors.

We also face an increasing need of reliability in electricity transmission. Optimization prob-

lems related to transmission networks have also been studied for a long time as the UC. These optimization problems increasingly tend to consider robustness to deal with reliability issues.

In this thesis, several optimization problems considering modern constraints related to the electricity supply chain are studied through MP. Several problems consider interactions between actors and are modelled through bi-level formulations. We illustrate how the difficulties introduced by the evolving context can be used to derive properties of the models considered to reformulate them into mixed integer linear programs. Efficient heuristic methods are obtained inspired by the exact formulations proposed, some of which being applicable to more general problems. An extensive analysis of the performance of the solving methods as well as the influence of the parameters of the problems introduced by modern constraints are presented.

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Mal nommer un objet, c'est ajouter au malheur de ce monde.

Albert Camus

Chapter 1

Introduction

New optimization problems are appearing for electricity generation companies due to increasing penetration of the use of electrical devices in our daily lives, the reliability needed and the possible production and trading options. The challenges related to the electricity supply chain are getting more complex with an increasing need of solving these challenges as our society is increasingly relying on electricity. This thesis studies various optimization problems related to the electricity supply chain, mainly considering new interactions appearing with new market regulations and technology. This introduction chapter is organized as follow:

Section 1.1 presents some evolutions in the electricity supply chain leading to new optimization problems. Section 1.2 provides a description of the optimization problems studied in this thesis. Section 1.3 presents some optimization tools used throughout this thesis.

1.1 Electricity supply chain evolution

Large scale distribution network problems appeared in the 19$^{\text{th}}$ century with the development of industry, railway, telecommunication and electricity. These distribution network problems have been getting larger and more complex since their introduction. Considering electricity distribution networks, the demand in electricity has been constantly increasing since 1950 [Meadows et al., 2004], new production methods have appeared and distribution is getting closer to its limit [Crappe, 2003]. The whole electricity supply chain is in evolution, from the production to the consumption.

The electricity supply chain had a vertical structure when first considered. All electricity was produced by a single *Generation Company* (GC), generally a national company, which had control over price and distribution. The only true actor in this context was the national GC,

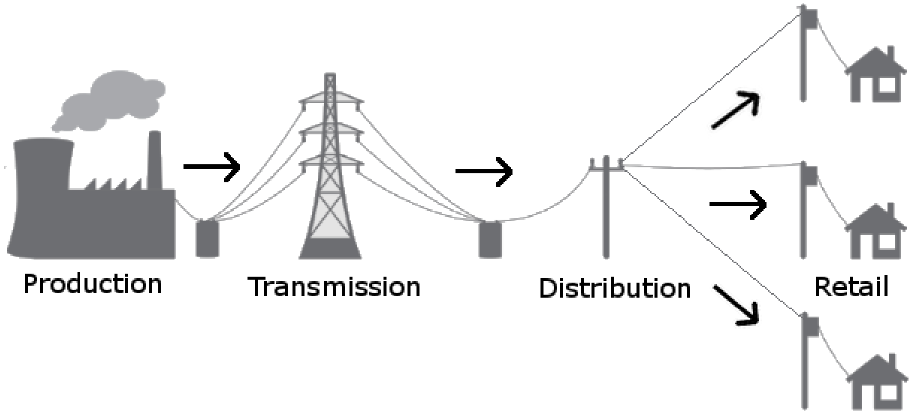

Figure 1.1: Vertical electricity supply chain

customers having only a limited influence through their demand that could be estimated statistically by the GC. Figure 1.1 illustrates the electricity supply chain at its first steps.

Electricity production has been studied for some time through optimization methods in *Unit Commitment* (UC) problems [Tahanan et al., 2015]. These problems consist in finding a minimum cost electricity production plan for a GC for a given time horizon and demand. Generators have specific costs such as startup cost, shut down costs and production costs as well as several physical constraints on their production capacity throughout time. This is a multi-period problem where the GC has to plan the production for several consecutive time periods. Many constraints of a UC problem cannot be modelled by simple linear inequalities making the problem challenging. A more detailed description of the UC problem is given in Section 2.2.

Due to limited computation power available when UC was first studied, the size of the models considered had to be relatively small. Therefore, the parameters of the models were often considered as deterministic, limiting their precision. For instance, the UC problem depends on the power that has to be produced at each time period which depends on the demand of customers. This demand can be estimated but remains uncertain. A GC needs a production plan that is financially interesting considering a demand. The demand is generally not know in advance and a GC must avoid not being able to satisfy it, pointing to out to the need of integrating this uncertainty in the problem. This usually increases considerably the difficulty of solving the UC problem. Some classical optimization methods integrating uncertainty are presented in Section 1.3.3 illustrating the impact of uncertainty on the size of the models considered.

The energy production technologies are evolving with the appearance of new production systems (solar panels, wind turbines, . . .), introducing uncertainty on the production side of the UC problem and increasing the difficulty of establishing a production plan [Morales et al., 2014]. Renewable energy sources are commonly much cheaper than fossil energy sources but their production capacity depends on the weather. Paradoxically, our society also relies more and more on electricity, needing a secure electricity production scheme, which is not the case with the developing renewable energies. A GC cannot be too optimistic about weather conditions, again, because of blackout issues. A GC needs to have an uncertainty measure in order to be able to integrate renewable energy sources. Again, integrating such risk measures complexifies the UC problem. Overall, the UC problem is much harder to model today than when the problem was first considered.

The liberal society of today also pushed the diversification of energy producers [Rifkin, 2011]. Some 20 years ago, an important modification in the electricity production scheme was introduced: the deregulation of the electricity markets. The European Commission started this deregulation in 1996 in order to allow new GCs to enter the market [Fact Sheets on the European Union, 2020]. As a consequence, the price of electricity could not be imposed by a single company anymore. The price of electricity would now be determined by a *Transmission System Operator* (TSO) based on bids made by all GCs in a *day-ahead market*. In such a market, each day, all GCs propose bids for the following day to the TSO. In their simplest form, bids are composed of a quantity of electricity and a unit cost. Once the bidding is closed, the TSO selects the bids maximizing the global welfare of producers and retailers, fixing the *spot price* of electricity for the next day. This breaks the vertical structure of price decision. The production traded by each GC with the TSO now depends on the bids of other GCs. The goal of deregulated electricity markets is to propose a market mechanism resulting in more competitive prices for customers. In Europe, local day-ahead markets are being grouped into a global market through the *Price Coupling of Regions* (PCR) project, attempting to harmonize the electricity prices in all the participating markets. The selection of bids by the TSO on the European market is a complex problem, mainly due to the numerous types of bids a GC can propose [Madani and Van Vyve, 2015]. More details on how day-ahead markets operate are provided in Section 2.3.

In a day-ahead market, a GC attempts to place bids maximizing its profit. Its profit depends on the production costs and on the reaction of the TSO who determines the spot price and the quantity traded. This introduces classical situations studied in game theory where a player has to make a move based on the possible moves of its opponents [Shubik, 1975]. Many problems in game theory have been studied through *bi-level programming* which shall be detailed in Section 1.3.2.

Nowadays, customers also have the possibility to produce electricity locally with renewable energy sources. With increasing connectivity, customers have the possibility to target at organizing themselves in *micro-grids* (MGs), independently from an external electricity provider. In a MG composed of multiple consumers, the demand of a consumer can be supplied by a pro-

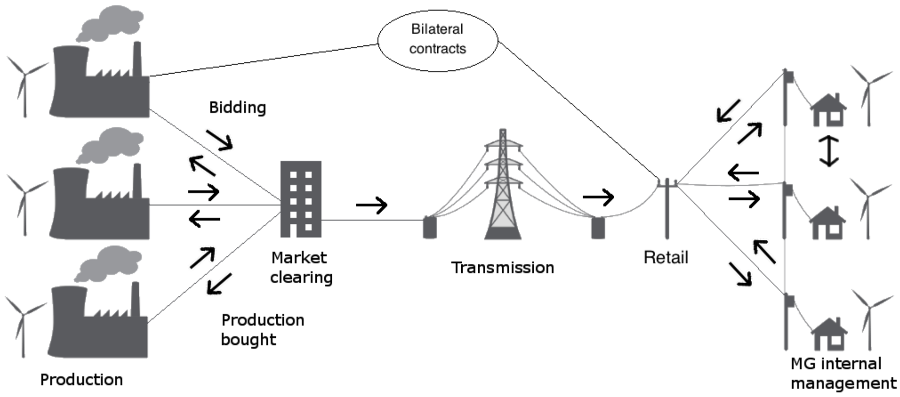

Figure 1.2: Horizontal electricity supply chain

duction device of another consumer rather than by a GC. A MG can also have a set of storage devices that store unused electricity produced by devices such as solar panels. The MG can use stored electricity when the production devices are not sufficient to cover the total consumption. All this is done through a central electricity management system that can also control when specific devices such as dish-washers should be working in order to optimize consumption. The MG model is still yet to reach. During the transition period, MGs will need external support to complete the production of their local production devices. GCs can propose bilateral contracts to micro-grids. The issue appearing in a UC problem for a GC trading with MGs is that the demand of a MG depends on how it organizes its consumption internally, independently of the GC. This contributes again in breaking the vertical structure of the electricity supply chain.

Figure 1.2 illustrates the horizontal structures in the current electricity supply chain considering deregulated electricity markets and local electricity production from the consumers. Deregulated electricity markets and micro-grids have one common goal: reduce the price of electricity for customers. While the initiative is good from a social point of view, it brings many new challenges in the electricity supply chain, many centered on interactions between various actors. These challenges need to be solved because of the formerly mentioned social organization relying increasingly on electricity.

1.2 Outline and Contributions

The next chapters of this thesis are organized as follow:

Chapter 2 This chapter gives a literature review of the fields related to the problems studied in this thesis. Section 2.1 provides elements related to *distribution networks*. Section 2.2 presents the *Unit Commitment* (UC) problem consisting in establishing a production of electricity at minimum cost given a set of generators. Section 2.3 presents how *day-ahead markets* work in Europe and how to model them mathematically. Finally, Section 2.4 presents the state-of-the-art methods for *bidding problems* in day-ahead markets for *Generation Companies* (GCs).

Chapter 3 The *Minimum Margin Problem*, introduced by Rossi et al. [2011], is studied in this chapter. This problem considers a set of customers that must be supplied in electricity by a set of feeders throughout an existing transmission network. The goal is to find an assignment of customers to feeders maximizing the production margin of the generators, preventing a blackout in case the demand of some customers increases. A maximum distance between each customer and its feeder is allowed, called *hop-constraint*, in order to integrate robustness in the transmission.

This thesis presents a formulation of the problem using *layered graphs*, improving significantly the computational time of previous studies. A general contribution is a serie of reductions that can be applied to layered graphs to reduce their size depending on the constraints of the problem considered. The use of hop-constraints is also extended to distance-constraints with numerical results illustrating the impact on computational time.

A paper co-written with B. Fortz on the Minimum Margin Problem has been published in EJOR [De Boeck and Fortz, 2017].

Chapter 4 The problem studied in the chapter was proposed by the company EDF at the European Study Groups with Industry 2016 conference in Avignon, France. The problem consists in a *Contract Proposition Problem* where a GC must choose a set of contracts to propose to micro-grids in order to be their external supplier. This is a two-stage decision problem, the GC decides which contracts to propose then the micro-grids choose the contract at their best advantage. The GC has to meet the demand of the micro-grids through a UC problem in addition to a fixed demand, independent of the migro-grids. The problem aims at maximizing the profit of the GC which is based on the consumption of the micro-grids which is not controlled by the GC.

The problem is formulated as a bi-level optimization problem. The formulation contains continuous and binary variables at the second level of the problem. Bi-level formulation are detailed in Section 1.3.2 but for now we can state that such types of formulations are very challenging.

A heuristic formulation, taking advantage of the binary variables at the second level, is proposed providing solutions close to optimality. The heuristic proposed does not increase significantly the computation time of a UC problem with a fixed demand.

A paper co-written with W. van Ackooij, B. Detienne, S. Pan and Michael Poss has been published in EJOR van Ackooij et al. [2018]. My personal contribution in this paper was firstly to obtain an exact reformulation providing the optimal solution of the problem in order to asses the quality of the heuristic solutions found, secondly to implement the formulation and perform numerical experiments.

Chapter 5 Bidding in day-ahead markets for a GC is tackled in this chapter. As we will present in Section 2.4, the constraints involving market regulations are very challenging to model and strong hypotheses are made in state-of-the-art studies. The bidding procedure is a two-stage decision problem. A GC attempting to maximize its profit first places bids on the day-ahead market. Then, the price of electricity and the quantities traded are defined in the day-ahead market. We consider in this chapter the bidding problem presented by Fampa et al. [2008] considering linear production costs for electricity, fixed bidding quantities for the GC and uncertainty in the bids of the competitors.

This thesis presents a new Dynamic Programming (DP) approach to solve the problem. Several variants of the problem are proposed which are solved through the DP approach. The combination of these variants leads to a heuristic method solving a more general version of the problem which does not consider fixed bidding quantities for the GC. A strong upper bound on the general problem is also found with the same DP approach. A study of the complexity of each variant is also performed.

The computational time to solve the problem is significantly decreased in comparison to previous solving methods, allowing to consider more uncertainty in the instances treated.

Chapter 6 This chapter also considers bidding in deregulated electricity markets for a GC but under different hypotheses than in the previous chapter. The bids of competitors are considered as known and a full UC problem is considered for the GC to model production costs. The problem also considers *price coupling of regions*, presented in Section 2.3, which consists in considering several day-ahead markets linked through a transmission network with a *Transmission System Operator* (TSO) coordinating the trading. In PCR, the price of electricity is the same in markets where the transmission network is not restrictive.

The problem is formulated as a bi-level problem. A linear reformulation is proposed introducing new valid inequalities through an extended formulation, tightening state-of-the-art formulations. A heuristic method is proposed, taking advantage of the equal price of electricity between markets in PCR when the transmission network is not restrictive. A general heuristic method is also proposed for MIP formulations containing *Special Ordered Sets of type 1*, narrowing the value of variables in such sets during a branch and bound procedure. All methods proposed can be extended for a retailer bidding in PCR with an internal problem different than the UC used for producers.

A paper co-written with L. Brotcorne and B. Fortz has been submitted to MMOR.

Chapter 7 This chapter presents general conclusions on the studied problems and some future directions of research.

1.3 Optimization tools overview

The general mathematical optimization tools used throughout this thesis are detailed in this section.

1.3.1 Mathematical programming

Mathematical program

A *Mathematical Program* (MP) is a mathematical representation of an optimization problem where the value of a variable $x \in \mathbb{R}^n$ minimizing or maximizing an objective function f must be found such that x is in a solution space $\mathscr{X}$ described by a finite set of constraints. For a maximization problem:

$$\begin{aligned} \max \quad & f(x) \\ \text{s.t.} \quad & x \in \mathscr{X} \end{aligned}$$

The way to tackle a MP depends on the type of objective function f used and the constraints describing set $\mathscr{X}$.

Linear program

A *Linear Program* is a MP considering a linear objective function and m linear constraints. Any LP, called *primal*, has a *dual* LP:

$$\begin{array}{rlrl} & \text{Primal} & & \text{Dual} \\ \max & c^t x & \min & b^t y \\ \text{s.t:} & Ax \leq b \;\; (y) & \text{s.t:} & A^t y \geq c \;\; (x) \\ & x \geq 0 & & y \geq 0 \end{array}$$

In a LP, each constraint, respectively variable, in the primal has an associated dual variable, respectively constraint. If a constraint in the primal is an equality, its corresponding dual variable is unsigned, otherwise the dual variable is non negative, and vice-versa.
There are several links between a primal and its dual. Consider the primal is a maximization problem:

1. *weak duality*: a feasible solution of the primal has a value smaller than or equal to the value of a feasible solution of the dual,

2. *strong duality*: if the primal and the dual problems are feasible, there exist optimal solutions of the primal and the dual having the same objective value,

3. *complementarity constraints*: solutions x and y of the primal and the dual are both optimal if and only if $(Ax-b)_i y_i = 0$ for all $i = 1, ..., m$ and $(A^t y - c)_j x_j = 0$ for all $j = 1, ..., n$.

The dual formulation of a LP if often used to prove optimality or provide bounds on the optimal value of the primal.
The simplex algorithm is the first algorithm designed to solve LPs. This algorithm was proposed by Dantzig [1951] and has an exponential complexity in the worse case. It finds an optimal solution by moving from vertex to vertex in the convex solution space of the problem, increasing the objective value.

A polynomial algorithm solving LPs was presented by Khachiyan [1980] using ellipsoid methods. Although improvements of this algorithms have been found, most LPs are still solved with the simplex algorithm as it is generally timewise more performant than polynomial algorithms.

An alternative to solve LPs are interior point methods. Interior points methods search for solutions inside the feasible solution space of the problem solved. The two main approaches are Primal-dual methods and barrier methods. In primal-dual methods [Mehrotra, 1991], constrained versions of the primal and dual formulations of a LP are iteratively solved until finding solutions with the same objective value for both formulations proving optimality by strong duality. If such solutions are not found, this approaches provides a feasible solution to the problem and the gap to optimality can be evaluated by the feasible solutions found for the dual problem. Barrier methods [Lustig et al., 1993] find solutions strictly contained in the solution space of the problem, considering each inequality defining the solution space as a barrier with an associated weight. The weight of a barrier indicates how much a solution is pushed away from this barrier. The weight of each barrier is updated while solving the problem, pushing the solution in the direction of an optimal solution.

Integer program

An *Integer Program* (IP) is a MP where $x \in \mathbb{Z}^n$. A *Mixed Integer Program* (MIP) is a MP combining continuous and integer variables. Finally, a *Mixed Integer Linear Program* (MILP) is a MIP with a linear objective function and linear constraints. If some variables are restricted to values 0 and 1, they are called *binary variables* or *decision variables*. General MILPs are NP-hard problems for which no polynomial algorithm exists unless $P = NP$.

The *LP relaxation* of an MILP is a LP with identical objective function and constraints where all variables are considered as continuous.
The present thesis focuses mainly on solving problems formulated through MILPs which can be written as follows:

$$\begin{aligned} \max \quad & c^t x + d^t y \\ \text{s.t.} \quad & Ax + Ey \leq b \\ & x \in \mathbb{R}_n^+ \\ & y \in \mathbb{Z}_n^+ \end{aligned}$$

Properties linking the primal and the dual formulations of a LP do not hold for MILPs in the general case. A MILP can be solved by iterating resolutions of its LP relaxation through a branch & bound algorithm [Wolsey, 1998]. This procedure can turn into an enumeration of all possible integer values in the worse case. Many other methods such as cutting plane algorithms [Mitchell, 2009], branch & cut and branch & price [M. Elf and Rinaldi, 2001],... have been developed to tackle MILPs, the choice of the method used depending on the structure of the problem.
MILPs can be solved through state of the art commercial solvers but the computational time is generally much higher than for LPs.

State of the art MP solvers

A wide range of commercial and open source MP solvers are available nowadays. In this thesis, the software CPLEX [2020] from IBM is used to perform numerical experiments as well as the Open Source framework JuMP [2020] for Julia [2020]. State-of-the-art methods to solve the LPs and MILPs treated in this thesis are integrated in these software.

1.3.2 Bi-level problem

A bi-level optimization problem models a situation involving two types of actors taking decisions one after the other at their own benefit [Colson et al., 2007]. This corresponds to a two-stage Stackelberg game where the first move is made by the *leader*, the second move by the *followers*. The goal is to optimize the objective of the leader under the constraint that the followers will optimize their objective based on the values of variables controlled by the leader. Consider the leader maximizes its objective function and controls a group of variables x and a follower needs to minimize its objective value and controls a group of variables y and all constraints are linear. Such a bi-level problem can be formulated as follows:

$$\begin{aligned} \max \quad & c^t x + d^t y \\ \text{s.t.} \quad & Ax \leq b \\ & x \geq 0 \\ & y \in \arg\min \; h^t x + i^t y \\ & \qquad \text{s.t.} \; Ex + Fy \geq g \\ & \qquad\qquad y \geq 0 \end{aligned} \tag{1.1}$$

Although constraints are linear and variables are continuous, this bi-level problems can have a non-convex solution space [Dempe, 2002]. Audet et al. [1997] showed links between linear bilevel problems and MILPs, illustrating linear bilevel problems are NP-hard.

One important observation on the bi-level formulation proposed is that it yields an *optimistic assumption*: if the follower has several optimal solutions, it will choose the most interesting one for the leader.

A simple technique can be used to rewrite this formulation as a single level formulation using strong duality. As variables x are controlled by the leader, the second level can be seen as a problem where variables x are fixed. If variables in the second level are continuous, the dual problem of the second level can be written depending on the value of x:

$$\begin{aligned} \max \quad & (g - Ex)^t z \\ \text{s.t.} \quad & F^t z \geq i \\ & z \geq 0, \end{aligned}$$

where z is the dual variable of constraint (1.1).

Using strong duality to guarantee optimality for the follower, the bi-level formulation can be rewritten into a MP:

$$\begin{aligned} \max \quad & c^t x + d^t y \\ \text{s.t.} \quad & Ax \leq b \\ & h^t x + i^t y = (g - Ex)^t z \\ & Ex + Fy \geq g \\ & F^t z \geq i \\ & x, y, z \geq 0 \end{aligned}$$

This reformulation yields a quadratic constraints. The way to tackle these constraints mainly depends on the studied problem. A commonly used technique are McCormick inequalities [1983], linearizing products of variables by introducing integer variables, transforming it into a generally challenging MILP formulation.

Another reformulation technique commonly used for linear bilevel problems are Karush-Kush-Tucker (KKT) conditions that use complementarity constraints of the second level problem to get a single level formulation [Karush, 1939, Kuhn and Tucker, 1950]. These conditions also introduce bilinear terms through complementarity conditions with the same difficulties when reformulating using strong duality as above.

Dual reformulation and KKT conditions are also valid if the first level contain integer variables. This does not hold if the second level contains integer variables as strong duality does not hold in general. Such problems are much more challenging to solve. The first general methods for obtaining optimal solutions bi-level mixed integer problems can be traced back to Moore and Bard [1990] and Bard and Moore [1992]. More recent works from Fischetti et al. [2016, 2017] have shown significant improvements using intersection cuts. These methods work only for bi-level problems having a specific structure. There is still no method to handle general bi-level mixed integer problems with integer and continuous variables at both levels. Such problems are generally solved by developing a method based on their specific structure.

Chapters 5 and 6 consider problems modelled as bi-level problems with continuous variables at the second level while another bi-level problem with binary variables at the second level is studied in Chapter 4.

1.3.3 Stochastic optimization

Stochastic optimization considers some parameters of an optimization problem as random variables. A way to treat such parameters in order to obtain a MP is to consider a set finite of scenarios S representing the probability distribution of the random variables. Consider a set of random variables A used as parameters. A scenario $s \in S$ is defined by a set of possible values for parameters A, denoted A_s. Each scenario s occurs with an associated probability p_s. In order to have an accurate modelization of the random variables, the scenarios considered must represent *almost surely* the distribution of the uncertain parameters [Birge and Louveaux, 2011]. That is, the probability of having a realization of parameters that corresponds to no scenario must be equal to zero. This is a heavy restriction in some optimization problems considering uncertainty due to the tremendous number of scenarios that should be considered. Practically, a limited number of scenarios trying to represent at best the distribution of uncertain parameters is considered in a stochastic optimization problem.

Given an MP formulation of a deterministic problem, a stochastic version can easily be derived based on a set of scenarios S. The variables of the deterministic MP are partitioned into *deterministic variables* x that have the same value in each scenario and *stochastic variables* y_s that can have different values in each scenario. A stochastic variable in the deterministic LP is replaced by $|S|$ variables in the stochastic reformulation, one for each scenario. Each constraint of the LP containing an uncertain parameters or a stochastic variable is replaced by $|S|$ constraints, one for each scenario. The objective function becomes a probabilistic measure of the initial objective function. An example of stochastic reformulation of a LP maximizing the expectation:

$$
\begin{array}{rl@{\qquad}rll}
\multicolumn{2}{c}{\text{Deterministic}} & \multicolumn{3}{c}{\text{Stochastic}} \\
\max & c_1^t x + c_2^t y & \max & c_1^t x + \sum_{s \in S} p_s c_2^t y(s) & \\
\text{s.t:} & A_1 x + A_2 y \leq b & \text{s.t:} & A_{1,s} x + A_{2,s} y(s) \leq b_s & \forall s \in S \\
& x, y \geq 0 & & x \geq 0 & \\
& & & y(s) \geq 0 & \forall s \in S
\end{array}
$$

Another probabilistic measure that is commonly used in stochastic optimization for the objective function is the *conditional value-at-risk*. The conditional value-at-risk (CVaR_ε with $\varepsilon \in [0,1]$) provides solutions avoiding extreme situations [Rockafellar and Uryasev, 2000]. For a random variable X representing a loss that must be minimized, it is defined as

$$\text{CVaR}_\varepsilon(X) = \mathbb{E}[X | X \geq \text{Var}_\varepsilon(X)],$$

where $\text{Var}_\varepsilon(X)$ is the ε-quantile of the distribution of X. A small value of ε represents a high aversion to risk.

If X is a random variable defined almost surely through a discrete set of scenarios S, then CVaR is computed by solving:

$$\mathrm{CVaR}_{\varepsilon}(X) = \min_{v}\{v + \frac{1}{1-\varepsilon}\sum_{s\in S} p_s(x(s)-v)^+\} \tag{1.2}$$

Terms $(x(s)-v)^+$ can be linearized by replacing them by variables w_s and adding the following constraints:

$$\begin{aligned} w_s &\geq x(s) - v, \\ w_s &\geq 0. \end{aligned}$$

An uncertainty measure that is generally used in economical problems is a Markowitz type measure 1952. This measure was introduced for financial problems and aims at preventing risk through diversification. A Markowitz type probabilistic measure μ_λ that will be considered in Chapter 4 is a convex combination of the expectation and the conditional value-at-risk of a random variable X proposed by Rockafellar and Uryasev [2013]:

$$\mu_\lambda(X) = \lambda\mathbb{E}[X] + (1-\lambda)\mathrm{CVaR}_{\varepsilon}(X),$$

with $\lambda \in [0,1]$. This allows to consider a tradeoff between risk and average performance.

Chapter 2

Literature Review

The present thesis focuses on new interactions between different fields of the electricity supply chain. This chapter provides an overview of the state-of-the-art models and techniques considered in the following chapters.

2.1 Distribution network

Power flow optimization in transmission networks have been widely studied since the 60's [Frank et al., 2012] but are getting more complex today. Failure in electricity systems have growing negative impacts, and distribution must become increasingly more reliable [Rifkin, 2011]. The energy production technologies are also evolving with the apparition of new production systems (solar panels, wind turbines, ...) meaning a larger number of energy producers to consider. In telecommunications distribution networks, the evolution is similar to electricity. Over the past years, data exchange has even been increasing at a much higher rate than electricity demands [Wigginton, 2016], and reliability is also an important issue.

Electricity and telecommunications *Distribution Network Configuration Problems* have similar constraints. In both problems, a set of feeders is given. Each feeder produces a specific resource with limited capacity. A set of customers must be assigned to the feeders through an existing network in order to satisfy the demands of the customers. Each customer is assigned to a single feeder and the customers assigned to a feeder must form a connected component. For electricity distribution networks, feeders are power generators, customers have an electricity demand and connectivity is needed to ensure the electricity transfer from feeders to customers. Several types of connectivity constraints might be considered for security purposes, such as enforcing that each customer must be connected through k distinct paths to its feeder [Botton et al., 2013]. In the context of telecommunications, distribution networks appear in multicast routing problems [Oliveira and Pardalos, 2005]. The feeders are data providers and the customers are data relays with a demand in data. Feeders can send data to customers in a certain range and customers can repeat the signal to transfer data to other customers. Connectivity is needed to ensure that a customer can receive the desired informations from its feeder or from a repetitor assigned to the same feeder. These problems are closely related to network design problems with relays and concentrators [Cabral et al., 2007, Contreras and Fernández, 2012, Gouveia and da Gama, 2006, Fortz, 2015]. Distribution networks can be represented on graphs and be subject to various types of constraints. Among these we consider in the following chapters capacity constraints, hop contraints and losses due to transportation distance.

2.1.1 Network flow problems

The transfer of ressources between feeders and customers is generally represented by a *flow optimization problem* where the flow throughout a graph materializes the transfer of ressources to customers which each have a specific demand. The demand of feeders is considered as negative to represent their production. A graph $G = (V, E)$ is used to represent a distribution network, each edge representing a transmission line, demands are denoted by d_i, $i \in V$. As the direction of a flow must be considered, edges E can be represented as a set of arcs A where each edge $\{i, j\} \in E$ is represented as two opposite directed arc $ij, ji \in A$. Arcs entering, respectively exiting, node i are denoted $\delta^-(i)$, respectively $\delta^+(i)$. For each arc $ij \in A$, a unit transfer cost c_{ij} and a maximum flow capacity C_{ij}^{max} are given. Variables f_{ij} define the flow from node i to node j. A simple minimum cost flow formulation from a source $s \in V$ can be modelled through a LP as follow:

$$\min \quad \sum_{ij \in A} c_{ij} f_{ij} \tag{2.1}$$

$$\text{s.t.} \quad \sum_{j \in \delta^-(i)} f_{ji} = d_i + \sum_{j \in \delta^+(i)} f_{ij} \qquad i \in V \setminus \{s\} \tag{2.2}$$

$$0 \leq f_{ij} \leq C_{ij}^{max} \qquad ij \in A \tag{2.3}$$

The objective function (2.1) to minimize is the flow cost. The flow conservation constraints (2.2) ensure that the amount of ressource entering a node equals the quantity of ressource exiting the node minus the demand of this node. Capacity constraints (2.3) limit the flow on each arc to their maximum capacity.

A *feasible flow* is a solution of the presented LP, the demand of all customers being satisfied. A classical algorithm to find a feasible flow is the *augmenting path algorithm* from Ford and Fulkerson [Ahuja et al., 1993]. Wayne [2002] presented an algorithm to solve the minimum cost flow problem in polynomial time.

Flow problems have been studied for a very long time [Shigeno, 2004, Frank et al., 2012] and can integrate many other types of constraints. For example, the minimum cost flow problem can integrate the design of the network where edges $\{i,j\} \in E$ can be installed with an associated installation cost $c'_{\{i,j\}}$. Introducing installation decision variables $x_{\{i,j\}}$ leads to the following formulation:

$$\min \quad \sum_{ij \in A} c_{ij} f_{ij} + \sum_{\{i,j\} \in E} c'_{\{i,j\}} x_{\{i,j\}} \tag{2.4}$$

$$\text{s.t.} \quad \sum_{j \in \delta^-(i)} f_{ji} = d_i + \sum_{j \in \delta^+(i)} f_{ij} \qquad i \in V \setminus \{s\} \tag{2.5}$$

$$0 \leq f_{ij} \leq C^{max}_{ij} x_{\{i,j\}} \qquad i,j \in A \tag{2.6}$$

$$x_{\{i,j\}} \in \{0,1\} \qquad \{i,j\} \in E \tag{2.7}$$

This type of simple network design problem already falls in the NP-hard category as the uncapacitated version reduces to a Steiner tree problem which is NP-hard [Biniaz et al., 2015]. Many other types of constraints such as multiple commodities representing several types of ressources are also often considered [Gendron and Frangioni, 1999].

These more general flow problems are out of the scope of this thesis. A simple flow problem consisting in finding a feasible flow in a more general problem is considered in Chapter 6.

2.1.2 Hop constrained problems

In several network related problems, a constraint on the maximum distance between a feeder and a customer can be imposed as well. The most common form of such type of constraints are *hop constraints* representing a maximum number of arcs H between a feeder and its customers [Gouveia, 1998, Balakrishnan and Altinkemer, 1992, Pirkul and Soni, 2003]. Hop constraints are used to model reliability issues and introduce robustness in the solutions proposed. Consider a customer c assigned to its feeder through a path P of length d. If there is a failure on a vertex or an edge of P, customer c would need to be supplied through another path. The larger the value of d is, the higher the risk of failure in distribution is [Ljubić and Gollowitzer, 2010]. The value of H can be fixed to limit the maximum probability of failure of transmission to customers. Furthermore, in telecommunications networks, there are data transfer delays due to distances between the transmitter and the receiver. Hop constraints can be used to limit these delays.

Adding a hop constraint on a flow optimization model can be done in different ways:

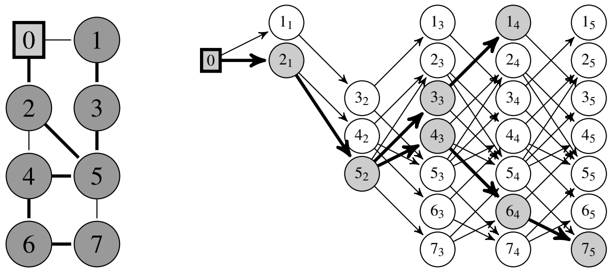

Figure 2.1: Layered graph

- By introducing variables keeping track of the distance between each customer and its feeder or by introducing variables representing each possible path of maximum length H between customers and feeders [Chvátal, 1983]. Both ways tend to make the formulation of the problem quite heavy leading to a large number of binary variables or an exponential number of constraints.

- The original graph can be extended into a layered graph, introduced by Gouveia [1998], as illustrated on Figure (2.1), to build an extended formulation of the problem. A layered graph derived from a graph G is a graph containing $H+1$ layers, layer 0 containing a root r and layer h containing vertices of G that can be reached through a path of length h from r. The number of hops between a feeder and a customer depends on which layer a customer is assigned to its feeder. Layered graphs are being used more and more in several types of hop constrained Steiner tree problems [Gouveia et al., 2014, 2011, Voß, 1999], facility location problems [Ljubić and Gollowitzer, 2010], survivable network design problems [Botton et al., 2013]...

2.1.3 Hop constrained reformulations on layered graphs

To illustrate the use of layered graphs for hop constrained problems, we consider the Steiner tree problem with multiple root nodes and the reformulation proposed by Gouveia et al. [2014] when including hop constraints. The Steiner tree problem is the problem of finding in a graph $G=(V,E)$ a tree that must include a given set of nodes $V' \subseteq V$. In the present case, the set V' is partitioned between root nodes R and terminal nodes T. All other nodes of the graph $S = V \backslash V'$ are Steiner nodes which can be used in the tree, or not, without any restriction. Adding a hop constraint limits the maximum distance between root and terminal nodes to a distance H.

First, we present a formulation for the Steiner tree problem without hop constraints. Each edge $\{i,j\} \in E$ has a cost $c_{\{i,j\}}$. Variable $x_{\{i,j\}}$ are equal to 1 only if the corresponding edge is used in the Steiner tree, 0 otherwise. Each Steiner node $i \in S$ has an associated variable y_i equal to 1 only if i is used in the Steiner tree and 0 otherwise.

$$\min \quad \sum_{\{i,j\}\in E} c_{\{i,j\}} x_{\{i,j\}} \tag{2.8}$$

$$\text{s.t.} \quad \sum_{\{\{i,j\}\in E | i\in V', j\notin V'\}} x_{\{i,j\}} \geq 1 \qquad V' \in C(V) \tag{2.9}$$

$$x_{\{i,j\}} \leq y_i \qquad i \in S, \{i,j\} \in E \tag{2.10}$$

$$\sum_{\{i,j\}\in E} x_{\{i,j\}} = |R| + |T| + \sum_{i\in S} y_i - 1 \tag{2.11}$$

$$\sum_{\{i,j\}\in E} x_{\{i,j\}} \geq 2y_i \qquad i \in S \tag{2.12}$$

$$x_{\{i,j\}} \in \{0,1\} \qquad \{i,j\} \in E \tag{2.13}$$

$$y_i \in \{0,1\} \qquad i \in S \tag{2.14}$$

The objective function (2.8) minimizes the building cost of the Steiner tree. Set $C(V)$ is the set of all subsets of V such that for each $V' \in V$, there exists at least one root node in V' and at least one terminal node in $V \backslash V'$. Constraint (2.9) ensures that the Steiner tree is connected from the route node by guaranteeing there is at least one edge in the Steiner tree along any cut between root and terminal nodes. The number of such constraints is exponential. Constraints (2.10) forces variables y_i to be equal to 1 if there exists an incident edge used in the Steiner tree. The number of edges used in a tree is equal to the number of nodes of the tree minus 1, which is imposed by constraint (2.11), guaranteeing there is no cycle in the solution together with constraint (2.9). Finally, constraints (2.12) avoid having Steiner nodes as leaves of the tree.

The formulation integrating a hop constraint H uses a layered graph [Gouveia et al., 2014]. For each root node $s \in R$, a directed layered graph $G_s^H = (V_s^H, A_s^H)$ containing all paths of length at most H rooted in s is defined. Graph G_s^H contains a vertices i_d for each vertex $i \in V$ such that there exists a path of length d from s to i in G for $0 \leq d \leq H$. A *layer* L_{sd} is the set of vertices i_d for a fixed $0 \leq d \leq H$. Set V_s^H is the set of all vertices of the layered graph are defined as

$$V_s^H = \{i_d | i \in V, 0 \leq d \leq H, i \in L_{sd}\}$$

and

$$A_s^H = \{i_d j_{d+1} | \{i,j\} \in E, 0 \leq d \leq H, i_d, j_{d+1} \in V_s^H\}$$

as the corresponding set of arcs.

The construction of layered graph G_v^H can be done in $O(n^2 H)$ by executing a Dijkstra algorithm up to H edges [De Boeck and Fortz, 2017]. A layered graph contains at most $O(nH)$ nodes and

$O(n^2H)$ arcs. We define the sets $P_{isd} = \{j \in L_{sd-1} | j_{d-1}i_d \in A_s^H\}$, with $1 \leq d \leq H$, as the sets of vertices that are predecessors of i_d in G_s^H.

A tree rooted in $s \in V$ can be represented as a tree in G_s^H if and only if the maximum distance between s and all leaves of the tree is at most H.

Variables X_{ij}^{sh} are associated to arcs $i_h j_{h+1} \in A_s^H$ and are equal to 1 if the corresponding arc is part of the rooted Steiner tree in G_s^H, for each $s \in R$. Variables Y_i^{sh} are associated to nodes $i_h \in V_s^H$, $1 \leq h \leq H$ and are equal to 1 if the corresponding node is part of the rooted Steiner tree in G_s^H, for each $s \in R$. The resulting MIP formulation is:

$$\min \quad \sum_{\{i,j\}\in E} c_{\{i,j\}} x_{\{i,j\}} \tag{2.15}$$

$$\text{s.t.} \quad \sum_{j \in P_{ish}} X_{ji}^{s,h-1} = Y_i^{sh} \qquad s \in R, i_h \in V_s^H, i \neq s \tag{2.16}$$

$$\sum_{h=1}^{H} Y_i^{sh} = 1 \qquad s \in R, i \in T \tag{2.17}$$

$$\sum_{h=1}^{H} Y_i^{sh} \leq 1 \qquad s \in R, i \in R \backslash \{s\} \tag{2.18}$$

$$\sum_{h=1}^{H} Y_i^{sh} \leq y_i \qquad s \in R, i \in S \tag{2.19}$$

$$\sum_{i_{h-1} j_h \in A_s^H, i \neq k} X_{ij}^{s,h-1} \geq X_{jk}^{sh} \qquad s \in R, j_h k_{h+1} \in A_s^H, j \neq s \tag{2.20}$$

$$\sum_{h=0}^{H-1} (X_{ij}^{sh} + X_{ji}^{sh}) \leq x_{\{i,j\}} \qquad s \in S, \{i,j\} \in E \tag{2.21}$$

$$X_{ij}^{sh} \in \{0,1\} \qquad s \in R, i_h j_{h+1} \in A_s^H \tag{2.22}$$

$$Y_i^{sh} \in \{0,1\} \qquad s \in R, i_h \in V_s^H \tag{2.23}$$

$$(2.10) - (2.14)$$

Indegree constraints (2.16) link arcs to node variables on each layer. Constraints (2.17)-(2.19) ensure each node in G is used at most once in each layered graph. The connectivity is ensured by constraints (2.20) by checking if there exists an incoming edge at the previous layer. Finally, constraints (2.21) link arc variables on the layered graphs to the edge variables in G. The hop constraint is guaranteed as terminal nodes are assigned to layers which are all at maximum distance H.

One interesting observation is that this formulation allows a compact reformulation of the problem. While the connectivity constraints (2.9) in the general problem without any hop constraint appear in exponential number, there is only a polynomial number of constraints in the hop constrained version (2.20).

Introducing variables X_{ij}^{sh} and Y_i^{sh} corresponds to a decomposition of variables x_{ij} and y_i, a technique used in extended formulation of a problem. Extended formulations are generally used in formulations where some constraints appear in exponential number in order to reduce the number of such constraints to a polynomial number [Conforti et al., 2010].

This type of extended formulation using layered graphs will be used in Chapter 3 on a new type of hop constrained problem.

2.2 Unit Commitment

The Unit Commitment consists in establishing an electricity production plan for a given time horizon and demand, this problem is a component of the problems treated in Chapters 4 and 5. Section 2.2.1 gives a general description of the problems and Section 2.2.2 provides two full deterministic formulations of the problem.

2.2.1 Problem description

The Unit Commitment problem (UC) problem consists in establishing a minimum cost electricity production plan for a fixed demand throughout a time horizon. The issue is to decide which generators must be online during each time period and how to dispatch the production. This is a challenging optimization problem due to the physical constraints of generators. To mention some of them:

- production costs that are non-linear expressions,
- startup and shut down costs, introducing decision variables and a non continuous objective function,
- ramping up and down capacities, limiting the variation of production from one time period to another for each generator, linking the time periods of the problem,
- minimum up and down times, forcing generators to be turned on or off for a minimum time period, linking again the time periods.

This problem has been widely studied in the literature [Padhy, 2004], and is challenging as it is non-convex.

The UC problem is a general problem which must be adapted to the type of generation units considered. Nuclear, thermal, hydraulic, solar, wind generation units, are some of the classical units used, each having their specific physical constraints. For instance, the cooling down of a nuclear generator is much longer than for a wind generator, implying a longer offline time. The generators used in the UC problems of the following chapters are thermal generators [Carrión and Arroyo, 2006] for which constraints are described in the following section. The current shift to renewable energy production tends to complexify the UC problem by increasing the uncertainty in the generation capacity of the production units used [Takriti et al., 1996, Tahanan et al., 2015].

The choice of the time windows is also an important factor. Production plans are usually set for 24 time periods of 1 hour, constraining the state of a generator to change only each hour. Choosing smaller time windows gives more flexibility on the state of generators but increases the number of decision variables and the difficulty of the model [Troy et al., 2012].

As a UC problem has several challenging constraints, several of them are generally dropped when the UC is a subproblem of a more general problem, as in previous studies of bidding in deregulated electricity markets [David, 1993, Fampa et al., 2008, Bakirtzis et al., 2007, Saleh et al., 2009, Sarkhani et al., 2014] which are studied in Chapter 5.

Simple formulations of UC problems consider only production and ramping constraints, leading to continuous models. The physical non-linear constraints of generators can generally be linearized, leading to a linear model. When considering startup and shut down costs, decision variables must be added to represent the state of the generators throughout the production plan, leading to MIP formulations that are much more challenging to solve.

2.2.2 Deterministic MILP formulations

This section describes two full deterministic UC MILP formulations considering a set thermal generation units J over a set of time periods $T = \{1, ..., 24\}$ composed of 24 periods of 1 hour. The formulations presented in the literature generally use one [Carrión and Arroyo, 2006, Al-Awami and El-Sharkawi, 2011, Baghdadi et al., 2011] or three [Chang et al., 2004, Dillon et al., 1978, Arroyo and Conejo, 2000] sets of binary variables. A formulation of each type is presented in this section. The formulation using a single set is proposed by Carrión and Arroyo [2006] and is denoted UC1 in the following, while the formulation using three sets of binary variables is proposed by Ostrowski et al. [2012] and is denoted UC2.

Formulation UC1 was originally an improvement of formulations using multiple types of binary variables, in order to reduce the number of binary variables used. The formulation presented by Ostrowski et al. [2012] leading to UC2 proposes valid inequalities tightening the formulations with multiple binary variables, compensating the larger number of binary variables present in such type of formulations. These two formulations are described in the following and are available in Appendix A.

Variables

A production unit is either on or off. The status of a production unit at a given time period is modelled by a binary variable :

$$v_j^t = \begin{cases} 1 & \text{if generation unit } j \in J \text{ is on at time period } t \in T, \\ 0 & \text{otherwise.} \end{cases}$$

These binary variables are mandatory in a full UC model. The discrete character of the status is a source of complexity when modelling a UC problem. These variables are used un UC1 and UC2.

Some formulations use two additional sets of binary variables representing the startup and shut down status of each unit at each time period as in UC2.

$$y_j^t = \begin{cases} 1 & \text{if generation unit } j \in J \text{ is started at time period } t \in T, \\ 0 & \text{otherwise.} \end{cases}$$

$$z_j^t = \begin{cases} 1 & \text{if generation unit } j \in J \text{ is shut down at time period } t \in T, \\ 0 & \text{otherwise.} \end{cases}$$

The production of generator $j \in J$ at period t in the production plan is denoted p_j^t and its maximum production capacity $\overline{p}_j^t$.

Objective function

A UC formulation generally minimizes the total generation cost which is composed of the production $c_j^t(p_j^t)$, the startup $c_j^u(t)$ and the shut down costs $c_j^d(t)$:

$$\min \sum_{t \in T} \sum_{j \in J} c_j^t(p_j^t) + c_j^u(t) + c_j^d(t)$$

This objective function is used in UC1 and UC2.

Production cost

The production p_j^t of a generator $j \in J$ which is turned on at period $t \in T$ depends on the amount of electricity produced which is bounded by a minimum $\underline{P_j}$ and maximum $\overline{P_j}$ production

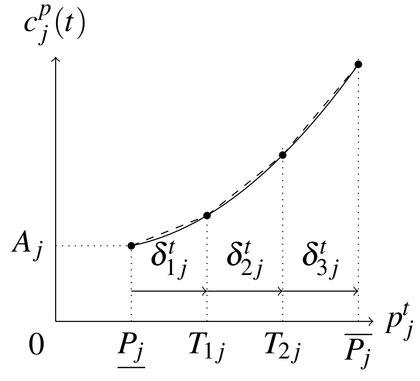

Figure 2.2: Production cost of a thermal generator

capacity. The cost $C_j^t(p_j^t)$ to produce p_j^t on generator j at a given period is typically a strictly increasing quadratic function:

$$C_j^t(p_j^t) = a_j(p_j^t)^2 + b_j p_j^t + c_j \qquad j \in J, t \in T \tag{2.24}$$

This function can be accurately approximated by a piecewise linear function [Bradley et al., 1977], as illustrated in Figure 2.2 where T_{ij} represents the total production at the i^{th} break in the piecewise linear function and δ_{ij}^t is the production on the i^{th} segment of the piecewise linear function.

The computation of $c_j^t(p_j^t)$ in a MILP formulation is approximated by:

$$c_j^t(p_j^t) = A_j v_j^t + \sum_{l=1}^{NL_j} F_{lj}\delta_{lj}^t \qquad j \in J, t \in T, \tag{2.25}$$

where A_j is a fixed cost for the generator when turned on, NL_j is the number of segments for production cost of unit j and F_{lj} is the unit production cost on segment $l \in NL_j$ of unit j. Note that this constraint is valid if and only if the MILP formulation minimizes the production cost and the unit production costs are increasing with the production.

Values of variables δ_{lj}^t at each time period are bounded by the production capacities of a unit and by the length of the segments in the linearized production cost function:

$$\delta_{1j}^t \le T_{1j} - \underline{P}_j \qquad j \in J, t \in T \tag{2.26}$$

$$\delta_{lj}^t \le T_{lj} - T_{l-1\,j} \qquad j \in J, t \in T, \in \{2, \dots, NL_j - 1\} \tag{2.27}$$

$$\delta_{NL_j\,j}^t \le \overline{P}_j - T_{NL_j-1\,j} \qquad j \in J, t \in T \tag{2.28}$$

The amount of electricity produced p_j^t by each unit j at each time period must satisfy :

$$p_j^t = \underline{P}_j v_j^t + \sum_{l=1}^{NL_j} \delta_{lj}^t \tag{2.29}$$

corresponding to the minimum production capacity if turned on in addition to the production on each segment of the linearized production cost function.

Demand and spinning reserve

In a classical UC problem, the demand is considered as know in advance and denoted D^t for each time period $t \in T$. A spinning reserve is also generally required, representing an additional production capacity that can be delivered if the demand is higher that the expected on delivery. The spinning reserve for period $t \in T$ is denoted R^t. The maximum production a generator can deliver at period $t \in T$ is denoted $\overline{p}_j^t$. Constraints on demand and spinning reserve are modelled as follow:

$$\sum_{j \in J} p_j^t \geq D^t \qquad t \in T \tag{2.30}$$

$$\sum_{j \in J} \overline{p}_j^t \geq D^t + R^t \qquad t \in T \tag{2.31}$$

Constraints (2.30) allow a higher production than the demand. When there is a financial loss when producing more than delivered, this constraint is considered as an equality or a penalty term can be added to the objective function.

In the problems studied in the following chapters, the demand is not known in advance. The demand and spinning reserve in constraints (2.30) and (2.31) will be considered as variables.

Startup and shut down costs

Startup costs occur when turning on a generator, the cost depends on how long the generation unit has been offline. As the time span is discretized into set T, this function can be approximated by a stepwise function [Wood and Wollenberg, 1996, Nowak and Roemisch, 2000] as illustrated in Figure 2.3 , where K_j^i represents the startup cost of generator j if it has been offline i time periods. Each generator $j \in J$ has a startup cost discretized in ND_j step. A shut down cost C_j represents a waste of fuel.

The constraints modelling the startup costs $c_j^u(t)$ and shut down costs $c_j^d(t)$ are modelled by the following constraints:

$$c_j^u(t) \geq K_j^t (v_j^t - \sum_{n=1}^{k} v_j^{k-n}) \qquad j \in J, t \in T, k \in \{1, ..., ND_j\} \tag{2.32}$$

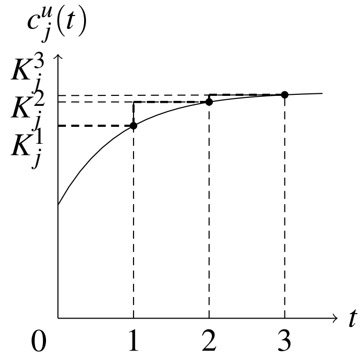

Figure 2.3: Startup costs of a thermal generator

$$c_j^u(t) \geq 0 \qquad j \in J, t \in T \tag{2.33}$$
$$c_j^d(t) \geq C_j(v_j^{t-1} - v_j^t) \qquad j \in J, t \in T \tag{2.34}$$
$$c_j^d(t) \geq 0 \qquad j \in J, t \in T \tag{2.35}$$

These constraints appear in UC1 and UC2. In UC2, an additional set of constraints is needed to ensure variables y_j^t and z_j^t take the appropriate values:

$$v_j^{t-1} - v_j^t + y_j^t - z_j^t = 0 \qquad j \in J, t \in T \tag{2.36}$$

Production constraints

The production p_j^t of a generator $j \in J, t \in T$ is bounded by minimum and maximum production capacities $\underline{P}_j$ and $\overline{P}_j$. The production and maximum production capacities are constrained as follow:

$$\underline{P}_j v_j^t \leq p_j^t \leq \overline{p}_j^t \qquad j \in J, t \in T \tag{2.37}$$
$$0 \leq \overline{p}_j^t \leq \overline{P}_j v_j^t \qquad j \in J, t \in T \tag{2.38}$$

If a generator is turned off at a given time period, these constraints force the production and ramping capacity to be equal to 0, otherwise, they are equivalent to $\underline{P}_j \leq p_j^t \leq \overline{p}_j^t \leq \overline{P}_j$.

A generator cannot deliver its maximum production capacity just after being turned on, it is subject to ramping constraints. These constraints limit the increase or decrease of production capacity from one period to another.

If a generator $j \in J$ is turned on, the maximum ramping up and down capacity from one period to the next are equal to RU_j and RD_j respectively. Next to this, a generator can be shut down only if the production at the previous time period is lower than SD_j and if a generator is turned on, it can deliver a production of at most SU_j at the corresponding time period.

This leads to the following ramping up constraints:

$$\overline{p}_j^t \leq p_j^{t-1} + RU_j v_j^{t-1} + SU_j(v_j^t - v_j^{t-1}) + \overline{P}_j(1 - v_j^t) \qquad j \in J, t \in T \tag{2.39}$$

$$\overline{p}_j^t \leq \overline{P}_j v_j^{t+1} + SD_j(v_j^t - v_j^{t+1}) \qquad j \in J, t \in T \backslash \{|T|\} \tag{2.40}$$

Similarly for ramping down constraints:

$$p_j^{t-1} - p_j^t \leq RD_j v_j^t + SD_j(v_j^{t-1} - v_j^t) + \overline{P}_j(1 - v_j^{t-1}) \qquad j \in J, t \in T \tag{2.41}$$

Constraints (2.39)-(2.41) are used in formulation UC1. When using variables representing when generators are started and shut down as in UC2, constraints (2.40) and (2.41) are replaced by:

$$\overline{p}_j^t - p_j^{t-1} \leq RU_j v_j^{t-1} + SU y_j^t \qquad j \in J, t \in T \tag{2.42}$$

$$\overline{p}_j^{t-1} - p_j^t \leq RD_j v_j^t + SD z_j^t \qquad j \in J, t \in T \tag{2.43}$$

Minimum up and down times

Generators are submitted to a minimum up and down time, representing a time interval during which the status of the generator cannot change. This can be caused by warm-up or cooling-down periods. Minimum up and down times of a generator $j \in J$ are denoted UT_j and DT_j.

