# Imperial College of Science, Technology and Medicine Department of Mechanical Engineering 

# Robotic Manipulators for In Situ Inspections of Jet Engines 

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## Declaration of Originality

I confirm that all the work presented in this dissertation is my own work, unless explicitly stated.

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#### Abstract

Jet engines need to be inspected periodically and, in some instances, repaired. Currently, some of these maintenance operations require the engine to be removed from the wing and dismantled, which has a significant associated cost. The capability of performing some of these inspections and repairs while the engine is on-wing could lead to important cost savings. However, existing technology for on-wing operations is limited, and does not suffice to satisfy some of the needs.

In this work, the problem of performing on-wing operations such as inspection and repair is analysed, and after an extensive literature review, a novel robotic system for the on-wing insertion and deployment of probes or other tools is proposed. The system consists of a finepositioner, which is a miniature and dexterous robotic manipulator; a gross-positioner, which is a device to insert the fine-positioner to the engine region of interest; an end-effector, such as a probe; a deployment mechanism, which is a passive device to ensure correct contact between probe and component; and a feedback system that provides information about the robot state for control. The research and development work conducted to address the main challenges to create this robotic system is presented in this thesis. The work is focussed on the fine-positioner, as it is the most relevant and complex part of the system.

After a literature review of relevant work, and as part of the exploration of potential robot concepts for the system, the kinematic capabilities of concentric tube robots (CTRs) are first investigated. The complete set of stable trajectories that can be traced in follow-the-leader motion is discovered. A case study involving simulations and an experiment is then presented to showcase and verify the work. The research findings indicate that CTRs are not suitable for the fine-positioner. However, they show that CTRs with non-annular cross section can be used for the gross-positioner. In addition, the new trajectories discovered show promise in minimally invasive surgery (MIS).

Soft robotic manipulators with fluidic actuation are then selected as the most suitable concept for the fine-positioner. The design of soft robotic manipulators with fluidic actuation is investigated from a general perspective. A general framework for the design of these devices


is proposed, and a set of design principles are derived. These principles are first applied in a MIS case study to illustrate and verify the work. Finite element (FE) simulations are then reported to perform design optimisation, and thus complete the case study. The design study is then applied to determine the most suitable design for the fine-positioner. An additional analytical derivation is developed, followed by FE simulations, which extend those of the case study. Eventually, this work yields a final design of the fine-positioner. The final design found is different from existing ones, and is shown to provide an important performance improvement with respect to existing soft robots in terms of wrenches it can support.

The control of soft and continuum robots relevant to the fine-positioner is also studied. The full kinematics of continuum robots with constant curvature bending and extending capabilities are first investigated, which correspond to a preliminary design concept conceived for the finepositioner. Closed-form solutions are derived, closing an open problem. These kinematics, however, do not exactly match the final fine-positioner design selected. Thus, an alternative control approach based on closed-loop control laws is then adopted. For this, a mechanical model is first developed. Closed-loop control laws are then derived based on this mechanical model for planar operation of a segment of the fine-positioner. The control laws obtained represent the foundation for the subsequent development of control laws for a full fine-positioner operating in 3D. Furthermore, work on path planning for nonholonomic systems is also reported, and a new algorithm is presented, which can be applied for the insertion of the overall robotic system.

Solutions to the other parts of the robotic system for on-wing operations are also reported. A gross-positioner consisting of a non-annular CTR is proposed. Solutions for a deployment mechanism are also presented. Potential feedback systems are outlined. In addition, methods for the fabrication of the systems are reported, and the electronics and systems required for the assembly of the different parts are described.

Finally, the use of the robotic system to perform on-wing inspections in a representative case study is studied to determine the viability. Inspection strategies are shortlisted, and simulations and experiments are used to study them. The results, however, indicate that inspection is not
viable since the signal to noise ratio is excessively low.

Nonetheless, the robotic system proposed, and the research conducted, are still expected to be useful to perform a range of on-wing operations that require the insertion and deployment of a probe or other end-effector. In addition, the trajectories discovered for CTRs, the design found for the fine-positioner, and the advances on control, also have significant potential in MIS, where there is an important need for miniature robotic manipulators and similar devices.

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To my parents

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#### Abstract

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## Chapter 1

## Introduction

Performing some of the required inspection and repair operations of jet engines while the engine is on-wing can provide significant cost savings to the aerospace industry. The technologies currently available to insert and deploy the probes and other tools for these on-wing operations are relatively limited, and cannot be used to address all needs. The development of new technologies for on-wing operations can satisfy some of these needs, providing significant cost savings to the aerospace industry, and represents the focus of the work presented in this thesis.

In this chapter, the industrial motivation behind this work is first introduced in section 1.1. The aim of the work is then described in section 1.2. The concept proposed in this work for a robotic system capable of on-wing inspections and other operations is outlined in section 1.3. The main requirements for the robotic system are summarised in section 1.4. The work required to develop the robotic system is then outlined in section 1.5. Finally, the thesis structure is also presented in section 1.5, together with a list of publications arising from this work.

### 1.1 Project background and motivation

### 1.1.1 Industrial relevance

Rolls-Royce plc (London, UK) is a manufacturer of jet engines with decades of tradition. In recent years, Rolls-Royce modified its business model, moving from selling engines and replacement components with the corresponding ad hoc servicing, to offering a TotalCare ${ }^{\Omega}$ package that includes the engine maintenance at a specific cost per operation hour. Such change in strategy has made lifelong aspects of the engine such as inspection, monitoring, and repair a part of the company's operations.

Inspection and repair operations can be conducted during overhauls, and a set of planned overhauls exist for aircraft engines. The cost of an overhaul or a similar operation that involves removing the engine from the aircraft wing and dismantling is significant. In this regard, there is a desire to maximise the period of time between overhauls, and to avoid any unplanned operations that require removing the engine from the aircraft wing.

Some of the required inspection and repair operations, however, can either be unpredictable, or require significantly shorter time periods between interventions than the remaining operations. Taking the engine off-wing to perform these operations that do not match the main, planned overhauls can have an important additional cost. Thus, there is a significant interest in the capability of performing these unplanned or higher frequency operations on-wing.

### 1.1.2 On-wing inspection technology

Miniature probes exist, which can be inserted into the engine to perform on-wing inspections. These probes can be used to inspect a component without damaging it relying on different techniques that belong to the field of non-destructive evaluation (NDE) [1]. The main NDE techniques are ultrasonic, electromagnetic, radiographic, visual, and thermographic, and these are briefly reviewed in Chapter 7.


Figure 1.1: Existing borescope. Image courtesy of General Electric.

The technologies currently available to insert and deploy probes (or other tools) in situ, however, are relatively limited. These mainly consist of passive tools (rigid or flexible); tools that can either be in a flexible state or a rigid state with a pre-defined shape; and borescopes such as that shown in Figure 1.1 or equivalent devices that can carry different end-effectors instead of a camera. These existing technologies are described in more detail in Chapter 2, and can be used to deploy probes (or other tools) in locations with relatively easy access, typically near the points of access into the engine. However, these technologies are not suitable to deploy probes or other tools in difficult to access locations.

### 1.1.3 General need for new on-wing inspection technology

Various applications that require the on-wing deployment of probes or other tools in difficult to access locations exist, and additional ones can be expected in the future. This general need for technologies to deploy probes and tools on-wing represents the motivation for this work. A certain degree of variability in the requirements for these on-wing inspections in difficult to access locations exists. However, they generally involve the insertion and deployment of probes or tools in locations relatively distant from the point of entry into the engine, through narrow, convoluted conducts, and without direct line of sight. In this regard, a reference on-wing inspection case is defined in the following to guide the development of new technology.


Figure 1.2: CAD rendering of a generic jet engine with main parts labelled. Image courtesy of Rolls-Royce.

The reference case requires the insertion of a probe into a chamber near the centre of the engine, where the high pressure compressor (HPC) discs are located, as shown in Figure 1.2, and the deployment of this probe on the various HPC discs to inspect them. The access route to reach the chamber of interest involves a relatively long and convoluted path, which is shown in Figure 1.3. This is composed of a first narrow part to enter towards the centre of the engine through a conduct with constant diameter and constant curvature; an open part with a turn of approximately 90 degrees; another narrow part to advance horizontally in the space between the shafts which is straight; and finally another open part in the chamber with the HPC discs. This route is illustrative of possible access routes that may be encountered in in situ inspections.

This reference use case is representative of the requirements of on-wing inspections in difficult to access locations, and is selected to be a particularly challenging one. It should be noted, however, that this is only an example of case used to guide the efforts in this work. The general aim of the work presented in this thesis is the development of technology to address the general on-wing inspection problem.

