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Threat Density Map Modeling for Combat Simulations

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ABSTRACT: *The modeling and simulation community has used probability threat maps and other similar approaches to address search problems and improve decision-making. Probability threat maps describe the probability of a location containing one or more enemy entities at a particular time. Although useful, they only describe the likelihood that the location is occupied without addressing the degree to which it is occupied. Thus, we investigate whether threat density maps that describe the searcher's expectation of seeing a number of target agents at a certain location in a given time interval are a viable method for improving synthetic behaviors in combat simulations. As a proof of principle, this paper introduces a probability model which quantifies the searcher agent's subjective belief about the number of enemy entities in a location, given the initial information described by a prior density function and the information provided by the assumed sensing model. In addition, this paper discusses a framework for initializing the model, as well as the model's key advantages and current limitations.*

1. Introduction

A probability threat map is a knowledge representation of the search environment as a discrete probability distribution, which provides a snapshot in time of unobserved threat locations. More specifically, probability threat maps are models of the perceived threat that describe the probability that any given one of a number of unseen entities that are moving independently is in a location (Darken, McCue, & Guerrero, 2010). They have been applied successfully to drive the synthetic behaviors for target scanning in military training simulations by prioritizing locations that are most likely to contain targets (Darken et al., 2010; Evangelista, Ruck, Balogh, & Darken, 2011).

Probability threat maps are derivatives of probabilistic occupancy maps used by game developers for opponent and target tracking (Isla & Blumberg, 2002; Isla, 2006); in addition, they use methods and techniques originally developed for mobile robotics designed to improve localization, search, navigation, and decision-making behaviors (Elfes, 1989; Thrun 2003). Others analogous approaches have been applied to investigate search problems with incomplete and uncertain information using unmanned aerial sensors and autonomous ground sensors (Bertucelli & How, 2005, 2006; Chung & Burdick, 2008, 2012; Chung, Kress, & Royset, 2009; Kagan & Ben-Gal, 2013).

Existing probability threat maps approaches for military simulations (Darken et al., 2010; Evangelista et al., 2011) provide simulated entities with subjective knowledge of likely enemy locations over a defined area, which is then used to carry out search decisions and search behaviors (e.g., select the next search area, modify movement, change tactical formations, path planning). These methods successfully improved the representation of search based on situational awareness and environmental factors in military simulations. However, there is a stated need and interest for expanding these methods essentially to enhance the representation of search, reasoning, and decision-making behaviors in combat simulations.

We believe that the current implementation of probability threat maps could be augmented with additional subjective knowledge of the threat necessary to model and simulate combat scenarios. Probability threat maps use statistical description of likely enemy locations but lack the ability to infer the number of the enemy from observed data and prior information. Ideally, the searcher should gain whatever information he can during the search process and then assess his subjective belief to infer the likely disposition (i.e. location and number of entities) of the threat.

A threat density map is a knowledge representation of the expected number of the enemy entities located inside each subdivision of the simulated area. More specifically, it quantifies the searcher agent's expectation of finding a number of enemy entities at a particular location in a time interval. The purpose threat density maps is to augment combat simulations with actionable subjective knowledge that can be exploited by the simulated entities for reasoning and decision-making in response to the threat and environment circumstances.

In contrast to probability threat maps, threat density maps provide the searcher agent with additional data needed in combat simulated scenarios to make better decisions amongst different courses of action consistent with the situation presented by the enemy forces (Pew & Mavor, 1998). For instance, depending on the size of the enemy forces the searcher agent can decide whether to defend, assault, attack, withdraw, avoid combat, or bypass. Such decisions would control other behaviors such as searching techniques, path planning, patrolling strategies, etc. In this context, simulated entities would have additional threat knowledge to reason and act upon.

In this paper, we introduce a threat density map model as a proof of principle. We build on current probability threat maps approaches to model the searcher's subjective belief regarding the threat size as a posterior density map instead of a discrete probability distribution. The main contribution of this paper is the formulation of the proposed threat density map model for combat simulations. This introductory section is followed, in Section 2, with a description of the problem and the model formulation. Section 3 describes the advantages and limitations of the current state of the model. Section 4 provides concluding remarks and discusses the direction of the future research.