Because the status of generators prior to the UC problem being solved must be considered, some generators must remain online or offline for a certain number of periods at the beginning of the UC planning. The number of time periods a generator $j \in J$ must remain online (offline) at the beginning of the planning is denoted $G_j(L_j)$. These constraints are modelled as follow:

$$\sum_{t=1}^{G_j} v_j^t = G_j \qquad j \in J \tag{2.44}$$

$$\sum_{t=1}^{L_j} v_j^t = 0 \qquad j \in J \tag{2.45}$$

When using only variables v_j^t as in UC1, minimum up time constraints are modelled as follow:

$$\sum_{n=t}^{t+UT_j-1} v_j^n \geq UT_j(v_j^t - v_j^{t-1}) \qquad j \in J, t \in \{G_j + 1 ... |T| - UT_j + 1\} \tag{2.46}$$

$$\sum_{n=t}^{|T|} (v_j^n - (v_j^t - v_j^{t-1})) \geq 0 \qquad j \in J, t \in \{|T| - UT_j + 2 ... |T|\} \tag{2.47}$$

Analogously for minimum down time constraints:

$$\sum_{n=t}^{t+DT_j-1} (1-v_j^n) \geq DT_j(v_j^{t-1}-v_j^t) \qquad j \in J, t \in \{L_j+1...|T|-DT_j+1\} \tag{2.48}$$

$$\sum_{n=t}^{|T|}(1-v_j^n-(v_j^{t-1}-v_j^t)) \geq 0 \qquad j \in J, t \in \{|T|-DT_j+2...|T|\} \tag{2.49}$$

With variables y_j^t and z_j^t as in UC2, the following set of constraints describe the convex hull of all feasible solutions in the minimum up and downtime polytope [Rajan and Takriti, 2005].

$$\sum_{t'=t-UT_j+1}^{t} y_j^{t'} \leq v_j^t \qquad j \in J, t \in G_j+1,\ldots,|T| \tag{2.50}$$

$$v_j(t) + \sum_{t'=t-DT_j+1}^{t} z_j^{t'} \leq 1 \qquad j \in J, t \in L_j+1,\ldots,|T| \tag{2.51}$$

Tightening of UC2

Some valid inequalities have been proposed to tighten the classical triple variable formulation by Ostrowski et al. [2012]. These constraints are used in UC2 and tend to exploit the impact of the status of the generators on the production capacity throughout the time periods due to ramping constraints and minimum up and downtime mainly. They are available in Appendix A.

2.3 Deregulated Electricity Markets

2.3.1 General context

The electricity market has strongly evolved during the past decades, mainly due to liberalization politics. Whereas the electricity market was ran by a single national Generation Company (GC) in each country during most of the 20^{th} century, with control over the production and the price of electricity, many GCs compete nowadays on the electricity market having many trading possibilities, among which, deregulated electricity markets.

Following directives of the European Commission, the production and transmission of electricity have been separated in order to avoid a monopoly of the transmission lines by the company owning them, allowing new GCs to use them. A deregulated electricity market is a structure where producers and retailers trade electricity daily and is ran by a *Transmission System Operator* (TSO). The TSO has the responsibility of finding a market equilibrium by coordinating the trading of electricity between actors and settling a uniform *spot price* for electricity. The growing advantages of spot pricing were presented by Littlechild [1988]. The spot price was

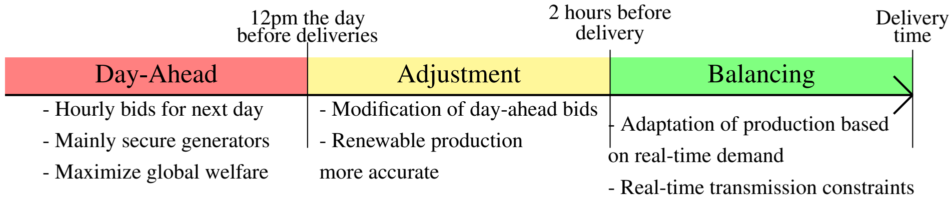

Figure 2.4: Different markets using in deregulated electricity market

described as follows:
"There exists a set of spot prices for electricity, varying from period to period according to changing conditions, such that demand and supply decisions taken by individual users and generators in the light of these prices are precisely the decisions that would be made by a single coordinating organization maximizing aggregate net benefits of the parties involved."
The TSO managing the the transmission network in Europe is ENTSOE ENTSO [2020].

Deregulated electricity markets contain several different markets. Electricity must be delivered based on the real-time demand. The trading process coordinated by the TSO starts the day before delivery in a *day-ahead* market and then adjusts through the *adjustment* and *balancing* markets.

In the day-ahead market GCs and retailers place hourly bids that will be honoured the next day. The TSO attempts to maximize the global welfare based on the bids received [O'Neill et al., 2005]. Two issues might occur on delivery the next day: GCs might not be able to produce the bidden quantities or the demand might be higher than expected. The adjustment market treats this issue about two hours before delivery time where another bidding game takes place to either complete the production that cannot be delivered by GCs or obtain more production in case of a higher demand. Again, these same issues might occur on delivery time. The balancing market then adjusts the production in real-time. The time line in Figure 2.4 summarizes this process. This thesis focuses on the day-ahead market.

The bidding in the day-ahead market generally closes around 12 p.m. in Europe before the delivery day before the TSO makes a selection of bids maximizing the global welfare. The bids that can be placed are of very different types. The simplest type of bids for a given time period are *step bids* composed of a unit production cost proposed to the TSO and the maximum quantity the producer/seller is willing to sell/buy and are fairly simple to model. The TSO can in this case buy any proportion of the corresponding bid. Other types of bids such as *block bids* put some additional constraints for the TSO, such as the obligation to accept the full quantity of the bid or nothing. *Complex bids* can also impose a minimum income for the GC. Each type of bids can also be placed over a set of time periods, imposing to the TSO to buy the same proportion of the bid throughout the time periods. All these more complex bids introduce non-convexity issues in the models representing day-ahead markets making them more difficulty to tackle. Several formulations in the literature make an abstraction of many rules to consider only step bids which can be modelled with continuons variables. Other types of bids generally require integer variables. The various types of bids can be found in the Euphemia description [2016], an algorithm used to solve real life bidding problems.

When considering only step bids the TSO can compute an aggregated production and a demand curve for each time period representing the total quantity that can be produced/bought for a given price. The spot price π^t for period t then lies at the intersection of the two curves as illustrated in Figure 2.5. If several spot price lie at the intersection of both curves, European regulations impose the highest one is considered [Hogan, 2012]. All production bids strictly under the spot price and all retailers bids strictly above the spot price are accepted by the TSO.

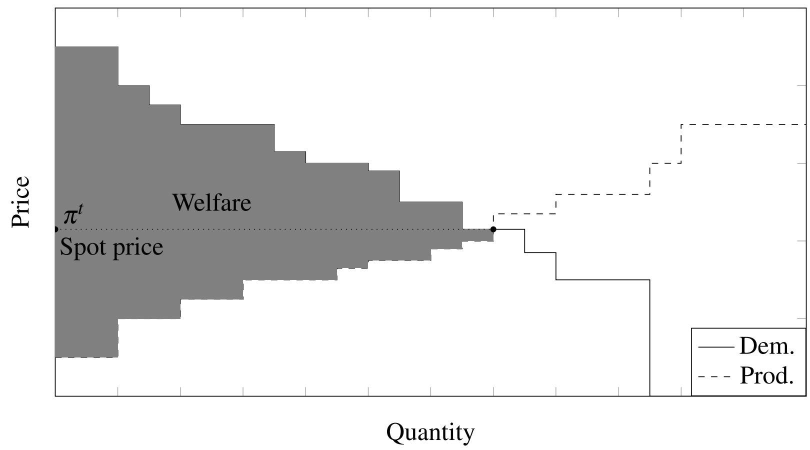

Figure 2.5: Production and demand curves on a day-ahead market and the resulting spot price

One main rule imposed to the GCs is that bids must be associated to generators. There are also restrictions on the bidding price that must be close to the marginal production cost of generators and a general maximum and minimum bidding price in order to avoid pure speculation. One main axis of the regulation around bidding in deregulated electricity market is fairness of actors and avoid an oligopolistic situation, see Madani [2017] for more details about the regulations.

When considering more general types of bids such as block bids, as these bids must be bought as a whole, some of them might be paradoxically rejected. This can occur if the whole bid would exceed the total traded quantity as illustrated in Figure 2.6 [Morales et al., 2014]. The first illustration presents a set of bids considering only step bids and the resulting spot price π^t where only a part of the last production bid is traded to meet the total demand Q. The second illustration has a higher demand, resulting in fully buying the last bid of the first illustration,

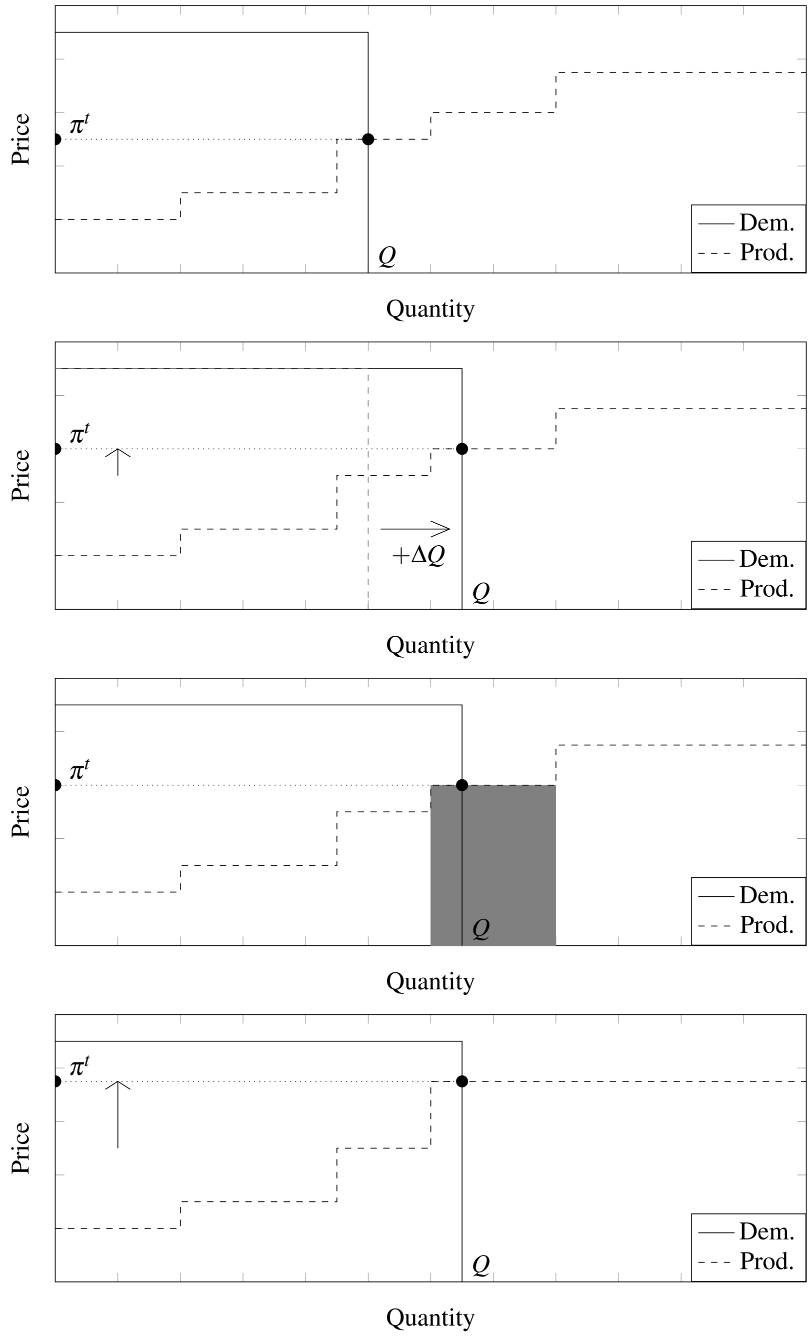

Figure 2.6: Paradoxal rejection of bloc bids

plus a part of the next production bid, resulting in a higher spot price. In the third illustration, we consider the last production bid in gray is a block bid, meaning it should be fully bought, which would exceed the total traded quantity. This bid must then be rejected as in the fourth illustration, increasing again the spot price above the price of the paradoxically rejected block bid.

In order to avoid rejecting bids at a price under the spot price, new bidding mechanism such as non-uniformity of prices are being assessed to avoid refusing bids when interesting from a global welfare point of view [Madani and Van Vyve, 2017].

Practically, several day ahead markets operated separately in Europe at first. Belgium, France and Holland were the first countries to group their markets into the TCL day-ahead market. Germany then joined these market to form the CWE market. Followed the *Price Coupling of Regions* [PCR, 2020] that exists today and extended to most countries of Europe aiming at having a unique spot price for electricity, the virtual ELIX spot price [2020]. The European day-ahead market is ran by a single TSO, managing the transmission network throughout the previously independent day-ahead markets (called *local day-ahead market* in the following). Many local day-ahead markets have joined the initiative but a common spot price is not always possible. This is partially due to the physical constraints of the transmission network. If the production is very cheap in one local day-ahead market, and the retailers offer a high price in another local market, the TSO will attempt to transfer electricity from the first market to the second. The goal of PCR is to have identical spot prices in local markets where no physical constraints limit the transfer of electricity between them. Otherwise the spot price will be cheaper in the exporting market than in the importing one.

Computationally, two of the best algorithms are from Cosmos [2011] and Martin et al. [2014]. The Euphemia algorithm [2016] is based on Cosmos [2011] and is used in the NWE region [NWE [2020]] to solve the problem associated with the coupling of the local day-ahead markets in the PCR region and settles the local hourly spot price of electricity and the electricity bought [Madani and Van Vyve, 2015]. This algorithm is a heuristic finding rapidly a first solution and then improving it in order to increase the global welfare. Considering state-of-the-art solving techniques, there is no formulation solving to optimality the maximization welfare problem integrating all market regulations and physical constraints on real size instances.

2.3.2 Generic formulation

This section presents an example of a generic formulation of the market equilibrium problem maximizing the global welfare proposed by Madani and Van Vyve [2015], contributors to the Euphemia algorithm.

The maximization welfare problem is defined over a set of time periods T, generally 24 periods of 1 hour. The problem considers several locations L representing the local day-ahead market connected through a transmission network. The problem considers step bids $i \in I$ associated to a location $l \in L$ and several time periods $t \in T$ and are represented by their price π^i and quantity $q^i_{l,t}$. Block bids $j \in J$ are represented similarly. The bids are placed by producers and retailers, producers bid negative quantities to simplify notations. Variables x_i and y_j, $i \in I, j \in J$ represent the proportion of a bid that is bought. Set N is a set indexing network constraints and K contains network elements. Note that this formulation considers step bids and block bids can be placed over several coupled time periods with the same price, meaning that if the TSO buys a proportion of a bid, it must buy the same proportion of this bid over all its time periods, linking the time periods of the problem.

The formulation is:

$$\max \quad \sum_{i \in I}\Big(\sum_{l \in L, t \in T} q^i_{l,t}\pi^i\Big)x_i + \sum_{j \in J}\Big(\sum_{l \in L, t \in T} q^j_{l,t}\pi^j\Big)y_j \tag{2.52}$$

$$\text{s.t.} \quad \sum_{i \in I} q^i_{l,t}x_i + \sum_{j \in J} q^j_{l,t}y_j = \sum_{k \in K} e^k_{l,t}n_k \qquad l \in L, t \in T \tag{2.53}$$

$$\sum_{k \in K} a_{m,k}n_k \leq w_m \qquad m \in N \tag{2.54}$$

$$0 \leq x_i, y_j \leq 1 \qquad i \in I \tag{2.55}$$

$$y_j \in \{0,1\} \qquad j \in J \tag{2.56}$$

The objective function (2.52) represents the global welfare. Constraints (2.53) are the balance constraints for each location and time period. The balance can be completed by power provided by the network which is modelled on the right-hand side of the constraints. Constraints (2.54) model generic transmission constraints of the network. For example, in the case of available-to-transfer capacity models, set K denotes the set of cross-border lines, variables n_k correspond to flows through these lines, and constraints (2.54) would then be capacity constraints on these flows. For flow-based models, they correspond to *critical network elements*. Further descriptions of these models are available in Madani and Van Vyve [2015]. Note that physical constraints related to electricity transmission networks are very complex to model. Many MPs tend to model only a part of them in order to keep a tractable model. Finally, constraints (2.55), (2.56) ensure any proportion of a step bid can be bought and blocs bids cannot be partially bought.

When considering only step bids, the spot price of a ressource in a given time period and location is the value of the dual variable associated to its balance constraint (2.53) [Baker and Taylor, 1979, Balachandran and Ramakrishnan, 1996]. In order to integrate the spot price in the formulation, the authors propose a reformulation using constraints of the dual problem and complementarity constraints [Madani and Van Vyve, 2015]. The complementarity constraints being valid only for continuous models, one of them is dropped in order to preserve a feasible model. Complementarity constraints being non-linear, they are generally difficult to handle if considered as such. A linear reformulation of the proposed model is given, solving much bigger instances of the problem.

In this thesis, the market equilibrium problem of the TSO will be considered as a subproblem of a bidding problem for a GC presented in the following section. State-of-the-art techniques for such problems generally consider step bids assigned to a single time period and simplified transmission constraints. These simplified formulations only make use of continuous variables and are generally decomposable by time period, lifting a lot of difficulties of the general market equilibrium problem of the TSO.

2.4 Bidding in deregulated Electricity Markets

A Generation Company (GC) has several options for trading electricity nowadays, among which bilateral contracts, used in a case study presented in Chapter 4, and trading in deregulated in electricity markets, used in two case studies presented in Chapters 5 and. 6.

When considering bilateral contracts, a GC must satisfy the demand of its customers. This demand can be fixed, bounded, uncertain,... but the price of electricity is defined through a contract and the task of the GC generally sums up to solving a generic UC problem. In deregulated electricity markets, the problem is more complex as the price of electricity depends on many actors.

For a GC bidding in deregulated electricity markets, bids must be made before the spot price of electricity or the production that will be bought by the TSO is known, this information is available only once the bidding is closed. This corresponds to classical problems in game theory where a player must make a move considering the possible moves of its opponents. This bidding problem is a two-stage Stackelberg game where the leader is the GC who first places bids, while the follower is the TSO that determines the spot prices and the actual trading of electricity once the bidding is closed. In order to compute its optimal bidding strategy, the GC must take into consideration the reaction of the TSO to its bids and have a feasible UC production plan in consequence. As the moves of the opponents are uncertain, a stochastic framework should be considered. Each scenario then represents a series of bids from competitor GCs.

Bidding problems under market constraints are part of a larger class of problems: *Mathematical*

Programs with Equilibrium Constraints (MPEC) [Luo et al., 1996]. The equilibrium constraints are defined by the market mechanism. In deregulated electricity market, the market equilibrium constraints correspond to the problem of the TSO maximizing the global welfare [Hobbs et al., 2000].

Two approaches to tackle bidding MPECs are Bertrand and Cournot methods [Hobbs and Helman, 2004, Hobbs et al., 2000, Ramos et al., 1999]. In these approaches, a competitor considers making moves changing only prices or only quantities respectively. An objective is to find a solution in a Bertrand-Cournot equilibrium where no improvement can be made without changing prices and quantities jointly. In bidding problems, a Bertrand model consists in bidding predetermined quantities while a Cournot model bids predetermined prices, mainly at marginal production costs of generators.

Bidding problems for a GC in deregulated energy markets have mainly been tackled through two approaches using a MP formulation: *price-taker* and *price-maker* formulations. The first approach considers the price of electricity is known in advance and the bidding GC does not influence the spot price of electricity, the second approach considers the spot price of electricity as a variable that is influenced by all bids proposed, including those of the bidding GC.

Finding an exact solution of a bidding problem is challenging due to all the constraints composing this problem: the physical production constraints, the transmission constraints, the market mechanism used, the types of bids considered, the uncertainty over power production and the bids of competitors. Many heuristic methods such as particle swarm optimization [Zhang et al., 2011], Monte Carlo simulations [Gountis and Bakirtzis, 2004, Saleh et al., 2009], genetic algorithms [Fampa and Pimentel, 2015], Cournot equilibriums [Badri and Rashidinejad, 2013], lagrangian relaxations [Fampa et al., 2008] or machine learning [Cocchi et al., 2018] have been proposed based on an exact formulation of the problem.

Note that this thesis considers bidding mechanisms used in European deregulated electricity market, but other mechanisms are used in other parts of the world for electricity trading. Two interesting surveys from Li et al. [2011] and Steeger et al. [2014] make a state-of-the-art presentation of the various approaches used to solve bidding problems with several types of market constraints and electricity production units. The following chapters consider bidding problems with a price-maker approach.

2.4.1 Price-taker formulation

Price-taker formulations consider that the spot price is known in advance and that the bids of the GC have no impact on the spot price. This hypothesis removes the game theory aspect of the bidding problem as the reaction of the TSO is ignored, simplifying significantly the problem. The spot prices throughout the time periods can be estimated using statistical methods [Angamuthu Chinnathambi and Ranganathan, 2016] or seen as random variables [Conejo et al., 2002].

When making a prediction over the spot prices $\tilde{\pi}^t$ for each period $t \in T$, and considering the GC bids a quantity p^t_g for each of its generators $g \in G$, p is a vector representing the bidden quantity at all time periods, $c(p)$ is the production cost of p and $\mathscr{P}$ is the set of feasible UC production plans for the GC, we get the following generic price-taker formulation:

$$\max_{p^t} \quad \sum_{t \in T} \tilde{\pi}^t \sum_{g \in G} p^t_g - c(p) \tag{2.57}$$

$$\text{s.t.} \quad p \in \mathscr{P} \tag{2.58}$$

The bidden price π^t_j is often the marginal production costs of the generators $j \in J$. This information is contained in the UC problem, either by forcing the bidden quantity to 0 or by paying a penalty of extra production if the marginal cost of a generator is above the expected spot price.

This is a reasonable approach if a GC has a relatively small penetration on the market. This approach is also often used when the production capacity of a GC contains uncertainty as with renewable energies to simplify the market part of the problem. Uncertain production capacities must not be neglected as a GC has to honour the bids accepted by the TSO on delivery time or pay penalties. Price-taker formulations being quite simple, uncertainty [Ferruzi et al., 2016] and risk measures [Beraldi et al., 2008] can easily be considered in the UC part of the problem. Revenue management has also been studied from a more global approach by considering bilateral contracts and bidding in a day-ahead market with a price-taker approach by Corchero and Heredia [2011]. Further, bidding in the day-ahead, balancing and adjustment markets as a price-taker is considered as a single problem by Triki et al. [2005].

The drawback of price-taker formulations is the too big abstraction of the market mechanism settling the spot price which is the threshold for selling bids. If the actual spot price is lower than expected, this can lead to negative profit or an infeasible UC production plan if a bid is not sold because of ramping up and down constraints.

2.4.2 Price-maker formulation

Price-maker formulations take into consideration the impacts of the bids of the GC on the spot price making more precise models with a higher degree of complexity as a drawback. These types of problems can be studied through bi-level formulations [Dempe et al., 2015], the leader being the GC, the follower being the TSO.

Simple bidding mechanism are considered in the literature for price-maker formulations, most of them without transmission constraints. Sellers $s \in S$, buyers $b \in B$ and generators $g \in G$ of the GC only place step bids which are associated to a single time period. A bid for a time period $t \in T$ is composed of a price π_c^t and a positive quantity q_c^t for competitors $c \in S \cup B$, a proportion x_c^t is accepted in a market equilibrium. The GC places bids for each generator $g \in G$ at price π_g^t and with a quantity p_g^t, the TSO trading a quantity q_g^t. The bidding prices are generally the marginal production costs of generators.

Once all bids are known, we obtain a continuous formulation for the welfare maximization problem of the TSO decomposable by time period with the following formulation for period $t \in T$:

$$\begin{aligned}
\max \quad & \sum_{b\in B} \pi_b^t q_b^t x_b^t - \sum_{s\in S} \pi_s^t q_s^t x_s^t - \sum_{g\in G} \pi_g^t q_g^t && && (2.59)\\
\text{s.t.} \quad & \sum_{b\in B} q_b^t x_b^t - \sum_{s\in S} q_s^t x_s^t = \sum_{g\in G} q_g^t && (\pi^t) && (2.60)\\
& 0 \le q_g^t \le p_g^t && g \in G && (2.61)\\
& 0 \le x_c^t \le 1 && c \in S \cup B && (2.62)
\end{aligned}$$

As mentioned in the previous section, the resulting spot price π^t is the value of the dual variable of constraint (2.60) in an optimal solution.

The resulting generic formulation for a bidding problem for a GC:

$$\begin{aligned}
\max_{p} \quad & \sum_{t\in T} \pi^t \sum_{g\in G} q_g^t - c(p) && && (2.63)\\
\text{s.t.} \quad & p \in \mathscr{P} && && (2.64)\\
& \forall t \in T \max_{q,x} \quad \sum_{b\in B} \pi_b^t q_b^t x_b^t - \sum_{s\in S} \pi_s^t q_s^t x_s^t - \sum_{g\in G} \pi_g^t q_g^t && && (2.65)\\
& \qquad\qquad \text{s.t.} \quad \sum_{b\in B} Q_b^t x_b^t - \sum_{s\in S} Q_s^t x_s^t = \sum_{g\in G} q_g^t && (\pi^t) && (2.66)\\
& \qquad\qquad\qquad 0 \le q_g^t \le p_g^t && g \in G && (2.67)\\
& \qquad\qquad\qquad 0 \le x_c^t \le 1 && c \in S \cup B \cup G, && (2.68)
\end{aligned}$$

where π_g^t, p, p_g^t, c(p) and $\mathscr{P}$ are defined as for price-taker formulations. This formulation is incomplete and given as an example as the objective function needs the value of the dual variable of constraint (2.66) in the second level problem. Constraints of the dual problem can be introduced to have π^t as a variable of the problem. In the case where the second level problem has several optimal solutions, the optimistic assumption leads to the following properties:

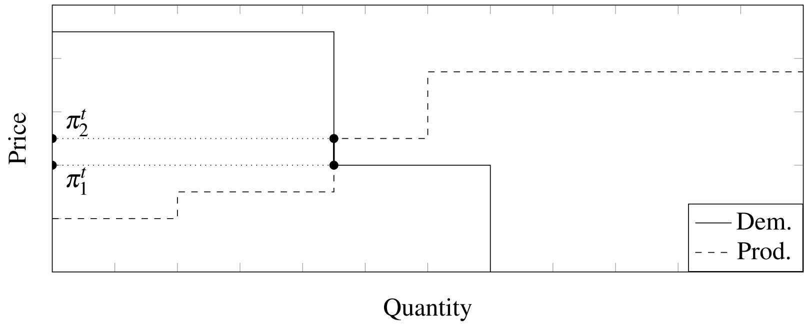

Figure 2.7: Multiple spot price possibilities

- If several spot-prices are possible, the highest one is chosen. This is not restrictive when modelling the problem as it corresponds to actual market mechanisms [Morales et al., 2014]. Several bidding prices are possible if the production and demand curve have a common step, as illustrated in Figure 2.7 where the spot price could lie in the interval $[\pi_1^t, \pi_2^t]$ without considering regulation which selects the highest possible one. The spot price is then said to be *degenerated* in an optimal solution of the dual of the maximization welfare problem.
- The bids of the GC have priority over the bids of the competitors in case the TSO has the choice between several bids to obtain a market equilibrium. If the GC is not producing at loss, its bids are traded first.

This formulation can already be challenging in itself, being a bi-level problem with bilinear terms in the objective function. As the follower problem is linear once the production of the leader is fixed, the bi-level formulation is generally rewritten as a single level problem using KKT conditions. This a reason why block bids are not considered in such bidding problems as they would introduce integer variables in the second level, making the bi-level problem much more challenging.

Price-maker approaches generally lack considering all real life constraints such as physical constraints of production units, various types of bids, full regulations constraints, transmission network constraints, uncertainty,... These approaches tend to start from a simple formulation and add real life constraints to the UC part of the problem or to the market mechanism. Most studies consider a deterministic approach by considering the information over the bids placed by the competitors of the GC is fully known. Real size instances are not yet considered through this approach.

The first study considering bidding in electricity market from a price-maker approach from David [1993] considers a bidding mechanism using only block bids and linear production costs for electricity. The selection of bids considers the quantity sold on the market can exceed the demand in order to avoid paradoxically rejected bids as illustrated in Figure 2.6. Allowing to buy an unneeded quantity of a block bid simplifies significantly the constraints induced by block bids. A dynamic programming is used to solve the problem by considering placing bids from the lowest to biggest production cost.

Other more recent studies consider a bidding mechanism only with step bids as with formulation (2.59)-(2.62) as a base. As it can be observed on Figure 2.5, considering only step bids allows to establish thresholds on the quantity a GC can bid before modifying the spot price. These thresholds are the differences between the quantity at the intersection of the aggregated production and demand curve and the different steps on both curves. Further details on these thresholds are presented in Chapter 5.

When considering only step bids and no transmission constraints in the deterministic case, the spot price of electricity on the market is a decreasing stepwise function depending on the bidden quantity of the GC. It can easily be modelled in a MIP formulation if the GC bids at the spot price as shown by de la Torre et al. [2002]. This allows to make abstraction of the role of the TSO as a follower and allows to consider the full bidden quantity is bought.

Many formulations focus on integrating a full UC formulation on the leader problem, reformulating the second level by using KKT conditions and McCormick envelopes to linearise bilinear terms [McCormick, 1983]. The McCormick inequalities introduce integer variables in the problem and tend to be weak regarding the gap between the LP relaxation and the optimal value. Another approach used to linearize bilinear terms which generally contain a continuous variable representing the spot price is to discretize the possible prices. A simple approach consists in doing a binary expansion of the spot price, decomposing the spot price in a sum of powers of two [Pereira et al., 2005]. This is a classical technique to reformulate continuous variables by using binary variables but is heavy because of the large number of binary variables needed for each continuous variable that is reformulated. Other studies consider a reduced discrete set of bidding price that is determined by the bids of the competitors [de la Torre et al., 2002] which reduces the number of binary variables in comparison to binary expansions.

One option offered to GCs through market regulations is to assign continuous piecewise bidding curves to generators where the steepness of each segment is based on marginal production costs at the breakpoint of the curve as illustrated in Figure 2.8. This has been studied by Bakirtzis et al. [2007] by considering that each generator can place several bids with the constraint that the bids must be bought from the cheapest to the most expensive.

Dalby [2017] uses the same curve bidding approach to minimize the difference between the profit and the production costs in order to recover start-up costs based on market equilibrium constraints presented by Bakirtzis et al. [2007].

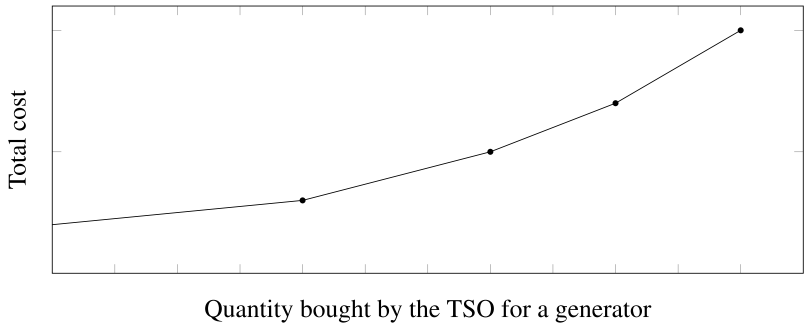

Figure 2.8: Bidding curve for generators

Bidding in the day-ahead, balancing and adjustment markets together has also been studied by Simoglou and Bakirtzis [2008]. Obviously, the bidding should be considered in all markets simultaneously as the same generators will produce energy for all markets. This problem is of course much more challenging and the UC formulation considered is simplified by considering only the marginal production costs, eliminating binary variables needed for start-up and shutdown costs.

The transmission constraint of the market managed by the TSO has not often been considered in the literature for price-maker approaches. This can be partially explained by the fact electricity constraints are complicated to model in a linear formulation. Still a couple of studies consider a transmission network without the full physical constraints. Kardakos et al. [2014] evaluate the impact of integrating a nodal formulation or a power transfer distribution factor formulation for the transmission problem at the TSO level, illustrating the complexity of such constraints. Ruiz and Conejo [2009] observe the identical spot price that can occur in local market when the transmission network is not restrictive, including the first and second Kirchhoff laws in their formulation which are common constraints in electricity transmission networks [Gómez-Expósito et al., 2008].

One last important element that is part of bidding problems in real life is the uncertainty in the bids of competitors. This has mainly been studied by Fampa et al. who propose a stochastic formulation of the problem where scenarios are sets of bids proposed by competitors. The introduction of stochasticity forces the strong hypothesis that production costs are linear. The problem is solved through heuristic methods [Fampa et al., 2008, Fampa and Pimentel, 2015] and an upper bound on the optimal value of the problem has been presented as well [Fampa and Pimentel, 2017].

Price-maker formulations that use the most accurate representation of the bidding mechanism are still far from integrating all constraints of bidding problems or considering real life instances, which can explain why price-taker formulations that ignore in some way the market mechanism are more commonly used.

Chapter 3

Production security

3.1 Introduction

The social organization has integrated the use of electricity in almost all of its activities nowadays. Failure in electricity systems have growing negative impacts and distribution must become increasingly more reliable [Rifkin, 2011]. Electricity production is appearing at a local level but customers still expect from generation compagnies (GCs) and the electricity transmission network to be able to supply their demand without risking a blackout. A GC needs to be able to trace the margin of production of its feeders on the network to evaluate to what extent they can support a variation of the demand and avoid having an overload for a feeder. There is also a necessity in guaranteeing a minimum security in transmission. The margin of feeders on a network can be studied through the *Minimum Margin Problem* (MMP) presented by Rossi et al. [2011] that considers *hop constraints* to add robustness in the transmission of electricity.

The MMP is an electricity distribution network configuration problem in which a set of electricity feeders have a maximum production capacity. A set of customers must be assigned to the feeders through an existing network. Each customer has a demand, is assigned to a single feeder and the customers assigned to a feeder must form a connected component. The goal of the MMP is to maximize the minimum margin over all feeders. The margin of a feeder is defined as the difference between its maximum production capacity and the total demand of customers assigned to this feeder.

Many distribution network configuration problems include hop constraints that limit the maximum number of transmission lines between a customer and its feeder to a given value H. Hop constraints are used to model reliability issues. Consider a customer assigned to its feeder through a path of length d. If there is a failure on a vertex or an edge of this path, a customer could needed to be supplied through another path. The bigger the value of d, the higher the risk of failure in distribution [Ljubić and Gollowitzer, 2010]. The value of H can be fixed to limit the maximum probability of failure of transmission to customers, introducing robustness in the problem.

Hop constraints were already considered in several network design problems by Balakrishnan and Altinkemer [1992] and Pirkul and Soni [2003] to mention some of them. These problems have often been studied using extended formulations with variables keeping track of the distances of vertices and/or edges from a root. Over time, layered graphs, introduced by Gouveia [1998] and presented in Section 2.1.3, have been used for problems including hop constraints to build extended formulations. A layered graph derived from a graph G is a graph containing $H+1$ layers, layer 0 containing a root r and layer h containing vertices of G that can be reached through a path of length h from r.

This Chapter is organized as follows. The *Minimum Margin Problem* is studied in Section 3.2. The problem definition and the formulation of Rossi et al. are given in Sections 3.2.1 and 3.2.2. A new extended formulation [Conforti et al., 2010] using layered graph and preprocessing techniques are presented in Section 3.2.3, reducing the size of the model and strengthening it. The different formulations as well as the impact of parameters of MMP are analyzed in Section 3.2.5. Two extensions of the MMP are presented in Section 3.3. Section 3.3.1 considers power losses which occur on transmission lines and Section 3.3.2 considers a maximum distance between feeders and customers in the case where the length of each transmission line must be considered.

A paper presenting the contributions from this chapter has been published in the European Journal of Operational Research [De Boeck and Fortz, 2017].

3.2 Minimum margin problem

3.2.1 Problem description

The *Minimum Margin Problem* (MMP), introduced by Rossi et al. [2011], considers an electricity transmission network represented by a graph $G=(V,E)$. Set V is partitioned into a set of feeders V_f and a set of customers V_c. Each customer $i \in V_c$ has a demand $dem_i \geq 0$ and each feeder $j \in V_f$ has a maximum capacity c_j. The margin M_j of feeder j is the difference between its maximum capacity and the demand of customers assigned to j. The minimum margin M_{min} is the minimum value M_j over all generators. The goal is to find an assignment of customers to feeders respecting hop constraints and maximizing the minimum margin of the feeders. Hop constraints limit the length of the shortest paths between each feeder and the customers assigned to it to a maximum value H. Consider a set $S = \{S_j\}_{j \in V_f}$ of disjoint subsets of nodes $S_j \subseteq V$. Set S is a feasible solution of MMP if:

- each set S_j contains feeder j and each customer $i \in V_c$ assigned to j,
- all customers $i \in V_c$ are assigned to exactly one set S_j,
- for each feeder $j \in V_f$, component $C_j^S = (S_j, E(S_j))$, with $E(S_j) = \{uv \in E | u, v \in S_j\}$, is connected and respects the hop contraints.

Rossi et al. [2011] showed that MMP is strongly *NP-hard* by reduction from 3-Partition [Garey and Johnson, 1979].
Consider a set of subtrees $\{T_j\}_{j \in V_f}$ of G, each one rooted in a different generator $j \in V_f$ and respecting the hop constraints. If S_j represents the vertices of tree T_j and $S = \{S_j\}_{j \in V_f}$ is a feasible solution, then T_j is a spanning tree in component C_j^S and satisfies the hop constraints. A same solution S can be built from different sets of subtrees $\{T_j\}_{j \in V_f}$ of G as each component C_j^S can have multiple spanning trees respecting hop constraints.
We denote by d_{ij} the minimum distance in G between feeder $j \in V_f$ and customer $i \in V_c$ without going through any other feeder, where the distance between two vertices is defined as the minimum number of edges in a path between these vertices. If no such path exists, $d_{ij} = +\infty$. Set $N_j = \{i \in V_c | d_{ij} \leq H\}$ is the set of customers that can potentially be assigned to j. The minimum feasible distance d_{min} is the minimum distance for which each customer in V_c can be assigned to at least one feeder.

3.2.2 Vertex formulation

Rossi et al. [2011] proposed a MIP formulation to solve MMP through a cutting plane algorithm. This *Vertex Formulation* (VF) uses assignment decision variables x_{ij}.

$$x_{ij} = \begin{cases} 1 & \text{if and only if } i \in V_c \text{ is assigned to } j \in V_f, \\ 0 & \text{otherwise.} \end{cases}$$

Based on values of variables x_{ij}, sets $\{S_j\}_{j \in V_f}$ are defined by $S_j = \{j\} \cup \{i \in V_c | x_{ij} = 1\}$. The VF formulation of Rossi et al. [2011] is:

$$\begin{aligned} \text{Max} \quad & M_{min} && && (3.1) \\ \text{s.t.} \quad & c_j - \sum_{i \in N_j} x_{ij} dem_i \geq M_{min} && \forall j \in V_f && (3.2) \\ & \sum_{j \in V_f} x_{ij} = 1 && \forall i \in V_c && (3.3) \end{aligned}$$

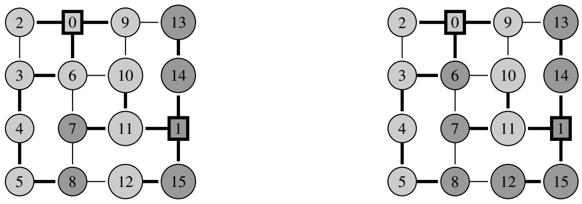

Figure 3.1: Solutions of (3.1)-(3.6)

$$x_{ij} = 0 \qquad \forall j \in V_f, i \notin N_j \tag{3.4}$$

$$x_{ij} \leq \sum_{k \in P_{ij}} x_{kj} \qquad \forall j \in V_f, d \in \{2, \ldots, H\}, i \in L^s_{j,d} \tag{3.5}$$

$$x_{ij} \in \{0,1\} \qquad \forall i \in V_c, j \in V_f \tag{3.6}$$

$$C^S_j \text{ is connected} \qquad \forall j \in V_f \tag{3.7}$$

$$\text{Distance between customer } i \text{ and its assigned feeder } j \text{ is at most } H \text{ in } C^S_j \qquad \forall i \in V_c \tag{3.8}$$

In this formulation, P_{ij} is the set of customers at distance $d_{ij} - 1$ of j in G. Set $L^s_{jd} = \{i \in V_c | d_{ij} = d\}$, is the set of customers i at shortest distance d from j in G. The number of constraints needed to ensure the connectivity of the components (3.7) and the hop constraints (3.8) is exponential. Rossi et al. proposed a cutting plane algorithm, starting only with margin constraints (3.2), assignment constraints (3.3), constraints (3.4) for customers that cannot be assigned to a feeder and *layered constraints* (3.5). These layered constraints represent the fact that if a customer i is at minimum distance $d > 1$ of a feeder j in G, i can be assigned to j only if another customer at minimum distance $d - 1$ is also assigned to j. Layered constraints are necessary for connectivity of solutions but not sufficient.

The cutting plane algorithm starts with constraints (3.1)-(3.6), relaxing the connectivity and hop constraints. Figure 3.1 illustrate two solutions of this initial formulation for $H = 4$ that are not feasible for MMP. Feeders are nodes 0 and 1 and customers are assigned to the feeder of the same color. In the left solution, customers 7 and 8 are not connected to their feeder and in the right solution customer 6 does not respect the hop constraint.

First, connectivity cuts are added in the cutting plane algorithm. When an integer solution is found, the connectivity of the subgraphs induced by each feeder and their respective customers is checked. The set of customers that are allocated to feeder $j \in V_f$ but that are not connected to it is denoted O_j. If O_j is none empty, connectivity cuts need to be added to the formulation. Let $\mathscr{C}_j$ be the set of connected components in O_j. A connectivity cut is added for each connected component in $\mathscr{C}_j$ of each feeder. Let CC be a connected component in C_j, and CS be the set of customers adjacent to CC but whose feeder is not j. A customer $i_{CC} \in CC$ whose distance to j is minimum in G is selected at random. At least one customer in CS must be assigned to feeder j to ensure i_{CC} is connected to j. Furthermore, all customers $k \in CS$ such that $d_{jk} + d_{ki_{CC}} > H$ can be removed from CS because of the hop constraint, d_{uv} representing the minimum distance in G between vertices u and v. This leads to the following connectivity cuts:

$$x_{i_{CC}j} \leq \sum_{k \in CS,\ d_{jk}+d_{ki_{CC}} \leq H} x_{kj} \qquad \forall CC \in \mathscr{C}_j, j \in V_f$$

In Figure 3.1, a connectivity cut is needed for component $\{7,8\}$ in the left solution. The closest customer to its feeder is customer 7 and $CS = \{6,11,12\}$ resulting in the following connectivity cut :

$$x_{7,1} \leq x_{6,1} + x_{11,1} + x_{12,1}$$

If an integer solution has empty sets O_j for each feeder $j \in V_f$, sets CC_j composed of feeder j and all customers assigned to it are connected components. The hop constraint is then checked by computing the distance to j in CC_j for each customer. Let F_j be the set of customers in CC_j at distance $H+1$ from j and CS_j the set of adjacent customers to CC_j assigned to a feeder different than j. If F_j is not empty, customers are violating the hop contraint. Constraint (3.4) guarantees a customer is assigned to a feeder only if its distance to the feeder is at most H in G. For each customer $i \in F_j$, there is at least one vertex $i' \in CS_j$ that must be assigned to j in order to respect the hop constraint, leading to the following distance cuts:

$$x_{ij} \leq \sum_{k \in CS_j} x_{kj} \qquad \forall i \in F_j, j \in V_f$$

In Figure 3.1, a distance cut is needed for customer 6 in the right solution. Set CS_1 = {3,5,9,10,11}, resulting in the following distance cut :

$$x_{6,1} \leq x_{3,1} + x_{5,1} + x_{9,1} + x_{5,10} + x_{9,11}$$

If no connectivity or distance cuts are found for an integer solution, it is feasible for MMP and the cutting plane algorithm terminates.

3.2.3 Layered Formulation

We introduce a compact extended formulation to solve MMP using layered graphs as well as preprocessing techniques reducing the size of the presented model. Layered graphs $G_j^H = (V_j^H, E_j^H)$ are defined for all feeder $j \in V_f$ using notations introduced in Section 2.1.3.

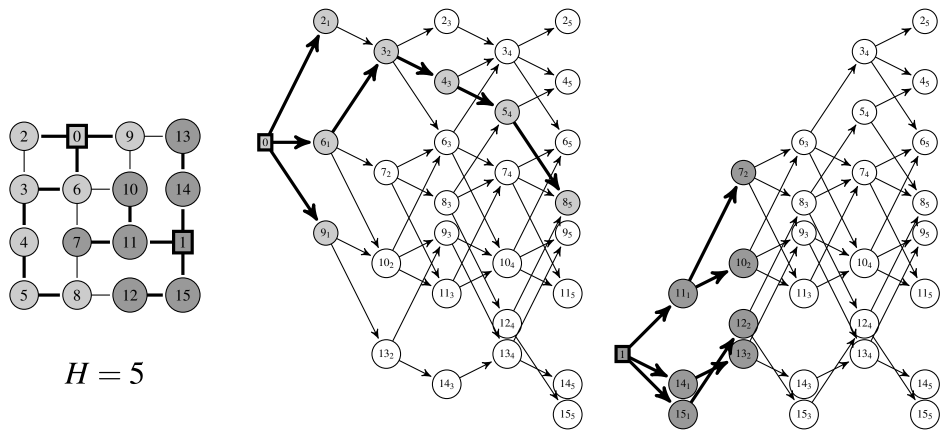

Figure 3.2: Solution of MMP of a network G on G^5

Feasible solutions of MMP can be represented on a set of m layered graphs $G_j^H = (V_j^H, A_j^H)$, one for each feeder $j \in V_f$. The layered graph G_j^H is built from G without using feeders different than j, that is, a feeder $j' \neq j$ will not appear in any layer L_{jd}. The set $G^H = \bigcup_{j \in V_f} G_j^H$ is the set of all layered graphs.

Lemma 1. *A feasible solution S of MMP can be represented on G^H if and only if $H \geq d_{min}$*

Proof. Consider a feasible solution S of MMP and all its connected components C_j^S on G. For any component C_j^S, consider a shortest path tree T_j^S. As C_j^S respects the hop constraints, T_j^S respects them as well. Tree T_j^S is representable on G_j^H if and only if the maximum distance between j and its assigned customers is at most H. By definition, d_{min} is the minimum distance for which all customers can be assigned to a feeder, so $d_{min} \leq H$. $\square$

Lemma 1 gives a lower bound for H, so we consider in the following $d_{min} \leq H$.

Figure 3.2 gives an example of a feasible solution of MMP on G^5. Square nodes 0 and 1 are two feeders and circle nodes 2 to 15 are customers.

An extended compact formulation of MMP is obtained using G^H and assignment-distance binary variables x_{ijd}:

$$x_{ijd} = \begin{cases} 1 & \text{if and only if } i \in V_c \text{ is assigned to } j \in V_f \text{ at a distance } 1 \leq d \leq H, \\ 0 & \text{otherwise.} \end{cases}$$

Variables x_{ijd} define the value of x_{ij} through $x_{ij} = \sum_{1 \leq d \leq H} x_{ijd}$. Assignments of customers at a distance satisfying the hop constraints are ensured by this relation and (3.3), which allows to remove hop constraints (3.8) from VF. Sets S_j are defined as in VF, $S_j = \{j\} \cup \{i \in V_c | x_{ij} = 1\}$. In a feasible solution S of MMP represented on G^H, each customer $i \in V_c$ is assigned to a single layered graph G_j^H in exactly one layer d. Moreover if a customer $i \in V_c$ is assigned to j at distance d, that is $x_{ijd} = 1$, then $i \in L_{jd}$ has at least one predecessor assigned to the same feeder in layer L_{jd-1} (if $d > 1$), otherwise C_j^S cannot be connected. Based on this observation, the exponential number of connectivity constraints (3.7) that is required in VF can be replaced by a polynomial number of connectivity constraints based on variables x_{ijd} and predecessor sets $P_{ijd} = \{k \in L_{jd-1} | k_{d-1} i_d \in A_j^H\}$:

$$x_{ijd} \leq \sum_{k \in P_{ijd}} x_{kjd-1} \quad \forall j \in V_f, d \in \{2, \ldots, H\}, i \in L_{jd}$$

These connectivity constraints ensure connectivity of components C_j^S for all feeders. The number of these constraints linearly depends on n, m and H.

This leads to a compact *Layered Formulation* (LF) of MMP:

$$\text{Max } M_{min} \tag{3.9}$$

$$\text{s.t. } c_j - \sum_{i \in N_j} \sum_{1 \leq d \leq H} x_{ijd} dem_i \geq M_{min} \qquad \forall j \in V_f \tag{3.10}$$

$$\sum_{j \in V_f} \sum_{1 \leq d \leq H} x_{ijd} = 1 \qquad \forall i \in V_c \tag{3.11}$$

$$x_{ijd} \leq \sum_{k \in P_{ijd}} x_{kj(d-1)} \qquad \forall j \in V_f, d \in \{2, \ldots, H\}, i \in L_{jd} \tag{3.12}$$

$$x_{ijd} = 0 \qquad \forall j \in V_f, d \in \{1, \ldots, H\}, i \notin L_{jd} \tag{3.13}$$

$$x_{ijd} \in \{0, 1\} \qquad \forall j \in V_f, i \in V_c, d \in \{1, \ldots, H\} \tag{3.14}$$

Constraints (3.10) bound M_{min} with the margins of the feeders, constraints (3.11) ensure customers are assigned to a single feeder at distance less than or equal to H. Constraints (3.12) ensure connectivity and constraints (3.13) set variables x_{ijd} to 0 when a customer i cannot be assigned at distance d to feeder j, that is, if $i \notin L_{jd}$. Variables set to 0 are removed from the formulation during preprocessing but we keep them here to allow comparaisons of the solution space of LF with further formulations.

Lemma 2. *Formulation LF is valid for MMP.*

Proof. Consider a feasible solution (x_{ijd}) of LF and the associated partition of nodes $S = \{S_j\}_{j \in V_f}$ of MMP where $S_j = \{i \in V_c : \sum_{1 \leq h \leq H} x_{ijd} = 1\}$. We need to show that each customer $i \in V_c$ is assigned and connected to its feeder at maximum distance H through S. Constraints (3.11) ensure that each customer i in V_c is assigned to exactly one feeder j at a certain distance d with $x_{ijd} > 0$. If d=1, i is a neighbour of j and connectivity is ensured. If $1 < d \leq H$, connectivity constraints (3.12) ensure there exists a variable $x_{i'jd-1} > 0$, with $i' \neq i$ and $ii' \in E$ so i is connected to i' in C_j^S. As $i' \in P_{ijd}$, i' is closer to j than i. Constraints (3.12) can be used to find a path connecting i to j in C_j^S. This procedure will terminate after at most $H-1$ iterations as d is decreased at each iteration, leading to paths of maximum length H. □

Constraints (3.11)-(3.14) guarantee the connectivity of solutions as well as hop constraints. The number of variables and constraints is $O(mnH)$. The value of H influences the size of LF. If H has the minimum value d_{min}, some customers can be assigned to a single feeder at a single distance, fixing some assignments during preprocessing. Increasing H might increase the number of feeders a customer could be assigned to leading to a combinatorially more complex problem and a possible better optimal value. We analyse the influence of H on the computation time and the optimal value in Section 3.2.5.

