This article has been accepted for publication in IEEE Wireless Communications Letters. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/LWC.2022.3218187

Non-Coherent OFDM Transmission via Off-the-Grid Joint Channel and Data Estimation

Masoud Bigdeli, Hamid Fathi, Iman Valiulahi, Christos Masouros, *Senior Member,* IEEE

***Abstract*—Pilot-aided channel estimation techniques are known to waste the spectral bandwidth. An off-the-grid blind estimator for time-variant orthogonal frequency division multiplexing (OFDM) systems is studied in this letter. In this regard, we propose a blind estimator based on atomic norm minimization (ANM) for OFDM systems. To do so, at the first transmission block, using a lifted ANM (LANM) and simple constraint on ℓ_2 norm of data, we simultaneously estimate the channel and data. For the subsequent blocks, we use a penalized ANM (PANM) to simultaneously track the channel's parameters and detect transmit signals. The proposed problems require an infinitedimensional search, hence are NP-hard. Therefore, we propose two semidefinite programs (SDPs) to implement them. We then derive the total computational complexity of the proposed estimator. The simulation results show the superiority of the proposed estimator to the state-of-the-arts.**

***Index Terms*—Blind OFDM channel estimation, lifted and penalized atomic norm minimization, semidefinite program (SDP).**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) systems are being used in telecommunications standards to achieve high rate transmissions. Signals can be transmitted using a large number of orthogonal subcarriers, which increases the symbol duration and decreases inter-symbol interference [1]. To achieve a coherent demodulation, sufficient knowledge of channel state information (CSI) is required. Hence, channel estimation is of paramount importance in OFDM systems [1]. Generally, channel estimation approaches can be divided into supervised and unsupervised methods [1]. In the supervised methods, CSI is estimated using pilot signals which are known at both the receiver and transmitter sides. However, this introduces transmission overheads. Unsupervised or blind techniques is a promising approach to circumvent the need for pilot signals and the relevant overheads.

The OFDM channel can be assumed as a combination of time dispersive channels, which includes a sparse structure at a high signal space dimension [1]. Thus, compressed sensing (CS) was leveraged in [2] to estimate the OFDM channel using pilot signals. The recovered time dispersive channels, however, were required to lie on a gird. This results in an unwanted basis mismatch since the time dispersive channels inherently belong to a continuous dictionary. This issue is solved using the atomic norm that promotes sparsity in continuous dictionaries [3]. The authors in [4] proposed the atomic lift technique for the blind super-resolution problem. In [5], a pilot-aided atomic norm minimization (ANM) is adopted to estimate OFDM channels. In [6], using a penalized ANM (PANM), the authors proposed a simple estimator to detect the time dispersive channels in presence of both dense and spiky noises.

Few works studied linear the linear precoding for blind OFDM detection [7], [8]. In particular, the authors in [7] applied the linear precoding method to the transmit symbols to estimate the OFDM channel and transmit data using a cross-correlation technique at the receiver side. The performance analysis of this linear precoding is derived in [8]. One issue with the above works is that when the precoded subcarrier is in a deep fade, the entire correlation matrix of the received signals is required, which increases the complexity at the receiver side. In addition, to resolve the inherent scalar ambiguity of the blind estimation, one might need to use pilot signals, which means that the estimation is not fully blind. Moreover, in [9], the authors studied rectangular differential coding with index modulation over the non-coherent OFDM time dispersive channels. The bits of information are first required to code, reducing the spectral efficiency. In addition, the channel states are assumed to be constant over two subsequent transmission blocks.

To address the above issues, we propose an off-the-grid blind estimator based on the ANM for a time-variant OFDM system where the channel parameters are not required to fall on a fine grid. Blind estimation is generally ill-posed without additional constraints because the blind deconvolution of two signals is a bi-linear inverse problem, hence is not tractable [4]. To tackle this challenging problem, for the first transmission block, where we have no knowledge about the channel, by assuming that the ℓ_2 norm of data is equal to one, we estimate the time dispersive channels, their corresponding amplitudes, and data using a lifted ANM (LANM). For the subsequent blocks, where we can use previous channel states, we propose a PANM to simultaneously track the channel's parameters and detect the data. Though the proposed problems are convex, they require an infinite-dimensional search, hence are NP-hard. To make the problems implementable, we propose two semidefinite programs (SDP)s. We then obtain the overall computational complexity of the proposed estimator. The simulation results are carried out to evaluate the performance of our proposed estimator and to show its superiority to the closest benchmarks.

Here, we introduce the notation used in this paper. Vectors and matrices are denoted by boldface lowercase and uppercase letters, respectively and scalars or entries are non-bold lowercase. The $\|\cdot\|_1$ and $\|\cdot\|_2$ are ℓ_1 and ℓ_2 norms, respectively. The operators $\mathrm{tr}(\cdot)$ and $(\cdot)^H$ are trace of a matrix, hermitian of a vector, respectively. In addition, $\mathrm{diag}(\boldsymbol{x})$ places $\boldsymbol{x} \in \mathbb{C}^{N\times 1}$ in the main diagonal of $\boldsymbol{X} \in \mathbb{C}^{N\times N}$.

