2022-09

REVISITING COORDINATED SUBMARINE TACTICS USING MODERN COMPUTATIONAL METHODS

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http://hdl.handle.net/10945/71111

NAVAL POSTGRADUATE SCHOOL

MONTEREY, CALIFORNIA

THESIS

REVISITING COORDINATED SUBMARINE TACTICS USING MODERN COMPUTATIONAL METHODS

by

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September 2022

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REPORT DOCUMENTATION PAGE				*Form Approved OMB No. 0704-0188*
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instruction, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188) Washington, DC 20503.				
1. AGENCY USE ONLY ***(Leave blank)***	**2. REPORT DATE** September 2022	**3. REPORT TYPE AND DATES COVERED** Master's thesis		
4. TITLE AND SUBTITLE REVISITING COORDINATED SUBMARINE TACTICS USING MODERN COMPUTATIONAL METHODS				**5. FUNDING NUMBERS**
6. AUTHOR(S) Spencer A. Kitten				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey, CA 93943-5000				**8. PERFORMING ORGANIZATION REPORT NUMBER**
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES) N/A				**10. SPONSORING / MONITORING AGENCY REPORT NUMBER**
11. SUPPLEMENTARY NOTES The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.				
12a. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release. Distribution is unlimited.				**12b. DISTRIBUTION CODE** A

13. ABSTRACT (maximum 200 words)

Current doctrine has largely discarded the use of coordinated submarine tactics (known as Wolfpack tactics) due to the complexity of inter-pack and intra-pack coordination. However, recent advancements in technology may greatly increase the feasibility of secure communication between submarines operating in a Wolfpack. Agent-based modeling is used to simulate the behavior of submarines operating in a wartime environment at sea. Three secure communication availabilities are represented: no communication between submarines, communication every 10 hours, and constant secure communication. Three types of wartime environments are considered: submarines hunting transiting merchants, submarines hunting transiting warships in an environment with neutral shipping, and submarines hunting transiting warships operating as a Surface Action Group (SAG) with neutral shipping. Effectiveness is measured as "yield," which is the average number of target kills as a function of the number of submarines in the Wolfpack. The simulation results stress that the success of Wolfpack tactics increasingly depends upon secure submarine communication and situational awareness with the growth of neutral shipping in the wartime environment.

14. SUBJECT TERMS submarine, Wolfpack, coordinated, attack, ASUW			**15. NUMBER OF PAGES** 113
			16. PRICE CODE
17. SECURITY CLASSIFICATION OF REPORT Unclassified	**18. SECURITY CLASSIFICATION OF THIS PAGE** Unclassified	**19. SECURITY CLASSIFICATION OF ABSTRACT** Unclassified	**20. LIMITATION OF ABSTRACT** UU

NSN 7540-01-280-5500 Standard Form 298 (Rev. 2-89) Prescribed by ANSI Std. 239-18

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REVISITING COORDINATED SUBMARINE TACTICS USING MODERN COMPUTATIONAL METHODS

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requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
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ABSTRACT

Current doctrine has largely discarded the use of coordinated submarine tactics (known as Wolfpack tactics) due to the complexity of inter-pack and intra-pack coordination. However, recent advancements in technology may greatly increase the feasibility of secure communication between submarines operating in a Wolfpack. Agent-based modeling is used to simulate the behavior of submarines operating in a wartime environment at sea. Three secure communication availabilities are represented: no communication between submarines, communication every 10 hours, and constant secure communication. Three types of wartime environments are considered: submarines hunting transiting merchants, submarines hunting transiting warships in an environment with neutral shipping, and submarines hunting transiting warships operating as a Surface Action Group (SAG) with neutral shipping. Effectiveness is measured as "yield," which is the average number of target kills as a function of the number of submarines in the Wolfpack. The simulation results stress that the success of Wolfpack tactics increasingly depends upon secure submarine communication and situational awareness with the growth of neutral shipping in the wartime environment.

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List of Acronyms and Abbreviations

AI Artificial Intelligence

AOU area of uncertainty

ASUW Anti-Surface Warfare

ASW Anti-Submarine Warfare

ASWORG U.S. Navy Antisubmarine Warfare Operations Research Group

CLT Central Limit Theorem

CAG coordinated attack groups

CDF Continuous Distribution Function

COIN Counterinsurgency Operations

DOD Department of Defense

GWOT Global War on Terror

HPC High Performance Computing

HVU High Value Unit

IJN Imperial Japanese Navy

ISR Intelligence, Surveillance, and Reconnaissance

MASINT measurement and signature intelligence

ML Machine Learning

MLE maximum likelihood estimator

NOLH Nearly Orthogonal Latin Hypercubes

NPS Naval Postgraduate School

OR Operations Research

PLA(N) Peoples Liberation Army (Navy)

PRNG pseudorandom number generator

SAG Surface Action Group

SEED Simulation Experiments & Efficient Designs

SONAR sound navigation and ranging

SORG Submarine Operations Research Group

SRWBR short range wide band radio

TCP Transmission Control Protocol

USN U.S. Navy

USSR Union of Soviet Socialist Republics

USW undersea warfare

UUV Unmanned Undersea Vehicle

WWI World War I

WWII World War II

Executive Summary

Current doctrine has largely discarded the use of coordinated submarine tactics (known as Wolfpack tactics) due to the complexity of inter-pack and intra-pack coordination. Recent advancements in autonomous undersea localization, communication, and other technologies have led to an increase in feasibility of coordinated operation between undersea vessels. The rise of such technologies requires rethinking the current warfighting doctrine around a more survivable and lethal undersea capability. Thoughtful explorations of the subject have already begun, such as Cares and Cowden (2021) careful analysis of the future of fleet tactics in the age of distributed warfare. Revisiting Wolfpack tactics will provide value to naval thinkers as they continue to search for optimal strategies in the coming naval warfighting landscape of increasingly autonomous unmanned systems.

Simulation is used to explore the effectiveness of Wolfpack tactics in seven different scenarios, modifying both the wartime environment and the frequency of information available to the submarines. Analysis of simulation output is performed to identify how the success of Wolfpack tactics varies under different constraints on secure communication between submarines and target environment. Three secure communication availabilities are considered: no communication between submarines, communication every 10 hours, and constant available communication. Three target environments are considered: submarines hunting transiting merchants, submarines hunting transiting warships in an environment containing neutral shipping, and submarines hunting transiting warships operating as a Surface Action Group (SAG) in an environment with neutral shipping. Submarines are assigned non-overlapping waterspaces, and each will not depart its assigned waterspace nor attempt to kill targets outside of their own waterspace. Vessels enter the first waterspace, then travel to the second waterspace and so on. A submarine approaches and classifies vessels in its waterspace as neutral or target. Classification is perfect and the submarine kills vessels classified as a target. Weapons fired by a submarine in one waterspace will not affect submarines in other waterspaces nor the behavior of shipping. Effectiveness is measured as the "yield", which is defined as the average number of successful target kills as a function of the number of submarines in the Wolfpack. Submarines have no logistic constraints, have no ammunition restrictions, and adversary warships will not attempt to destroy any submarines.

When submarines communicate, they communicate perfect information to an off-hull entity with no loss of stealth or chance of interdiction. A submarine only communicates to the off-hull entity when the submarine detects a target but is unable to prosecute it before the target leaves its waterspace. The information passed is the locations of and future paths of targets that the submarine cannot prosecute. Only the submarine in the next waterspace receives the information from the off-hull entry; it accesses the information at deterministic times. No information concerning neutral shipping is passed. As a result, though there exists incentive to communicate, the effect of communication on the average number of Targets killed is limited. Communication most improved the performance of the Wolfpack in a wartime environment with a very small Target to neutral Merchant arrival rate ratio, allowing a submarine to better act on destroying a surface action group concealed in a large amount of neutral shipping traffic.

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Acknowledgments

I would like to thank my wife, Rosemary Kitten, for all of the love and support she has given me to continue my education and help raise our two wonderful children, Ella Kitten and Brooks Kitten. I would also like to thank the Naval Postgraduate School (NPS) faculty and staff who were not directly involved with my thesis, but lent their time and consideration anyway: Stephen Upton of Simulation Experiments & Efficient Designs (SEED) Center for Data Farming, for being an enormous help with getting me started with Hamming. Dr. Schuchard, for greatly improving my acumen for writing software. Dr. Sanchez for her knowledgeable support with understanding the Data Farming process. And CAPT Kline USN (Ret.) and CAPT Cares USN (Ret.) for initially showing me this very interesting topic, and for keeping me thinking operationally. Finally, I would like to graciously thank my advisor, Dr. Louis Chen, for his guidance and patience as this topic has sporadically evolved into the polished product it is now, and to my second reader, Dr. Patricia Jacobs, for her observant eye, attention to detail, high standards, and assistance with portions of this work.

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CHAPTER 1: Introduction: The Role of Submarine Wolfpack Tactics in the Larger Undersea Warfare Picture

The strategy of the United States (U.S.) military in the emerging world of great power competition heavily depends on its capabilities in the undersea warfare (USW) domain. The importance of USW is reflected in recent thought, planning, analysis, and strategy from various Department of Defense (DOD) groups and affiliated support agencies. The JP 3-32 (Joint Chiefs of Staff 2021) specifies that the need to control "the undersea portion of the operational area is vital to the success of joint operations" and the *Renewed Great Power Competition* discussion by Congressional Research Service (2022) clearly defines that the undersea domain as a specific area of interest alongside space and cyberspace.

At the fall of the Soviet Union in 1991, the U.S. Navy (USN) suddenly enjoyed a significant advantage in the undersea domain. Construction of an entire class of advanced fast-attack submarines were halted (Mizokami 2020), important acoustic detection systems were mothballed or transferred to civilian use (Whitman 2005), and the once active Anti-Submarine Warfare (ASW) capability of the USN found itself with no capable near-peer competitors (Benedict 2005). The submarine force slowly adapted itself to the world of Intelligence, Surveillance, and Reconnaissance (ISR) operations in littoral areas, supporting the new Counterinsurgency Operations (COIN) focus during the era of Global War on Terror (GWOT) operations. This is not to imply the complete degradation of capability of the U.S. military in the USW domain as some key capabilities of the U.S. Submarine Force still remain strong (Zhao 2018). However, Clark (2015) suggests that the advantage enjoyed by the USN may come to an end in the near future with the rise of near-peer competitors such as the Peoples Liberation Army (Navy) (PLA(N)).

An important feature of USW is submarine warfare, and a famous but now abandoned tactic of submarine employment is the use of submarine Wolfpack Tactics. At its most basic level, Wolfpack Tactics are tactics employed by a group of coordinated submarines against an opponent. With limited and rare exceptions (Submarine Operations Research Group 1944; Frost 1980), there has been little work done on practicing or advancing the idea of

Wolfpack tactics. There is good reason for hesitance; coordinating multiple warships at sea is no trivial matter. Submarines typically are not in constant communication and typically can only communicate off-hull at any significant bandwidth by coming up in depth shallow enough to expose an antenna above the sea surface, placing the boat in great peril. Torpedo employment comes with an inherent risk of friendly fire without careful coordination, which would be absolutely intolerable in the modern era of multi-billion dollar submarines. Also, most modern military submarines are significantly quiet vehicles, exposing the submarines to a risk of collision with other members of the Wolfpack if their waterspace is not managed effectively.

Recent advancements in autonomous Unmanned Undersea Vehicle (UUV) localization (Howarth II 2022), acoustic and laser undersea communication (Mclaughlin 2015), and other technologies have led to an increase in feasibility in coordinated operation between undersea vessels. The rise of such technologies will require rethinking the current warfighting doctrine around a more survivable and lethal undersea capability. Thoughtful explorations of the subject have already begun, such as Cares and Cowden (2021), who provide careful analysis of the future of fleet tactics in the age of distributed warfare. It is suggested that revisiting Wolfpack tactics using data farming methods will provide value to naval thinkers as they continue to search for optimal strategies in the coming naval warfighting landscape of increasingly autonomous unmanned systems.

1.1 A Bit of History

Before exploring the utility of Wolfpack tactics on the modern battlefield, it is crucial to understand the history of submarine warfare. The tactics used by submarines have evolved through history, guided mostly by changes in technology. These changes have evolved the submarine from a commerce raiding torpedo boat, to operating as a group against an organized foe, to the modern solitary and mysterious attack submarine. War has changed the submarine as much as the submarine has changed warfare.

1.1.1 WWI to the Interwar Period: Unrestricted Submarine Warfare and Strategic Planning

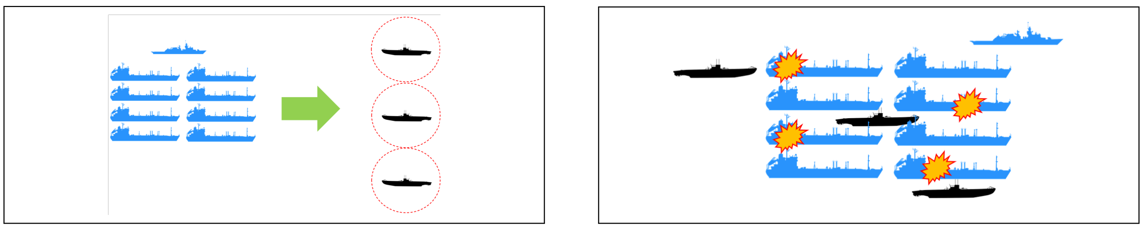
Near the end of World War I in 1918, the German submarine UB-68 was operating in the Mediterranean. After suffering from technical difficulties, the beleaguered vessel surfaced and was ordered scuttled in order to keep it out of enemy hands. The boats captain, the infamous young Karl Dönitz, would spend the remainder of the war in Redmires prison. He contemplated deeply improvements to the anti-convoy tactics utilized by the German Navy. Unrestricted submarine warfare was used by the German Navy, allowing any vessel flying under the flag of the adversary to be torpedoed by the submarine from a concealed position. This had taken a significant toll on the British war effort but was not able to completely isolate the enemy (Rohwer 2015). Insistence in the idea of the "freedom of the seas" by the Wilson administration during the war required that the German Navy move from unrestricted submarine warfare to restricted submarine warfare by surfacing and forcing the merchant to surrender before engaging. The submarine would then search the vessel and allow time for the crew and passengers to disembark before sinking the ship, a highly impractical maneuver that was met by surprise attacks from the merchant on more than one occasion. Germany did not elect to satisfy the U.S. for long and eventually resumed unrestricted submarine warfare, believing it could achieve victory prior to significant U.S. involvement. This would prove to be a costly mistake. The captured German submarine Captain was not alone, as Holwitt (2009) elucidates an entire interwar history as even the U.S. would come to realize the possible utility of unrestricted submarine warfare and quietly add it the official U.S. war plans just before World War II (WWII). Germany would prepare their own submarine fleet during the interwar period, according to the vision of the now Grand Admiral, Karl Dönitz.

The British would also be conducting their own analysis of the effectiveness of submarine tactics against their own naval strategy, and would begin a pivot towards having materiel transit by merchants operating in large convoys. "By October [1917] over 1,500 merchant ships in about 100 convoys had reached the British Isles. Only ten ships were lost to U-boats while sailing in the convoys: one ship out of 150. By comparison, the loss rate for ships sailing independently (inbound or otherwise) was one in ten" (Blair 2000). Though transiting merchants found safety in numbers, it was thought that there could be a large increase in effectiveness by having the attackers also operate in larger numbers. Dönitz would revive an older idea from a different German Naval Officer, Hermann Bauer, to group several submarines together and have them operate as a "Wolfpack" (Rudeltaktik) in order to engage the convoy effectively.

1.1.2 Wolfpack Tactics: From Early Struggles, to "The Happy Times," to a Bitter End

In 1939, Dönitz gets his chance to test Wolfpack tactics, now as Commander of the Submarine Fleet (*Konteradmiral*) for the German Navy. The size of the Wolfpack would be six U-boats, and the results of the first attempt yielded two drastically different interpretations. One competing viewpoint by Blair (2000) held that "in reality, the first pack was so far a disaster: three of its six U-boats sunk; only four ships of the Caribbean convoy [HG3] positively sunk, one of them a prohibited passenger liner". The situation appeared differently to Dönitz. "From the German perspective the assault on HG32, which warrants no notice in British accounts, was a qualified success: proof of the basic concept of pack attack convoys" (Milner 2003). The U-boats were able to achieve some of their intended goal, and the three lost U-boats were sunk en route to the battlespace vice as a result of combat action while operating as a Wolfpack. What was needed was a few more reattempts in order to hone in the best possible strategy for the U-boats to use operating in Wolfpacks.

The second attempt would be less promising than the first, to both parties: it was less successful overall; there were significant challenges with command and control of the group; and it appeared that U-boats would be more successful operating independently in a target-rich environment (Rohwer 2015). The target rich environment would soon dwindle as the British began operating in protected convoys in earnest following the Battle of Dunkirk, but still left the issue of command and control. Instead of one of the U-boats having tactical control of the situation, with the group suffering from a lack of coordination if the leader was forced under or sunk, it was decided by Dönitz to have the control of the Wolfpack coordinated onshore and communicated to the submarines at once (those astute in history will recognize here the introduction of the Enigma Machine). As shown in Figure 1.1, U-boats would typically form a "barrier" at sea, wait until contact is made with an approaching convoy, inform and receive direction from German High Command via encrypted communications, and finally commence the coordinated attack. The last key to executing this strategy was the preference of German submarines to attack while surfaced and at night. This would afford the submarine commander the maximum likelihood of successfully sinking its intended target. “Dönitz’s wolf pack strategy brought them together, but once in contact with the enemy, each U-boat skipper attacked independently” (Symonds 2018). This scheme would prove to be immensely effective against Allied shipping, and would send tens of millions of tons of badly needed materiel to the bottom of the sea.



(a) Barrier of U-Boats encounter incoming convoy
(b) Wolfpack, under external direction, converges and attacks

Figure 1.1. Stages of Wolfpack Attack

U-boat successes would continue to mount until March of 1943, where they would reach their apex: “the U-boats came nearest to their aim of interrupting the lines of communication between the Old World and the New when they sank 39 merchant ships out of four successive convoys” (Rohwer 2015). Ultimately, continued advancements in technology, tactics, and organization by the Allies would result in the nullification of the element of stealth and surprise of the U-boat. High-frequency direction finding (huff-duff) would raise the cost of communication by a U-boat to instant counter-detection, and cryptologic efforts by Turing’s team at the Bletchley Park would break the enigma machine, exposing the vital communication between the U-boat and German High Command. No less than two months later would the German Navy find themselves reeling from the loss of forty-one submarines, to include the loss of Dönitz’s own son (Milner 2003). Dönitz would declare the Battle of the Atlantic lost and withdraw his U-boats from the mid-Atlantic.

1.1.3 Experimentation by the United States

Although the decision to wage unrestricted submarine warfare against Japan in the opening stages of WWII had been made by American naval strategists prior to the commencement of hostilities, the USN had yet to fully appreciate the effect that a submarine (or group of submarines) could have in a war at sea. It was still the era of Mahan-ian style fleet concentration and the idea of decisive battles. The Imperial Japanese Navy (IJN) had certainly paid attention to Mahan, and commenced the War in the Pacific with an opening attack at Pearl Harbor, concentrating effort against the USN's battleships. The aircraft carriers were not present for the air raid and the submarine piers were untouched (Miller 1997). It would be a costly mistake by the IJN; USN submarines across the pacific would immediately be put to sea to wage *guerre de course* against the Empire of Japan. The effort of the USN submarine force would not immediately come to fruition due to entrenched risk-adverse tactics, faulty torpedoes, and an unclear vision of the submarine as a tool of war. These early failures would lead the submarine force to lag significantly behind in tonnage sunk as compared to the German Navy (Hoffman 2016).

Under the leadership of Admiral Lockwood, and after the other significant mechanical and tactical mindset issues were corrected within the submarine force, American experiments in Wolfpack tactics began in late 1943. Instead of Wolfpacks, groups of American submarines were called coordinated attack groups (CAG). Doctrine was modified slightly from the operational methods of the German Navy, particularly in terms of command and control. Instead of coordinated from ashore, the American submarine group would grant one of the afloat officers tactical control of the group. Also, Roscoe (1949) shows that instead of converging attack as a group, various submarines within the CAG would be sent sequentially into action so as to avoid friendly fire.

The last major submarine CAG attempt was made by Captain Earl Hydeman, which is retold in detail by Smith (2003). Nine submarines, operating as three Wolfpacks, would depart for the Tsushima strait. Outfitted with brand-new sound navigation and ranging (SONAR) technology, the CAGs were able to successfully penetrate past Japanese minefields and sink 27 Japanese vessels at the cost of one American submarine. Overall, a total of 65 different Wolfpacks deployed from Hawaii with great effect (Hoffman 2016).

1.1.4 The Modern Submarine and the Retirement of Wolfpack Tactics

The end of WWII would naturally bring a reduction in military expenditures. Grateful servicemembers would return home to their families and many of the now unneeded platforms that they served on would be retired from service. However, the world would not return to a state of peace, and the way submarines would be used would change dramatically. From the defeated Germany would come the modern fleet submarine that never was: the Type XXI, a submarine that would have had a major impact in the Battle of the Atlantic. Just two of these advanced submarines would be given to the U.S. for study, with twelve given to the Union of Soviet Socialist Republics (USSR). These would later become the basis of the ubiquitous USSR Whiskey-class diesel submarine (United States Submarine League et al. 2002).

The submarine of the future would have to improve in many areas to evolve from commerce raiding torpedo boats to a modern submarine. These innovations would include submarines that could dive deeper, drive faster, and remain undetected in the new Cold War era of espionage. Advancements in hull design would allow operating depths to dramatically increase. Friedman and Christley (1994) provides a rich history of the enhancements in acoustic technology provided to submarines, from detection methods to changes in machinery alignment to facilitate quieting. The most crucial development of all however, would come from the brainchild of Admiral Rickover's effort. In 1955 the USS Nautilus (SSN - 571) would signal that she was "Underway on Nuclear Power" and later become the first vehicle to transit over the North Pole, and do so submerged underneath the thick sea-ice above (United States Submarine League et al. 2002). Submarine design would continue to evolve in a direction that would lead it back to operating as an independent vehicle: from carrier-escort to a true fast-attack submarine, capable of ASW, Anti-Surface Warfare (ASUW), and ISR. Figure 1.2 shows the changes in hull shape from WWII to the modern attack submarine.

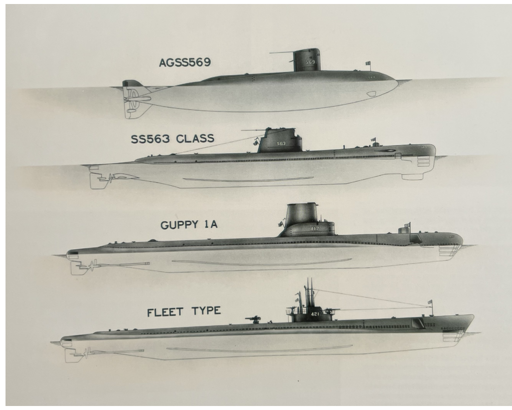


Figure 1.2. The evolution of the post-WWII diesel submarine from fleet boat (bottom) to revolutionary fast-attack submarine. Source: (United States Submarine League et al. 2002).