The number of binary variables in LF depends on H and can be considerably larger than in VF. To reduce the number of such variables, variables x_{ij} are reintroduced and a relaxation is performed over variables x_{ijd} in the *Layered Formulation Relaxation* (LFR):

$$\text{Max } M_{min} \tag{3.15}$$

$$\text{s.t. } c_j - \sum_{i \in N_j} x_{ij} dem_i \geq M_{min} \qquad \forall j \in V_f \tag{3.16}$$

$$\sum_{j \in V_f} x_{ij} = 1 \qquad \forall i \in V_c \tag{3.17}$$

$$x_{ij} = \sum_{1 \leq d \leq H} x_{ijd} \qquad \forall j \in V_f, i \in N_j \tag{3.18}$$

$$x_{ijd} \leq \sum_{k \in P_{ijd}} x_{kj(d-1)} \qquad \forall j \in V_f, d \in \{2, \dots, H\}, i \in L_{jd} \tag{3.19}$$

$$x_{ijd} = 0 \qquad \forall j \in V_f, d \in \{1, \dots, H\}, i \notin L_{jd} \tag{3.20}$$

$$x_{ij} \in \{0,1\} \qquad \forall j \in V_f, i \in N_j \tag{3.21}$$

$$0 \leq x_{ijd} \leq 1 \qquad \forall j \in V_f, d \in \{1, \dots, H\}, i \in L_{jd} \tag{3.22}$$

Lemma 3. *Formulation LFR is valid for MMP.*

Proof. The result follows from the fact that there always exist a feasible solution of LFR where x_{ijd} are binary if x_{ij} are binary. Indeed, given a solution (x_{ij}, x_{ijd}) of LFR with x_{ijd} fractional, a integer solution (x_{ij}, x'_{ijd}) with the same cost can be derived by setting $x'_{ijd} = 1$ for the smaller d such that $x_{ijd} > 0$ and $x_{ijd'} = 0$ for $d' \neq d$. It is then easy to see that x'_{ijd} is a solution of LF, hence of MMP. □

Let $\mathscr{S}^F$ be the solution space of a formulation F and $\overline{F}$ be the LP-relaxation of F. When substituting variables x_{ij} by $\sum_{1\leq d\leq H} x_{ijd}$ in the LP-relaxation of LFR using constraint (3.18), we get the LP-relaxation of LF, leading to $proj_{x_{ijd}}(\mathscr{S}^{\overline{\text{LF}}})$ $=proj_{x_{ijd}}(\mathscr{S}^{\overline{\text{LFR}}})$. The number of binary variables in LFR is the same than in VF and at most the same than in LF as each variable x_{ij} in LFR corresponds to at least one variable x_{ijd} in LF. Computational results obtained for LF and LFR with state-of-the-art MIP solvers are reported in Section 3.2.5.

In these experiments, a set of instances on transmission networks represented as bipartite graphs will be used. Layered graphs derived from bipartite graphs can have a lower number of vertices than when using random graphs.

Lemma 4. *If $G = (V,E)$ is a bipartite graph, then for any $u, v \in V, u \neq v$, u cannot appear in two consecutive layers of G^H_v.*

Proof. In a bipartite graph $G = (V,E), V = V_1 \cup V_2$, paths between vertices in the same set V_i are of even length and paths between vertices in different sets V_i are of odd length. Suppose vertex u appears in two consecutive layers d and $d+1$, that is there exists a path of length d and another of length $d+1$ from v to u in G, this contradicts the bipartite hypothesis. □

To illustrate Lemma 4, the graph used in Figure 3.2 is bipartite. We can observe each customer does not appear in two consecutive layers of G^5_0 or G^5_1.

3.2.4 Preprocessing of layered graphs

A feasible solution $S = \{S_j\}_{j\in V_f}$ of MMP defines a set of connected components C^S_j in a graph $G = (V,E)$. All components can be rebuilt from a spanning tree $T^S_j \subseteq C^S_j$ satisfying hop constraints, in particular from any shortest path tree of C^S_j. When searching in G^H_v for the representation of a shortest path tree $T_v \subseteq G$ of a subset of vertices $V' \subseteq V$, some reductions can be applied to G^H_v to remove some arcs and vertices that cannot appear in a shortest path tree. Let $\delta^-(u_d)$ be the number of incoming arcs of u_d in G^H_v and $G(V') = (V', E(V'))$ be the induced subgraph of G for a set of vertices $V' \subseteq V$, $E(V') = \{uv \in E | u, v \in V'\}$.

We define a *redundant* vertex/arc of G^H_v as a vertex or arc that cannot appear in the representation on G^H_v of a shortest path rooted in v in G', for any induced subgraph $G' = G(V')$ where $V' \subseteq V$ contains v.

Redundant vertices and arcs can be removed from G^H_v as they will not appear in the representation on G^H_v of any shortest path tree of a subgraph $G(V'), V' \subseteq V$ containing v. All vertices and arcs that are accessible in G^H_v only through a redundant vertex or arc can also be eliminated from G^H_v. We introduce in the following a series of reductions that eliminate redundant vertices and arcs. These reductions eliminate form G^H the possibility of representing some spanning

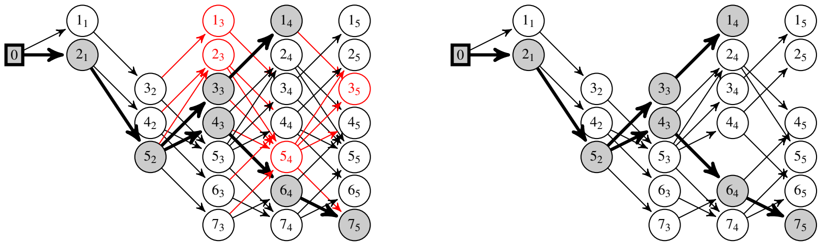

Figure 3.3: Simple path reductions

trees of components C_j^S that are not shortest path trees. In the following figures used to illustrate reductions techniques, red vertices and arcs in the left-hand side graph are those eliminated from the original layered graph by a specific reduction technique, and the right-hand side graph contains the modified tree representation of the tree in Figure 2.1 with possibly shorter paths.

Simple path reductions

In a tree T_v of G rooted in v, all paths between v and a vertex of T_v are simple paths, that is, paths where each vertex of G appears at most once. In G_v^H, if a vertex u_d has only one incoming arc from a vertex u'_{d-1}, the outgoing arc going to u'_{d+1} going back to u' will never be used in a shortest path tree T_v rooted in v. *Simple Path Reductions* (SPR) remove such arcs from G_v^H, as well as vertices that have no more incoming arc after eliminating these arcs.

Lemma 5. *Let u be a node of in-degree 1 at level d (i.e. $\delta^-(u_d) = 1$) and let u'_{d-1} be the unique predecessor of u_d in G_v^H. Then arc $u_d u'_{d+1}$ is redundant.*

Proof. Assume there exists a node u such that $\delta^-(u_d) = 1$, $u'_{d-1}u_d \in A_v^H$ and $u_d u'_{d+1} \in A_v^H$ is not redundant. Then there exists a subgraph $G' = G(V')$, where $V' \subseteq V$ contains v, and a shortest path P from v to u' in G' rooted in v such that arc $u_d u'_{d+1}$ belongs to P. Since $u'_{d-1}u_d$ is the only arc incoming in u_d, it also belongs to P. But then the subpath of P from v to u'_{d-1} is shorter than P, contradicting the fact that P is a shortest path. Hence $u_d u'_{d+1}$ is redundant.

□

Figure 3.3 gives the layered graph G_v^H with SPR of the graph considered in Figure 2.1. The SPR can be applied to a layered graph G_v^H in $O(n^2 H)$ [De Boeck and Fortz, 2017].

Root neighbour reductions

In any path P rooted in v in G, the second vertex is in any case a neighbour of v. These neighbours of v can only appear in layer 1 in the representation of P on G_v^H when considering P is a shortest path. *Root Neighbour Reductions* (RNR) remove from G_v^H all vertices u_d such

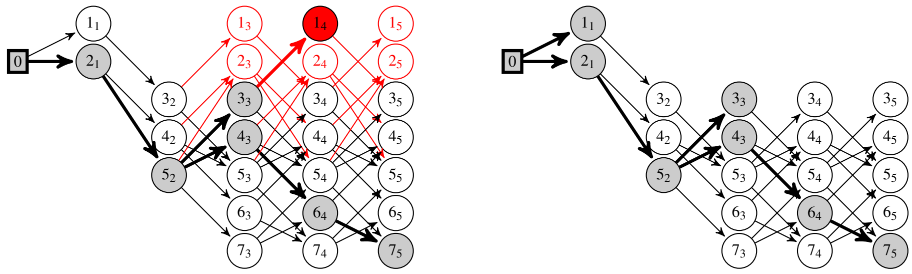

Figure 3.4: Root neighbour reductions

as $vu \in E$ and $d > 1$, as well as all vertices that have no more incoming arcs after removal of neighbours of v.

Lemma 6. *All vertices $u_d \in V_v^H$ such that $vu \in E$ and $d > 1$ are redundant.*

Proof. Consider a subgraph $G' = G(V')$, where $V' \subseteq V$ contains v, a shortest path P in G' rooted in v of maximum length H, its representation P^H on G_v^H and a vertex $u_d \in V_v^H$ with $d > 1$ such as $vu \in E$. Vertex $u_d \notin P^H$, otherwise P is not a shortest path as vertices from position 1 to $d-1$ could be eliminated. Thus vertex u_d is redundant. $\square$

In Figure 3.4, considering the layered graph G_v^H of Figure 2.1, vertices 1 and 2 are eliminated from all layers except layer 1.

The RNR can be applied to a layered graph G_v^H in $O(n^2H)$ [De Boeck and Fortz, 2017].

Triangle reductions

If an induced subgraph $G' \subseteq G$ contains some triangle $u\,u'\,u''$, a shortest path P from $v \in V$ to a another vertex of G' will not contain the three vertices of the triangle. If the first vertex of $u\,u'\,u''$ in P is at distance $d-1$, then the two other vertices are reachable at distance d. *Triangle Reductions* (TR) remove redundant arcs in triangles, as well as all vertices that have no more incoming arcs.

Lemma 7. *If G contains a triangle $u\,u'\,u''$, $\delta^-(u_d) = 1$ and $u''_{d-1}u_d \in A_v^H$, then arc $u_d u'_{d+1}$ is redundant.*

Proof. Assume there exists a triangle $u\,u'\,u''$ in G such that $\delta^-(u_d) = 1$, $u''_{d-1}u_d \in A_v^H$ and $u_d u'_{d+1} \in A_v^H$ is not redundant. Then there exists a subgraph $G' = G(V')$, where $V' \subseteq V$ contains v, and a shortest path P from v to u' in G' rooted in v such that arc $u_d u'_{d+1}$ belongs to P. Since $u''_{d-1}u_d$ is the only arc incoming in u_d, it also belongs to P. As $u\,u'\,u''$ form a triangle in G, $u''_{d-1}u'_d \in A_v^H$ and the path obtained by adding $u''_{d-1}u'_d$ to the subpath of P from v to u''_{d-1} is shorter than P, contradicting the fact that P is a shortest path. Hence $u_d u'_{d+1}$ is redundant.

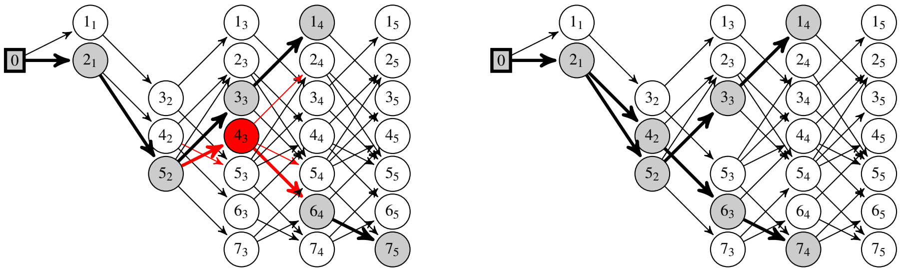

Figure 3.5: Triangle reductions

□

The TR can be applied in $O(n^2H)$ [De Boeck and Fortz, 2017]. Figure 3.5 illustrates TR on the graph of Figure 2.1. In the initial graph, consider triangle 2-4-5.

In the layered graph of G if 2_1 is in T_0^H, vertices 4_2 and 5_2 can be in T_v^H. There is no reason to consider path $2_1 - 5_2 - 4_3$ in G_0^H and as 5_2 can only be reached through 2_1, arc $5_2 4_3$ will not appear in a shortest path from v.

Shortest path tree reductions

A combination of reductions SPR, RNR and TR, can be applied through *Shortest Path Tree Reductions* (SPTR) to reduce the size of G_v^H, leading to a graph G_v^{HR}. Set $G^{HR} = \bigcup_{j \in V_f} G_j^{HR}$. In reductions SPR and TR, a necessary condition to remove an arc is that its source must have only one incoming arc. After applying SPR or TR, the number of incoming arcs of some vertices decrease, leading to possible new arc elimination if reductions are performed once again. Consider as an example the layered graph from Figure 2.1. Arc $6_4 7_5$ is not eliminated when applying once SPR or TR in Figures 3.3 and 3.5. After applying SPR and TR, vertex 6_4 has only one incoming arc left, coming from 7_3, thus arc $6_4 7_5$ will be removed if SPR are performed once again.

To perform SPTR, first RNR are applied before iterating SPR and TR until no more arc is removed. Performing an iteration of SPR and TR is made in $O(n^2H)$. If at an iteration of SPR-TR some arcs have been removed, consider the minimum distance d for which an arc $u_d u'_{d+1}$ has been removed. Iterating SPR-TR is interesting in the case there are new vertices with only one incoming arc. As there are no new vertices with only one incoming arc in L_{vd} after the last iteration of SPR-TR, no arc between layers d and $d+1$ will be removed after a new iteration. This leads to a maximum of $H-1$ iterations of SPR-TR and a worst case complexity of $O(n^2H^2)$ for SPTR. When working with bipartite graphs, SPTR have a complexity of $O(n^2H)$ as, after RNR, only a single iteration of SPR is performed because there are no triangles to apply TR.

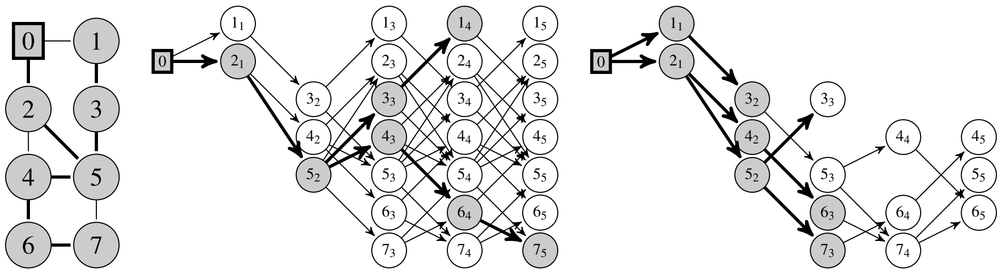

Figure 3.6: Shortest path tree reductions applied on a layered graph

The complexity computed above is for applying reductions in a single layered graph. The overall complexity to apply reductions to all the layered graphs is therefore $O(mn^2H^2)$.

Figure 3.6 gives the graph G of Figure 2.1 with layered graphs G_0^5 and G_0^{5R}. A total of 11 vertices (42%) and 29 arcs (63%) are eliminated with SPTR from the initial representation of G_0^H.

Lemma 8. *All feasible solutions of MMP can be represented on G^{HR}.*

Proof. Consider a feasible solution S of MMP and its components C_j^S. For each component C_j^S consider a shortest path tree T_j^S. As vertices and arcs removed from G_j^H with SPTR are redundant, T_j^S can be represented on G_j^{HR}. □

The solution spaces of LF and LFR are possibly reduced by SPTR as for each vertex i eliminated from L_{jd}, variable x_{ijd} is set to 0 with constraint (3.13) or (3.20). For each eliminated arc $i'_{d-1}i_d$, the set of predecessors P_{ijd} of the target of this arc is reduced and its associated connectivity constraints (3.12) or (3.19) become tighter in both formulations as the right-hand side will contain less positive variables. In the following, we shall denote by LF-R, respectively LFR-R, formulation LF, respectively LFR, using G^{HR} rather than G^H. As for LF and LFR, LP-relaxations of LF-R and LFR-R are identical. Lemma 8 indicates that any feasible solution of LF-R, respectively LFR-R, is a feasible solution of LF, respectively LFR, thus $\mathscr{S}^{\text{LF-R}} \subseteq \mathscr{S}^{\text{LF}}$ and $\mathscr{S}^{\text{LFR-R}} \subseteq \mathscr{S}^{\text{LFR}}$, the same inclusion holds for LP-relaxations. A study of the computation times needed to solve different formulations is presented in Section 3.2.5. The objective values obtained when solving the LP-relaxations of LFR and LFR-R are compared in Section 3.2.5.

The reductions presented in this section can be applied to other hop constrained optimization problems, in particular if an optimal solution can be represented by a set of shortest path trees. For instance, SPR can be applied when searching a solution containing simple paths. On hop constrained spanning or Steiner trees [Gouveia, 1998, Gouveia et al., 2011], the RNR and TR can also be used on instances where weight of arcs respect the triangular inequality, that is, the weight of an arc uv is at most the sum of weights of arcs of any path going from u to v. In the following section, we analyse the computation time needed to apply the reductions to layered graphs for MMP as well as the performance gain obtained by solving LF-R or LFR-R rather than LF or LFR.

3.2.5 Numerical results

After describing the instances used in this Section, we analyze the computation time needed to build layered graphs and perform reductions. The impact of the reductions on the size of the layered graphs is studied. We report the performances of VF, LF, LFR, LF-R and LFR-R, and their LP-relaxations, before analyzing the impact of parameters H, n and m.

All the solution methods were implemented using Java 1.8.0 and ILOG CPLEX 12.6 Java API. Tests were made on a 12-core i7-4930K 3.40 GHz processor limiting RAM memory to 4Gb and computation time to 1800s. The RAM is limited as none of the tests performed encounter memory issues. The average computation times provided include the time of instances that are not solved to optimality in 1800s.

Instances

Instances from Rossi et al. [2011] were kindly provided by the authors and are used to compare VF, LF and LFR. Rossi et al. use mainly two types of instances: *mimetic* and *square*. These instances are derived from grid-graphs [Itai et al., 1982]. A grid graph $G_{m,m}$ is a graph having vertices at all possible integer coordinates (x,y), with $0 \leq x < m$, $0 \leq y < n$. An edge uv, $u = (x_u, y_u)$ and $v = (x_v, y_v)$, is in $G_{m,m}$ if $u, v \in G_{m,m}$ and $|x_u - x_v| + |y_u - y_v| = 1$. Grid-graphs are bipartite graphs and have a structure close to real electricity distribution networks [Enacheanu et al., 2009]. We refer to mimetic instances as *bipartite* instances in the remainder of this section. Additional bigger bipartite instances have been generated as well as *diagonal* instances to evaluate solving of MMP on non bipartite graphs. All instances are of the following types:

- *square*: G is a complete square grid-graph,
- *bipartite*: G is a connected subgraph of a *square* instance,
- *diagonal*: modification of *bipartite* instances, where some edges have been transformed into diagonals.

Diagonal instances can contain triangles or cycles of odd length, none of the diagonal instances used are bipartite. Instances are available at `https://github.com/jdeboeck/MMP`.

Instances are illustrated on Figure 3.7, square nodes represent feeders. The feeders are placed on a circle centered in the middle of the graph. The density of bipartite and diagonal instances,

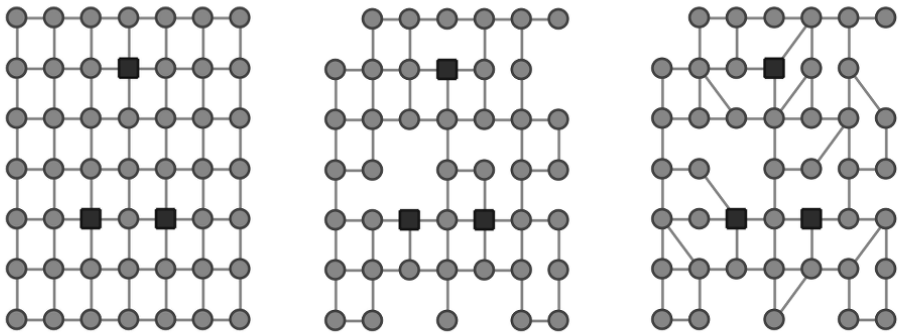

Figure 3.7: Overview of *square*, *bipartite* and *diagonal* instances

representing the proportion of nodes that are kept from the original grid-graph, is on average 60%. All these instances are connected and the average degree of the nodes is between 2.6 and 2.9. The circle nodes are the customers and have an integer random demand between 0 and 100. The capacity of the feeders is sufficient so that any feeder could supply the whole network to guaranty M_{min} is positive. Each configuration *type-n-m* is tested over 10 instances. For bipartite instances, those with up to 100 customers and 5 feeders are from Rossi et al. [2011].

Lemma 9 allows us to consider only instances having the same capacity for all feeders.

Lemma 9. *An instance of MMP can be transformed in linear time into an instance where all feeders have the same capacity and M_{min} is identical.*

Proof. Consider an instance $\mathscr{I}$ of MMP and its graph $G = (V, E)$. An instance $\mathscr{I}'$ having the same capacity c for all feeders and its graph $G' = (V', E')$ can be built from $\mathscr{I}$ as follows:

1. Start with $G' = G$ and consider the feeder of $\mathscr{I}$ having the maximum capacity $c = \max\{c_j | j \in V_f\}$.
2. For each feeder $j \in V'_f$ such as $c_j < c$:
   (a) add a customer i to V' and an edge ij to E',
   (b) set demand of i to $c - c_j$,
   (c) set capacity of j to c.

Capacities are considered as arbitrarily high in the instances used as the aim is to test the solution times of the formulations. Values of capacities only have a translation effect on the solution space.

From a feasible solution S' of $\mathscr{I}'$, a feasible solution S of $\mathscr{I}$ can be built be deleting the customers added during construction of $\mathscr{I}'$ from S'. As the customers added can be assigned

Instance				G^H			SPR		RNR		TR		SPTR = G^{HR}			
type	n	m	H	time	vert.	arcs	vert.	arcs	vert.	arcs	vert.	arcs	time	it.	vert.	arcs
bip	100	10	6	0.01	622	1166	-9%	-28%	-13%	-24%	-	-	<0.01	1.0	- 23 %	- 45 %
	100	10	10	0.01	1705	3611	-7%	-25%	-10%	-16%	-	-	<0.01	1.0	- 20 %	- 40 %
	100	20	6	<0.01	913	1589	-15%	-35%	-22%	-32%	-	-	<0.01	1.0	- 35 %	- 54 %
	100	20	10	0.01	2335	4624	-17%	-35%	-20%	-26%	-	-	<0.01	1.0	- 39 %	- 56 %
	500	10	12	0.01	3671	8716	-2%	-14%	-6%	-10%	-	-	0.01	1.0	- 9 %	- 25 %
	500	10	16	0.02	6608	16254	-1%	-12%	-4%	-8%	-	-	0.01	1.0	- 7 %	- 21 %
	500	20	12	0.02	6406	14543	-3%	-17%	-6%	-10%	-	-	0.01	1.0	- 11 %	- 27 %
	500	20	16	0.04	12477	29617	-2%	-15%	-4%	-7%	-	-	0.03	1.0	- 7 %	- 22 %
diag	100	10	6	0.01	1258	2691	-7%	-23%	-21%	-31%	-6%	-13%	0.01	4.6	- 42 %	- 64 %
	100	10	10	0.02	3643	9039	-4%	-18%	-16%	-21%	-5%	-9%	0.02	5.7	- 36 %	- 56 %
	100	20	6	0.01	1782	3472	-13%	-31%	-34%	-43%	-8%	-14%	<0.01	3.9	- 58 %	- 74 %
	100	20	10	0.02	5160	11691	-10%	-27%	-29%	-34%	-7%	-12%	0.02	5.5	- 56 %	- 73 %
	500	10	12	0.03	8318	23141	-1%	-10%	-7%	-11%	-2%	-5%	0.07	6.4	- 13 %	- 31 %
	500	10	16	0.06	15810	45796	-1%	-4%	-3%	-8%	-3%	-7%	0.15	6.4	- 9 %	- 24 %
	500	20	12	0.04	13407	33791	-2%	-13%	-8%	-11%	-3%	-6%	0.10	6.9	- 20 %	- 39 %
	500	20	16	0.09	26846	70878	-1%	-11%	-5%	-8%	-2%	-4%	0.25	7.3	- 13 %	- 29 %

Table 3.1: Elimination of vertices and arcs in layered graphs with SPR, RNR and TR

to one feeder only, margins are identical in S and S' for each feeder. □

Shortest path tree reductions

Reductions SPR, RNR, TR and SPTR are tested on G^H of bipartite and diagonal instances with various values for parameters m, n and H. The proportion of vertices and arcs removed in the resulting layered graphs are given in Table 3.1. The computation time to generate layered graphs and perform reductions, as well as the number of iterations of SPTR, are also reported. The number vertices in the layered graphs G^H corresponds to the number of variables x_{ijd} in LF and LFR not initially fixed to zero. The computation time needed to generate G^H is similar to the time needed to perform the preprocessing needed to build VF. To perform SPTR in bipartite instances, as only one iteration of RNR and SPR is performed and complexities to build G^H and perform SPTR with a single iteration are identical, the time required is more or less the same as to build G^H. On diagonal instances, several iterations are made during SPTR, leading to a higher computation time and more vertices and arcs eliminated. As there are at most $H-1$ iterations during SPTR, the computation times stays reasonable. We will observe in the following that the preprocessing times for all formulations are negligible compared to computation times of the MIP.

On average, 25% of the vertices and arcs are removed by SPTR. The number of vertices and arcs removed with SPTR is generally bigger than applying SPR, RNR and TR separately, even for bipartite instances where a single iteration of SPTR is performed. This can be explained by the fact that some reductions enable some further reductions of other types. Also note that bipartite instances have less vertices and arcs than diagonal ones as expected with Lemma 4.

Instance				VF		LF				LFR			
						G^H		G^{HR}		G^H		G^{HR}	
Type	n	m	H	#sol.	time	#sol.	time	#sol.	time	#sol.	time	#sol.	time
bip	50	3	6	10	0.13	10	0.06	10	**0.04**	10	0.08	10	0.09
	50	5	6	10	1.46	10	0.13	10	0.16	10	0.26	10	**0.12**
	100	3	6	10	0.13	10	0.10	10	**0.09**	10	0.11	10	0.12
	100	5	6	10	0.84	10	0.13	10	0.14	10	0.15	10	**0.12**
squ	49	3	6	10	3.5	10	0.29	10	0.24	10	0.19	10	**0.18**
	49	5	6	10	130.05	10	12.86	10	10.68	10	10.00	10	**5.93**
	100	3	6	10	0.63	10	0.23	10	0.22	10	0.20	10	**0.15**
	100	5	6	10	52.17	10	1.71	10	**1.50**	10	2.32	10	1.75
bip	100	10	6	7	548.95	10	4.04	10	3.96	10	24.06	10	**3.86**
	100	20	6	10	143.85	10	0.37	10	**0.34**	10	0.63	10	0.39
	200	10	8	8	369.16	10	2.09	10	1.89	10	2.73	10	**0.89**
	200	20	8	10	195.59	10	1.07	10	0.85	10	2.04	10	**0.71**
	500	10	12	1	1756.87	8	394.43	8	**355.71**	8	441.25	9	398.48
	500	20	12	0	1800.0	7	583.48	7	**548.28**	7	561.42	7	551.91
diag	100	10	6	5	980.78	10	20.67	10	26.33	10	110.06	10	**18.14**
	100	20	6	8	437.07	10	0.52	10	**0.47**	10	1.26	10	1.01
	200	10	8	4	1087.69	9	291.4	9	233.69	9	296.16	10	**89.46**
	200	20	8	6	777.66	9	204.47	9	**183.92**	8	368.16	9	187.12
	500	10	12	0	1800.0	5	1070.78	5	1011.41	3	1402.0	5	**934.81**
	500	20	12	0	1800.0	4	1083.72	5	**1008.04**	4	1266.77	4	1089.53

Table 3.2: Comparaison of VF, LF and LFR

Comparison of the formulations

Table 3.2 compares the number of instances solved and the average computation times of all instances, including layered graph construction and preprocessing techniques of formulations VF, LF, LF-R, LFR and LFR-R. Values H used for hop constraints are the same as in Rossi et al. [2011] and are all greater than or equal to d_{min}. The best computation time for each set of instances is indicated in bold. When VF is not solved to optimality, no feasible solution is found as it is a cutting plane algorithm. When not solved to optimality, all layered formulations found at least one feasible solution. The final gap of instances not solved to optimality is given in Table 3.3 with the LP-relaxation gap of different formulations. The formulation that has the best computation times is LFR-R for 11 of the 20 tested configurations. Figure 3.8 shows the proportion of solved instances with respect to time for each formulation. It is interesting to observe that although LFR(-R) has significantly less binary variables than LF(-R), it is beaten on almost half of the configurations by LF(-R). In Figure 3.8, we observe that LF is globally better than LFR. This can be explained by the powerful preprocessing and cutting techniques that have been developed for models using integer and binary variables, from Gomory's cuts end of the 50's [Gomory, 1958] to recent work [Gamrath et al., 2015, Lu et al., 2014], that are integrated in state-of-the-art MIP solvers. On average, the preprocessing of CPLEX eliminates 32% more variables in LF compared to LFR. Concerning the efficiency of SPTR, computation times are on average 23% lower for LF-R compared to LF and 47% lower for LFR-R compared

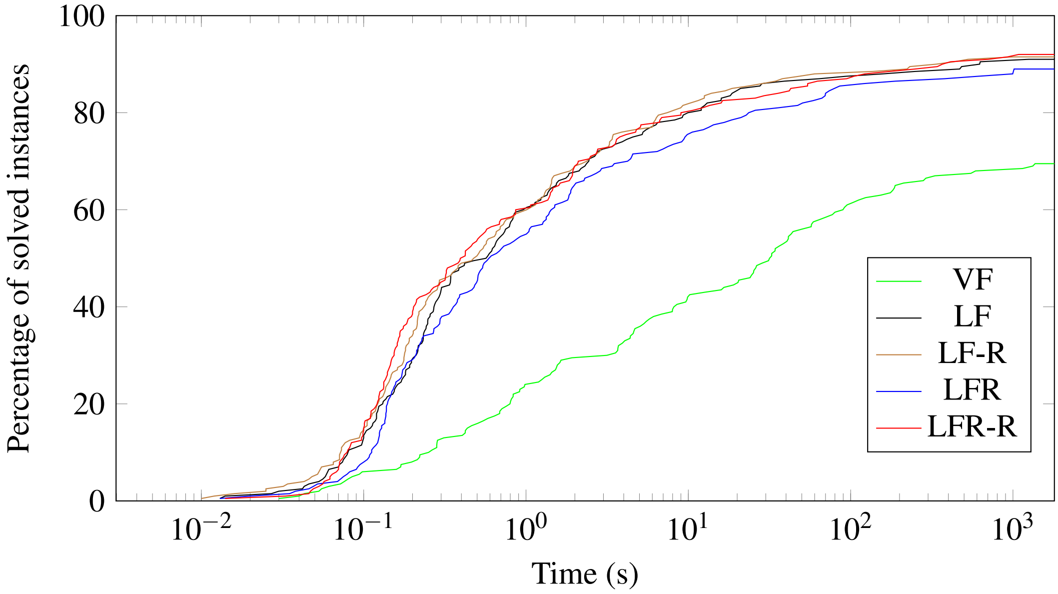

Figure 3.8: Percentage of solved instances with respect to time

to LFR. Formulations LF and LFR have globally much smaller computation time than VF, especially for larger instances where the number of connectivity and distance cuts that need to be added to VF gets much larger. A high standard deviation can be observed in Table 3.2 for layered formulations on computation times of some instances. When solving *diag-500-10-12* using LFR-R, five instances are not solved in 1800s and the other five instances are solved on average in 69s. It can also be observed that the computation times tend to be drastically smaller for bipartite instances than for diagonal instances. This can be a consequence of the small number of vertices in layered graphs of bipartite graphs as shown in Lemma 4.

Table 3.3 reports the gaps of the LP-relaxations of each formulation as well as the gap after solving the root node of the branch & bound tree (with cuts added by CPLEX) and the final gaps of instances not solved to optimality.

Instance				Best obj.	LF				LF-R				Closed gap		
Type	n	m	H		LP gap	root node gap	#sol.	final gap	LP gap	root node gap	#sol.	final gap	LP gap	root node gap	final gap
bip	100	10	6	1136.8	12.62	1.82	10	-	12.39	1.71	10	-	2 %	4%	-
	100	20	6	414.8	100.17	2.07	10	-	76.82	1.7	10	-	23%	17%	-
	200	10	8	2376.6	34.94	2.85	10	-	24	2.14	10	-	31%	23%	-
	200	20	8	1000.4	71.13	5.4	10	-	26.11	8	10	-	63%	-47%	-
	500	10	12	6741.2	21.57	6.07	8	1.35	13.48	4.72	8	1.28	38%	17%	5%
	500	20	12	2938	65.81	8.23	7	2.94	40.84	7.93	7	2.64	37%	1%	15%
diag	100	10	6	1084.9	37.54	1.95	10	-	32.44	1.91	10	-	13%	2%	-
	100	20	6	442	72.58	4.64	10	-	56.75	4.64	10	-	21%	0%	-
	200	10	8	2383.5	27.65	4.53	9	2.38	11.44	4.49	9	1.67	58%	-1%	29%
	200	20	8	1029.4	42.21	13.38	9	1.75	36.03	10.91	9	1.22	14%	18%	50%
	500	10	12	6450.7	10.32	4.52	5	2.58	7.74	3.87	5	1.29	26%	17%	40%
	500	20	12	3038.1	67.31	18.53	4	6.38	62.89	17.92	5	4.25	7%	3%	32%

Instance				Best obj.	LFR				LFR-R				Closed gap		
Type	n	m	H		LP gap	root node gap	#sol.	final gap	LP gap	root node gap	#sol.	final gap	LP gap	root node gap	final gap
bip	100	10	6	1136.8	12.85	4.55	10	-	12.39	4.09	10	-	4 %	10 %	-
	100	20	6	414.8	104.03	11.37	10	-	76.82	5.48	10	-	26 %	52 %	-
	200	10	8	2376.6	35.89	5.47	10	-	24	5.23	10	-	33 %	4 %	-
	200	20	8	1000.4	70.73	16.01	10	-	26.11	9.7	10	-	63 %	39 %	-
	500	10	12	6742.3	22.92	6.07	8	1.35	13.48	6.74	9	1.35	41 %	-11 %	0 %
	500	20	12	2938.4	73.17	12.05	7	10.87	41.43	10.58	7	6.17	43 %	12 %	43 %
diag	100	10	6	1084.9	37.54	12.26	10	-	32.76	10.96	10	-	13 %	11 %	-
	100	20	6	442	73.77	31.91	10	-	56.75	6.14	10	-	23 %	81 %	-
	200	10	8	2385.1	26.95	7.63	9	5.25	11.45	6.2	10	-	58%	19 %	100 %
	200	20	8	1048.8	43.94	15.42	8	9.23	36.71	10.38	9	5.03	16 %	33 %	46 %
	500	10	12	6450.1	10.32	6.45	3	3.87	8.39	4.52	5	2.58	19 %	30 %	33 %
	500	20	12	3028.3	84.09	21.01	4	306.46	72.07	18.17	4	23.92	14 %	14 %	92 %

Table 3.3: Gap analysis of layered formulations

The ≪Best obj.≫ column reports the average objective of the best solutions found over all formulations. Final gap averages are computed over unsolved instances only. As the capacity of the feeders in the tested instances is arbitrarily high, the objective value of MMP is also arbitrarily high. Therefore we report absolute gaps rather than relative gaps that are all very low. The absolute gap is the difference between the upper bound and the objective value of the best feasible solution found over all formulations. In order to interpret these values, it can be related to the number n of customers and the average demand of 50 for each customer. The ≪Closed gaps≫ columns report the proportion of the gap of G^H that is closed with G^{HR}. On average, 29% of the LP gap is closed, going up to 63% closed at best, while 45% of the final gap is closed for unsolved instances.

Impact of the hop limit value

The value of the objective function and the computation time depending on the value of H has been analysed by De Boeck and Fortz [2017] using LFR-R. Some are illustrated in Figure 3.9 which gives the proportion of solved instances depending on time for all values of H used. In this Figure, instances used are the bipartite and diagonale ones with at least 100 customers and 10 feeders. The computation times strongly increase with H increasing. Results presented

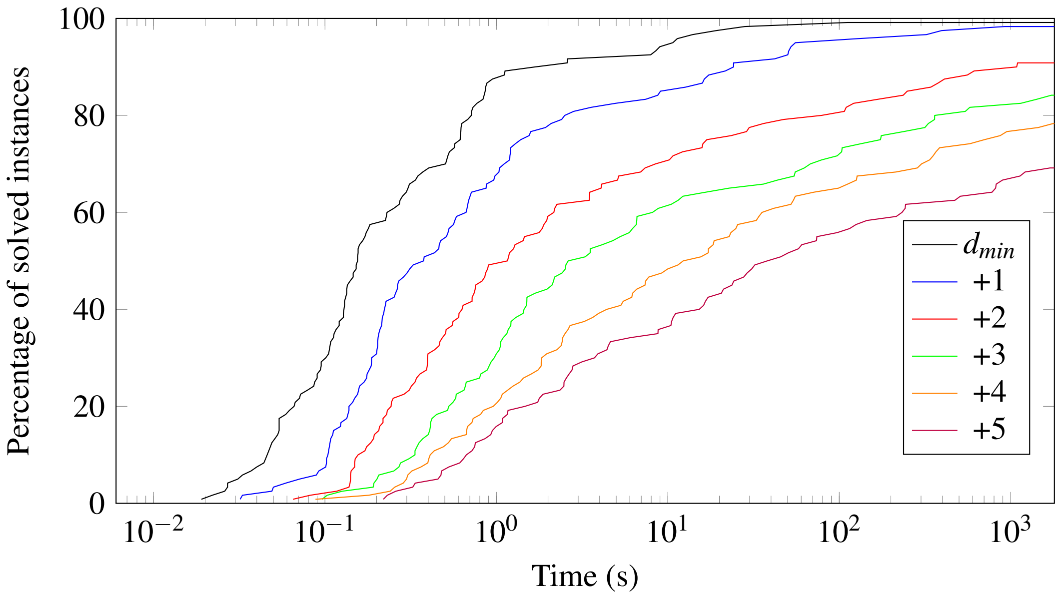

Figure 3.9: Percentage of instances solved depending on time with LFR on G^{HR}

by De Boeck and Fortz [2017] report the optimal objective value also increases with H and if the value of H used is too big, there is a risk of finding a worse solution than for $H-1$ after a fixed computing time.

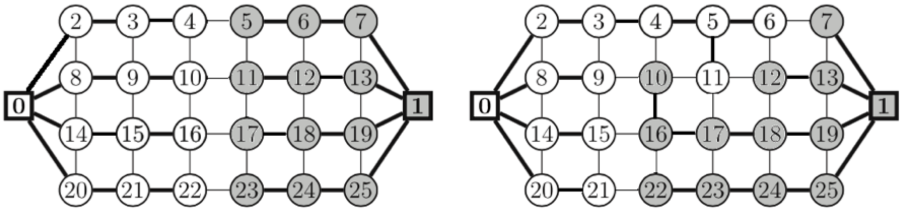

Figure 3.10: Truncated optimal solutions of MMP considering hop losses

3.3 Extensions of the problem

3.3.1 Distribution networks with hop losses

Problem definition

Distribution networks can incur losses during transportation, as it is the case in electricity networks. This is one of the motivation of Rossi et al. [2011] for adding hop constraints to MMP. Indeed, limiting the distance between feeders and customers indirectly limits the power loss.

Our objective in this section is to present a model allowing to explicitly consider losses due do transportation. Consider the MMP on the graph in Figure 3.10. All terminal nodes are customers with a demand equal to 1, $c_0 = c_1 = 24$ and $H = 5$, two optimal solutions of MMP are given with $M_{min} = 12$.

The second solution contains longer transportation distances and is worse than the first one when considering power losses. If there is a 5% loss of power per hop, the effective minimum margin 10.70 for the first solution and 10.38 for the second one. Solutions of MMP may have different effective minimum margins considering losses.

Let us assume we are given a *loss function* l that indicates the amount by which the demand of a customer assigned at distance d must be multiplied to compensate the power loss. It is a discrete increasing function such that $l(0) = 1$. If a customer is assigned at distance d of its feeder, its *adapted demand* is $l(d)dem_i$. The power losses are independent and only depend on the distances at which customers are assigned. We define the *Minimum Margin Problem considering Losses* (MMP$_L$) as the problem of finding a feasible solution of HDNCP maximizing the minimum margin of the feeders M^L_{min} that takes in consideration the power losses due to transportation. The margin of a feeder is now defined as the difference between its capacity and the sum of the adapted demands of customers assigned to it at specific distances. In the special case where $l(d) = 1$ for all $1 \leq d \leq H$, MMP$_L$ reduces to MMP.

Reformulation of LFR

If MMP_L is solved with LFR, the optimal value obtained only provides an upper bound on the optimal minimum margin of MMP_L. In order to model the effective margins of the feeders taking into consideration the losses due to transportations, margin constraints (3.16) are replaced by

$$c_j - \sum_{i \in N_j} \sum_{1 \leq d \leq H} x_{ijd} dem_i l(d) \geq M^L_{min} \; \forall j \in V_f$$

leading to the *Layered Formulation Relaxed with Losses (LFRL):*

$$\begin{aligned}
&\text{Max } M^L_{min} && && (3.23)\\
&\text{s.t. } c_j - \sum_{i \in N_j} \sum_{1 \leq d \leq H} x_{ijd} dem_i l(d) \geq M^L_{min} && \forall j \in V_f && (3.24)\\
&\sum_{j \in V_f} x_{ij} = 1 && \forall i \in V_c && (3.25)\\
&\sum_{j \in V_f} x_{ij} \leq 1 && \forall i \in V_s && (3.26)\\
&x_{ij} = \sum_{1 \leq d \leq H} x_{ijd} && \forall j \in V_f, i \in N_j && (3.27)\\
&x_{ijd} \leq \sum_{k \in P_{ijd}} x_{kj(d-1)} && \forall j \in V_f, d \in \{2, \dots, D_{max}\}, i \in L_{jd} && (3.28)\\
&x_{ijd} = 0 && \forall j \in V_f, d \in \{1, \dots, H\}, i \notin L_{jd} && (3.29)\\
&x_{ij} \in \{0, 1\} && \forall j \in V_f, i \in N_j && (3.30)\\
&0 \leq x_{ijd} \leq 1 && \forall j \in V_f, d \in \{1, \dots, D_{max}\}, i \in L_{jd} && (3.31)
\end{aligned}$$

Formulation LFRL applied G^{HR} is denoted LFRL-R. As power losses are considered, some symmetries in MMP are broken in MMP_L as illustrated in Figure 3.10, meaning potentially better computation times.

Numerical results

We tested LFRL on the same instances as in Section 3.2.5. We used the loss function $l(d) = (1-p)^{-d}$ that corresponds to a loss of a fraction p per hop. Table 3.4 presents the number of instances solved and the average computation times for algorithm LFRL-R with $H = d_{min} + 3$ for different values of p. Figure 3.11 shows the percentage of instances solved depending on time for all tested values of p.

Instance				$p = 0$		$p = 0.02$		$p = 0.05$	
type	n	m	H	#sol	time	#sol	time	#sol	time
bip	50	3	6	10	0.09	10	0.07	10	0.06
	50	5	6	10	0.11	10	0.15	10	0.09
	100	3	6	10	0.12	10	0.13	10	0.16
	100	5	6	10	0.13	10	0.22	10	0.15
squ	49	3	6	10	0.18	10	0.21	10	0.20
	49	5	6	10	5.91	10	4.83	10	1.20
	100	3	6	10	0.15	10	0.15	10	0.14
	100	5	6	10	1.74	10	1.52	10	1.26
bip	100	10	6	10	3.86	10	1.81	10	1.51
	100	20	6	10	0.39	10	0.35	10	0.29
	200	10	8	10	0.92	10	1.14	10	0.55
	200	20	8	10	0.7	10	0.62	10	0.45
	500	10	12	9	398.84	10	360.01	10	9.31
	500	20	12	7	551.91	7	577.06	7	544.96
diag	100	10	6	10	28.17	10	8.83	10	7.39
	100	20	6	10	1.01	10	0.88	10	1.13
	200	10	8	10	89.17	9	185.05	10	7.69
	200	20	8	9	187.08	10	123.55	10	33.71
	500	10	12	5	1033.42	6	889.53	6	895.27
	500	20	12	4	1089.46	4	1083.08	4	1081.99
Averages				9.2	169.668	9.3	161.9595	9.35	129.37

Table 3.4: p impact solving LFRL-R with $H = d_{min} + 3$

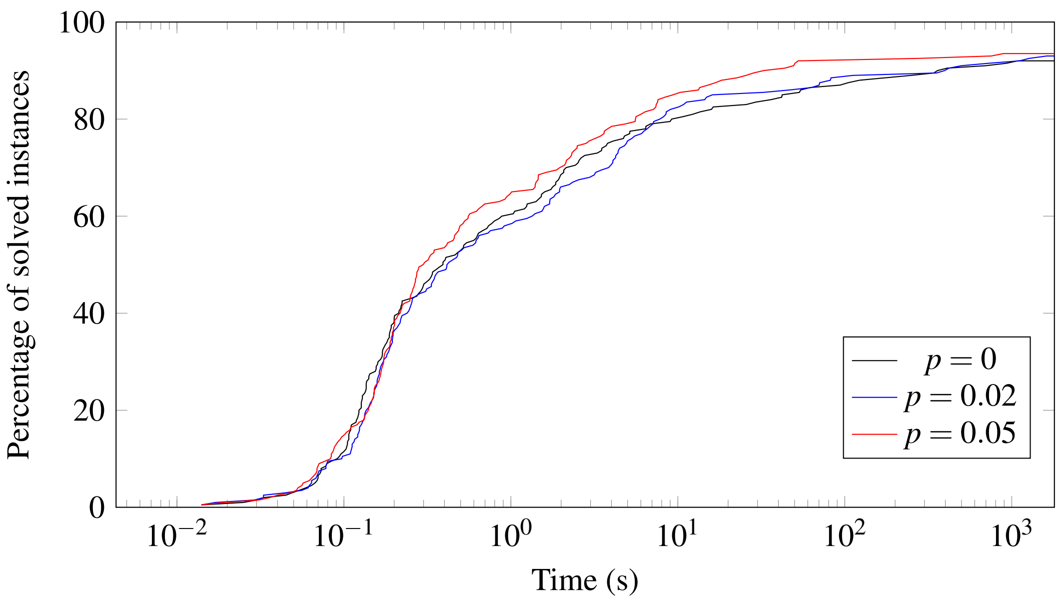

Figure 3.11: Percentage of instances solved depending on time

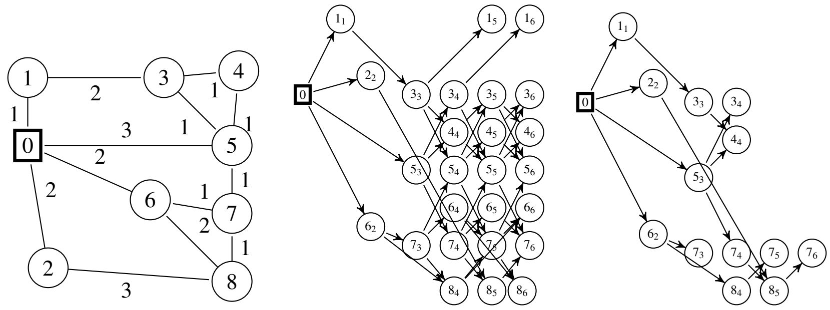

Figure 3.12: Layered graph of an edge-weigted graph G with $D = 6$ with and without SPTR

The computation times generally decrease as p increases. This is probably explained by the fact that assigning a customer to different feeders results in different margins as the distance is likely not the same to each feeder. This breaks a lot of symmetries in the branching tree and therefore reduces the computing time.

3.3.2 Distance constrained model

Another application of distribution network design that can be modeled as a variant of HDNCP arises in the context of multicast routing in telecommunications. In multicast routing, feeders send data to terminal nodes that have a given demand in data. These nodes are also used as relay to transfer data to other terminal nodes. Some delay is incurred by traversing the path from the feeder to the terminal node. In applications involving streaming data (such as audio or video), such delays must be limited [Oliveira and Pardalos, 2005].

Delays are often proportional to the physical length of links between nodes. We define the Multicast Routing Problem (MRP) as a variant of HDNCP where hop constraints are replaced by *delay constraints*. We consider a graph $G = (V, E)$ where edges have an integer length in $\{1, \ldots, L\}$ such that the triangular inequality is satisfied, and we want to impose a maximum length D on each path linking a feeder to a terminal node. Considering as before the minimum margin as objective to maximize, we denote this variant as MMP_D.

To solve MMP_D, we can use formulations introduced before by using a modified definition of layered graphs taking the length of arcs into account. We construct a layered graph G_v^D rooted in v such that if an edge has length d, its endpoints will appear in layers that are separated by $d-1$ layers rather than in consecutive layers, as illustrated on Figure 3.12.

When solving MMP_D, the layered graph reductions SPR can be applied, as well as RNR and TR if the triangular inequality is satisfied, leading to reduced layered graphs G_v^{DR}. Models LF(-R) and LFR(-R) can be used to solve MMP_D.

We evaluate the efficiency of LFR on reduced layered graphs $\{G_j^{DR}\}_{j \in V_f}$ (LFR-DR). The bi-

Instance			$L=1$		$L=2$		$L=10$		$L=20$		$L=30$		$L=40$		$L=50$	
type	n	m	#sol	time	#sol	time	#sol	time	#sol	time	#sol	time	#sol	time	#sol	time
bip	100	10	10	2.98	10	154.24	9	248.11	8	454.33	8	373.77	9	350.01	7	679.23
	100	20	10	0.15	10	11.37	10	41.76	10	15.42	10	23.08	10	14.11	10	76.06
	200	10	10	0.66	10	22.87	9	232.05	8	521.28	8	572.36	7	439.65	9	270.08
	200	20	10	0.21	10	1.11	10	3.90	10	14.44	10	15.78	10	26.05	10	149.85
diag	100	10	10	2.79	10	83.76	8	494.86	6	551.41	6	727.19	7	805.03	7	627.82
	100	20	10	0.28	10	0.51	10	14.32	10	11.54	10	37.66	10	28.72	9	208.08
	200	10	10	3.71	9	263.09	7	831.05	6	823.87	5	949.25	6	942.92	5	1085.47
	200	20	10	0.78	9	192.73	8	369.27	8	442.07	9	311.26	8	377.60	8	367.64
Averages			10	0.98	9.8	91.21	8.9	279.41	8.3	354.28	8.3	376.29	8.4	373.01	8.1	433.03

Table 3.5: Time solving with LFR using various edge length L

partite and diagonal instances of Section 3.2.5 are used with random lengths in $\{1,\ldots,L\}$ for each edge such that triangular inequality is satisfied. The limit on the path lengths is set to $D = d_{\min} + \lceil \frac{3L}{2} \rceil$, where $d_{\min}$ is the minimum feasible (weighted) distance. Table 3.5 reports the number of solved instances and the average computation times for different values of L and Figure 3.13 presents the percentage of solved instances depending on time.

The computation times increase strongly for values of L from 1 to 10 but tend to increase slower as the value of L increases, as observed on Figure 3.13.

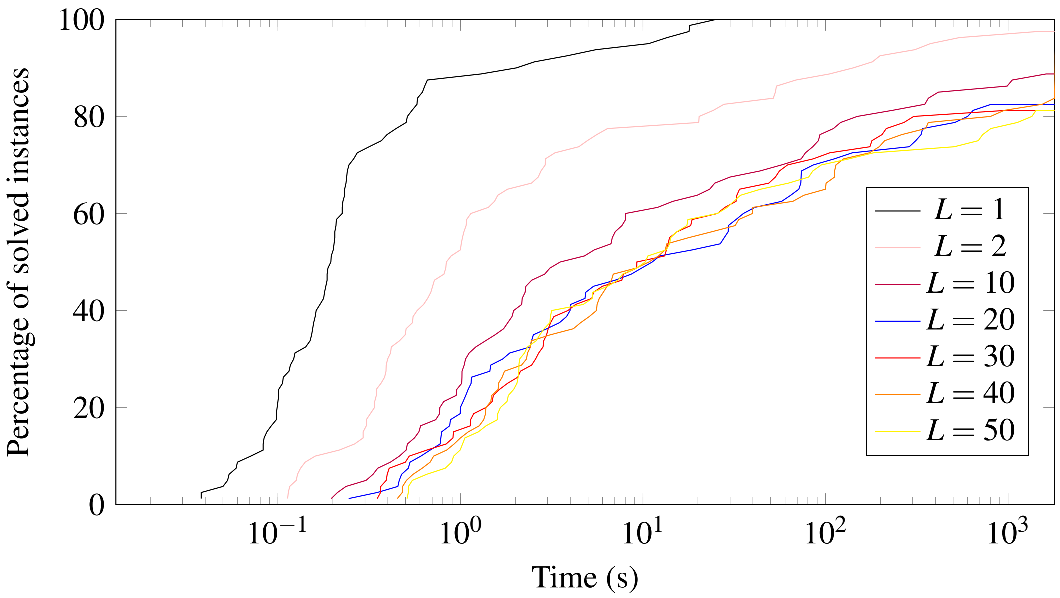

Figure 3.13: Percentage of instances solved depending on time

3.4 Conclusion

This chapter has presented efficient MIP formulations using state-of-the-art techniques to solve the Minimum Margin Problem. The methods proposed use layered graphs to derive a strong and compact descriptions of the solution space using extended formulations. From computational experiments, we can conclude that layered graphs, when used in conjunction with reduction techniques, lead to good extended formulations of hop constrained problems.