II. Problem Formulation

We study an OFDM system with N subcarriers and cyclic prefix (CP) length of L_{cp} over M transmission blocks. At the transmitter side, for each block, the data symbols are passed through a serial to parallel converter to obtain $N \times 1$ frames. The N-point inverse discrete Fourier transform (IDFT) of frames modulates the signal on N subcarriers. The CP of length L_{cp} is then added in order to prevent inter symbol interference (ISI). The resulting signal is converted to serial data. A filter $g_T(\tau)$ is exploited to shape the signal to be transmitted over a multi-path environment where it's base-band model contains s Dirac delta function as [1]

$$h(\tau) = \sum_{i=1}^{s} \alpha_i \delta(\tau - \tau_i), \tag{1}$$

where τ_i and $\alpha_i = |\alpha_i| e^{j\theta_i}$ are the delay of the time dispersive channel and its corresponding complex amplitude at the i-th path for the first block, respectively. In addition, $\delta(\tau - \tau_i)$ is the Dirac delta function located at τ_i. Assuming perfect synchronization in both time and frequency domains, the received signal is passed through a matched receive filter $g_R(\tau)$. After the serial to parallel conversion and removing the cyclic prefix, the N-point DFT can be used to obtain the frequency domain signal $y_{n,1}$ at the first transmission block as

$$y_{n,1} = x_{n,1}\, h_{n,1} + w_{n,1},\ \forall n \in \mathcal{N} \tag{2}$$

where $\boldsymbol{x}_1 \in \mathbb{C}^{N\times 1}$ is the transmitted data vector, and $\boldsymbol{w}_1 \in \mathbb{C}^{N\times 1}$ is a zero mean circularly symmetric complex Gaussian noise with the variance of σ_h^2 for each entry. $g_T(\tau)$ and $g_R(\tau)$ are band-limited root-raised-cosine functions and $h_{n,1} = g_n h'_{n,1}$, $n \in \mathcal{N}$, where g_n and $h'_{n,1}$ can be sampled from DTFT of the discretized version of $g(\tau) = g_T(\tau) * g_R(\tau)$ and (1), respectively. We can assume that $g_n = 1$ and write the DTFT of $h_{n,1}$ as $h_{n,1} = \sum_{i=1}^{s} \alpha_i\, e^{-j2\pi \frac{n}{N}\frac{\tau_i}{T_s}}$, which constructs $\boldsymbol{h}_1 \in \mathbb{C}^{N\times 1}$ in which T_s is the sampling interval.

We assume that the channel states are constant over a transmission block, then in a subsequent block, a small variation in channel states at the Fourier domain occurs, i.e., $\boldsymbol{h}_m = \boldsymbol{h}_{m-1} + \Delta_m, \forall m \in \{2, \cdots, M\}$ where $\boldsymbol{h}_m \in \mathbb{C}^{N\times 1}$ is the channel states related to the $m-$th transmission block and Δ_m is a zero-mean circularly symmetric complex Gaussian number with the variance σ_c^2 at the $m-$the transmission block, where is independent to the Gaussian noise. We assume that all the sub-carriers experience the same noise. However, our model can be generalized to the case where each sub-carrier has its noise in the cost of more notations.

Assuming the CP time as T_{cp}, then $0 \le \tau_{i,m} \le T_{cp} + T_s - T_g$ where $T_g = \frac{4}{6T_s}$ is the duration of $g(\tau)$ to satisfy the effects of the temporal conditions [1]. Then, the received signal can be recast as

$$\begin{aligned} y_{n,m} &= x_{n,m} h_{n,m} + w_{n,m}, \\ &= x_{n,m} \sum_{i=1}^{s} \alpha_i\, e^{-j2\pi \frac{n}{N}\frac{\tau_i}{T_s}} + w_n,\ \forall m \in \{2, \cdots, M\}. \end{aligned} \tag{3}$$

where $\boldsymbol{w}_m \in \mathbb{C}^N$ is a zero-mean circularly symmetric complex Gaussian noise with the variance of $\sigma^2 = \sigma_h^2 + m\sigma_c^2$ for each entry at the m-th block. Let us define $f_i := \frac{1}{N}\frac{\tau_i}{T_s}$, therefore the received signal at the m-th block in the vector form can be written as

$$\boldsymbol{y}_m = \mathrm{diag}(\boldsymbol{x}_m)\boldsymbol{h}_m + \boldsymbol{w}_m, \quad \forall m \in \{2, \cdots, M\}, \tag{4}$$

where $\boldsymbol{h}_m$ is the channel vector at the m-th block with entries $h_{n,m} = \sum_{i=1}^{s} \alpha_i\, e^{-j2\pi f_i n} + \sum_{j=1}^{m} \Delta_j$, $0 \le f_i \le 1$. Now, the problem is to estimate the transmitted signal and channel parameters form the observed signal $\boldsymbol{y}_m$ of all blocks, which we call it blind channel estimation. To solve this problem, we first estimate the channel and data at the first block using the LANM, then we use the obtained channel for tracking the channel changes and estimating the signal for the remaining transmitting blocks using PANM.