The ocean as a battlefield has changed. Tournadre (2014) suggests a fourfold increase in global shipping traffic between the years 1992 and 2012, concentrated mostly in the Indian Ocean and China Seas. The need for throughput of a warfighter, or its ability to destroy a target quickly and move on to the next target, has coincided with the rapid advancement of technology. The rise of autonomous and semi-autonomous systems has created a need to explore new strategies (Johnson and Selby 2021). The war at sea is becoming a battle of networks, requiring the management of flow of the communication between platforms and the application of force from those platforms at the enemy (Cares and Cowden 2021). A submarine can no longer afford to operate individually in a transparent ocean (Kallenborn 2019).

1.2 Operations Research and Data Farming

To evaluate how new technologies can shape the battlefield, Operations Research (OR) methods can be used to inform decision-makers and designers in the research and development process. OR began as a necessity of war, created to assist in utilizing the newly invented radar to locate enemy aircraft for the Royal Air Force shortly before WWII (Holstein et al. 2018). At the advent of war, OR divisions were founded in each of the Armed Forces of the United States. On the subject of Wolfpacks, one of the first major successes of the U.S. Navy Antisubmarine Warfare Operations Research Group (ASWORG) was addressing the optimal size of merchant convoys (Gass and Assad 2005). Innovative mathematical techniques would continue to be developed over time such as Queuing Theory, Search Theory, and Monte Carlo simulation.

This thesis reports results of a computational method used extensively in OR known as "data farming". From Cambridge University Press (2022), data farming is "the process of using computers to create large amounts of data, which can then be looked at in order to find out useful information about complex systems". Lucas et al. (2015) suggests that simulation methods are computationally cheap enough and accurate to become methods of "first resort" when tackling the analysis of complex systems. A key strength of data farming is flexibility; if some new factor or feature need be introduced, the experiment can be modified and re-run in short order to provide rapid insight. For example, data farming methods could be used to suggest solutions to other USW challenges, such as the optimal number and placement of UUVs. This research will utilize that strength to explore how the effectiveness of a Wolfpack can change in seven different scenarios, modifying both the wartime environment and the information available to the submarine.

1.3 Study Objective

In this research, data farming methods are used to study the effectiveness of submarines operating as a Wolfpack to interdict desired targets. Data farming methods are only as accurate as the simulation and design of the experiment, and while careful diligence has been taken in removing possible sources of error, difficult to detect sources of error may still exist (such as artifacts related to the Bertrand paradox or the inspection paradox). This approach relies on approximating submarine behavior through conditional logic and employing the simulation in a few, tightly defined situations. Moreover, this approach relies on making approximations to the distribution of some features, such as detection ranges, speeds of vehicles, and torpedo effectiveness. Simulation output is summarized using simple linear regressions and neural networks. The summaries give insight to features that make a Wolfpack in the defined scenario effective.

1.4 Thesis Organization

Chapter II provides a prospective of the overall workflow and details of the simulation. Chapter III is an analysis of the performance of the simulation against several different known situations. Chapter IV presents the results from the various experiments. Chapter V concludes the thesis by presenting the overall conclusions for the work and highlights possible areas of future work.

CHAPTER 2:
Formulation and Methodology

2.1 Simulation Assumptions and Design

This simulation is a discrete-time agent-based simulation with the ability to run as a terminating simulation or as a non-terminating simulation. Discrete-time refers to the property of the simulation to simulate objects in an environment down to a time resolution, in this case one second. Other simulations may be constructed as discrete-event simulations that increment on each new event as opposed to each unit time. Discrete-time is used in this research, but it is recommended to explore the advantages of discrete event simulation as well in future work. The property of agent-based refers to the conditional logic that governs the behavior of the submarine and other vessels; the submarine will make decisions based on the conditional logic outlined in Section 2.1.1. Terminating and non-terminating refer to how long the simulation will be allowed to run. Terminating simulations terminate upon encountering a preset condition, such as total vessels simulated. Non-terminating simulations run for a sufficiently long amount of time such that long term stochastic behavior of the system can be approximated. **All parameters associated with submarine performance are either gathered from unclassified sources or assigned convenient values for experimentation.**

2.1.1 Simulation Assumptions

Three distinct cases will be considered: all arriving vessels are targets, mixtures of arriving target vessels and neutral vessels, and a Surface Action Group (SAG) in an environment of neutral vessels. A SAG is a collection of tightly packed warships around a High Value Unit (HVU), as shown in Figure 2.3. Within some cases, the effect of changing the frequency of communication occurrences between submarines, f_c will also be evaluated as shown in Table 2.1.

Table 2.1. Cases and Settings

Case	No Communication	f_C = 10hrs	f_C=1sec
All Vessels Targets	✓		
Some Vessels Targets	✓	✓	✓
SAG Transit	✓	✓	✓

In this simulation, a 100nm by 200nm waterspace is assigned to each submarine. If there is one sub the targets are vulnerable in the 100 nm by 200 nm waterspace; if there are 2 submarines the targets are vulnerable in a waterspace 100 nm by 400 nm, and so on. Targets and Merchants enter the simulation from the westernmost boundary of the first submarine's waterspace from some point uniformly distributed along its height, and travel directly East at an experimentally set speed. The waterspace boundaries are shared from one submarine to the next along Longitudinal lines as in Figure 2.1. The submarine will start at 100nm from the eastern boundary of its waterspace and at a uniformly distributed random point along the height of the waterspace. At the start of the simulation, the submarine will commence a barrier search to the North or South with probability 0.5. The submarine will continue until it reaches within 40,000 yds from a boundary, where it will reverse direction. Submarines do not exit their assigned waterspace, nor does the geometry or location of the waterspace change in any way during the simulation. Submarines classify the identity of any unknown vessel with perfect precision and after a constant amount of time. Submarines have no logistic constraints, have no ammunition restrictions, and surface ships will not attempt to destroy any submarines. A full overview is given by Figure 2.3.

Submarines (green) perform a barrier search to the North and South at the center of their assigned waterspace, traveling at 12kts. Detections of any vessel by a submarine occurs with probability of detection $P_d = 1$ at 40,000yds. When detecting any vessel at a range of 40,000yds, the submarine will immediately change course to the vessel's location and increase speed to 30kts. When the submarine arrives within a range of 20,000yds, it slows to 17kts and begins attempting to classify the vessel. During classification, the submarine will choose no other vessel to pursue. Correct target classification happens with probability $P_{correct} = 1$ after 20min of attempted classification. A submarine will always need to perform this classification procedure for each new vessel. If the vessel is a Neutral ship (blue), the submarine disengages tracking and will not need to reclassify the neutral ship. If the vessel is a Target (red), the submarine attempts a shot every minute, killing the target with an experimentally set probability of kill P_{kill}. The submarine has perfect battle damage assessment capabilities. Upon destroying its intended Target, or classifying a vessel as a neutral, Submarines will return to the center of its waterspace to resume barrier search. Submarines are always able to detect new vessels, and will break from returning to its central position to pursue a newly detected vessel.

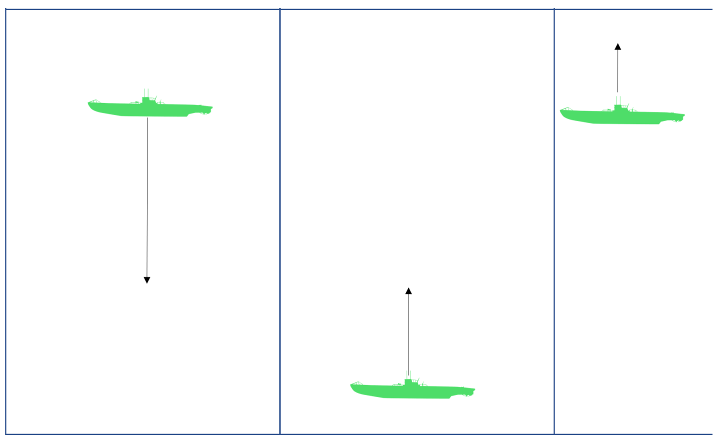


Figure 2.1. Submarine Waterspace Arrangement and Patrolling Behavior

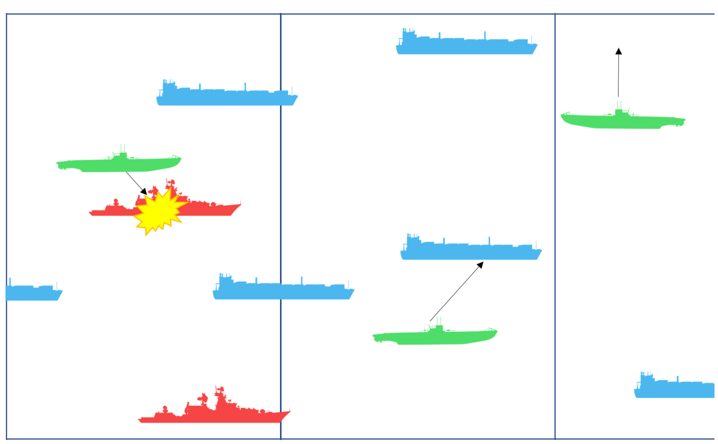


Figure 2.2. Simulation Operating with a Mixture of Targets and Neutral Shipping

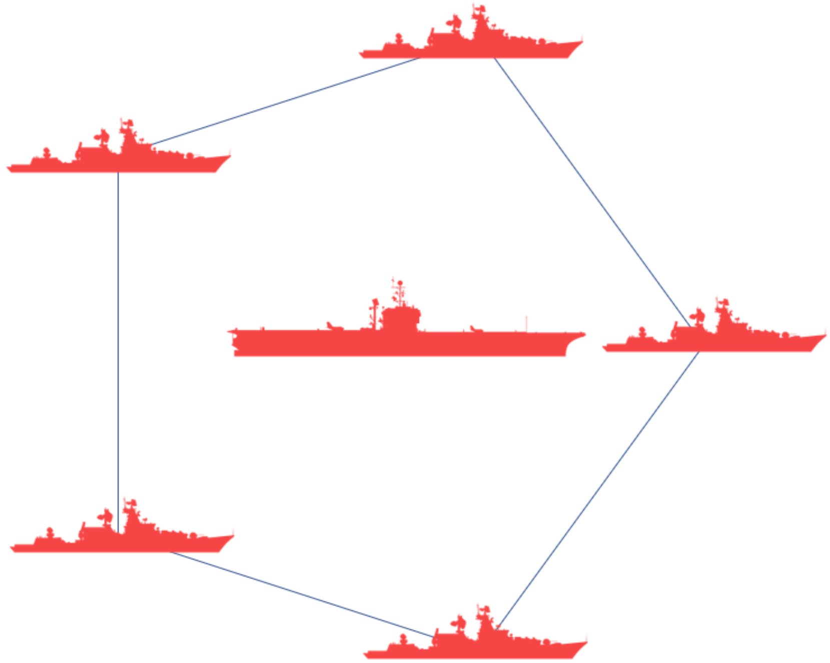


Figure 2.3. SAG Arrangement

2.1.2 Communication and Intercept

If a detected target reaches the rightmost waterspace boundary for a given submarine before the submarine can successfully destroy it, the submarine will communicate. There are two parts to how a submarine communicates: uploading new information to the off-hull entity and downloading new information from the off-hull entity. When submarines communicate, they communicate perfect information to an off-hull entity with no loss of stealth or chance of interdiction. The information communicated is the target's last known position and the time of the observation. This information is only available to the subsequent submarine in the next waterspace in the Wolfpack. For example, if the first submarine transmits information about a target to the off-hull entity, only the second submarine will be able to receive this information. The frequency at which another submarine downloads off-hull tasking is given in Table 2.1. For $f_C = 10hrs$, the time a submarine waits before initially recovering information from the off-hull entity is given by multiplying 10hrs by a uniformly distributed pseudo-random number ($U(0, 1)$). The submarine will communicate every 10 hours after. This ensures the submarines are not all communicating at the same time. The future path of the target is known perfectly and an interception point will be calculated from this data. A submarine will only act on tasking from the previous submarine if it is able to intercept the target (equation 2.2). A submarine in receipt of tasking will change course to an intercept point using equation 2.1, and change speed to 30kts as shown in Figure 2.4. A submarine will ignore all other targets until it reaches the optimal intercept point. Once reaching the intercept point, the submarine will still classify the vessel to ensure it is a target. If the submarine is unable to intercept the target, information concerning the target is not forwarded to the next submarine.

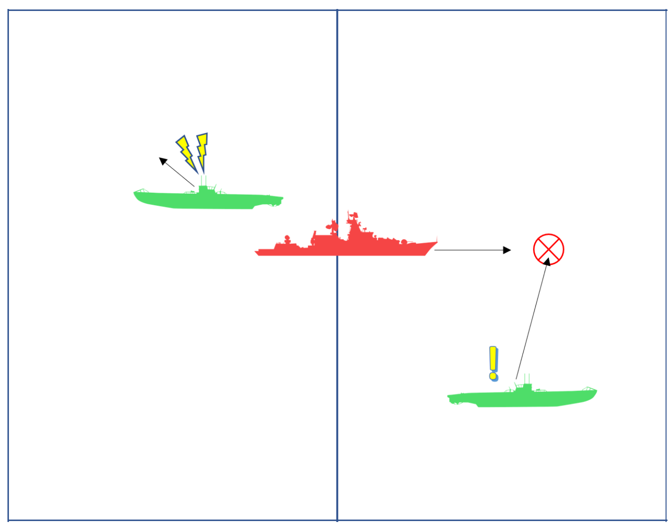


Figure 2.4. Interdiction of an Escaped Target by Submarine

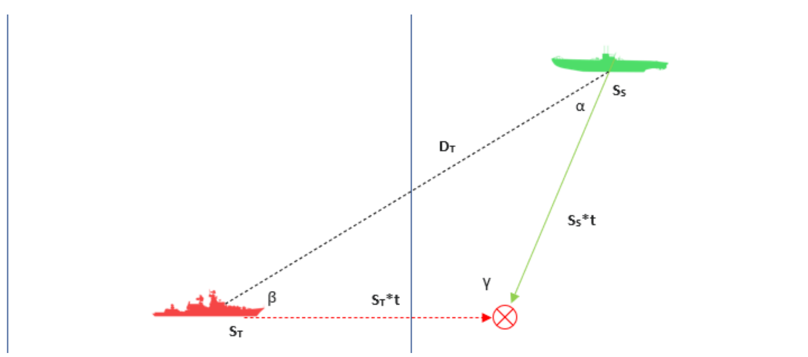


Figure 2.5. Optimal Course for Interdiction given by Equation

From Figure 2.5, the parameter α is derived by the Law of Sines:

$$\begin{aligned} \frac{\sin\alpha}{S_T t} = \frac{\sin\beta}{S_S t} = \frac{\sin\gamma}{D_T} \\ \alpha = \arcsin\left[\frac{S_T}{S_S}\sin\beta\right] \end{aligned} \tag{2.1}$$

If the target is at a lower latitude than the submarine, α is subtracted from the bearing to the target from the submarine BRG_{TS} to get the optimal intercept course CRS_o, otherwise it is added ($CRS_o = BRG_{TS} \mp \alpha$). The time until intercept t_{int} can be found also from Equation 2.2.

$$t_{\text{int}} = \frac{\sin\beta}{\sin\gamma S_S} D_T \tag{2.2}$$

By calculating the known position of the target at time $t_{\text{now}}+t_{\text{int}}$, the submarine can determine if transitioning to interdiction is justified. For example, if the submarine will be unable to reach the target before it leaves the eastern boundary of the submarines operating area, the submarine will not pursue and instead allows the next submarine to attempt interdiction. Also, if the optimal intercept point is located outside its waterspace to the West, and the next submarine has the information about the target, the next submarine will position itself to catch the target as it enters the next submarines assigned waters.

Once all the behaviors, limitations, and inputs are all carefully specified, a simulation may be run. Visualization is a costly feature, but this simulation can create both images and ".gif" files to give an analyst insight (Figure 2.6). Red points correspond to Targets, blue points to neutral Merchants, and green to Submarines.

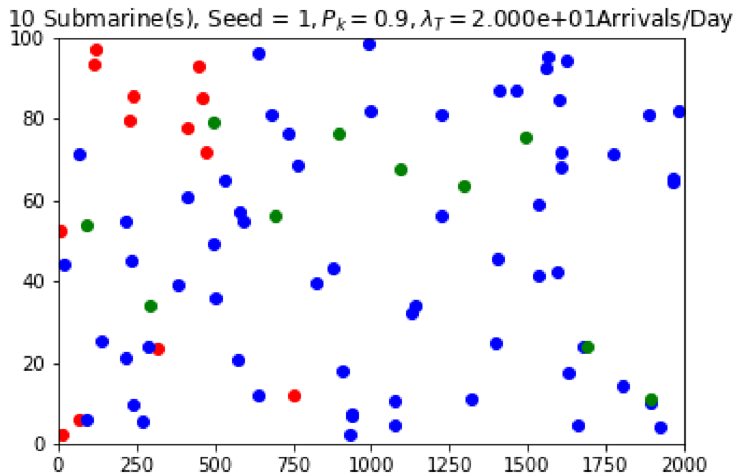


Figure 2.6. Screen Capture From an Example Scenario

2.1.3 Simulation Design

The arrival rate for each vessel is implemented into the experimental inputs (2.2) and denoted by λ_T and λ_M, respectively.

The times between arrivals of vessels to the region patrolled by the submarines are independent having exponential distribution with rate $\lambda_{TOT} = \lambda_T + \lambda_M$. In the simulation the interarrival time is smallest integer greater than or equal to the simulated interarrival time. The type of the arriving vessel is determined by comparing the value of a random number having a uniform distribution on [0,1]; if it is less than or equal to $\lambda_T/(\lambda_T + \lambda_M)$ the vessel is a target; otherwise it is a neutral merchant. The entering position has a uniform distribution on the interval [0, 100nm]. All non-submarine vessels travel in straight lines from their entering positions through the waterspace at an experimentally set speed. The simulation is run for 1,000,0000 seconds in non-terminating mode.

2.2 High Performance Computing

The Hamming supercomputer at the Naval Postgraduate School (NPS) High Performance Computing (HPC) center was utilized to execute this Data Farming experiment. In order to properly execute, the simulation needed two additional program files created: *RUN.py* and *runjobs.pbs*. The function of *RUN.py* is to take input from *test-jobs.pbs* (Section A.2), execute *Simulation.py*, and save the data for exporting to *.csv* files under the name *Killed-Targets-N.csv* where N is the associated job number. The function of *run-jobs.pbs* is to allow the execution of a large number of simulation experiments by Hamming. Once all required files were uploaded, the command is inputted to run the entire batch.

This generates 990 separate simulation jobs and executes them all in parallel, as resources were available. Together, most simulation experiments completed inside of three days, though this is highly variable depending on available resources and the intended length of the experiment. Simulation execution statistics were not recorded in this study.

2.3 A Brief Overview of the Data Farming Process

As shown in Figure 2.7, the simulation starts with the creation of design parameters through the use of Nearly Orthogonal Latin Hypercubes (NOLH) designs.

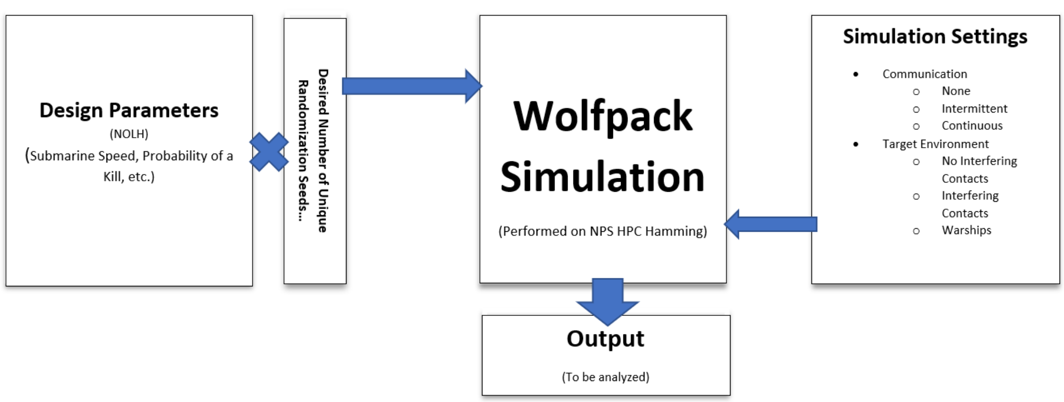


Figure 2.7. The Data Farming Workflow

A NOLH is created given the range of possible values desired for each of the inputs of the simulation. The following terms will be used to describe a NOLH created for a simulation experiment: a design is a matrix of inputs, a *factor* is a column in this matrix, a *design point* is any single row of the design, and a *level* is the value of a single parameter. A visual representation of this terminology can be found in Figure 2.8. For example, a factor would correspond to an experimental parameter like submarine patrolling speed and a level would correspond to the specific value of the speed being tested, such as 12kts.

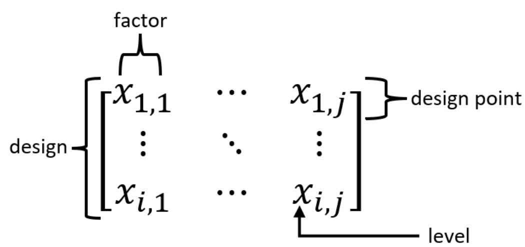


Figure 2.8. Terminology

The created NOLH will be crossed with a different design containing the desired number of unique seeds for randomizing the simulation (Section 2.4.1). This entire process will be referred to as "intelligent experimental design." The resulting intelligent experimental design is then used as input for the simulation. The simulation is run for each of the many design points created by the intelligent experimental design, utilizing the Naval Postgraduate School's HPC Center's resource known as the "Hamming" supercomputer, and the desired outputs are saved to a Microsoft Excel spreadsheet. Three distinct wartime environments are considered: all arriving vessels are targets, mixtures of arriving target vessels and neutral vessels, and a SAG with neutral vessels. A SAG is a collection of tightly packed warships around a HVU, as shown in Figure 2.3. Those resulting outputs are then analyzed, looking for indications that particular behaviors or conditions impact the efficiency of a submarine Wolfpack.

2.4 Intelligent Experimental Design Through the Use of Nearly Orthogonal Latin Hypercubes

The study of a small number of factors and their interactions can generate experiments with millions of design points, so an analyst must be careful with design construction. Searching for a justifiable method of reducing the number of tested outcomes may also be dangerous as an analyst certainly does not want to exclude a particular possibility from experimentation. Information on NOLHs can be found in the appendix. Also, more extensive discussion can be found in the work by Sanchez and Wan (2021).

This research will study the influence of the following factors on the success of Wolfpack tactics: Submarine Patrolling Speed (S_S), Probability of a Successful Kill (P_K), Average Arrival Rate of Targets (λ_T), Average Arrival Rate of Merchants (λ_M), and the Speed of Transiting Non-Submarine Vessels (S_T). A NOLH with 33 design points was creating from the settings on Table 2.2, using the NOLH generator provided by Sanchez (2011). For experiments involving only targets, the level of λ_M will be 0 for all design points.

Table 2.2. Design Parameters

Factor	High Level	Low Level	Number of Decimals
S_S	20	3	0
P_K	1.00	0.00	2
λ_T	20	1	0
λ_M	180	1	0
S_T	30	1	0

2.4.1 Crossed Designs

The generated NOLH is then crossed with thirty different randomization seeds. Each design point has 30 simulation replications. The simulation replications for different design points are correlated since they use the same 30 random number seeds.