2. Problem Description and Formulation

A threat density map, $\boldsymbol{tm}$, is a random variable defined over a finite set of locations, X, which assigns a score to each individual cell $x_i \in X$, $i = 1, \dots, C$, at a certain time step t describing the expected number of enemy entities in each cell. The set of locations, X, represents the area of operations discretized into a two-dimensional grid comprising C total cells, which can either be unoccupied or occupied by one or more enemy entities. The random variable $\boldsymbol{tm} = (tm_1, \dots, tm_C)$ denotes the state of the threat density map, where the random variable tm_i indicates the number of enemy entities in cell x_i. Let $k \in \mathbb{Z}^+$ be the grand total number of enemy entities across all cells in X, namely, $k = \sum_{i=1}^{C} tm_i$.

Our fundamental problem is to infer the unknown value of tm_i, namely the unknown number of enemy entities located in the individual cells, based on a sequence of sensing outcomes and assumptions about the success of those sensing actions. To accomplish this, we first initialize each cell with a prior density function, $p(tm_i)$, based on how the searcher believes the enemy is spatially distributed and the certainty of prior information available. This prior information is then combined with the data from our assumed sensing model, $p(s_i^t|tm_i)$, which is the probability density function of the number of enemy entities sensed in cell x_i at time step t, s_i^t, conditional on tm_i. Finally, for each individual cell we update the prior $p(tm_i)$ to the posterior, $p(tm_i|\delta_i^t)$, with the data from the sensing model, $p(s_i^t|tm_i)$, and infer the expected number of enemy entities through successive Bayesian updates.

It is important to define key assumptions required for our framework. First, the total number of enemy entities in the set of locations is a priori unknown but bounded by k enemy entities. Second, the spatial distribution of the enemy entities across the set of locations can be represented with a prior density function. Third, the number of enemy entities in any given cell is independent of the number of entities in all other cells. Lastly, sensing actions within the same cell are conditionally independent from other sensing actions whether in the same cell or in other cells. Clearly, the assumptions of independence and conditional independence may not be realistic as the knowledge that a cell is occupied or not at a particular time can help figure out the state of it and other cells at the current and future times. However, these assumptions, commonly used in related literature, reduce computational complexities and allow us to decompose the problem for solving threat density maps for the individual cells independently (Thrun, 2003; Merali & Barfoot, 2013).

2.1. Initializing Threat Density Maps

To initialize tm_i at time step $t = 0$, we choose a prior density function, $p(tm_i)$, for every cell to represent the searcher's subjective belief about the enemy's spatial distribution in the location set previous to initiating the search. This prior density function summarizes the probability that the random variable tm_i takes on any given values n, which we can write explicitly as $p(tm_i = n)$.

Defining sensible prior density functions varies by the type of prior information (i.e. specific, vague, insufficient) about the enemy and the unknown parameter tm_i. Information regarding the enemy (e.g., size, composition, known or suspected locations, likely formations and movement) normally exists in military scenarios for combat simulations and should be used to initialize priors for each cell. Exact or credible intelligence data available (e.g., intelligence reports, situation reports, satellite imagery) of the enemy and the environment can be useful to define k and strong priors and perhaps to define other aspects of the world (e.g., likely movement routes, probable employment areas, key terrain, obstacles). On the other hand, with vague intelligence data we might have to assume a prior based on general considerations (e.g. most probable or most dangerous enemy disposition).

In brief, we have distinct cases of prior information available (i.e. specific, vague, and no prior information available) to consider when specifying $p(tm_i)$. The inclusion of prior information into the prior $p(tm_i)$ is one of the benefits of our approach because it leads to stronger inferences about tm_i. Regardless of the level of certainty, we can specify a prior to quantify uncertainty around the spatial distribution of the enemy entities and express what is believed or known about tm_i before inspecting any cell $x_i \in X$. Below we discuss a discrete prior density function that can be used to initialize the $p(tm_i)$ with prior information.