III. LANM at the First Transmission Block

As discussed, our target is blind estimation of the channel vector at the first transmission block when the receiver does not know the transmit signal. Generally speaking, a blind detection is extremely ill-posed, indeed, it can not be solved without additional constraints [4], [10]. Thus, we assume that $\boldsymbol{x}_1$ lies in a known low-dimensional dictionary [10], as $\boldsymbol{x}_1 = \boldsymbol{B}\boldsymbol{g}$, where $\boldsymbol{B} \in \mathbb{C}^{N\times L}$ is a Gaussian random matrix, which is known at both the transmitter and receiver sides and $\boldsymbol{g} \in \mathbb{C}^L$ is unknown constrained on $L \ll N$ and $\|\boldsymbol{g}\|_2 = 1$ in which L the size of the recovered data. This is indeed a linear coding such that $\boldsymbol{g}$ is passed through coding matrix $\boldsymbol{B}$. We denote the n-th column of matrix $\boldsymbol{B}^T \in [\,\boldsymbol{b}_1, \cdots, \boldsymbol{b}_N\,]$, with $\boldsymbol{b}_n \in \mathbb{C}^L$, and rewrite (2) at the first block as

$$\begin{aligned} y_{n,1} &= \boldsymbol{b}_n^T \boldsymbol{g} h_{n,1} + w_{n,1} = \boldsymbol{b}_n^T \boldsymbol{g} \boldsymbol{e}_n^T \boldsymbol{h}_1 + w_{n,1} \\ &= \boldsymbol{e}_n^T (\boldsymbol{h}_1 \boldsymbol{g}^T) \boldsymbol{b}_n + w_{n,1}, \end{aligned} \tag{5}$$

where $\boldsymbol{e}_n$ is the n-th standard basis vector of $\mathbb{R}^N$, i.e., all components of $\boldsymbol{e}_n$ are zero, except the n-th component which equals one. Let us define $\boldsymbol{Z} = \boldsymbol{h}_1\, \boldsymbol{g}^T$, then using the lifting technique [10], we can rewrite (6) as a linear measurements of $\boldsymbol{Z}$,

$$y_{n,1} = \boldsymbol{e}_n^T \boldsymbol{Z} \boldsymbol{b}_n + w_{n,1} = \langle \boldsymbol{Z}, \boldsymbol{e}_n \boldsymbol{b}_n^H \rangle_F + w_{n,1}, \tag{6}$$

where $\langle \boldsymbol{X}, \boldsymbol{Y} \rangle_F = \mathrm{tr}(\boldsymbol{X}^H \boldsymbol{Y})$ is the Frobenius inner product. Thus, $\boldsymbol{y}_1$ can be written as a linear measurement of $\boldsymbol{Z}$ contaminated by Gaussian noise as

$$\boldsymbol{y}_1 = \mathcal{X}(\boldsymbol{Z}) + \boldsymbol{w}_1, \tag{7}$$

where $\mathcal{X}(\cdot) : \mathbb{C}^{N\times L} \mapsto \mathbb{C}^N$ is a linear operator.

Regarding the fact that $\boldsymbol{Z}$ is a rank-one matrix and $\|\boldsymbol{g}\|_2 = 1$, $\boldsymbol{h}_1$ and $\boldsymbol{g}$ can be precisely obtained from $\boldsymbol{Z}$ using it's left and right singular vectors, respectively. Thus, the goal is to estimate $\boldsymbol{Z}$ from measurement vector $\boldsymbol{y}_1$. To do so, we first write $\boldsymbol{Z}$ in the form of a sparse combination of the complex sinusoidal atoms $\boldsymbol{c}(f,\theta) = \left[e^{-j2\pi f + j\theta}, \cdots, e^{-j2\pi N f + j\theta}\right]^T$ for $f \in [0,1]$, as $\boldsymbol{Z} = \boldsymbol{h}_1 \boldsymbol{g}^T = \sum_{i=1}^{s} |\alpha_i|\, \boldsymbol{c}(f_i, \theta_i)\, \boldsymbol{g}^T \in \mathbb{C}^{N\times L}$. Now, we can define a set of atoms in the form $\mathcal{A}_L = \{\boldsymbol{A}(f, \boldsymbol{v}, \theta) = \boldsymbol{c}(f,\theta)\boldsymbol{v}^H \in \mathbb{C}^{N\times L} | f \in [0,1], \|\boldsymbol{v}\|_2 = 1, \theta \in$