A randomization seed is a number taken as an input to a pseudorandom number generator (PRNG). Python utilizes an algorithm known as the "Mersenne Twister," which was originally the work of Matsumoto and Nishimura (1998). A seed will govern how the behavior of the PRNG evolves during a simulation. For the randomization seeds, the integers

1 through 30 were selected. To apply the randomization seed values to the existing design, an operation called "crossing" will be applied to the two matrices. Crossing the vector of randomization seeds with the previously created NOLH design creates a new design that repeats the first design at each value of the second. For these experiments, n_d refers to the number of design points from starting design, m refers to the number of simulation replications being evaluated at the second design, and k refers to the number of factors being evaluated. For one column vector of seeds ($k = 1$) with a range of 30 values ($m = 30$), the total number of design points (T_d) is 990. If a larger number of seeds are desired for experimental resolution, they will evolve as shown in equation 2.3.

$$
\begin{aligned}
T_d &= n_d \times m^k \\
T_d &= 33 \times 30^1 \\
T_d &= 990
\end{aligned}
\tag{2.3}
$$

There exists other methods for boosting the desired resolution from a given design, such as stacking methods, but they are not used in this work. The final crossed design is modified slightly by the analyst to conform to the required inputs of *RUN.py* and saved as *test-jobs.pbs*.

2.5 Data Analysis

The final part of data farming is data analysis. This analysis functionally begins at the design of the simulation. An analyst may need to add more existing features in an experiment than the ones chosen in this research. For the Python programming language, any required additional data would be as straightforward as modifying a few lines of code. After the data is collected and aggregated, JMP (JMP 2021) was used to perform the analysis of the simulation output. Linear regressions and one-layer neural networks will be used to summarize how the input factors influence the results of the simulation, such as the average cumulative number of targets killed by a Wolfpack.

2.5.1 Data Preparation

In order to properly analyze the output of the simulation, first the data must be prepared. For each experimental run, the appropriate inputs must be matched to the outputs. Moreover, the outputs will need to be averaged (by simple arithmetic mean). To simplify the process of data preparation, the program *design_prep.py* was created. This program iterates over every output file, averaging each factor, matches it to the appropriate inputs, and appends that to a larger output file. This program is flexible to changes in the number of metrics measured in the output file, so more or less properties may be measured without workflow interruption. This larger output file has the format as shown in Figure 2.9, and is ready for analysis.

1 kills	1 tracking	2 kills	2 tracking	3 kills	3 tracking	4 kills	...	n_targets	n_mercha	n_submar	seeds	max_sam	speed_sul	P_k	Lam_T	Lam_M	speed_tar
46	4355.97	28.4636	4385.87	7.03636	4240.13	5.02727	...	1	1	10	19	1	20	0.09	9	35	23
41	4786.21	24.2857	5080.15	13.2571	5381.55	1.67619		1	1	10	16	1	20	0.09	9	35	23
88	2458.73	55.5355	2520.91	12.891	2549.65	14.7867		1	1	10	19	1	13	0.88	20	74	24
65	5483.04	28.6082	5776.73	25.2577	4816.65	4.86598	...	1	1	10	16	1	6	0.41	13	23	23
62	5779.29	32.8764	6543.4	12.6067	5211.64	5.96629		1	1	10	3	1	6	0.41	13	23	23

Figure 2.9. Example Output Layout

2.5.2 Regression Analysis

Regression techniques will be utilized to summarize simulation output. The following is an example estimating a response of interest Y. If we let $X_1, ..., X_k$ denote the k factors in a given experiment, regression coefficients are unbiased estimators of the β_i, and the ϵ are independent random errors with mean zero. The linear regression model used appears in equation 2.4 and includes linear effects, quadratic effects, and second degree factorial effects:

$$Y = \beta_0 + \sum_{i=1}^{k} \beta_i X_i + \sum_{i=1}^{k} \beta_{i,i}(X_i - \overline{X_i})^2 + \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \beta_{i,j}(X_i - \overline{X_i})(X_j - \overline{X_j}) + \epsilon \tag{2.4}$$

These methods will be used to identify the relative importance of various factors on the average number of Targets killed from Wolfpack operation. The ability of the regression to summarize the simulation output is evaluated by R^2.

2.5.3 Machine Learning: Neural Networks

Machine learning is a rapidly growing field with many applications across a variety of fields, including OR (Jordan and Mitchell 2015). This research utilized one of the major types of machine learning tools, single layer neural networks, which is a nonlinear regression, in order to summarize simulation output. The ability of the neural networks to summarize the simulation output is evaluated by R^2. The purpose of creating such a model would be to allow decision-makers to make rapid estimates of average number of targets killed using measurements and estimates of the input parameters. The neural network is created using the neural network tool in JMP (2021), and Figure 2.10 gives the construction of the neural network used in this research.

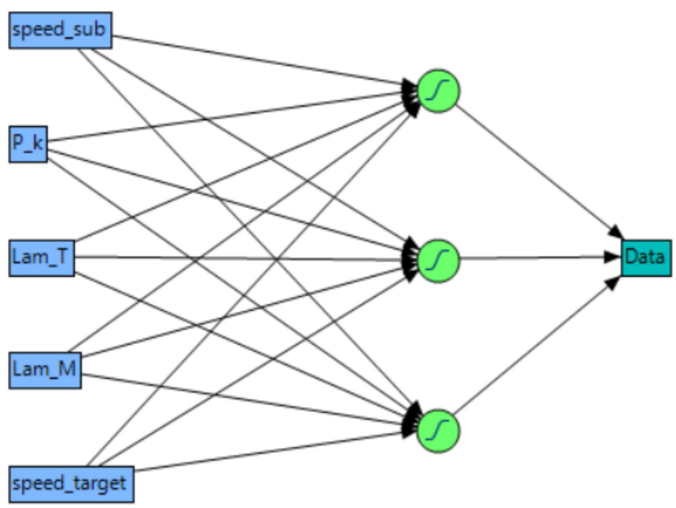


Figure 2.10. Neural Network Layout

All constructed neural networks were made with the settings shown in Figure 2.11. See JMP (2021) Learning Library: Neural Networks for the meaning of the settings. There exists a complex relationship between the number of nodes in a network, number of hidden layers, and method of optimization against the predictive power of a neural network. Analysis was not performed on the optimal configuration of nodes or layers that produced the most optimal configuration, though it is suggested that a single layer with few nodes is sufficient for this application (Wanas et al. 1998).

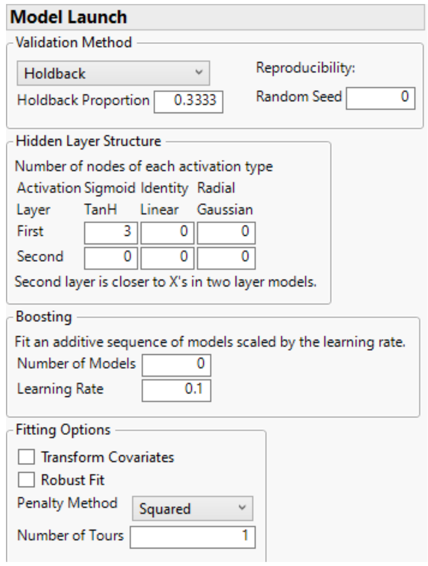


Figure 2.11. Neural Network Settings

CHAPTER 3: Simulation Evaluation

Once formulation and methodology is set, the simulation may now be subject to experimentation. Comparing the output of the simulation to that of other models can assist in assessing the reasonableness of portions of the simulation. The following evaluations of the simulation demonstrate that the simulation reasonably represents these situations under the conditions considered.

3.1 Applied Search Theory and Detection

Wagner et al. (1999) work elucidates many of the characteristic mathematical problems that a naval officer could face, such as investigating the Probability of Detection (P_d) of a target traveling at a given speed by a searching platform over variable speed. For this test, only the initial submarine is considered. The submarine is assigned a waterspace that is 100nm tall and 200nm wide. The submarine will start at 100nm from the eastern boundary and at a uniformly distributed random point along the height of the waterspace. At the start of the simulation, the submarine will commence a barrier search to the North or South with probability 0.5. The submarine will continue until it reaches within 40,000yds from a boundary, where it will reverse direction. Targets enter the simulation at a uniformly distributed random point along the easternmost boundary and travel directly east at 16kts. The simulation is run such that only one target initializes in a replication and attempts to cross the submarine's barrier search path. Either positive detection by the submarine or the target reaching the end of the submarine's patrol area undetected results in simulation termination. Design construction consisted of sampling from a submarine searching speed at every 1/3 knot from 0 to 30 knots and a target closing speed of 16 knots. Five-hundred replications at each speed are created by crossing the initial design with a design containing the same five-hundred random number seeds. Then, for each value in speed, the proportion of successful detections is found through a simple arithmetic mean, assigning the value of 1 to a successful detection and 0 to an unsuccessful one. This estimator is compared to the computed values of P_d given by the available models of Wagner et al. (1999) and Sözen and Craparo (2016).

The equations from Wagner et al. (1999) and Sözen and Craparo (2016) are given by Equation 3.1 and 3.2. The height of the barrier search is L units, with the Submarine traveling at speed v. The Submarine has a cookie cutter detection radius of R. Targets move with speed u.

$$P_d = \begin{cases} 1 - \left(\frac{L}{R} - \sqrt{(\frac{v}{u})^2 + 1} - 1\right)^2 \frac{R^2}{L(L-2R)} & \text{if} \quad Rv \le u\sqrt{L(L-2R)} \\ 1 & \text{otherwise.} \end{cases} \tag{3.1}$$

$$P_d = \begin{cases} \frac{2R}{L}\sqrt{(\frac{v}{u})^2 + 1} + \frac{R^2 v(\frac{\pi}{2} - \arctan\frac{u}{v} - \frac{v}{u})}{(L-2R)uL} & \text{if} \quad \frac{Rv^2}{(L-2R)u\sqrt{u^2+v^2}} < 1 \\ 1 + \frac{R^2 v \arcsin\frac{(L-2R)u}{Rv}}{(L-2R)uL} - \frac{2R}{L} + \frac{\sqrt{R^2v^2-(L-2R)^2u^2}}{Lv} & \text{otherwise.} \end{cases} \tag{3.2}$$

The Wolfpack simulation output trends well with Equations 3.1 and 3.2, which did not deviate significantly from each other within the desired testing ranges and is shown by Figure 3.1. At higher speeds, the simulation seems to represent the probability of detection of the Sözen and Craparo (2016) model better than the model from Wagner et al. (1999).

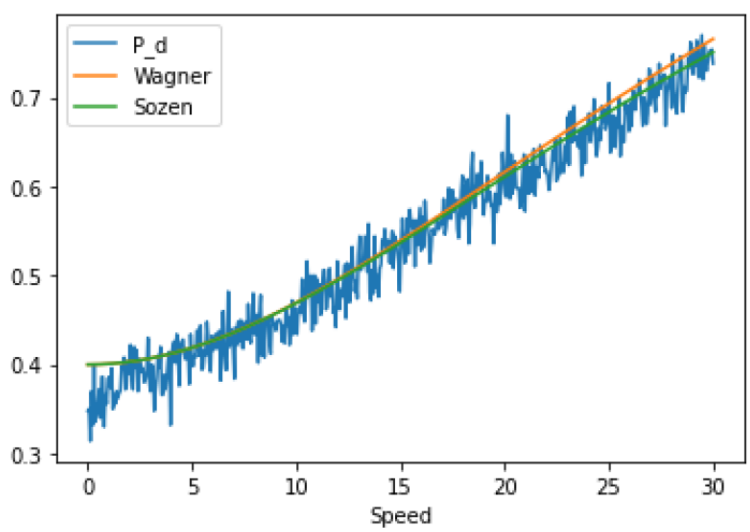


Figure 3.1. 500 replications

3.2 Submarine Operations Research Group's General Equations

Submarine Operations Research Group (1944) authored a report for the Commander In Chief, United States Fleet to assist with creating generalized models for predicting the success of a group of submarines. These models perform well for predicting the basic behavior of a submarine Wolfpack operating without intercommunication. The report continues to elaborate the behavior of multiple submarines converging and attacking at once, similar to the preferred method of attack by the Kriegsmarine at the time. This research will only make comparisons to the models that mimic the scenario outlined in Chapter 2.1.

3.2.1 Model Evaluation

Table 3.1 gives definitions to the used parameters.

Table 3.1. Design Parameters

Parameter	Definition
S	Number of distinct ships sighted per month by N submarines
N	Number of submarines
C	Constant in Eq. (3.4)
r	Effective search width in a submarine
W	Effective width of a shipping route
F	Traffic along shipping route, in ships per month
P	Probability that one submarine will sink a ship which has been sighted
H_i	Number of ships sunk per month by N independent submarines
H_g	Number of ships sunk per month by N group submarines
R	Ratio of group yield to independent yield
N_s	Number of submarines required to contact every ship passing along the assigned shipping route
D	Days between contacts by one submarine
K, k	Homing probabilities

From Table 3.1, the following equations are given.

$$S = CN \tag{3.3}$$

$$C = \frac{rF}{W} \tag{3.4}$$

$$H_i = CN_iP \tag{3.5}$$

$$S = CN \tag{3.6}$$

$$H_g = CN\left[1 - (1 - P)^N\right] \tag{3.7}$$

The results of H_g may now be directly compared to the output of the simulation. A terminating simulation was created with the following settings: 10 submarines, all vessels are targets, 200 targets generated, targets arrive in accordance with a Poisson arrival process, and the simulation terminates when the last target leaves the waterspace of the 10th submarine. These settings are applied to the simulation design and crossed with same 30 different randomization seeds for the PRNG. Targets enter the region through a uniformly distributed random point along the easternmost boundary and travel East at 16kts. The number of Targets killed by each submarine is recorded by the simulation, and averaged over the 30 replications. The cumulative sum of the number of targets killed by N submarines is then plotted in blue over the plot of H_g to show fit.

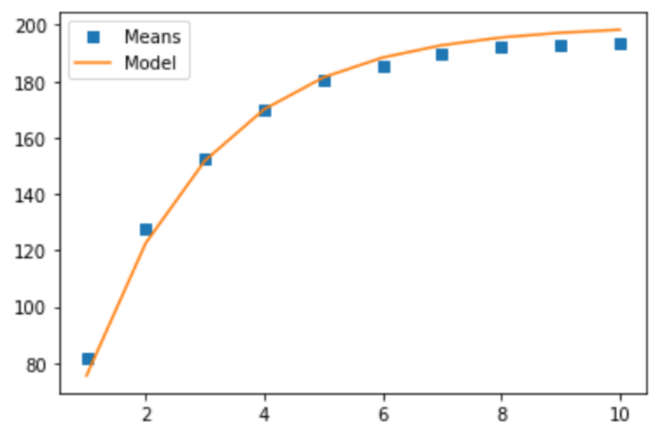


Figure 3.2. Averaged Model Generated Yield (Blue) and Submarine Operations Research Group (1944) (Orange) vs Number of Submarines Operating in Wolfpack

3.3 A Deterministic Queuing Model for Submarine Interdiction of Surface Targets

The last model, from Jacobs (2022), is constructed to predict the fraction of Targets that survive an encounter by the first submarine operating as described in chapter 2.1.1. It is assumed that there is a mixture of both Target and Neutral vessels and that no Targets escape destruction by the submarine once detected. P_k is the probability of a successful kill, S_S is the speed of the patrolling submarine, S_T is the speed of the transiting Neutral and Target vessels, λ_N is the arrival rate of Neutral vessels, and λ_T is the arrival rate of Target vessels. The following experimental parameters are set:

Table 3.2. Simulation Settings

P_k	0.5
S_S	16kts
S_T	12kts
λ_T	10/day
λ_N	10/day

These parameters are used in addition to the following parameters for the deterministic model. $\frac{1}{\theta_N}$ and (respectively $\frac{1}{\theta_T}$) are the expected time to close and identify a Neutral and (respectively a Target vessel). The time to close and identify a target are deterministic times of 20min each, set by the simulation. Therefore, $\frac{1}{\theta_N}$ is given by $20min + 20min = 40min$. The time to successfully kill a Target once identified is given by equation 3.9.

$$E[Time\ Until\ Hit] = 10min \cdot P_K + (1 - P_K)(10min + E[Time\ Until\ Hit]) \tag{3.8}$$

$$E[Time\ Until\ Hit] = 20min \tag{3.9}$$

Therefore, $\frac{1}{\theta_T}$ is then given by $20min + 20min + 20min = 1hr$.

The values of the average number of vessels in a region, as well as the expected time to detect a target if there is one vessel in the region $\frac{1}{\delta}$ are estimated by running a set of 30 simulations, each with different randomization seeds (chapter 2.4.1), and with the settings from Table 3.2. The submarine is located inside a 100nm by 200nm waterspace, starting at 100nm from the eastern boundary and at a uniform randomly distributed point along the height of the waterspace. The submarine will travel North or South with probability 0.5 and flip direction of travel when it encounters a boundary. The Targets and Merchants enter from the eastern most boundary at a random point uniformly distributed along the height of the waterspace and travel East at 16kts.

Table 3.3. Model Parameters

Average Number of Vessels in Waterspace	6.12
$\frac{1}{\delta}$	2613 sec

The number of samples is given by n and the value of sample i is given by x_i. The mean and standard deviation from n samples are $\hat{\mu}_n$ and $\hat{\sigma}_n$.

$$\hat{\mu}_n = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{3.10}$$

$$\hat{\sigma}_n^2 = \frac{1}{n}\sum_{i=1}^{n} (x_i - \hat{\mu})^2 \tag{3.11}$$

The following functions are defined:

Table 3.4. Variable Functions

$U(t)$	expected number of undetected neutrals in the region at time t
$B(t)$	expected number of undetected targets in the region at time t
$Z(t)$	expected number of targets that leave the region during $(0, t]$

Let:

$$E[\text{Time Until Vessel Detected}] = \frac{1}{\delta[U(t) + B(t)]} \tag{3.12}$$

$$E[\text{Expected Time Serving Target}] = \frac{1}{\theta_T}\frac{B(t)}{U(t) + B(t)} \tag{3.13}$$

$$E[\text{Expected Time Serving Neutral}] = \frac{1}{\theta_N}\frac{U(t)}{U(t) + B(t)} \tag{3.14}$$

$$\tag{3.15}$$

The following ordinary differential equations are then suggested to model the population of $U(t)$, $B(t)$, and $S(t)$:

$$\frac{d}{dt}U(t) = \lambda_N - \mu_N U(t) - \left[\frac{\frac{1}{\theta_N}\frac{U(t)}{U(t)+B(t)}}{\omega\left(U(t), B(t)\right)} \times \frac{U(t)+B(t)}{1+U(t)+B(t)} \right] \tag{3.16}$$

$$\frac{d}{dt}B(t) = \lambda_T - \mu_T B(t) - \left[\frac{\frac{1}{\theta_B}\frac{B(t)}{U(t)+B(t)}}{\omega\left(U(t), B(t)\right)} \times \frac{U(t)+B(t)}{1+U(t)+B(t)} \right] \tag{3.17}$$

$$\frac{d}{dt}Z(t) = \mu_T B(t) \tag{3.18}$$

An estimate for the fraction of Targets that survive and exit the submarine's operating area at time t is given by $\frac{Z(t)}{\lambda_T t}$ for $t > 0$. Euler's method was applied to numerically solve the system of equations (3.16-3.18) with initial conditions $U(0) = B(0) = Z(0) = 0$. The simulation was run for a maximum time of $1e5$ seconds and for 30 different PRNG values, consisting of the integers from 1 to 30. For each simulation replication the number of targets to exit the region during $1e5$ seconds divided by the number of targets to enter the region during $1e5$ seconds is computed. From the simulation, the average survival rate and the 95% normal confidence interval is 0.535±0.174 using Equations 3.10 and 3.11. The computed survival rate from Equations 3.16-3.18 at time $1e5$ seconds is 0.514, falling well within the confidence interval.

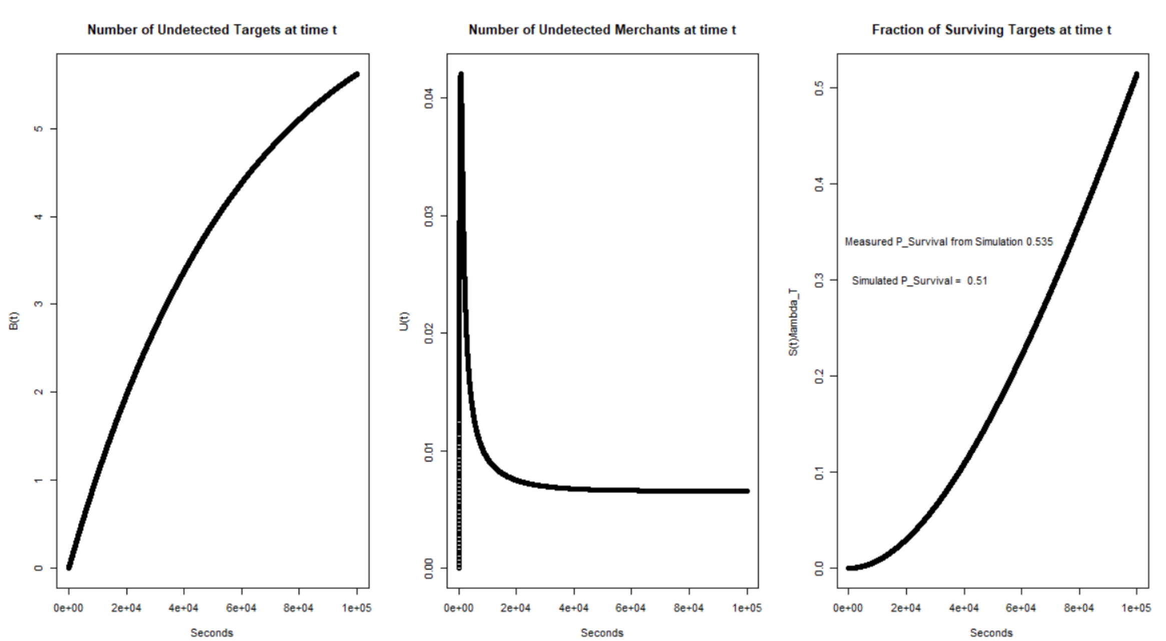


Figure 3.3. Plots of $B(t)$, $U(t)$, and $\frac{Z(t)}{\lambda_T t}$

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CHAPTER 4:
Results

The simulation was run for 1,000,000 seconds for each design point with 10 Submarines in "non-terminating" mode. The output of the simulation is organized as described by section 2.5.1. For each submarine i, the average number of targets killed is calculated by taking the arithmetic mean over all design points. These means are then averaged over each of the replications. These means are then cumulatively summed for each number of submarines i and represented by $\bar{Y}_i$. For example, $\bar{Y}_5$ is the average cumulative number of Targets killed by the first 5 Submarines. Y_i denotes the average cumulative kills by i submarines for a single replication. For each submarine i, Y_i is represented on the following plots as points and changes in $\bar{Y}_i$ will be shown as a smoothed line. The smoothed line is created using the default settings for the Spline method of the JMP (2021) Smoother function. See JMP (2021) Learning Library: Graph Builder for the meaning of the settings.