Other examples of applications that may require new on-wing inspection technology, even


Figure 1.3: Access route selected in an illustrative reference case to deliver probes to the chamber with HPC discs on-wing. The locations of interest for probe deployment are circled in blue.
though with less complex access restrictions, also exist. One of them is the detection of potential defect-like indications in turbine blade roots. Blade roots can be difficult to access and, even though they could be inspected using ultrasonic probes [2,3] or eddy-current probes [4,5], the location of the potential defect-like indications makes it difficult to insert and deploy probes on-wing. Another set of examples of challenging on-wing inspections are those relying on radiographic techniques. These can require inserting radioactive beads and detection films in difficult to access locations inside the engine. Lastly, inspections relying on enhanced visual techniques $[6,7]$ may also require new on-wing capability. These inspections can be complex to perform in situ since they involve applying visible liquids on the component so that they seep into potential cracks, cleaning excess liquid from the surface, and inserting a camera or optic fibre to visually detect any potential defect. The insertion and deployment of the required probes and inspection devices in many of these applications cannot be done using existing rigid tools or borescope-type devices, so new technology is required.

### 1.2 Aim and preliminary considerations

Responding to the industrial need presented in the previous section, the aim of the work reported in this thesis is the development of a robotic system capable of inserting and deploying probes and other end-effectors on-wing in order to perform general in situ inspections. A reference use case, defined to help guide the development of this robotic system, is the insertion of probes into the chamber with HPC discs and the subsequent deployment on the HPC discs. However, the robotic system should also be capable of other on-wing inspections. In this regard, the robotic system should be a versatile device capable of inserting and deploying an end-effector (typically a probe) for general on-wing inspections, such as those mentioned in the previous section, or other general operations.

The deployment of probes and other end-effectors inside the engine involves navigating the engine geometry through complex, narrow routes to reach the region of interest, positioning the probes in the desired location, and ensuring a correct and stable deployment to perform the inspection. This generally requires the application of controllable forces and moments on the end-effector to insert it and accurately deploy it. Considering the forces in nature, together with the problem context in terms of surrounding engine, in this work the most reliable option is considered to be the use of contact forces applied by a structure on the probe. These forces must be transmitted through the structure to a support point, and this structure must fit through the entire access route. Other alternatives such as the use of propelling forces from miniature flying robots to deliver the end-effector were discarded since they are considered significantly less reliable and practically difficult to achieve. The use of non-contact electromagnetic forces to bring the end-effector to the desired location, such as those proposed for medical interventions [8], was also discarded since jet engines include an important number metallic components.

The majority of components inside a jet engine are also non-ferromagnetic, and the engine has few support points where a robot could reliably hold to advance. Thus, untethered robot locomotion to navigate inside the engine is difficult in practice. In this regard, the best option a priori is considered to be the use an elongated, narrow manipulator such as a snake-robot
to insert and deploy the end-effector. The development of such snake-robot with the mobility and size required for general on-wing inspections represents the main part of the development of the robotic system.

In the reference case defined in the previous subsection 1.1.3 to serve as guidance for this work, the access route selected is that shown in Figure 1.3, which is illustrative of a complex onwing inspection. The probes need to be entered through this route, and subsequently deployed on the HPC discs, which are circled in blue in Figure 1.3. The route is relatively long with curvature variations, multiple turns, and limited room to manoeuvre. However, the route has the advantage of being independent of the rotation of the HPC discs, which could allow deploying a probe on a HPC disc, and rotating the disc while maintaining the probe in a fixed position in order to perform a full disc inspection. Thus, the main objective for this example inspection is to deploy a probe on the HPC discs.

In order to reach the discs, the robotic system needs to enter the chamber with the HPC discs through a hole in the shaft, as shown in Figure 1.3. This manoeuvre requires the robotic system to perform a relatively sharp turn. The robotic system should therefore be capable of either articulating or bending with significant flexibility to perform the turn. Furthermore, once part of the robotic system is already inserted inside the chamber with the HPC discs, the rest of it should be capable of continuing advancing through the hole so that the probe can be delivered to the HPC discs.

After passing through the hole, the probes need to be deployed on the various HPC discs. The deployment on the first disc encountered when accessing the HPC chamber through the hole can be performed by directing the robotic system directly towards this disc. Once this is inspected, the robotic system should manoeuvre to pass through the gap between this first disc and the shaft, and then reach the following disc, where it should deploy the probe to inspect it. The procedure is then intended to be repeated to reach each of the following discs and inspect them. The height of the gap between the first disc and the shaft is similar to the height of the gap between shafts, so a robotic system capable of entering through the access route is expected to be capable of passing through the gap under the disc. It should be noted that the manoeuvre
to pass through the gap between each disc and the shaft is expected to require significant dexterity, and thus to involve the full robotic system manoeuvring to complete it. Once the robot has passed through the gap, this gap can help in anchoring or bounding the motion of part of the body of the robotic system so that it can deploy the probe on the subsequent disc.

In the access route shown in Figure 1.3, any device inserted can be guided by reaction forces from the engine in the constrained parts, which are therefore relatively simple to follow, but requires steering in the open spaces, and accurate control in the final part. The curvature in the first narrow conduct is constant, but presents variations throughout the rest. As a consequence, a rigid structure can only be entered through the first conduct, but the rest of the route requires a device capable of moving in exact or approximate follow-the-leader motion [9], i.e. the body of the device following the path selected by the distal end. This implies that the robotic system must be capable of either articulating or bending with a flexible structure. Finally, the entry hole is selected to be 6 mm in diameter, which represents one of the main constraints on the robotic system.

### 1.3 Concept proposed

The robot concept proposed in this work for the on-wing insertion and deployment of a probe (or other end-effector) is schematised in Figure 1.4, and consists of two main parts: a grosspositioner and a fine-positioner. The gross-positioner is a low mobility device capable of reaching the region of interest inside the engine with a low accuracy. The fine-positioner is a miniature, accurate robotic manipulator that can be coupled at the end of the gross-positioner, and can position its distal end to a desired pose in space using a set of degrees of freedom (DOFs). The fine-positioner thus serves to deploy the probe (or other end-effector) in the target location, using its dexterity to compensate for any deviations from the gross-positioner.

The robot system also has two additional parts. One of them is a deployment mechanism, which is a passive mechanism placed between the distal end of the fine-positioner and the probe that is conceived to tolerate small misalignments from the robot and ensure a correct probe


Figure 1.4: Conceptual drawing of the proposed robotic system for on-wing inspections, comprising a gross-positioner, a fine-positioner, a deployment mechanism, a feedback system, and an end-effector (in this instance a probe).
deployment. It should be noted that the deployment mechanism refers to all passive mechanisms used to correct for misalignments on the end-effector; any active DOFs incorporated on the overall system correspond to the fine-positioner. The other additional part of the robotic system is a feedback system, which is a system that provides information about the robot state to control it.

This robot concept, schematised in Figure 1.4, is conceived to exploit the reaction forces from contact with the engine in order to support and guide the robot. This is viable in on-wing inspections since the engine parts are relatively robust, and implies that the robot needs not be self-supporting. Such a robot concept contrasts with some of the existing trends in the development of robots for inspections, and facilitates the development of the robot since it reduces the required number of DOFs and the force that the robot structure must support. In particular, it implies that the gross-positioner can be a device with very limited mobility, or even passive in some instances, which significantly reduces complexity. In addition, it broadens the regions inside the engine that can be reached with the limited number of DOFs achievable with miniature diameter robotic manipulators.

This robot concept, composed of a gross-positioner plus a fine-positioner, is intended to be applicable to a broad range of on-wing inspections. These inspections generally require accessing the engine region of interest through a convoluted route in a first instance, where reaction forces can be exploited, and then deploying a probe or other end-effector, which requires DOFs and accuracy provided by the fine-positioner. In this regard, the development of a robot system like that schematised in Figure 1.4 matches the aim of versatility described in the previous section.

In the reference case of on-wing inspection defined in the previous subsection 1.1.3, the grosspositioner should be capable of reaching the chamber with the HPC discs. The main challenge in reaching this chamber is the 90 degree turn in open space. In this instance, the mobility of the gross-positioner combined with the dexterity of the fine-positioner are expected to be sufficient to execute the turn. After the turn, the gross-positioner only needs to advance in the narrow gap between shafts to reach the chamber of interest, which is considered to be a relatively simple operation, even with a low-mobility gross-positioner. The only envisaged potential issues when advancing between the shafts are lateral deviations from the desired path, and friction with the engine complicating the advancement. However, the fine-positioner is expected to be capable of helping in correcting for deviations. In addition, some fine-positioner concepts, such as the one selected in Chapter 2, can help in the advancement, overcoming frictional forces by actively contributing to self locomotion, as described in Chapter 6.

In order to enter the chamber with the HPC discs, a combination of the motion of the finepositioner and the gross-positioner is expected to be required. The distal end of the finepositioner should be steered towards the hole in the shaft, while the gross-positioner and potentially the proximal part of the fine-positioner are used to advance the entire robotic system. This procedure should continue until the entire fine-positioner is inside the chamber with the HPC discs. At this point, a length of a few millimetres of the distal end of the gross-positioner is expected to be entering into the chamber with the HPC discs.

The pose at the distal end of the gross-positioner when entering into the chamber is expected to present significant variation. In this regard, a fine-positioner with a high dexterity is expected to be necessary to deploy the probes in the desired location on the first disc. After inspecting
the first disc, the fine-positioner is expected to be used to steer the robotic system into the gap between the disc and shaft, while the gross-positioner is used to advance it. A combined manoeuvre of the fine-positioner and gross-positioner is expected to be necessary to pass through the gap between the disc and shaft. This procedure is indended to be repeated to inspect all subsequent HPC discs.