$[0, 2\pi)\}$, hence the lifted atomic norm for the arbitrary matrix $\boldsymbol{Z}$, over the $\mathcal{A}_L$ can be written as

$$\|\boldsymbol{Z}\|_{\mathcal{A}_L} = \inf\{t > 0 : \boldsymbol{Z} \in t\mathrm{conv}(\mathcal{A}_L)\},$$

where $\mathrm{conv}(\mathcal{A}_L)$ is the convex hull of set $\mathcal{A}_L$ and t is a positive constant variable or $\|\boldsymbol{Z}\|_{\mathcal{A}_L} = \inf_{f_i\in[0,1),\boldsymbol{v}\in\mathbb{C}^N,\|\boldsymbol{v}\|_2=1} \left\{ \sum_i \alpha_i \middle| \alpha_i A(f_i, \boldsymbol{v}, \theta_i), \alpha_i \geq 0, \theta_i \in [0, 2\pi) \right\}$. Thus, we can conclude that (8) is a sum of sparse signals over the atom set $\mathcal{A}_L$ and dense noise. Thus, the off-the-grid CS can be considered a promising solution [3]. Below, we propose a LANM that achieves a low-dimensional representation of $\boldsymbol{Z}$ constrained on the noise power density as

$$\min_{\boldsymbol{Z}\in\mathbb{C}^{P\times L}} \|\boldsymbol{Z}\|_{\mathcal{A}_L} \quad \text{s.t.} \quad \|\boldsymbol{y}_1 - \mathcal{X}(\boldsymbol{Z})\|_2 \leq \sigma_h^2. \tag{8}$$

Implementing such a problem is generally challenging since an infinite-dimensional search for the objective value is required. To tackle this issue, using [3], we provide an SDP characterization for the lifted atomic norm as

$$\|\boldsymbol{Z}\|_{\mathcal{A}_L} = \inf_{\boldsymbol{u}\in\mathbb{C}^N,\boldsymbol{W}\in\mathbb{C}^{L\times L}} \left\{ \frac{1}{2}\mathrm{tr}(\mathrm{Toep}(\boldsymbol{u})) + \frac{1}{2}\mathrm{tr}(\boldsymbol{W}) \middle| \begin{bmatrix} \mathrm{Toep}(\boldsymbol{u}) & \boldsymbol{Z} \\ \boldsymbol{Z}^H & \boldsymbol{W} \end{bmatrix} \succeq 0 \right\}, \tag{9}$$

where $\mathrm{Toep}(\boldsymbol{u})$ is the Toeplitz matrix with the first column $\boldsymbol{u}$ and $\boldsymbol{W}$ is an $L \times L$ matrix. Thus, the problem can be written in the SDP form as below

$$\begin{aligned} \min_{\boldsymbol{Z},\boldsymbol{u}\in\mathbb{C}^N,\boldsymbol{W}\in\mathbb{C}^{L\times L}} \ & \frac{1}{2}\mathrm{tr}(\mathrm{Toep}(\boldsymbol{u})) + \frac{1}{2}\mathrm{tr}(\boldsymbol{W}) \\ \text{s.t.} \ & \begin{bmatrix} \mathrm{Toep}(\boldsymbol{u}) & \boldsymbol{Z} \\ \boldsymbol{Z}^H & \boldsymbol{W} \end{bmatrix} \succeq 0, \\ & \|\boldsymbol{y}_1 - \mathcal{X}(\boldsymbol{Z})\|_2 \leq \sigma^2. \end{aligned} \tag{10}$$

where can be efficiently solved using off-the-shelf convex solvers such as CVX [11]. Since $\hat{\boldsymbol{Z}}$ is low-rank, the singular value decomposition can be used to simultaneously extract $\boldsymbol{h}_1$ and $\boldsymbol{g}$. More precisely, $\hat{\boldsymbol{h}}_1 = \hat{\sigma}_1\hat{\boldsymbol{u}}_1$ and $\hat{\boldsymbol{g}} = \hat{\boldsymbol{v}}_1$, where $\hat{\sigma}_1$ is the largest singular value of $\hat{\boldsymbol{Z}}$. In addition, $\hat{\boldsymbol{u}}_1$ and $\hat{\boldsymbol{v}}_1$ are the left and right singular vectors corresponding to $\hat{\sigma}_1$. The authors in [10] have shown that there is a logarithmic relation between the sparsity level, number of observed samples and the coding length for the blind convolution problem. It is beneficial to follow the proposed approach in [10] to see if there exists a similar relation between s, L, and N. In the next section, we show that the estimated channel at the first transmission block can be employed to detect the transmit signal and consequently the channel at the second transmission block. Then, we show that this technique can be repeated for all the remaining blocks.

IV. PANM for the Remaining Transmission Blocks

In this section, we propose an optimization problem based on PANM to estimate the channel and data for all the remaining blocks. Assuming that we have the channel at the $m-1$-th transmission block for $m \geq 2$, we can estimate the transmit signal at the m-th block by solving the following least square problem

Algorithm 1: Non-Coherent OFDM Detection.