Another important observation from computational experiments is that relaxing a large number of binary variables of a MIP formulation does not necessarily improve the computation time using state-of-the-art MIP solvers. This is due to the efficiency of multiple preprocessing techniques and cutting-plane methods available in MIP solvers for dealing with binary variables.

The reductions proposed for layered graphs could be adapted to hop constrained problems with a different connectivity structure than those studied in this paper, e.g. edge-disjoint paths [Botton et al., 2013], or to deal with formulations where edge variables need to be created explicitly (e.g. hop constrained Steiner tree problems [Gouveia et al., 2014]).

Layered graphs allow easily to integrates extensions of the problem as constraints related to the number of hops or considering distance constraints rather than hop constraints. On numerical results, an interesting observation is that the solution time tends to increase relatively slowly with the distances considered for the edges and the maximum distance allowed.

Chapter 4

Power generation in presence of micro-grids

4.1 Introduction

In the upcoming future, the energy landscape will significantly change from the current picture by incorporating more and more decentralized elements such as micro-grids (MG). These are subparts of the electricity network with an advanced energy management system interacting with programmable elements in the grid including good monitoring and control functions, a pervasive communication system and specific items such as smart meters, programmable loads, switchable storage systems and a variety of controllable energy sources including solar, wind and wave generators. Some of the major changes introduced are the following:

- Micro-grids: smaller nearly isolated sub-grids that interact only with the global system when a load/offer mismatch occurs. Most importantly these sub-grids can optimize their electricity consumption independently of the financial interest of an external electricity provider.
- Storage systems of increasing performance. These storage devices can partially mitigate the intermittency of local decentralized production such as wind / solar generation.
- Demand management tools: use advanced information technology to pilot electricity use. For instance, shut down electrical heating, reprogram hot water tank recharging etc...

Considering these changes, it becomes of great interest to examine the interaction with more traditional elements composing the power system. For instance what will be the new role for large centralized generation assets such as nuclear, thermal or hydro generation? It also becomes of interest to examine how classical energy management questions such as Unit Commitment (UC) should evolve to account for this new context.

In this chapter we consider a bilateral contract interaction between a Generation Company (GC) owning a set of centralized assets and a set of micro-grids. We also account for potential competition at the centralized level, but only in a simplified way. We assume that any competitors to the GC have a fixed predetermined interaction with the micro-grids. In this work the GC can offer contracts to the micro-grids that detail the price of buying/selling electricity to the network. The problem is modelled through a bi-level formulation considering binary variables at the second level, problems which are known to be hard to solve to optimality.

A reformulation into a single level problem is proposed by removing the optimistic assumption encountered in bi-level problems as detailed in Section 1.3.2. This relaxation considers the followers represented by the micro-grids do not necessarily choose a solution at the best interest of the GC among their optimal power management plans. This heuristic reformulation is tractable thanks to a preprocessing step that is not harder than solving the original power management problem of the micro-grids. An exact reformulation of the problem is proposed as well in order to assess the quality of the solutions found with the heuristic method on small instances.

Extensive numerical experiments confirm the interest of the suggested heuristic reformulation. Essentially, the reformulation proposed takes inspiration, and to some degree is equivalent with, the value function approach originally designed by Outrata [1990].

This chapter is organised as follows. Our assumptions are described in Section 4.2, where we present the ingredients of the formulation proposed in a deterministic setting. Section 4.3 deals with the suggested reformulation technique for the bilevel problem. In Section 4.4, we show how the proposed formulation can be adapted to handle uncertainty. We perform computational experiments in Section 4.5 on a case study built from realistic data. Some details of the model underlying the case study are provided in Appendix B. Finally, we conclude the chapter in Section 4.6.

A paper presenting the contributions from this chapter has been published in the European Journal of Operational Research [van Ackooij et al., 2018].

4.2 The problem

In this chapter we study an electrical system where several micro-grids (MG) interact with classical generation companies through bilateral contracts. Contracts define hourly buying and selling prices throughout a week. We will make the assumption that micro-grids typically have a set of production units mostly composed of renewable intermittent sources (wind, solar).

Remaining energetically independent of the rest of the system is then possible, up to a certain extent, by also using a set of batteries. The remaining time, i.e., in case of production surplus or lack of generation, an interaction with a classical generation company will provide useful back up to meet the total demand in energy.

The *Contract Proposition Problem* (CPP) for a GC consists of selecting a set of bilateral contracts to propose to a set of micro-grids in order to maximes its profit. The profit of the GC is defined by the contracts subscribed by the micro-grids and amount of electricity traded. The GC considers a UC problem with a fixed demand in addition to the electricity traded with the micro-grids. Competitor generation companies are considered in the contract proposal system. We assume the contracts proposed by the competitors are known.

Each micro-grid $q \in Q$ consists in a small subnetwork that has highly volatile generation capacities (solar, wind), and two types of demands to be attended, which we denote as hard and elastic demands. On the one hand, hard demands must be met strictly at all times. On the other hand, elastic demands (heating up water, recharging electric cars, ...) can be shifted within a certain time window. This models the fact that environmentally aware users may be willing to postpone or anticipate an essential electrical consumption in order to reduce the total cost. Since this use is essential it will take place somewhere within a given time window. Micro-grids are assumed to be relatively autonomous in terms of energy. However, due to the uncertain nature of their production, they need to buy or sell electricity from a GC by choosing a contract minimizing their power management expenses.

Each contract proposed to a micro-grid by a GC specifies the costs of buying/selling electricity from/to the GC for each period of the time horizon T. Specifically, each contract $k \in K$ is specified by (i) the fixed price of contract c_k paid by the micro-grid to the GC that proposes it, (ii) a unit cost f_{kt} for buying electricity during time period t, and (iii) a unit price g_{kt} for selling electricity during time period t. We denote by K_0 the subset of contracts possibly proposed by the GC whose decisions are being optimized, while K also contains contracts of competing companies.

Let $x \in \mathbb{R}^{Q\times T}$ represent the electricity production of the GC. We are mainly interested here in the interaction between the GC and the micro-grids, so the value x_{qt} represents the electricity produced by the GC and fed into micro-grid q during period t. Similarly, we can define y_{qt} as the amount of electricity bought by the GC from micro-grid q during time period t. We denote by $F(x,y)$ the cost of producing $x-y$. Hence, one can think of the problem

$$\min_{x,y\geq 0} F(x,y) \tag{4.1}$$

as a compact abstract representation for the combination of unit-commitment, nuclear power plant maintenance planning, hydro power generation and other related problems in addition to feeding the connected micro-grids with the power described by $x-y$. The mapping F can take the value ∞ for given vectors x,y if certain constraints cannot be met. For instance, whenever $x-y$ exceeds the generation capacity of the system. With this convention any constraints on generation (or other constraints) can be readily incorporated in the framework by adding the characteristic function of these constraints to F. Let us note that computing $F(0,0)$ amounts to solving the power generation problem without considering micro-grids. In particular, computing F is therefore as difficult as solving a UC problem with a fixed demand. We also remark that in view of the above discussion, assuming that the GC problem is feasible without micro-grids means that $F(0,0) < \infty$.

Vectors x and y indicate the power flow between the micro-grids and the GC. These variables are related to the internal functioning of the micro-grids and in particular their internal constraints. To this end we introduce an abstract constraint set $\mathscr{M}_q$ for each $q \in Q$, modelling in particular the demand-power balance. These power flows induce a certain cost governed by a contract. The GC has to decide which contracts it proposes to the micro-grids. We introduce an additional binary variable Z_{qk} that is equal to 1 if contract k is offered to micro-grid q and 0 otherwise. The index set K will denote the set of all contracts and $K_0 \subseteq K$, the set of contracts suggested by the GC, i.e., $K \setminus K_0$ is the set of contracts suggested by the competitors of the GC. We denote the set of contracts that the GC can offer as $\mathscr{Z} \subseteq \{0,1\}^{|Q|\times|K_0|}$. For instance, $\mathscr{Z}$ could contain all binary vectors, where the GC offers a fixed number of contracts N_q to each micro-grid $q \in Q$. In that case, we would have $\mathscr{Z} = \{Z \in \{0,1\}^{|Q|\times|K_0|} : \sum_{k\in K_0} Z_{qk} \geq N_q, \ \forall q \in Q\}$.

For each micro-grid $q \in Q$, the binary variable z_{qk} indicates if micro-grid q subscribes to contract k. Notice that micro-grid q interacts with the GC only if it subscribes to one of its contracts, that is, if $\sum_{k\in K_0} z_{qk} = 1$. We introduce local versions of the variables x, y, called $\tilde{x}$ and $\tilde{y}$ that describe locally the power status. In particular, $x = \tilde{x}$ and $y = \tilde{y}$ only if a contract with the GC is subscribed, otherwise $x = y = 0$.

Summarizing, the deterministic bilevel problem below considers the following optimization variables:

- Z_{qk}: 1 if contract k is offered to micro-grid q (leader)
- z_{qk}: 1 if contract k is subscribed by micro-grid q (follower)
- $\tilde{x}_{qt}$: power consumed by micro-grid q during period t (follower)
- $\tilde{y}_{qt}$: power produced by micro-grid q during period t (follower)
- x_{qt}: power consumed by micro-grid q during period t from the *GC* (follower)
- y_{qt}: power produced by micro-grid q during period t for the *GC* (follower)

The CPP can now be modelled through a nonlinear bilevel formulation:

$$\min \quad F(x,y) - \sum_{q\in Q}\sum_{k\in K_0} z_{qk}(c_k + \sum_{t\in\mathscr{T}} \big(f_{kt}x_{qt} - g_{kt}y_{qt}\big)) \tag{4.2a}$$

$$\text{s.t.} \quad Z \in \mathscr{Z} \tag{4.2b}$$

$$(x_q, y_q, z_q) \in \arg\min_{z_{q\cdot}\in\{0,1\}^K, \tilde{x}_q, \tilde{y}_q} \sum_{k\in K} z_{qk}(c_k + \sum_{t\in\mathscr{T}} \big(f_{kt}\tilde{x}_{qt} - g_{kt}\tilde{y}_{qt}\big)), \quad \forall q \in Q \tag{4.2c}$$

$$\text{s.t.} \ (\tilde{x}_q, \tilde{y}_q) \in \mathscr{M}_q, \tag{4.2d}$$

$$x_q = \tilde{x}_q \sum_{k\in K_0} z_{qk}, \tag{4.2e}$$

$$y_q = \tilde{y}_q \sum_{k\in K_0} z_{qk}, \tag{4.2f}$$

$$z_{qk} \leq Z_{qk}, \quad k \in K \tag{4.2g}$$

$$\sum_{k\in K} z_{qk} = 1, \tag{4.2h}$$

where the constraints (4.2b), (4.2d)-(4.2h) are as explained below. The objective function of the leader, (4.2a), minimizes the production cost already mentioned in (4.1) combined with transaction costs related to the micro-grids. Similarly, the objective functions of the micro-grids, (4.2c), amounts to minimizing their total transaction costs with respect to all GCs, not only the one represented by the leader. Hence, (4.2c) involves all contracts in K while (4.2a) considers only the contracts in K_0. Constraints (4.2b) and (4.2d) restrain, respectively, the set of contracts offered by the GC and the feasible power consumption/production for the micro-grids. Constraints (4.2e) and (4.2f) model that $x_q = \tilde{x}_q$ and $y_q = \tilde{y}_q$ if and only if micro-grid q subscribes to one of the contracts offered by the GC. Constraints (4.2g) impose that only a contract offered by a GC can be subscribed by a micro-grid, while constraints (4.2h) forces each micro-grid to select exactly one contract. One challenging aspect of this formulation is the presence of binary variables at the second level as mentioned in Section 1.3.2.

4.3 MILP reformulations

In this section we present two reformulations of the bilevel formulation (4.2a)-(4.2h) of CPP. In Section 4.3.1 a first reformulation removes the usual optimistic assumption of bi-level formulations and is a heuristic to the original problem. If for some micro-grid $q \in Q$, their exists several optimal values for (x_q, y_q, z_q) in 4.2c, this heuristic does not guarantee the solution used will be optimal for the GC. We also suggest an exact reformulation in Section 4.3.2 for comparative purposes. In particular the numerical experiments carried out in Section 4.5 show that results from both reformulations differ, hence implying that the arg min in (4.2c) is indeed not unique and justifying that care should be taken in the interpretation of the results.

4.3.1 Heuristic reformulation

The bilinear optimization problem described in the previous section involves binary variables at the second level and non-linear constraints, all that being built on top of the already difficult optimization problem (4.1). Hence, the problem is addressed heuristically rather than exactly and we show below how it is possible to exploit the somewhat simple linking constraints (4.2g) to provide a heuristic reformulation for the bilinear problem.

We propose below a one-level reformulation of the bilevel problem that may not validate the previously mentioned optimistic assumption. The key aspect of the reformulation proposed relies on preprocessing. Specifically, for each $k \in K$ and $q \in Q$, we solve the restricted follower problem where z_{qk} is equal to 1, namely:

$$\begin{aligned} \min_{\tilde{x}_q, \tilde{y}_q} \quad & c_k + \sum_{t \in \mathscr{T}} (f_{kt} \tilde{x}_{qkt} - g_{kt} \tilde{y}_{qkt}) \\ \text{s.t.} \quad & (\tilde{x}_{qk}, \tilde{y}_{qk}) \in \mathscr{M}_q, \end{aligned}$$

An optimal solution of this problem represents an optimal power management plan if micro-grid q chooses contract k. Let $(\overline{x}_{qk}, \overline{y}_{qk})$ be an optimal solution of the above problem and $\overline{C}_{qk}$ be its value. Then, a heuristic solution to the above bilevel problem, which ensures the optimality of the second level, can be found by solving

$$\min_{z,Z} \quad F(x,y) - \sum_{q \in Q} \sum_{k \in K_0} \overline{C}_{qk} z_{qk} \tag{4.3a}$$

$$\text{s.t.} \quad x_{qt} = \sum_{k \in K_0} \overline{x}_{qkt} z_{qk}, \quad \forall q \in Q, t \in \mathscr{T} \tag{4.3b}$$

$$y_{qt} = \sum_{k \in K_0} \overline{y}_{qkt} z_{qk}, \quad \forall q \in Q, t \in \mathscr{T} \tag{4.3c}$$

$$z_{qk} \leq Z_{qk}, \quad \forall q \in Q, k \in K \tag{4.3d}$$

$$\sum_{k \in K} z_{qk} = 1, \quad \forall q \in Q \tag{4.3e}$$

$$z_{qk} \leq 1 - Z_{q\ell}, \quad \forall k \in K, l \in K, q : \overline{C}_{qk} > \overline{C}_{ql} \tag{4.3f}$$

$$Z \in \mathscr{Z} \tag{4.3g}$$

$$z \in \{0,1\}^{Q \times K}$$

The objective (4.3a) is obtained from (4.2a) by replacing the micro-grid cost for the GC by $\overline{C}_{qk} z_{qk}$. Similarly, (4.3b) and (4.3c) are obtained from (4.2e) and (4.2f) by replacing the variables $\tilde{x}$ and $\tilde{y}$ with their fixed values $\overline{x}$ and $\overline{y}$ computed in the preprocessing phase. It is exactly in these constraints that lies the heuristic aspect of this reformulation since the followers no longer choose the optimal solution that most benefits the leader, but instead take the one computed in the preprocessing phase, when (4.2c) admits several solutions. Finally, constraint (4.3f) ensures that the micro-grids choose the contracts that lead to the cheapest solutions. Indeed if $Z_{ql} = 1$ and the cheap solution $\bar{C}_{ql}$ is available, any costlier solution is ruled out by equation (4.3f).

4.3.2 Comparative exact formulation

To assess the quality of the heuristic approach, the bi-level formulation is reformulated into an exact one-level problem where (4.3b) and (4.3c) are replaced by non-linear constraints involving the decision variables $\tilde{x}$ and $\tilde{y}$ that are restricted by the sets $\mathscr{M}_q$ for each $q \in Q$. In addition, the formulation contains restrictions enforcing $\tilde{x}$ and $\tilde{y}$ to be optimal for the subproblems. Values $\overline{C}_{qk}$ are defined as in the heuristic reformulation.

$$\min_{z,Z,\tilde{x},\tilde{y}} \quad F(x,y) - \sum_{q\in Q}\sum_{k\in K_0} \overline{C}_{qk} z_{qk} \tag{4.4a}$$

$$\text{s.t.} \quad (4.3\text{d}) - (4.3\text{g})$$

$$x_{qt} = \sum_{k\in K_0} \tilde{x}_{qkt} z_{qk}, \quad \forall q \in Q, t \in \mathscr{T} \tag{4.4b}$$

$$y_{qt} = \sum_{k\in K_0} \tilde{y}_{qkt} z_{qk}, \quad \forall q \in Q, t \in \mathscr{T} \tag{4.4c}$$

$$\overline{C}_{qk} = c_k + \sum_{t\in\mathscr{T}} (f_{kt}\tilde{x}_{qkt} + g_{kt}\tilde{y}_{qkt}), \quad \forall q \in Q, \forall k \in K \tag{4.4d}$$

$$(\tilde{x}_{qk}, \tilde{y}_{qk}) \in \mathscr{M}_q \tag{4.4e}$$

$$z \in \{0,1\}^{Q\times K}$$

Constraints (4.4b) and (4.4c) contain bilinear terms that can be linearized using classical techniques. Specifically, let us introduce variables X_{qkt} and Y_{qkt}, respectively equal to products $\tilde{x}_{qkt} z_{qk}$ and $\tilde{y}_{qkt} z_{qk}$. These variables can be substituted in constraints (4.4b) and (4.4c), adding also the constraints

$$X_{qkt} \le \tilde{x}_{qkt}$$
$$X_{qkt} \le M z_{qk}$$
$$X_{qkt} \ge \tilde{x}_{qkt} - M(1 - z_{qk}),$$

that models the products of variables through big-M constraints, and similarly for variables Y_{qkt}. The difficulty of the resulting MILP is that big-M constraints often lead to numerical instability and weak LP relaxations.

4.4 Stochastic extension of the model

Power generation problems that involve renewable intermittent energy like wind and solar are subject to uncertainty, since the output of the renewable power plants depends on the weather conditions. This power output is therefore only partially known and should in principle be considered uncertain. In this section, we propose a model that accounts for uncertainty in generation. The resulting bilevel stochastic optimization problem becomes tractable due to a reformulation similar to the one in the deterministic case.

The classical stochastic reformulation presented in Section 1.3.3 for LPs can be adapted to the bi-level deterministic formulation (4.2a)-(4.2h). We assume decent weather forecasts an available for the time period considered in order to generate a set of scenarios S. Each scenario $s \in S$ defines an abstract constraint set $\mathscr{M}_q(s)$ representing the demand-power balance of $q \in Q$ in scenario $s \in S$. As the GC proposes contracts and MGs choose them before the realization of a scenario, variables z and Z are considered as deterministic. Variables $x, \tilde{x}, y$ and $\tilde{y}$ are considered as stochastic as they represent the input and output of electricity of the MGs during the realization of a scenario. The introduction of uncertainty affects both the objective function and the constraints of the follower problems, as well as the objective function of the leader. All constraints of the follower problem can be reformulated as in Section 1.3.3. We consider the expectation as probabilistic measure for the follower and a convex combination of the expectation and the conditional value-at-risk presented in Section 1.3.3 for the leader. Let

$$X = F(x(s), y(s)) - \sum_{q \in Q} \sum_{k \in K_0} z_{qk}(c_k + \sum_{t \in \mathscr{T}} \big(f_{kt} x_{qt}(s) - g_{kt} y_{qt}(s)\big)),$$

and

$$Y = \sum_{k \in K} z_{qk}(c_k + \sum_{t \in \mathscr{T}} (f_{kt} x_{qt}(s) - g_{kt} y_{qt}(s))),$$

be the random variables representing the objectives of the leader and the follower. This leads to the following reformulation:

$$\begin{aligned}
\min \quad & \lambda \mathbb{E}[X] + (1-\lambda)\mathrm{CVaR}_\varepsilon[X] && (4.6)\\
\text{s.t.} \quad & Z \in \mathscr{Z} && (4.7)\\
& (x_q(s), y_q(s), z_q) \in \arg\min_{z_{q.} \in \{0,1\}^K, \tilde{x}_q, \tilde{y}_q} \mathbb{E}[Y], \quad q \in Q && (4.8)\\
& \qquad \text{s.t. } (\tilde{x}_q(s), \tilde{y}_q(s)) \in \mathscr{M}_q(s), s \in S && (4.9)\\
& \qquad x_q(s) = \tilde{x}_q(s) \sum_{k \in K_0} z_{qk}, s \in S && (4.10)\\
& \qquad y_q(s) = \tilde{y}_q(s) \sum_{k \in K_0} z_{qk}, s \in S && (4.11)\\
& \qquad z_{qk} \le Z_{qk}, \quad k \in K_0, s \in S && (4.12)
\end{aligned}$$

$$\sum_{k \in K} z_{qk} = 1, s \in S \tag{4.13}$$

$$\tag{4.14}$$

The heuristic and comparative reformulations introduced for the deterministic case can be applied to formulation (4.6)-(4.13) as in Sections 4.3.1 and 4.3.2. During preprocessing, for each $k \in K$ and $q \in Q$, we solve the restricted follower problem minimizing the expectation of cost considering z_{qk} is equal to 1 to obtain values $\overline{C}_{qk}$. The objective of the heuristic and comparative reformulations contain $\text{CVaR}_{\varepsilon}$ which can be linearized as detailed in Section 1.3.3 in order to obtain MILP formulations.

4.5 Case study based on thermal power unit-commitment

We assess below the heuristic reformulation proposed on a case study where function F represents the production cost of a UC problem which is modelled by formulation UC1 proposed by Carrión and Arroyo [2006] and presented in Section 2.2. The full formulation of UC1 is also provided in Appendix A. We first describe the data used before presenting numerical results.

4.5.1 Data

General system

The time horizon is one week, each day is split in periods of one hour, i.e., $T = 168$. The data used for the unit-commitment part is provided by the original paper of Carrión and Arroyo [2006], 50 generators are used giving a daily fixed load of 135,5 GWh.

Considered contracts

Several types of contracts are proposed combining the possibility of lower prices during nighttime and/or during the weekend. This leads to a total of four types of contracts. The competitors offer one contract of each type to each micro-grid. The GC generates four possible contracts of each type, thus $|K_0| = 16$ and $|K| = 20$. For all contracts, the average cost of 1kW is about 0,25 EUR [RTE, 2020].

Description of the micro-grids

In what follows we define devices as an abstract group of objects fulfilling a purpose of consumption, production or storage under time constraints within a micro-grid. The devices con-

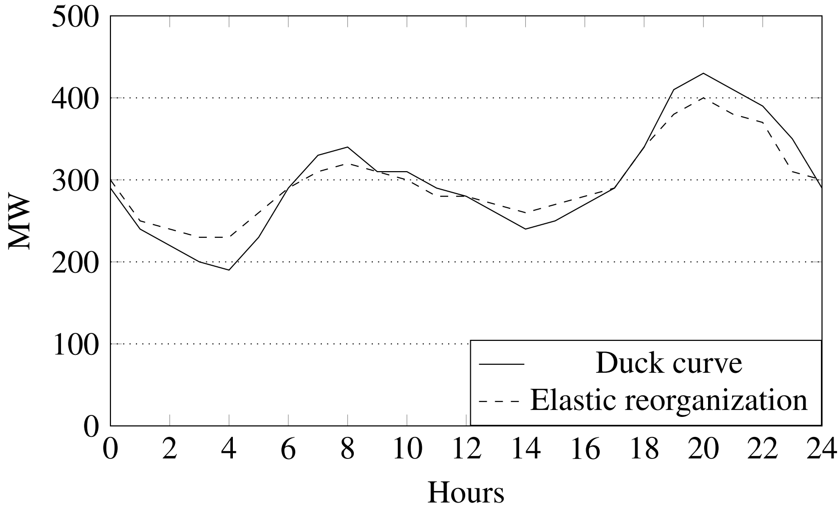

Figure 4.1: Duck curve example for a micro-grid with 10000 devices

sidered, their consumption and the time periods where they are used fit numbers reported in a recent study of electricity demand/offer equilibrium in France [RTE, 2020].

- Consumption devices are devices appearing in a common household, which we split into three further sub-classes:
  - Constant: these devices consume power constantly, representing 30% of the daily consumption.
  - Comfort: devices that consume power during a predetermined period each day, representing 30% of the daily consumption.
  - Elastic: devices for which predetermined windows of usage are defined for each day as well as a total daily load, representing 40% of the daily consumption.

  The periods where comfort devices are used and elastic devices can be used as well as their consumptions are randomly generated such that the total consumption of a micro-grid fits a classical duck curve for each day. The average consumption of 200 devices over one week is 1GWh, which corresponds to the consumption of 5000 common (European) households. Consumption of elastic devices can be reorganized to reduce costs by taking advantage of low hourly prices, production devices and storage devices. Figure 4.1 illustrates the hourly demand of the micro-grid composed of 10000 devices.

- Production devices are equally split into solar panels and wind turbines. The production capacity of each production device is generated randomly. For each micro-grid, their production capacity is sufficient to satisfy on average 50% of the consumption of micro-grids.

- Storage devices are of a single type, their capacity is sufficient to store on average 30% of the daily electricity load of a micro-grid. We consider that during the storage process there is a total loss of 20% of energy due to physical constraints.

In the largest instance, 90,000 devices are considered, corresponding to over $2,000,000$ households. The total consumption of devices in this instance is on average 64 GWh per day. As on average 50% of the daily load is supplied by production devices, about 32 GWh must be provided from the global network each day.
The penetration of the MGs in the total load of the GC is defined as

$$penetration = \frac{load\ of\ the\ MGs}{total\ load\ for\ the\ GC}$$

If all MG considered choose a contract from the GC, the penetration is about 19%.

The constraints of the internal management of a MG $\mathscr{M}_q$ is provided in Appendix B.

Uncertainty

Four simple independent scenarios are considered based on the possibility of good sunshine and good wind that influence the production of micro-grids. All scenarios have the same probability. Weather conditions are considered as identical each day of the week in a scenario. In bad conditions (no sunshine or no wind), a production device produces only 50% of its maximum production capacity. The value of ε used in CVaR in equal to 0.1, representing a high aversion to risk.

4.5.2 Numerical results

All the methods were implemented in Java 1.8 and ILOG CPLEX 12.6 Java API. Tests were made on a 4-core i7 2.30 GHz processor with 16Go of RAM memory. Maximum computation time is set to 3600s. Instances are available at `https://github.com/jdeboeck/CPP`.

Table 4.1 presents the average computation times for solving the follower problems. Times and objectives are averaged over the 20 possible contracts for the 5 instances of micro-grids of each size. The objectives are the average consumption costs for micro-grids of a given size which is also the average income for a GC supplying those micro-grids. They are provided in euros. The standard deviation (σ) of the objective value is also provided. These results show that solving the restricted follower problem can be done fairly quickly. The objective value for all contracts is positive and for all instances considered, there exists at least one contract of the GC that is cheaper and one that is more expensive than those of the competitors. Each micro-grid is financially interesting for the GC if the additional production costs are not too high.

# Devices	Avg. Obj	σ	time(s)
100	34.99	2.25	0.15
200	37.41	3.67	0.32
1000	207.51	14.97	1.91
2000	332.93	21.32	3.95
5000	653.61	37.36	14.57
10000	957.65	40.56	24.01

Table 4.1: Average computation time and objective value for the micro-grid power planning preprocessing step.

Table 4.2 presents results solving small instances with the exact (Exact) and the heuristic (Heuristic) bilevel reformulations. Value of λ is set to 1 in order to optimize the expectation. The first two columns report the number of devices in the considered micro-grids and the number of micro-grids of each size. Hence, the first line considers one micro-grid with 100 devices and the last line considers five micro-grids with 100 devices plus five micro-grids with 200 devices.

It has been observed in the results that for each instance, both approaches (Heuristic and Exact) select the same contracts for each micro-grid. The difference between optimal values lies in the extra production cost there is for the GC to satisfy the demand of the micro-grids who subscribed a contract. Specifically, in Exact, the planning of elastic devices of those micro-grids can be reorganized to reduce the cost of the UC problem, following the optimistic assumption of bi-level formulations that optimizes the objective value of the leader. Column $\Delta(F-UC)$ reports the gap in production costs between both formulations. Let us denote by (x_{opt}, y_{opt}) and (x_{heur}, y_{heur}) the power generation returned by Exact and Heuristic, respectively. Columns $F(x_{opt}, y_{opt}) - F(0,0)$ and $F(x_{heur}, y_{heur}) - F(0,0)$ correspond to GC's extra production costs due to the consumption of the micro-grids in the solutions of Exact and Heuristic respectively. The value

$$\Delta(F-UC) = \frac{F(x_{heur}, y_{heur}) - F(x_{opt}, y_{opt})}{F(x_{opt}, y_{opt}) - F(0,0)}$$

is the relative gap between the costs of extra production in Heuristic and Exact. The reported computation times of Heuristic do not include the preprocessing times.

Computation times of Heuristic (without considering preprocessing) are quite stable while those of Exact tend to grow very quickly and the last instance considering 10 micro-grids cannot be solved in one hour. Concerning the instance with a single micro-grid with 200 devices, the extra production cost is 0 because the micro-grid chose a contract of another company. The gap on extra production cost is very low in all tests. Table 4.3 reports the results of Heuristic

Test set		Exact		Heuristic		$\Delta(F-UC)$ (%)
# Devices	nb	$F(x_{opt}, y_{opt}) - F(0,0)$	time(s)	$F(x_{heur}, y_{heur}) - F(0,0)$	time(s)	
100	1	8.134968	76.26	8.146005	26.9	0.136
100	2	29.413995	202.3	29.438447	28.63	0.083
100	3	67.440871	312.46	67.471015	28.07	0.045
100	4	52.290345	678.9	52.334735	26.67	0.085
100	5	24.543927	690.63	24.588317	27.22	0.181
200	1	0	135.01	0	21.16	-
200	2	71.09899	242.34	71.1142	25.88	0.021
200	3	128.69007	635.02	128.717851	28.05	0.022
200	4	188.672761	1684	188.715209	25.85	0.022
200	5	171.455653	1919.43	171.498219	25.95	0.025
100-200	1	60.079001	223.74	60.090038	25.7	0.018
100-200	2	100.510032	893.82	100.550933	25.93	0.041
100-200	3	196.12615	1697.23	196.184324	27.19	0.030
100-200	4	240.957646	3520.43	241.045643	27.38	0.037
100-200	5	-	> 3600	196.081488	28.7	-

Table 4.2: Comparing Exact and Heuristic on small instances.

on larger instances. We consider three values for λ: 1, 0.5 and 0. When set to 1 (resp. 0), the formulation optimizes the expectation (resp. CVaR_ε). Value ε is set to 0.1, representing a high aversion to risk. Again, in each instance, five micro-grids of each size are used. Columns E report the expectation of the optimal solutions, columns CVaR report their conditional value-at-risk, both are given in thousands of €. Columns sold report the number of contracts of the GC chosen by the MG in the optimal solution found. These values are reported only for $\lambda = 1$ as they were identical for other values of λ after rounding. The first line considers only the original UC problem while the last line considers 20 micro-grids (5 of each size) with a maximum possible penetration of 19%. Notice the negative solution cost of the last line, which is obtained from a revenue from the micro-grids that is higher than the production cost. Computation times tend to grow slowly when adding micro-grids and stay close to the computation time of the original UC problem when $\lambda = 1$. For $\lambda = 0.5$, the expectation is similar, the CVaR decreases significantly for some instances and the computation time increases of about 20%. With $\lambda = 0$, results for the expectation and CVaR are similar than for $\lambda = 0.5$ and computation times are more than doubled for most instances in comparison of $\lambda = 1$.

The heuristic formulation does not increase much the difficulty of the UC problem on the tested instances when minimizing the expectation. Integrating CVaR in the objective function with $\lambda = 0.5$ seems interesting as it is not very time consuming and reduces CVaR almost to its optimal value.

Test set	$\lambda = 1$					$\lambda = 0.5$			$\lambda = 0$		
	E	CVaR	time(s)	sold	penetration	E	CVaR	time(s)	E	CVaR	time(s)
UC	3599	-	20.98	0	0	3599	-	20.98	3599	-	20.98
1000	3221	3580	26.31	5	1.2%	3221	3250	30.09	3221	3250	47.41
2000	2973	3304	22.98	4	2.1%	2973	3018	26.37	2973	3018	46.71
5000	1373	1526	24.38	5	6.0%	1374	1510	27.33	1375	1509	46.54
10000	-421	768	24.98	5	10.9%	-421	-144	29.87	-420	-145	56.54
1000-2000	2595	2884	25.00	9	3.3%	2595	2670	28.09	2595	2670	50.53
1000-2000-5000	378	960	25.61	14	9.1%	379	589	30.24	379	589	50.63
1000-2000-5000-10000	-2055	145	32.42	10	12.6%	-2055	-1768	44.19	-2053	-1769	82.29

Table 4.3: Solution of Heuristic for larger instances.

The number of contracts sold per instance reflects the quadratic aspect of production costs considered in the formulation for the UC [Carrión and Arroyo, 2006]. Summing the numbers of contracts sold in instances where all micro-grids have the same size (lines 2 to 5 of Table 4.3), a total of 19 contracts are sold to the 20 micro-grids. When solving the problem with all 20 micro-grids together (last line of the table), only 10 contracts are sold. In this solution, respectively 2, 1, 4 and 3 contracts are sold to the MGs of size 1000, 2000, 5000 and 10,000. The results of this last instance let us suppose the GC cannot afford having a penetration higher than about 13% with the contracts proposed by the competitors. Even the small MG with 1000 devices go to competitors illustrating that the GC can only propose contracts more expensive than those of competitors.
If many micro-grids must be supplied with electricity, the average production costs increases for the GC in the UC problem. At some point, the GC cannot afford proposing low price contracts to additional micro-grids, otherwise it would produce at a loss. This approach prevents the GC from falling in this situation by proposing contracts that are more expensive than those of the competitors to micro-grids that are not financially interesting.

For each instance, the same MGs choose a contract of the GC for each value of λ considered but the contracts sold are not all identical. Table 4.4 illustrates the solution of the instance with 5 micro-grids of size 5000. The contracts are represented in the columns and are grouped by type. Constant price, daytime-nighttime price, week-weekend prices and daytime-nighttime plus week-weekend prices. A blank dot appears when a contract is proposed to a micro-grid by the GC, a black dot appears when a micro-grid selects a contract. The five micro-grids select a contract from the GC for all values of λ but the contracts proposed and chosen for the fourth and fifth micro-grid vary. Integrating CVaR can discredit contracts that present a high risk in some scenarios.

4.6 Conclusion

We have addressed a variant of power generation optimization problems where the GC interacts with micro-grids through bilateral contracts by buying and selling electricity. We have modelled the *Contract Proposition Problem* as a bilevel stochastic optimization problem, built on the top of the already difficult unit commitment problem that the GC must solve to produce its energy. For realistic size data, solving the bilevel problem exactly is out of reach. The heuristic reformulation proposed lifting the optimistic assumption in bi-level problems leads to a much simpler model. Although numerical results exhibit that the resulting solution is indeed not optimal, it has a small gap to optimality on the smallest instances. Interestingly enough, we could link this difference between the optimistic bilevel solution and the solution obtained

Test set		Cst price				DN price				WWE price				DN and WWE price			
MG	λ	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	○				○		○		○	○	○	○	○		○	●
	0.5	○				○		○		○	○	○	○	○		○	●
	0	○				○		○		○	○	○	○	○		○	●
2	1	○				○		○	●		○	○	○			○	
	0.5	○				○		○	●		○	○	○			○	
	0	○				○		○	●		○	○	○			○	
3	1	○				○		○		○	○	○	○	○		○	●
	0.5	○				○		○		○	○	○	○	○		○	●
	0	○				○		○		○	○	○	○	○		○	●
4	1	○				○		○	●	○	○	○	○	○		○	○
	0.5	○				○		○	●	○	○	○	○	○		○	○
	0	○				○		○		○	○	○	○	○		○	●
5	1	○				○		○	○	●	○	○	○	○	○	○	○
	0.5	○	●			○		○	○		○	○	○	○	○	○	○
	0	○	●			○		○	○		○	○	○	○	○	○	○

Table 4.4: Contracts proposed and selected to five MG with $|D| = 5000$.

to the existence of load-shifting devices within the micro-grid that are piloted differently in the optimistic situation.

The heuristic reformulation has been used in a case-study based on a thermal unit commitment problem using realistic data. The numerical results have confirmed the tractability of the heuristic approach, since the solution time of the heuristic reformulation is at most 50% higher than the one required for solving the unit commitment problem. In contrast, the solution of an exact reformulation of the bilevel problem increases the solution times by more than 10 000% in the presence of only 8 micro-grids. The computational experiments also confirm the quality of the approximation provided by our heuristic, which provides a solution very close to the optimistic solution. An interesting venue for future research would seek to extend the kind of reformulations proposed to other bilevel optimization problems with integer variables at the second level, which is a class of problems that are notoriously difficult to solve exactly.

Chapter 5

Bidding in day-ahead markets under uncertainty

5.1 Introduction

With the creation of deregulated electricity markets introduced in Section 2.3, new electricity Generation Companies (GCs) appeared next to national companies. All production companies are in competition to sell the biggest amount of electricity at the best possible price as presented in Section 2.4. This competition takes place in a day-ahead market controlled by a Transmission System Operator (TSO). Every day, each GC makes bids to the TSO for the following day. The TSO selects the bids maximizing the global welfare.

A stochastic bidding problem has been introduced by Fampa et al. [2008] under a price-maker Bertrand approached introduced in Section 2.4. As mentioned in Section 2.4.2, price-maker approaches require strong hypothesises to be tackled. In the present case, a simplistic UC model is considered with linear production costs, bidden quantities are fixed and the demand on the market is considered as constant. These strong assumptions allow to consider a stochastic framework for the bids of the competitors. Linear production costs allow to decompose the problem by time period. Note that the constant demand is not restrictive as shown in Section 5.3.4.

This problem has been tackled through a primal-dual heuristic and a genetic algorithm [Fampa et al., 2008, Fampa and Pimentel, 2015]. The quality of these heuristic methods was validated through a heavy MIP formulation of the problem used to compute the optimal value. A method providing an upper bound has also been found by Fampa and Pimentel [2017] to asses the quality of a solution found through a heuristic.

In this chapter the *single-period Stochastic Bidding Problem* (SBP) is studied, generalizing the problem presented by Fampa et al. to variable bidding quantities. A dynamic programming approach is used to tackle the problem as well as some variants among which the problem presented by Fampa et al. is solved to optimality.

This chapter is structured as follows. Section 5.2 describes SBP. The model used to clear the day-ahead market and some general properties of SBP are presented in Section 5.3 with a proof that SBP is NP-hard. Section 5.4 presents a dynamic programming (DP) approach to tackle SBP used in Section 5.5 on several variants of the problem providing an upper bound in polynomial time, solving the problem presented by Fampa et al. [2008] to optimality and proposing a heuristic for SBP. Numerical results are presented in Section 5.6.

This problem was proposed by M. Labbé who obtained preliminary results with É. Marcotte on a dynamic programming approach on a variant of SBP presented in Section 5.5.1 providing an upper bound in polynomial time.

5.2 Problem Description

The *single-period Stochastic Bidding Problem* (SBP) for a GC consists in proposing a set of bids to a *transmission system operator* (TSO) on an electricity day-ahead market considering a single time period. Bids are associated to generators, production costs are considered as linear, and bids of competitors as well as the demand on the market are considered as random variables. The goal is to maximize the expected profit of the GC by choosing the bidding prices and quantities of its generators.

The GC owns a set of generators G, $|G| = m$ and makes bids (π_g, q_g) for each generator $g \in G$ on a day-ahead market, π_g representing the bidden unit price of electricity and q_g the bidden quantity. A maximum unit price $\overline{\pi}$ is defined by the TSO, All generators $g \in G$ have a production capacity $q \in [0, \overline{q}_g]$ and a unit production cost c_g. We consider $c_g < \overline{\pi}$ for all $g \in G$. Generators in G are indexed by increasing price, that is, $c_g \leq c_{g'}$ if $g, g' \in G, g < g'$. The TSO considers only step-bids, meaning any percentage of a bidden quantity can be bought.

The competitors place bids with a set of generators G^c. The price and quantity of each bid as well as the total demand on the market are considered as random variables. A stochastic framework presented in Section 1.3.3 is used to represent these bids through a set of discrete scenarios S. Each scenario $s \in S$ is composed of:

- p_s: the probability of scenario s,
- d_s: the total demand on the market,

- $\{(\tilde{\pi}_g^s, \tilde{q}_g^s)\}_{g \in G^c}$: the bids of the competitors, with $0 < \tilde{\pi}_j^s \leq \overline{\pi}$ and $0 < \tilde{q}_j^s$. We consider the total production of the competitors is strictly greater than the demand d_s in order to ensure the demand can be met in each scenario, $\sum_{g \in G^c} \tilde{q}_g^s > d_s$ for all $s \in S$. Without loss of generality, competitor bids are placed at different prices in each scenario.

In a deregulated market, a single GC is likely to represent only a small proportion of the total number of generators bidding in this market. We consider in the following $|G| < |G^c|$. The spot price defined in each scenario $s \in S$ is denoted π^s.

As an example, consider an instance where the GC has three generators with $\overline{q_1} = 2$, $\overline{q_2} = 2$ and $\overline{q_3} = 3$ and $c_1 = 1$, $c_2 = 3$ and $c_3 = 5$. Value $\overline{\pi} = 14$ and there are three scenarios with equal probability and demand, respectively $\frac{1}{3}$ and 10. Bids of competitors in scenario 1 are $\{(4,1),(6,5),(10,2),(12,3)\}$, $\{(2,4),(5,4),(10,3)\}$ in scenario 2 and $\{(8,3),(10,4),(11,2),(14,2)\}$ in scenario 3. The situation is represented in Figure 5.1, spot prices being at the highest possible one at the intersection of the production and demand curves. A feasible solution to BP is

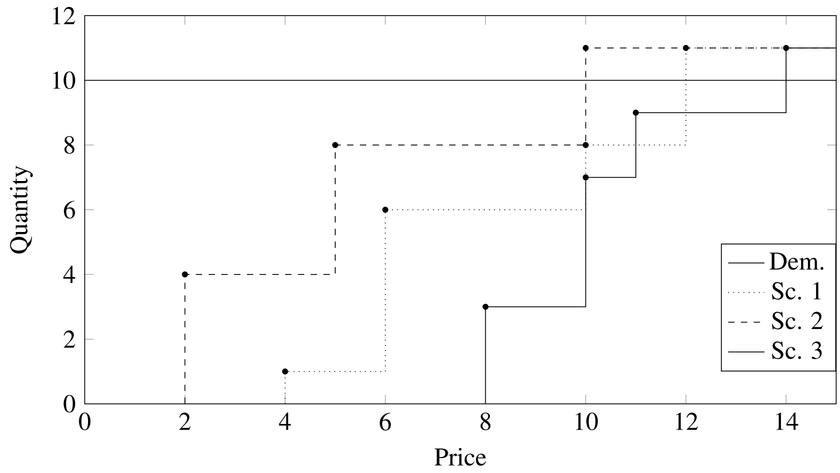

Figure 5.1: BP instance

$((4,2),(8,1),(10,3))$ and is illustrated in Figure 5.2 where the bids of the GC are represented in red for each scenario.

5.3 General properties

5.3.1 Day-ahead spot price and bidding prices

In this section, we show the spot prices in a market equilibrium can be discretized to the bidden prices on the market. As a consequence, the GC can restrict its bidding prices to the bidding prices of competitors in SBP.

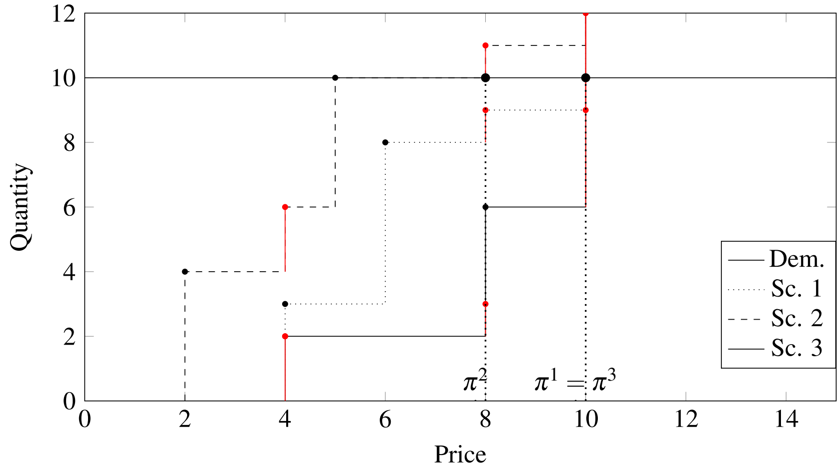

Figure 5.2: BP feasible solution

The spot price is settled once the TSO received all bids of the day-ahead market. In SBP, this computation takes place once a scenario $s \in S$ occurs. The market mechanism considers a fixed demand d_s and step bids. The TSO does not make any difference between the bidding GCs. It considers a set of bids $\{(\pi_g, q_g)\}_{g \in G^{TSO}}$, where G^{TSO} is the set of bidding generators ordered by increasing bidding price. All prices are considered as different. If the TSO receives several bids at the same price, it considers them as one aggregated bid in the market clearing procedure. It then has the responsability of dispatching the total quantity traded of each aggregated bids to each separate bid, which does not influence the spot price. The TSO trades quantities q_g^s, $g \in G^{TSO}$ in the maximization welfare problem that can be formulated as follow:

$$
\begin{aligned}
(Spot - Primal) \quad \min \quad & \sum_{g \in G^{TSO}} \pi_g q_g^s \\
\text{s.t.} \quad & \sum_{g \in G^{TSO}} q_g^s = d_s \qquad (\pi^s) \qquad (5.1)\\
& 0 \le q_g^s \le q_g, \quad \forall g \in G^{TSO} \quad (\alpha_g)
\end{aligned}
$$

This formulation is a variant of the one presented in Section 2.4.2 by considering a fixed demand and a single time-period. The initial objective function $\max \overline{\pi} d_s - \sum_{g \in G^{TSO}} \pi_g q_g^s$ maximizing the welfare corresponds to minimizing $\sum_{g \in G^{TSO}} \pi_g q_g^s$. As mentionned in the literature review, the spot price of electricity is the value of the dual variable π^s of constraint (5.1) in an optimal solution of the dual problem [Baker and Taylor, 1979, Balachandran and Ramakrishnan, 1996]. As a consequence, all bids strictly under the spot price are bought.

Solving Spot-Primal is equivalent to solving a relaxation of a Knapsack problem. An optimal solution is obtained by taking the full quantity of the cheapest bids such that their total production is less or equal to d_s in addition to a fraction of the quantity of the cheapest remaining bid in order to meet d_s. Let $g^* = \min\{g \in G^{TSO} | \sum_{g' \in G^{TSO}, \pi_{g'} \leq \pi_g} q_g > d_s\}$ be the first generator such that the production of all generators up to g^* exceeds d_s. Set $G^* = \{g \in G^{TSO} | \pi_g < \pi_{g^*}\}$ is the set of all generators in bids cheaper price than π_{g^*}. The price of bids associated to generators in G^* are lower or equal to the spot price as their total production does not exceed the demand. An optimal solution of Spot-Primal is:

$$
\begin{aligned}
q_g^s &= q_g && g \in G^*, \\
q_{g^*}^s &= d_s - \sum_{g \in G^*} q_g, && \\
q_g^s &= 0 && g \in G^{TSO} \backslash \{G^* \cup g^*\}
\end{aligned}
$$

The full quantity of generators in G^* is bought and generator g^* completes the demand, minimizing the objective value of Spot-Primal to

$$\sum_{g \in G^*} \pi_g q_g + \pi_{g^*}(d_s - \sum_{g \in G^*} q_g) \tag{5.2}$$

The dual of Spot-Primal is:

$$
\begin{aligned}
(Spot-Dual) \quad \max \quad & d_s \pi^s - \sum_{g \in G^{TSO}} q_g \alpha_g \\
\text{s.t.} \quad & \pi^s - \alpha_g \leq \pi_g && g \in G^{TSO} \\
& \alpha_g \geq 0 && g \in G^{TSO}
\end{aligned} \tag{5.3}
$$

We can observe from Spot-Dual that the spot price π_s in an optimal solution can be restricted to the bidden prices. Values of variables α_g can be optimized based on a given value of π^s by observing the objective function and constraint (5.3):

- if $g \in G^*$, as $\pi_g \leq \pi^s$, we have $\alpha_g = \pi^s - \pi_g$,
- if $g \in G \backslash G^* : \alpha_g = 0$.

Consider an optimal solution where π^s is different of all bidden prices. A new solution can be built by increasing π_s to π_{g^*} with a value δ, increasing variables $\alpha_{g'}, g' \in G^*$ of δ as well in order to preserve a feasible solution. The set of generators in G^* would not be changed under such a modification. To prove optimality of the new solution, the objective value of Spot-Dual can be rewritten as:

$$d_s \pi_{g^*} - \sum_{g \in G^*} q_g(\pi_{g^*} - \pi_g) = \pi_{g^*}(d_s - \sum_{g \in G^*} q_g) + \sum_{g \in G^*} q_g \pi_g, \tag{5.4}$$

which is equal to the objective value of the Spot-Primal in (5.2), proving optimality of the new solution of the dual by using strong duality. Any optimal solution of Spot-Dual can thus be transformed into one where the spot price π^s is in the set of bidden prices, more specifically, equal to π_{g^*}. This spot price is the highest possible one. Otherwise, the full bidden quantity of generators in $G^* \cup \{g^*\}$ would have to be traded, which is not possible as the production would strictly exceed the demand. This leads to the following Lemma:

Lemma 10. *Given a set of bids $\{(\pi_g, q_g)\}_{g \in G^{TSO}}$, the highest spot price π^s in an optimal solution of Spot-Dual is π_{g^*} with $g^* = \min\{g \in G^{TSO} : \sum_{g' \in G^{TSO}, \pi_{g'} \leq \pi_g} q_g > d_s\}$.*

The highest spot price is thus the smallest price such that the total production available exceeds the demand and can be computed in a straightforward way.

As a consequence of Lemma 10, the bidding price of a GC can be discretized based on the bidding prices of the competitors in all scenarios of S. Let Λ be the set of increasing and distinct prices of bids of competitors throughout all scenarios in S where the values 0 and $\overline{\pi}$ are added if not present in the scenarios, that is, $\Lambda = \{\tilde{\pi}_j^s | s \in S, j \in J^c\} \cup \{0, \overline{\pi}\}$. Set I is the set of price indices in Λ with $n = |I| - 1$. The i^{th} price in Λ is denoted λ_i with $\lambda_0 = 0$ and $\lambda_i < \lambda_{i'}, i, i' \in I$. We consider that if the GC does not make a bid for some given generator $g \in G$, then $\pi_g = \overline{\pi}$ and $q_g = 0$.

Lemma 11. *Assuming the GC has priority over competitors, there exists an optimal solution $\{(\pi_g, q_g)\}_{g \in G}$ of SBP with $\pi_g \in \Lambda$ and $c_g < \pi_g$ for all $g \in G$.*

Proof. Consider an optimal set of bids such that there exists a generator $g \in G$ for which $\pi_g \notin \Lambda$. This means $\pi_g < \lambda_n = \overline{\pi}$. Let $\lambda_i = \min\{\lambda_{i'} \in \Lambda | \pi_g < \lambda_{i'}\}$ be the smallest price in Λ greater than π_g. For each scenario $s \in S$, either:

- $\pi_g < \pi^s$, in which case $\lambda_i \leq \pi^s$ and bid (π_g, q_g) is bought by the TSO. Increasing π_g to λ_g preserves the quantity traded because of priority over competitors and increases the selling price.

- $\pi_g > \pi^s$, the demand is met with bids having a price lower than π_g. The bid of the GC not being sold, price π_g can be increased to λ_i without changing profit.

- $\pi_g = \pi^s$, meaning the spot price is on a bid of the GC. As no competitor made a bid at price π_g, the TSO needs this bid to meet the demand. By increasing π_g to λ_i, the same quantity will be traded, again because of priority over competitors, and increases the selling price.

An optimal solution will stay optimal by increasing π_g to λ_i as it cannot decrease the profit in any scenario. This can be iterated for all bids with in $\pi_g \notin \Lambda$.