### 1.4 Basic requirements

Each on-wing inspection entails a set of requirements on the robotic system. Even though the specific requirements can vary to some extent for each specific inspection, an important part of the main requirements in terms of size, kinematics, dynamics, accuracy or systems compatibility are common in general on-wing inspections. In this regard, the majority of requirements used for the development of the robotic system in this work are common requirements for general on-wing inspections, and in instances where values specific for each scenario are required, the requirements of the reference case defined in subsection 1.1.3 are used. These requirements are established in the following subsections.

### 1.4.1 Fine-positioner

The fine-positioner is the most complex part of the robotic system, as well as the most challenging to develop. Its requirements are thus described in more detail in the following subsections.

## Geometry

The fine-positioner must fit through the entry port and through the access route, which generally represents one of the most determinant constraints on the design. In the reference case used as example in this work, this implies that the maximum diameter cannot exceed 6 mm . This constraint is representative of typical on-wing inspections.

The required length depends on the specific capabilities of the gross-positioner in terms of proximity to the regions to be inspected that can be achieved reliably, which is generally not specified. In the reference case defined in the previous subsection 1.1.3, on a first instance the required length of the fine-positioner can be established to be approximately 10 cm based on the distance between HPC discs, the distance between the point of entry into the chamber with HPC discs and the first disc, and the space to manoeuvre inside the chamber with HPC discs. The fine-positioner must also be capable of negotiating the curved access route, and therefore the capacity to bend or articulate in order to navigate in the route shown in Figure 1.3 is essential.

## Kinematics: DOFs

The fine-positioner must be capable of moving the end-effector to a desired location with a specific pose. The variation in the approximation by the gross-positioner must be corrected by the fine-positioner through a set of DOF in order to ensure an accurate positioning of the probes. In the reference case defined in the previous subsection 1.1.3, the HPC chamber is accessed through a small aperture in a specific location, as can be seen in Figure 1.3. This implies that the position of the inserted device is known with a small variation at that point. However, the orientation can present significant variations in all three independent directions.

These orientation deviations at the chamber entry point translate as pose deviations at the distal end of any fine-positioner device, and are amplified by the distance from the entry point. The fine-positioner must compensate for all these deviations, acting as a manipulator in 3D space. This generally requires 6 DOF.

In the reference case previously defined in subsection 1.1.3, the HPC discs are axially symmetric, and therefore the probes can be deployed in any circumferential position, which reduces the complexity of the problem by 1 DOF. This advantage, however, does not particularly simplify the design of most fine-positioner concepts since most of the existing robotic manipulators identified in the literature review as candidates for the fine-positioner are composed of 2 DOF segments. In addition, the position of the HPC discs with respect to the entry point into the
chamber implies that a typical fine-positioner design can only reach a limited set of deployment locations on the disc circumference. Thus, the reduction of 1 DOF in the requirements is not always possible, and depends on the specific robot design.

The gross-positioner is expected to be capable of providing 1 DOF corresponding to the advancement of the proximal end of the fine-positioner. This can reduce by another 1 DOF the kinematic requirements of the fine-positioner. However, it should be noted that the use of the gross-positioner to provide 1 DOF may not be always reliable or accurate.

On the other hand, the inspection may require avoiding obstacles, such as those in the HPC chamber in the reference case, or negotiating the geometry in other scenarios. This can require additional DOFs. Lastly, it is desirable for the robotic system to be as versatile as possible. As a result, the desirable number of DOFs for the fine-positioner is established to be 6 , with the option of having 5 DOF if it is not possible to achieve 6 .

## Dynamics

The main dynamic requirement is for the fine-positioner to be capable of supporting its own weight, the weight and forces associated to the payload, and the forces required to ensure a correct probe deployment on the inspected component. Inertial forces are not considered in the requirements since the final objective is to position the probe in a static deployment location.

In inspections conducted using the most frequent NDE techniques for on-wing inspections, which are ultrasound with a coupling medium and eddy currents, the force to deploy a probe on the component is negligible. In the reference case for this work, the technique selected is ultrasound with a coupling medium, as described in Chapter 7, which also requires practically zero deployment force.

The main force that the fine-positioner needs to support apart from its own weight is then that associated to the payload. Considering the weight of typical miniature probes used for on-wing inspection, the stiffness of the corresponding cables, and the required bending of these, in a first instance it is estimated that the fine-positioner should be capable of applying the equivalent of
0.25 N at its distal end.


#### Abstract

Accuracy

The fine-positioner should be capable of a high accuracy in the positioning of the end-effector. The required orientation accuracy depends on the capability of the deployment mechanism to tolerate misalignments. The required position accuracy in the direction perpendicular to the inspected component depends on whether reaction forces from the component can be used in the probe deployment. If reaction forces are possible, the accuracy only needs to be sufficient to ensure that contact is achieved and that the reaction forces are below the limits that the robot can cope with. If reaction forces are not possible, the accuracy typically needs to be sufficient to ensure that the probe is within the acceptable stand-off for the inspection, which is typically sub-millimetric. The required position accuracy in the direction parallel to the inspected component depends on the effect of the position errors on the inspection performance, which is specific to each application.


In the reference case defined this work, reaction forces are possible. Thus, the required accuracy in the direction perpendicular to the inspected component only needs to be sufficient to ensure that the probe is in contact with the component without damaging the fine-positioner selected. The required absolute accuracy in the direction parallel to the surface of the component is not critical since the proposed inspection involves scanning the entire HPC disc. Thus, the accuracy only needs to be sufficient to ensure that the component can be scanned consistently without gaps in the scan. In this regard, a certain error in absolute accuracy can be accepted, provided that the relative error between positions during a scan is small enough to ensure a full scan coverage.

## Compatibility

The fine-positioner must be compatible with the rest of the robotic system. It must therefore be capable of accommodating wires from the payload or microtubes to deliver a coupling medium
to the probe. Moreover, it must be attachable to the gross-positioner; it must be possible to attach a deployment mechanism to the fine-positioner; and it must be possible to incorporate a feedback system on the fine-positioner, leading to an integrated robot.

### 1.4.2 Gross-positioner

The gross-positioner must be capable of navigating the access route and reaching the engine region of interest. In the reference case defined in the previous subsection 1.1.3, this implies that it needs to be capable of reaching lengths over 1 m , with a maximum outer diameter (OD) of 6 mm , and it must be capable of either bending or articulating to fit through the access route shown in Figure 1.3. It also needs to be capable of at least 1 DOF to advance, and potentially additional DOFs to negotiate obstacles and follow the access route. However, the required number of DOFs depends on the gross-positioner concept selected.

The required force for the gross-positioner depends on the weight of the fine-positioner and the rest of the robot parts, as well as on the reaction forces on the fine-positioner and end-effector. The development of the fine-positioner is the most challenging part of the robotic system so, to simplify its development, it needs to be selected first. The chosen fine-positioner together with the payload and other robot parts then determine the required force on the gross-positioner.

The gross-positioner must also be capable of accommodating any elements to actuate and control the fine-positioner, as well as the payload and the feedback system.

### 1.4.3 Deployment mechanism

The deployment mechanism should be capable of correcting the misalignments from the finepositioner. The magnitude of the corrections depends on the accuracy of the fine-positioner selected. Since the development of the fine-positioner is the most challenging part, this should be developed first aiming to minimise the misalignments. Then the misalignments of the robot can be experimentally estimated, and the deployment mechanism can be adapted accordingly.

The deployment mechanism should also ensure a correct contact between probe and component, forcing the probe to conform in the case of non-rigid probes. In the case of inspections performed with a couplant medium, the deployment mechanism should ensure a correct coupling. Finally, the deployment mechanism should be attachable to the fine-positioner, compatible with the probe, and should have a minimal weight.

### 1.4.4 Feedback system

The feedback system should be capable of providing sufficient information about the state of the robot to enable control and accurate positioning of the end-effector. The information required depends on the type of robot selected for the fine-positioner and for the gross-positioner. However, in general the feedback system should provide information that allows at least for the determination of the end-effector pose.

In the reference case previously defined in subsection 1.1.3, there is generally no direct line of sight to the robot, and the robot must navigate inside the jet engine, which includes many metallic components that act as a Faraday cage. This implies that, in general, a proprioceptive feedback system compatible with these conditions is required. The feedback system also needs to fit through the 6 mm diameter entry hole, and must be compatible with the rest of the robotic system. These requirements are representative of typical on-wing inspections.

The feedback system should have a sufficient accuracy to enable a correct and reliable deployment of the probe on the desired location, as well as the insertion of the robot through the access route. The required accuracy of the feedback system depends on the type of robot and accuracy required for the fine-positioner. Thus, the accuracy of the feedback system must be such that the fine-positioner can meet the requirements described in subsection 1.4.1.

### 1.4.5 End-effector

The end-effector must fit through the access route. In the reference case, this implies that it must fit through 6 mm diameter holes. The end-effector must also be capable of operat-
ing inside a jet engine, which includes many metallic components, the majority of which are non-ferromagnetic. In addition, the end-effector should be compatible with the deployment mechanism, and the rest of the robotic system.