1: Set $m = 1$ and solve (10) and obtain $\hat{\boldsymbol{h}}_1$ and $\hat{\boldsymbol{g}}$.
2: **Repeat**
3: Set $m = m + 1$ and solve (11) to obtain $\tilde{\boldsymbol{x}}_m$.
4: Solve (15) to obtain $\hat{\boldsymbol{h}}_m$ and $\hat{\boldsymbol{e}}_m$.
5: Obtain a better estimation of $\boldsymbol{x}_m$ using $\hat{\boldsymbol{x}}_m = \min_{\boldsymbol{x}_m\in\mathcal{M}} \|\boldsymbol{x}_m - (\tilde{\boldsymbol{x}}_m + \hat{\boldsymbol{e}}_m)\|_2$.
5: **Until** $m = M$
Result: $\hat{\boldsymbol{g}}$, $\hat{\boldsymbol{x}}_m$, $m \geq 2$ and $\hat{\boldsymbol{h}}_m, \forall m$.

$$\tilde{\boldsymbol{x}}_m = \min_{\boldsymbol{x}_m\in\mathcal{M}} \|\boldsymbol{y}_m - \mathrm{diag}(\boldsymbol{x}_m)\hat{\boldsymbol{h}}_{m-1}\|_2, \tag{11}$$

where $\mathcal{M}$ is a predefined constellation. Then, we can define $\boldsymbol{e}'_m = \boldsymbol{x}_m - \tilde{\boldsymbol{x}}_m$ as an error vector and rewrite the observation signal in (3) as

$$\begin{aligned} y_{n,m} &= (\hat{x}_{n,m} + e'_{n,m})h_{n,m} + w_{n,m}, \\ &= \hat{x}_{n,m}h_{n,m} + e_{n,m} + w_{n,m} \end{aligned} \tag{12}$$

where $e_{n,m} = e'_{n,m}h_{n,m}$. Consequently, the observed signal at the receiver side can be written in the vector form as

$$\boldsymbol{y}_m = \mathrm{diag}(\tilde{\boldsymbol{x}})\boldsymbol{h}_m + \boldsymbol{e}_m + \boldsymbol{w}_m, \ \forall m \geq 2. \tag{13}$$

In practice, regarding the fact that the channel states do not vary fast between two consecutive transmission blocks, the demodulation error, $\boldsymbol{e}'_m$, is sparse. Consequently, $\boldsymbol{e}_m$ is sparse. This means that the observed signal is a combination of two sparse vectors, $\boldsymbol{h}_m$ and $\boldsymbol{e}_m$ in different domains, which are contaminated by dense noise. Following [12], we propose the following PANM that obtains a low-dimensional representation for the channel with respect to the sparsity of $\boldsymbol{e}_m$ and the power of AWGN noise

$$\begin{aligned} \min_{\boldsymbol{h}_m,\boldsymbol{e}_m} \ & \|\mathrm{diag}(\tilde{\boldsymbol{x}})\boldsymbol{h}_m\|_{\mathcal{A}} + \lambda\|\boldsymbol{e}_m\|_1 \\ \text{s.t.} \ & \|\boldsymbol{y}_m - \mathrm{diag}(\boldsymbol{x})\boldsymbol{h}_m - \boldsymbol{e}_m\|_2 \leq \sigma^2, \ \forall m \geq 2, \end{aligned} \tag{14}$$

where $\lambda > 0$ is a regularization parameter, balancing the sparsity of $\boldsymbol{h}_m$ and $\boldsymbol{e}_m$. The atom set for the above problem can be written as $\mathcal{A} = \{A(f, \theta) = \boldsymbol{c}(f, \theta) \in \mathbb{C}^{N\times 1} | f \in [0, 1], \theta \in [0, 2\pi)\}$, consequently, the atomic norm over set $\mathcal{A}$ is defined by $\|\boldsymbol{h}\|_{\mathcal{A}} = \inf_{f_i\in[0,1)} \left\{ \sum_i \alpha_i \middle| \alpha_i A(f_i, \theta_i), \alpha_i \geq 0, \theta \right\}$. Problem (14) requires an infinite dimensional search over the interval $[0, 1)$, $\forall f_i$, hence is NP-hard and challenging to implement. However, one can employ the SDP for atomic norm minimization which was first proposed in [3, Proposition II.1] over the observation set $\mathcal{N}$ as $\|\boldsymbol{h}\|_{\mathcal{A}} = \inf_{\boldsymbol{u},t} \left\{ \frac{1}{2N}\mathrm{tr}(\mathrm{Toep}(\boldsymbol{u})) + \frac{t}{2} \middle| \begin{bmatrix} \mathrm{Toep}(\boldsymbol{u}) & \boldsymbol{h} \\ \boldsymbol{h}^H & t \end{bmatrix} \succeq 0 \right\}$, where $\mathrm{Toep}(\boldsymbol{u})$ denotes the Toeplitz matrix whose first column is equal to u and t is a constant variable. Indeed, the proposed semidefinite programming turns the infinite dimensional