Furthermore, if a generator $g \in G$ such that $c_g \geq \pi_g$ bids a positive quantity, the generator is not generating profit. Its bit can be set to $(\overline{\pi}, 0)$ in order to remove its bid without decreasing the objective value of a solution. $\square$

5.3.2 Bi-Level Formulation

The goal of BP is to maximize the expectation of the profit of the GC. The profit depends on the quantities of the bids of the GC the TSO will select, the resulting production costs and the spot price defined by all bids. This leads to the following bi-level formulation presented by Fampa et al. [2008]:

$$
\begin{aligned}
(BP-BL) \quad \max \quad & \sum_{s \in S} p_s \sum_{g \in G} (\pi^s - c_g) q_g^s \\
\text{s.t.} \quad & 0 \leq \pi_g \leq \overline{\pi}, \quad \forall g \in G \\
& 0 \leq q_g \leq \overline{q}_g, \quad \forall g \in G \\
& (q_g^s, \pi^s) \in \arg\min \sum_{s \in S} \Big(\sum_{g \in G} \pi_g q_g^s + \sum_{g \in G^c} \tilde{\pi}^s q_g^s \Big) && s \in S \\
& \qquad \text{s.t.} \quad \sum_{g \in G \cup G^c} q_g^s = d_s && (\pi^s) \\
& \qquad 0 \leq q_g^s \leq q_g && g \in G \\
& \qquad 0 \leq q_g^s \leq \tilde{q}_g^s && g \in G^c
\end{aligned}
$$

A classical reformulation presented in Section 1.3.2 can be applied to BP-BL in order to reformulate it as a single level MIP integrating the spot prices in all scenarios. This reformulation contains bilinear terms in the objective function and the constraints, making it hard to tackle.

Still, some observations can be made following the optimistic assumption of the bi-level formulation mentioned in Section 1.3.2:

- if the dual of Spot-Primal is degenerated, the TSO sets the spot price to the highest possible value,
- if the GC proposes bids at the spot price, the TSO buys quantities of these bids maximizing the profit of the GC. The GC thus has priority over competitors, allowing to restrict the bidding prices to Λ by Lemma 11.

Another observation that can be made from the objective function of the leader is that it is profitable for the GC to sell a quantity $q_g^s > 0$ with generator g if and only if $c_g < \pi^s$. To avoid selling at loss we bid generators at a price higher than their unit production cost.

5.3.3 Complexity

The complexity class of SBP is studied in this section. The *decision version* of SBP (DBP) that consists in determining wether SBP has an optimal value equal to some value V is considered. We show that DBP is NP-complete by a reduction from the 3-Partition problem that is *NP*-complete in the strong sense [Garey and Johnson, 1979]. The 3-Partition problem consists in determining if a set A of $3n$ positive integers $\{a_1,...,a_{3n}\}$, for which $\frac{B}{4} < a_j < \frac{B}{2}$ for a given value B for all $j \in \{1,...,3n\}$ and $\sum_{j=1}^{3n} a_j = nB$, can be partitioned into n sets $A_1,...A_n$ such that $\sum_{a \in A_i} a = B$ for all $i \in \{1,...,n\}$. Sets A_i contain 3 elements because of the bounds imposed on values a_j.

Lemma 12. *Problem SBP is NP-hard in the strong sense.*

Proof. The complexity class is proven by reductions 3-Partition $\propto$ DBP $\propto$ SBP. Given a instance of 3-Partition, an instance of DBP can be built as follows:

- A generator is considered for each value in A. For each $g \in G$, the maximum production $\overline{q}_g$ is equal to the g^{th} value a_g in A and the production cost $c_g = 0$
- $2n$ scenarios $S = \{s_i^1, s_i^2 | i \in \{1,...,n\}\}$. For each couple of scenario $s_i = (s_i^1, s_i^2)$:
  - $p_{s_i^1} = p_{s_i^2} = \frac{1}{2n}$, $d_{s_i^1} = d_{s_i^2} = iB$
  - In s_i^1, there is a unique bid $(i, iB+1)$ from competitors
  - In s_i^2 there is a unique bid $(i+1, iB+1)$ from competitors

  Thus, $\Lambda = \{0, 1, ..., n+1 = \overline{\pi}\}$
- $V = \frac{B}{2n} \sum_{i=1}^{n} i(2i+1)$

Figure 5.3 gives an illustration of two pairs of scenarios. If the GC does not bid, the spot prices in the two scenarios of s_i are respectively λ_i and λ_{i+1}.

Consider a set of bids of the GC, Q_i as the total quantity bidden up to price λ_i included. The sum of maximum possible profits in scenarios s_i^1 and s_i^2 based on a set of bids is denoted $\overline{R_{s_i}}$. We have

$$\overline{R_{s_i}(\mathcal{Q})} = \begin{cases} \lambda_i Q_i + \lambda_{i+1} iB & \text{if } Q_i < iB, \\ \lambda_i iB + \lambda_{i+1} iB & \text{if } Q_i = iB, \\ \lambda_i iB + \lambda_i iB & \text{if } Q_i > iB. \end{cases}$$

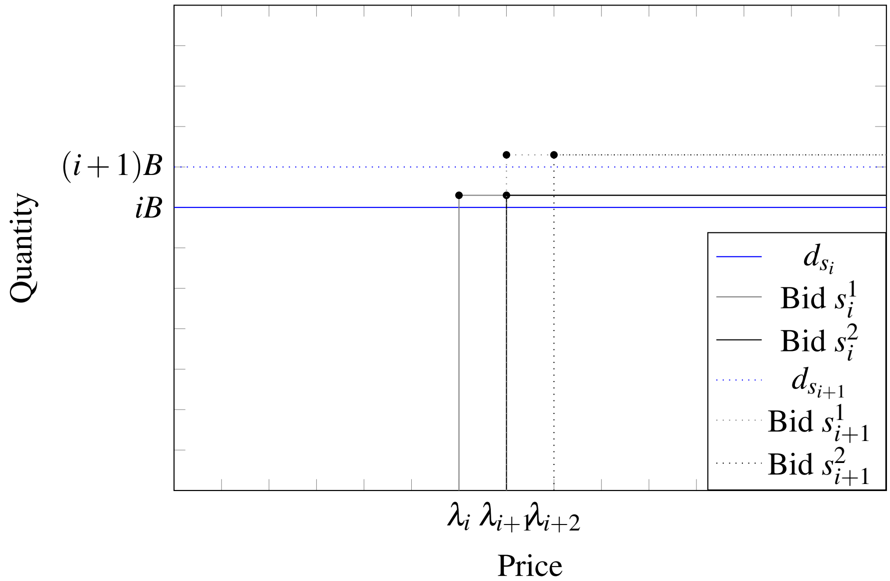

Figure 5.3: Scenarios s_i and s_{i+1} generated from 3-Partition

In the first case, the GC sells the full quantity Q_i in s_i^1 because of priority over competitors and in s_i^2, the GC could bid the rest of its production at price λ_{i+1}, selling a total quantity iB. In the second case, the situation is the same in s_i^1 but selling a higher quantity and in s_i^2 the GC must not make a bid at price λ_{i+1} to preserve the spot price at λ_{i+1}. In the last case, the GC sells a quantity iB, decreasing the spot price in s_i^2.

Profit $\overline{R_{s_i}}$ has maximal value if and only if $Q_i = iB$ as $\lambda_i < \lambda_{i+1}$. An upper bound on the optimal value of SBP is obtained considering the maximum profit is obtained in all scenarios. As $\lambda_i = i$, this maximum possible average revenue is equal to value V :

$$V = \sum_{i=1}^{n} \frac{(2i+1)iB}{2n}$$

An instance of DBP built from a 3-Partition instance has a solution if and only if $Q_i = iB$ for all $i \in \{1,...,n\}$, meaning the total production bidden at price λ_i is equal to B for all $i \in \{1,...,n\}$. If such a solution exists, all generators are bidding their maximum capacity as $\sum_{g=1}^{3n} \overline{q}_g = nB$. The quantities of generators $g \in G$ bidden at price λ_i, $i \in \{1,...,n\}$ correspond to the integers values a_g composing set A_i in a solution of the 3-Partition problem. This proves reduction 3-Partition $\propto$ DBP. Reduction DBP $\propto$ SBP follows trivially. $\square$

5.3.4 Constant demand generalization

The problem treated in this chapter considers only bids from producers and a fixed demand in the day-ahead market, under the assumption that the demand is exceeded when considering all bidden quantities. This hypothesis is not restrictive as it is a generalization of a bidding problem considering bids of producers and retailers.

Consider the demand is not fixed but is composed of retailer bids $\{(\tilde{\pi}_g^s, \tilde{q}_g^s)\}_{g \in R^{TSO}}$ proposed to the TSO where R^{TSO} is the set of retailers. The bids of producers are made with generators in P^{TSO}. The formulation of Spot-Primal would then be:

$$\max \quad \sum_{g \in R^{TSO}} \pi_g q_g^s - \sum_{g \in P^{TSO}} \pi_g q_g^s \tag{5.5}$$

$$\text{s.t.} \quad \sum_{g \in P^{TSO}} q_g^s = \sum_{g \in R^{TSO}} q_g^s \tag{5.6}$$

$$0 \leq q_g^s \leq q_g, \quad \forall g \in P^{TSO} \cup R^{TSO} \tag{5.7}$$

In this formulation, each bid $\{(\tilde{\pi}_g^s, \tilde{q}_g^s)\}$ of a retailer $g \in R^{TSO}$ can be transformed into a bid $\{(\tilde{\pi}_g^s, \tilde{q}_g^s)\}$ of a producer and considering the demand of this additional production bids $\tilde{q}_g^s$ must be satisfied. Set G^{TSO} is composed of the original production bids and the retailer bids seen as production bids. The total bidden quantity from retailers bids transformed into production bids $d_s = \sum_{g \in R^{TSO}} \tilde{q}_g^s$ must be satisfied, representing a fixed demand. The objective function of (5.5) representing the global welfare becomes

$$\max \sum_{g \in R^{TSO}} \pi_g q_g^s - \sum_{g \in P^{TSO}} \pi_g q_g^s = \max\ d_s - \sum_{g \in G^{TSO}} \pi_g q_g^s = d_s - \min \sum_{g \in G^{TSO}} \pi_g q_g^s,$$

which has the same objective function than in Spot-Primal. Constraint (5.6) is equivalent to constraint (5.1) considering all retailers bids must be satisfied when considering them as production bids.

An illustration of a scenario considering producers and retailers as well as the transformation into a fixed demand is given in Figure 5.4. In this example, the production bids are (2,1), (4,2), (6,2), (8,1) and the retailer bids are (3,1),(6,1),(7,4),(12,4) with a total demand of 10. The bids added to the competitors bids in SBP are (3,1),(6,1) and (7,4) and consider a fixed demand equal to 10. With producers and retailers, the spot price is 7 for a traded quantity equal to 5 with bids (2,1), (4,2) and (6,2). In SBP the spot price is also 7 with a total traded quantity equal to 10. Bids (2,1), (3,1),(4,2), (6,1), (6,2) are totally sold and bid (7,4) is partially sold. The production bids traded with producers and retailers are traded as well in SBP, representing a quantity equal to 5. The additional bids traded in SBP are bids coming from retailers bids that are sold.

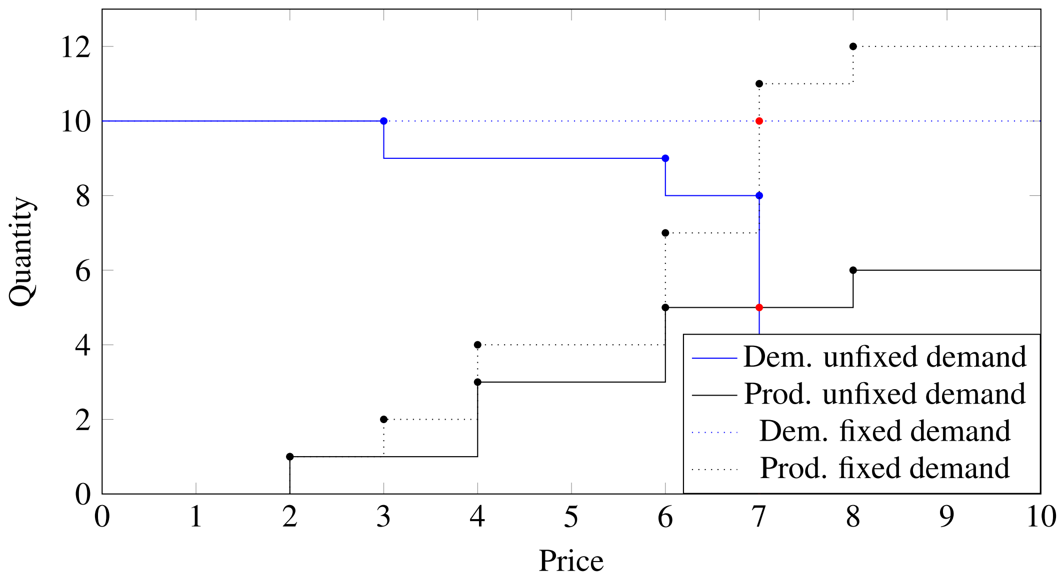

Figure 5.4: Scenario example of BP and the corresponding CDBP

5.4 Dynamic programming approach for SBP

This section presents a general dynamic programming approach to build a solution of SBP, bidding by increasing price in Λ. By Lemma 11, the bidding prices can be restricted to set Λ, composed of all bidding prices of competitors by increasing value, and $\pi_g > c_g$ to avoid production at loss. Set I is the set of price indices in Λ, the i^{th} bidding price in Λ is denoted λ_i. Given a set of bids $B = \{(\pi_g, q_g)\}_{g \in G}$, the following notations are defined representing mainly informations over the bids of the GC per bidding price:

- $\pi^s(B), s \in S$, the spot price in scenario s for a solution B.
- $q_i = \sum_{g \in G, \pi_g = \lambda_i} q_g, i \in I$, is the quantity bidden at price λ_i.
- $Q_i = \sum_{g \in G, \pi_g \leq \lambda_i} q_{i'}, i \in I$, is the cumulative quantity bidden up to price λ_i.
- $G_i = \{g \in G | \pi_g = \lambda_i\}, i \in I$, the generators with a bid at price λ_i.
- $\mathcal{G}_i = \{g \in G | \pi_g \leq \lambda_i\}, i \in I$, the generators with a bid up to price λ_i.
- $b_i = (G_i, q_i), i \in I$, the aggregated bids bidden at price λ_i.
- $B_i = (\mathcal{G}_i, Q_i)$, the bids at a price less than or equal to λ_i.
- $c(G', q)$ is the minimum cost to produce quantity q with generators in G' with their full capacity available.
- $\overline{q}^{G'} = \sum_{g \in G'} \overline{q}_g$ is the maximum production capacity of generators $G' \subseteq G$.

- $\overline{q}_i^{G'} = \sum_{\substack{g \in G' \\ c_g < \lambda_i}} \overline{q}_g$ is the maximum production capacity at price λ_i with generators $G' \subseteq G$ without producing at loss.

- $r_{s,i} = d_s - \sum_{\substack{g \in G^c \\ \tilde{\pi}_g^s < \lambda_i}} \tilde{q}_g^s, s \in S, i \in I$ is the residual demand at price $\lambda_i \in \Lambda$ and scenario $s \in S$ and represents the maximum quantity the GC can sell up to price λ_i before demand is met at price λ_i. If $r_{s,i} \leq 0$ the demand is met by competitor bids strictly under price λ_i. We consider $r_{s,n+1} = -\infty$ for all $s \in S$.

- $r_i^{max} = \max_{s \in S}\{r_{s,i}\}$ and $r_i^{min} = \min_{s \in S}\{r_{s,i}\}$, are the maximum and minimum residual demand at price λ_i over all scenarios.

Notice function $c(G', q)$ is piecewise linear as the generator have linear production costs, generators in G' produce quantity q from the cheapest to the most expensive.

The following lemma proposes a straightforward way to compute the spot price in a given scenario based on the bids of the GC and residual demands.

Lemma 13. *In SBP, given the set of cumulative bidden quantities $Q_i, i \in I$ at each price in Λ and residual demands $r_{s,i}$, the spot price π^s can be computed as follows in $O(\log|\Lambda|)$:*

$$\pi^s = \min\{\lambda_i \in \Lambda : Q_i > r_{s,i+1}\} \tag{5.8}$$

Proof. The optimistic assumption of the bi-level formulation of BP forces the spot price to take the highest possible value. Lemma 10 proves that the highest possible spot price in scenario s is the price π_{g^*} such that $g^* = \min\{g \in G^{TSO} : \sum_{g' \in G^{TSO}, g' \leq g} q_g > d_s\}$. Recall bids given to the TSO at the same price are aggregated in G^{TSO}. We have

$$\sum_{g' \in G^{TSO}, g' \leq g} q_g > d_s \Leftrightarrow \sum_{g \in G, \pi_g \leq \lambda_i} q_g + \sum_{g \in G^c, \pi_g \leq \lambda_i} \tilde{q}_g^s > d_s$$

$$\Leftrightarrow Q_i > d_s - \sum_{g \in G^c, \pi_g \leq \lambda_i} \tilde{q}_g^s \Leftrightarrow Q_i > r_{s,i+1}$$

The bidding price is thus λ_i with $i = \min\{\lambda_i \in \Lambda : Q_i > r_{s,i+1}\}$ which can be computed in logarithmic time based on residual demands and cumulative quantities ordered by increasing price. $\square$

Considering the example in Figure 5.1, the set of prices $\Lambda = \{0, 2, 4, 5, 6, 8, 10, 11, 12, 14\}$, $n = 10$ and residual demands are:

i	0	1	2	3	4	5	6	7	8	9
λ_i	0	2	4	5	6	8	10	11	12	14
Q_i	0	0	2	2	2	3	6	6	6	6
$r_{1,i+1}$	10	10	9	9	4	4	2	2	-1	-∞
$r_{2,i+1}$	10	6	6	2	2	2	-1	-1	-1	-∞
$r_{3,i+1}$	10	10	10	10	10	7	3	1	1	-∞

The highest possible spot prices in the three scenarios are $\pi_1 = \pi_3 = 10$ and $\pi_2 = 8$. In scenario 1, bids of generators 1 and 2 are fully sold and one unit of the third bid is sold, and the profit is $(10-1).2+(10-3).1+(10-5).1 = 30$. In scenario 2, only the bid of generator 1 is sold, the profit is $(8-1).2 = 16$. In scenario 3, all bids are sold, and the profit is $(10-1).2+(10-3).1+(10-5).3 = 40$. The expected profit is $\frac{86}{3}$.

When placing bids by increasing price, the following lemma allows to determine easily the variation of the expected profit.

Lemma 14. *Consider a set of bids $B_{i-1} = \{(\pi_g, g_q)\}_{g\in G'\subset G}$ with $\pi_g < \lambda_i$ for all $g \in G'$. The impact on the expected profit adding an aggregated bid $b_i = (G_i, q_i), G_i \subset G\backslash G'$ depends on residual demands, Q_{i-1}, G_i and $q_i = \sum_{g\in G_i} q_g$.*
Furthermore, if $Q_{i-1} < r_{s,i}$, then $\pi^s(B_{i-1}\cup\{b_i\}) = \min_{i'\geq i}\left\{\lambda_{i'} \in \Lambda | r_{s,i'+1} < Q_i\right\}$.

Proof. After adding b_i, in each scenario $s \in S$, either:

- $Q_{i-1} \geq r_{s,i}$: the bids at price lower than λ_i are sufficient to satisfy the whole demand. The new bid is not bought by the TSO and profit stays constant.
- $Q_{i-1} < r_{s,i}$: all bids up to λ_{i-1} are sold and a quantity $q^*_{s,i} = \min\{q_i, r_{s,i} - Q_{i-1}\}$ of the new bids is sold in order to avoid exceeding the demand. The difference in profit depends on the additional quantity $q^*_{s,i}$ sold, its production costs with generators in G_i and $\pi^s(B_{i-1}\cup\{b_i\})$. As q_i is the quantity bidden at maximum price, values $Q_{i+1},\ldots,Q_n$ are equal to $Q_i = Q_{i-1}+q_i$. The new spot price, which either stays constant or decrease, can be computed based on Q_i and the residual demands, $\pi^s = \min_{i'\geq i}\left\{\lambda_{i'} \in \Lambda | r_{s,i'+1} < Q_i\right\}$.

The variation of profit can thus be computed based on residual demands, Q_{i-1}, G_i and q_i. □

Quantity $q^*_{s,i}$ is the additional quantity sold in scenario $s \in S$ when adding a bid $b_i, i \in I$ to a set of bids B_{i-1} as defined in the proof of Lemma 14.

Lemma 14 allows to formulate the difference of profit in each scenario $s \in S$ by adding a bid $b_i, i \in I$ to a set of bids B_{i-1} as a function $\Delta_s(Q_{i-1}, b_i)$. Consider $\pi^s_1 = \pi^s(B_{i-1})$ and $\pi^s_2 = \pi^s(B_{i-1}\cup\{b\})$ as the spot prices before and after adding b_i. The difference on profit is computed as follows:

$$\Delta_s(Q_{i-1}, b_i) = \begin{cases} 0 & \text{if } r_{s,i} < Q_{i-1}, \\ \pi_2^s q_{s,i}^* + (\pi_2^s - \pi_1^s)Q_{i-1} - c(G_i, q_{s,i}^*) & \text{if } Q_{i-1} \leq r_{s,i}. \end{cases} \tag{5.9}$$

In the first case the spot price is lower or equal to λ_{i-1} and the new bid is not sold. In the second case, the new bid can be sold and influences the spot price. The income depends on the influence of q_i on the spot price and quantities $q_{s,i}^* = \min\{q_i, r_{s,i} - Q_{i-1}\}$. As spot prices π_s^1 and π_s^2 are at least equal to λ_i in this case, their computation can be based on Q_{i-1} and Q_i as explained in the proof of Lemma 14:

$$\pi_s^1 = \min_{i' \geq i}\{\lambda_{i'} \in \Lambda : Q_{i-1} > r_{s,i'+1}\} \; ; \; \pi_s^2 = \min_{i' \geq i}\{\lambda_{i'} \in \Lambda : Q_i > r_{s,i'+1}\}$$

The expectation of the difference of profit can be formulated as follow:

$$\Delta(Q_{i-1}, b_i) = \sum_{s \in S} p_s \Delta_s(Q_{i-1}, b_i)$$

The value of a feasible solution B of SBP is defined by:

$$R(B) = \sum_{i=1}^{n} \Delta(Q_{i-1}, b_i)$$

with $Q_0 = 0$. It is the sum of differences considering placing bids at each price from λ_1 to λ_n.

Consider $R_i^*(\mathscr{G}_i, Q_i)$ as the maximum expected profit bidding up to price $\lambda_i, i \in I$, a total quantity Q_i with generators $\mathscr{G}_i$. By Lemma 14, $R_i^*(\mathscr{G}_i, Q_i)$ can be computed based on $R_{i-1}^*(\mathscr{G}_{i-1}, Q_{i-1})$ and some optimal aggregated bid $b_i = (G_i, q_i)$. In order to find such an aggregated bid, we consider a *candidate function* $\Theta_i(\mathscr{G}_i, Q_i)$ that provides a finite number of aggregated bids $b_i = (G_i, q_i)$ among which at least one appears in a solution of value $R_i^*(\mathscr{G}_i, Q_i)$. The sets of generators in bids returned by $\Theta_i(\mathscr{G}_i, Q_i)$ can trivially be considered as subsets of $\mathscr{G}_i$, respecting production cost constraints. A finite number of quantities for bids on the other hand is harder to obtain as quantities are continuous variables. For now, we leave the definition of the candidate function to the following sections. Still, some candidates returned by $\Theta_i(\mathscr{G}_i, Q_i)$ can be trivially excluded:

- if $Q_i \leq r_i^{min}, i \in I$, the minimum residual at price λ_i, the bids before price λ_i can be made at price λ_i without changing profit thus $\Theta_i(\mathscr{G}_i, Q_i) = \{(\mathscr{G}_i, Q_i)\}$,
- if $Q_i = Q_n$ for $i \in I$, there is no interest in bidding quantities such that $r_{i-1}^{max} < Q_{i-1} < Q_i$ as q_i would not be sold in any scenario, allowing to exclude such candidates from $\Theta_i(\mathscr{G}_i, Q_i)$.

This leads to the following recursive formula to compute $R_i^*(\mathscr{G}_i, Q_i)$ by using formula (5.9) and a candidate function:

$$R_i^*(\mathscr{G}_i, Q_i) = \max_{(G_i, q_i) \in \Theta_i(\mathscr{G}_i, Q_i)} R_{i-1}^*(\mathscr{G}_i \backslash G_i, Q_i - q_i) + \Delta(Q_i - q_i, (G_i, q_i)), i > 0, \tag{5.10}$$

$R_0^*(\mathscr{G}_0, Q_0) = 0.$

The optimal value of SBP is equal to $R_n^*(G, Q_n)$ for some cumulative quantity Q_n. Without loss of generality, we consider all generators are bidden in an optimal solution as they can be bidden at price $\overline{\lambda}$ without producing at loss or influencing the spot price. But the cumulative quantity bidden Q_n in an optimal solution is harder to establish, again because of the continuous character of quantities. We consider a *cumulative quantity function* $\Phi(G)$ as a function that provides a finite set of values out of which at least one is equal to the cumulative quantity bidden in an optimal solution of SBP. As for the candidate function, we leave the definition of the candidate function to the following sections. This leads to generic algorithm SBP^{DP} solving SBP:

Lemma 15. *Given a candidate function $\Theta_i, i \in I$, a cumulative quantity function Φ and formula (5.10), an optimal solution of SBP can be found with algorithm SBP^{DP} computing:*

$$\max_{Q_n \in \Phi(G)} \{R_n^*(G, Q_n)\}.$$

Figure 5.5 illustrates how a solution of SBP can be considered as a path in a directed graph. Nodes represent aggregated bids B_i composed of a set of generators $\mathscr{G}_i$ and a cumulative quantity Q_i bidden at most at price λ_i, an arc represents an aggregated bid b_i. The nodes of the graph used in SBP^{DP} are determined by the candidate and final quantity functions. At price λ_n, there is a node for each quantity returned by the final quantity function. All these nodes are composed of all generators. From these nodes, incoming arcs can be found with the candidate function providing nodes at a lower price for each bid returned. This can be iterated until price λ_0. The weight of each arc is the impact on the expected profit of adding the corresponding bid computed with formula (5.9). Finding the optimal value of SBP corresponds to finding the longest path from λ_0 to a node at price λ_n.

5.5 Variants of SBP solved by dynamic programming

Several variants of SBP are studied in this section, proposing a candidate and cumulative quantity function for each of them to be used in algorithm SBP^{DP}. A summary of the variants presented are listed in Table 5.1, problem SBP-R providing an upper bound for SBP, all others providing a feasible solution. These variants are used in a heuristic method presented at the end of this section aiming at finding a solution in a Bertrand-Cournot equilibrium.

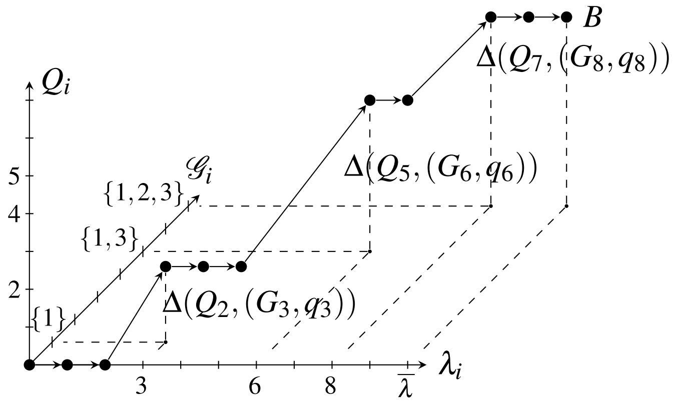

Figure 5.5: SBP solution $B = \{(3,2),(8,1),(6,2)\}$

Problem	Specificity	Optimization of
SBP-R	Multiple bids allowed for each generator	Bidden quantities
SBP-Q	Bidding with fixed quantities	Bidding prices
SBP-S	Bidding at most one generator per price	Bidden prices and quantities
SBP-2	Bidding with 2 generators	Bidden prices and quantities
SBP-P	Bidding with fixed prices	Bidden quantities

Table 5.1: Variants of SBP

5.5.1 Upper bound

Problem description and properties

In SBP, each bid must be assigned to a generator. This limits the bidding opportunities for the GC as if it places a bid with some generator not producing at full capacity, it looses the remaining production capacity of this generator in the bidding procedure. These assignments are also restrictive on production costs, the GC cannot wait the end of the bidding procedure to assign the production traded with the TSO to its cheapest generators.

The *single-period Stochastic Bidding Problem Relaxation* (SBP-R) considers the GC is free to make as many bids it wants, independently from its number of generators. A solution of SBP-R is a set of bids $B = \{(\pi_j, q_j)\}_{j \in J}$ where J is the set of indices of bids. Bids are placed at different prices and $|J| \leq n$. The total bidden production is dispatched to generators only once the TSO cleared the market and determined the total quantity to trade with the GC. This quantity is produced with generators from the cheapest to the most expensive, minimizing the production costs of the GC. As bids are not assigned to the generators, sets G_i used in the dynamic approach for SBP are not relevant. Only the bidden quantities $q_i, i \in I$ at each price in Λ need to be determined to find an optimal solution of SBP-R. In this section, bids $b_i, i \in I$ are determined only by the quantity bidden at price λ_i.

The SBP-R is a relaxation of SBP as a solution of SBP is trivially feasible for SBP-R and its value in SBP-R will be higher or equal than in SBP as production costs can potentially be decreased.

As the TSO selects the quantity it buys without associating them to generators, formula (5.9) can be simplified as follows:

$$\Delta_s(Q_{i-1}, b_i) = \begin{cases} 0 & \text{if } r_{s,i} < Q_{i-1}, \\ R_s(Q_i) - R_s(Q_{i-1}) & \text{if } Q_{i-1} \leq r_{s,i}, \end{cases} \tag{5.11}$$

where $R_s(Q_i)$ is the *single bid profit* made in scenario s placing a single bid (λ_i, Q_i). In such situation $Q_{i'} = 0$ if $i' < i$ and $Q_{i'} = Q_i$ otherwise. The value of $R_s(Q_i)$ can be computed as follows:

$$R_s(Q_i) = \begin{cases} 0 & \text{if } r_{s,i} < 0, \\ \lambda_i r_{s,i} - c(G, r_{s,i}) & \text{if } 0 \leq r_{s,i} < Q_i, \\ \min\left\{\lambda_{i'} \in \Lambda : r_{s,i'+1} < Q_i\right\} Q_i - c(G, Q_i) & \text{if } Q_i \leq r_{s,i}. \end{cases} \tag{5.12}$$

The spot price is computed in $O(\log n)$. Production costs $c(G, Q_i)$ are computed in $O(\log m)$ if maximum cumulative production capacities and production price of generators are precomputed. As we consider $m = |G| < |G^c| \leq n$, the worst case computation time of $R_s(Q_i)$ is in $O(\log n)$.

In the following section, we present candidate and cumulative quantity functions allowing to compute a finite set of cumulative values Q_i to consider when searching for in an optimal solution. The single bid profits $R_s(Q_i)$ are precomputed based on these values in order to compute $\Delta_s, s \in S$ in constant time.

Candidate and cumulative quantity function

The cumulative quantity function is trivial in SBP-R. As $c_g < \overline{\pi}$ for all $g \in G$, the full production capacity can be considered as bidden, at worse at price $\overline{\pi}$ in order to avoid production at loss. We have $\Phi(G) = \{\overline{q}^G\}$.

The following Lemma provides a finite number of candidates for quantities q_i in an optimal solution of SBP-R based on residual demands.

Lemma 16. *There exists a set of optimal bids $B = \{(\pi_j, q_j)\}_{j \in J}$ of SBP-R that has all cumulative quantities Q_i equal to one of the following candidates:*

- *the maximum production capacity without producing at loss $\overline{q}_i^G$,*

- *a residual demand for the next higher bid price, $r_{s,i+1}, s \in S$, if lower than $\overline{q}_i^G$, if $i < n$,*
- *Q_{i+1}, if $i < n$.*

Proof. Consider an optimal solution where some values Q_i are not in the set of candidates and the smallest index $i \in I$ of these values. As $Q_{i+1} > Q_i$, the value of Q_i is increased up to its nearest candidate preserving optimality by transferring some quantity δ from Q_{i+1} to Q_i. As bidding at loss is not allowed, $Q_i < \overline{q}_i^G$. The variation on profit obtained by increasing Q_i will depend on the impact on the spot price in each scenario $s \in S$. Several situations can occur:

- $\pi^s < \lambda_i$: all the demand is already met and the quantity δ transferred is not sold.
- $\pi^s > \lambda_{i+1}$: all the bidden quantity up to price λ_{i+1} included is sold. Transferring a quantity δ leaves Q_{i+1} unchanged. As a consequence, the spot price is unchanged as well as the profit.
- $\pi^s = \lambda_i$: the demand needs all bids lower than price λ_{i-1} in addition to some production at price λ_i. We face 2 cases:

  - $Q_i < r_{s,i} \rightarrow$ Increasing Q_i to the nearest candidate preserves the same spot price as the demand will not be exceeded. The quantity sold increases as well as profit.
  - $Q_i \geq r_{s,i} \rightarrow$ The demand is already met with the bidden quantity and the quantity δ transferred is not sold. The spot price and profit remain unchanged after increasing Q_i.

- $\pi^s = \lambda_{i+1}$: the spot price could decrease if $Q_i + \delta$ exceeds the following residual demand $r_{s,i+1}$. This cannot occur as $r_{s,i+1}$ is a candidate. The spot price remains constant and the quantity sold can increase, increasing profit.

For all scenarios, increasing Q_i to its next candidate cannot decrease profit, proving optimality of the new solution. □

The candidate function $\Theta_i(Q_i)$ used to solve SBP-R is based on candidates proposed by Lemma 16. Note parameter $\mathscr{G}_i$ is omitted in Θ_i as generators are not assigned to bids in SBP-R. The bids returned by Θ_i consist in quantities q_i that can be bidden at price λ_i. Values $q_i \in \Theta_i(Q_i)$ are such that

$$Q_{i-1} \in \{\max\{\overline{q}_{i-1}^G, Q_i\} \cup \{r_{s,i'} | s \in S, i' \geq i, r_{s,i} \leq \max\{\overline{q}_{i-1}^G, Q_i\}\}\}, \quad Q_{i-1} = Q_i - q_i.$$

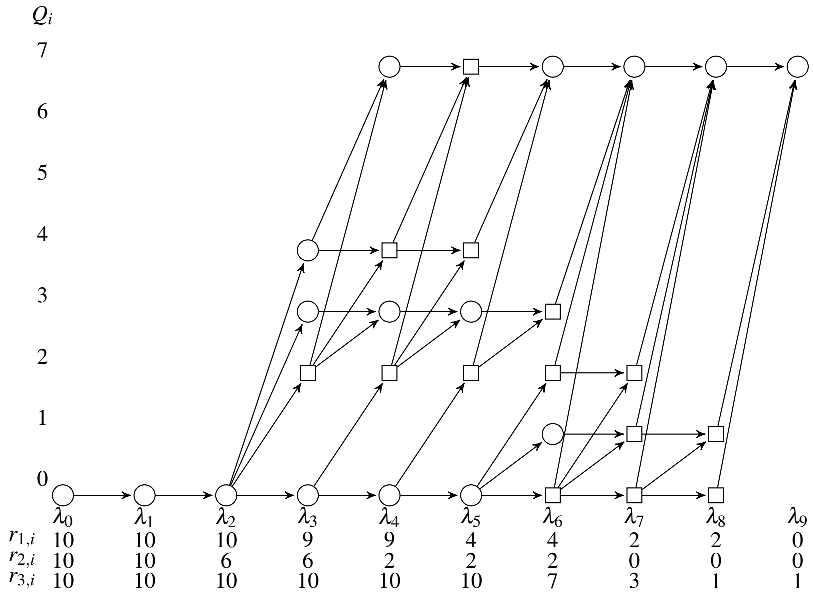

Figure 5.6: Graph of SBP-R instance with $\overline{q_1} = 2$, $\overline{q_2} = 2$ and $\overline{q_3} = 3$ and $c_1 = 1$, $c_2 = 3$ and $c_3 = 5$.

The exclusion of some candidates for SBP mentioned in Section 5.4 applies to this candidate function. The candidate function returns $O(|S||G^c|)$ candidates as for each scenario the residual demand has $O(|G^c|)$ different values, one for each competitor bid.

Solving SBP-R with algorithm SBP^{DP} can be interpreted as searching for the longest path in an oriented graph. The nodes of the graph are the candidates for all possible prices and there is an arc between two nodes representing candidates Q_{i-1} and Q_i. The weight of an arc from Q_{i-1} to Q_i is equal to $\Delta(Q_{i-1}, q_i)$. The path to determine goes from price λ_0 with $q_0 = 0$ to candidate $\overline{q}^G$ at price λ_n. An illustration of the graph corresponding to the instance in Figure 5.1 is given in Figure 5.6, the square nodes represents residual demands at the next price $r_{s,i+1}$.

Complexity of SBP^{DP}-R

Algorithm SBP^{DP}-R solves SBP-R using SBP^{DP} and the candidate and cumulative quantity function presented for SBP-R.

The preprocessing computes the following informations:

- the ordered set Λ in $O(|S||G^c| + n \log n)$,
- residual demands $r_{s,i}$ for each price in $O(n|S|)$,
- maximum production capacities q_i^G for each price in $O(n)$,
- all candidates quantities generated by the candidate and cumulative quantity functions

and its corresponding single bid profit. The cumulative quantity is Q_n. The candidate quantities at each price λ_i are the maximum production capacity and the residual demands in all scenarios at higher prices. There are $|G^c|$ possible residual demands in each scenario, each of these values can appear as candidate at lower prices, leading to a total of $O(n|S||G^c|)$ candidates. A single bid profit in a scenario is computed in $O(\log n)$, all single bid profits are computed in $O(n|S|^2|G^c|\log n)$.

The overall complexity of preprocessing is in $O(n|S|^2|G^c|\log n)$.

Given preprocessing information, for each candidate quantity Q_i, $\Theta_i(Q_i)$ provides $O(|S||G^c|)$ candidates at the previous price. The difference in profit adding a bid $q_i \in \Theta_i(Q_i)$ in a given scenario is computed in constant time knowing single bid profits of each candidate with formula (5.11). All the differences in profit are computed in $O(n|S|^3|G^c|^2)$. An optimal solution of SBP can be built when computing these differences in formula (5.10), leading to a polynomial algorithm to solve SBP-R.

5.5.2 Bidding with fixed quantities

Problem description

The *single-period Stochastic Bidding Problem with fixed Quantities* (SBP-Q) is a constrained version of SBP where bidding quantities of generators are fixed. This corresponds to a Bertrand model where a GC tries to optimize its profit by acting only on prices. Without loss of generality we consider $q_g = \overline{q}_g$ for all $g \in G$. Solving BP-Q consists in finding an optimal set of bidding prices for generators. This problem has already been studied through heuristic methods by Fampa et al. [2008] and Fampa and Pimentel [2015, 2017].

If the GC makes a bid $b_i = (G_i, q_i), i \in I$ at price λ_i, then $q_i = \sum_{g \in G_i} \overline{q}_j$. Aggregated bids b_i in this section are determined by the set of generators G_i bidden at price λ_i.

Lemma 17. *Problem BP-Q is NP-hard in the strong sense.*

Proof. The reduction presented in Theorem 12 can be easily converted to this case. □

Candidate and cumulative quantity function

As for SBP-R, as all generators are such that $c_j < \overline{\pi}$, there exists an optimal solution of BP-Q such that $Q_n = \overline{q}^J$. We use $\Phi(G) = \{\overline{q}^J\}$.

The candidate function considers subsets of generators of $\mathscr{G}_i$ avoiding generators producing at loss at price λ_{i-1}. The candidate set $\Theta_i(\mathscr{G}_i) = \{G' \subset \mathscr{G}_i | c_g < \lambda_{i-1}, g \in G_i \backslash G'\}$ contains $O(2^m)$

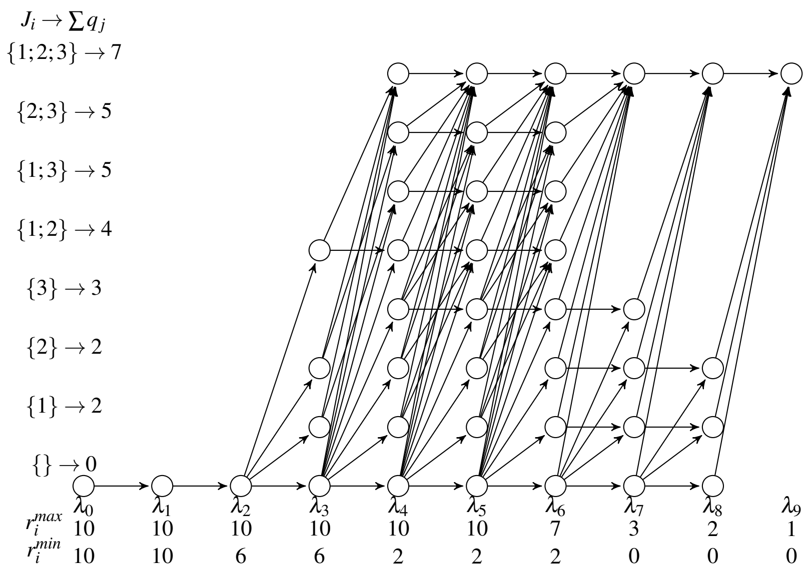

Figure 5.7: Graph of SBP-Q instance with $\overline{q_1} = 2$, $\overline{q_2} = 2$ and $\overline{q_3} = 3$ and $c_1 = 1$, $c_2 = 3$ and $c_3 = 5$.

candidates in the worse case. Parameter Q_i is omitted from Θ_i as this cumulative quantity can be found based on $\mathscr{G}_i$.

Figure 5.7 illustrates the graph of SBP-Q of the instance of Figure 5.1 excluding the candidates mentioned in Section 5.4. As the candidate function for SBP-Q returns an exponential number of candidates, the graph is larger and denser than for SBP-R when increasing the number of generators.

Complexity of SBP^{DP}-Q

As for the resolution of SBP-R, SBP^{DP}-Q solves SBP-Q using SBP^{DP} as a base with the candidate and cumulative quantity functions presented for SBP-Q.

The preprocessing computes the following informations:

- the order set Λ in $O(|S||G^c| + n \log n)$,
- residual demands $r_{s,i}$ for each price in $O(n|S|)$,
- maximum production capacities q_i^G for each price in $O(n)$,
- all possible spot prices π_1^s and π_2^s used in formula $\Delta_s, s \in S$ (5.9) for all prices in Λ in $O(n|S|2^m|\log n)$ as there are $O(2^m)$ candidate quantities to consider at each price.

The overall complexity of preprocessing is in $O(n|S|2^m \log n)$.

There are $O(n2^m)$ candidate quantities. For each candidate $\mathscr{G}_i$, the candidate function returns

$O(2^m)$ sets G_i. The difference in profit bidding G_i is computed in $\log m$, representing the computation of production costs in formula (5.9), spot prices being precomputed. An optimal solution is found using algorithm SBP^{DP} in $O(n|S|2^{2m}\log m)$.

Lemma 18. *SBP-Q a polynomial problem for a fixed number of generators.*

5.5.3 Bidding single-price generators

Problem description

The *single-period Stochastic Bidding Problem with Single-price generators* (SBP-S) is a variant of SBP consisting in finding an optimal set of bids under the constraint that a single generator can be bidden at each price in Λ. If, for an instance of SBP, there exists an optimal solution with at most one generator bidden at each price, the optimal value of SBP and SBP-S are equal. Practically, as the number of generators is much smaller than the number of bidding prices and the GC has an interest in diversification when many scenarios are considered, there is a high chance that an optimal solution of SBP-S is optimal for SBP. To give an idea of these numbers, in the instances considered further in the numerical experiments, the number of generators is between 2 and 10 and the number of prices between 100 and 250.

A function $\Delta_{i_1,i_2}(\delta), 0 \leq i_1 < i_2$ is defined for a feasible solution of SBP-S and represents the impact on the expected profit increasing q_{i_1} by δ and decreasing q_{i_2} by δ, respecting production capacities of bidden generators. This corresponds to transferring δ unit of power from G_{i_2} to G_{i_1}. With this transfer, cumulative quantities $Q_i, i_1 \leq i < i_2$ increase by δ, others stay constant. Function $\Delta^s_{i_1,i_2}(\delta), i_1 < i_2$ represents the impact on profit in scenario $s \in S$. We consider $\delta \in [\delta^{min}, \delta^{max}]$ where values δ^{min} and δ^{max} respect production capacities at prices λ_{i_1} and λ_{i_2} and are bounded by residual demands as follows:

- $\delta^{min} = \max\{\{-q_{i_1}, q_{i_2} - \overline{q}_{i_2}^{G_{i_2}}\} \cup \{r_{s,i''} - q_{i'} | i_1 < i' < i_2, i'' \geq i', q_{i'} \geq r_{s,i''}, s \in S\}\}$ is the minimum value that can be added to q_{i_1} such that no value $Q_{i'}, i_1 \leq i' < i_2$ goes strictly below another residual demand at higher price in some scenario.
- $\delta^{max} = \min\{\{\overline{q}_{i_1}^{G_{i_1}} - q_{i_1}, q_{i_2}\} \cup \{r_{s,i''} - q_{i'} | i_1 < i' < i_2, i'' \geq i', q_{i'} \leq r_{s,i''}, s \in S\}\}$ is the maximum value that can be added to q_{i_1} such that no value $Q_{i'}, i_1 \leq i' < i_2$ goes strictly above another residual at higher price demand in some scenario.

If $\delta \in]\delta^{min}, \delta^{max}]$, then a transfer cannot change the spot price in any scenario. For $n' > n$, we consider $q_{n'} = +\infty$ and $q_{n'} - \overline{q}_{n'}^{G_{n'}} = -\infty$ in order to eliminate bounds on δ^{min} and δ^{max} coming from a limitation of production capacity at price $\lambda_{n'}$. Price $\lambda_{n'}$ is set to $\overline{\pi}+1$.

Lemma 19. *For all* $0 \leq i_1 < i_2 \leq n+1$, *function* Δ_{i_1,i_2} *for a solution of SBP-S has a maximal* *value reached in δ^{min} and/or δ^{max}.*

Proof. All functions $\Delta^s_{i_1,i_2}(\delta)$ are linear for $\delta \in]\delta^{min}, \delta^{max}]$ as the spot price does not change in any scenario after adding δ, the difference in production sold is either δ, either 0, and production costs are linear as $|G_i| \leq 1, i \in I$. If $\delta = \delta^{min}$, either:

- the spot price is constant in every scenario,
- the spot price increases in some scenario s, and $\Delta^s_{i_1,i_2}(\delta^{min})$ is greater or equal to

$$\lim_{\delta \to \delta^{min}, \delta > \delta^{min}} \Delta^s_{i_1,i_2}(\delta).$$

In all cases $\Delta_{i_1,i_2}(\delta)$ is either linear on $[\delta^{min}, \delta^{max}]$ either linear on $]\delta^{min}, \delta^{max}]$ and has a local maximum in δ^{min}. □

Lemma 19 leads to the following description of candidates in an optimal solution of SBP-S.

Lemma 20. *There exists an optimal solution of SBP-S such that cumulative quantities Q_i are equal to one of the following candidates:*

- $r_{s,i'}, i' \geq i, s \in S$, *if* $r_{s,i'} \leq \overline{q}_i^{\mathcal{G}_i}$
- Q_{i-1} *or* $Q_{i-1} + \overline{q}_i^{G_i}$
- Q_{i+1} *or* $Q_{i+1} - \overline{q}_{i+1}^{G_{i+1}}$

Proof. Consider an optimal solution of SBP-S such that there exists a cumulative quantity Q_i that is not at any candidate. This implies a bid is made at price λ_i as $Q_i > Q_{i-1}$ and at price λ_{i+1} as $Q_i < Q_{i+1}$ and generators G_i and G_{i+1} are not producing at full capacity. Thus some production can be transferred between G_i and G_{i+1}. By Lemma 19, function $\Delta_{i,i+1}$ has maximum value setting Q_i at one of the proposed candidate. Bidden quantities that are not at any candidate can be adapted from the smallest to highest bidding price. □

Lemma 21. *There exists an optimal solution of SBP-S such that if there exist two indices i_1 and i_2, $i_1 < i_2$ with Q_{i_1} and Q_{i_2} not equal to a residual demand at a strictly higher price, $0 < q_{i_1} < \overline{q}^{G_{i_1}}$ and $0 < q_{i_2} < \overline{q}^{G_{i_2}}$, then there is a quantity $Q_i, i_1 < i < i_2$ equal to some higher residual demand.*

Proof. Consider an optimal solution of SBP-S violating Lemma 21 for indices i_1 and i_2, $i_1 < i_2$. By Lemma 19, adding the value δ maximizing Δ_{i_1,i_2} to q_{i_1} and subtracting it from q_{i_2} leads either to a value $Q_i, i_1 \leq i < i_2$ equal to some higher residual demand, either puts production of

G_{i_1} or G_{i_2} to 0 or full capacity. In both cases i_1 and i_2 do not violate Lemma 21 anymore. This procedure can be iterated until the modified optimal solution of BP-S respects Lemma 21. $\square$

Lemma 21 ensures that there exists an optimal solution of SBP-S such that if two non consecutive cumulative quantities Q_i and $Q_{i'}$ are equal to a residual demand, than there exists an price index $i < i'' < i'$ with $G_{i''} \neq \emptyset$ such that either:

- $Q_{i''}$ is equal to a residual demand
- $\lambda_{i''}$ is the only price between λ_i and $\lambda_{i'}$ where the quantity bidden is positive but under the maximum production capacity $0 < q_{i''} < \overline{q}_{i''}^{G_{i''}}$. The associated bid $(G_{i''}, q_{i''})$ is referred to as a *jump*.

Consider sets $\Phi_i^{G'}$, $0 \leq i \leq n, J' \subseteq J$:

$$\Phi_i^{G'} = \{r_{s,i'} | i' > i, s \in S, r_{s,i'} \leq \overline{q}_i^{G'}\} \cup \{\phi + \overline{q}_g | g \in G', c_g < \lambda_{i-1}, \phi \in \Phi_{i-1}^{G' \setminus \{g\}}\} \cup \Phi_{i-1}^{G'},$$

$\Phi_0^{G'} = \Phi_i^{\emptyset} = \{0\}$ and $\Phi_i^{G'} = \{\}$ if $i < |G'|$. Set $\Phi_i^{G'}$ contains quantities $q < \overline{q}_i^{G'}$ equal to a residual demand plus the full production of a subset of generators of G' bidden under price λ_i. These quantities allow to reach all cumulative quantity Q_{i-1} coming from a residual demand and followed by generators bidden at maximum capacity. By Lemma 21, there exists an optimal solution of SBP-S such that a jump (G_i, q_i) is made from a quantity in $\Phi_{i-1}^{G_i \setminus g}$ for some $g \in G_i$.

Lemma 22. *There exists an optimal solution of SBP-S such that Q_n is in Φ_n^G.*

Proof. The possible quantities suggested for Q_n can be reached from some residual demand and adding the full capacity of a subset of G. Consider an optimal solution of SBP-S such that Q_n is not in Φ_n^G and i as the maximum index such that Q_i is equal to a residual demand at higher price. All cumulative quantities $Q_{i'}, i' > i$ are not equal to a residual demand and some generators in $G \setminus G_i$ are not producing at maximum capacity. Let $i^* > i$ be the lowest price such that the generators bidden are not producing at maximum capacity. By Lemma 19, function $\Delta_{i^*,n+1}$ has maximum value for a value δ setting Q_{i^*} to a higher residual or setting its production to 0 or to $\overline{q}_{i^*}^{G_{i^*}}$. This procedure can be iterated until $Q_n \in \Phi_n^G$. $\square$

There are $O(n2^m)$ sets $\Phi_i^{G'}$ to be computed from λ_0 to λ_n to obtain Φ_n^G. As values in $\Phi_{i-1}^{G'}$ are replicated in $\Phi_i^{G'}$ if $G' \subset G$, the number of values in Φ_n^G depends exponentially on n and m,

Lemmas 20-22 and sets $\Phi_i^{G'}$ lead to candidate and cumulative quantity functions for SBP-S that are used in algorithm SBP^{DP}-S that uses SBP^{DP} as a base.

Candidate and cumulative quantity function

By Lemma 22, the cumulative quantity function $\Phi(G)$ for SBP-S contains all values in Φ_n^G.

Lemma 21 leads to a candidate function $\Theta_i(\mathscr{G}_i, Q_i)$ for SBP-S. Candidates are such that either no generator is bidden at price λ_i, either the full capacity of one generator is removed from Q_i, either a jump is made to a value in $\Phi_{i-1}^{G'}$. Let $G' = \{g \in \mathscr{G}_i | c_g < \lambda_{i-1}\}$. The candidate function is defined as follows:

- if $|G'| < |G_i| - 1$, $\Theta_i(\mathscr{G}_i, Q_i) = \{\}$,
- if $|G'| = |G_i| - 1$, let $g = G_i \backslash G'$, $\Theta_i(\mathscr{G}_i, Q_i)$ contains candidates (G', Q_{i-1}) at price λ_{i-1} such that Q_{i-1} is equal to $\max\{0, Q_i - \overline{q}_g\}$, $\min\{Q_i, \overline{q}_{i-1}^{G'}\}$ or any quantity in $\Phi_{i-1}^{G'}$ between those values,
- otherwise, $\Theta_i(\mathscr{G}_i, Q_i)$ contains candidates $(G_i \backslash \{g\}, Q_{i-1}), g \in \mathscr{G}_i$ at price λ_{i-1} such that Q_{i-1} is equal to $\max\{0, Q_i - \overline{q}_g\}$, $\min\{Q_i, \overline{q}_{i-1}^{\mathscr{G}_i \backslash \{g\}}\}$ or any quantity in $\Phi_{i-1}^{\mathscr{G}_i \backslash \{g\}}$ between those values.