### 1.4.6 Integrated inspection system

The requirements of the different parts comprising the full robotic system for on-wing inspections are interrelated. This implies that the performance of one of the parts can affect the requirements of another, and vice versa. In general, as noted in the previous subsections, the most challenging part in the development of the full robotic system is the development of the fine-positioner. Thus, this should be selected first with the minimum possible requirements from the other parts. Then, the specifications of the chosen fine-positioner dictate the requirements on the other parts.

### 1.5 Project parts and thesis outline

The development of the complete robotic system for on-wing inspections, schematised in Figure 1.4, requires work in all different parts of the system: the fine-positioner, the gross-positioner, the deployment mechanism, and the feedback system. In addition, the application of the robotic system to the reference case previously defined in subsection 1.1.3 requires a study of the inspection to select the most suitable NDE technique and probe, determine the most suitable inspection strategy, and evaluate the expected performance to determine the viability.

As previously mentioned, the most important and challenging part of the work is the development of the fine-positioner. The second most relevant part of the work is the development of the gross-positioner. These two parts complement each other, and combined lead to the main robotic manipulator that can insert and position an end-effector. In this regard, the development of the fine-positioner combined with the gross-positioner represents the core of the work presented in this thesis. The other parts of the robotic system, which are the deploy-
ment mechanism and feedback system, together with the inspection study, are considered to be complementary.

The development of the fine-positioner and gross-positioner requires first a literature review to select the most suitable types of robot to be explored and developed. Then the development of the selected type of fine-positioner requires detailed work on analysis and design of the robot, fabrication, and control, which includes work on mechanical modelling, kinematics, and closedloop control laws. The development of the gross-positioner requires similar work on analysis, design, fabrication, and control. However, in the case of the gross-positioner, the required level of complexity is significantly lower since it is a simpler device with less DOFs and lower accuracy requirements.

The rest of this thesis is then structured as follows. A literature review on robotic manipulators and technologies relevant to on-wing operations is presented in Chapter 2. This literature review leads to the selection of two types of robot as the most relevant to be explored for either the fine-positioner and gross-positioner. These are concentric tube robots (CTRs), and soft robotic manipulators with fluidic actuation.

A new, general study of the kinematic capabilities of CTRs is presented in Chapter 3. The study is focussed on the follow-the-leader capabilities of these robots since, as elucidated in this work, CTRs are one of the few snake-robots capable of perfect follow-the-leader motion, which is desirable when advancing inside cluttered environments like a jet engine. The analysis is general and considers all possible robot designs and controls to achieve follow-the-leader motion. The result of the study identifies the complete follow-the-leader capabilities of CTRs, which closes an open question. However, one of the conclusions from the study is that these robots can generally not be used for the fine-positioner, since the viable lengths between the robot distal end and the actuation box for it are excessively short, suggesting that soft robotic manipulators need to be used instead. Nonetheless, the work on CTRs also shows that using a CTR with non-annular cross section it is possible to create a robotic device of any required length, which is the concept selected for the gross-positioner.

Research conducted on analysis and design of soft robotic manipulators with fluidic actuation is
described in Chapter 4. The development of a novel, general framework for the design of these robots is presented first. This leads to a set of design principles, and to new insights into the behaviour of these robots. The development of a non-dimensional analysis is then also briefly reported. The application of the design principles, new insights, and non-dimensional analysis to find the most suitable design of the fine-positioner is finally presented which, together with numerical methods for optimisation, yields the final design of the fine-positioner.

The work completed on control of the soft robotic manipulator to be used as the fine-positioner is described in Chapter 5. The derivation of new, closed-form solutions to the full kinematics of the robot assuming a set of deformation modes is presented first. The work conducted on mechanical modelling is then summarised. Finally, the derivation of closed-loop control laws based on this mechanical model is presented for planar operation of a segment of the fine-positioner, laying the foundation for the development of general control laws.

In Chapter 6, the work conducted on the analysis and development of the gross-positioner, deployment mechanism, and feedback system is described. The fabrication of the different systems in the complete robotic system for on-wing inspections is also described in Chapter 6 , particularly in terms of fine-positioner and gross-positioner. Research completed on path planning, which can be used for navigation of the robot inside the engine, is also presented. Finally, the proposed integration and expected operation of the complete on-wing inspection system is outlined.

All considerations related to the inspection in the reference case defined in subsection 1.1.3 to guide and illustrate the development efforts in this work are presented in Chapter 7. The inspection requirements and selection of the most suitable technique and probe are introduced first. The study of the inspection in both planar and 3D case is then described. Finally, the resulting expected performance of the inspection is reported.

Concluding remarks summarising the most relevant parts of the research reported in this thesis are presented in Chapter 8. Future work to complete and translate the robotic system proposed for on-wing inspections of jet engines to industry is also outlined in Chapter 8.

### 1.6 List of publications

The main parts of the research reported in this thesis have been published in the following set of publications.

- A. Garriga-Casanovas and F. Rodriguez y Baena. Complete follow-the-leader kinematics using concentric tube robots. International Journal of Robotics Research, 37.1, pp. 197222, 2018.
- A. Garriga-Casanovas, I. Collison, and F. Rodriguez y Baena. Towards a Common Framework for the Design of Soft Robotic Manipulators with Fluidic Actuation. Soft Robotics, 5.5, pp. 622-649, 2018.
- F. Liu, A. Garriga-Casanovas, R. Secoli, and F. Rodriguez y Baena. Fast and Adaptive Fractal Tree-Based Path Planning for Programmable Bevel Tip Steerable Needles. Robotics and Automation Letters, 1.2, pp. 601-608, 2016. © 2016 IEEE.
- A. Garriga-Casanovas, A. A. M. Faudzi, T. Hiramitsu, F. Rodriguez y Baena, and K. Suzumori. Multifilament Pneumatic Artificial Muscles to Mimic the Human Neck. IEEE International Conference on Robotics and Biomimetics, 2017.
- A. Garriga-Casanovas and F. Rodriguez y Baena. Kinematics of Continuum Robots with Constant Curvature Bending and Extension Capabilities. Journal of Mechanisms and Robotics, 11.1, 011010, 2018.
- A. Garriga-Casanovas and F. Rodriguez y Baena. Manipulator, Patent Application Number 1812408.1. Patent Application, 2018.


## Chapter 2

## Literature Review on Miniature Robotic Manipulators

A myriad of robotic manipulators and similar devices have been proposed in the literature. The robotic manipulators and devices relevant to the insertion and deployment of probes on-wing are reviewed in this Chapter. The scope of the review and the division of existing devices into a set of categories are first described in section 2.1. The review of devices separated into six different categories is reported in sections 2.2 to 2.8 . The most promising devices selected for the fine-positioner and gross-positioner are finally presented in section 2.9.

### 2.1 Preliminary considerations and classification

The aim of the review is to identify the most promising concepts to satisfy the needs of the finepositioner and gross-positioner, outlined in the previous chapter. Even though the devices in this review are predominantly robotic, the review is general and considers all types of devices. In addition, the review is not limited to devices conceived as manipulators, but includes all devices that could be used or adapted as manipulators, or that could be combined with other devices to make a manipulator, or that could be relevant to create a fine-positioner or a grosspositioner.

The requirements for the fine-positioner and gross-positioner outlined in the previous chapter are similar to the requirements for operations in minimally invasive surgery (MIS). As a consequence, a noticeable part of the devices in this review are originally conceived for medical applications.

In the review, passive devices that do not offer any active DOFs are first briefly described in section 2.2. Devices that can provide some mobility in the form of 1 or 2 DOFs are then briefly presented in section 2.3. Then, devices that can be used to achieve 5 or 6 DOFs are reviewed in more detail sections 2.4 to 2.8 , which represent the core of the review.

There is a significant number of concepts in the literature that can lead to devices with 5 or 6 DOFs, and these present very different characteristics. To avoid having to consider the suitability of each device individually, it is helpful to classify them into into a set of categories that are relevant to the aim of the review, so that general features corresponding to each category can be extracted, enabling a more general analysis. The classification proposed in this work is based on the actuation, as it is considered to be the most determining factor. This results into five categories of devices that can achieve 5 or 6 DOFs: tendon-driven devices with multiple segments, concentric tube robots, devices actuated by a pressurised fluid, devices with electromagnetic actuation, and devices actuated by shape-memory alloys (SMA).

### 2.2 Passive devices

### 2.2.1 Description and review

Passive devices refer to all rigid tools or flexible tools that can be used to insert and deploy probes in situ. The simplest and most common tool used for on-wing inspections is a rigid stick with the probe mounted at the distal end. Rigid tools with more complex geometries to avoid obstacles and bring probes to a desired location also exist, which are similar to that conceptually illustrated in Figure 2.1 (left). These can be used in on-wing inspections, and they can be created by bending a rod or tube to the required shape.


Figure 2.1: Illustrative example of a rigid tool with a complex pre-defined geometry to insert and deploy any end-effector attached at the tip (left), and sword tool developed by WesDyne Sweden AB (right), with a probe mounted on it including all the wires for the probe. Image on the right courtesy of WesDyne Sweden AB.