This article has been accepted for publication in IEEE Wireless Communications Letters. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/LWC.2022.3218187

search issue to a tractable SDP. Consequently, we can write problem (14) as

$$\begin{aligned}\min_{\boldsymbol{h}_m,\boldsymbol{e}_m,\boldsymbol{u},t}\ & \frac{1}{2N}\mathrm{tr}(\mathrm{Toep}(\boldsymbol{u}))+\frac{t}{2}+\lambda\|\boldsymbol{e}_m\|_1\\ \text{s.t.}\ & \begin{bmatrix}\mathrm{Toep}(\boldsymbol{u}) & \mathrm{diag}(\tilde{\boldsymbol{x}})\boldsymbol{h}_m\\ \mathrm{diag}(\tilde{\boldsymbol{x}}^H)\boldsymbol{h}_m^H & t\end{bmatrix}\succeq 0,\\ & \|\boldsymbol{y}-\mathrm{diag}(\tilde{\boldsymbol{x}})\boldsymbol{h}_m-\boldsymbol{e}_m\|_2\leq\sigma^2,\ \forall m\geq 2,\end{aligned}\tag{15}$$

which is convex and can be solved by CVX. Then, using the Vandermonde decomposition of $\mathrm{Toep}(\hat{\boldsymbol{u}})$, where $\hat{\boldsymbol{u}}$ is the solution of problem (15) which was first proposed in [3], the channel can be estimated. It is worth noting that a more accurate estimation of the transmit signal can be obtained using the following optimization $\hat{\boldsymbol{x}}_m=\min_{\boldsymbol{x}_m\in\mathcal{M}}\|\boldsymbol{x}_m-(\tilde{\boldsymbol{x}}_m+\hat{\boldsymbol{e}}_m)\|_2$, where $\hat{\boldsymbol{e}}_m$ is the solution of problem (15). The above procedure can be done for all the blocks. The overall procedure is summarized in Algorithm 1.

Since the proposed problems are convex, they can be implemented using the interior point method with Newton steps, which is leveraged by off-the-shelf convex solvers such as CVX [11]. The total computational complexity for this method is given by $\mathcal{O}((E+F)^{1.5}E^2)$, where E and F are the numbers of variables and constraints in the optimization problem, respectively [11]. Hence, the overall complexity of the proposed problem in (10) and (15) are approximately

$$\mathcal{O}((N+L^2+3)^{1.5}(N+L^2)^2)\approx\mathcal{O}(N^{3.5}+L^7),\tag{16}$$

and

$$\mathcal{O}((3N+3)^{1.5}(3N+1)^2)\approx\mathcal{O}(46N^{3.5}),\tag{17}$$

respectively. From these derivations, one can understand that the proposed problems can be implemented for a moderate number of OFDM subcarriers in practice. We also show this by numerical simulations in Section V. For large OFDM subcarrier scenarios, one might need to use the greedy-based methods [13] or alternating direction method of multipliers (ADMM) [14].

V. Simulation Results

In this section, we evaluate the performance of the proposed blind estimator using 1000 Monte-Carlo simulations for an OFDM system with $N=128$ over $M=20$ transmission blocks. The parameters of the OFDM system are set according to [1]. We define the normalized mean-square error (NMSE) as $\mathbb{E}[\|\boldsymbol{H}-\hat{\boldsymbol{H}}\|_F^2/\|\boldsymbol{H}\|_F^2]$ where $\boldsymbol{H}=[\boldsymbol{h}_1,\cdots,\boldsymbol{h}_M]$ in which $\|\cdot\|_F$ is the Frobenius norm and the symbol error rate (SER) as

$$\mathrm{SER}=\mathbb{E}\left[\frac{1}{L}\sum_{k=1}^{L}\mathbb{I}_{(g_k-\tilde{g}_k)}+\frac{1}{N(M-1)}\sum_{m=2}^{M}\sum_{k=1}^{N}\mathbb{I}_{(x_{k,m}-\tilde{x}_{k,m})}\right],$$

where $\mathbb{I}$ is a binary indicator such that $\mathbb{I}=0$ if $x_{n,m}=\tilde{x}_{n,m}$ ($g_k=\tilde{g}_k$), otherwise, $\mathbb{I}=1$, to investigate the channel and symbol estimation performances, respectively. Note that the first and second terms of SER are for $m=1$ and $m\geq 2$, respectively. At the first transmission block, we uniformly and independently generate 3 time dispersive channels at the interval $\tau_i\in[T_s,NT_s)$ where $T_s=1$ microsecond (μs) and

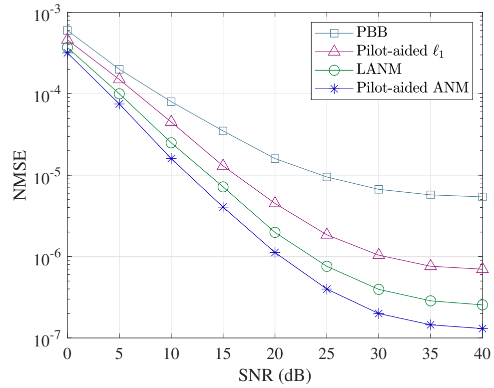

Fig. 1: A Comparison of NMSE of the LANM-based channel estimation with the benchmarks for $N=128$, $L=12$.