2.1.1. Discrete Prior Density Function

Consider the case in which the search agent lacks or has vague prior information. A common practice in such situation is to define a conventional prior, such as the discrete uniform, that does not favor any particular value. However, as previously mentioned prior information for combat simulation scenarios is typically available. Therefore, it is then sensible to define a prior density function that can account for a broad range of possibilities fundamental to combat simulated scenarios.

For this particular problem, with no idea about the distribution of tm_i we define a discrete prior assuming that any cell in X could contain up to k targets evenly distributed but more likely for the enemy to be nonexistent in some cells. Accordingly, tm_i is a discrete random variable with a finite range bounded by k, i.e., $\{0, 1, \dots, k\}$. Further, we assume that the prior is defined for each cell such that $p(tm_i) = 1/k$ for $n = 1, \dots, k$. However, our prior subjective belief inclines us to anticipate that many cells will be empty rather than occupied because enemy forces tend to cluster together whether they operate as cohesive large element or as smaller dispersed elements. To represent such belief we define the parameter ε such that each of the values in the range $1 \le n \le k$ occurs with probability $\varepsilon(1/k)$ and $(1-\varepsilon)$ for $n = 0$. That is, the unconditional prior probability distribution for an individual cell is given by the following probability density function:

$$p(tm_i = n) = \begin{cases} \varepsilon\left(1/k\right), & n = 1,\ 2,\ \dots,\ k \\ 1-\varepsilon, & n = 0 \\ 0, & \text{otherwise} \end{cases} \qquad (1)$$

The expected value of the random variable tm_i for cell x_i at time step $t = 0$ is:

$$\mu_{tm_i} = E(tm_i) = \sum_{n=1}^{k} n\varepsilon\left(1/k\right) = \varepsilon\left(k+1/2\right) \qquad (2)$$

Although the choice of ε is subjective it is also suitable to initialize $p(tm_i)$ when specific prior information is available. For instance, suppose we know the mean number of enemy entities for some specific cells. In this case, we do not have any difficulty incorporating this information in $p(tm_i)$. We simply solve Eq. (2) for ε, i.e., $\varepsilon = 2\mu/(k+1)$, for $\varepsilon \in [0,1]$, and use this value to define the prior of tm_i for those particular cells.

2.2. Sensing Model

Sensing actions, namely, observing or inspecting cells, are knowledge-producing events that changes the searcher's subjective belief of the threat. The searcher's ability to observe enemy entities in a cell is modeled using the combat simulation's target detection model, which specifies the probability of detecting a target, P_d, as a function of the brightness of the target, the brightness of the target's background, and the subjective size of the target given that one or more targets are present in the location. Although P_d varies by type of target, it is generally constant for targets of the same type and size, and against a particular background.

In our framework, sensing actions represent binomial trials with $(k+1)$ possible outcomes (i.e. observing between zero and k enemy entities) of the actual number of entities in the cell. They return the number of enemy entities sensed, s_i^t, in cell x_i at time step t. Therefore, we specify a binomial sampling model, $p(s_i^t|tm_i)$, which describes the searcher's ability to gain subjective knowledge regarding tm_i. This sampling model provides the conditional probability that s_i^t is b conditioned on tm_i and given P_d, i.e., $p(s_i^t|tm_i) = p(s_i^t = b|tm_i = n)$, expressed as

$$p\left(s_i^t = b \middle| tm_i = n\right) = \frac{n!}{b!\left(n-b\right)!}\left(P_d\right)^b\left(1-P_d\right)^{n-b} \qquad (3)$$

for $b = 0, 1, \dots, k$ and $0 \le P_d \le 1$. In Eq. (3) the binomial coefficient $n!/b!\,(n-b)!$ describes the number of combinations of n things taken b at a time without regard of their order; $(P_d)^b$ is the likelihood of b detections given P_d; and $(1-P_d)^{n-b}$ is the probability of missing $(n-b)$ of the possible detections.