Flexible tools to bring probes to locations that cannot be reached with rigid tools have also been proposed. These flexible tools are typically designed to bend in a pre-defined manner when in contact with structures in the environment, which serve to guide the tool. The most prominent concept is a tool resembling a sword developed by WesDyne Sweden AB (Taby, Sweden) [10], shown in Figure 2.1 (right). This is a slender tool designed to bend with a low stiffness in a given direction, but to present a high stiffness in the orthogonal directions. Thus, it can deform but it generally remains in a plane. It is therefore suitable for narrow environments where contact forces deform and guide the tool, but where it is necessary for the tool to remain in a given plane.

### 2.2.2 Analysis, discussion and applicability

In general, these passive tools offer 0 active DOFs by definition. Thus, the pose at the distal end of the tool is determined by the pose at the proximal end, and any external wrenches acting on the tool. This means that, for rigid tools, the pose of the end-effector can be controlled by imposing the pose at the proximal end. For flexible tools, the end-effector pose depends on the forces from the environment, so it is necessary to know the effect of the environment on the tool to insert or deploy any probe on the end-effector.

Passive tools are simple, reliable, low cost, and can be developed in a short time. In general, the applicability of passive tools for on-wing inspections is difficult to define $a$ priori, as it depends
on the specific geometry of the obstacles in each application, and on the target end-effector pose relative to the access point. Thus, it needs to be considered on a case by case basis. Nonetheless, passive tools are typically suitable to insert and deploy probes in locations with relatively easy access, commonly those with direct line of sight or accessible via a single turn with significant room to manoeuvre and proximity to the access point. In addition, passive tools that are flexible can reach locations accessible via a narrow conducts that can guide a flexible tool without openings that require steering the tool. In general, passive tools cannot not advance along routes with significant lengths involving multiple turns.

Passive tools are therefore not suitable for the reference on-wing inspection case defined in subsection 1.1.3. The access route shown in Figure 1.3 is long and presents multiple turns to reach the engine region of interest, so a rigid tool cannot be inserted through it, and a typical flexible tool cannot perform the first 90 degree turn in an open chamber.

### 2.3 Low mobility devices

Low mobility devices comprise all existing devices designed to actively provide 1 or 2 DOFs, or to adopt or grow to a pre-defined shape when desired. The majority of low mobility devices are borescope-type devices, such as that shown in Figure 2.2 (centre), but there are also devices that can transition between a limp state and a rigid state, such as that shown in Figure 2.2 (left), and robots that can grow to advance in a desired direction [11].

### 2.3.1 Borescope-type devices

## Description and review

Borescope-type devices, such as that shown in Figure 2.2 (centre), generally consist of a slender structure with a circular cross section of nearly constant diameter. These devices have an active segment capable of 1 or 2 DOFs at the distal part, and a passive body that is typically flexible. The distal part can generally bend and is actuated by a set of 2,3 or 4 tendons that


Figure 2.2: (left) PretzelFlex ${ }^{T M}$ which is a device that can transition between limp state and rigid state with a pre-defined shape (image courtesy of Surgical Innovations ltd); (centre) borescope-type device (image courtesy of Olympus - copyright remains the property of Olympus); and (right) diagram of a standard borescope-type device.
are distributed circumferentially, and are routed through the body of the device to an actuation box at the base. The tendons force the distal segment to bend when tensioned, as illustrated in Figure 2.2 (right). Controlling the tension or displacement in the different tendons allows controlling the magnitude and direction of bending of the distal end. This provides 1 DOF in devices with 2 tendons, which bend in a plane, and 2 DOFs in devices with 3 or 4 tendons, which can bend in space. The end-effector is typically attached at the distal end of the device, and the passive, flexible body serves to insert the device in long routes.

## Analysis, discussion and applicability

Borescope-type devices offer some mobility of the end-effector, but cannot be used to position it to any desired pose in space since they only offer a maximum of 2 DOFs. The insertion of the borescope and the rotation at the proximal end can be used in practice to provide additional DOFs. However, these additional DOFs are generally not reliable for accurate positioning of a probe in in situ inspections. Reaction forces from the environment can be used to aid in the mobility, such as forces applied on the distal end by pressing it against a concave corner of a component. In this manner, it is possible to constrain certain displacement DOFs and use the available control to modify the orientation of the device. However, operations involving reaction forces to aid in mobility are typically strongly dependent on the operator skill and are
unreliable.

Borescope-type devices are therefore generally difficult to use alone for the deployment of probes on-wing since they do not have sufficient DOFs. Borescope-type devices are generally only suitable to deploy probes in cases where the pose at the proximal end of the distal, active segment of the device is controllable and known, and the desired deployment location can be reached by simply bending the distal end. However, these cases are not frequent in onwing inspections. The use of borescope-type devices in terms of NDE is primarily for visual inspections, where the exact positioning of the camera is not crucial, and only the orientation of the camera's field of view is relevant, which can be provided by the 2 DOFs of the device. Borescopes for visual inspections are available from companies such as GE or Olympus (Tokyo, Japan), as the one shown in Figure 2.2 (centre).

Borescope-type devices can be used as gross-positioners in some cases to insert an end-effector or another tool for manipulation at their distal end. However, the viability as gross-positioners is also difficult to define a priori. Borescope-type devices can advance in cluttered environments found in on-wing inspections by using the active distal part to steer, and using the environment to support the flexible body, which can then keep the device advancing. However, they cannot manoeuvre in open spaces, they can present difficulty to follow through sharp turns, and friction can prevent their advancement in long routes with with curvature variations. Thus, the specific application of borescope-type devices for the insertion of tools must be considered individually in each case.

In the reference inspection case defined in subsection 1.1.3, borescope-type devices are not suitable since they cannot be used to accurately deploy a probe. In addition, they cannot be used as gross-positioners either as they can generally not perform the first 90 degree turn in open space.

### 2.3.2 Shape locking devices

## Description and review

The main device among those that can transition between a pre-defined shape and a limp state is the PretzelFlex ${ }^{T M}$ by Surgical Innovations ltd (Leeds, UK) [12], shown in Figure 2.2 (left). It consists of a structure divided into rigid elements that are linked together by an elastic material. The structure has a cable routed through that, when tensioned, forces the individual elements in contact, which makes the structure adopt a pre-defined geometry and hold it with a relatively high stiffness. When the cable is not tensioned, the device presents a limp state.

Alternative shape locking devices can be conceived, for example using SMAs that can return to a pre-defined geometry by the application of heat. However, the strain that can be recovered using SMAs is limited [13]. Thus, alternatives to the PretzelFlex ${ }^{T M}$ can generally not be used as manipulators.

## Analysis, discussion and applicability

Shape locking devices can only be in a limp state or in a rigid state with a pre-defined geometry, as that shown in Figure 2.2 (left). In this regard, the pose at the distal end is either determined by the external forces in the limp state, or fully specified by the pre-defined shape of the device and the pose at the proximal end, as in rigid devices. Shape locking devices therefore have 0 DOF in general. In cases where the transition between limp state and rigid state can be controlled, they can be considered to have 1 DOF.

The applicability of shape locking devices in terms of on-wing inspections is generally limited to cases where the pose at the proximal end of the device can be controlled exactly, so that the probe at the distal end can be deployed accurately. These cases are generally limited to inspections in locations near the access point, or where the proximal end of the shape locking device can be inserted with a rigid tool. In this regard, shape locking devices can be considered an augmentation of rigid tools that enables reaching additional locations inside
the engine thanks to the change in shape between limp state and rigid state. However, shape locking devices cannot manoeuvre to negotiate obstacles, and thus their applicability in on-wing inspections is limited.

In the particular reference case defined in subsection 1.1.3, shape locking devices are not suitable since the access route requires manoeuvring. In addition, a shape locking device over 1 m long is difficult to create in practice. The use of a shape locking device to act as a fine-positioner is not suitable, since these cannot provide 5 or 6 DOFs.

### 2.3.3 Vine-like robot

## Description and review

Vine-like robots are a singular type of robot that can grow with a desired shape while maintaining a constant diameter [11], as shown in Figure 2.3 (A, B, C, D). These robots consist of a tubular membrane made of a soft plastic that can be pressurised. The membrane is initially stored in a reel at the base of the robot. This is arranged in such a manner that, as the tubular membrane is pressurised and the material is released, this emerges in a process known as eversion 2.3, shown in Figure 2.3 (A, B, C, D). In this manner, the robot can grow, and the material is mechanically fed to the distal end through the eversion process.

Vine-like robots can steer as they grow thanks to a set of latches at their sides, which can release an additional amount of membrane that is initially pinched, as shown in Figure 2.3 (E). This works by lengthening the membrane at one side of the robot when the latch is activated, making the robot steer. The release of material from a latch, however, is permanent, and thus the steering of the robot is permanent. In this regard, once the robot has grown a given length to a given shape, it cannot reposition its distal end as it does not have active DOFs.


Figure 2.3: Concept of vine-like robot and implementation as it grows using emersion (A, B, C, D). Steering mechanisms of vine-like robots using a set of latches that pinch material and can be released when desired (E). Images reproduced from [11] with permission of the rights holder, the American Association for the Advancement of Science.

## Analysis, discussion and applicability

Vine-like robots are applicable as gross-positioners as they can advance following complex routes in a relatively accurate follow-the-leader motion while maintaining a constant diameter. In addition, thanks to the eversion mechanism, they create no friction as they advance, and they can adapt to the environment, passing through small openings. However, they are not suitable as fine-positioners as they do not offer active DOFs.