As the candidate function uses values in sets $\Phi_i^{G'}$, the number of candidate returned is exponential in i.

5.5.4 Bidding with 2 generators

Problem description

SBP can be solved to optimality when considering at most 2 generators (SBP-2) by using Lemmas used to solve SBP-S. Consider an instance of SBP with $|G| \leq 2$. If there exists an optimal solution such that the two generators have different bidding prices, the optimal value of SBP-2 is equal to the optimal value of SBP-S. If such a solution does not exist, then two generators are bidden at the same price λ_i. In this case, Q_i can be restricted to residual demands respecting production capacities or to $\overline{q}_i^G$ when searching for an optimal solution. If Q_i is not equal to such a value in an optimal solution, then Q_i can either be increased to the first higher residual demand increasing the quantity sold without changing the spot price, either be increased to $\overline{q}^G$ if a residual cannot be reached, without decreasing profit.

Candidate and cumulative quantity function

The cumulative quantity function of SBP-S can be used to solve SBP-2 as it includes the possible cumulative quantity for optimal solutions with two generators bidden at the same price if $n > 1$.

The candidate function of SBP-S can be adapted for SBP-2. Set $\Theta_i(\mathscr{G}_i, Q_i)$ contains all candidates considered in BP-S and candidates (G, Q_i) if $\mathscr{G}_i = G$.

5.5.5 Heuristic searching a Bertrand and Cournot equilibrium

A heuristic attempting to find a solution of SBP in a Bertrand-Cournot equilibrium is presented in this section. To recall, a Bertrand approach aims at maximize profit by adjusting prices of a ressource while a Cournot approach maximizes profit by adjusting quantities. A solution to an economical problem in a Bertrand-Cournot equilibrium is a solution such that the profit cannot be increased without adjusting prices and quantities simultaneously. A Bertrand approach for SBP has been presented in Section 5.5.2. A Cournot approach bidding with fixed price is presented in the following, leading to a heuristic method alternating these two approaches.

Bidding with fixed prices

The *single-period Stochastic Bidding Problem with fixed Prices* (BP-P) is similar to SBP-Q fixing prices π_g of generators $g \in G$ and searching for optimal quantities to bid. Sets $G_i, i \in I$ are provided, respecting production costs and values q_i must be found. Without loss of generality, we can consider all generators of the GC are bidden. A heuristic method is presented in this section using SBP^{DP} as a base with candidate and cumulative quantity functions that do not guarantee an optimal solution.

The cumulative quantity function $\Phi(G)$ is the same than in SBP-S. The candidate function $\Theta_i(Q_i)$ is defined as follow:

- if $G_i = \emptyset$, then $\Theta_i(Q_i) = \{0\}$ as no quantity can be bidden at price λ_i,
- otherwise, $\Theta_i(Q_i)$ contains values q_i such that $q_i = 0$, $q_i = \overline{q}^{G'}, G' \subseteq G_i$ representing the maximum production capacity of a subset of generators in G_i or $Q_i - q_i = Q_{i-1} \in \{r_{s,i'} | i' \geq i, s \in S, r_{s,i'} \leq \overline{q}^{\mathscr{G}_i}\}\}$ is equal to residual demand to avoid decreasing a spot price π_s under a certain value.

These candidates are considered because when bidding a generator, two main possibilities observed for SBP-S occur:

- bid its maximum production capacity to sell as much as possible,
- bid under its maximum capacity in order to reach a residual demand to avoid decreasing the spot price under a certain value.

This leads to a number of candidates exponential in the number of generators and n at each price.

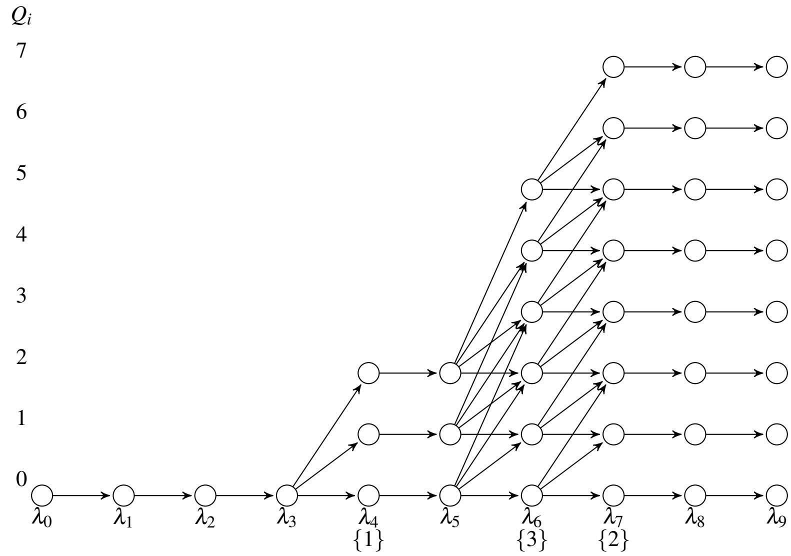

Figure 5.8: Graph of SBP-P instance with $J_4 = \{1\}$, $J_6 = \{3\}$ and $J_7 = \{2\}$.

Figure 5.8 illustrates the graph obtained using the candidate and cumulative quantity functions on the instance presented in Section 5.2. Prices where no generator is bidden could be eliminated from this graph as the incoming arcs all have a weight equal to 0.

Note that the solution found is optimal if at most one generator is bidden per price. The candidate and cumulative quantity function used can be proven to return values in an optimal solution in a similar way than in the lemmas used to solve SBP-S.

Gauss-Seidel heuristic

The optimal value of SBP-R provides an upper bound on the optimal value of SBP and SBP-Q provides a feasible solution using a Bertrand approach. This feasible solution can be improved by using a Cournot approach that aims at optimizing quantities for the prices found with SBP-Q by solving SBP-P. If the optimal value of SBP-P is higher than for SBP-Q, better quantities are found through SBP-P for the bids and these quantities can be used as fixed quantities to solve SBP-Q again. This iterative procedure solving SBP-Q and SBP-P can be made until a stopping criterion is met. This *Heuristic for SBP* (HSBP) using Bertrand and Cournot models is illustrated in Figure 5.9. The input is an instance of SBP and initial quantities $q_i, i \in I$ for the generators to solve SBP-Q. Value z_R^* is an upper bound obtained solving SBP-R. As a heuristic method is considered to solve SBP-P, the stopping criterion of HSBP is either to find a solution previously found or have 4 iterations without improvement on the objective value. Note that the best solution returned is in a Bertrand-Cournot equilibrium if it had been found at a previous iteration and bidding prices are all different, in which case profit could not be increased adjusting quantities or prices separately.

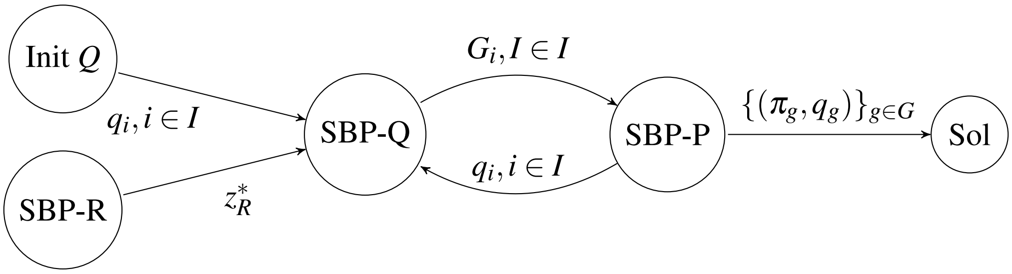

Figure 5.9: SHBP

HSBP^k

The solution provided by HSBP depends on the initial quantities used to solve SBP-Q. Algorithm HSBP^k first runs HSBP using maximum production capacities as initial capacities. A feasible solution of SBP with bidden quantities q_g for all generators $g \in G$ is found. Algorithm HSBP is then run k times in parallel. The initial quantities of generator $g \in G$ are generated with a Gaussian distribution $\mathcal{N}(q_g, \frac{q_g}{10})$ such that initial quantities are in $[0, \overline{q}_g]$ to respect production capacities. Algorithm HSBP^k is a simple enhancement of HSBP in order to analyse the influence of initial quantities in HSBP.

5.6 Numerical results

All the algorithms are implemented in Java 1.8.0 and numerical experiments are made on a 4-core i7 2.30 GHz processor with 16Go of RAM memory.

5.6.1 Instances

The instances used by Fampa and Pimentel [2015] were kindly provided the authors for our numerical experiments. This data is related to electricity trading in Brazil in 2008. These instances consider $|J| = 6$, $|J^c| = 108$ and $|S| \in \{10, \dots, 70\}$. The focus in the present numerical results is mainly to consider a more important number of scenarios going up to 200 scenarios and 10 generators. These instances are built from the initial ones by clustering or splitting the production of generators and generating new scenarios by duplicating existing scenarios and modifying of at most 10% the bidden prices and quantities. For each set of values $(|J|, |S|)$, five instances are considered. The instances are available at `https://github.com/jdeboeck/SBP`.

5.6.2 Optimality Gap

Only instances of SBP with two generators can be solved to optimality using SBP^{DP}-2. This section analyses the quality of the solutions of the variants of SBP with respect to the optimal value of SBP. Table 5.2 provides the computational results of the different variants of SBP considered with two generators. Results are averaged over five instances. Times are in given in seconds, gaps are in percents. The gaps are relative to the optimal value. Solution times of HSBP do not include the solution time of SBP-R.

Instance		SBP-R		SBP-2		SBP-S		HSBP		SBP-Q	
$\lvert G^c \rvert$	$\lvert S \rvert$	gap	time	z^*	time	gap	time	gap	time	gap	time
50	10	1.91	0.03	387689	1.01	0.0	0.72	0.49	0.04	2.98	0.0
50	20	3.61	0.18	419588	4.51	0.0	3.37	0.0	0.07	0.79	0.0
50	30	3.51	0.61	365623	11.19	0.0	8.38	0.0	0.15	0.99	0.01
50	40	2.73	1.69	428025	23.48	0.0	17.04	0.03	0.26	0.84	0.02
50	50	2.75	3.85	375486	39.38	0.0	29.16	0.0	0.37	0.32	0.03
108	10	3.23	0.02	376115	1.19	0.0	0.9	0.33	0.03	0.85	0.0
108	20	2.84	0.2	393069	5.92	0.0	4.45	0.0	0.08	0.48	0.0
108	30	2.12	0.47	378072	13.09	0.0	10.17	0.0	0.24	0.11	0.01
108	40	1.11	1.51	423856	27.63	0.0	20.86	0.0	0.42	0.15	0.02
108	50	1.82	2.68	385641	43.42	0.0	32.83	0.0	1.09	0.13	0.02

Table 5.2: Numerical results with 2 generators

Algorithms to solve SBP-2 and SBP-S always obtain the same optimal value. Both algorithms have solution times that are much larger than for other variants. Computation times are larger for SBP-2 than SBP-S as more candidates are returned by functions Θ_i. All other problems are solved in a much shorter time and provide relatively small gaps, especially for HSBP that finds an optimal solution for most instances.

Figure 5.10 provides the relative gaps of the 50 instances considered in Table 5.2.

Table 5.3 provides an example of the iteration process of HSBP with the evolution of the solution of an instance with 6 generators. The variation of quantities and prices are highlighted. The objective value is indicated next to the algorithm iteration followed by the bids composing the solution. The gap after the first iteration of SBP-Q is 3.56 % and falls down to 0.15%.

Table 5.4 considers instances of SBP with 3 or 4 generators. These instances cannot be solved to optimality with the presented methods. The gaps provided are the relative gaps to the optimal value of SBP-S. Although SBP-S provides the best feasible solution of SBP, the algorithm

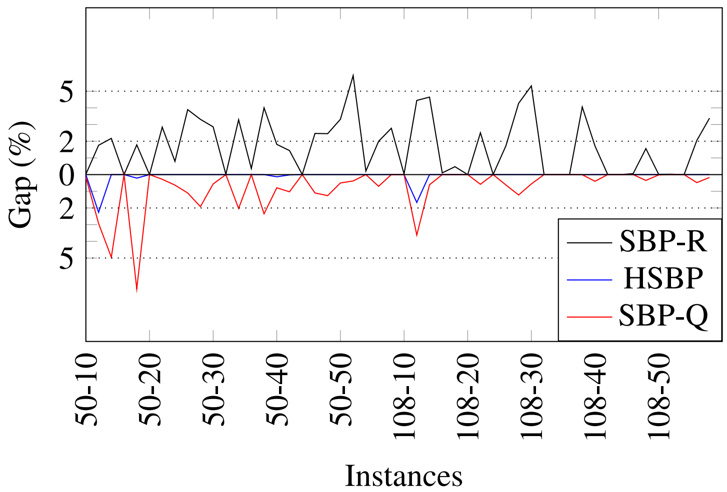

Figure 5.10: Gaps to optimality

```
SBP-R upper bound :  350566.5
Iteration 1:
SBP-Q : 338074.4 ; (166,67) (164,529) (165,1300) (366,22) (366,1261) (367,2557)
SBP-P : 349989.03 ; (166,'55') (164,529) (165,'1182') (366,22) (366,1261) (367,'546')
Iteration 2:
SBP-Q : 349989.03 ; ('165',55) ('166',529) (165,1182) (366,22) (366,1261) (367,546)
SBP-P : 349991.0 ; (165,'67') (166,'518') (165,'1181') (366,22) (366,1261) (367,546)
Iteration 3:
SBP-Q : 349991.0 ; (165,67) (166,518) ('166',1181) (366,22) (366,1261) (367,546)
SBP-P : 350013.03 ; (165,67) (166,'529') (166,'1170') (366,22) (366,1261) (367,546)
Iteration 4:
SBP-Q : 350013.03 ; ('166',67) (166,529) (166,1170) (366,22) (366,1261) (367,546)
SBP-P : 350013.03 ; (166,67) (166,529) (166,1170) (366,22) (366,1261) (367,546)
```

Table 5.3: Iterations of HSBP

used is again very time consuming compared to others. This can be explained by the number of candidates to consider and the numerous values provided by the cumulative quantity function. It can be noticed that the gap of the upper bound provided by SBP-R is smaller than when considering two generators. Also, solving HSBP and HSBP^5 seems to improve quite significantly the feasible solution of SBP found by solving SBP-Q. Although the solution time of HSBP is small, it is much larger than SBP-Q. This is due to the exponential number of candidates considered in the heuristic used to solve SBP-P and its cumulative quantity function and the number of iterations of HSBP. This number of iterations of HSBP will be presented in results on larger instances.

Instance		SBP-R		SBP-S		HSBP^5		HSBP		SBP-Q	
J	$\|S\|$	gap	time	z^*	time	gap	time	gap	time	gap	time
3	10	0.29	0.02	399931	7.87	0.0	0.33	1.84	0.04	2.28	0.0
3	20	0.79	0.17	397459	54.11	0.0	1.1	0.64	0.2	2.24	0.01
3	30	0.1	0.48	375649	137.02	0.26	1.42	0.29	0.26	0.29	0.02
3	40	0.96	1.56	419294	352.46	0.19	6.01	0.36	0.57	1.41	0.03
3	50	0.29	2.9	403834	613.76	0.49	8.91	0.87	0.84	1.87	0.04
4	10	0.16	0.02	407724	65.01	0.06	0.37	1.12	0.04	1.4	0.0
4	20	0.47	0.17	400995	434.05	0.01	1.61	0.02	0.48	1.99	0.02
4	30	0.22	0.46	374344	1095.93	0.27	3.79	0.36	0.48	0.51	0.03
4	40	0.71	4.14	431630	2715.63	0.93	6.8	1.27	0.78	1.72	0.05
4	50	0.23	8.23	401999	4477.54	0.3	31.8	0.8	4.51	1.48	0.06

Table 5.4: Numerical results with 3 and 4 generators and $|J^c| = 108$

5.6.3 Scenario and generator influence

The complexity analysis of algorithms used to solve SBP-R and SBP-Q shows the solution time mainly depends on the number of scenarios and generators of the GC and the competitors. Recall, the complexity of SBP-R and SBP-Q are respectively $O(n|S|^2|G^c|\log n + n|S|^3|G^c|^2)$ and $O(n|S|2^m \log n + n|S|2^{2m} \log m)$ counting preprocessing and computation. The impact of $|S|$ and m on the computation time is analysed in this section.

Table 5.5 and 5.6 analyse the impact of the number of scenarios and generators on the computation time. The gaps are the relative gaps with the upper bound provided by SBP-R and not gaps to optimality. Columns c. gap provide the closed gap of the solution of HSBP, respectively HSBP^k, compared to the solution of SBP-Q, respectively HSBP. The solution time of HSBP does not include the solution time of SBP-R. Column iter. of HSBP gives the number of iteration of the algorithm, that is the number of times SBP-Q and SBP-P are solved before termination of the heuristic.

Instance	SBP-R		HSBP^5		HSBP				SBP-Q	
$\|S\|$	z_R^*	time	gap	c. gap	gap	c. gap	iter.	time	gap	time
25	361736	0.22	0.49	29.52	0.88	48.05	3.2	1.97	1.8	0.18
50	380260	1.68	0.59	37.79	1.31	34.51	2.8	4.89	2.1	0.47
100	318915	14.73	0.13	25.79	0.28	55.15	3.2	16.73	1.03	1.06
150	305972	47.14	0.08	30.12	0.6	22.9	2.4	12.28	0.77	1.56
200	295788	103.86	0.21	8.81	0.26	12.32	2.8	39.85	0.49	2.13

Table 5.5: Impact of number of scenarios, $|G| = 6, |G^c| = 108$

Instance	SBP-R		HSBP^5		HSBP				SBP-Q	
$\|J\|$	z_R^*	time	gap	c. gap	gap	c. gap	iter.	time	gap	time
2	392752	1.73	1.74	0.0	1.74	10.83	2.0	0.68	1.87	0.03
4	402934	1.74	0.53	20.64	1.02	18.98	2.0	0.65	1.7	0.06
6	380260	1.64	0.47	42.19	1.31	34.51	2.8	4.97	2.1	0.59
8	380288	1.71	0.24	48.38	0.55	25.98	3.6	25.27	0.7	4.94
10	383738	1.82	0.38	35.41	0.81	32.77	3.2	285.63	1.25	65.75

Table 5.6: Impact of number of generators, $|S| = 50, |G^c| = 108$

As one could expect by observing the complexity, the solution time increases more than linearly for SBP-R and linearly for SBP-Q as the number of scenario increases. Increasing the number of generators has no impact on SBP-R but increases exponentially the solution time of SBP-Q. Figure 5.11 illustrates the relative gap to the upper bound provided by solving SBP-R of all instances in Tables 5.5 and 5.6. The heuristic HSBP^5 tends to improve quite significantly the solution provided by HSBP with maximum capacities for all generators at the first run of SBP-Q. The relative gap of the feasible solution of HSBP with the upper bound is generally under 1%. Using 5 random quantities near the solution of HSBP for HSBP^5 closes the gap to the upper bound of about 30%. These results also illustrate the quality of the upper bound provided by SBP-R as the gap with the best solution found for SBP is on average at most at 0.5%. This gap tends to decrease as m and $|S|$ increase illustrating the efficiency of this heuristic approach in terms of solution quality.

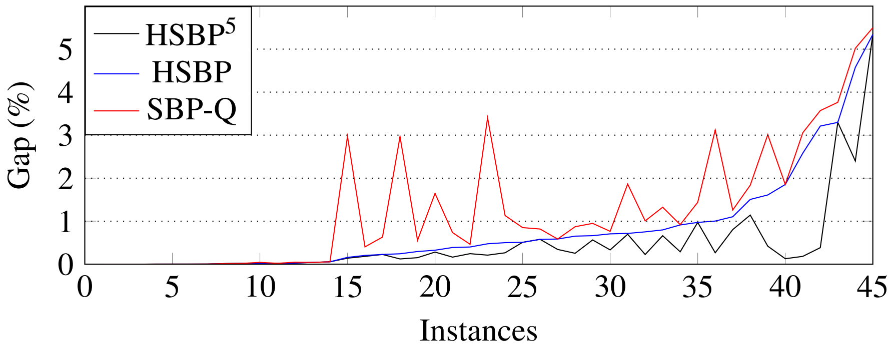

Figure 5.11: Gaps to z_R^*

5.7 Conclusion

The dynamic programming approach presented in Section 5.4 is used to solve many variants of SBP, providing feasible solutions for SBP close to optimality with a good upper bound found in polynomial time. Problem SBP has been shown to be NP-hard, just as SBP-Q, but it was also shown that the difficulty of SBP-Q lies in the number of generators of the GC as the algorithm proposed is exponential in their number.

Problem SBP-Q has been solved to optimality much faster than in previous studies [Fampa et al., 2008, Fampa and Pimentel, 2015, 2017]. When a large number of generators is considered, problem SBP-R can provide a good quality upper bound in limited time. All proposed methods seem quite resistant when increasing the number of scenarios. As the heuristic method is less resistant when increasing the number of generators, an alternative is to consider virtual power plants as Pandžić et al. [2013] which use aggregation of generators into a single bids in order to reduce their number in the bidding procedure.

As HSBP^5 improves significantly the solution of HSBP by generating random initial quantities based on an initial solution, a local search mechanism could be used in future research in order to improve the initial quantities used when iterating HSBP.

Chapter 6

Bidding in Price Coupled Regions

6.1 Introduction

In this chapter we study the problem of bidding in day-ahead markets for a GC as in the previous chapter but under a different approach. In Chapter 5, importance was given at considering uncertainty in bids of competitor GCs and a strong hypothesis of linear production cost was made. In this chapter, a deterministic bidding approach is used, integrating accurate production costs and market mechanisms by studying the *Bidding Problem integrating Unit Commitment* (BPUC).

This problem is studied under a price-maker approach. The PCR, introduced in Section 2.3 coupling several day-ahead markets through a transmission network is considered in the market mechanism. An effect of coupling day-ahead markets is the presence of possible local spot prices. Bids from different areas and transmission constraints influence local spot prices making it harder for GCs to estimate their profit when bidding. Considering the bidding information of competitors as known is a strong hypothesis but methods have been presented for such estimation through statistics [Morales et al., 2014] or machine learning [Chen et al., 2018].

When considering bidding problems under strong assumptions as in price-maker formulations, it is of great importance to try to keep track of their impact on the results. An analysis of the challenges introduced by PCR on bidding problems shall be performed in this chapter.

Some authors have studied coupling day-ahead markets. Transmission constraints make the models very complex. To compensate the difficulty added by the transmission network, Ruiz and Conejo [2009] consider a UC problem composed of only ramping up and down constraints, avoiding start-up and shut-down cost and Kardakos et al. [2014] consider that local demands are fixed in advance and no ramping constraints in the UC model are used. In this chapter, a full UC formulation as presented in Section 2.2 is used to model production costs and only capacity constraints are considered for the transmission network.

The BPUC is presented as bi-level problem and reformulated into an MPEC by introducing constraints representing a market equilibrium based on complementarity constraints arising from KKT conditions. These constraints are linearized and tightened by using an extended formulation describing the spot prices by using *special ordered sets of type 1* (SOS) [Beale and Forrest, 1976]. The equilibrium constraints presented are independent of the first level constraints and can be used when bidding at marginal costs or even in bidding problems for retailers.

Two heuristic methods are proposed to solve large size instances in moderate CPU times. The properties of the primal and dual formulations of the TSO are analyzed in order to derive an Iterative Aggregation-Disaggregation algorithm (IAD) [Rogers et al., 1991]. The SOS constraints are also used to run a SOS-narrowing heuristic that narrows the values of the spot prices during the branch and bound procedure in order to limit the size of the branching tree. This SOS-narrowing heuristic is generic and can be adapted to any formulation containing SOS of type 1.

Numerical results put into highlight the quality of the formulation and solution methods developed in this chapter. An analysis of different market mechanisms is provided by considering different regulations for bidding prices or the impact of the transmission network on the problem. These results illustrate the importance of considering an accurate bidding mechanism if reliable results are desired.

This chapter is organized as follows. Section 6.2 provides a definition of the BPUC problem along with a bilevel formulation and a single level reformulation as a MPEC. An extended linearized formulation obtained by discretizing the spot prices is presented in Section 6.3. The MILP formulation is based on a discretization of possible bidding prices which are studied in Section 6.4 in order to limit the number of binary variables. An adaptation of the proposed formulation of BPUC, imposing to bid at marginal costs of generators, is presented in Section 6.5. Heuristic methods are proposed in Section 6.6, out of which the IAD algorithm in Section 6.6.2 and the SOS-narrowing in Section 6.6.3. Computational results of the proposed formulations and the impact of the capacitated transmission network are analyzed in Section 6.7. A final conclusion is given in Section 6.8.

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6.2 Problem definition

The *Bidding Problem integrating Unit Commitment* (BPUC) is a multi-period price-maker problem where a GC maximizes its profit under unit commitment and market equilibrium constraints in a capacitated transmission network. The GC and competitors propose bids to the TSO who returns the spot prices of each time period and the quantities traded between each actor. A bid is composed of a unit price and a quantity and the TSO can select any proportion of a bid proposed by a producer or retailer. Bids of the competitors are considered as known.

Several day-ahead markets in different regions are represented as a set of nodes N connected through a transmission network and the TSO maximizes the global welfare. The TSO imposes fixed minimum and maximum bidding prices $\underline{\pi}^t$ and $\overline{\pi}^t$ for each time period $t \in T$. It fixes the local spot π_n^t price for each period $t \in T$ and node $n \in N$ such that local seller bids under the spot price are fully bought and bids above are not bought and conversely for local buyer bids. The bids that are bought are said to be *in-the-money*, those that are not are *out-of-the-money* in market regulations [EUPHEMIA, 2016]. All actors have to bid in their day-ahead market and power exchanges are possible through a set of capacitated transmission lines E between the day-ahead markets.

A solution of BPUC is represented by a set of bids $\{(\pi_n^t, p_n^t)\}_{t\in T, n\in N}$ for *Virtual Power Plants* (VPP) of a GC such that p_n^t is the bidden quantity at node n, meaning the GC aims to bid at the spot prices by taking into consideration the reaction of the market to its bid. A VPP is an aggregation of generators as studied by Pandžić et al. [2013], where the bidden production of several generators is aggregated into a single bid. We consider that the full bidden production of the GC must be dispatched to retailers by the TSO.

We denote by P_n the set of feasible solutions of the UC in node n, where $p_n = \{p_n^t\}_{t\in T} \in P_n$ is a vector of resulting quantities offered on market n throughout the time horizon. The cost for producing p_n is denoted by $c(p_n)$. Note that the GC is free to produce more than the quantities offered on the market. In order to guarantee feasibility of BPUC, we assume for each period $t \in T$ and node $n \in N$ that the maximum production capacity of the GC, $\overline{q}_n^t$, is smaller than the total demand in n and that there exists a feasible solution when the GC does not participate in market n.

In the following, we first present a linear formulation for the TSO problem with p_n^t as parameter to provide a model integrating a fixed bidden production by a GC that must be dispatched by the TSO during the market clearing procedure. A bilevel formulation of BPUC is then provided and reformulated into a bilinear MPEC by using complementarity constraints of the TSO problem.

6.2.1 Unit commitment model

The UC is a challenging problem to solve in itself. In previous studies only few UC specific components have been considered in price-maker bidding problems constrained by a transmission network. Only start-up and shut-down costs are considered by Kardakos et al. [2014] while Ruiz and Conejo [2009] integrate only ramping-up and down constraints.

A full UC formulation is composed of several specific constraints such as:

- non-linear production costs,
- startup and shut down costs, introducing decision variables and a non continuous objective function,
- ramping up and down capacities, limiting the variation of production from on time period to another for each generator, linking the time periods of the problem,
- minimum up and down times, forcing generators to be turned on or off for a minimum time period, linking again the time periods.

Even production costs cannot be modelled through linear expressions and are often approximated through linearization techniques.

In the present chapter, we propose general market equilibrium constraints for a GC bidding in day-ahead markets for which any generic formulation of a UC can be integrated. In the computational experiments, a full state-of-the-art deterministic UC formulation presented by Ostrowski et al. [2012] is used and is provided in Appendix A.

6.2.2 Market equilibrium problem

The actors in coupled day-ahead markets are divided into buyers B and sellers S, partitioned into sets B_n and S_n, $n \in N$ of buyers and sellers by node. Every actor bids in its local day-ahead market. Each buyer $b \in B$ defines a bid (π_b^t, q_b^t) composed of a price π_b^t and a strictly positive quantity q_b^t. When clearing the market, the TSO buys a proportion x_b^t of bid $b \in B$ at period t. The same notations apply for sellers $s \in S$. The transmission network is represented by a graph (N, E), where a maximum capacity $C_{nm}^{max} > 0$ is associated with each edge nm. The set of neighbors of a node $n \in N$ is denoted Θ_n. Set A is the set of arcs obtained by replacing each edge in E by two arcs in opposite directions. The flow $f_{nm}^t, nm \in A, t \in T$ corresponds to the flow from node n to node m at period t. The objective of the TSO is to maximize the global welfare of all actors.

In a day-ahead market without transmission constraints and accepting only step bids composed of a unit price and a quantity, the market clearing procedure can be performed by searching the intersection between the aggregated production and demand curves as previously illustrated in Figure 2.5.

The marked equilibrium problem considered in this chapter is a simplified version of the model proposed by Madani and Van Vyve [2015] considering only step bids. As the GC intents to bid at the spot price and sell the integrality of the quantity offered on the market, we consider that the full production is dispatched to retailers by the TSO. The market equilibrium problem is decomposable by time period. Given the quantities p_n^t offered by the GC in period t, it is formulated as follows:

$$
\begin{aligned}
(ME^t)\ \max \quad & \sum_{n\in N}\Big(\sum_{b\in B_n} \pi_b^t q_b^t x_b^t - \sum_{s\in S_n} \pi_s^t q_s^t x_s^t\Big) && && (6.1a)\\
\text{s.t.} \quad & \sum_{b\in B_n} q_b^t x_b^t - \sum_{s\in S_n} q_s^t x_s^t + \sum_{m\in\Theta_n} (f_{nm}^t - f_{mn}^t) = p_n^t && n\in N \quad (\pi_n^t) && (6.1b)\\
& 0 \le x_b^t \le 1 && n\in N, b\in B_n \quad (y_b^t) && (6.1c)\\
& 0 \le x_s^t \le 1 && n\in N, s\in S_n \quad (y_s^t) && (6.1d)\\
& 0 \le f_{nm}^t \le C_{nm}^{max} && nm\in A\ (r_{nm}^t) && (6.1e)
\end{aligned}
$$

where dual variables are indicated next to constraints. The objective function (6.1a) corresponds to the global welfare. Constraints (6.1b) are the balance constraints at each node, imposing the production offered by the GC is bought. Constraints (6.1c)-(6.1e) are bounds on variables. A noticeable difference with a market without transmission constraints is that the TSO may be limited in its choice of bids because of transmission constraints. Note that under the proposed assumptions, ME^t admits a feasible solution for each time period for any production of the GC.

A market equilibrium corresponds to an optimal solution of ME^t. The spot price in a period t and market n is given by the optimal value of dual variable π_n^t [Baker and Taylor, 1979, Balachandran and Ramakrishnan, 1996].

The dual of ME^t for period t is given by:

$$
\begin{aligned}
(MED^t)\ \min \quad & \sum_{n\in N}\Big(p_n^t \pi_n^t + \sum_{b\in B_n} y_b^t + \sum_{s\in S_n} y_s^t + \sum_{m\in\Theta_n} C_{nm}^{max} r_{nm}^t\Big) && && (6.2a)\\
\text{s.t.} \quad & q_b^t \pi_n^t + y_b^t \ge \pi_b^t q_b^t && n\in N, b\in B_n (x_b^t) && (6.2b)\\
& -q_s^t \pi_n^t + y_s^t \ge -\pi_s^t q_s^t && n\in N, s\in S_n (x_s^t) && (6.2c)\\
& \pi_n^t - \pi_m^t + r_{nm}^t \ge 0 && nm\in A (f_{nm}^t) && (6.2d)\\
& y_b^t, y_s^t, r_{nm}^t \ge 0 && && (6.2e)
\end{aligned}
$$

Variables y_b^t and y_s^t represent the welfare obtained from a bid. Some observations can be made from the primal and dual of the TSO about the values of variables in a market equilibrium. Let us consider a market equilibrium and two adjacent nodes n and m. If $r_{nm}^t > 0$, then $f_{nm}^t = C_{nm}^{max}$ by complementarity constraints as r_{nm}^t is the dual variable of (6.1e). Variable r_{nm}^t only appears in (6.2d) in the constraints of the dual and must be minimized in the objective function (6.2a), thus $r_{nm}^t = \pi_m^t - \pi_n^t$ if $\pi_m^t - \pi_n^t \geq 0$. Variables r_{nm}^t represent the spot price difference between nodes m and n if this difference is positive and 0 otherwise. Constraint (6.2d) can be strengthened as follow as at most one term out of r_{nm}^t and r_{mn}^t is positive:

$$\pi_n^t - \pi_m^t + r_{nm}^t - r_{mn}^t = 0 \tag{6.3}$$

Two nodes n and m can have different spot prices if and only if $|f_{nm}^t - f_{mn}^t| = C_{nm}^{max}$. Furthermore, if $r_{nm}^t > 0$, line $nm \in A$ is said to be *saturated*. In this case, $\pi_n^t < \pi_m^t$, meaning that when considering two nodes linked by a transmission line, the exporting one has the lowest spot price.

When considering transmission constraints, the market clearing procedure is more complex than finding the intersection of two curves. For instance, if the production bids are very cheap in a bidding area n, the TSO might not be able to buy all of them because of the limited capacity of transmission lines potentially creating different spot prices in different bidding areas. Figure 6.1 illustrates the impact of the transmission network on a given time period t for the following bidding data:

- $N = \{1,2\}; E = \{(1,2)\}$
- $C_{1,2}^{max} = 3\ GWh$
- The GC is bidding in node 1
- $B_1 = \{(80,0.5),(75,0.5),(60,1),(37,0.5),(25,0.5)\}$
- $S_1 = \{(10,1),(20,1),(30,1.5),(35,0.5),(40,0.5)\}$
- $B_2 = \{(90,1),(70,1.5),(63,0.5),(58,0.5),(50,1),(43,0.6),(41,0.4)\}$
- $S_2 = \{(25,1),(33,1),(38,0.5),(47,1),(52,1.5)\}$

Bids in node 1 and node 2 are represented respectively in blue, and red. Demand and production bids are represented respectively with a full line and dashed line. The upper graphic represents the aggregated demand and production curves by node without linking them with a transmission network and the resulting market equilibriums. The spot price in node 1 is 30 €/MW and a

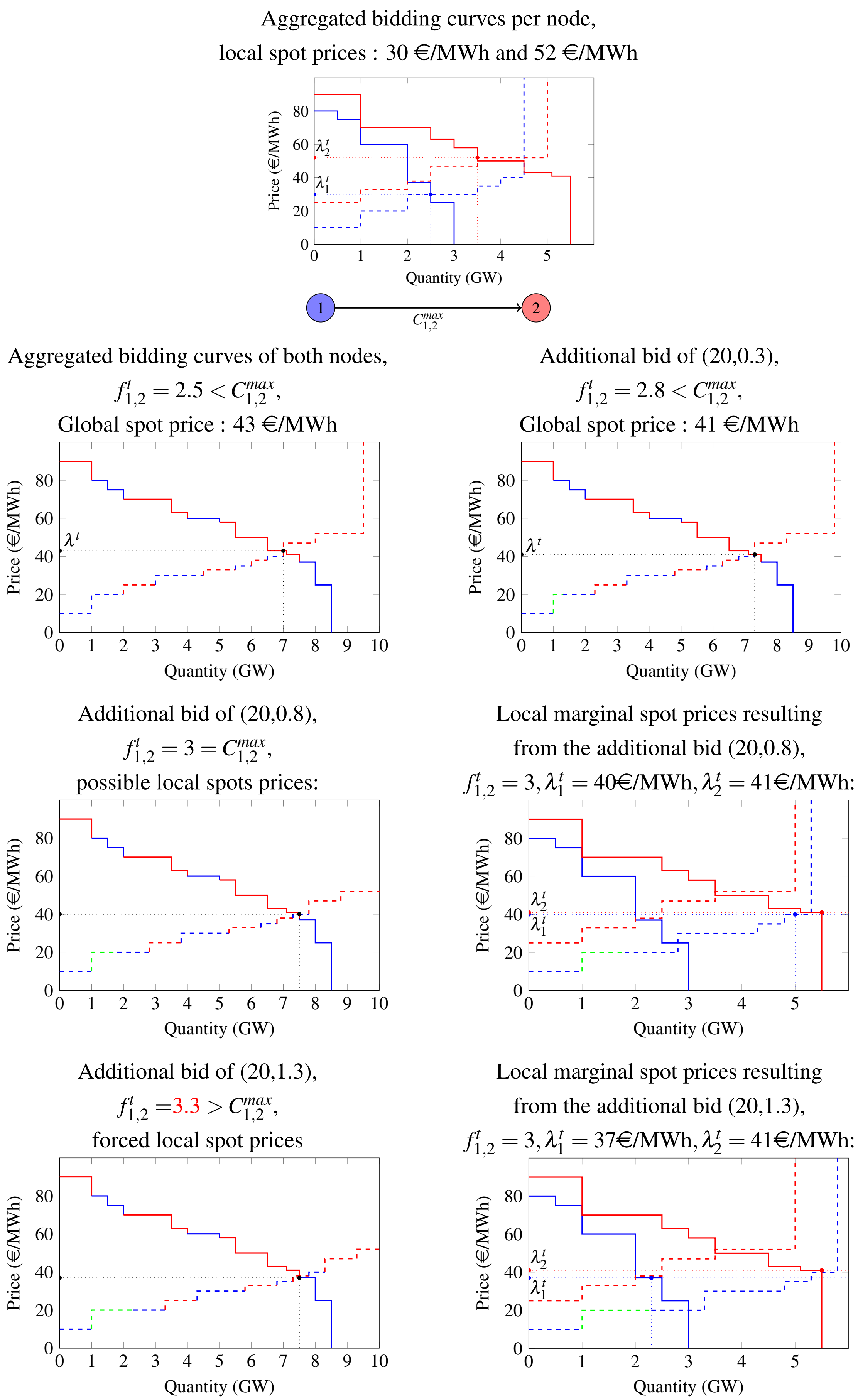


Figure 6.1: Market equilibrium examples

spot price in node 2 is 52 €/MW. The upper-left graphic represents the aggregated curves considering both nodes linked by the transmission network. The resulting spot price is 43 €/MW in both nodes. A demand of 2GW and a production of 4.5 GW bought in node 1. The exceeding production of 2.5 GW is sent to node 2 to complete the local demand, respecting the capacity of the transmission network. The upper-right graphic represents the aggregated curves considering both nodes and an additional production bid (20,0.3) placed in node 1 and represented in green. The exceeding production bought in node 1 is of 2.8 GW and can be sent through the transmission network to node 2, resulting in a market equilibrium with a global spot price of 41 €/MW. The extra bid is fully bought and the demand bid (41,0.5) in node 1 is now only partially bought. The middle-left graphic consider an additional production bid of (20,0.8), reaching the maximum transmission capacity of the transmission line, breaking the obligation of having equal spot prices in nodes 1 and 2. As node 1 is exporting, the spot price in node 1 is smaller or equal than in node 2. The resulting local spot prices illustrated in the middle-right figure are of 40 €/MW in node 1 and of 41 €/MW in node 2. In both nodes, all production bids under the local spot price are fully bought as all demand bids are above the spot price. Bids at the spot price are partially bought up to the dot on the corresponding curve. The bottom-left graphic represents the aggregated curves with an extra bid (20,1.3) placed in node 1. The exceeding production in node 1 is now of 3.3 GW, which exceeds strictly the capacity of the transmission network. The capacity limiting the objective value of the TSO, variable $r^t_{1,2} > 0$ and the spot price cannot be equal in both nodes. The bottom-right graphic represents the market equilibrium obtained with the additional bid. As without the additional bid, the spot price in node 2 stays at 41 €/MW, but falls to 37 €/MW in node 1. Notice that once the transmission line from 1 to 2 is at its maximum capacity, the spot price in node 2 cannot decrease when increasing the production bidden in node 1. This can play at the advantage of the GC if some production is bidden in node 2 and can be sold at a higher price than if all bids are aggregated in a single bidding area.

In a market equilibrium, connected nodes having the same spot price at a given time period are defined as a *group*. At each time period t, the nodes N are partitioned into a set of groups $\mathscr{G}^t$ in which $r^t_{nm} = 0$ for all $n,m \in G, G \in \mathscr{G}^t$.

Given the production of competitors, the spot price can be bounded considering the offer of the GC on the market.

Lemma 23. *Consider a set of quantities p^t_n for the market equilibrium problem in period t, the resulting market equilibrium at period t with groups $\mathscr{G}^t$ and the spot prices π^t_n. Increasing a quantity p^t_n cannot increase the spot price in any group.*

Proof. Consider a bid $s \in S_n$ in a group $G \in \mathscr{G}^t$ such that increasing q^t_s of a quantity q in period

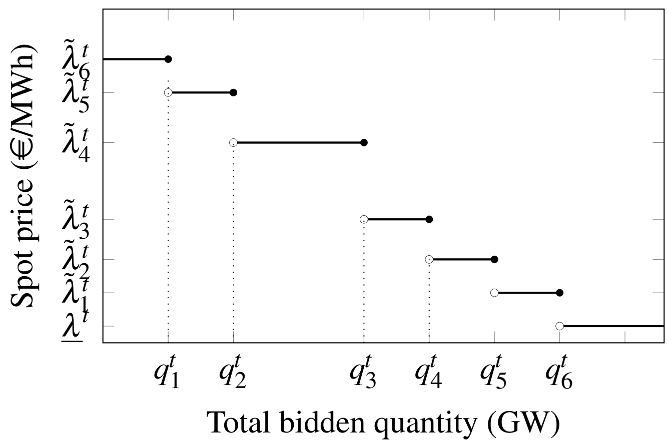


Figure 6.2: Spot price without transmission network

t increases the spot price in node n' in group $G' \in \mathscr{G}^t$. If $\pi^t_s > \pi^t_n$, then the extra quantity q is not bought and the spot prices remain unchanged. Otherwise, $\pi^t_s = \pi^t_n$ and the spot price $\pi^t_{G'}$ can increase only if after modifying the bidden quantities, quantity q is fully bought in addition to a positive quantity q' not previously sold in G. Buying q' is done if and only if it increases the global welfare. If so, the quantity q' bought in G' could have been bought without increasing q^t_s contradicting the initial market equilibrium hypothesis. □

It follows from Lemma 23 and the assumption that the spot price is maximal, that π^t_n can be expressed as an upper step-wise decreasing function in p^t_m for all $m \in N$ as illustrated in Figure 6.2. Values q^t_i are quantities at which the spot price decreases. These values are fixed parameters when considering a single day-ahead market as shown by de la Torre et al. [2002] but vary depending on all quantities $p^t_n, n \in N$ when considering coupled markets.

6.2.3 Market equilibrium constraints

BPUC is modeled as a bilevel problem where GC is the leader maximizing its profit and the TSO is the follower:

$$(BPUC-BL) \max_{p^t_n} \quad \sum_{n\in N}\left(\sum_{t\in T}\pi^t_n p^t_n\right) - c(p_n) \tag{6.4a}$$

$$\text{s.t.} \quad p_n \in P_n \qquad n \in N \tag{6.4b}$$

$$\forall t \in T \min_{\pi^t, y^t_b, y^t_s, r^t_{nm}} \quad \sum_{n\in N}(p^t_n\pi^t_n + \sum_{b\in B_n} y^t_b + \sum_{s\in S_n} y^t_s + \sum_{m\in\Theta_n} C^{max}_{nm} r^t_{nm}) \tag{6.4c}$$

$$\text{s.t.} \quad q^t_b\pi^t_n + y^t_b \geq \pi^t_b q^t_b \qquad n \in N, b \in B_n \tag{6.4d}$$

$$-q^t_s\pi^t_n + y^t_s \geq -\pi^t_s q^t_s \qquad n \in N, s \in S_n \tag{6.4e}$$

$$\pi_n^t - \pi_m^t + r_{nm}^t - r_{mn}^t = 0 \qquad nm \in A \qquad (6.4f)$$

$$y_b^t, y_s^t, r_{nm}^t \geq 0 \qquad (6.4g)$$

The leader in the first level controls the quantities p_n^t produced and offered on the market, as solution of problem (6.4b). The follower in the second level controls the remaining variables, among which, the spot prices π_n^t, and is formulated by MED^t for each time period. Under the hypotheses considered in the previous section regarding the bids of competitors, the optimal value of the second level is well defined for any production levels p_n^t as the problem is a feasible linear program.

Assuming that $c(p_n)$ is an increasing lower semi-continuous function in p_n, the optimal value of BPUC is well defined. Classical unit commitment models respect this assumption since i) production costs are generally quadratic, ii) discontinuities appear with start-up costs and iii) the spot prices are upper semi-continuous functions. The objective function of (6.4) is thus an upper semi-continuous function in p_n^t for all $n \in N$ admitting a global maximum.

Several optimal solutions can exist at the second level if several bids are made at the spot price and the TSO can choose any subset of these bids to satisfy the demand. Furthermore, being the dual variables of the balance constraints, the spot prices can be degenerated, i.e. several different values are possible in an optimal solution. Therefore, the second level of the proposed bilevel formulation is not a point-to-point map for values of variables p_n^t and an optimistic assumption is made. This assumption considers that the follower always chooses the best solution for the leader among the set of optimal solutions of the second level. This yields the following properties for BPUC-BL:

- the GC has priority over the competitors when bidding at the same price, which can practically be ensured by decreasing the optimal bidding prices by a small amount,
- the TSO maximizes spot prices in optimal solutions of market equilibriums, which satisfies the assumption made on the TSO problem.

The second level is an equilibrium problem where bids are chosen in order to maximize the global welfare. Reformulating a bilevel model of this type into a single level problem can be done by finding a set of constraints representing the solution space of the second level, that is, all possible market equilibriums, in order to obtain an MPEC [Colson et al., 2007].

The follower problem can be reformulated into a set of equilibrium constraints by using complementarity constraints of the TSO primal and dual formulations:

$$x_b^t \left(q_b^t \pi_n^t + y_b^t - \pi_b^t q_b^t \right) = 0 \qquad t \in T, n \in N, b \in B_n \qquad (6.5a)$$

$$x_s^t\left(-q_s^t\pi_n^t + y_s^t + \pi_s^t q_s^t\right) = 0 \qquad t \in T, n \in N, s \in S_n \qquad (6.5b)$$
$$y_b^t\left(1 - x_b^t\right) = 0 \qquad t \in T, n \in N, b \in B_n \qquad (6.5c)$$
$$y_s^t\left(1 - x_s^t\right) = 0 \qquad t \in T, n \in N, s \in S_n \qquad (6.5d)$$
$$(C_{nm}^{max} - f_{nm}^t)r_{nm}^t = 0 \qquad t \in T, nm \in A \qquad (6.5e)$$

The bilinear terms $x_b^t y_b^t$ and $x_s^t y_s^t$ can be replaced by y_b^t and y_s^t by (6.5c) and (6.5d). We can then rewrite constraints (6.5a) and (6.5b) as:

$$y_b^t = -q_b^t\pi_n^t x_b^t + \pi_b^t q_b^t x_b^t \qquad t \in T, n \in N, b \in B_n \qquad (6.6a)$$
$$y_s^t = q_s^t\pi_n^t x_s^t - \pi_s^t q_s^t x_s^t \qquad t \in T, n \in N, s \in S_n \qquad (6.6b)$$

Let us replace r_{nm}^t in (6.5e) by $\pi_m^t - \pi_n^t$:

$$C_{nm}^{max} r_{nm}^t = f_{nm}^t(\pi_m^t - \pi_n^t) \qquad t \in T, nm \in A \qquad (6.7)$$

Lemma 24. *Constraints (6.7) are valid for BPUC-BL.*

Proof. Constraints (6.5e) can be rewritten as:

$$C_{nm}^{max} r_{nm}^t = f_{nm}^t r_{nm}^t$$

As previously mentioned r_{nm}^t represents the spot price difference between nodes m and n, $\pi_m^t - \pi_n^t$, if this difference is positive and 0 otherwise. Thus,

- if $r_{nm}^t = 0$, then both sides equal 0,
- if $r_{nm}^t > 0$, then $r_{nm}^t = \pi_m^t - \pi_n^t$.

□

In this chapter constraints (6.6) and (6.7) are called the reduced complementarity constraints. Let P^1 be the solution space of constraints of ME^t (6.1) and MED^t (6.2) combined with the complementarity constraints (6.5), which defines the optimal solution space of ME^t, and P^2 be the solution space of the same constraints ME^t (6.1) and MED^t (6.2) combined with the reduced complementarity constraints (6.6) and (6.7).

Lemma 25. *The solution spaces defined by P^1 and P^2 are equal.*

Proof.
- $P^1 \subseteq P^2$: consider a solution $(x, y, \pi, r, f) \in P^1$. This solution satisfies (6.7) as described in Lemma 24. The solution also satisfies (6.6) as these constraints are derived from (6.5) which are satisfied by (x, y, π, r, f).

- $P^2 \subseteq P^1$: consider a solution $(x,y,\pi,r,f) \in P^2$. By substituting y_b^t in (6.2b) using (6.6a), we obtain:

$$(-\pi_n^t + \pi_b^t)q_b^t x_b^t \geq (-\pi_n^t + \pi_b^t)q_b^t \tag{6.8}$$

  If

  * $\pi_b^t > \pi_n^t$, constraints (6.1c) and (6.8) lead to $x_b^t = 1$,
  * $\pi_b^t = \pi_n^t$, constraint (6.6a) implies that $y_b^t = 0$,
  * $\pi_b^t < \pi_n^t$, constraint (6.6a) implies that $x_b^t = 0$ and $y_b^t = 0$ as these variables are positives.

  Thus for any $t \in T, b \in B$, either $y_b^t = 0$ or $x_b^t = 1$, and constraints (6.5c) are satisfied by (x,y,π,r,f). Similarly, constraints (6.5d) are implied by (6.1d), (6.8) and (6.6b).
  As constraints (6.5c) and (6.5d) are valid for P^2, y_b^t, y_s^t can be replaced by $x_b^t y_b^t$ and $x_s^t y_s^t$ in constraints (6.6a) and (6.6b) respectively, leading to constraints (6.5a) and (6.5b).
  Constraint (6.5e) is also satisfied by (x,y,π,r,f):

  – if $r_{nm}^t = 0$, then this constraint is trivially satisfied,
  – if $r_{nm}^t > 0$, then $r_{nm}^t = \pi_m^t - \pi_n^t$. Constraints (6.5e) are equivalent to constraints (6.7) by substitution.

□

The reduced complementarity constraints (6.6)-(6.7) of Lemma 25 combined with those of the primal and dual of the TSO can replace the second level problem of BPUC-BL. Variables y_b^t, y_s^t can be substituted by using (6.6a) and (6.6b). This results in the following MPEC:

$$(BPUC-MPEC) \ \max \quad \sum_{n\in N}\left(\sum_{t\in T}\pi_n^t p_n^t\right) - c(p_n) \tag{6.9a}$$

$$\text{s.t.} \quad p_n \in P_n \qquad n \in N \tag{6.9b}$$

$$\sum_{b\in B_n} q_b^t x_b^t - \sum_{s\in S_n} q_s^t x_s^t + \sum_{m\in\Theta_n}(f_{nm}^t - f_{mn}^t) = p_n^t \qquad t\in T, n\in N \tag{6.9c}$$

$$(\pi_n^t - \pi_b^t)(1-x_b^t) \geq 0 \qquad t\in T, n\in N, b\in B_n \tag{6.9d}$$

$$(-\pi_n^t + \pi_s^t)(1-x_s^t) \geq 0 \qquad t\in T, n\in N, s\in S_n \tag{6.9e}$$

$$(-\pi_n^t + \pi_b^t)x_b^t \geq 0 \qquad t\in T, n\in N, b\in B_n \tag{6.9f}$$

$$(\pi_n^t - \pi_s^t)x_s^t \geq 0 \qquad t\in T, n\in N, s\in S_n \tag{6.9g}$$

$$\pi_n^t - \pi_m^t + r_{nm}^t - r_{mn}^t = 0 \qquad t\in T, nm\in A \tag{6.9h}$$

$$C_{nm}^{max} r_{nm}^t = f_{nm}^t(\pi_m^t - \pi_n^t) \qquad t \in T, nm \in A \qquad (6.9i)$$
$$0 \le x_b^t \le 1 \qquad t \in T, n \in N, b \in B_n \qquad (6.9j)$$
$$0 \le x_s^t \le 1 \qquad t \in T, n \in N, s \in S_n \qquad (6.9k)$$
$$0 \le f_{nm}^t \le C_{nm}^{max} \qquad t \in T, nm \in A \qquad (6.9l)$$
$$r_{nm}^t \ge 0 \qquad t \in T, nm \in A \qquad (6.9m)$$

Constraints (6.9c)-(6.9m) define market equilibrium constraints for all $t \in T$. Constraints (6.9c) are the balancing contraints, constraints (6.9d)-(6.9g) ensure bids in-the-money are bought and bids out-of-the-money are rejected. Constraints (6.9h)-(6.9i) link the flows with the differences of spot prices between the bidding areas. Note that the equilibrium only depend on bidden quantity at the first level and are valid when considering a retailer bidding in PCR.