Each factor from the input design (Table 2.2) is analyzed for effects on $\bar{Y}_{10}$. The analyzed cases will start with no interfering contacts and no communication. The next cases are analyzed for all the communication schemes and are ones with interfering contacts, finishing with SAG. Each case will also be compared for effects across communication schemes. Analysis methods will include both linear regressions and single layer neural networks to identify the most important factors.

Additional analysis includes determining the number of submarines required to achieve 80% and 90% the average cumulative kills of a Wolfpack of 10 Submarines. It is noted that the average cumulative number of kills for a Wolfpack of size 10 ($\bar{Y}_{10}$) behaves erratically as P_K increases. Figure 4.1 displays the high variability of average cumulative number of kills for each replication Y_{10}, plotted as black dots, as P_K increases. This is due to limitations from the relatively small NOLH design and could be eliminated by increasing the size of the NOLH or implementing stacking methods.

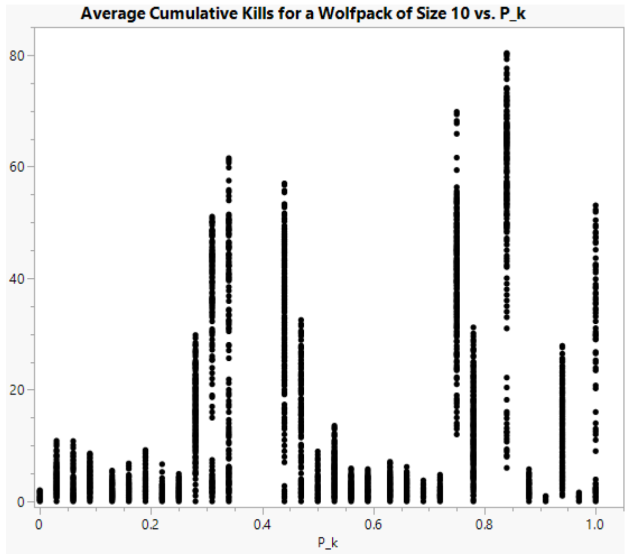


Figure 4.1. Average Cumulative Kills for a Wolfpack of Size 10 vs P_K

4.1 No Interfering Contacts

For this experiment, all vessels that entered were Target vessels and so communication between the submarines is not needed. The output of the simulation was expected to agree most closely to the results modeled by Submarine Operations Research Group (1944) as shown in Chapter 3.2.

4.1.1 No Communication

Figure 4.2 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". How changes in λ_T effects the $\bar{Y}_i$ for a submarine i are shown in color. A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function. Larger values of λ_T are indicated in shades of red where smaller values are shown in shades of blue. Changes in λ_T drive large changes in the value of $\bar{Y}_i$. The line has a similar shape to the provided by Submarine Operations Research Group (1944), equation 3.7.

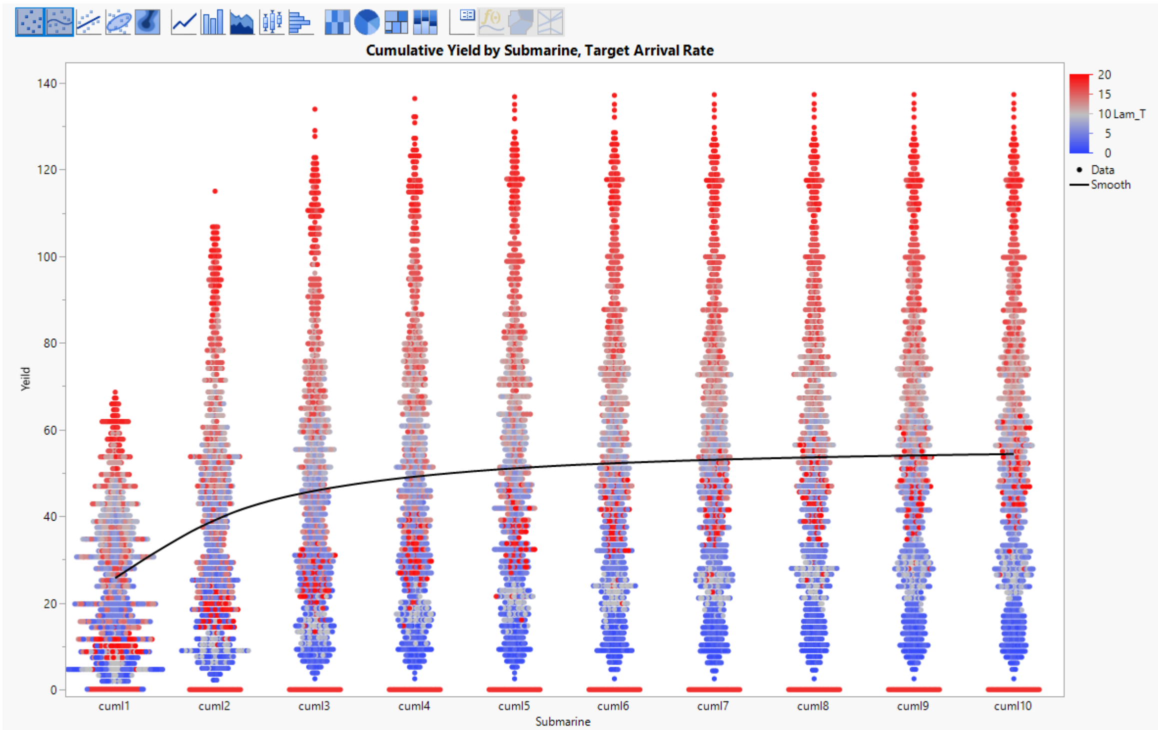


Figure 4.2. Cumulative Yield by Submarine, Target Arrival Rate, No Interfering Contacts, No Communication

Figure 4.3 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". How changes in S_T affects the $\bar{Y}_i$ for a submarine i are shown in color. Larger values of S_T are indicated in shades of red where smaller values are shown in shades of blue. Larger values of S_T are indicated in shades of red where smaller values are shown in shades of blue. A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function.

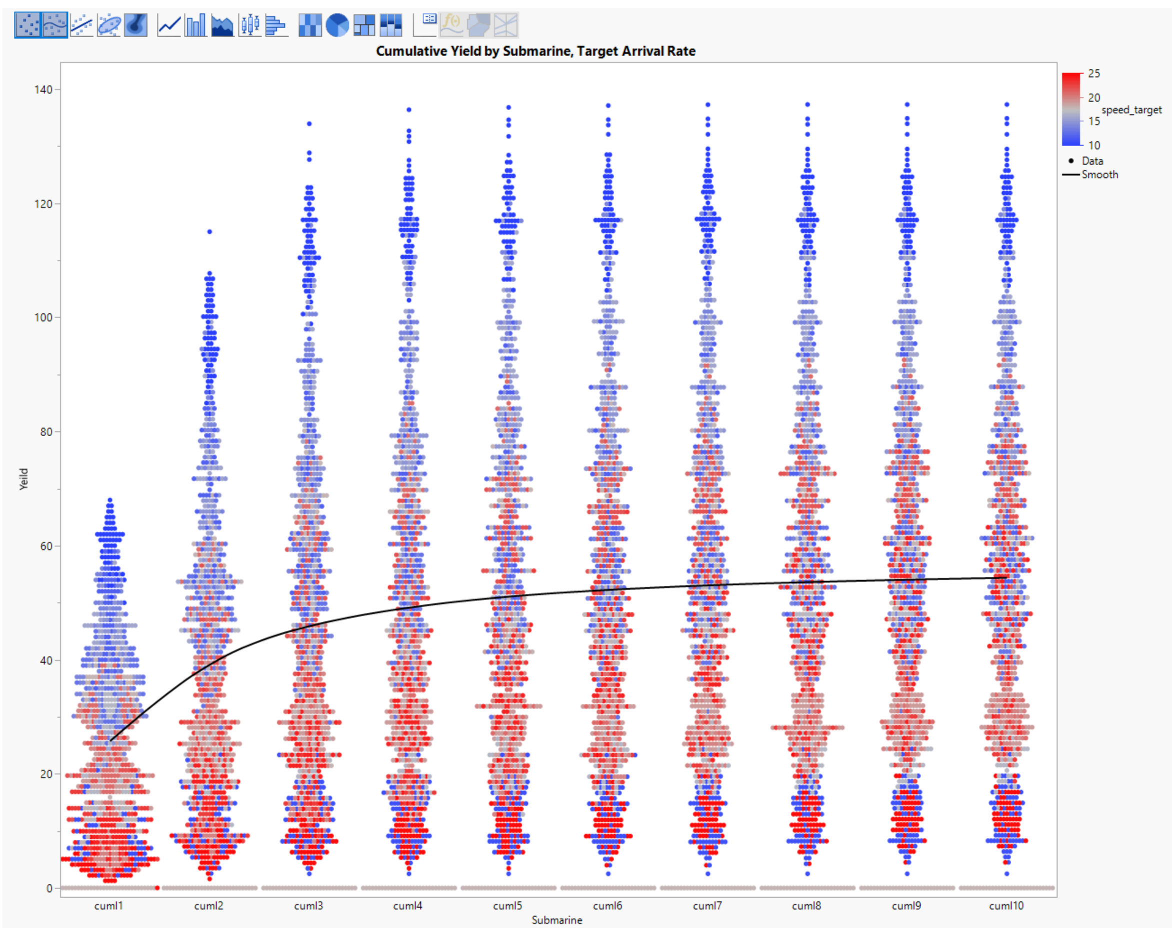


Figure 4.3. Cumulative Yield by Submarine, Target Arrival Rate, No Interfering Contacts, No Communication

Figure 4.4 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". The graph is additionally grouped into ranges of P_K. How changes in S_T effects the $\bar{Y}_i$ for a submarine i are shown in color. A line is plotted to summarize changes in each $\bar{Y}_i$using the Spline method of the JMP (2021) Smoother function. Larger values of S_T are indicated in shades of red where smaller values are shown in shades of blue. The large differences in the magnitude of the yield curves are shown from changes in S_T. The eye should be drawn to the extremes of these plots; a $P_K = 0.5$ has the largest amount of variability.

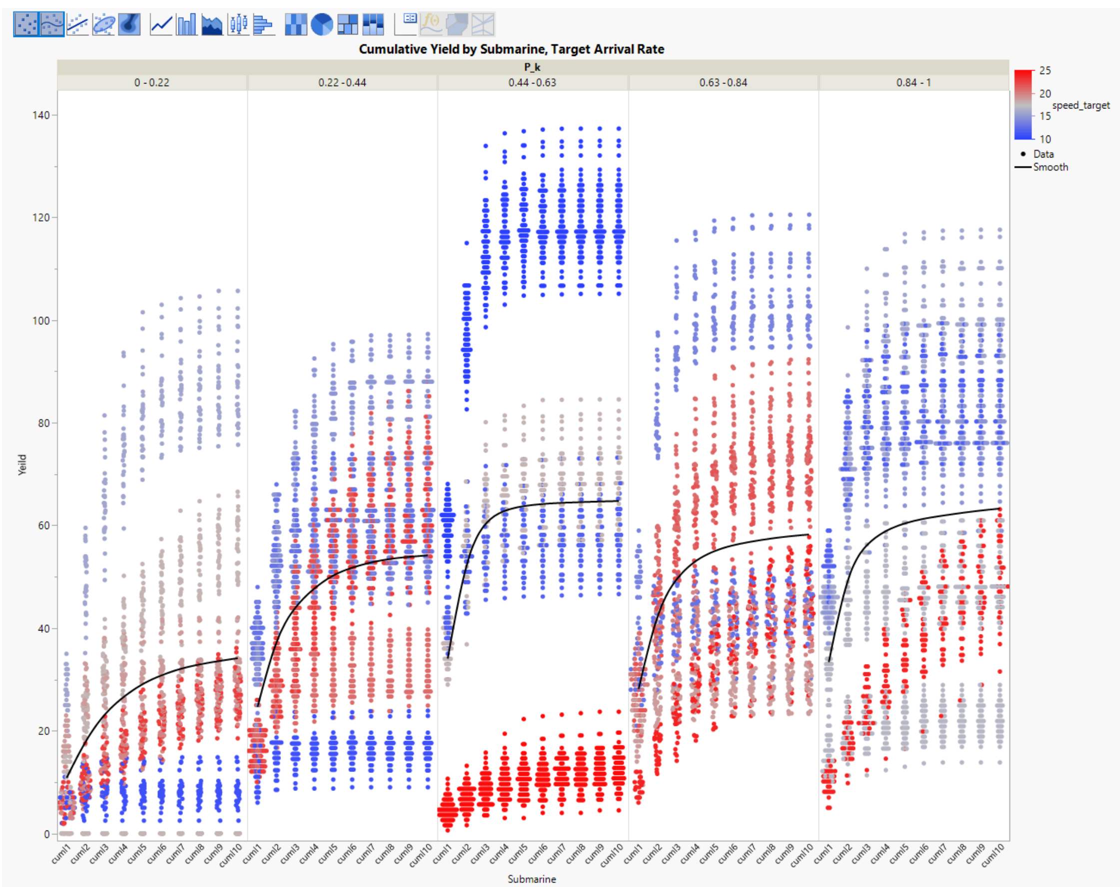
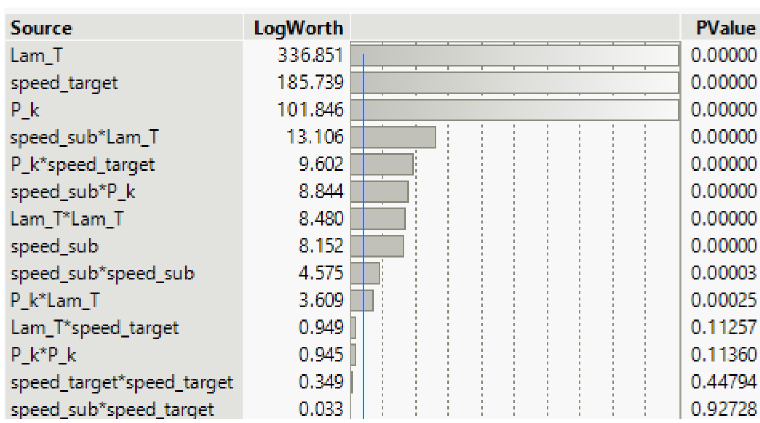


Figure 4.4. Cumulative Yield by Submarine, Target Arrival Rate, No Interfering Contacts, No Communication

A linear regression model was used to summarize the association between simulation average number of Targets killed by all 10 submarines and the simulation factors . The regression model includes main effects, second degree quadratic effects, and effects of each factor to degree two. The model achieved an $R^2 \approx 0.89$ and principally emphasized the importance of the main effects from λ_T, S_S, and P_K as shown in Figure 4.5. A one-layer neural network created from the settings shown in Figure 2.11 to predict the $\bar{Y}_{10}$ results in a model with an $R^2 \approx 0.96$. Similar to Figure 4.5, λ_T, S_T, and P_K are the most important identifiers to performance.



Source	LogWorth	PValue
Lam_T	336.851	0.00000
speed_target	185.739	0.00000
P_k	101.846	0.00000
speed_sub*Lam_T	13.106	0.00000
P_k*speed_target	9.602	0.00000
speed_sub*P_k	8.844	0.00000
Lam_T*Lam_T	8.480	0.00000
speed_sub	8.152	0.00000
speed_sub*speed_sub	4.575	0.00003
P_k*Lam_T	3.609	0.00025
Lam_T*speed_target	0.949	0.11257
P_k*P_k	0.945	0.11360
speed_target*speed_target	0.349	0.44794
speed_sub*speed_target	0.033	0.92728

Figure 4.5. Regression Including Full Factorial Effects to Degree 2, No Interfering Contacts, No Communication

The number of Submarines needed to achieve 80% and 90% of the average cumulative kills as a Wolfpack of 10 Submarines ($\bar{Y}_{10}$) is considered. As an example, to calculate the number of submarines N_{SUB} required to achieve $0.8\bar{Y}_{10}$ for a given P_K, the value of $\bar{Y}_{N_{SUB}}$ must be strictly greater than $0.8\bar{Y}_{10}$. As shown in Table 4.1, about 3 submarines are all that is needed to achieve 90% of $\bar{Y}_{10}$ for the same P_K.

Table 4.1. Efficiency for Different Fixed Parameters, No Interfering Contacts, No Communication

Selected P_K	$\bar{Y}_{10}$	Std Err Mean	N_{SUB} for $0.8\bar{Y}_{10}$	N_{SUB} for $0.9\bar{Y}_{10}$
0.25	31.70	0.78	2	4
0.50	69.14	1.32	3	3
0.75	103.99	1.33	3	3

4.2 Interfering Contacts

This section will contain the results from simulations run with a mixture of Targets and Merchants passing through the submarine operating area. All three communication schemes are considered: No Communication, Intermittent Communication, and Constant Communication.

4.2.1 No Communication

Figure 4.6 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". Changes in P_K are shown in color. A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function.

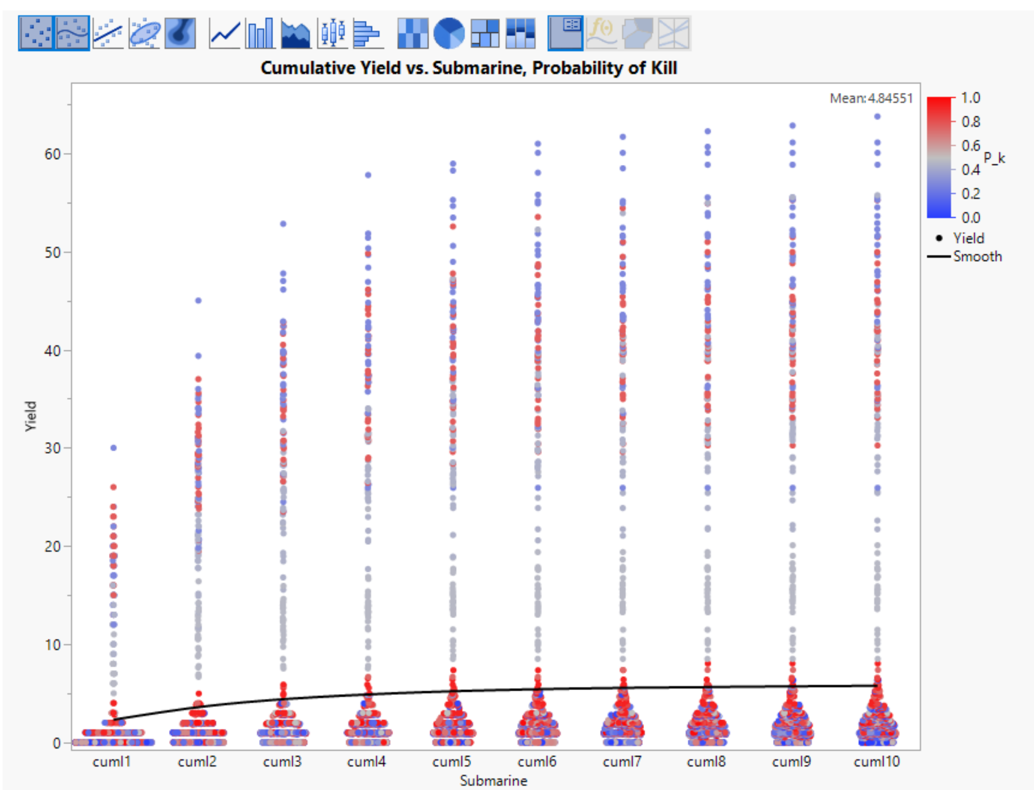


Figure 4.6. Cumulative Yield by Submarine, P_K, Interfering Contacts, No Communication

Figure 4.7 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". Changes in S_S are shown in color. A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function.

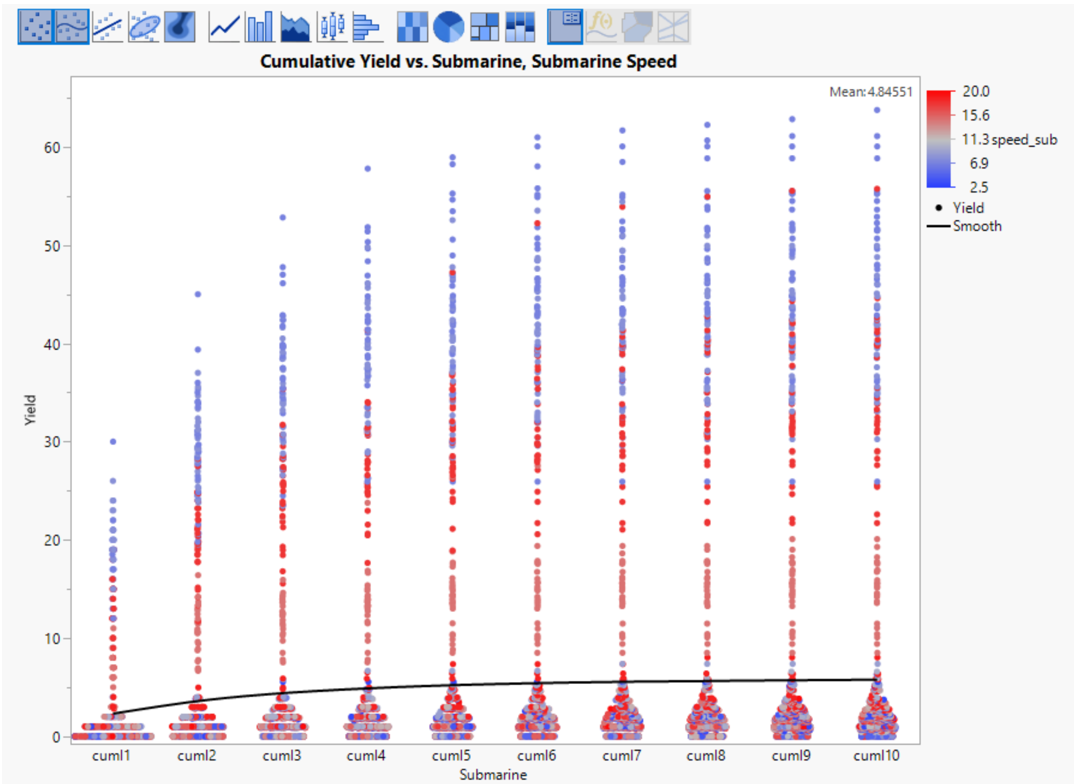


Figure 4.7. Cumulative Yield by Submarine, S_S, Interfering Contacts, No Communication

Figure 4.8 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". Changes in S_T are shown in color. A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function. The cumulative yield curve appears to be sensitive to even small values of S_T.