2.3. Multiple Sensing Actions

Above we focused on the probability for a single sensing action at time step t. However, our goal is to infer tm_i based on all cell inspections through time step t. Let $p(\delta_i^t|tm_i)$ indicate the distribution of the sensing outcomes for cell x_i up to time step t and let $\delta_i^t = \{s_i^{\tau_1}, \dots, s_i^{\tau_j}\}$ denote the history of the number of enemy entities sensed through time step t. Assuming multiple inspections of cell x_i at different time steps, $s_i^{\tau_j}$, $j = 1, \dots, t$, represents the number of enemy entities sensed at time τ_j.

As previously stated, the probability of each sensing action is conditionally independent of other sensing actions; specifically, s_i^t and δ_i^{t-1} are conditionally independent given tm_i. In other words, if tm_i is known, additional knowledge of δ_i^{t-1} does not change the searcher's belief about how many enemy entities he will see at the next observation (s_i^t). Therefore, the probability of the data set (i.e. history of the enemy entities sensed) is given by:

$$p\left(\delta_i^t \middle| tm_i\right) = p\left(s_i^t, \delta_i^{t-1} \middle| tm_i\right) = p\left(s_i^t \middle| tm_i\right) p\left(\delta_i^{t-1} \middle| tm_i\right) \quad (4)$$

2.4. Updating Threat Density Maps

We now discuss how to update probabilities after a new sensing action is performed. According to Bayesian inference, we can estimate the posterior through time step t, $p(tm_i|\delta_i^t)$, given a prior on tm_i and the data resulting from the sensing model, $p(s_i^t|tm_i)$. Applying Bayes rule to the terms $p(\delta_i^t|tm_i)$ and $p(\delta_i^{t-1}|tm_i)$ in Eq. (4) and with the conditional independence assumption of the sensing actions results in the posterior density function $p(tm_i|\delta_i^t)$ of tm_i given the history of enemy entities sensed in cell x_i through time step t. The posterior density is given by

$$p\left(tm_i \middle| \delta_i^t\right) = \frac{p\left(s_i^t \middle| tm_i\right) p\left(tm_i \middle| \delta_i^{t-1}\right)}{p\left(s_i^t \middle| \delta_i^{t-1}\right)} \quad (5)$$

where $p(s_i^t|tm_i)$ is obtained from the sensing model in Eq. (3); $\mathrm{p}(tm_i|\delta_i^{t-1})$ represents either a prior at time step $t = 0$, i.e., $p(tm_i)$, or a posterior without the most recent sensing action result; and $p(s_i^t|\delta_i^{t-1})$ is the normalization factor resulting from marginalizing over tm_i and applying the foregoing conditional independence assumption of sensing actions given tm_i:

$$p\left(s_i^t \middle| \delta_i^{t-1}\right) = \sum_{n=0}^{k} p\left(s_i^t \middle| tm_i = n\right) \, p\left(tm_i = n \middle| \delta_i^{t-1}\right). \quad (6)$$

Substituting Eq. (6) in Eq. (5), the individual cell beliefs can be updated using the following:

$$p\left(tm_i \middle| \delta_i^t\right) = \frac{p\left(s_i^t \middle| tm_i\right) p\left(tm_i \middle| \delta_i^{t-1}\right)}{\sum_{n=0}^{k} p\left(s_i^t \middle| tm_i = n\right) \, p\left(tm_i = n \middle| \delta_i^{t-1}\right)}. \quad (7)$$

Eq. (7) results in a distribution of the unknown number of enemy forces in the cell conditioned on the observed sample data. Thus we have a probability model that quantifies the searcher's new state of subjective belief about tm_i, given the initial information described by the prior $p(tm_i)$ and the information provided by the sensing model $p(s_i^t|tm_i)$.