Vine-like robots could be used as gross-positioners in the reference on-wing inspection case defined in subsection 1.1.3. However, they are considered to require a fine-positioner to accurately deploy any probe, which needs to be mounted at the distal end of the vine-like robot. Moreover, the steering capability and accuracy of vine-like robots is limited as it depends on a set of latches. Thus, they could present difficulties to reach the point of entry into the chamber with the HPC discs. It should also be noted that vine-like robots were proposed during the third year of the work reported in this thesis, hence they were not considered in the initial selection of the devices to be developed in the work.


Figure 2.4: Typical tendon-driven robot composed of two segments with a flexible backbone and cables distributed circumferentially (left) (image taken from [14], © 2003 IEEE); and possible arrangements of cables in the cross-section of tendon-driven devices (right) (image taken from [15]).

### 2.4 Tendon-driven devices with multiple segments

### 2.4.1 Description and review

Tendon-driven devices correspond to all devices actuated by means of a set of cables that can be tensioned to move and control the robot. The most common layout consists on a slender structure acting as a backbone, which can be flexible or articulated, and which has a set of cables distributed circumferentially, as shown in Figure 2.4 (left). The cables can be arranged in different manners, as shown in Figure 2.4 (right), but in general they serve to create a deflection on the backbone structure when tensioned, allowing control of the device. The borescopetype devices presented in section 2.3 .1 can be considered to be a simple type of tendon-driven device, with one segment that can deflect in any direction providing 2 DOFs. However, tendondriven devices can be composed of multiple segments stacked serially, allowing multiple DOFs, typically with 2 DOFs per segment. In this section, tendon-driven devices composed of multiple segments or capable of more than 2 DOFs are reviewed.

Tendon-driven devices are the most popular type of snake-robot, and they have existed for decades. Pioneering work started in the late 1960s [16], as shown in Figure 2.5 (a), it followed in subsequent decades [17], with robots such as that shown in 2.5 (b), and it continues in more recent years, e.g. $[15,18,19]$. The devices proposed in the literature range broadly in size and mobility, from 3 mm OD catheters with 3 DOF [20] to 100 mm OD hyper-redundant robots [21], as shown in Figure 2.5 (c) and (d), respectively.


Figure 2.5: Illustrative examples of a broad range of tendon-driven robots corresponding to (a) a device proposed in the late 1960s (image taken from [16]), (b) a robot developed in the 1990s (image courtesy of Victor Andersen, originally published in ASME Transactions, and reproduced from the original publication [17]), (c) a recent 3 mm OD catheter with 3 DOFs (image taken from [20], © 2013 IEEE), and (d) a 100 mm OD manipulator with 32 DOFs (image taken from [21]), © 1999 IEEE.

The use of a flexible rod as backbone is popular in tendon-driven robots, and relevant examples of such devices are [14], shown in Figure 2.4 (left), and [22], shown in Figure 2.6 (left). These robots offer 2 DOFs per segment of the robot, they can be miniaturised, and are well-suited to MIS. However, the force they can support in miniature size is limited.

To improve the force of tendon-driven robots with a flexible rod as backbone, an improved robot concept is proposed in [23-25], known as the distal dextrous unit (DDU), shown in Figure 2.6 (right). The main particularity of this design is that, instead of using cables as tendons, it uses microtubes made of nitinol arranged in a co-located manner (Figure 2.4 (right)), which can transmit both tension and compression forces. Thus, to generate bending in a segment of the robot as that shown in Figure 2.6 (centre) and (right), the DDU employs tension in the microtube corresponding to one side of the cross-section, and compression in the microtubes at the opposite side $[26,27]$. As a result, the DDU can support 1 N forces with a design that is 4 mm OD. The existing prototype [28-30], shown in Figure 2.6 (centre), is composed of 2 segments, and provides 4 DOFs.

One general limitation in the use of a flexible rod as backbone is the limited support it provides to torsional forces. This issue can be addressed by using a notched structure as backbone, and this has been used in designs from Stanford University [31,32], Johns Hopkins University [33-35], or Habrin Institute of Technology [36, 37], shown in Figure 2.7 (left), (centre), and (right), as some relevant examples. The issue with torsion is also identified in a robot being


Figure 2.6: Tendon-driven robots with a flexible rod as backbone, which correspond to a medical device initially developed by Hansen Medical Inc. (left) (image taken from [22]), a prototype of the DDU (centre) (image taken from [25], © 2004 IEEE), and the concept for the DDU (right) (image taken from [26], © 2006 IEEE).


Figure 2.7: Tendon-driven robots with a notched structure as backbone from Stanford University (left) (image taken from [31], © 2008 IEEE), Johns Hopkins University (centre) (image taken from [33], © 2011 IEEE), and Habrin Institute of Technology (right) (image taken from [36]).
developed at Nottingham University [38,39], where the use of a notched structure as backbone has been recently adopted. This robot is relevant as it is aimed at in situ inspections and repairs [40], and has a significant number of DOFs that provide it with hyperredundancy [41]. The design of this robot and work published to date in terms of analysis and control [38, 42] are also in line with other devices introduced above, e.g. [27,31,32].

Alternative designs of flexible backbones also exist. The most relevant are a design with a backbone made of a pressurized air tube [43], which provides more adaptability, a design with an extensible backbone that provides one additional DOF per segment [44], and a design with segments made of a granular medium that can be jammed with the application of vacuum [45], which can support significant loads when jammed.

Tendon-driven robots with backbones made of rigid links and articulated joints also exist.


Figure 2.8: Tendon-driven robots with backbones made of rigid links and articulated joints corresponding to ViaCath design (a) and (b) (images taken from [47], © 2007 IEEE), and EndoWrist (c) and (d) (images taken from [48, 49], respectively).

Prominent ones are devices developed for MIS such as the Endowrist [46], by Intuitive Surgical (Sunnyvale, USA), and Viacath [47], initially developed by Hansen Medical (Mountain View, USA), shown in Figure 2.8 (a) and (b), respectively. These two devices provided the foundation for the current EndoWrist [48, 49], shown in Figure 2.8 (c) and (d). Rigid links are also included in some tendon-driven robots developed by the company OC Robotics [50] (Bristol, UK), currently part of GE, although the company also has robots with flexible elements [51,52].

### 2.4.2 Analysis, discussion and applicability

The tendon-driven devices introduced in previous paragraphs are generally composed of segments that provide 1 or 2 DOFs, each actuated with 3 or 4 cables. Multiple segments are then stacked serially to create a manipulator. The cables actuating the distal segments are routed through the proximal segments, which leads to a certain coupling in the tension and displacement applied to control the different cables, and complicates the analysis and control of the robots. This complex analysis and control was already identified in [14], and is considered in the majority of devices presented in previous paragraphs. Reference work for the quasistatic analysis and control of tendon-driven robots was developed at Stanford University and is presented in [31,32], and more recent work on static and dynamics in the general case is reported in [18].

Tendon-driven robots are generally capable of 6 DOF or more and can have a diameter of a few millimetres, e.g. [20, 32, 48]. Moreover, their design generally offers a working channel to accommodate other instruments, and they can support payloads of near 100 g with diameters of less than $1 \mathrm{~cm}[33,39]$, which makes them well-suited devices as fine-positioners. It should
be noted, however, that tendon-driven robots require an accurate control of the tension in the cables for accurate positioning, which complicates coupling to other devices. The use of tendondriven robots as gross-positioners is more difficult due to the fact that they are complicated to control when interacting with the environment, despite recent progress [53]. In addition, they are difficult to create with long lengths, high mobility, and small diameters (one of the most slender devices manufactured is [41]).

In the reference on-wing inspection case previously defined in subsection 1.1.3, a gross-positioner over 1 m long and with a 6 mm OD is required, which is difficult to achieve with tendon-driven robots. A tendon-driven fine-positioner would be an interesting option if it was possible to accurately control the tension in the cables. However, the cables of the fine-positioner must be routed through a long gross-positioner, which introduces friction, elasticity, uncertainty and potentially a certain degree of slack in the system. This complicates significantly the control of any tendon-driven fine-positioner in practice, compromising their viability.

### 2.4.3 Singular devices

Two other tendon-driven robots exist, which differ significantly from the other designs and need to be reviewed separately. The first is the FLEX System [54,55], which is composed of two rigid tubes with a high number of articulations arranged concentrically. The outer tube has three cables threaded through and equally spaced circumferentially, whereas the inner tube has one central cable threaded through [56]. Thus, tensioning and releasing the cables of the outer and inner tube allows advancing them reciprocally in a follow-the-leader motion. The three cables of the outer tube allow steering the distal section of the robot to select the desired path $[57,58]$. The FLEX System is attractive for operation in cluttered environments like a jet engine, especially to be used as a complete robot that acts as an accurate gross-positioner and does not require a fine-positioner. However, the existing design is 300 mm long and 10 mm OD, and the required length extension or diameter reduction for use in the reference case defined in subsection 1.1.3 are not possible, as confirmed in correspondence with the inventor of the device.