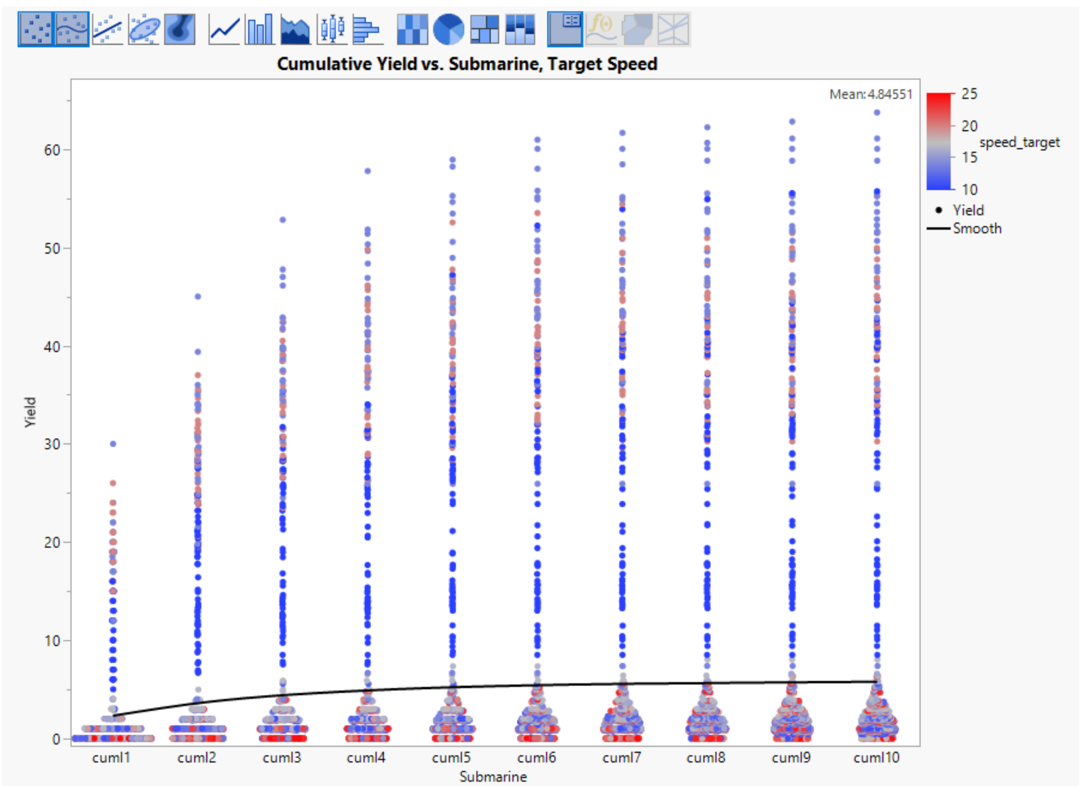


Figure 4.8. Cumulative Yield by Submarine, S_T, Interfering Contacts, No Communication

Figure 4.9 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". Changes in λ_T are shown in color. A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function.

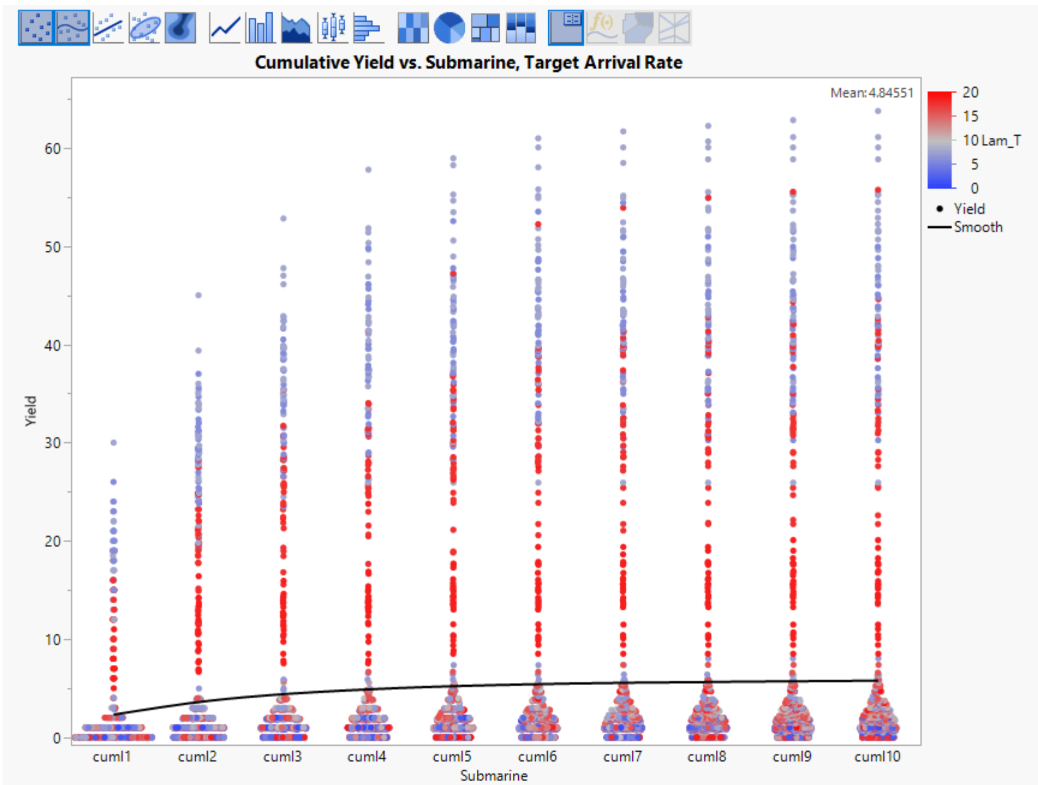


Figure 4.9. Cumulative Yield by Submarine, λ_T, Interfering Contacts, No Communication

Figure 4.10 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". Changes in λ_M are shown in color. A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function. The cumulative yield curve appears to also be sensitive to moderate values of λ_M

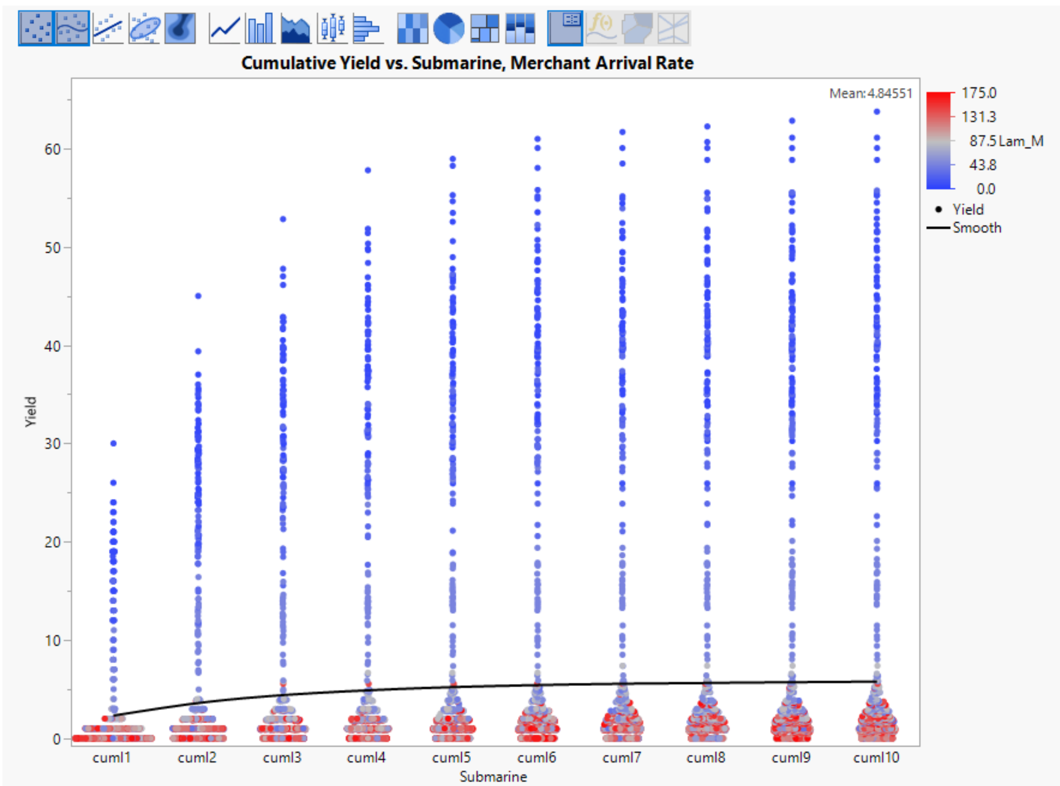


Figure 4.10. Cumulative Yield by Submarine, λ_M, Interfering Contacts, No Communication

A linear regression model was used to summarize the association between simulation average number of targets killed by all 10 submarines and the simulation factors . The regression model includes main effects, second degree quadratic effects, and effects of each factor to degree two. The model achieved an $R^2 \approx 0.90$ and principally emphasized the importance of the main effects from S_T and λ_M (Figure 4.11). A one layer network has an $R^2 \approx 0.95$. As in the case with no interfering contacts, both the linear model and the neural network remain excellent tools to estimate factor importance and model outcomes.

Source	LogWorth	PValue
speed_target	119.824	0.00000
Lam_M	73.445	0.00000
P_k*Lam_M	16.506	0.00000
Lam_M*Lam_M	13.069	0.00000
P_k	13.057	0.00000
P_k*Lam_T	11.493	0.00000
P_k*P_k	11.480	0.00000
Lam_T*Lam_T	8.317	0.00000
Lam_T*speed_target	8.129	0.00000
speed_target*speed_target	6.798	0.00000
P_k*speed_target	3.984	0.00010
Lam_M*speed_target	3.408	0.00039
speed_sub*speed_sub	3.235	0.00058
speed_sub*P_k	2.678	0.00210
speed_sub	1.298	0.05039
Lam_T*Lam_M	0.375	0.42174
speed_sub*speed_target	0.245	0.56879
Lam_T	0.215	0.60993
speed_sub*Lam_M	0.147	0.71231
speed_sub*Lam_T	0.001	0.99850

Figure 4.11. Regression Including Full Factorial Effects to Degree 2, Interfering Contacts, No Communication

The number of Submarines needed to achieve 80% and 90% of the average cumulative kills as a Wolfpack of 10 Submarines ($\bar{Y}_{10}$) is considered. As an example, to calculate the number of submarines N_{SUB} required to achieve $0.8\bar{Y}_{10}$ for a given P_K, the value of $\bar{Y}_{N_{SUB}}$ must be strictly greater than $0.8\bar{Y}_{10}$. The introduction of interfering contacts suggests at least 2 more submarines are needed to achieve the same 80% and 90% $\bar{Y}_{10}$ as compared to the "No Interfering Contacts" case (Table 4.1).

Table 4.2. Efficiency for Different Fixed P_K, Interfering Contacts, No Communication

Selected P_K	$\bar{Y}_{10}$	Std Err Mean	N_{SUB} for $0.8\bar{Y}_{10}$	N_{SUB} for $0.9\bar{Y}_{10}$
0.25	0.52	0.10	7	10
0.50	1.68	0.17	5	6
0.75	2.85	0.29	5	6

4.2.2 Intermittent Communication

Each submarine in the Wolfpack was allowed to receive information every 10 hours. A description of communication assumptions as well as interdiction methods are provided in Section 2.1.2. Figure 4.12 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". Changes in S_T are shown in color. A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function.

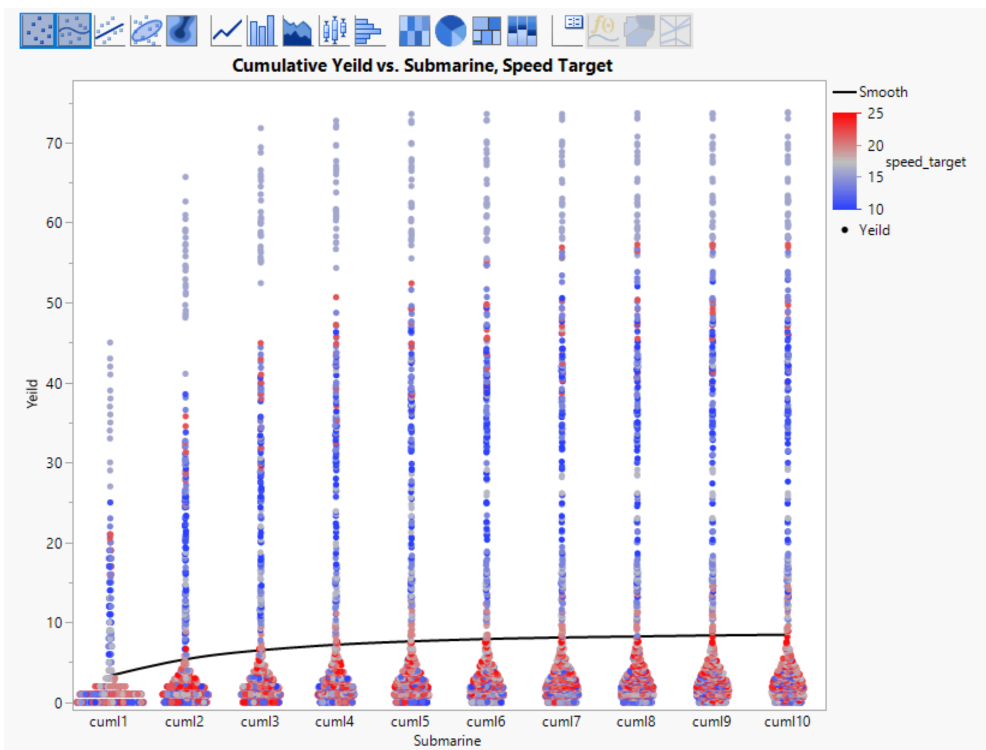


Figure 4.12. Cumulative Yield by Submarine, S_T, Interfering Contacts, Intermittent Communication

Figure 4.13 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". Changes in S_S are shown in color. A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function.

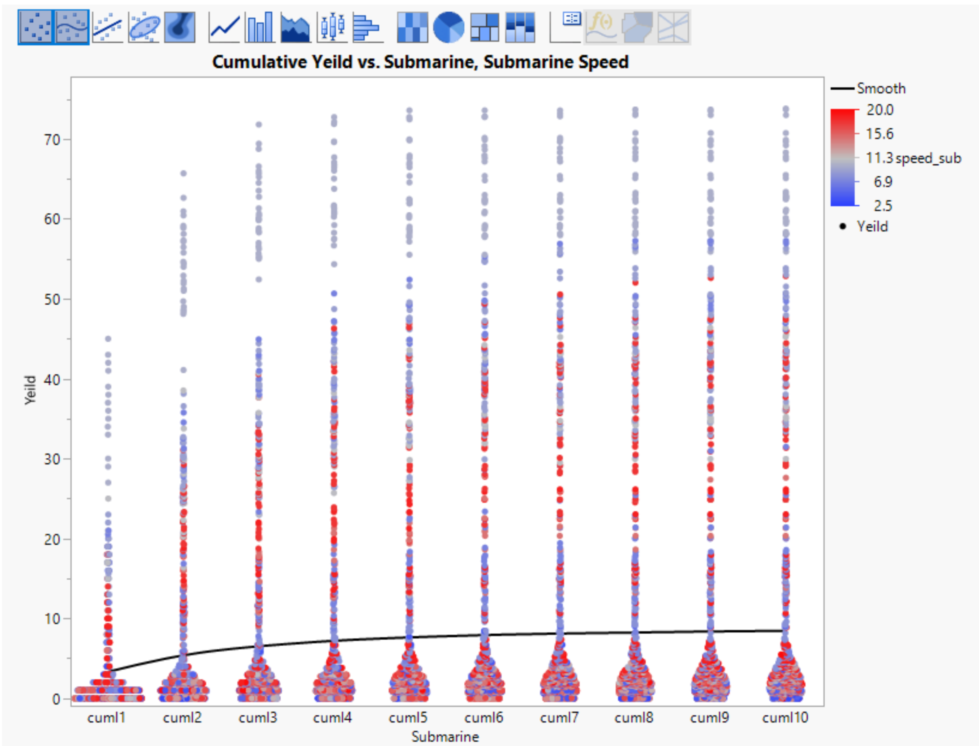


Figure 4.13. Cumulative Yield by Submarine, S_S, Interfering Contacts, Intermittent Communication

Figure 4.14 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". Changes in P_K are shown in color. A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function.

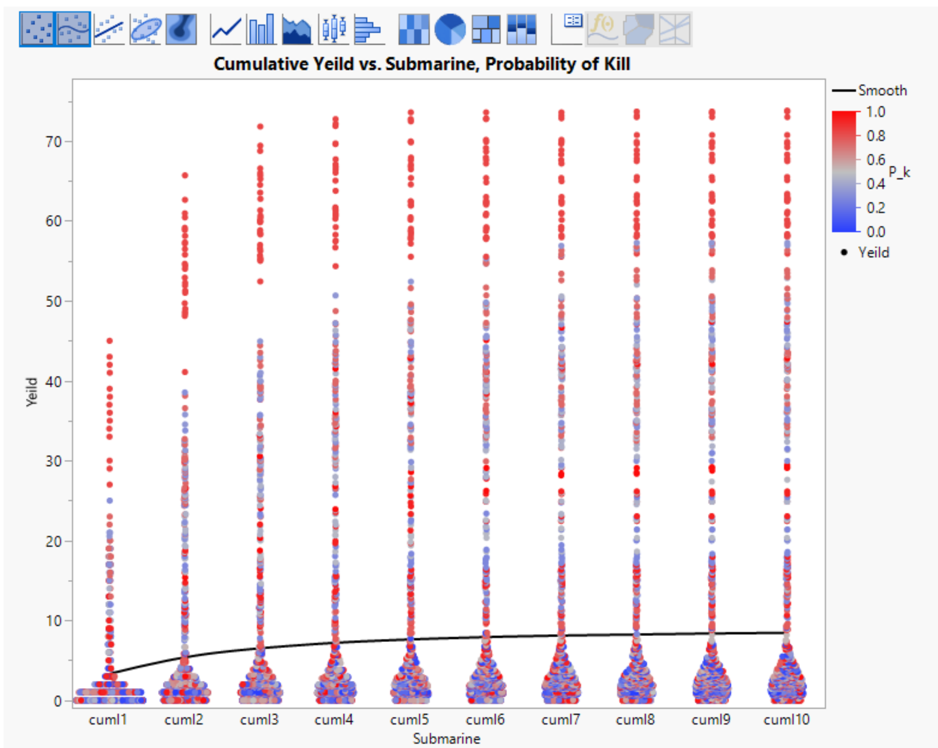


Figure 4.14. Cumulative Yield by Submarine, P_K, Interfering Contacts, Intermittent Communication

Figure 4.15 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". Changes in λ_T are shown in color. A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function.

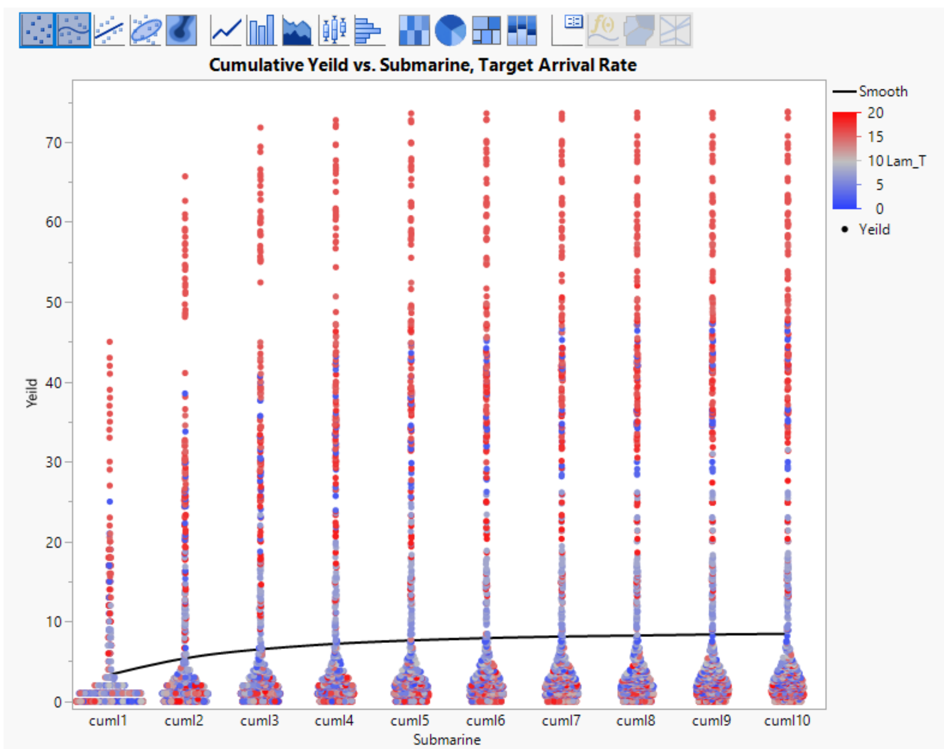


Figure 4.15. Cumulative Yield by Submarine, λ_T, Interfering Contacts, Intermittent Communication

Figure 4.16 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". Changes in λ_M are shown in color. A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function.

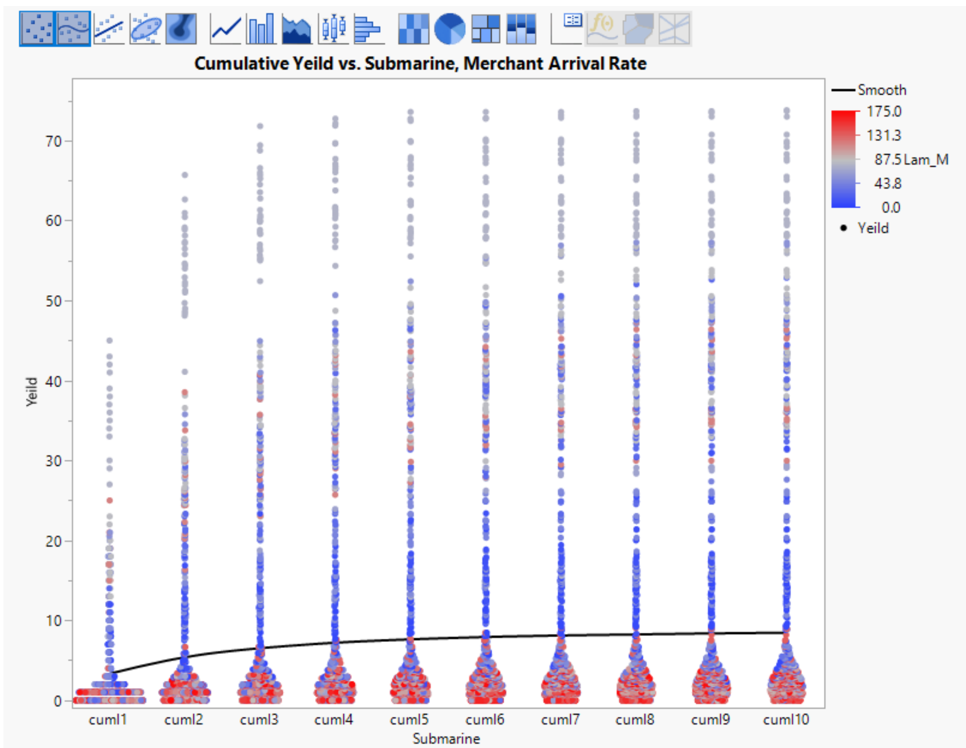
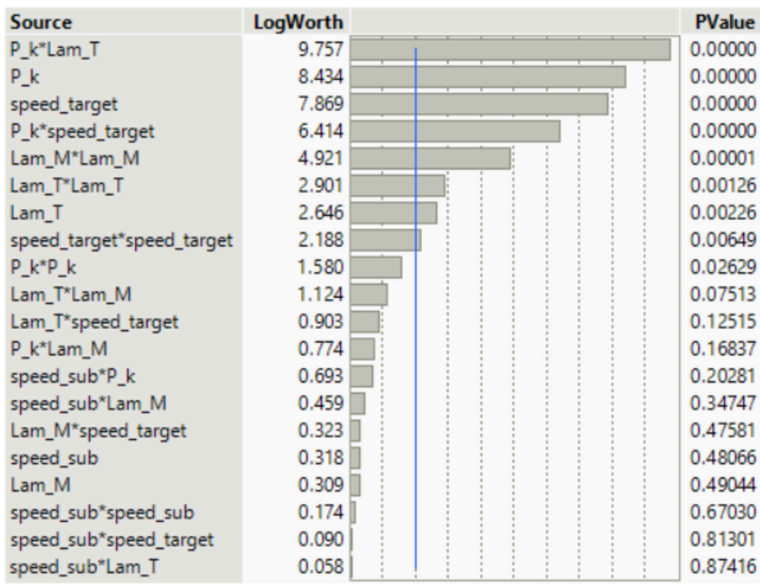


Figure 4.16. Cumulative Yield by Submarine, λ_M, Interfering Contacts, Intermittent Communication

A linear regression model was used to summarize the association between simulation average number of targets killed by all 10 submarines and the simulation factors . The regression model includes main effects, second degree quadratic effects, and effects of each factor to degree two. The model achieved an $R^2 \approx 0.49$ (Figure 4.17) and emphasized a larger mixture of factors than the previous linear models. A one-layer neural network results in a model with an $R^2 \approx 0.68$. In this case, the linear model and the neural network were not able to define as clear of a relationship between the factors and the simulation output, though the neural network performed with a higher R^2.