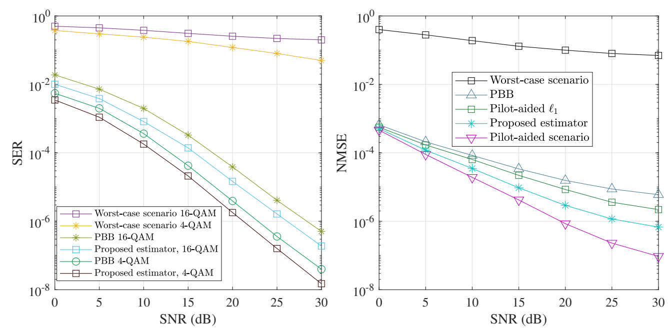

Fig. 2: A Comparison of SER and NMSE between the proposed estimator and benchmarks in Figs. 2 (a) and (b) for $N=128$, $L=12$, and $s=3$, respectively.

their corresponding amplitudes are i.i.d. zero-mean circularly symmetric complex Gaussian variables with zero-mean and unit variance. For the subsequent blocks, we consider the dependency of $\boldsymbol{h}_m=\boldsymbol{h}_{m-1}+\Delta_m$ with $\sigma_c^2=0.01$. To have a fair balance between the sparsity of the demodulation error and OFDM channel, we set $\lambda=1$ in (14) as there is no prior knowledge about the channel or noise sparsity. We use [7] which is based on the linear precoding and cross-correlation techniques as the comparative benchmark and call it as PBB. For a fair comparison, we assume that J in [7] that is the number of OFDM blocks with the same channel states is one. To evaluate the channel estimation performance of the proposed estimator, we compare its NMSE with the pilot-aided ℓ_1 minimization [2] and ANM [5].

We commence our evaluations by comparing the NMSE of LANM-based channel estimation with PBB, pilot-aided ℓ_1 minimization and ANM in Fig. 1 for different SNR values. For this simulation, we set $L=12$. As shown in this figure, by increasing the SNR thresholds, the NMSE of all approaches decreases. This figure demonstrates that although the LANM estimates the channel and transmit signal simultaneously, it performance is comparable to the pilot-aided ANM and outperforms the conventional channel estimations methods. Since the transmit signal is known at the receiver side for the pilot-aided ANM, its NMSE severs as a lower bound for

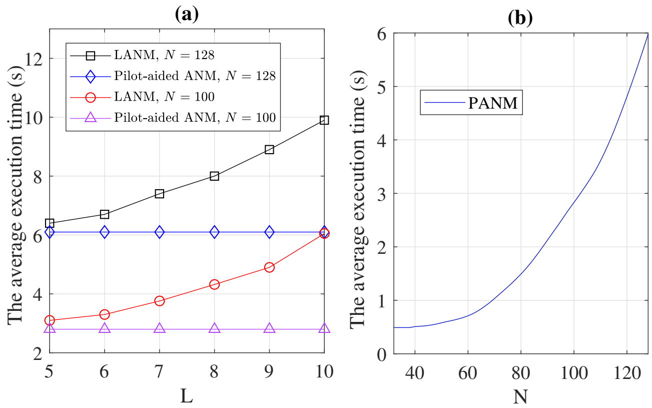

Fig. 3: The average execution time of the proposed LANM and PANM in Fig. 3 (a) and (b), respectively.

the LANM. In addition, the NMSE of proposed LANM is less than the PBB and pilot-aided ℓ_1 minimization for all the SNR values. The LANM outperforms the PBB and pilot-aided ℓ_1 minimization since is more robust against the additive noise and recovers the time dispersive channels in a continuous dictionary, respectively.

We compare the SER of proposed estimator for $L = 12$ with PBB in Fig. 2 (a) and its NMSE with PBB, pilot-aided ℓ_1 minimization and ANM in Fig.2 (b). We consider the worst-case scenario where the channel is estimated once at the first transmission block and then use for the consecutive blocks. One can understand from Fig. 2 that the SER and NMSE decrease by increasing the SNR thresholds. Fig. 2 (a) demonstrates that the performance obtained in terms of SER of the proposed estimator outperforms the PBB for the same quadrature amplitude modulation (QAM) constellation. The QAM singling is adapted from [15]. Moreover, we can understand from this figure that by enlarging the QAM constellation, the SER increases for both approaches. Similar to Fig. 1, Fig. 2 (b) reveals that the NMSE of the proposed estimator is comparable to the pilot-aided ANM and outperforms the PBB and pilot-aided ℓ_1.