Figure 2.9: Singular tendon-driven robots corresponding to the FLEX System (left) (image taken from [56]), and an interlaced robot consisting of two conventional tendon-driven robots arranged concentrically (right), courtesy of [59].

The second singular robot is the combination of two standard tendon-driven robots with flexible backbones arranged concentrically [59]. The robot is designed for a reciprocal actuation of the two composing devices, such that at each instant of time one of them is maintained fixed and thus used to preserve the shape of the robot, while the other advances one section and steers. In this manner, the robot can advance in follow-the-leader motion. The existing prototype, however, presents a significant diameter and a limited length, which make it unsuitable as a gross-positioner in the reference on-wing inspection case defined in the previous subsection 1.1.3. In addition, it requires relatively accurate control of the tension in the cables as in standard tendon-driven robots, which makes it difficult to use as a fine-positioner.

### 2.5 Concentric tube robots

### 2.5.1 Description and review

Concentric tube robots consist of a set of precurved, super-elastic tubes arranged concentrically, as shown in Figure 2.10 (left) and (right). The geometry of the robot is thus determined by the elastic equilibrium of the tubes that compose it. The control of the relative insertion and rotation of the tubes (typically with an actuation box at the robot's proximal end) enables control of the robot's motion, generally with 2 DOFs associated to each tube. It should be noted


Figure 2.10: Sketch of a general CTR composed of three tubes illustrating the concept (left), and example of CTR robot in practice (right). The right image is taken from [60], © 2009 IEEE.
that the motion achievable by a specific robot depends on its design in terms of precurvature and stiffness of the tubes that comprise it.

CTRs were initially proposed over a decade ago [61], [62], and since then research on different aspects of CTRs has been reported in the literature. The mechanical analysis of these robots is well established, with traditional approaches assuming no external loads and no friction, such as in [63] and [64], and subsequent studies considering external forces, as in [65] and [66], and also including friction between tubes, with [67]. As a result, accurate control of the robots is possible ( [68], [69]), and stable paths can be planned ( [60], [70], [71]). In addition, feedback systems based on Fibre-Bragg Gratings (FBG) have been proposed and incorporated into the robots ( [72]), enabling closed-loop control with proprioceptive sensing. All this established research has allowed applications in MIS, including [73-77], which showcase the capabilities of CTRs.

One of the main attractions of concentric tubes robots is the kinematics that they can achieve. CTRs are capable of follow-the-leader motion, as well as moving in general directions oblique to their centreline, which allows repositioning and compensating for external forces. The known capabilities for follow-the-leader motion at the start of the work reported in this thesis, however, were limited to robot designs composed of piecewise constant curvature tubes, and it was unknown whether CTRs were capable of follow-the-leader motion in other designs and trajectories. Finding the full capabilities for follow-the-leader motion is particularly relevant to determine the applicability of these robots to operate in cluttered environments like a jet


Figure 2.11: Example of CTR composed of two tubes with non-annular cross section to prevent relative rotation of the tubes and thus prevent snap-through instability. Image taken from [79].
engine.

The practical application of concentric tubes robots can be limited by an instability known as snap-through [78], which is inherent of these robots. Such instability is a consequence of the finite torsional stiffness of the tubes. It occurs when a variation in the torsional deformation of the tubes can lead to a lower energy state of the overall robot, at which point the tubes adopt the new configuration abruptly, changing their torsional deformation and thus the shape of the robot. This can make the robot unstable at certain configurations. An analytical solution predicting the snap-through instability exists for CTRs with tubes with constant curvature $[69,78]$, which indicates that the unstable configurations increase with robot length and with the curvature of the tubes. However, general solutions are not available.

The use of tubes with non-annular cross section, as shown in Figure 2.11, has been proposed to overcome the snap-through instability [79, 80], where the non-annular cross section prevents any relative rotation of the tubes, and thus any snap-through. The main disadvantage of employing non-annular cross section is that each tube only provides 1 DOF, which implies either a lower mobility or greater number of tubes required, and therefore greater diameter, for an equivalent capacity. At larger diameters, the risk of kinking is higher, and outer tubes have significant stiffness, which dominate the behaviour of the robot and can be undesirable for robot mobility.

Nonetheless, CTRs with non-annular cross section can achieve practically any desired length without instabilities.

### 2.5.2 Analysis, discussion and applicability

In general, CTRs can offer 6 DOFs or more, and typically have millimetric diameters. This, together with the ability to move in a follow-the-leader manner, makes CTRs well suited for on-wing operations. They could be used as gross-positioners, as fine-positioners, or as complete robots that combine both. The main disadvantage of standard CTRs with annular cross section is the snap-through instability, which can limit their length and maximum curvature. However, the general capabilities for follow-the-leader and corresponding snap-through were unknown at the start of this work. In addition, CTRs with non-annular cross section can be used to overcome the snap-through instability in cases where significant length is required, at the expense of mobility.

In the reference on-wing inspection case previously defined in subsection 1.1.3, CTRs could be considered as both gross-positioners or fine-positioners a priori. In the case of using a CTR as fine-positioner only, the tubes of the robot need to be routed through the gross-positioner to an actuation box outside the engine, where the insertion and rotation of the tubes is controlled. In the case of using CTRs as gross-positioners, a non-annular cross section may be necessary to prevent snap-through considering the required length of over 1 m . The final design and viability of CTRs, however, depends on the full capabilities for follow-the-leader achievable, and on the corresponding effects of the snap-through instability, which need to be studied in detail.

### 2.6 Fluidic actuation devices

Fluidic actuation devices include all manipulators and similar devices that rely on a pressurised fluid to actuate and control them. An important part of fluidic actuation devices are soft robots actuated by a pressurised fluid. These are devices made of soft materials and with easily


Figure 2.12: Examples of devices with fluidic actuation corresponding to (a) manipulator from Festo (Sankt Ingbert, Germany) (image taken from [81], © 2014 IEEE), (b) soft robot segment from Lyon University (image taken from [82]), (c) bending segment from Harvard University (image taken from [83], © 2014 IEEE), and (d) soft robotic manipulator from the Massachusetts Institute of Technology (image taken from [84]).
deformable structures, which comprise a set of chambers that can be pressurized to achieve structural deflection, and thus generate motion [85]. The field of soft robotics has received significant attention in recent years [86, 87], and a myriad of soft robots with fluidic actuation have been proposed, such as those shown in Figure 2.12. As a consequence, an important part of the devices in this section are soft robots with fluidic actuation.

The fluidic actuation devices relevant to the aims of this review generally need to be capable of bending or articulating to provide DOFs and navigate inside cluttered environments. This bending can generally be achieved in two elementary ways: with an extension in the part of the structure that is pressurised, or with a contraction in the part of the structure that is pressurised. Fluidic actuation devices can then be divided into two subcategories: extending devices and contracting devices. This division is further elucidated in Chapter 4.

Extending devices with fluidic actuation are the most popular. The most common layout consists on a deformable structure with one or multiple chambers, an inextensile region at one side of the chamber, and a region that can easily extend at the opposite side [89, 89-97], three examples of which are shown in Figure 2.13. Thus, the structure tends to bend when a chamber is pressurised, which provides 1 DOF associated to the pressurised chamber. Combining two or three chambers in a structural segment, this can provide various DOFs such as that shown in Figure 2.13 (c), and stacking multiple segments serially can lead to a robotic manipulator,


Figure 2.13: Extending devices corresponding to (a) a segment with one chamber and an inextensible layer at one side that produces bending when pressurised (image taken from [88]), (b) a prototype of a similar device (image taken from [89]), and (c) a segment with three chambers capable of bending in any direction in space (image taken from [90]).


Figure 2.14: Sketch of segment of FMA (left) and detail of outer wall of FMA made of rubber with circumferential fibres (centre) (images taken from [98], © 1992 IEEE), and robotic manipulator made of two FMA segments (right) (image taken from [99]).
such as [84] shown in Figure 2.12 (d).
A pioneering and prominent soft robot with fluidic actuation designed to work as an extending device is the flexible micro-actuator (FMA) [98,100,101]. The concept for a segment of the FMA is shown in Figure 2.14 (left). It consists on a cylinder with three longitudinal chambers that can be pressurised independently. The structure is made of rubber, and it has circumferential fibres on the outer wall, shown in Figure 2.14 (centre), which prevent it from expanding radially while allowing it to extend longitudinally. Thus, a differential pressure in the chambers leads to bending, whereas an increased pressure in all chambers leads to extension. As a result, a segment of FMA can bend in any direction and also extend, providing 3 DOFs. A manipulator can then be created by stacking multiple segments serially, as shown in Figure 2.14 (right).

The FMA has been applied to make robotic fingers [102], legs [103], manipulators [99], snake-


Figure 2.15: CAD of a segment of Stiff-Flop in frontal section view (left) and top view (centre) (images taken from [113], © 2015 IEEE), and prototype of segment of Stiff-Flop (right) (image taken from [115]).
robots locomoting in pipes [104, 105], and colonoscopy instruments [106], with sizes ranging between 1 mm OD and 20 mm OD [101]. In addition, the FMA inspired work in multiple similar devices [107-111]. A general drawback of all these devices is the limited force they can support. A robot concept aimed to address this issue is the Stiff-Flop robot [112, 113], shown in Figure 2.15, which is a robotic manipulator aimed at MIS $[114,115]$. It has a similar layout as the FMA, with three chambers per segment, but also includes a granular jamming element that can make a segment stiff when activated [116], as shown in Figure 2.15. However, the Stiff-Flop concept only offers higher force when jammed, but not when moving its segments to position its distal end, which limits its advantages. In addition, both the Stiff-flop and the FMA are designed to have a constant cross section where the area of the pressurised chambers is relatively low. This can limit their force, as elucidated in the study in Chapter 4, and is an area of potential improvement.