2.5. Inference about the Number of Enemy Entities

During initialization we estimate the expected number of enemy entities for every cell from the prior probabilities and maintain this during runtime until the cell posterior distribution is updated after a sensing action. Once the posterior is computed, we utilize Eq. (8) to determine the expected number of entities x_i.

$$E\left(tm_i \middle| \delta_i^t\right) = \sum_{n=0}^{k} n p\left(tm_i = n \middle| \delta_i^t\right). \quad (8)$$

3. Advantages of Threat Density Maps

In this section we discuss the advantages of providing simulated entities with threat density maps as well as the limitations of the current state of the model. Simply put, the main advantage of the proposed approach as compared to probability threat maps is that a threat density map provides a probability distribution of the unknown number of enemy entities and the expected number of enemy entities in a cell, which can be influenced by a detailed prior distribution. To conceptualize the notion of threat density maps applied to combat simulations and to demonstrate its practicality and advantages, we coded and implemented in a rudimentary JavaScript simulation the aforementioned threat density map and for comparison, an adaptation of the probability threat maps (see Appendix 1) discussed in Darken et al. (2010).

The notional scenario consists of a simulated infantry soldier (searcher) searching for an enemy fireteam to either engage them or to report their disposition, location, and actions. From intelligence data the searcher knows that enemy fireteam (targets) is not moving and consists of three entities close together and one scout far ahead. Figure 2(a) shows the targets actual distribution, i.e., $\{x_{11} = 1,\ x_{16} = 3\}$, which is unknown to the searcher. Based on their doctrinal spatial dispersion and the size of a cell we initialized threat density maps for the individual cells assuming that any cell could contain one or three targets but not two or four, yet being free of enemy entities is even more probable than occupation by one or three. Figure 1 shows an example of this prior for a single cell. Finally, we assumed a uniform prior to initialize the probability threat map and the probability of detection remained constant for the simulation, i.e., $P_d = 0.65$.

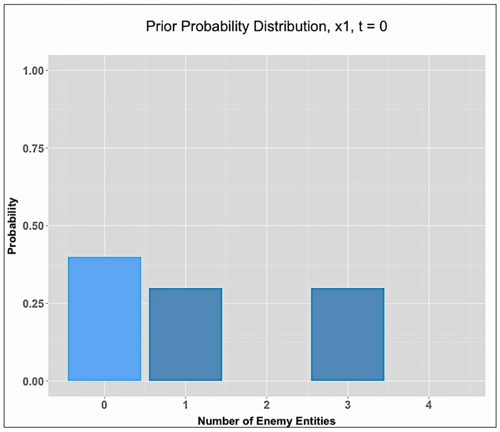

Figure 1: Discrete prior distribution of tm_1 for cell x_1 with $\varepsilon = 0.4$ assuming that it is more likely that the cell is occupied (containing either one or three targets) than empty.

One of the main advantages of our Bayesian approach to threat density maps is the availability of a posterior distribution of the unknown number of targets in a cell rather than a single value as in the probability threat map approach. For example, consider the situation shown in Figure 2 in which the searcher sensed zero targets after inspecting cell x_1. The low probability value in the probability threat map [Figure 2(b)] indicates that cell x_1 is less likely to contain one or more targets when compared to the other cells. However, the searcher lacks knowledge about the degree to which the cell x_1 is occupied, when in fact it can be empty or occupied by one or three targets because cell inspections are not perfect. The coarse threat knowledge provided by the probability threat map, although useful for search decisions is not sufficient for making decisions related to tactical courses of action.

On the other hand, the threat density map posterior distribution summarizes the state of knowledge about the unknown number of targets in the cell conditional on the prior and sensing data. In contrast to the probability threat map, the threat density map in Figure 2(d) suggests that although cell x_1 is more likely to be empty there is still a chance to find one or three targets in the cell. In this situation, the posterior distribution of tm_1 provides the searcher with a more accurate picture of the likely state of cell x_1. This more detailed representation of threat knowledge provides the searcher the basis for a more confident course of action selection.