6.3 MILP reformulation

BPUC-MPEC contains a continuous bilinear objective and several bilinear constraints where all bilinear terms contain a spot price variable. The linearization of the bilinear terms is challenging since all products involve continuous variables.

Let Λ^t be the ordered set of all bidding prices of competitors at period t over all bidding areas including the minimum and maximum bidding prices $\underline{\pi}^t$ and $\overline{\pi}^t$ allowed by the TSO. Prices in Λ^t are denoted by $\tilde{\lambda}_i^t, i \in I^t$ where I^t is the set of price indices of Λ^t.

Lemma 26. *There exists an optimal solution of BPUC such that $\pi_n^t \in \Lambda^t$ for all $n \in N, t \in T$.*

Proof. Consider an optimal solution with a spot price π_n^t that is not in Λ^t. Increasing π_n^t to $\tilde{\lambda}_i^t = \min\{\lambda \in \Lambda^t | \lambda > \pi_n^t\}$ preserves validity of constraints (6.9d)-(6.9g) and potentially increases the objective value. For all $m \in \Theta_n$:

- if $r_{nm}^t = 0$, π_m^t is increased to $\tilde{\lambda}_i^t$ in order to preserve validity of constraints (6.9h) and (6.9i).
- if $r_{nm}^t > 0$, r_{nm}^t is decreased by the same amount as π_n^t is increased in order to satisfy constraints (6.9h) and (6.9i). Variable r_{nm}^t may become negative in this procedure if $\pi_m^t \in]\pi_n^t, \tilde{\lambda}_i^t[$. In this case, π_m^t is not in Λ^t either and can also be increased to $\tilde{\lambda}_i^t$.

Once the increase of π_n^t is propagated to r_{nm}^t and π_m^t for each adjacent node m, the procedure can be iterated until all spot prices are in Λ^t. □

It follows from Lemma 26 that relevant spot prices values π_n^t in an optimal solution can be discretized to values in Λ^t by using Special Ordered Sets (SOS) of type 1 [Beale and Forrest, 1976]. Consider binary variables $z_{in}^t, i \in I^t$,

$$z_{in}^t = \begin{cases} 1 & \text{if and only if } \pi_n^t = \tilde{\lambda}_i^t \\ 0 & \text{otherwise.} \end{cases}$$

The following constraints restrict variables π_n^t in BPUC-MPEC to values in Λ^t:

$$\sum_{i \in I^t} z_{in}^t = 1 \qquad t \in T, n \in N \tag{6.10a}$$

$$\pi_n^t = \sum_{i \in I^t} \tilde{\lambda}_i^t z_{in}^t \qquad t \in T, n \in N \tag{6.10b}$$

Sets $\{z_{in}^t\}_{i \in I^t}$ for all $t \in T$ and $n \in N$ are SOS of type 1, meaning exactly one variable in each set must be different from 0. Constraints (6.10a)-(6.10b), lead to an extended formulation of BPUC-MPEC where the continuous spot price variables are substituted by binary variables. All products of continuous variables in BPUC-MPEC can be rewritten as the product of a binary and a continuous variable which can easily be linearized. The following variables and inequalities are used for linearization for all $t \in T, n \in N, i \in I^t$:

$$P_{in}^t = z_{in}^t p_n^t \qquad 0 \le P_{in}^t \le \overline{q}_n^t z_{in}^t \qquad P_{in}^t \le p_n^t \qquad P_{in}^t \ge p_n^t - \overline{q}_n^t(1 - z_{in}^t) \tag{6.11a}$$

$$X_{ib}^t = z_{in}^t x_b^t \qquad 0 \le X_{ib}^t \le z_{in}^t \qquad X_{ib}^t \le x_b^t \qquad X_{ib}^t \ge x_b^t + z_{in}^t - 1 \qquad b \in B_n \tag{6.11b}$$

$$X_{is}^t = z_{in}^t x_s^t \qquad 0 \le X_{is}^t \le z_{in}^t \qquad X_{is}^t \le x_s^t \qquad X_{is}^t \ge x_s^t + z_{in}^t - 1 \qquad s \in S_n \tag{6.11c}$$

$$\overline{F}_{inm}^t = z_{in}^t f_{nm}^t \qquad 0 \le \overline{F}_{inm}^t \le C_{nm}^{max} z_{in}^t \qquad \overline{F}_{inm}^t \le f_{nm}^t \qquad \overline{F}_{inm}^t \ge f_{nm}^t - C_{nm}^{max}(1 - z_{in}^t) \qquad m \in \Theta_n \tag{6.11d}$$

$$\underline{F}_{inm}^t = z_{in}^t f_{mn}^t \qquad 0 \le \underline{F}_{inm}^t \le C_{nm}^{max} z_{im}^t \qquad \underline{F}_{inm}^t \le f_{nm}^t \qquad \underline{F}_{inm}^t \ge f_{nm}^t - C_{nm}^{max}(1 - z_{im}^t) \qquad m \in \Theta_n \tag{6.11e}$$

Linearization constraints as (6.11) usually introduce a large LP gap due to the introduction of additional variables that are weakly linked to the initial model. Valid inequalities linking these new variables and the initial variables can tighten the formulation. By multiplying constraint (6.10a) by variables p_n^t, x_b^t, x_s^t and f_{nm}^t respectively we obtain the following constraints:

$$\sum_{i \in I^t} P_{in}^t = p_n^t \qquad t \in T, n \in N \tag{6.12a}$$

$$\sum_{i \in I^t} X_{ib}^t = x_b^t \qquad t \in T, n \in N, b \in B \tag{6.12b}$$

$$\sum_{i \in I^t} X_{is}^t = x_s^t \qquad t \in T, n \in N, s \in S \tag{6.12c}$$

$$\sum_{i \in I^t} \overline{F}^t_{inm} = f^t_{nm} \qquad t \in T, nm \in A \tag{6.12d}$$

$$\sum_{i \in I^t} \underline{F}^t_{inm} = f^t_{nm} \qquad t \in T, nm \in A \tag{6.12e}$$

These constraints illustrate that the new variables are a disaggregation by price of the initial ones. The balance constraint (6.9c) can also be disaggregated by price by multiplying them by the corresponding variables z^t_{in}:

$$\sum_{b \in B_n} q^t_b X^t_{ib} - \sum_{s \in S_n} q^t_s X^t_{is} + \sum_{m \in \Theta_m} (\overline{F}^t_{inm} - \underline{F}^t_{imn}) = P^t_{in} \quad t \in T, n \in N, i \in I^t \tag{6.13}$$

Multiplying constraints by binary variables as done for constraints (6.12) and (6.13) is similar to the RLT procedure proposed by Sherali and Adams [1994] to tighten the linear programming relaxation of model with binary variables. The formulation derived from BPUC-MPEC by adding constraints (6.10), (6.11), (6.12) and (6.13) is denoted BPUC-MILP and is available in Appendix C.

6.4 Price elimination

The discretization of π^t_n through sets I^t is heavy as it introduces $|T|(|B|+|S|)$ binary variables to the formulation in addition to the continuous variables resulting from linearization. The prices to consider at each node can be restricted to a smaller set than Λ^t. We define in this section sets I^t_n containing the indices of prices in Λ^t to consider as possible spot prices in an optimal solution.

Lemma 23 allows to bound the spot prices at each node. The highest, respectively lowest, possible spot prices in an optimal solution are those obtained when solving the TSO problem with the GC bidding no capacity, respectively its full capacity, at each node and period. These bounds on the spot prices are obtained by solving two times MED^t for each time period, first fixing $p^t_n = 0$, secondly fixing $p^t_n = \overline{q}^t_n$. At each node n and time period t, let $\underline{i}^t_n$, respectively $\overline{\imath}^t_n$, be the index in Λ^t of the minimum, respectively maximum, possible spot price. The indices of the spot prices to consider in sets I^t_n can be restricted to $\{\underline{i}^t_n, \dots, \overline{\imath}^t_n\}$.

Indices in sets I^t_n can also be restricted based on potential groups in a market equilibrium. Spot prices can be equal or different between nodes depending on weither they are in the same group or not. Consider an index $i \in \{\underline{i}^t_n, \dots, \overline{\imath}^t_n\}$ such that $\tilde{\lambda}^t_i \in \Lambda^t$ corresponds to a price bidden in a node $m \neq n$. Then the spot price at node n can be equal to $\tilde{\lambda}^t_i$ if and only if it is equal to $\tilde{\lambda}^t_i$ at node m and n and m are in the same group. This allows to eliminate from I^t_n indices $i \in [\underline{i}^t_n, \overline{\imath}^t_n]$

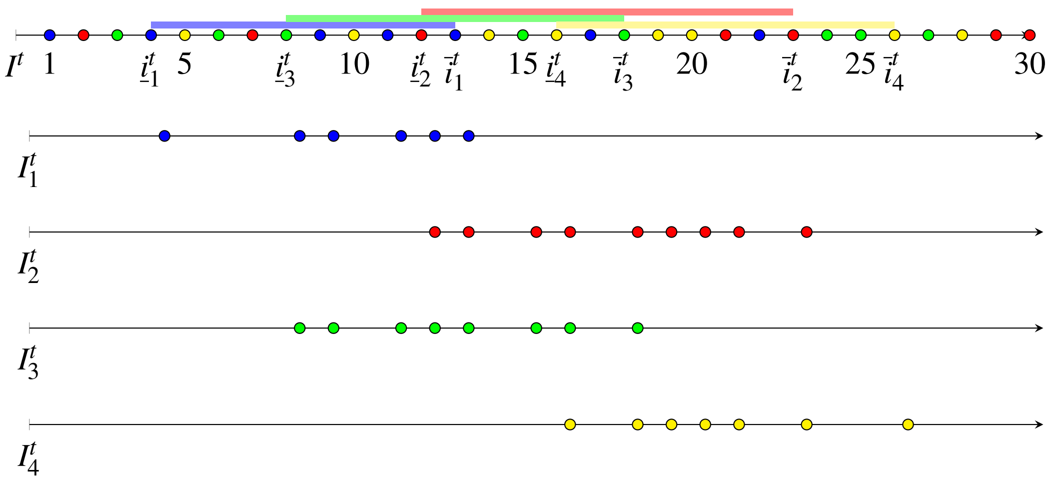


Figure 6.3: Price discretization by node for a given period

such that $\tilde{\lambda}_i^t$ is bidden at a node $m \neq n$ and $i \notin [\underline{i}_m^t, \bar{i}_m^t]$. Let $\tilde{I}_n^t$ be the set of all indices of prices in Λ^t bidden at node n at period t included in $\{\underline{i}_n^t, \ldots, \bar{i}_n^t\}$. Then I_n^t is defined as follow:

$$I_n^t = \bigcup_{m \in N} (\tilde{I}_m^t \cap \{\underline{i}_n^t, \ldots, \bar{i}_n^t\})$$

Figure 6.3 provides an illustration of the potential spot prices by node for a time period considering 30 bidding prices in a network of 4 nodes. All bids are represented by a color associated to a node on the first line. The indices $\underline{i}_n^t$ and $\bar{i}_n^t$ are indicated for each node. The price indices considered in I_n^t are local price indices in $\{\underline{i}_n^t, \ldots, \bar{i}_n^t\}$ in addition to the prices indices bidden in another node m that are in $\{\underline{i}_n^t, \ldots, \bar{i}_n^t\} \cap \{\underline{i}_m^t, \ldots, \bar{i}_m^t\}$. The resulting average number of prices per node is of 7.5. Note that in this example, there does not exist any common spot price for all nodes meaning there are at least two groups in an optimal solution.

When using sets I_n^t, the proportion bought for some bids in B and S in a feasible solution can be trivially fixed. For all bids $b \in B_n$:

- if $\pi_b^t > \tilde{\lambda}_{\bar{i}_n^t}^t$, then $x_b^t = 1$,
- if $\pi_b^t < \tilde{\lambda}_{\underline{i}_n^t}^t$, then $x_b^t = 0$,

and conversely for bids in S.

6.5 Bidding at marginal costs

The literature on price-maker bidding considers two approaches: bid at unconstrained prices [de la Torre et al., 2002, Bakirtzis et al., 2007, Ruiz and Conejo, 2009] or bid at marginal production costs [Dalby, 2017, Kardakos et al., 2014]. The MPEC formulation proposed for BPUC can easily be adapted to bid at marginal production cost in a problem BPUC^M. We consider each generator makes a single bid (π_j, p^t_{jn}) at a fixed marginal production cost π_j for each generator $j \in J_n$ at each time period. Set J_n is composed of the generators at node n.

In BPUC, it is assumed that the GC can sell its full bidden production. The same does not hold for BPUC^M as some generators might have a marginal cost above the spot price and can therefore not sell any production. Constraints (6.9g) rejecting such bids in BPUC-MPEC can be added for the bids of the GC, leading to the following formulation derived from BPUC-MPEC:

$$
\begin{aligned}
(BPUC^M - MPEC)\ \max\quad & \sum_{n\in N}\sum_{j\in J_n}\left(\sum_{t\in T}\pi^t_n p^t_{jn}\right) - c(p_{jn}) && && (6.14a)\\
\text{s.t.}\quad & p_{jn} \in P^j_n && n\in N, j\in J_n && (6.14b)\\
& \sum_{b\in B_n} q^t_b x^t_b - \sum_{s\in S_n} q^t_s x^t_s && t\in T, n\in N && (6.14c)\\
& + \sum_{m\in\Theta_n}(f^t_{nm} - f^t_{mn}) = \sum_{j\in J_n} p^t_{jn} && && \\
& (\pi^t_n - \pi^t_b)(1-x^t_b) \geq 0 && t\in T, n\in N, b\in B_n && (6.14d)\\
& (-\pi^t_n + \pi^t_s)(1-x^t_s) \geq 0 && t\in T, n\in N, s\in S_n && (6.14e)\\
& (-\pi^t_n + \pi^t_b)x^t_b \geq 0 && t\in T, n\in N, b\in B_n && (6.14f)\\
& (\pi^t_n - \pi^t_s)x^t_s \geq 0 && t\in T, n\in N, s\in S_n && (6.14g)\\
& (\pi^t_n - \pi_{jn})p^t_{jn} \geq 0 && t\in T, n\in N, j\in J_n && (6.14h)\\
& \pi^t_n - \pi^t_m + r^t_{nm} \geq 0 && t\in T, nm\in A && (6.14i)\\
& C^{max}_{nm} r^t_{nm} = \pi^t_m f^t_{nm} - \pi^t_n f^t_{nm} && t\in T, nm\in A && (6.14j)\\
& 0 \leq x^t_b \leq 1 && t\in T, n\in N, b\in B_n && (6.14k)\\
& 0 \leq x^t_s \leq 1 && t\in T, n\in N, s\in S_n && (6.14l)\\
& 0 \leq f^t_{nm} \leq C^{max}_{nm} && t\in T, nm\in A && (6.14m)\\
& r^t_{nm} \geq 0 && && (6.14n)
\end{aligned}
$$

where P_n^j is the feasible solution space for generator j in node n and constraint (6.14h) rejects bids from generators with a marginal cost above the spot price. The same reformulation technique used for BPUC-MPEC can be applied by introducing variables $P^t_{ijn} = p^t_{jn} z^t_{in}$. The spot prices are discretized in the same way adding marginal production costs to sets Λ^t. The linearized formulation BPUC^M-MILP is available in Appendix C.

6.6 Heuristic methods

In this section we describe two heuristic solution methods based on the structure of the problem.

6.6.1 Iterative price-taker algorithm

A simple price-taker formulation with estimation of spot prices $\tilde{\lambda}^t_n$ is considered as follow:

$$(BP-PT)\ \max \quad \sum_{n\in N}\sum_{t\in T} \tilde{\lambda}^t_n p^t_n - c(p_n) \tag{6.15a}$$

$$\text{s.t.} \quad p_n \in P_n \tag{6.15b}$$

A feasible solution of a bilevel problem can be computed by iteratively solving a leader and a follower subproblem. In the leader subproblem, variables of the follower are fixed and variables of the leader are optimized and conversely for the follower subproblem. For BPUC, the leader subproblem is BP-PT fixing spot prices $\{\pi^t_n\}_{t\in T, n\in N}$ as in a price-taker formulation and the follower subproblem is MED^t updating the spot prices based on the new new bidden quantities. A feasible solution of BPUC is found with algorithm *BP-start* as follow:

1. Set $p^{t*}_n = p^t_n = p^t_{n,0}$, $t\in T, n\in N$,
2. Initialize $\tilde{\lambda}^t_n$ by solving MED^t for each time period,
3. $z^* = \sum_{n\in N}\sum_{t\in T}\left(\tilde{\lambda}^t_n p^t_{n0}\right) - c(p_{n0})$,
4. Update p^t_n by solving BP-PT,
5. Update $\tilde{\lambda}^t_n$ by solving MED^t for each time period,
6. Set $z = \sum_{n\in N}\sum_{t\in T}\left(\tilde{\lambda}^t_n p^t_n\right) - c(p_n)$
7. If $z > z^*$, set $z^* = z$, $p^{t*}_n = p^t_n$, go to step 4
8. Return z^* and p^{t*}_n

where $p^t_{n,0}$ is an initial bidden quantity and p^{t*}_n is the best bidden quantity found. In the following, BP-start provides an initial solution for BPUC by setting $p^t_{n,0}$ to 0. This algorithm can be

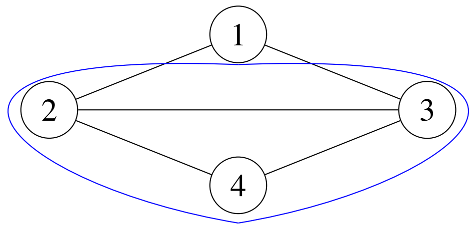


Figure 6.4: Partial aggregation of nodes

adapted to obtain a feasible solution of BPUC^M by setting $p^t_{nj} = 0$ for all generators having a marginal cost above the spot price $\tilde{\lambda}^t_n$ before step 6.

6.6.2 Iterative Aggregation Disaggregation algorithm

In a solution of BPUC, the spot price in a group at a given time period is based on bids within this group, the imported/exported quantities of this group and the production of the GC. The spot price is independent of the structure of the network within this group. As already illustrated in Figure 6.1, a spot price is much simpler to compute when the transmission constraints are not blocking the selection of bids of the TSO. We already presented some conditions under which two nodes cannot be in the same group in Section 6.4 through the spot prices in sets I^t_n. Based on these observations, an efficient clustering of nodes can reduce the difficulty of solving BPUC.

Bids in different nodes of the same group in a market equilibrium can be aggregated before clearing the market leading to the same market equilibrium as illustrated in Figure 6.4 where nodes 2 to 4 are considered in a single group. Aggregating the bids in a group of nodes can reduce significantly the size of the problem by considering fewer spot prices, removing the transmission constraints within groups and reducing the number of lines between groups to consider. In Figure 6.4, aggregating the nodes 2,3 and 4 results in a two nodes network with only two spot prices to establish (rather than four) and a single transmission line whose capacity is the sum of the capacities $C^{max}_{1,2}$ and $C^{max}_{1,3}$. The drawback of performing an aggregation of nodes is the loss of information of the orignal problems that can lead to non-optimal or infeasible solutions. A solution to improve or repair a solution of an aggregated formulation is to retrieve and use some aggregated information in a disaggregation procedure. This scheme corresponds to classical aggregation and disaggregation techniques used to solve large scale problems described by Rogers et al. [1991].

Consider a set of groups over time periods $\mathscr{G} = \bigcup_{t \in T} \mathscr{G}^t$. When aggregating the nodes at period $t \in T$, the transmission network is modified into a graph $(\mathscr{G}^t, E^t)$. Edges between two groups G_1 and G_2 in the original transmission network are aggregated as a single edge G_1G_2

whose capacity is the sum of the capacities of the aggregated edges. Set A^t is the set of arcs corresponding to edges in the graph $(\mathscr{G}^t, E^t)$. Consider the following aggregated formulation of BPUC with groups $\mathscr{G}$ where π^t_G is the spot price in group $G \in \mathscr{G}^t$:

$$
\begin{aligned}
(BPUC-\mathscr{G})\ \max \quad & \sum_{G\in\mathscr{G}^t}\sum_{n\in G}\left(\pi^t_G\sum_{t\in T}p^t_n - c(p_n)\right) && && (6.16a)\\
\text{s.t.} \quad & p_n \in P_n && n\in N && (6.16b)\\
& \sum_{b\in B_G} q^t_b x^t_b - \sum_{s\in S_G} q^t_s x^t_s && t\in T, G\in\mathscr{G}^t && (6.16c)\\
& \quad + \sum_{GG'\in A^t}(f^t_{GG'} - f^t_{G'G}) = \sum_{n\in G} p^t_n && && \\
& (\pi^t_G - \pi^t_b)(1-x^t_b) \geq 0 && t\in T, G\in\mathscr{G}^t, n\in G, b\in B_n && (6.16d)\\
& (-\pi^t_G + \pi^t_s)(1-x^t_s) \geq 0 && t\in T, G\in\mathscr{G}^t, n\in G, s\in S_n && (6.16e)\\
& (-\pi^t_G + \pi^t_b)x^t_b \geq 0 && t\in T, G\in\mathscr{G}^t, n\in G, b\in B_n && (6.16f)\\
& (\pi^t_G - \pi^t_s)x^t_s \geq 0 && t\in T, G\in\mathscr{G}^t, n\in G, s\in S_n && (6.16g)\\
& \pi^t_{G_1} - \pi^t_{G_2} + r^t_{G_1G_2} \geq 0 && t\in T, G_1G_2\in A^t && (6.16h)\\
& C^{max}_{G_1G_2} r^t_{G_1G_2} = \pi^t_{G_2} f^t_{G_1G_2} - \pi^t_{G_1} f^t_{G_1G_2} && t\in T, G_1G_2\in A^t && (6.16i)\\
& 0 \leq x^t_b \leq 1 && t\in T, n\in N, b\in B_n && (6.16j)\\
& 0 \leq x^t_s \leq 1 && t\in T, n\in N, s\in S_n && (6.16k)\\
& 0 \leq f^t_{G_1G_2} \leq C^{max}_{G_1G_2} && G_1G_2\in A^t && (6.16l)\\
& r^t_{G_1G_2} \geq 0 && G_1G_2\in A^t && (6.16m)
\end{aligned}
$$

This formulation corresponds to aggregating the balance constraints of BPUC-MPEC in constraint (6.16c), ignoring flow constraints within each group and can be reformulated into a MILP similar to BPUC-MPEC. A feasible solution of BPUC-$\mathscr{G}$ is feasible for BPUC if and only if there exist flows within each group satisfying the demand at each node. This is done by checking for all groups $G \in \mathscr{G}^t, t \in T$ the feasibility of the initial demand and transmission constraints :

$$
\begin{aligned}
(FLOW^t_G)\ & \sum_{n\in G}\sum_{b\in B_n} q^t_b x^{t*}_b - \sum_{n\in G}\sum_{s\in S_n} q^t_s x^{t*}_s + \sum_{m\in\Theta_n, m\in G}(f^t_{nm} - f^t_{mn}) && \\
& \quad + \sum_{G_2\in\Theta_{G_1}}(f^{t*}_{G_1G_2} - f^{t*}_{G_2G_1}) = p^{t*}_n && (6.17a)\\
& 0 \leq f^t_{nm} \leq C^{max}_{nm} \qquad nm\in A, n,m\in G && (6.17b)
\end{aligned}
$$

where $x^{t*}_b, x^{t*}_s, f^{t*}_{G_1G_2}, p^{t*}_n$ are the values in the feasible solution of BPUC-$\mathscr{G}$.

In an optimal solution of BPUC-$\mathscr{G}$, if there exists a group $G \in \mathscr{G}^t$ such that $FLOW_G^t$ is not feasible, then G is partioned to restore violated transmission constraints in BPUC-$\mathscr{G}$. The partitioning is performed by running an augmenting path algorithm and partitioning nodes in G along cuts where all lines are at their maximum capacity.

The *Iterated Aggregation-Disaggregation* algorithm (IAD) solves iteratively formulation BPUC-$\mathscr{G}$ until a feasible solution of BPUC is found, starting with $\mathscr{G}^t = N$ for all $t \in T$ and disaggregating groups $\mathscr{G}^t$ each time $FLOW_G^t$ is not feasible. Algorithm IAD iterates at most until all groups are partitioned into single nodes in which case BPUC-$\mathscr{G}$ is equivalent to BPUC-MPEC. The initial groups $\mathscr{G}$ of IAD can be improved considering observations made in Section 6.4. In a group $G \in \mathscr{G}^t$, feasible prices are prices that are feasible for all nodes in the group, that is $I_G^t = \cap_{n \in G} I_n^t$. If there exists a group $G \in \mathscr{G}^t$ such that $I_G^t = \emptyset$, then the optimal solution of BPUC-$\mathscr{G}$ cannot be feasible for BPUC when discretizing prices. Such a group G is partitioned by removing a node n such that $I_{G \setminus n}^t \neq \emptyset$. If no such node exists, nodes are removes from G at random until there exists a feasible price for the remaining nodes of G. In the example of Figure 6.3, there exists no feasible price for a group containing all four nodes. The resulting partitioning can be $\{1,2,3\}$ and $\{4\}$ or $\{1\}$ and $\{2,3,4\}$.

In order to retrieve a feasible repaired solution of BPUC at each iteration of IAD where the solution is not feasible for BPUC, the productions p_n found at the current iteration are used as initial production in BP-start to obtain a feasible solution of BPUC. The flowchart of algorithm IAD is presented in Figure 6.5. Step *Cut groups (1)* corresponds to the partitioning of groups if there exists a group G such that $I_G^t = \emptyset$, step *Cut groups (2)* corresponds to the partitioning of a group in an optimal solution of B-$\mathscr{G}$ where the flow is not feasible.

When solving IAD, if there exists a time period $t \in T$ such that $\mathscr{G}^t = \{N\}$, then flow constraints disappear from BPUC-$\mathscr{G}$ in this period. The production of the GC at period t is then only constrained by (6.16c)-(6.16g) regarding the market equilibrium constraints. This leads to a single node formulation as presented by de la Torre et al. [2002] where the spot price $\pi_{\{N\}}^t$ can then be expressed as a piece-wise linear function depending on the total production of the GC at period t, as already illustrated in Figure 6.2. Let r_i^t be the residual demand at price $i \in I_{\{N\}}^t$, that is, the production the GC can sell with priority over competitors at price $\tilde{\lambda}_i^t$:

$$r_i^t = \sum_{b \in B : \pi_b^t \geq \tilde{\lambda}_i^t} q_b^t - \sum_{s \in S : \pi_s^t < \tilde{\lambda}_i^t} q_s^t$$

On a single node, if the production of the GC lies in an interval $]r_{i+1}^t ; r_i^t]$ then the spot price π_n^t is equal to $\tilde{\lambda}_{i+1}^t$, as showed by de la Torre et al. [2002]. After price discretization, constraints (6.16c)-(6.16i) of BPUC-$\mathscr{G}$ for periods $t \in T$ such that $\mathscr{G}^t = \{N\}$ can be replaced by

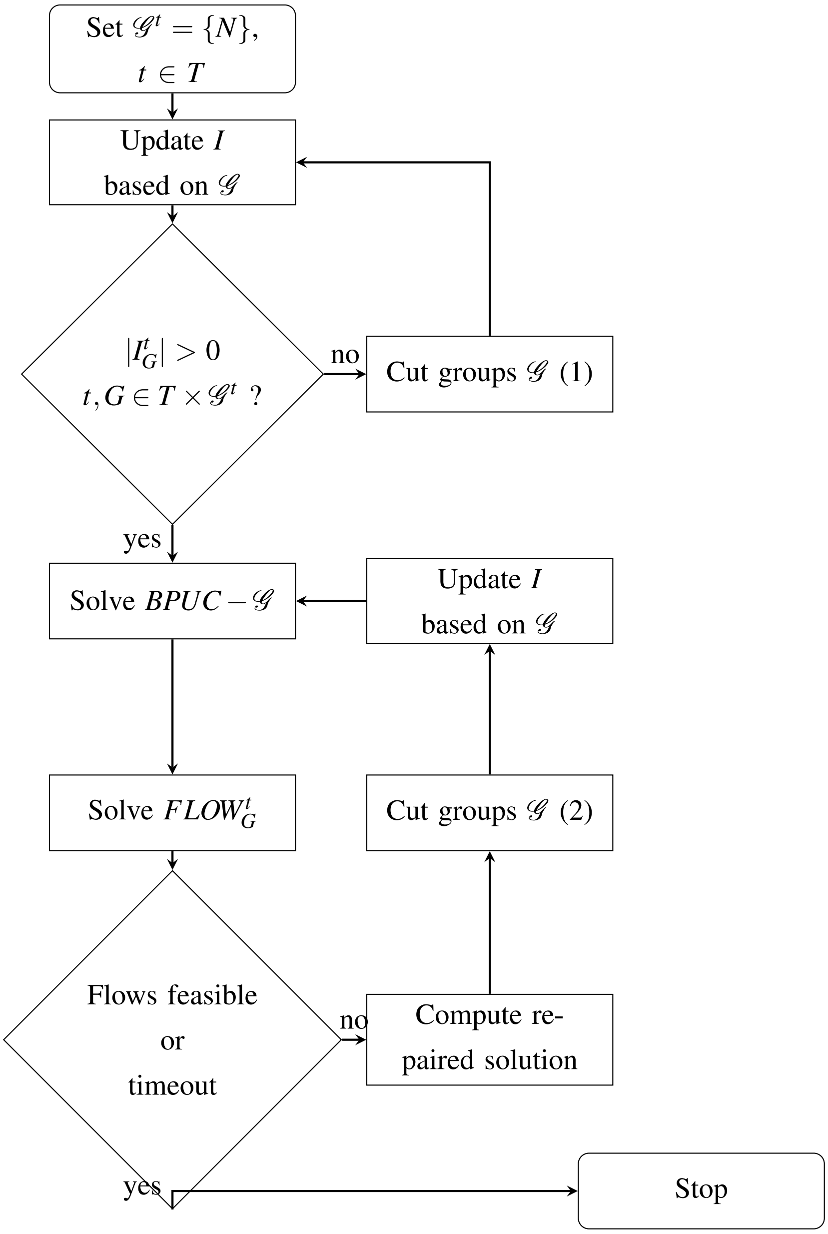


Figure 6.5: IAD flowchart

the following constraints:

$$\sum_{i\in I^t} z^t_{i\{N\}} = 1 \tag{6.18a}$$

$$\sum_{i\in I^t} P^t_{i\{N\}} = \sum_{n\in N} p^t_n \tag{6.18b}$$

$$P^t_{i\{N\}} \leq r^t_i z^t_{i\{N\}} \qquad i \in I^t_{\{N\}} \tag{6.18c}$$

$$P^t_{i\{N\}} \geq r^t_{i+1} z^t_{i\{N\}} \qquad i \in I^t_{\{N\}} \tag{6.18d}$$

$$0 \leq P^t_{i\{N\}} \leq \overline{q}^t_n z^t_{i\{N\}} \qquad i \in I^t_{\{N\}} \tag{6.18e}$$

$$P^t_{i\{N\}} \geq \sum_{n\in N} p^t_n - \sum_{n\in N} \overline{q}^t_n (1 - z^t_{i\{N\}}) \qquad i \in I^t_{\{N\}} \tag{6.18f}$$

$$z^t_{i\{N\}} \in \{0,1\} \qquad i \in I^t_{\{N\}} \tag{6.18g}$$

Formulation BPUC$-\{N\}$ considering all nodes are aggregated at all time periods is available in Appendix C.

6.6.3 SOS-narrowing

For each time period and node, a special ordered set composed of binary variables $\{z^t_{in}\}_{i\in I^t_n}$ is used to represent the possible spot prices. Fractional values of these variables obtained during the branching process may give some indication on prices that are unlikely to be optimal spot prices. Consider the following values at a node of the branching tree for variables $\{z^t_{in}\}_{i\in I^t_n}$ with $I^t_n = \{1,\dots,10\}$:

i	1	2	3	4	5	6	7	8	9	10
z^t_i	0	0.02	0.06	0.61	0.20	0.08	0.03	0	0	0

These values can be seen as a discrete probability distribution. Prices up to price π^t_3 and after π^t_6 can be considered as unlikely to be an optimal bidding price. These price can be eliminated in the remaining subtree of the branch and bound tree in order to narrow the value of the potential prices considered. Consider an elimination coefficient $\alpha \in [0, 0.5]$ and the following indices:

$$i^t_m = \min\{i \in I^t : \sum_{i'\in I^t, i'\leq i} z^t_{i'} \geq \alpha\} \;\; ; \;\; i^t_M = \max\{i \in I^t : \sum_{i'\in I^t, i'\geq i} z^t_{i'} \geq \alpha\}$$

As $\sum_{i'\in I^t} z^t_i = 1$, $z^t_{i_m} \leq z^t_{i_M}$. At each node of the branching tree, the prices at indices out of $\{i^t_m,\dots,i^t_M\}$ are eliminate from the subtree by setting the corresponding variables to 0 through local cuts. These cuts reduce the number of binary variables and thus the size of the solution space during the branching procedure but may also eliminate optimal solutions from the model.

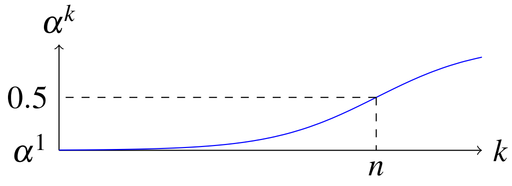


Figure 6.6: Sigmoid curve for α^k in SOS-N

In order to avoid eliminating optimal solution too early in the branching tree and to limit the size of the branch and bound tree, the value of α is increased from an initial value up to 0.5 following a sigmoid curve as the number of nodes explored increases. This increases the number of prices eliminated in the subtrees as the number of nodes explored increases. Consider α^1 as an initial value for α, n as the maximum number of nodes that can be explored and α^k as the elimination coefficient used at node k. Then,

$$\alpha^k = \frac{1}{1+e^{-\frac{\delta k}{n}+\delta}} \quad ; \quad \delta = \ln\frac{1-\alpha^1}{\alpha^1}$$

We have $\alpha^k \leq 0.5$ for all $k \in [1,n]$ and $\alpha^{n+1} > 0.5$. Figure 6.6 illustrates the value of α^k throughout the iterations of the heuristic.

In the following the SOS narrowing heuristic with values α and n is referred to as SOS-n(α,n).

6.7 Numerical results

We present in this section computational results to assess the quality of models and solution methods defined in this chapter. We first provide a description of the characteristics of the instances. Some preliminary computational experiments are then performed on small instances with exact methods. Finally larger instances are only solved through the heuristic methods to prove their scalability. An analysis of the impact of PCR on the profit of the GC is performed. The impact of restricting bidding prices to marginal production cost and the resulting impact on the spot prices is also analysed. All the results reported are averaged over five instances.

Tests are performed on a 8-core i7-4790K 4.00 GHz with 32 Gb of RAM memory and the computation time is limited to 1800 s. All average computation times provided include the time of instances that are not solved to optimality in 1800s. All methods are implemented using Julia 1.0 with packages JuMP 0.18.5 and CPLEX 0.4.1 interfaced with ILOG CPLEX 12.7. The default parameters of CPLEX are used. The SOS parameter of CPLEX available for special ordered sets is deactivated as no improvement in the solution time has been observed during the tests performed.

6.7.1 Data

The instances used in BPUC are built from several sources [Epexspot, 2020, ENTSO, 2020] and consider 24 time periods. Each bid of a competitor represents a generator similar to those of the GC in terms of capacity. The bidding prices considered reflect bids observed on the EPEX market. Figure 6.7 illustrates the hourly amount of electricity traded and the average spot price and demand over instances with 200 bids without considering bids of the GC. The

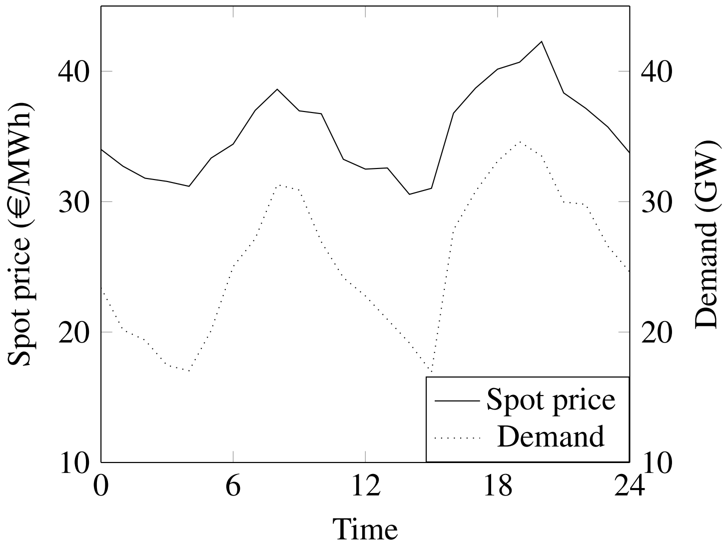


Figure 6.7: Average demand (GWh) and spot-price (€/MWh) of instances with 200 bids

instances are available on the github repository `https://github.com/jdeboeck/BPUC`.

Formulation UC2 presented in Section 2.2.2 is used to model the UC problem. The generators considered are thermal units in a bus to bus system. This model integrates linearized quadratic production costs, start-up and shut-down costs related to the online/offline time, ramping up and down constraints and minimum online and offline time. The data for generators is provided by Carrión and Arroyo [2006]. Ten generators types are given. The instances considered replicate these generators to obtained the desired number of generators. In case $(|J|-5)$ mod 5 = 0, the five generators at odd indices are added.

The transmission network is composed of 4 nodes representing the Netherlands, Belgium, France and Germany. Daily average spot prices on the EPEX and BELPEX are similar for Belgium, France and Germany. The total bidden quantity of competitors are randomly partitioned in each country in the average following proportions:

Netherlands	Belgium	France	Germany
15%	15%	35%	35%

The transmission network represented in Figure 6.8 contains transmission lines between each

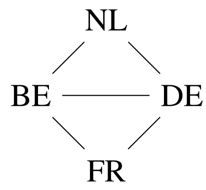


Figure 6.8: Transmission network

pair of bordering countries. It has been observed in EPEX that hourly spot prices are often equal between France and Germany. The values for capacities of the transmission lines have been chosen to be compatible with the EPEX data regarding the ratio of equal spot prices between regions over the time horizon.

6.7.2 Exact formulations for BPUC

Table 6.1 presents numerical results obtained with algorithm BP-start on small instances as well as the impact of adding the obtained starting solution to BPUC-MILP. Gaps are relative to the best bound found with BPUC-MILP with a starting solution.

Instance			Start heuristic			BPUC-MILP			
						No start		With start	
$\|S\|$	$\|J\|$	Count.	Time (s)	Gap (%)	Iter.	Time (s)	Gap (%)	Time (s)	Gap (%)
100	5	BE	0.63	0.85	1.45	191.11	0	118.55	0
100	10	BE	0.97	3.35	2.11	1097.45	1.19	864.91	0
200	5	BE	0.85	0.97	1.58	298.11	0	99.16	0
200	10	BE	1.23	3.02	2.23	1800	10.85	1653.31	0.74

Table 6.1: Start solution impact on BPUC-MILP

Sets S and J are the set of competitor bids and the set of generators of the GC. Column Count. are the countries where the GC is bidding and Iter. is the number of iterations of BP-start. Feasible solutions with a small gap to optimality can be found with BP-start in a short time. Adding a starting solution to BPUC-MILP improves significantly the solution time and the end gap for unsolved instances.

Figure 6.9 shows the evolution with respect to time of the relative gap to the best solution found throughout all methods on an unsolved instance of Table. Bounds are in the upper half of the figure.

The upper bound on BPUC falls down to a value close to optimality in a short time before stagnating. Finding integer solution seems to be difficult. Without using a starting solution,

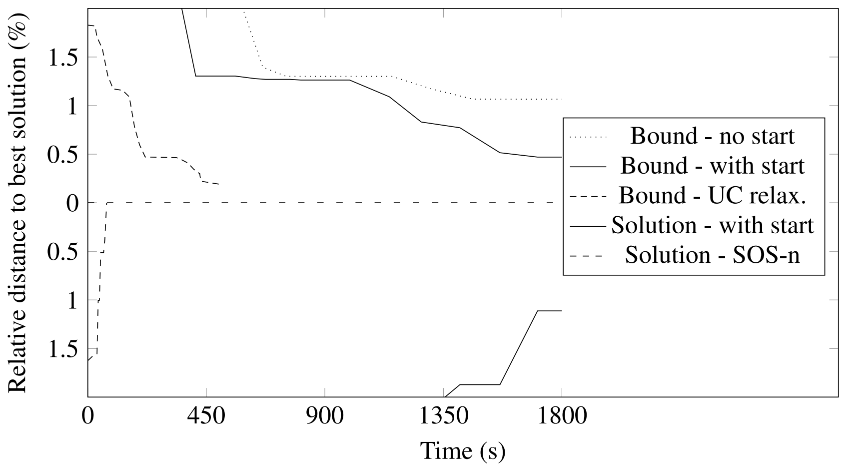


Figure 6.9: Gap evolution of BPUC-MILP for an instance with $|S| = 200, |J| = 10$

Instance			BPUC-MILP - without (6.12) and (6.13)			BPUC-MILP		
$\lvert S\rvert$	$\lvert J\rvert$	Count.	LP gap (%)	End gap (%)	Time (s)	LP gap (%)	End gap (%)	Time (s)
100	5	BE	605.42	298.34	1800	12.48	0	118.55
200	5	BE	351.12	217.53	1800	3.37	0	99.16
100	10	BE	1303.62	665.22	1800	23.38	0	864.91
200	10	BE	459.84	341.62	1800	8.2	0.75	1653.31

Table 6.2: Strengthening BPUC-MILP

the first integer solution with a gap under 20% is found after 931 seconds, the final gap being 12.81% with only three other integer solutions found. With a starting solution, a total of six feasible solutions are found, the best one having a relative gap of 1.09%.

Curve Bound - UC relax. is obtained solving BPUC-MILP and relaxing the integrality constraint of the UC variables, which provides an upper bound for BPUC in the smallest time. This partial relaxation of the integer variables of BPUC-MILP is denoted BPUC-MILP$_R$ in the following. The relative gap of SOS-n(0.01,2000) is also illustrated, the heuristic method finds the best feasible solution in a very short time.

In all the following results we initialize all solution methods with a starting solution obtained with BP-start .

Some insight on the tightness of constraints (6.12) and (6.13) used to strengthen BPUC-MILP after linearizing the extended formulation of BPUC-MPEC is reported in Table 6.2. All gaps reported are relative to the best solution found with the full BPUC-MILP formulation. From the LP gaps we can observe that the strengthening constraints are significantly tightening the solution space. Without these constraints, no feasible solution of decent quality can be found.

Numerical results of formulation BPUC-MILP on larger instances are reported in Table 6.3. When the GC is bidding in two countries, the number of generators is equally split in both

Instance			LP-relaxation		BPUC-MILP				BPUC-MILP$_R$	
$\|S\|$	$\|J\|$	Count.	LP gap (%)	Time (s)	Opt.	End gap (%)	Time (s)	Nodes	Bound gap (%)	Time (s)
200	10	BE	8.2	2.93	2	0.75	1653.31	4283	0.83	448.27
300	10	BE	5.74	3.01	0	0.85	1800.41	4257	0.9	180.43
400	10	BE	3.94	3.15	1	0.31	1651.59	3419	0.35	130.91
200	20	BE	39.74	10.24	0	25.88	1800.44	3	25.21	1822.9
300	20	BE	17.83	11.3	0	8.46	1800.22	18	8.15	1827.11
400	20	BE	11.27	11.1	0	4.46	1800.26	0	3.3	1805.42
200	10	BE-FR	6.52	3.12	2	0.6	1661.77	2782	0.67	329.0
300	10	BE-FR	3.99	3.41	1	0.41	1726.39	744	0.31	149.78
400	10	BE-FR	3.52	3.15	0	0.6	1800.26	113	0.23	197.14
200	20	BE-FR	17.61	12.63	0	6.54	1800.12	0	5.67	1803.96
300	20	BE-FR	9.79	14.65	0	3.06	1800.21	0	2.59	1801.87
400	20	BE-FR	7.49	12.97	0	2.42	1800.31	0	1.64	1803.87

Table 6.3: Numerical results for BPUC-MILP

countries. All gaps are relative to the best solution found with BPUC-MILP. Column Nodes is the number of nodes explored in the branch and bound tree. Only very few instances are solved to optimality and some do not start the branching procedure. The bound gap reported for BPUC-MILP$_R$ is the relative gap of its best bound to the best solution found with BPUC-MILP. Formulation BPUC-MILP$_R$ provides on average a better upper bound on BPUC than BPUC-MILP in a generally shorter time. During the branching procedure, the gaps tend to get low quite quickly but decrease very slowly afterwards as illustrated in Figure 6.9. We can observe that the size of the UC formulation strongly influences the difficulty of the instances. The penetration of the GC on the market defined by $\frac{|J|}{|S|}$ is also correlated to the difficulty of solving an instance. The higher the penetration, the more the GC influences the spot prices resulting in more potential spot prices to consider in sets I_n^t and more binary variables in formulation BPUC-MILP.

Figure 6.10 illustrates the local spot spices over time on an instance where the GC is bidding in two nodes. This spot prices represented with a full line are those of the best solution found with BPUC-MILP. The dotted line represents the spot prices obtained without the bids of the GC, in which case the spot prices are equal in all nodes in 10 time periods over 24 and spot prices are always identical between France and Germany. With the bids of the GC, 8 time periods admit a global spot price and France and Germany have the same spot price in 17 periods. Recalling the exporting/importing situation of a node can be deduced from local spot prices, one can observe it fluctuates over time, mainly between The Netherlands and Belgium on one side and France and Germany on the other side.

Figure 6.11 gives the number of price indices in sets I_n^t to illustrate the impact of price eliminations presented in Section 6.4. Without any price elimination, there would be 400 binary variables for bid prices at each time period. The largest number of prices after eliminations is 26 and lies in Belgium as the generators of the GC have the highest local penetration in this

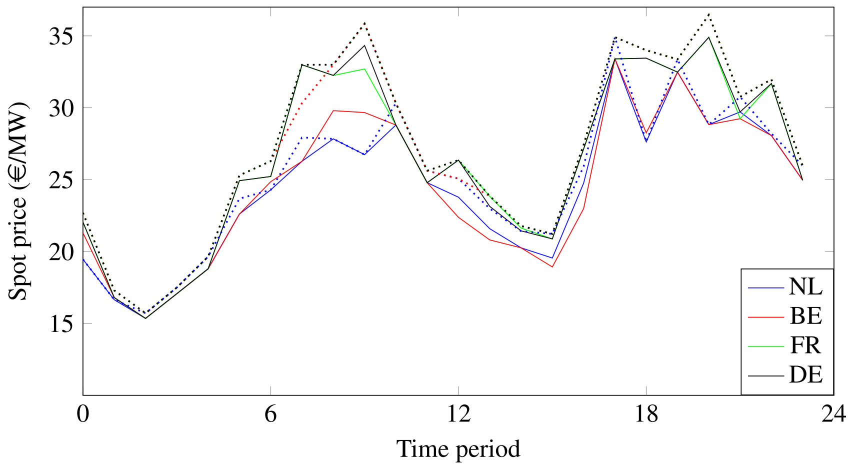


Figure 6.10: Spot prices at each time period and node for $|S| = 400$, $|J| = 20$, country = BE-FR

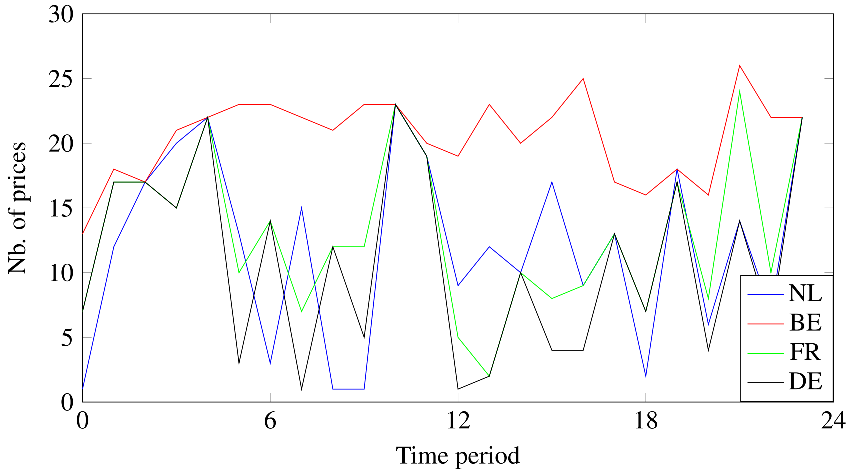


Figure 6.11: Number of indices in sets I_n^t for $|S| = 400$, $|J| = 20$, country = BE-FR

Instance			IAD				
$\|S\|$	$\|J\|$	count.	Gap	C. gap (%)	Time (s)	Iter.	Nb. gr.
200	10	BE	0.45	39.19	86.73	2.8	1.8
300	10	BE	0.39	53.57	65.89	2.6	2.15
400	10	BE	0.27	10.0	56.14	2.0	1.99
200	20	BE	4.8	76.17	2244.08	3.0	2.15
300	20	BE	4.65	38.25	1458.44	2.0	1.93
400	20	BE	3.08	3.75	2121.34	2.0	2.12
200	10	BE-FR	0.42	30.0	197.34	2.4	1.92
300	10	BE-FR	0.15	51.61	116.58	2.2	1.98
400	10	BE-FR	0.13	43.48	148.53	2.4	2.05
200	20	BE-FR	5.29	1.49	2120.24	1.6	2.22
300	20	BE-FR	1.87	26.09	1428.44	1.0	1.71
400	20	BE-FR	1.65	-2.48	2458.29	1.6	2.19

Table 6.4: Numerical results for IAD

country. The number of prices in France is also generally more important than in Germany as the GC also bids in France. In several time periods, the number of local prices to consider is equal to one, fixing the local spot prices on these periods.

6.7.3 Heuristic methods

Table 6.4 gives numerical results for IAD. Gaps are relative to the best bound found with BPUC-MILP or BPUC-MILP$_R$. Column C. gap gives the closed gap of the best solution z^* of IAD on the best solution z^{MILP} of BPUC-MILP relatively to the best upper bound found $\bar{z}$:

$$C.\,gap = 1 - \frac{\bar{z} - z^*}{\bar{z} - z^{MILP}}$$

Column Iter. is the number of iterations of IAD before finding a feasible solution or reaching the time limit and column Nb. gr. is the average number of groups per time period in the solution returned by IAD. Except for a couple of instances, IAD improves significantly the value of the best solution found as well as the solution time. The number of iterations is quite reduced before finding a feasible solution by solving BPUC-$\mathscr{G}$, the groups being rapidly partitioned. The average gaps and time per iteration over the 60 instances of Table 6.4 are given in Figure 6.12. The numbers shown in brackets next to the iteration indices are the number of instances that did not terminate before the given iteration. The repaired solutions computed improve over the iterations, illustrating the importance of initial starting productions in BP-start. Figure 6.13 gives the average number of groups per time period per iteration. The number of groups considered in the first iteration is often different from 1 as the network can be partitioned from the beginning to have at least one price in each set I^t_G. The most important partitioning is then made at the end of the first iteration. At the end of further iterations, only one or two groups are generally partitioned.