In Fig. 3, we investigate the average execution time for the proposed problems. As shown in (16), the total complexity of the proposed LANM is a function of N and L. Thus, in Fig. 3 (a), we evaluate the average execution time of LANM versus L for the different values of N. For performing simulations, we use an Intel Core i7 $-$ 6700 2.6GHz CPU computer. It is observed from this figure that the average execution time is increasing by growing L or N and lower bounded by the pilot-aided ANM. Moreover, in Fig. 3 (b), we plot the average execution time of PANM for each transmission, which shows that the implementation time increases when N grows. From Fig. 3, one can observe that the proposed problems can be efficiently implemented for a moderate number of OFDM subcarriers in practice. For large subcarrier scenarios, one might develop greedy-based methods [13] or ADMM [14].

VI. Conclusions and Future Research Direction

A blind estimator based on ANM for a time-variant OFDM system is proposed in this letter. The channel is estimated at the first block using LANM. Then, using PANM, the channel and transmit signal are estimated at the subsequent blocks. The total computational complexity of the proposed problems was derived, showing that they can be implemented for a moderate number of OFDM subcarriers in practice. The simulation results support the validation of the proposed estimator and illustrate its superiority in terms of SER and NMSE compared to the benchmarks. The performance of the proposed estimator relies on the channel error power, σ_c^2. Though we show that when σ_c^2 is small, the proposed estimator can be efficiently employed. An interesting research direction would be developing a robust ANM problem which was first studied in [16] to evaluate the effects of σ_c^2 on the estimation performance.

VII. Acknowledgment

This project has received funding from the European Union's Horizon 2020 research and innovation program under the Marie Skodowska-Curie grant agreement No 812991.

References

[1] O. Simeone, Y. Bar-Ness, and U. Spagnolini, "Pilot-based channel estimation for ofdm systems by tracking the delay-subspace," *IEEE transactions on wireless communications*, vol. 3, no. 1, pp. 315–325, 2004.

[2] P. Cheng, Z. Chen, Y. Rui, Y. J. Guo, L. Gui, M. Tao, and Q. Zhang, "Channel estimation for ofdm systems over doubly selective channels: A distributed compressive sensing based approach," *IEEE Transactions on Communications*, vol. 61, no. 10, pp. 4173–4185, 2013.

[3] G. Tang, B. N. Bhaskar, P. Shah, and B. Recht, "Compressed sensing off the grid," *IEEE transactions on information theory*, vol. 59, no. 11, pp. 7465–7490, 2013.

[4] Y. Chi, "Guaranteed blind sparse spikes deconvolution via lifting and convex optimization," *IEEE Journal of Selected Topics in Signal Processing*, vol. 10, no. 4, pp. 782–794, 2016.

[5] S. Pejoski and V. Kafedziski, "Estimation of sparse time dispersive channels in pilot aided ofdm using atomic norm," *IEEE Wireless Communications Letters*, 2015.

[6] I. Valiulahi, F. Parvaresh, and A. A. Beheshti, "Eliminating impulsive noise in pilot-aided ofdm channels via dual of penalized atomic norm," *IEEE Communications Letters*, vol. 23, no. 11, pp. 2059–2062, 2019.

[7] A. Petropulu, R. Zhang, and R. Lin, "Blind ofdm channel estimation through simple linear precoding," *IEEE Transactions on Wireless Communications*, vol. 3, no. 2, pp. 647–655, 2004.

[8] R. Lin and A. P. Petropulu, "Linear precoding assisted blind channel estimation for ofdm systems," *IEEE Transactions on Vehicular Technology*, vol. 54, no. 3, pp. 983–995, 2005.

[9] L. Xiao, G. Liu, P. Xiao, and T. Jiang, "Error probability analysis for rectangular differential ofdm with index modulation over dispersive channels," *IEEE Wireless Communications Letters*, vol. 10, no. 12, pp. 2795–2799, 2021.

[10] A. Ahmed, B. Recht, and J. Romberg, "Blind deconvolution using convex programming," *IEEE Transactions on Information Theory*, vol. 60, no. 3, pp. 1711–1732, 2013.

[11] M. Grant and S. Boyd, "Cvx: Matlab software for disciplined convex programming, version 2.1," 2014.

[12] I. Valiulahi, F. Haddadi, and A. Amini, "Robustness of two-dimensional line spectral estimation against spiky noise," *IEEE Transactions on Signal Processing*, vol. 67, no. 23, pp. 5998–6008, 2019.

[13] N. Rao, P. Shah, and S. Wright, "Forward–backward greedy algorithms for atomic norm regularization," *IEEE Transactions on Signal Processing*, vol. 63, no. 21, pp. 5798–5811, 2015.

[14] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via the alternating direction method of multipliers," 2011.

[15] J. Proakis and M. Salehi, "Digital communications," *McGraw-Hill*, 2014.

[16] B. N. Bhaskar, G. Tang, and B. Recht, "Atomic norm denoising with applications to line spectral estimation," *IEEE Transactions on Signal Processing*, vol. 61, no. 23, pp. 5987–5999, 2013.