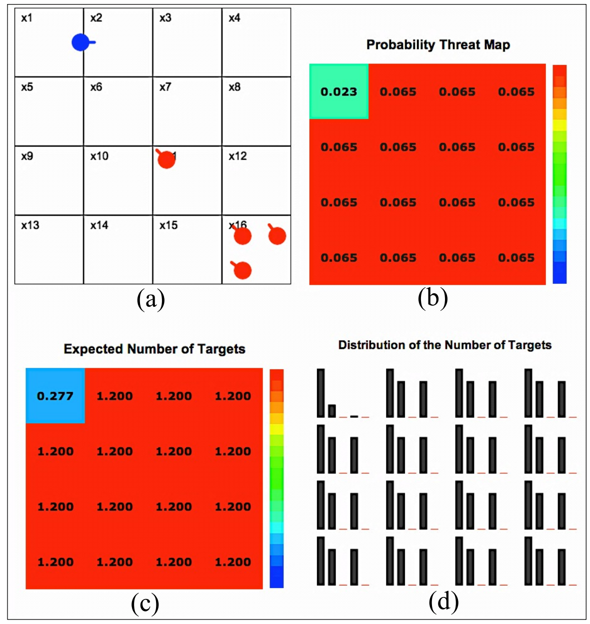

Figure 2: Screenshot of the simulated scenario at time step $t = 0.25$ where the searcher is depicted in blue and the targets are depicted in red (a), the probability threat map (b) and threat density map consisting of the expected number of targets (c) and the related probability distributions of the number of targets (d).

Consider the situation in Figure 3 in which the searcher after inspecting several cells sensed two targets (dark red entities) in cell x_{16}. For such situation, it would be difficult for the searcher to select a course of action that provides the best possibility of success based solely on the probability threat map. Therefore, it is appealing to quantify the searcher's expectation of finding a number of targets at the cell. Updating the threat density map's prior information with sensed data, provides interpretable answers, such as the event that tm_{16} equals three has probability of one [Figure 3(d)] thus, the searcher could expect to see three targets in the cell [Figure 3(c)]. Then, he can exploit this subjective knowledge to make reasonable decisions consistent with the likely state of the threat, for example, decide to search the cell for the unobserved target or to move out of the cell and avoid combat.

Likewise, threat density map data can also be used to support reasoning. Consider a separate simulation run (Figure 4) in which the searcher sensed one target (dark red entity) in cell x_{11} given $P_d = 0.9$. Based on the threat density map the searcher could assume with a high degree of certainty that he found the scout entity of the enemy fireteam and hence could use this belief for identifying the neighboring cell that could contain the remaining three targets and to determine how he deploys, orient, and engages the remaining targets.

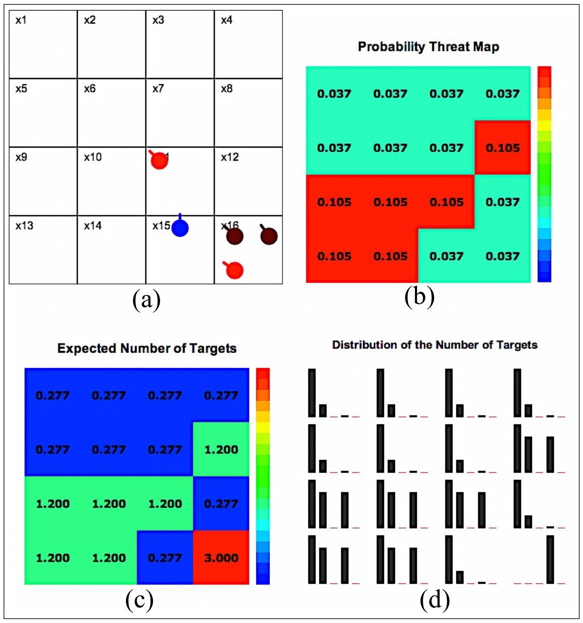

Figure 3: Screenshot of the scenario and the state of subjective threat knowledge in which the searcher sensed two targets (depicted in dark red) in cell x_{16}.