Source	LogWorth	PValue
P_k*Lam_T	9.757	0.00000
P_k	8.434	0.00000
speed_target	7.869	0.00000
P_k*speed_target	6.414	0.00000
Lam_M*Lam_M	4.921	0.00001
Lam_T*Lam_T	2.901	0.00126
Lam_T	2.646	0.00226
speed_target*speed_target	2.188	0.00649
P_k*P_k	1.580	0.02629
Lam_T*Lam_M	1.124	0.07513
Lam_T*speed_target	0.903	0.12515
P_k*Lam_M	0.774	0.16837
speed_sub*P_k	0.693	0.20281
speed_sub*Lam_M	0.459	0.34747
Lam_M*speed_target	0.323	0.47581
speed_sub	0.318	0.48066
Lam_M	0.309	0.49044
speed_sub*speed_sub	0.174	0.67030
speed_sub*speed_target	0.090	0.81301
speed_sub*Lam_T	0.058	0.87416

Figure 4.17. Regression Including Full Factorial Effects to Degree 2, Interfering Contacts, Intermittent Communication

The number of Submarines needed to achieve 80% and 90% of the average cumulative kills as a Wolfpack of 10 Submarines ($\bar{Y}_{10}$) is considered. As an example, to calculate the number of submarines N_{SUB} required to achieve $0.8\bar{Y}_{10}$ for a given P_K, the value of $\bar{Y}_{N_{SUB}}$ must be strictly greater than $0.8\bar{Y}_{10}$. While still requiring a larger number of submarines to achieve a proportional yield as compared to Table 4.1, allowing the submarines to communicate lowered the N_{SUB} by about 1 or 2 Submarines to achieve 80% and 90% of $\bar{Y}_{10}$ as compared to Table 4.2. A $P_K = 0.75$ is needed for communication to make a difference on the value of $\bar{Y}_{10}$.

Table 4.3. Efficiency for Different Fixed P_K, Interfering Contacts, Intermittent Communication

Selected P_K	$\bar{Y}_{10}$	Std Err Mean	N_{SUB} for $0.8\bar{Y}_{10}$	N_{SUB} for $0.9\bar{Y}_{10}$
0.25	1.57	0.16	5	6
0.50	0.96	0.26	5	6
0.75	43.62	1.18	4	5

4.2.3 Constant Communication

Each submarine now is downloading information from an off-hull entity every second. A description of communication assumptions as well as interdiction methods are provided in Section 2.1.2. Figure 4.18 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". Changes in S_T are shown in color. A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function.

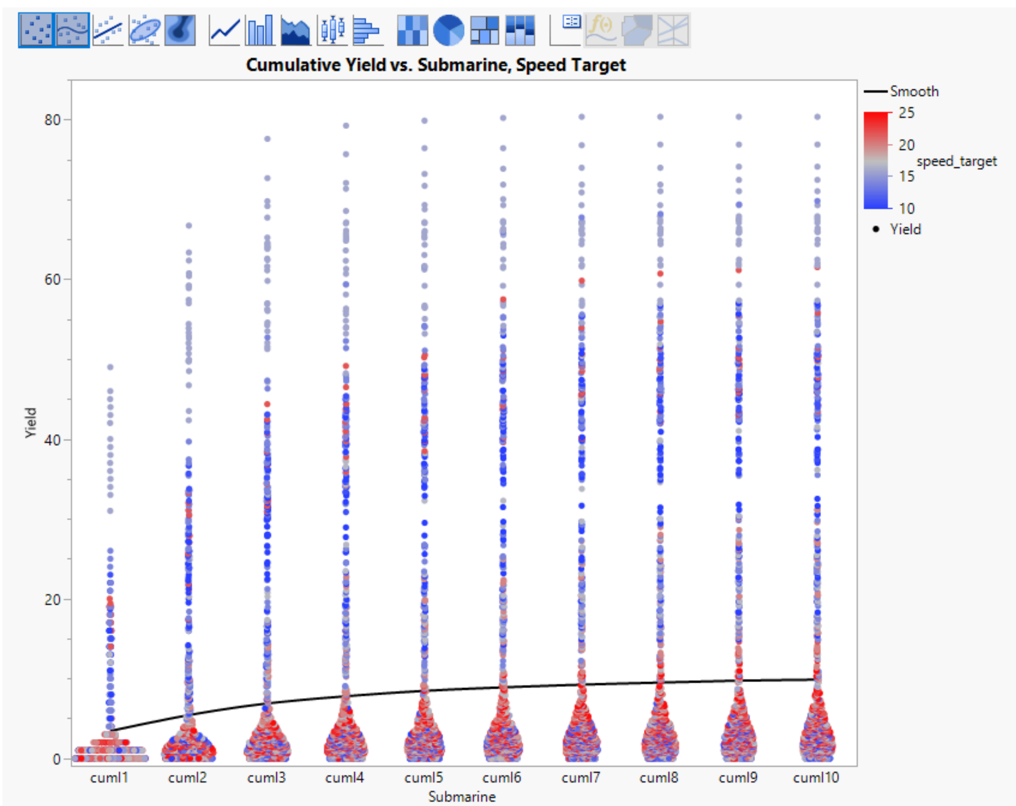


Figure 4.18. Cumulative Yield by Submarine, S_T, Interfering Contacts, Constant Communication

Figure 4.19 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". Changes in S_S are shown in color. A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function.

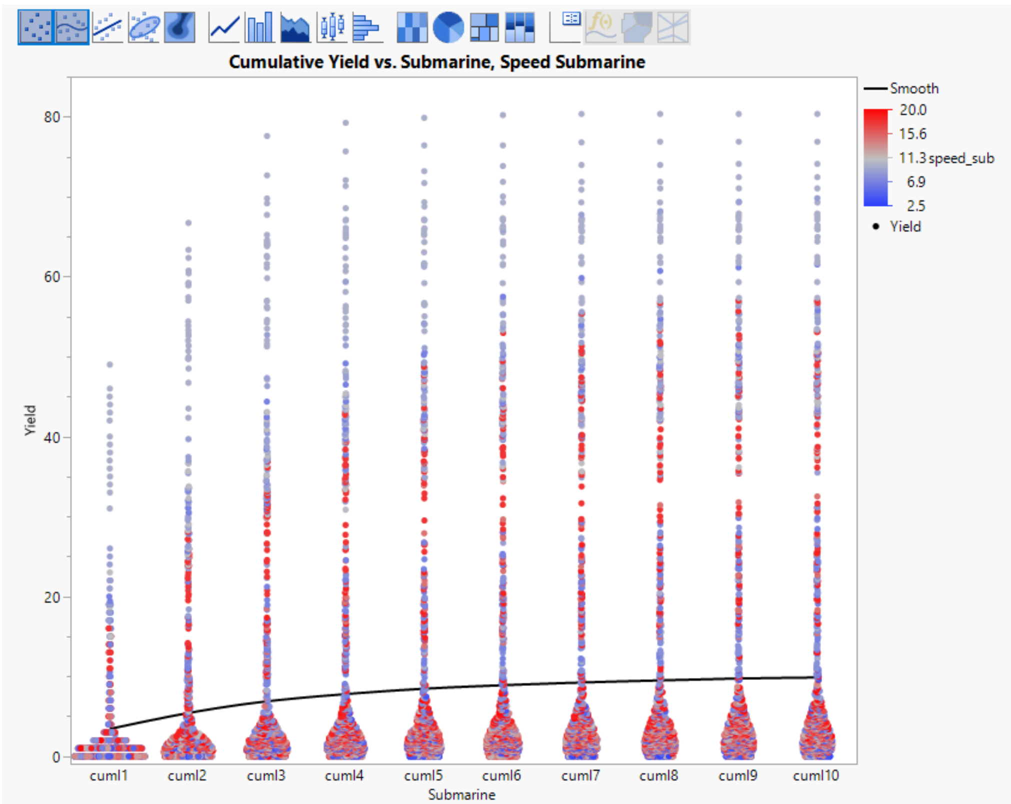


Figure 4.19. Cumulative Yield by Submarine, S_S, Interfering Contacts, Constant Communication

Figure 4.20 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". Changes in P_K are shown in color. A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function.

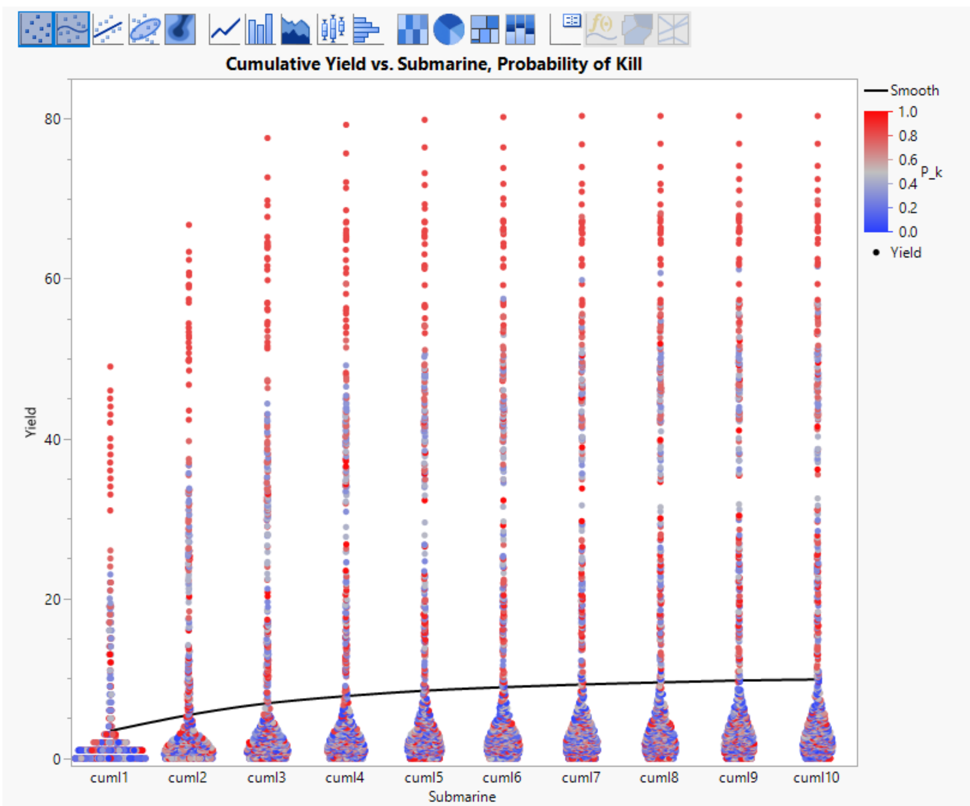


Figure 4.20. Cumulative Yield by Submarine, P_K, Interfering Contacts, Constant Communication

Figure 4.21 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". Changes in λ_T are shown in color. A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function.

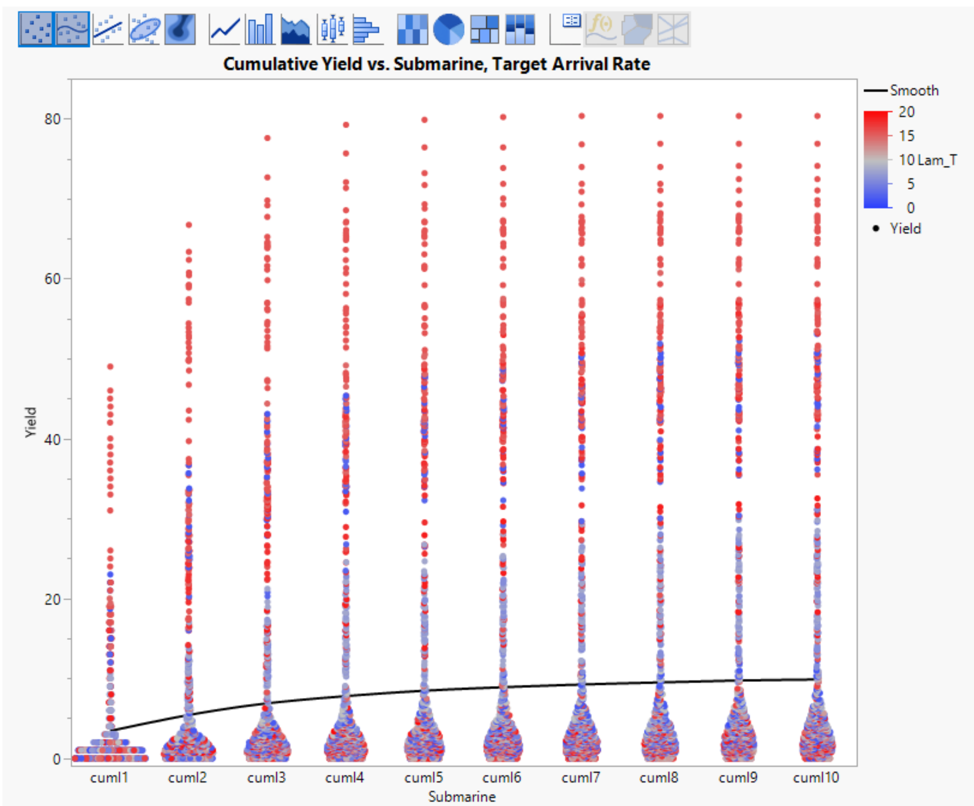


Figure 4.21. Cumulative Yield by Submarine, λ_T, Interfering Contacts, Constant Communication

Figure 4.22 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". Changes in λ_M are shown in color. A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function.

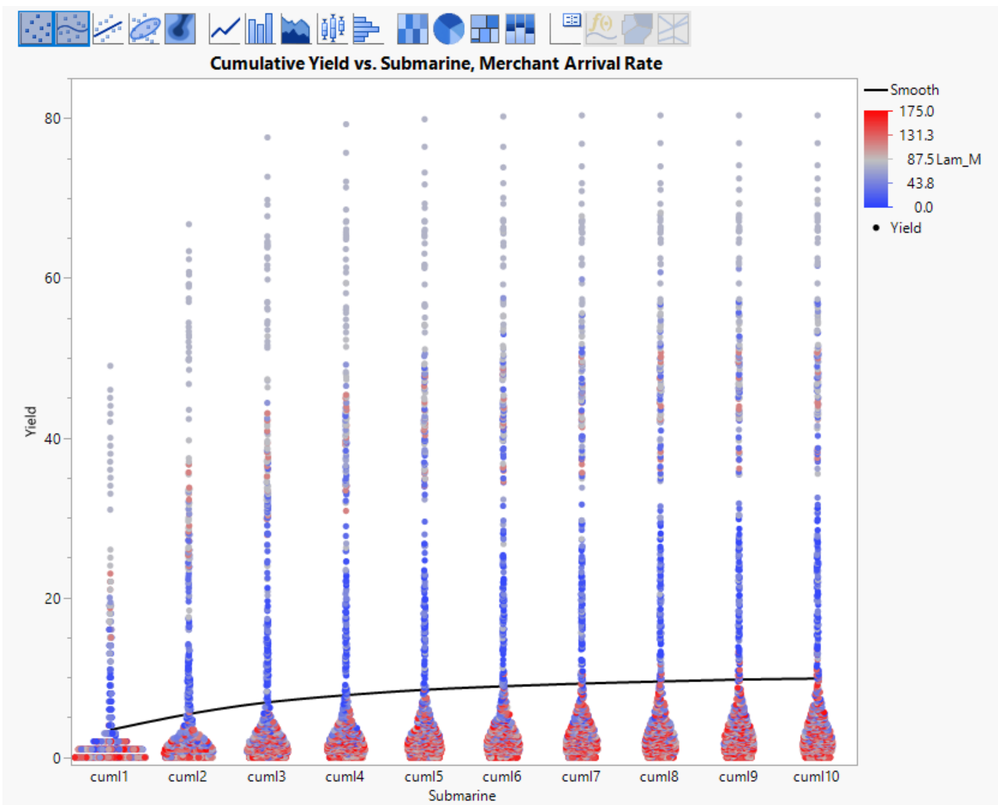


Figure 4.22. Cumulative Yield by Submarine, λ_M, Interfering Contacts, Constant Communication

A linear regression model was used to summarize the association between simulation average number of targets killed by all 10 submarines and the simulation factors . The regression model includes main effects, second degree quadratic effects, and effects of each factor to degree two. The model achieved an $R^2 \approx 0.48$ (Figure 4.17). A one-layer neural network results in a model with an $R^2 \approx 0.65$. The neural network continues to outperform the linear model as measured by R^2.

Source	LogWorth	PValue
P_k	13.640	0.00000
P_k*speed_target	12.805	0.00000
Lam_T*Lam_M	6.333	0.00000
speed_sub*P_k	6.137	0.00000
Lam_T*Lam_T	5.672	0.00000
speed_target*speed_target	5.556	0.00000
P_k*Lam_T	5.523	0.00000
Lam_M*Lam_M	5.220	0.00001
speed_target	4.085	0.00008
Lam_T	3.176	0.00067
speed_sub*Lam_T	2.200	0.00631
speed_sub*speed_target	2.026	0.00941
P_k*P_k	1.510	0.03091
Lam_M*speed_target	1.167	0.06801
speed_sub*speed_sub	0.639	0.22946
P_k*Lam_M	0.539	0.28905
Lam_M	0.381	0.41591
Lam_T*speed_target	0.262	0.54689
speed_sub*Lam_M	0.223	0.59844
speed_sub	0.173	0.67149

Figure 4.23. Regression Including Full Factorial Effects to Degree 2, Interfering Contacts, Constant Communication

The number of Submarines needed to achieve 80% and 90% of the average cumulative kills as a Wolfpack of 10 Submarines ($\bar{Y}_{10}$) is considered. As an example, to calculate the number of submarines N_{SUB} required to achieve $0.8\bar{Y}_{10}$ for a given P_K, the value of $\bar{Y}_{N_{SUB}}$ must be strictly greater than $0.8\bar{Y}_{10}$. There is not improvement in the N_{SUB} as compared to Table 4.3. Also, it is noted a $P_K = 0.75$ is needed for communication to make a difference on the value of $\bar{Y}_{10}$. The values of $\bar{Y}_{10}$ are also comparable to the those displayed in intermittent communication on Table 4.3.

Table 4.4. Efficiency for Different Fixed P_K, Interfering Contacts, Constant Communication

Selected P_K	$\bar{Y}_{10}$	Std Err Mean	N_{SUB} for $0.8\bar{Y}_{10}$	N_{SUB} for $0.9\bar{Y}_{10}$
0.25	2.02	0.23	7	8
0.50	1.15	0.39	7	8
0.75	47.76	1.25	4	5

4.2.4 Comparing Across Communication Schemes for Interfering Contacts

Figure 4.24 shows changes in $\bar{Y}_i$ for each additional submarine i in a Wolfpack for each communication scheme. The intervals are plotted to show each $\bar{Y}_i$, as well as their associated 95% confidence intervals, for each communication scheme using the Line of Fit method of the JMP (2021) Graph Builder. Communication scheme 1 is communicating constantly, communication scheme 2 is every 10 hours, and communication scheme 3 is no available communication. There is a 46% increase in average cumulative number of kills for a Wolfpack of size 10 by communicating every 10 hours instead of not communicating at all. Communicating every second instead of every 10 hours amounted to little benefit as measured by $\bar{Y}_i$, especially for a low number of submarines.

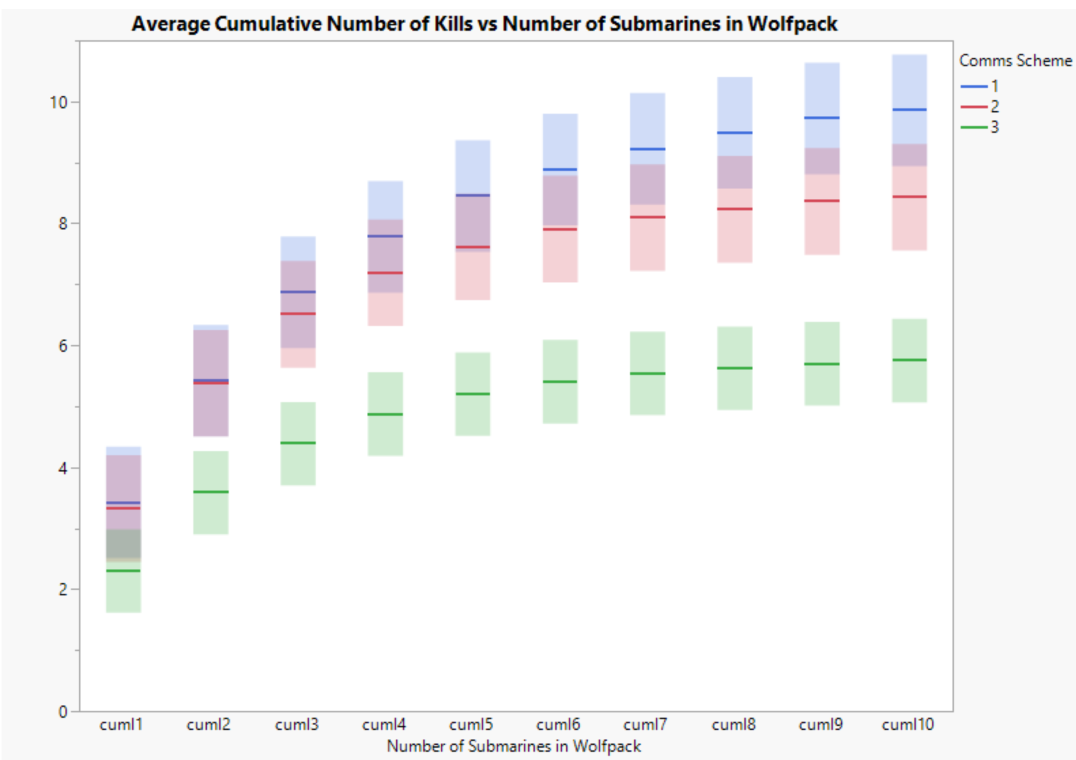


Figure 4.24. Average Cumulative Nubmer of Kills vs Number of Submarines in Wolfpack, Interfering Contacts

4.3 Surface Action Group

A SAG configuration is shown in Figure 2.3. All members of the SAG all have equal priority to be destroyed. It is expected that ten submarines are more than sufficient to find and destroy a six ship SAG, so analysis will be given to finding factors that improve the efficiency of the Wolfpack,

4.3.1 No Communication

Each member of the Wolfpack must detect, classify, and destroy the SAG (or portions of the SAG) independently. Figure 4.25 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". Changes in S_S are shown in color. A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function.