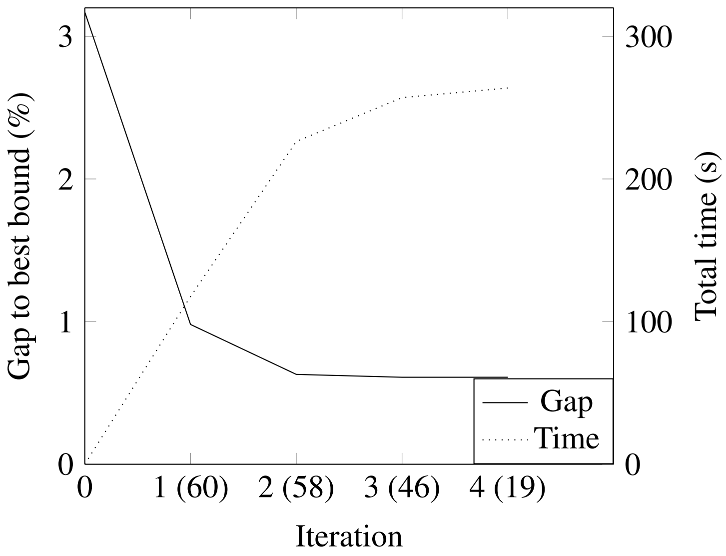


Figure 6.12: Evolution of repaired solution in IAD

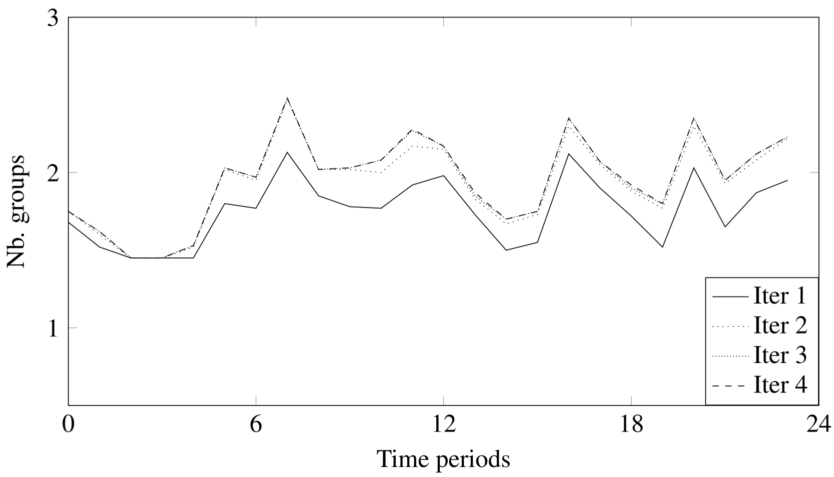


Figure 6.13: Evolution of groups in IAD

Instances			BPUC-MILP - SOS-n				IAD - SOS-n		
$\|S\|$	$\|J\|$	count.	Gap (%)	C. gap (%)	Time (s)	Nodes	Gap (%)	C. gap (%)	Time (s)
200	10	BE	0.57	22.97	96.13	911	0.6	-33.33	57.21
300	10	BE	0.45	46.43	75.28	708	0.27	30.77	48.35
400	10	BE	0.28	6.67	53.19	485	0.27	0.0	39.64
200	20	BE	11.08	44.99	1298.25	1980	6.74	-40.42	2040.89
300	20	BE	3.98	47.14	843.71	1723	3.33	28.39	948.72
400	20	BE	2.53	20.94	945.83	1455	2.48	19.48	1324.32
200	10	BE-FR	0.43	28.33	77.81	496	0.43	-2.38	88.13
300	10	BE-FR	0.2	35.48	64.87	496	0.15	0.0	96.75
400	10	BE-FR	0.15	34.78	64.62	256	0.17	-30.77	107.18
200	20	BE-FR	3.77	29.8	1445.91	1833	4.28	19.09	1877.62
300	20	BE-FR	1.32	47.83	1142.27	1576	0.86	54.01	1293.67
400	20	BE-FR	1.26	21.74	1215.05	1418	1.27	23.03	1521.87

Table 6.5: Numerical results for SOS-n(0.01,2000)

Numerical results for the SOS-n(0.01,2000) heuristic are given in Table 6.5. This heuristic is tested on BPUC-MILP and on IAD. Gaps are relative to the best bound found with BPUC-MILP or BPUC-MILP$_R$. The closed gaps are the gaps closed by the best solution found with SOS-n on the best solution found with the corresponding solving method without SOS-n relatively to the best upper bound found. The initial value for $\alpha = 0.01$ and the limitation to 2000 nodes are the parameters reporting the best results. Increasing the number of nodes improves slightly the solution and increases slightly the solution time and conversely for the value of α.

When used on BPUC-MILP, SOS-n improves significantly the solution time as well as the best solution found. The largest instances that do not exit the root node of the branch and bound tree in BPUC-MILP have over a thousand nodes with SOS-n, illustrating some binary variables are eliminated starting from the root node. On IAD, SOS-n improves the average solution time but the solution is sometimes of lower quality than without SOS-n. Still, IAD - SOS-n finds on average better solutions than BPUC-MILP - SOS-n.

Figure 6.14 summarizes the performance of the different solving methods presented per instance. Gaps are relative to the best bound found with BPUC-MILP or BPUC-MILP$_R$. Negative gaps are associated with feasible solutions. Overall, formulation BPUC-MILP$_R$ provides the best upper bound and the best feasible solution is found by IAD - SOS-n. As already observed, the difficulty of the instances is strongly correlated with the penetration of the GC and the size of the UC formulation. For a given number of generators and nodes, the gap tends to decrease when the number of bids increases.

6.7.4 Market impact

This section provides some insight on the impact on the profit of the GC when considering a transmission network or restricting bidding prices to marginal prices of generators.

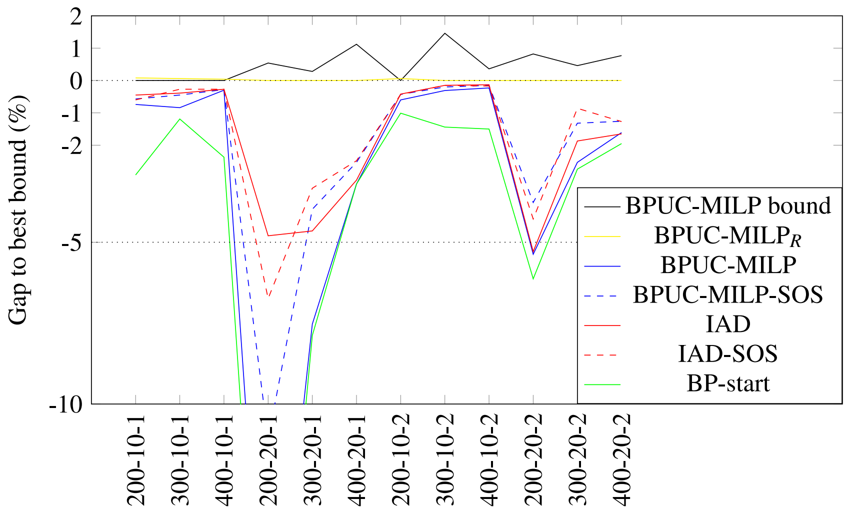


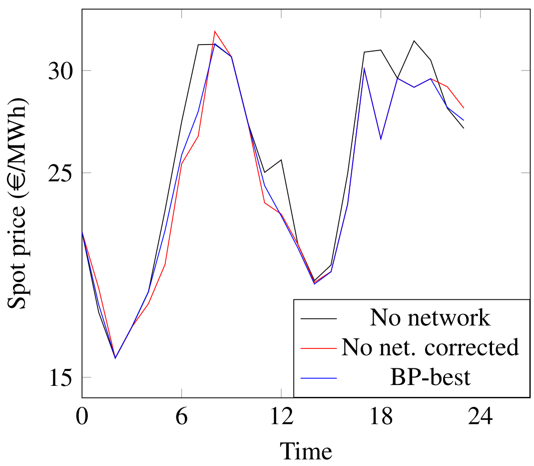
Figure 6.14: Solving methods gap comparaison

Instances			de la Torre et al.		BPUC-{N}					B. gap (%)
$\|S\|$	$\|J\|$	count.	LP gap (%)	Time	LP gap (%)	Time (s)	Gap to BPUC (%)	C. gap 1 (%)	C. gap 2 (%)	
200	10	BE	2.26	4.88	1.86	1.28	-5.42	3.01	22.75	0.57
300	10	BE	1.34	5.26	1.13	1.7	-3.86	1.02	43.19	0.27
400	10	BE	1.34	5.0	1.17	2.11	-3.03	0.39	66.95	0.27
200	20	BE	3.46	95.25	3.07	36.25	-14.37	22.35	24.5	6.67
300	20	BE	1.77	18.16	1.56	6.37	-10.2	7.39	35.49	3.33
400	20	BE	1.66	45.03	1.44	11.3	-8.69	3.31	24.82	2.48
200	10	BE-FR	1.97	4.12	1.58	1.38	-3.14	1.05	52.27	0.43
300	10	BE-FR	1.37	3.95	1.13	1.62	-0.2	0.34	72.24	0.15
400	10	BE-FR	1.1	2.84	0.94	1.43	-2.46	0.53	72.75	0.15
200	20	BE-FR	3.24	41.45	3.0	29.78	-3.78	4.83	44.94	3.77
300	20	BE-FR	1.7	11.71	1.51	9.78	-0.72	1.94	75.44	0.86
400	20	BE-FR	1.61	23.8	1.43	21.02	-2.7	1.35	61.88	1.26

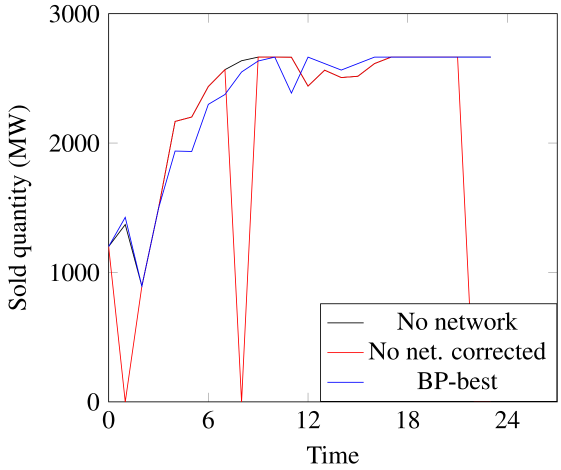
Table 6.6: Comparaison with single node model

Table 6.6 compares the formulation proposed by de la Torre et al. [2002] for BPUC considering a single day-ahead market with the equivalent formulation BPUC-$\{N\}$.

The columns under de la Torre et al. provide the LP gaps and the solution times to optimality. The columns under BPUC-$\{N\}$ also reports LP gaps and the total solution time that are both slightly improved. The constraints of the transmission network being removed, BPUC-$\{N\}$ is a relaxation of BPUC-MILP. Column Gap to BPUC reports the gap of the optimal value of BPUC-$\{N\}$ relatively to the best upper bound found for BPUC. These gaps are reported as negative as the optimal value of BPUC is overestimated when ignoring the transmission constraints. C. gap 1 reports the first corrected gap, that is the profit for the GC considering it sells its full quantity at the spot prices obtained considering the transmission network relatively to the best upper bound found. Considering this *optimistic* correction where the GC sells everything without worrying about bidding prices, the solution provided by BPUC-$\{N\}$ is of



Spot prices



Production

Figure 6.15: Single node network results for $|S| = 300$, $|J| = 20$, country = BE

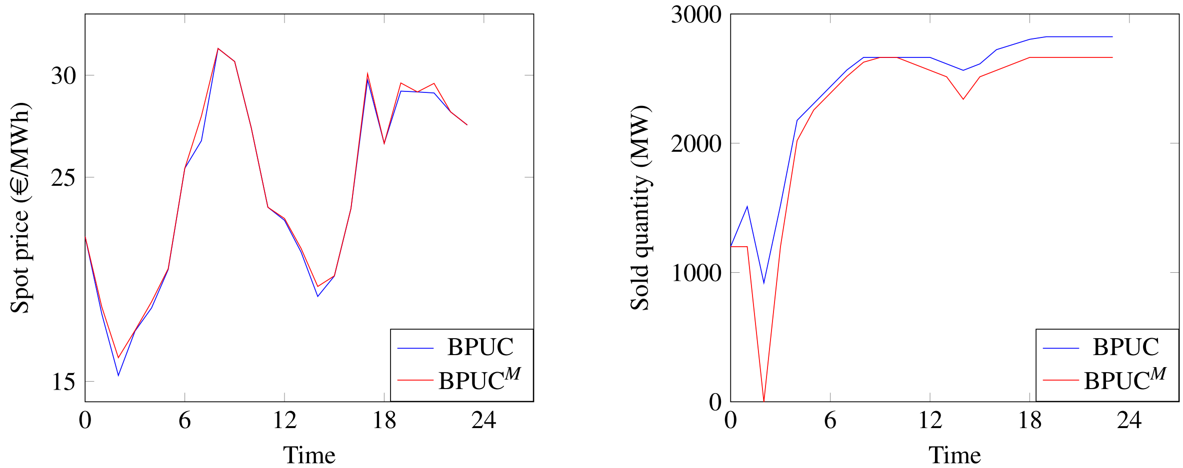
significantly lower quality than the best solution obtained with the former methods, the best gap obtained considering the network being reported in column B. gap. A more realistic correction is to consider the GC places bids according to the spot prices and quantities computed in BPUC-$\{N\}$. The risk in this case is to place a bid over the accurate local spot price and not sell any production at some time periods as it can be the case in price-taker formulations. The gap of the *realistic* correction, considering bids over the spot price are not sold, relatively to the best upper bound found is reported in column C. gap2. The gaps obtained are very large, illustrating the importance of integrating an accurate computation mechanism of the spot prices to avoid bidding at a too high price and not selling.

Figure 6.15 illustrates the spot prices and quantities sold by the GC with no network before and after the realistic correction and compares it to the best solution found for BPUC. The curves reported for the spot prices correspond to the local spot price of the node where the GC is bidding. On the left figure, we can observe the spot prices found without considering the transmission constraints can be pretty far from the corrected local spot prices. Furthermore, in the four periods where the spot price is over-estimated, the corresponding quantity sold is equal to zero if the GC bids at the computed spot price.

A comment is now given on the effect of constraining bidding prices to the marginal cost of the associated generator as in BPUC^M. Numerical results of formulation BPUC^M are given in Table 6.7. The gap to BPUC is the relative gap of the value of the best solution found by BPUC^M on the best solution found by BPUC-MILP. As bidding prices are fixed in BPUC^M, its optimal value is at most the optimal value of BPUC. The end gaps are slightly bigger than when solving BPUC-MILP, this is explained by the additional difficulty in BPUC^M to track if

Instances			BPUC^M-MILP		
$\|S\|$	$\|G\|$	count.	End gap (%)	Time (s)	Gap to BPUC (%)
200	10	BE	1.97	1797.47	6.99
300	10	BE	1.05	1800.2	5.4
400	10	BE	0.49	1701.41	4.7
200	20	BE	22.87	1800.55	1.85
300	20	BE	13.49	1800.48	4.89
400	20	BE	12.7	1800.22	7.72
200	10	BE-FR	1.75	1786.01	4.05
300	10	BE-FR	2.12	1800.34	3.54
400	10	BE-FR	1.52	1782.56	2.38
200	20	BE-FR	5.75	1800.27	2.29
300	20	BE-FR	9.98	1807.93	8.79
400	20	BE-FR	7.67	1800.27	7.76

Table 6.7: Bidding at marginal costs



Spot prices

Production

Figure 6.16: Marginal bidding results for $|S| = 300$, $|G| = 20$, country = BE

generators can sell their production depending on their marginal cost and the spot prices. The gaps to BPUC is on average of 5% and the end gaps of the formulation are slightly bigger than for BPUC-MILP. Figure 6.16 illustrates the local spot prices of each bidding market as well as the sold quantity per time period. In the results of this instance, the same generators are turned on until period 13 in BPUC and in BPUC^M. The bidden production of the GC is limited by the spot prices in periods 2 and 14 when bidding at marginal costs. The production of the GC is smaller is these periods in BPUC^M than in BPUC to limit the production at loss. When the spot prices are not limiting the bidden production, one can expect that the optimal bidden quantities are identical in BPUC and BPUC^M as start-up and shut-down costs are identical until period 13 included. But because of ramping up constraints, the total production in BPUC^M catches up the total production of BPUC only in period 9 and 10 before decreasing again to avoid a too important production at loss in period 14. At period 14 in BPUC^M, the GC can either decide to

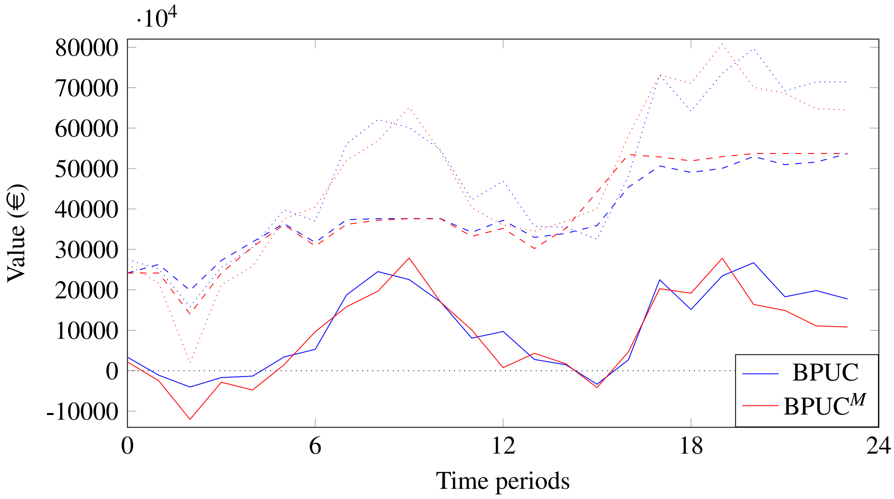


Figure 6.17: Profit, income and production cost for BPUC and BPUC^M on $|S| = 300$, $|J| = 20$, country = BE

reduce the production of the generators over the spot price and keep it turned on, producing at loss but avoiding future start-up cost, or shut-down the generators to avoid producing at loss but adding eventual start-up costs in the future. In this instance, the involved generator is turned off at period 14 in BPUC^M and is not turned back on during the remaining times periods, start-up cost being too important. In order to compensate a lower production during the last periods in BPUC^M, the generators that are turned on in period 14 in BPUC^M produce more than in BPUC at a higher unit cost. The restriction of limiting bidding prices to marginal production costs can significantly modify the UC production plan.

Finally, we illustrate how the profit of the GC is split into income and production for solutions of BPUC and BPUC^M in Figure 6.17. The instance used is the same as for Figure 6.16. The dotted line represents the income, the dashed line the production cost and the full line the profit of the GC. In period 4, the income decreases significantly regarding the income in BPUC^M while the production cost decreases only slightly. This is a consequence of the GC not bidding any production with some generators with a marginal cost under the spot price but keeps producing power with them to reach a desired quantity in future time periods. In period 15, the production cost of BPUC^M surpasses those in BPUC, consequence of the GC deciding to shut down a generator in period 14 and producing more with the other generators in future periods. One can also observe that in both formulations, the GC has a negative profit in certain time periods. This is explained by the ramping up constraints that limit the increase and decrease of the total production. In order to be able to sell a lot of production during periods with a high spot price, the GC must start the production several periods in advance even if this results in a production at loss for certain periods, illustrating an interest in producing at loss.

6.8 Conclusion

This chapter presents a Mathematical Program with Equilibrium Constraints formulation and a tight Mixed Integer Linear reformulation of the bidding problems. A set of constraints defining a market equilibrium in a capacitated transmission network is provided as well as a linearization using a limited number of binary variables by exploiting the possible values of spot prices based on bids of competitors. Instances with a limited number of generators and a larger number of bids are solved at less than 1% from optimality. Further study over the quality of the market equilibrium constraints proposed could be performed by using them in a bidding problem for a retailer rather than a producer.

Numerical results have illustrated the complexity introduced by regulations such as PCR when searching for optimality. Still, this problem could be addressed by exploiting properties enforced on local spot prices in PCR in an Iterative Aggregation-Disaggregation heuristic, aggregating the bidding areas with a potentially equal spot price throughout the time periods to reduce the number of binary variables. The gap to optimality is significantly reduced in comparison of the best solutions found with the Mixed Integer Linear formulation. This heuristic is particularly interesting if many areas are likely to have equal spot prices in a market equilibrium and can be adapted to other bidding problems with a transmission constrained network studied in the literature.

The SOS-n heuristic, reducing the number of binary variables to consider in Special Ordered Sets during a branch and bound procedure, has shown a particularly good efficiency in improving the gap to optimality in limited time on the Mixed Integer Linear formulation. This generic heuristic can be adapted to any formulation containing a special ordered set of type 1 represented by binary variables. Note that the SOS-n heuristic was more efficient than when using the Special Ordered Set of type 1 parameter of CPLEX on the Mixed Integer Linear formulation.

Numerical results have also shown the importance of considering a price-maker formulation with a spot price computation as close as possible to reality. A bad evaluation of the spot price might result in bids that have a price too high and are not sold when clearing the market, leading to important losses.

Chapter 7

Conclusions

In this thesis, we studied several problems related to the electricity supply chain arising from new technologies or market regulations. The problems studied focus on the increasing interaction and interdependency between actors of the electricity market. We integrated these new constraints in mathematical formulations, that illustrate the complexity of these new optimization problems. Still, specific solving methods have been developed for each problem considered by taking advantage of the inherent structure of each problem, allowing to provide some contributions for more general optimization problems.

Increasing dependance to electricity implies an increasing need for reliability of the transmission network and power supply. To this purpose, the *Minimum Margin Problem* (MMP) is studied in Chapter 3, aiming at guaranteeing a close electricity supply to customers while managing the power reserve of the generators of the network. New electricity production devices and the increasing penetration of interconnected power management let us foresee the organization of customers into local independent power networks as micro-grids. Micro-grids are composed mainly of renewable electricity devices. External *Generation Companies* (GCs) can interact with micro-grids in case of excessive or insufficient internal power production. Due to the internal power management of micro-grids, these can have more influence on a GC than when considering customers separately. This interaction is studied through a *Contract Proposition Problem* (CPP) in Chapter 4 where a GC must choose a set of bilateral contracts to propose to micro-grids. Next to bilateral contracts, GCs also have the possibility to trade electricity in day-ahead markets. The GCs interact in this case with a *Transmission System Operator* (TSO) through a bidding procedure, the TSO fixing the spot price of electricity based on the bids received. Such bidding problems being challenging, they are studied under two different perspectives. In Chapter 5 we consider uncertainty in the market mechanism and linear production costs for the GC in the *Stochastic Bidding Problem* (SBP). In Chapter 6, a full *Unit*

Commitment problem (BPUC) is considered for production costs with deterministic bidding data.

Common aspects of these problems are reliability, growing uncertainty and interactions between actors. These problems have been solved in this thesis mainly by using exact methods aiming at finding optimal solutions while heuristic methods have been introduced for the more challenging problems. When heuristic methods are proposed, methods proposing bounds on the optimal solutions of the problem studied are proposed as well to keep track of the quality of the solutions returned by the heuristic.

It is to note that strong hypotheses are made for most of the applied problems studied, as in most of the related literature, without being able to keep track of the impact of these hypotheses on the quality of the solutions found. A questioning on the purpose of such applied problems can be made if they are not appliable as named. But the solutions found can in certain cases be interpreted as bounds in real life or a specific study can lead to more general solving methods.

The first problem studied is the MMP introduced by Rossi et al. [2011] considering reliability in power supply throughout a transmission network. To deal with reliability issues, the MMP aims at maximizing the production margin of feeder on the network while satisfying the demand of a set of customers. A *hop constraint* is introduced to limit the distance between customers and feeders and limit the probability of transmission failure. The problem was tackled by using *layered graphs*. Numerical results have illustrated again how these graphs are efficient when modelling hop constrained problems. Interesting preprocessing techniques having been found for the mathematical approach but the problem has a lot of underlying hypotheses. It uses a transmission network without considering any physical constraints which are known to be challenging. Still, as these physical constraints are likely to reduce the solution space if modelled, the solution of this problem can be considered as an upper bound in a real life applications.

The three other problems involve interactions between actors. All are two-stage Stackelberg games where a leader makes a decision to which a set of followers respond and are modelled through bi-level formulations.

The CPP is the first bi-level problem studied in this thesis. A GC must choose a set of contracts to propose to micro-grids to be their external electricity provider. The micro-grids optimize their internal electricity management based on their production units and the prices of the contracts subscribed. The GC optimizes a generic problem, such as a UC, considering the electricity traded with the micro-grids who subscribed a contract. The bi-level formulation of this problem contains binary variables at the second level, formulation which are known to be challenging. A novel heuristic method is proposed to solve such type of problems, taking advantage of these binary variables and relaxing the *optimistic assumption* made in bi-level formulations. Computational experiments assess the quality of the solutions found. Despite the quality of the solution found, the time horizon and number of scenarios considered is far from modelling real life situation accurately leading to difficult interpretations of the quality of a solution in real life. Still, studying this problem proposed a general approach for contrat propositions with other actors than a GC and micro-grids.

The two last bidding problems studies make strong hypothesis too, as all studies considering price-maker approaches as we do. Price maker approaches take into consideration the interaction between the bidding GC and the TSO. These problems are generally modelled as bi-level problems with the GC as the leader who places bids and the TSO as the follower who fixes the spot price of electricity and the quantity of electricity traded. The strong hypotheses are justified by the numerous contraints in real life situations considering market regulations and production constraints.

The first bidding problem, SBP, considers uncertainty in the bids of competitors and linear production costs. A constrained version of the problem, SBP-Q, was introduced by Fampa et al. [2008]. We introduce a novel approach for this problem based on dynamic programming. Several variants of SBP are studied. SBP-Q is solved to optimality in a significantly smaller time than past studies. An upper bound for SBP computed in polynomial time and a heuristic method that provides solutions near optimality using Bertrand and Cournot approaches are proposed as well. The number of scenarios used is larger than in previous work, assessing the efficiency of the methods proposed regarding uncertainty.
Again, interpretations of the solutions are limited in real life because of the linear production costs that make a total abstraction. Extending the dynamic programming approach to more general production is unfortunately hard to foresee as most of the method relies on linear production costs.

The second bidding problem, BPUC, considers a full UC for production costs and deterministic bidding data from the competitors. Such types of problems have been studied for 20 years, always with a very simplified version of the bidding mechanism. The *Price Coupling of Regions* (PCR) is integrated in the problem, considering several day-ahead markets linked through a transmission network in order to increase the global welfare. A set of market equilibrium constraints have been presented that are equivalent to the follower problem in the bi-level formulation. These constraints can be used for other actors in day-ahead market such as retailers. Two heuristic method have been presented which find good quality solutions in much smaller time than exact methods. The first one uses properties of PCR while the second, a SOS-narrowing heuristic, is a novel method for MILP formulations containing *Special Ordered Sets of type 1*. The efficiency of the SOS-narrowing heuristic should be assessed as a general method in future

research.
A focus is put on the impact of different hypotheses in the numerical results of this problem. The impact of ignoring some regulations on bidding prices or the PCR as done in previous studies can lead to solutions underestimating or overestimating the profit by over 100%. As only few physical constraints are considered on the transmission network in BPUC, the same misestimation could be observed when integrating additional physical constraints.

It is also to not that the bidding mechanism used in both bidding problems is also relatively simple as in all similar studies. Block bids are not considered, although they can have a significant impact on the spot price of electricity. For these two problem, there are only little possibilities of interpretations solutions because of the hypotheses made.

Overall, the impact of considering new reliability, technological or regulation issues force the introduction of strong hypotheses in previously studied optimization problems. Efficient solving methods providing solutions of good quality have been found for all theoretical problems, all keeping trace of the gap to optimality. But the gap to reality is often not traceable. The hypotheses made often depend on the solving method used rather than the problem studied in itself. This has been illustrated in bidding problems where block bids are not considered as they would add to much complexity in the bi-level formulations which generally used to solve such problems. From this arises two questions. Is the solving method considered well adapted or is the real life mechanism useable optimally ? The two bidding problems studied illustrate the complexity of these problems under the strong hypotheses considered which are still still far from real life situations. Using mathematical programming is maybe not the best path, as illustrated in Chapter 5 where dynamic programming was more efficient, or maybe the structure of the market makes the problem practically not solvable to optimality.

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Appendix A

Unit commitment

The two UC formulation used throughout this thesis are provided in this section. The notations and constrained are described in Section 2.2.2.

A.1 Formulation UC1

This first UC formulation is proposed by Carrión and Arroyo [2006] and used in Chapter 4. It considers a single set of binary variables to model the state of each generator throughout the time horizon. In Chapter 4, a UC problem is considered with a fixed demand and an amount of electricity $x-y$ to produce for micro-grids. At period t, the amount of electricity to produce for micro-grids is denoted $(x-y)^t$ and the fixed demand d^t. The total production cost is denoted $F(x,y)$.

Link with market equilibrium variables:

$$F(x,y) = \sum_{t\in T}\sum_{j\in J} c^t_j(p^t_j) + c^u_j(t) + c^d_j(t)$$

$$d^t + (x-y)^t \leq \sum_{j\in J} p^t_j \qquad t\in T$$

Production cost:

$$c^t_j(p^t_j) = A_j v^t_j + \sum_{l=1}^{NL_j} F_{lj}\delta^t_{lj} \qquad j\in J, t\in T$$

$$p^t_j = \underline{P}_j v^t_j + \sum_{l=1}^{NL_j} \delta^t_{lj} \qquad j\in J, t\in T$$

$$\delta^t_{1j} \leq T_{1j} - \underline{P}_j \qquad j\in J, t\in T$$

$$\delta_{lj}^t \leq T_{lj} - T_{l-1\,j} \qquad j \in J, t \in T, \in \{2,\ldots,NL_j - 1\}$$

$$\delta_{NL_j\,j}^t \leq \overline{P}_j - T_{NL_j-1\,j} \qquad j \in J, t \in T$$

Start-up / shut-down costs:

$$c_j^u(t) \geq K_j^t(v_j^t - \sum_{n=1}^{k} v_j^{k-n}) \qquad j \in J, t \in T, k \in \{1,...,ND_j\}$$

$$c_j^u(t) \geq 0 \qquad j \in J, t \in T$$

$$c_j^d(t) \geq C_j(v_j^{t-1} - v_j^t) \qquad j \in J, t \in T$$

$$c_j^d(t) \geq 0 \qquad j \in J, t \in T$$

Production capacities:

$$\underline{P}_j v_j^t \leq p_j^t \leq \overline{p}_j^t \qquad j \in J, t \in T$$

$$0 \leq \overline{p}_j^t \leq \overline{P}_j v_j^t \qquad j \in J, t \in T$$

Ramping up and down :

$$\overline{p}_j^t \leq p_j^{t-1} + RU_j v_j^{t-1} + SU_j(v_j^t - v_j^{t-1}) + \overline{P}_j(1 - v_j^t) \qquad j \in J, t \in T$$

$$\overline{p}_j^t \leq \overline{P}_j v_j^{t+1} + SD_j(v_j^t - v_j^{t+1}) \qquad j \in J, t \in T \setminus |T|$$

$$p_j^{t-1} - p_j^t \leq RD_j v_j^t + SD_j(v_j^{t-1} - v_j^t) + \overline{P}_j(1 - v_j^{t-1}) \qquad j \in J, t \in T$$

Minimum up / down times:

$$\sum_{n=t}^{t+UT_j-1} v_j^n \geq UT_j(v_j^t - v_j^{t-1}) \qquad j \in J, t \in \{G_j + 1 ... |T| - UT_j + 1\}$$

$$\sum_{n=t}^{|T|} (v_j^n - (v_j^t - v_j^{t-1})) \geq 0 \qquad j \in J, t \in \{|T| - UT_j + 2 ... |T|\}$$

$$\sum_{n=t}^{t+DT_j-1} (1 - v_j^n) \geq DT_j(v_j^{t-1} - v_j^t) \qquad j \in J, t \in \{L_j + 1 ... |T| - DT_j + 1\}$$

$$\sum_{n=t}^{|T|} (1 - v_j^n - (v_j^{t-1} - v_j^t)) \geq 0 \qquad j \in J, t \in \{|T| - DT_j + 2 ... |T|\}$$

$$\sum_{t=1}^{G_j} v_j^t = G_j \qquad j \in J$$

$$\sum_{t=1}^{L_j} v_j^t = 0 \qquad j \in J$$

Variables:

$$c_j^p(t), c_j^u(t), c_j^d(t), \delta_l(j,t) \geq 0 \qquad j \in J, t \in T$$

$$v_j(t) \in \{0,1\} \qquad j \in J, t \in T$$

A.2 Formulation UC2

This second UC formulation is proposed by Ostrowski et al. [2012] and used in Chapter 6. Two additional sets of binary variables are used to model when generators are turned on or off. The UC must produce a minimum quantity p^t for each time period $t \in T$. The total production cost is denoted $c(p)$.

Link with market equilibrium variables:

$$c(p) = \sum_{t\in T}\sum_{j\in J} c_j^t(p_j^t) + c_j^u(t) + c_j^d(t)$$

$$p^t \leq \sum_{j\in J} p_j^t \qquad t \in T$$

Production cost:

$$c_j^t(p_j^t) = A_j v_j^t + \sum_{l=1}^{NL_j} F_{lj}\delta_{lj}^t \qquad j \in J, t \in T$$

$$p_j^t = \underline{P}_j v_j^t + \sum_{l=1}^{NL_j} \delta_{lj}^t \qquad j \in J, t \in T$$

$$\delta_{1j}^t \leq T_{1j} - \underline{P}_j \qquad j \in J, t \in T$$

$$\delta_{lj}^t \leq T_{lj} - T_{l-1\,j} \qquad j \in J, t \in T, \in \{2, \ldots, NL_j - 1\}$$

$$\delta_{NL_j\,j}^t \leq \overline{P}_j - T_{NL_j-1\,j} \qquad j \in J, t \in T$$

Start-up / shut-down costs:

$$c_j^u(t) \geq K_j^t(v_j^t - \sum_{n=1}^{k} v_j^{k-n}) \qquad j \in J, t \in T, k \in \{1, ..., ND_j\}$$

$$c_j^u(t) \geq 0 \qquad j \in J, t \in T$$

$$c_j^d(t) \geq C_j(v_j^{t-1} - v_j^t) \qquad j \in J, t \in T$$

$$c_j^d(t) \geq 0 \qquad j \in J, t \in T$$

$$v_j^{t-1} - v_j^t + y_j^t - z_j^t = 0 \qquad j \in J, t \in T$$

Production capacities:

$$\underline{P}_j v_j^t \leq p_j^t \leq \overline{p}_j^t \qquad j \in J, t \in T$$

$$0 \leq \overline{p}_j^t \leq \overline{P}_j v_j^t \qquad j \in J, t \in T$$

Ramping up and down :

$$\overline{p}_j^t \leq p_j^{t-1} + RU_j v_j^{t-1} + SU_j(v_j^t - v_j^{t-1}) + \overline{P}_j(1 - v_j^t) \qquad j \in J, t \in T$$

$$\overline{p}_j^t - p_j^{t-1} \leq RU_j v_j^{t-1} + SU y_j^t \qquad j \in J, t \in T$$

$$\overline{p}_j^{t-1} - p_j^t \leq RD_j v_j^t + SD z_j^t \qquad j \in J, t \in T$$

Minimum up / down times:

$$\sum_{t'=t-UT_j+1}^{t} y_j^{t'} \leq v_j^t \qquad j \in J, t \in G_j+1,\ldots,|T|$$

$$v_j(t) + \sum_{t'=t-DT_j+1}^{t} z_j^{t'} \leq 1 \qquad j \in J, t \in L_j+1,\ldots,|T|$$

$$\sum_{t=1}^{G_j} v_j^t = G_j \qquad j \in J$$

$$\sum_{t=1}^{L_j} v_j^t = 0 \qquad j \in J$$

Tightening:

$$p_j^t \leq \overline{P}_j v_j^{t+K(t)} + \sum_{i=1}^{K(t)} (SD_j + (i-1)RD_j) z_j^{t+i} - \sum_{i=1}^{K(t)} \overline{P}_j y_j^{t+i}$$

$$j \in J, t \in T$$

$$p_j^{t-1} - p_j^t \leq RD_j v_j^t + SD_j z_j^t - (RD_j - SU_j + \underline{P}_j) y_j^{t-1} - (RD_j + \underline{P}_j) y_j^t$$

$$j \in \{k \in J | RD_k > SU_k - \underline{P}_k, UT_k \geq 2\}, t \in T$$

$$p_j^{t-1} - p_j^t \leq RD_j v_j^{t+1} + SD_j z_j^t + RD_j z_j^{t+1} - (RD_j - SU_j + \underline{P}) y_j^{t-1} - (RD_j + \underline{P}_j) y_j^t - RD_j y_j^{t+1}$$

$$j \in \{k \in J | RD_k > SU_k - \underline{P}_k,\ UT_k \geq 3, DT_j \geq 2\}, t \in \{1,\ldots,|T|-1\}$$

$$p_j^{t-2} - p_j^t \leq 2RD_j v_j^t + SD_j z_j^{t-1} + (SD_j + RD_j) z_j^t - 2RD_j y_j^{t-2} - (2RD_j + \underline{P}) y_j^{t-1} - (2RD_j + \underline{P}) y_j^t$$

$$j \in J, t \in \{2,..,|T|-2\}$$

$$p_j^t - p_j^{t-1} \leq RU_j v_j^t - \underline{P}_j z_j^t - (RU_j - SD_j + \underline{P}_j) z_j^{t+1} + (SU_j - RU_j) y_j^t$$

$$j \in \{k \in J | RU_k > SD_k - \underline{P}_k \text{ and } UT_k \geq 2\}, t \in \{1,\ldots,|T|-1\}$$

$$p_j^t - p_j^{t-2} \leq 2RU_j v_j^t - \underline{P}_j z_j^{t-1} - \underline{P}_j z_j^t + (SU_j - RU_j) y_j^{t-1} + (SU_j - 2RU_j) y_j^t$$

$$j \in \{k \in J | RU_k > SD_k - \underline{P}_k, UT_k \geq 2 \text{ and } DT_k \geq 2\}, t \in \{2,\ldots,|T|-2\},$$

where $K(t) = \max\{k \in \{1,...,UT_j\} | SD_j + (k-1)RD_j < \overline{P}_j \text{ and } k+t \leq |T|\}$.

Variables:

$c_j^p(t), c_j^u(t), c_j^d(t), \delta_l(j,t) \geq 0 \qquad j \in J, t \in T$

$v_j(t), y_j(t), z_j(t) \in \{0,1\} \qquad j \in J, t \in T$

Appendix B

Micro-grid internal management

This section provides the full description of the micro-grid internal management problem appearing at the bi-level formulation proposed in Section 4.2.

For simplicity, a bus model is considered for power flows, that is, the power network of a micro-grid is not taken into consideration. We define two types of unit components in the micro-grid, which are called devices in the following. Storage devices typically represent batteries, whose status may switch between online (connected to the grid) and offline during the time horizon. During its offline periods, a storage device can be unloaded: for example, a battery car is loaded during the night. During this time period, it can be used to store and serve power, but it must be fully loaded at the end of the night. During the day, the car is used and its battery is emptied so that its storage level is low when it is back online. Each storage device is associated to a set of online time intervals. At the beginning of such a time interval, the charge level is a stochastic input parameter (for example, it depends on how much the car has been used during the day). The required charge level at the end of the online period is modelled through more general parameters giving minimum and maximum acceptable charge levels for each online time step. We also assume that each storage device has limited capacity, charging and discharging speed and a power loss factor that is the proportion of power stored to the power consumed during the charge.

The regular devices come with stochastic consumptions and productions of power during each time step. Some of their consumption can be partially delayed (elastic demand). We model this feature by defining a set of time intervals for each device (for example, a water heater must heat the water during the night). During each of them, the required total power consumption is known, as a stochastic input data. The maximum power consumption of devices is limited during each time step.

The decisions to be taken in the micro-grid problem are, first, to choose a contract among those proposed by the GCs. Then, for each regular device and each time step, the amount of elastic power consumption must be determined. For each storage device, the amounts of power consumed (to charge) and released must be fixed. We provide below a model for the problems faced by each one of the micro-grids.

Parameters

Scenario-independent parameters:

- K: set of available contracts, defined by c_k, f_k and g_k. We assume that $\forall t \in \mathscr{T}$, $f_{kt} \geq g_{kt}$.
- D: set of non-storage devices. For each device $d \in D$:
  - Θ_D^d: set containing sets of time periods defining elastic consumption slots
- S: set of storage devices. For all $d \in S$:
  - $\bar{s}^d$: capacity of storage device d
  - $\bar{\ell}^d$: maximum power used to reload d during one time period
  - $\bar{u}^d$: maximum power released by d during one time period
  - α^d: power loss factor when charging d
  - Θ_S^d: set of time intervals when d is online (can be charged or discharged). We note, for all $\theta \in \Theta_S^d$, $\theta = [t^-(\theta), t^+(\theta)]$.
  - $\underline{S}_t^d, \bar{S}_t^d$: minimum and maximum charge level for d at time t.

Scenario-dependent parameters: For all scenario $\xi \in \Xi$

- For each device $d \in D$:
  - $\forall t \in \mathscr{T}$:
    - $b_t^d(\xi)$: power production of d during period t in scenario ξ
    - $r_t^d(\xi)$: power consumption of d during period t in scenario ξ
    - $\bar{w}_t^d(\xi)$: maximum possible elastic consumption of d during t
  - $\forall \theta \in \Theta_D^d$
    - $e_\theta^d(\xi)$: total elastic power demand of d during θ in scenario ξ

- For each storage device $d \in S$ and online interval $\theta \in \Theta_S^d$: $I_\theta^d(\xi)$ is the initial stock level when d is plugged in.

Decision variables

- Stage 0:
  - $\forall k \in K$: $z_k = 1$ if contract k is chosen by the micro-grid, 0 otherwise
- Stage $t, t \in \mathscr{T}$:
  - $x_t(\xi)$: power consumed by the micro-grid during period t
  - $y_t(\xi)$: power produced by the micro-grid during period t
    * $\forall d \in D$, $w_t^d(\xi)$: elastic power consumed by d during t
    * For all $d \in S$:
      · $s_t^d(\xi)$: power stock in d at the end of t
      · $\ell_t^d(\xi)$: power consumed to charge device d during t
      · $u_t^d(\xi)$: power released by discharging device d during t

Formulation

$$
\begin{aligned}
\min \quad & \mathbb{E}\left[\sum_{k\in K}\sum_{t\in\mathscr{T}} (f_{kt}x_t(\xi) + g_{kt}y_t(\xi) + c_k)\, z_k\right] && \text{(B.1a)}\\
\text{s.t.} \quad & \sum_{k\in K} z_k = 1 && \text{(B.1b)}\\
& x_t(\xi) - y_t(\xi) = \sum_{d\in D}\left(r_t^d(\xi) + w_t^d(\xi) - b_t^d(\xi)\right) + \sum_{d\in S}\left(\ell_t^d(\xi) - u_t^d(\xi)\right) \quad \forall t,\xi && \text{(B.1c)}\\
& s_t^d(\xi) = s_{t-1}^d(\xi) + \alpha^d \ell_t^d(\xi) - u_t^d(\xi) \quad \forall d\in S, \theta\in\Theta_S^d, t\in\theta - \{t^-(\theta)\}, \xi && \text{(B.1d)}\\
& s_{t^-(\theta)}^d(\xi) = I_\theta^d(\xi) + \alpha^d \ell_{t^-(\theta)}^d(\xi) - u_{t^-(\theta)}^d(\xi) \quad \forall d\in S, \theta\in\Theta_S^d, \xi && \text{(B.1e)}\\
& \sum_{t\in\theta} w_t^d(\xi) = e_\theta^d(\xi) \quad \forall d\in D, \theta\in\Theta_D^d, \xi && \text{(B.1f)}\\
& w_t^d(\xi) \le \bar{w}^d \quad \forall d\in D, t, \xi && \text{(B.1g)}\\
& \underline{S}_t^d \le s_t^d(\xi) \le \bar{S}_t^d \quad \forall d\in S, t, \xi && \text{(B.1h)}\\
& \ell_t^d \le \bar{\ell}^d \quad \forall d\in S, t && \text{(B.1i)}\\
& u_t^d \le \bar{u}^d \quad \forall d\in S, t && \text{(B.1j)}
\end{aligned}
$$

$$\ell_t^d(\xi) = u_t^d(\xi) = 0 \quad \forall d \in S, \theta \notin \Theta_S^d, t \in \theta \tag{B.1k}$$

$$y_k \in \{0,1\} \quad \forall k \in K \tag{B.1l}$$

$$x, y, w, r, s, l, u \geq 0 \tag{B.1m}$$

The objective of the problem (B.1a) is to minimize the total cost for the micro-grid, which is composed of the fixed cost of the contract, the cost of buying power from the GCs minus the income obtained from selling the over-production. Constraint (B.1b) states that exactly one contract must be chosen. Constraints (B.1c) ensure that the power flow into/out of the micro-grid is equal to its production/consumption during each time step. In the right-hand-side, the summation over D (resp. S) represents the total consumption/production of regular (resp. storage) devices. Constraints (B.1d) and (B.1e) define the level of power stock for each device and time step. Constraints (B.1f) fix the correct total amount of power that must be consumed by a device during an elastic consumption interval. The instantaneous power consumed by a device is limited by constraints (B.1g). The acceptable stock levels are bound by constraints (B.1h). Constraints (B.1i) and (B.1j) define maximum charging and discharging speeds for the storage devices, while constraints (B.1k) are just a way to state that an offline device cannot be charged or discharged (the corresponding variables may as well be omitted in the model). The domains of the variables are given in constraints (B.1l) and (B.1m).

Appendix C

Bidding in Price Coupled Regions

The full MILP formulation of three problems consider in Chapter 6 are provided bellow. Notations are as defined as in the corresponding chapter.

C.1 BPUC-MILP formulation

Full formulation of BPUC considering the GC bids at the spot price.

$$
\begin{aligned}
\max \quad & \sum_{n\in N}\left(\sum_{t\in T}\sum_{i\in I_n^t}\tilde{\lambda}_i^t P_{in}^t\right) - c(p_n) && && \\
\text{s.t.} \quad & p_n \in P_n && n\in N && \text{(C.1a)}\\
& \sum_{b\in B_n} Q_b^t X_{ib}^t - \sum_{s\in S_n} Q_s^t X_{is}^t && && \\
& \quad + \sum_{m\in\Theta_m}(\overline{F}_{inm}^t - \underline{F}_{imn}^t) = P_{in}^t && t\in T, n\in N, i\in I^t && \\
& \sum_{i\in I_n^t}\tilde{\lambda}_i^t(z_{in}^t - X_{ib}^t) - \pi_b^t(1-x_b^t) \geq 0 && t\in T, n\in N, b\in B_n && \\
& -\sum_{i\in I_n^t}\tilde{\lambda}_i^t(z_{in}^t - X_{is}^T) + \pi_s^t(1-x_s^t) \geq 0 && t\in T, n\in N, s\in S_n && \\
& -\sum_{i\in I_n^t}\tilde{\lambda}_i^t X_{ib}^t + \pi_b^t x_b^t \geq 0 && t\in T, n\in N, b\in B_n && \\
& \sum_{i\in I_n^t}\tilde{\lambda}_i^t X_{is}^t - \pi_s^t x_s^t \geq 0 && t\in T, n\in N, s\in S_n && \\
& \sum_{i\in I_n^t}\tilde{\lambda}_i^t z_{in}^t - \sum_{i\in I_m^t}\tilde{\lambda}_i^t z_{im}^t + r_{nm}^t - r_{mn}^t = 0 && t\in T, nm\in A && \\
& x_b^t = 0 && t\in T, n\in N, b\in B_n : \pi_b^t < \tilde{\lambda}_{\underline{i}_n^t}^t &&
\end{aligned}
$$

$$
\begin{aligned}
& x_b^t = 1 && t \in T, n \in N, b \in B_n : \pi_b^t > \tilde{\lambda}_{\bar{i}_n^t}^t \\
& x_s^t = 0 && t \in T, n \in N, b \in S_n : \pi_s^t > \tilde{\lambda}_{\bar{i}_n^t}^t \\
& x_s^t = 1 && t \in T, n \in N, b \in S_n : \pi_s^t < \tilde{\lambda}_{\underline{i}_n^t}^t \\
& C_{nm}^{max} r_{nm}^t = \sum_{i \in I_n^t} \tilde{\lambda}_i^t \underline{F}_{inm}^t - \sum_{i \in I_n^t} \tilde{\lambda}_i^t \overline{F}_{inm}^t && t \in T, nm \in A \\
& \sum_{i \in I_n^t} z_{in}^t = 1 && t \in T, n \in N \\
& \sum_{i \in I_n^t} P_{in}^t = p_n^t && t \in T, n \in N \\
& \sum_{i \in I_n^t} X_{ib}^t = x_b^t && t \in T, n \in N, b \in B \\
& \sum_{i \in I_n^t} X_{is}^t = x_s^t && t \in T, n \in N, s \in S \\
& \sum_{i \in I_n^t} \overline{F}_{inm}^t = f_{nm}^t && t \in T, nm \in A \\
& \sum_{i \in I_n^t} \underline{F}_{inm}^t = f_{nm}^t && t \in T, nm \in A \\
& 0 \leq P_{in}^t \leq \overline{Q}_n^t z_{in}^t && t \in T, n \in N, i \in I_n^t \\
& P_{in}^t \leq p_n^t && t \in T, n \in N, i \in I_n^t \\
& P_{in}^t \geq p_n^t - \overline{Q}_n^t (1 - z_{in}^t) && t \in T, n \in N, i \in I_n^t \\
& 0 \leq X_{ib}^t \leq z_{in}^t && t \in T, n \in N, b \in B_n, i \in I_n^t \\
& X_{ib}^t \geq x_b^t + z_{in}^t - 1 && t \in T, n \in N, b \in B_n, i \in I_n^t \\
& 0 \leq X_{is}^t \leq z_{in}^t && t \in T, n \in N, s \in S_n, i \in I_n^t \\
& X_{is}^t \geq x_s^t + z_{in}^t - 1 && t \in T, n \in N, s \in S_n i \in I_n^t \\
& 0 \leq \overline{F}_{inm}^t \leq C_{nm}^{max} z_{in}^t && t \in T, nm \in A, i \in I_n^t \\
& \overline{F}_{inm}^t \geq f_{nm}^t - C_{nm}^{max} (1 - z_{in}^t) && t \in T, nm \in A, i \in I_n^t \\
& 0 \leq \underline{F}_{inm}^t \leq C_{nm}^{max} z_{im}^t && t \in T, nm \in A, i \in I_n^t \\
& \underline{F}_{inm}^t \geq f_{nm}^t - C_{nm}^{max} (1 - z_{im}^t) && t \in T, nm \in A, i \in I_n^t \\
& 0 \leq x_b^t \leq 1 && t \in T, n \in N, b \in B_n \\
& 0 \leq x_s^t \leq 1 && t \in T, n \in N, s \in S_n \\
& 0 \leq f_{nm}^t \leq C_{nm}^{max} && t \in T, nm \in A \\
& 0 \leq r_{nm}^t && t \in T, nm \in A \\
& z_{in}^t \in \{0, 1\} && t \in T, n \in N, i \in I_n^t
\end{aligned}
$$

C.2 BPUC^M-MILP formulation

Variant of BPUC where the GC bids at the marginal production costs of generators. Consider $\overline{Q}^t_{jn}$ as the maximum production capacity of generator j at node n in period t and variables P^t_{ijn} as a disaggregation of variables P^t_{in} by generator.

BPUC^M-MILP is obtained from BPUC-MILP, replacing constraints (C.1a) by

$$p_{nj} \in P^j_n \qquad n \in N, j \in J_n,$$

and by adding the following constraints:

$$\begin{aligned}
& p^t_n = \sum_{j \in J_n} p^t_{jn} && t \in T, n \in N \\
& P^t_{in} = \sum_{j \in J_n} P^t_{ijn} && t \in T, n \in N, i \in I^t_n \\
& \sum_{i \in I^t_n} \tilde{\lambda}^t_i P^t_{ijn} - \pi^t_{jn} p^t_{jn} \geq 0 && t \in T, n \in N, j \in J_n \\
& \sum_{i \in I^t_n} P^t_{ijn} = p^t_{jn} && t \in T, n \in N, j \in J_n \\
& 0 \leq P^t_{ijn} \leq \overline{Q}^t_{jn} z^t_{in} && t \in T, n \in N, j \in J_n, i \in I^t_n \\
& P^t_{ijn} \leq p^t_{jn} && t \in T, n \in N, j \in J_n, i \in I^t_n \\
& P^t_{ijn} \geq p^t_{jn} - \overline{Q}^t (1 - z^t_{in}) && t \in T, n \in N, j \in J_n, i \in I^t_n
\end{aligned}$$

C.3 BPUC-$\{N\}$ formulation

Relaxation of constraints related to the transmission network in BPUC.

$$\begin{aligned}
\max \quad & \sum_{t \in T} \sum_{i \in I^t} \tilde{\lambda}^t_i P^t_i - c(p) \\
\text{s.t.} \quad & p \in P \\
& \sum_{i \in I^t} z^t_i = 1 && t \in T \\
& \sum_{i \in I^t} P^t_i = p^t && t \in T \\
& P^t_i \leq r^t_i z^t_i && t \in T, i \in I^t \\
& P^t_i \geq r^t_{i+1} z^t_i && t \in T, i \in I^t \\
& 0 \leq P^t_i \leq \overline{Q}^t z^t_i && t \in T, i \in I^t
\end{aligned}$$

$$
\begin{aligned}
&P_i^t \leq p^t && t \in T, i \in I^t \\
&P_i^t \geq p^t - \overline{Q}^t(1 - z_i^t) && t \in T, i \in I^t \\
&z_i^t \in \{0, 1\} && t \in T, i \in I^t
\end{aligned}
$$