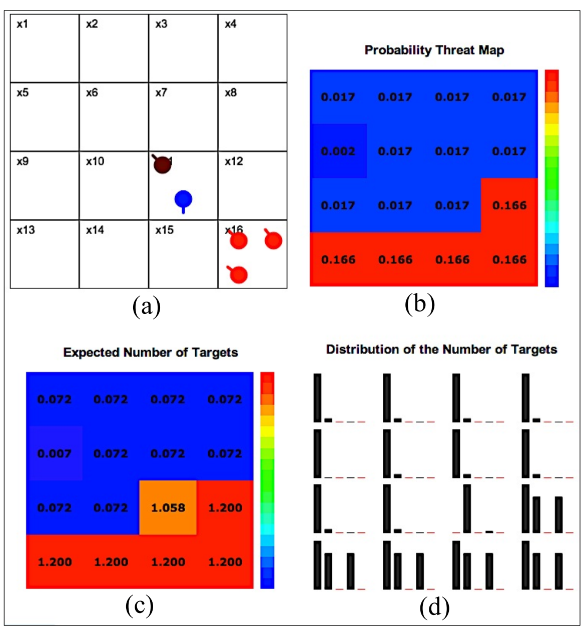

Figure 4: Screenshot of the scenario and the state of subjective knowledge in which the searcher sensed one target (depicted in dark red) in cell x_{11}.

3.1. Integrating Prior Information

The incorporation of a prior density function for tm_i with prior information is the final favorable feature of the threat density map that differentiates it from the probability threat map. As previously mentioned, intelligence data or prior information is typically available for combat simulated scenarios. Regardless of the level of certainty of the prior information, we can use the aforementioned discrete prior density function or other suitable discrete distributions to describe uncertainty for tm_i in a mathematical model. However, from a modeling perspective the difficulty is in how to effectively integrate prior information from different sources (e.g., intelligence, doctrine, environment) using a prior density function (Blasco, 2007).

In Figure 1 above we already demonstrated an example for initializing threat density maps given prior information and intelligence data (i.e. the total number of targets and their tactical formation). Below we briefly discuss two cases of prior information available common to combat simulated scenarios for initializing threat density maps.

First, presume that the prior information available consists only of the total number of enemy entities (a fireteam of four entities) and their posture (not moving) but neither their actual location nor their tactical formation is known. In this situation of vague prior information is sensible to assume that any cell could contain up to four enemy entities and logically we can expect that many cells will be empty instead of occupied. Accordingly, we could set the value of ε to be 0.75 and utilize Eq. (1) to initialize threat density maps for each cell $x_i \in X$. Figure 5 shows the prior distribution for cell x_1.

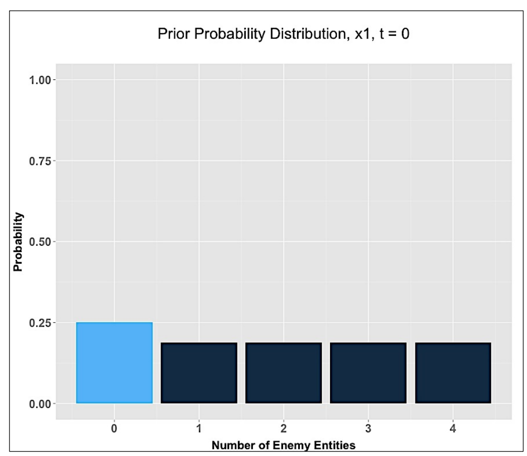

Figure 5: Discrete prior distribution of tm_1 for cell x_1 assuming that it is more probable to be empty and equally likely to be occupied by at least one and no more than four targets.

The plot in Figure 5 shows that it is more likely for a cell to be unoccupied and equally possible to be occupied by one, two, three, or four enemy entities. The expected number of enemy entities in each cell is 1.875, thus the searcher can expect to find approximately two enemy entities in any particular cell at the next time step.