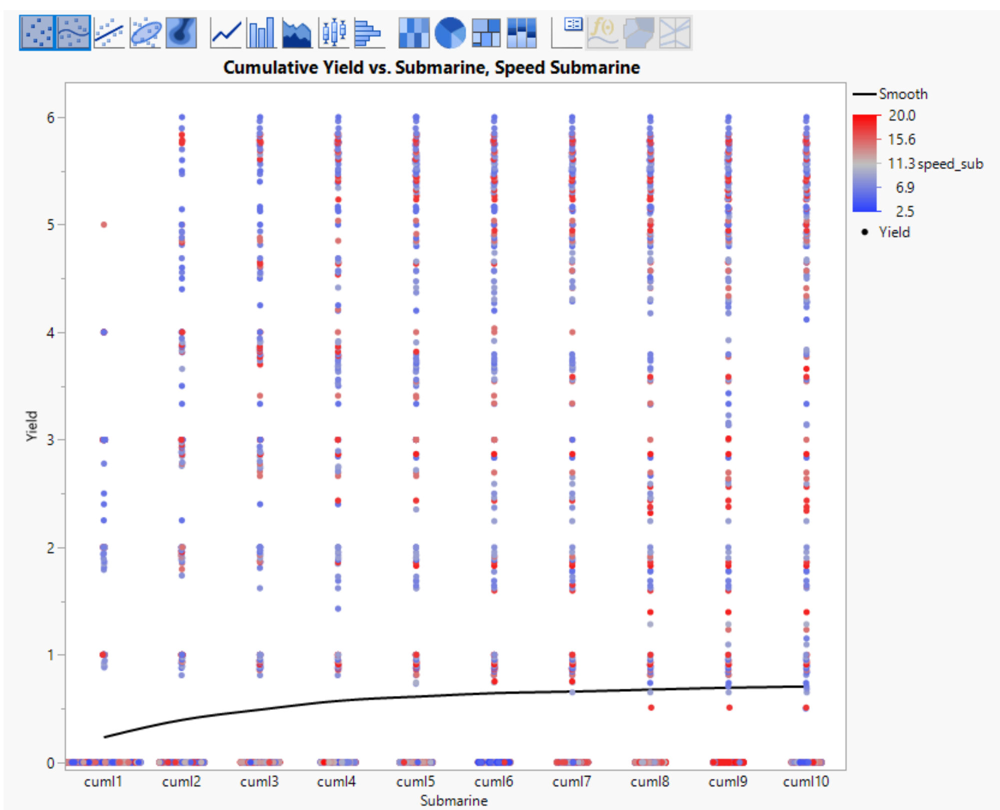


Figure 4.25. Cumulative Yield vs. Submarine, S_S, Surface Action Group, No Communication

Figure 4.26 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". Changes in S_T are shown in color. A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function.

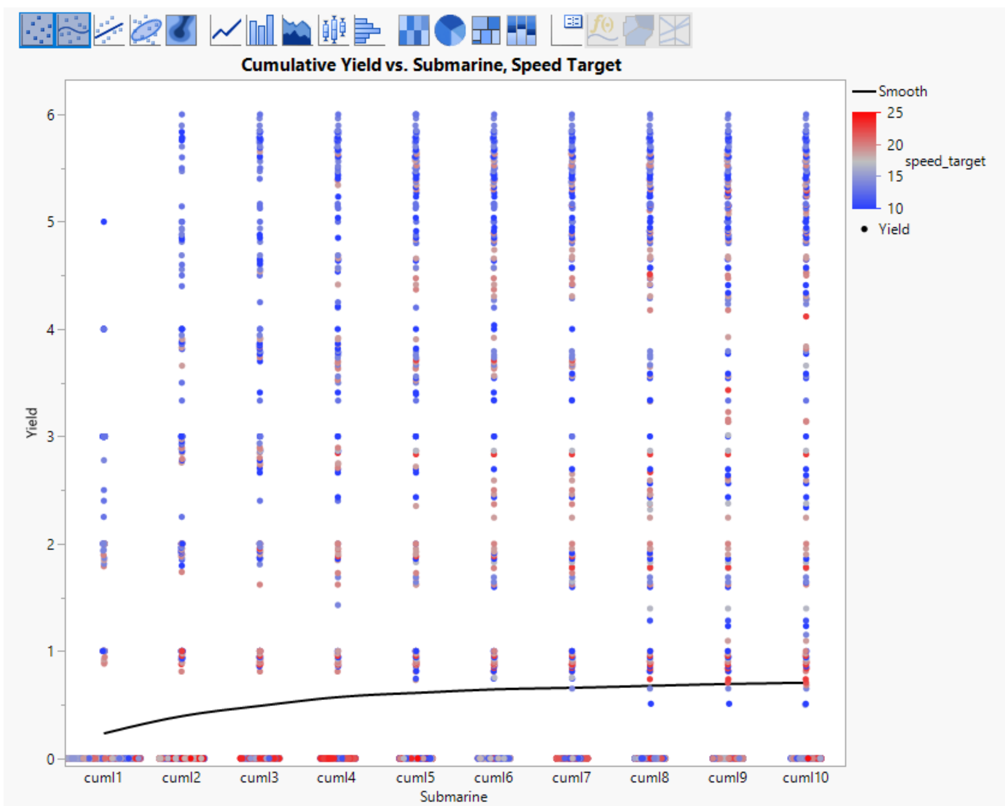


Figure 4.26. Cumulative Yield vs. Submarine, S_T, Surface Action Group, No Communication

Figure 4.27 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function. This graph is grouped by ranges of λ_M, and with changes in P_K shown in color. This is done to highlight how changes in λ_M can drastically affect the yield of destroying even a few members of the SAG.

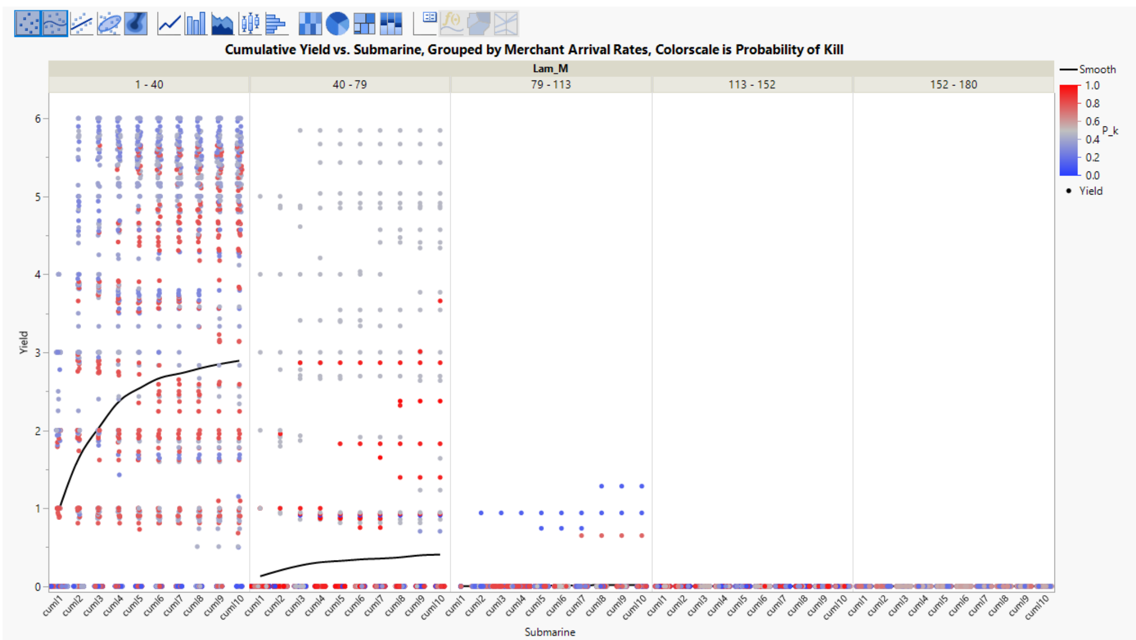


Figure 4.27. Cumulative Yield vs. Submarine, Grouped by Merchant Arrival Rate, Colorscale is P_K, Surface Action Group, No Communication

A linear regression model was used to summarize the association between simulation average number of targets killed by all 10 submarines and the simulation factors . The regression model includes main effects, second degree quadratic effects, and effects of each factor to degree two. The model achieved an $R^2 \approx 0.70$ (Figure 4.17). A one-layer neural network results in a model with an $R^2 \approx 0.67$.

Source	LogWorth	PValue
P_k	13.640	0.00000
P_k*speed_target	12.805	0.00000
Lam_T*Lam_M	6.333	0.00000
speed_sub*P_k	6.137	0.00000
Lam_T*Lam_T	5.672	0.00000
speed_target*speed_target	5.556	0.00000
P_k*Lam_T	5.523	0.00000
Lam_M*Lam_M	5.220	0.00001
speed_target	4.085	0.00008
Lam_T	3.176	0.00067
speed_sub*Lam_T	2.200	0.00631
speed_sub*speed_target	2.026	0.00941
P_k*P_k	1.510	0.03091
Lam_M*speed_target	1.167	0.06801
speed_sub*speed_sub	0.639	0.22946
P_k*Lam_M	0.539	0.28905
Lam_M	0.381	0.41591
Lam_T*speed_target	0.262	0.54689
speed_sub*Lam_M	0.223	0.59844
speed_sub	0.173	0.67149

Figure 4.28. Regression Including Full Factorial Effects to Degree 2, Surface Action Group, No Communication

4.3.2 Intermittent Communication

Figure 4.29 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". Changes in S_S are shown in color. A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function.

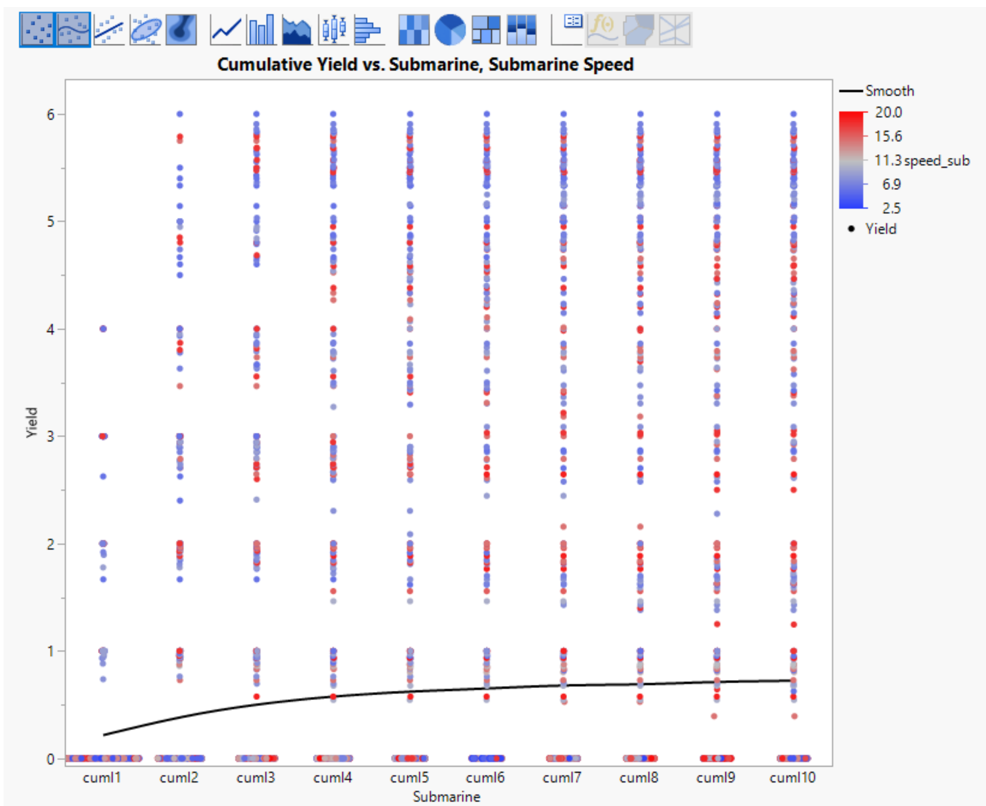


Figure 4.29. Cumulative Yield vs. Submarine, S_S, Surface Action Group, Intermittent Communication

Figure 4.30 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". Changes in S_T are shown in color. A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function.

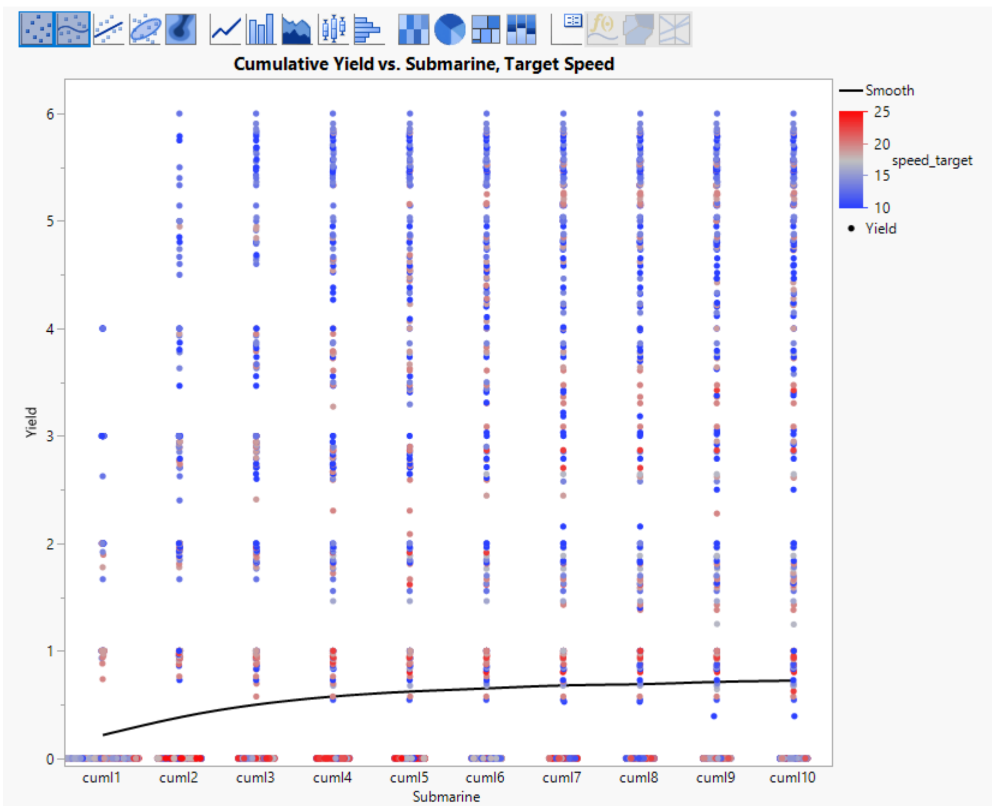


Figure 4.30. Cumulative Yield vs. Submarine, S_T, Surface Action Group, Intermittent Communication

Figure 4.31 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". Changes in P_K are shown in color. A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function.

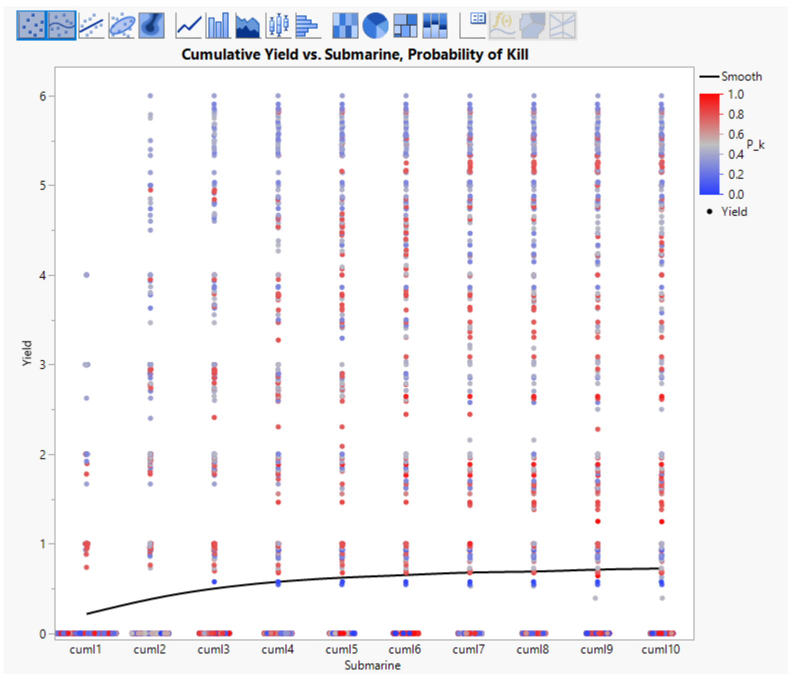


Figure 4.31. Cumulative Yield vs. Submarine, P_K, Surface Action Group, Intermittent Communication

Figure 4.32 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function. This graph is grouped into ranges of λ_M. While this case is still highly sensitive to changes in λ_M, the cumulative yield does not drop to zero as in Figure 4.27.

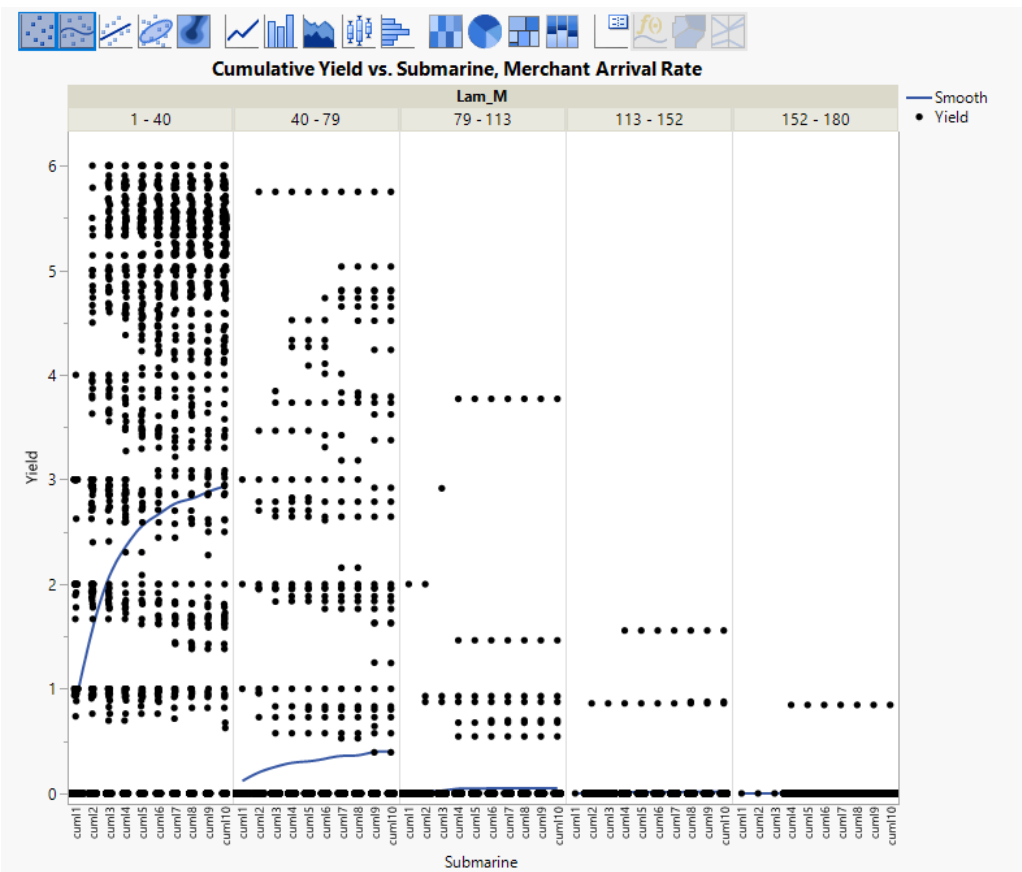


Figure 4.32. Cumulative Yield vs. Submarine, λ_M, Surface Action Group, Intermittent Communication

A linear regression model was used to summarize the association between simulation average number of targets killed by all 10 submarines and the simulation factors . The regression model includes main effects, second degree quadratic effects, and effects of each factor to degree two. The model achieved an $R^2 \approx 0.74$ (Figure 4.17). A one layer neural network results in an $R^2 \approx 0.80$.

Source	LogWorth	PValue
Lam_M	176.654	0.00000
speed_target	62.126	0.00000
speed_sub	15.836	0.00000
Lam_M*Lam_M	10.231	0.00000
Lam_M*speed_target	9.906	0.00000
speed_sub*speed_sub	1.635	0.02319
speed_sub*Lam_M	1.224	0.05964
speed_sub*P_k	0.708	0.19571
speed_sub*speed_target	0.379	0.41789
P_k*speed_target	0.320	0.47811
P_k	0.187	0.64986
P_k*Lam_M	0.133	0.73704
speed_target*speed_target	0.068	0.85418
P_k*P_k	0.050	0.89062

Figure 4.33. Regression Including Full Factorial Effects to Degree 2, Surface Action Group, Intermittent Communication

4.3.3 Constant Communication

Figure 4.34 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". Changes in S_S are shown in color. A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function.

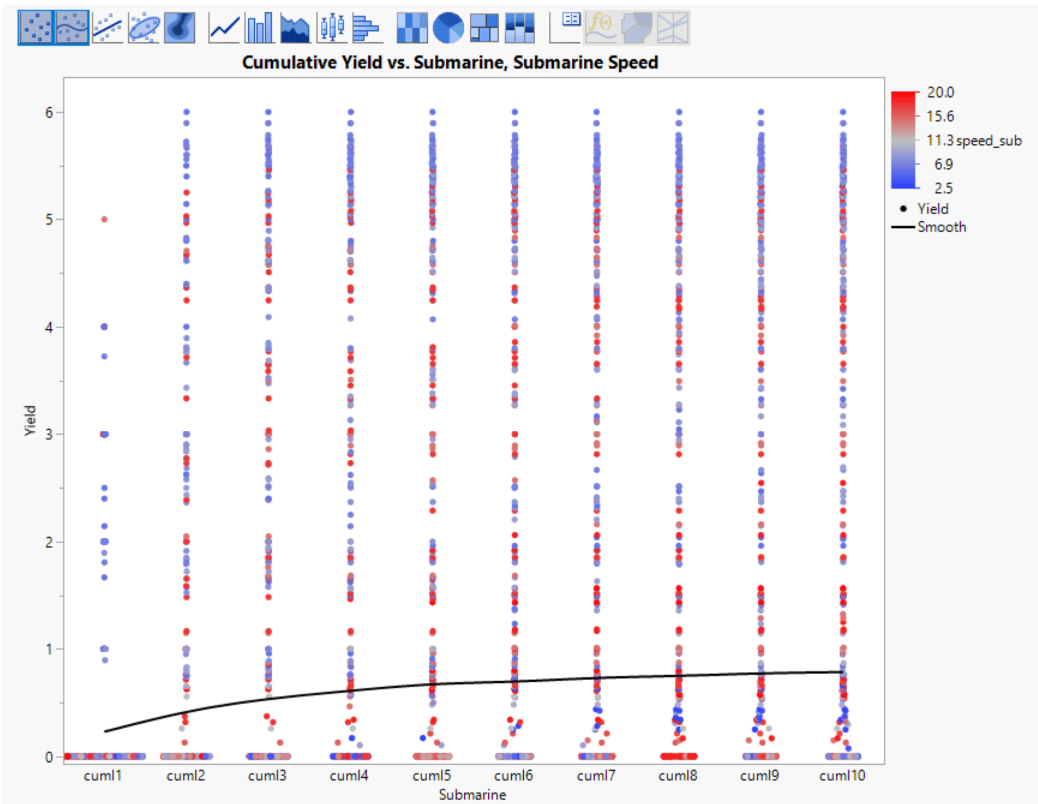


Figure 4.34. Cumulative Yield vs. Submarine, S_S, Surface Action Group, Constant Communication

Figure 4.35 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". Changes in S_T are shown in color. A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function.

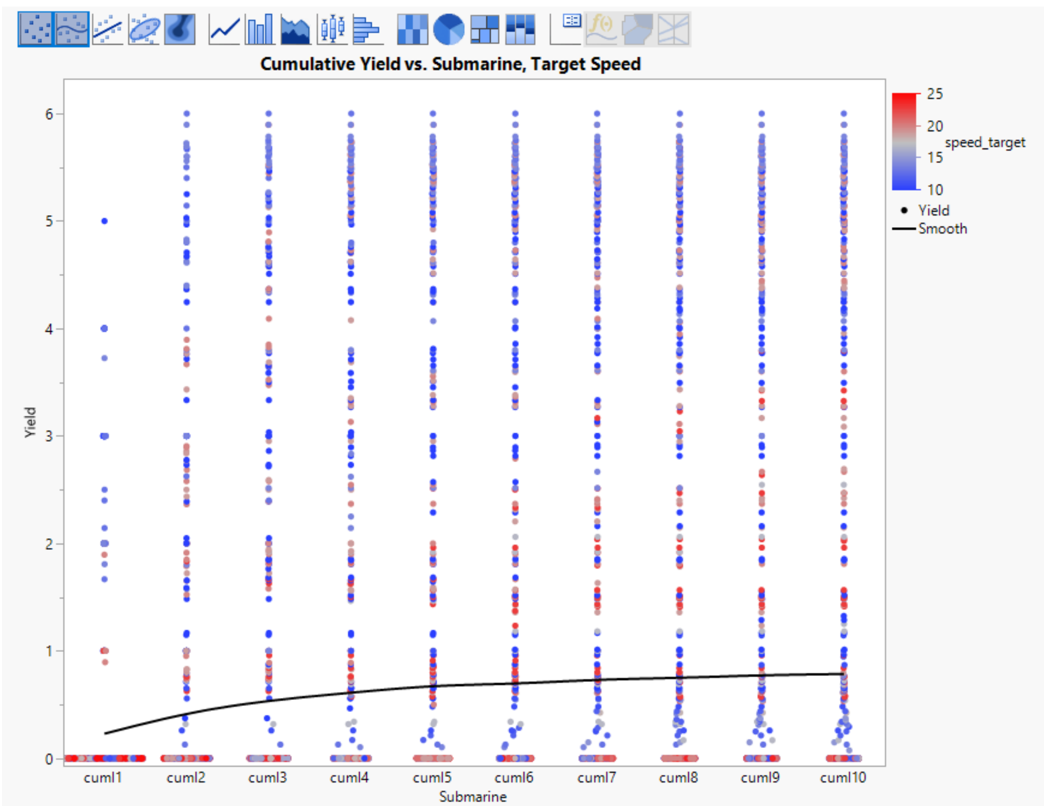


Figure 4.35. Cumulative Yield vs. Submarine, S_T, Surface Action Group, Constant Communication

Figure 4.36 displays $\bar{Y}_i$ for each number of submarines i, labeled "cuml1, cuml2,..., cuml10". Changes in P_K are shown in color. A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function.

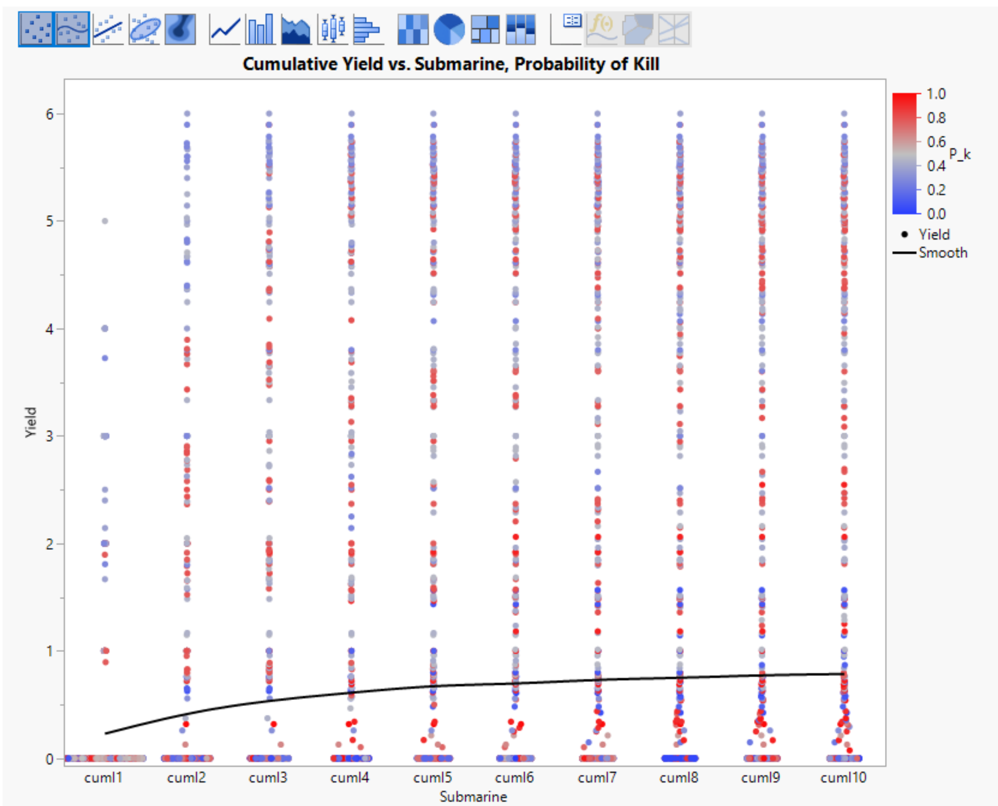


Figure 4.36. Cumulative Yield vs. Submarine, P_K, Surface Action Group, Constant Communication

Figure 4.37 displays $\bar{Y}_i$ for each number of submarines i. A line is plotted to summarize changes in each $\bar{Y}_i$ using the Spline method of the JMP (2021) Smoother function. This graph is grouped into ranges of λ_M While this case is still highly sensitive to changes in λ_M, the cumulative yield appears even more resilient to changes in λ_M than Figure 4.32.

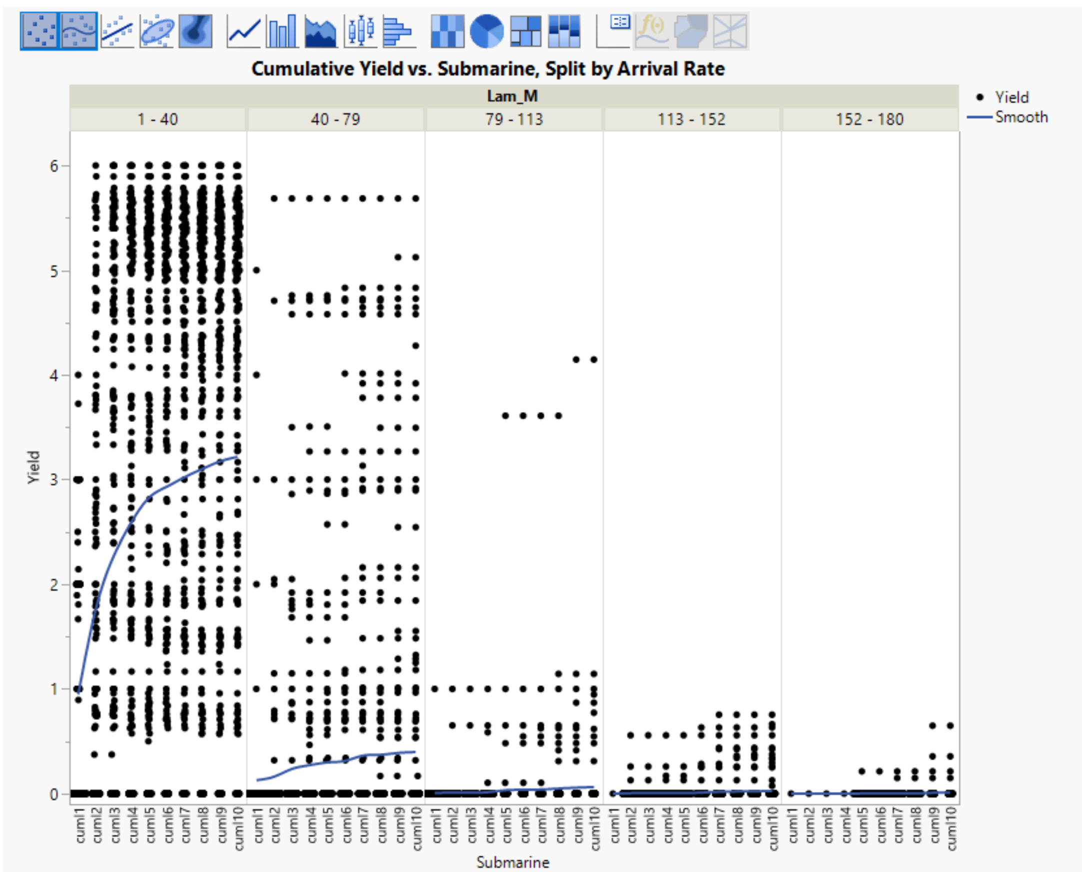


Figure 4.37. Cumulative Yield vs. Submarine, λ_M, Surface Action Group, Constant Communication

A linear regression model was used to summarize the association between simulation average number of targets killed by all 10 submarines and the simulation factors . The regression model includes main effects, second degree quadratic effects, and effects of each factor to degree two. The model achieved an $R^2 \approx 0.78$ (Figure 4.22). A one-layer neural network results in a model with an $R^2 \approx 0.87$.

Source	LogWorth	PValue
Lam_M	212.997	0.00000
speed_target	59.061	0.00000
speed_sub	28.112	0.00000
Lam_M*Lam_M	16.063	0.00000
Lam_M*speed_target	5.531	0.00000
speed_sub*speed_sub	2.899	0.00126
speed_sub*Lam_M	1.914	0.01220
speed_sub*P_k	1.382	0.04153
P_k*speed_target	0.937	0.11562
P_k	0.543	0.28648
speed_sub*speed_target	0.402	0.39617
speed_target*speed_target	0.277	0.52822
P_k*P_k	0.276	0.52959
P_k*Lam_M	0.235	0.58254

Figure 4.38. Regression Including Full Factorial Effects to Degree 2, Surface Action Group, Constant Communication

4.3.4 Comparing Across Communication Schemes for SAG

Differences in yield are difficult to compare with very low number of Target vessels. There are small differences in cumulative average number of Targets killed for each number of submarines between the different communication schemes, however, there are not enough experiments performed to establish any difference in these values. It is instead suggested to compare average number of Targets killed for each number of submarines across both λ_M and communications scheme. It becomes apparent that communication offers greater resilience in situations where there exists a very small Target to Merchant ratio. Communication scheme 1 is communicating constantly, communication scheme 2 is communicating every 10 hours, and communication scheme number 3 is not communicating.

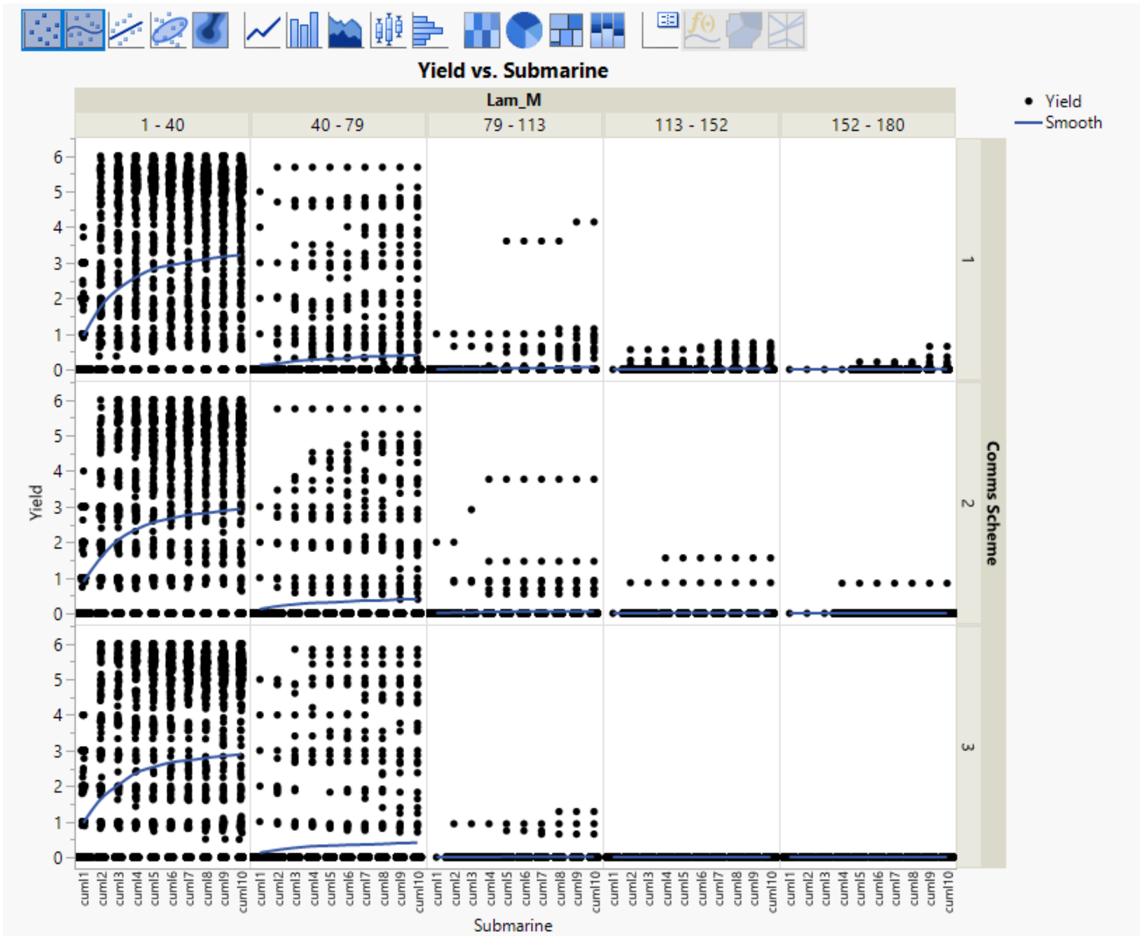


Figure 4.39. Yield vs. Submarine, Surface Action Group

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CHAPTER 5: Conclusions

This research presents applying the data farming process to study the effectiveness of Wolfpack tactics in an environment with targets and neutral shipping under the most advantageous conditions. Communication between submarines is supposed to be free of error and not expose the submarine to danger. There are no logistic constraints. A myopic interdiction method is proposed to facilitate cooperative battle for a Wolfpack of submarines in non-overlapping waterspaces. This research results in the relationships between communications schemes and average number of targets killed for different numbers of submarines in the Wolfpack.

This research is intended to be a launching point for further investigation of coordinated battle. Objects in this simulation may be thought of as surface ships or even aircraft in environments with degraded communications with simple modifications to the existing code. Different types of vessels or weapons may be added, such as mines. More behaviors may be accounted for, such as necessitating a minimum standoff distance from any vessel by a submarine, evasion techniques, and the ability of a submarine to engage multiple Targets. The flexibility of the data farming process allows investigation of many different kinds of processes with relative ease.

5.1 Analysis

For situations involving the ability to destroy any entering vessel, the work of Submarine Operations Research Group (1944) remains a useful and accurate model for determining optimal Wolfpack size as well as expected number of Targets killed for a submarine Wolfpack where there is no neutral shipping. About three submarines in a Wolfpack with a P_K higher than 0.5 will be able to achieve 90% the average cumulative number of targets killed of ten submarines $\bar{Y}_{10}$, and will also not need to communicate. For situations involving a mixture of Targets and neutral shipping, the regression models considered became increasingly poor predictors of the average cumulative number of targets killed by a Wolfpack of 10 Submarines; single layer neural networks considered had marginally better results.

Communication between submarines is only possible if a submarine detects a Target but is unable to prosecute it before the Target leaves it waterspace. As a result, the effect of communication is limited. There exists an incentive to communicate, but communicating more frequently than once every 10hrs had diminishing returns. For the SAG, the low number of Targets in experiments made direct comparison through average number of Targets killed difficult. Instead it was observed that communicating allowed a Wolfpack to locate and destroy a SAG more effectively in environments with a large number of neutral vessels as compared to not communicating.

5.2 Future Work

We suggest using data farming methods is an effective way to evaluate new strategies. The scenario demonstrated is not a one-size fits all evaluation for Wolfpack tactics in every battlespace and instead should be reevaluated in a case-by-case scenario, such as one with multiple threat axis or unsymmetrical waterspace allocations. This work only considered a situation in which the waterspace owned by each submarine is fixed and does not allow overlap. It is suggested that future work consider submarines or UUVs that can operate in stratum. It also is recommended to utilize data farming methods with discrete-event simulations, which can be significantly computationally faster. Building on this work, future experiments should consider utilizing simulation techniques to inform strategic development such as through the use of simulated annealing.

APPENDIX

A.1 Latin Squares

To arrive at a NOLH, we must first construct a Latin Square. Brualdi (2010) provides the following definition of a Latin Square: "Let *n* be a positive integer and let *S* be a set of *n* distinct elements. A Latin square of order *n*, based on the set *S*, is an *n*-by-*n* array, each of whose entries is an element of *S* such that each of the *n* elements of *S* occurs once (and hence exactly once) in each row and once in each column." The popular game of Sudoku (Figure A.1a) is an excellent example of this property, where the numbers 1 through 9 can be arranged in a matrix in many different permutations while still abiding by the given restrictions.

1	8	4	9	6	3	7	2	5
5	6	2	7	4	8	3	1	9
3	9	7	5	1	2	8	6	4
2	3	9	6	5	7	1	4	8
7	5	6	1	8	4	2	9	3
4	1	8	2	3	9	6	5	7
9	4	1	3	7	6	5	8	2
6	2	3	8	9	5	4	7	1
8	7	5	4	2	1	9	3	6

(a) Example Sudoku Puzzle

$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 2 \\ 2 & 3 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 2 & 3 \\ 3 & 2 & 1 & 0 \\ 1 & 0 & 3 & 2 \\ 2 & 3 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} (0,0) & (1,1) & (2,2) & (3,3) \\ (1,3) & (0,2) & (3,1) & (2,0) \\ (2,1) & (3,0) & (0,3) & (1,2) \\ (3,2) & (2,3) & (1,0) & (0,1) \end{bmatrix}$$

(b) Source: Brualdi (2010), Orthogonality Between Two Matrices

Figure A.1. Latin Squares

If two distinct solved Sudoku puzzle are taken and superimposed onto one another, such that each element now makes up an ordered pair of the two puzzles, the two puzzles can be thought of as "orthogonal" to each other (Figure A.1b). The property of orthogonality between the two puzzles is more than just a neat property of the arrangement of numbers. If a given design is organized with this property in mind, the differences in variation will be minimized and conclusions can be better drawn between the given data resulting from simulation output.

A.2 Nearly Orthogonal Latin Hypercubes

There exists one more property of designs that is desired for proper experimentation: space-filling. If one wanted to test the result of two actions having binary inputs, and their interaction with each other, one would need to construct a “factorial” design. Thinking geometrically, this can be thought of needing to test each of the vertices of a square. As an experimenter added one additional factor to the experiment, all the vertices of a cube would now need to be tested. This would continue through any hypercube shape and leave an experimenter with 2^k vertices to test, where k is the number of factors. There exist methods of cutting down on the immense number of created design points, such as by disregarding high order interactions (see fractional factorial design discussion by Sanchez and Wan (2021)). Another method is to use the previously discussed Latin Hypercubes. One weakness of Latin Hypercubes is that a given random arrangement may have poor space-filling qualities, which require an experiment to have large number of design points as compared to factors. As a simple example, take a plot of two factors together from a given design like any given square from Figure A.2. An example of a poor space-filling design would have the points clustered in a corner of that square, unlike the spread out points as shown from a NOLH design. The NOLH's space filling properties married with the low required number of inputs required to achieve an informative result makes NOLH's perfect for simulation experiments (Cioppa and Lucas 2007). This research will utilize the NOLH generator provided by Sanchez (2011).

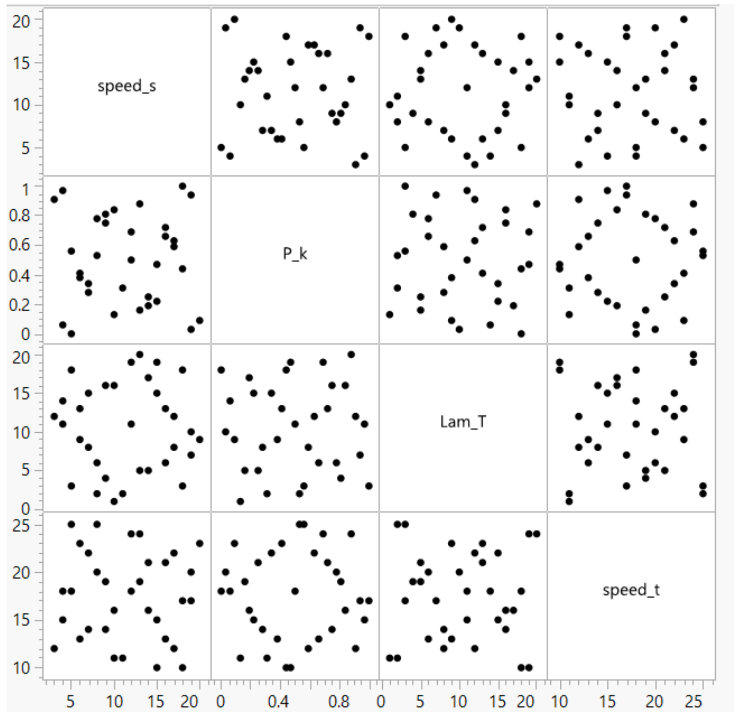

Figure A.2. Geometry of NOLH Visible When Viewed as a Scatterplot Matrix Between Several Factors

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