Second, specific prior information can easily be incorporated through the prior density function. For example, suppose that from the most current intelligence data available it is known that there is a squad-size element in a linear defense, arrayed from the southwest corner of the area of operations to the northeast corner, heavily concentrated in cell x_{11}, defending the southeast sector of the area of operations, as depicted in Figure 6. Incorporating this prior information into the model can be done in a flexible manner and inferences can be compared under different priors in order to choose a prior that characterizes the most likely threat situation. One alternative, for example, is to set the value of ε equal to one for the cells known to be occupied and zero otherwise. Such an approach can be efficient but it does not account for the possibility that the situation could change before the searcher reaches any of these cells. Therefore, one could select other values of ε for the cells of interest. Perhaps another alternative is to deduce from doctrine and terrain data the maximum number of enemy entities that can occupy a cell and other relevant factors to initialize the priors for each cells that produce $E\left(tm_{8,14}\right) = 2$, $E(tm_{11} = 5)$, and zero otherwise.

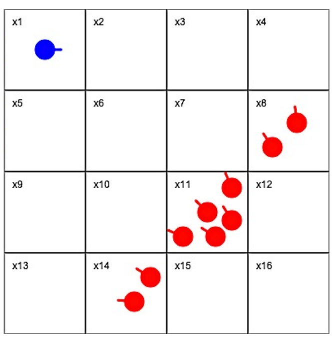

Figure 6: Screenshot of the location set with ground truth data. The searcher is depicted in blue and the targets in a linear defense, heavily concentrated in cell x_{11}, are depicted in red.

3.2. Current Limitations of Threat Density Maps

As we have seen in the previous examples, there are significant advantages of augmenting combat simulated scenarios with threat density maps as they provide simulated entities with actionable subjective knowledge to make course of action decisions, which in turn determines other search, movement, and path planning behaviors. However, the proposed approach has some fundamental limitations. While the assumptions of independence and conditional independence, described in Section 2, allows us to solve the threat density maps for the individual cells, the model excludes features for modeling spatial dependencies and temporal effects. This limitation is evident in Figure 3(c) and 3(d) as the model properly estimates the expected number of enemy entities in the cell, i.e. $E(tm_{16}) = 3.0$, essentially due to the inclusion of prior information into the model; however, it fails to exploit this information for estimating tm_i for the other cells.

4. Conclusions and Future Directions

In this paper we proposed a threat modeling approach for estimating the number of the enemy entities at a certain location in a given time interval. The model estimates the expected number of enemy entities as a posterior density map, can be initialized with intelligence reports and prior information, and works for any number of enemy entities and their spatial distribution. Although a threat density map approach is not required for all combat simulation models and scenarios, they offer several important advantages over probability threat maps that make them suitable for implementation in combat simulations for improving the representation of search, reasoning, and decision-making behaviors.

Efforts are underway to introduce probability distributions that can model threat movement. Furthermore, future work will focus on addressing known limitations and extending the proposed model by introducing spatial and temporal dependencies and interactions, and developing hierarchical threat density map representations. Finally, we plan to experiment with and characterize the utility of the model for improving the capabilities of simulated entities in a combat simulation scenario for different threat conditions.

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Appendix 1: Probability Threat Map Adaptation

In this section we briefly describe our basic adaptation of the probability threat maps approach discussed in Darken et al. (2010).

Let q_i be the conditional probability that an unseen enemy entity is present in cell $x_i \in X$ and after inspecting cell x_j, where $x_i \neq x_j$, $\tilde{q}_i$ is the estimated probability before inspecting cell x_i, and P_d is the probability of detecting a target (see Section 2.2). According to the axioms of probability theory, $0 \leq q_i \leq 1$ and the total probability over all C cells is $\sum_{i=1}^{C} q_i = 1$. Suppose the searcher inspects cell x_j, assuming that cell inspections are independent of neighboring cells, then, q_i takes the form

$$q_i = \frac{\tilde{q}_i I_i + \tilde{q}_j \left(1 - P_d\right)\left(1 - I_i\right)}{\sum_{i'=1}^{C} \tilde{q}_{i'} I_{i'} + \tilde{q}_j \left(1 - P_d\right)} \qquad (9)$$

where the term I_i is an indicator function that equals to zero if $x_i = x_j$ and equals to one otherwise.

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