Another layout of extending devices consists on three extensible tubes that can be pressurised, arranged in parallel between two platforms, as shown in the examples in Figure 2.16. Thus, a differential pressure in the tubes generates bending, whereas an increased pressure leads to expansion. These designs can be considered equivalent to a scaled-up version of the FMA, since they comprise three parallel chambers that are designed to extend. These devices have been proposed to create robotic manipulators such as [117-121], shown in Figure 2.16 (a)-(e). More recently, improved versions have also been reported in the literature [122], where a tendon is included inside each extensible tube to control contraction of the tube and thus improve accuracy of the manipulator. In general, however, the size of these devices is a few centimetres in diameter, and miniaturisation tends to lead to a design equivalent to the FMA layout in


Figure 2.16: Examples of designs consisting on three extensible tubes arranged in parallel between two platforms, corresponding to the design of a robot known as Amadeus (a), (b) (images taken from [117,118], respectively, © 1997 IEEE and © 2001 IEEE), a robotic manipulator (c) (image taken from [119], © 2001 IEEE), a prototype of a segment designed to provide bending (d) (image taken from [120], © 2004 IEEE), and a prototype of slender segment design capable of bending and extending (e) (image courtesy of [121], © 2012 IEEE).
terms of principle of operation.

Contracting devices with fluidic actuation also exist. These generally rely on pneumatic artificial muscles (PAMs), which are elongated flexible structures with a bladder designed to expand radially when pressurised, and as a consequence to produce a longitudinal contraction [123,124]. In the majority of cases, a segment of a contracting device consists on 3 or 4 PAMs arranged in parallel and attached to a flexible structure like a rod or a notched cylinder [125-133], as shown in Figure 2.17 (left) and (right). Thus, when one or more of the PAMs are pressurised, these contract, making the device bend. Multiple segments can be stacked serially to create a manipulator such as that shown in Figure 2.17 (left). Contracting devices with other layouts also exist, such as that shown in Figure 2.17 (centre), which consists on a set of balloons attached between two sides of articulated joints [134]. Thus, when the balloons are pressurised, they expand radially, creating tension in their structure that actuates the joint.

### 2.6.1 Analysis, discussion and applicability

Devices with fluidic actuation are generally robust, compliant, and miniaturisable to diameters of a few millimetres thanks to their design simplicity. They can provide multiple DOFs, and manipulators with 5 or 6 DOFs exist [100,129]. In addition, they are versatile in design, so they can accommodate payloads in their structure, and they can be coupled to other devices, since


Figure 2.17: Contracting devices with fluidic actuation corresponding to prototype of manipulator with three segments (left) (image taken from [129], © 2015 IEEE), concept of design made of articulated joints and balloons that can be pressurised (centre) (image taken from [134]), and CAD of device made of four PAMs arranged in parallel inside a notched cylinder (right) (image taken from [131], © 2015 IEEE).
they only require a set of tubes delivering pressure for actuation and control. Moreover, they can collapse on themselves with the application of vacuum, which enables insertion through very confined spaces. Both extending and contracting devices exist, although their relative advantages and disadvantages are currently not well established. Some new insights are presented in the study in Chapter 4.

The main limitation in the devices introduced in the previous subsection is that they have low force, which limits the applicability in miniature size. Even in one of the most relevant existing designs, the FMA, the force that one segment can support with a 6 mm OD is only a few grams. Work to explore improvements on the design of the robots exist [135-137], but in general their force remains low. Stiff-Flop is the only design that can support significant forces when it is fixed, but it loses the force when it needs to move. Another general limitation of devices with fluidic actuation, particularly of soft robots, is their difficult control, which limits their accuracy.

Devices with fluidic actuation can therefore be suitable as fine-positioners. In particular, designs similar to the FMA can be used to create manipulators with 5 or 6 DOFs with diameters of a few millimetres. Devices with fluidic actuation, however, are not suitable as gross-positioners that need to manoeuvre in space as they generally cannot support significant forces to carry a fine-positioner and end-effector. Nonetheless, they can be considered as gross-positioners in very cluttered environments where they can be supported by the surrounding structures since


Figure 2.18: Robots with electromagnetic actuation composed of a set of rigid links and joints actuated by electric motors, corresponding to (a) a snake-robot for pipe inspection (image taken from [138], © 2010 IEEE), (b) a manipulator resembling an elephant trunk (image taken from [139]), (c) a versatile snake-robot (image taken from [140]), and (d) a manipulator for MIS (image taken from [141], © 2011 IEEE).
they are compliant and thus adapt to complex geometries.

In the reference inspection case defined in subsection 1.1.3, devices with fluidic actuation and particularly FMA-type robots could be suitable solutions as fine-positioners. They can meet the size and mobility requirements, they are easily attachable to any gross-positioner, and they can incorporate the payload and other systems. However, they cannot be used as gross-positioners since they cannot navigate the open parts of the access route.

### 2.7 Electromagnetic actuation devices

### 2.7.1 Description and review

Electromagnetic actuation devices comprise all devices that rely on electromagnetic forces for actuation. Solutions involving electromagnetic fields external to a device, such as [8, 142], are not included since they are not viable inside jet engines. The devices in this category are devices with embedded electromagnetic actuation, chiefly in the form of electromagnetic motors.

The most common design consists on a set of rigid links and articulated joints stacked serially, with electric motors embedded in the links that actuate the joints. Thus, robotic manipulators with multiple DOFs can be created, as well as hyper-redundant robots with the ability of self-locomotion. Various examples of such robots exist, including a manipulator resembling an


Figure 2.19: Design of i-snake robot with 6 DOFs (a), CAD rendering of a segment of i-snake (b), and detail of a universal joint of i-snake (c). Images courtesy of [146, 147], © 2011 and 2012 IEEE, respectively.
elephant trunk [139], a snake-robot for pipe inspection [138], a robotic manipulator for MIS named i-snake [141], a water-proof snake-robot design [143], and a versatile robot capable of self-locomotion in difficult terrain [140, 144, 145]. The most representative of these are shown in Figure 2.18 (a-d).

From these robots, the most relevant to this review is the i-snake [146]. It has a 13 mm OD, and comprises a set of rigid links and articulated joints with 1 or 2 DOFs [147], as shown in Figure 2.19 (a) and (c). Each link incorporates the motors for joint actuation, as shown in Figure 2.19 (b), as well as micro-intertial sensors, which provide feedback of the robot state for control [148, 149].

### 2.7.2 Analysis, discussion and applicability

The devices with electromagnetic actuation introduced in the previous subsection are generally modular and can provide 6 DOFs or more. They are generally made of joints that can rotate 45 degrees or more, and they can offer a working channel to incorporate payload, as in [146]. However, the existing devices have a significant diameter, and miniaturisation is generally difficult since the reduction in motor torque is significantly greater than the weight reduction. The existing robot with the smallest diameter is the i-snake, but its outer diameter is still 13 mm , and with this size it can barely support its own weight in motion against gravity for designs with 5 or 6 DOFs. Thus, diameter reduction is considered practically impossible, as
confirmed with researchers that led the i-snake project.

The modularity and DOFs provided by devices with electromagnetic actuation implies that they could be used as gross-positioners in some scenarios. However, these devices are generally made of rigid links, which limits their adaptability and manoeuvrability, and they have diameters of over 1 cm , so they can only be used in applications with significant room. Devices with electromagnetic electromagnetic actuation could also be used as fine-positioners in some cases since they can provide 6 DOFs, they can be easily coupled to a gross-positioner, and they can incorporate payload. However, their diameter is significant, so they are only viable in applications where the access route has an opening of well over 1 cm .

In the reference on-wing inspection case previously defined in subsection 1.1.3, devices with electromagnetic actuation are not suitable as fine-positioners nor as gross-positioners since the entry hole has a 6 mm diameter. Diameter reductions of existing designs such as the i-snake to 6 mm OD while maintaining sufficient mobility and capability of carrying some payload are considered practically inviable.

### 2.7.3 Singular devices

A set of singular devices with electromagnetic actuation that do not rely on motors to create a manipulator also exist. These are briefly reviewed in this subsection.

The first is a snake-robot, shown in Figure 2.20 (c), which is capable of swimming and was developed in the LAMPETRA project [150]. It consists on a set of links and articulated joints, and employs pairs of magnets in each joint, the relative orientation of which can be modified, to generate actuation. However, miniaturisation of this concept is not considered to be possible as it offers relatively limited specific force. The device is designed to operate in water with its weight supported by forces, but is generally not applicable as a gross-positioner or finepositioner.

The second is an inspection robot developed by Alstom Inspection Robotics (currently part of GE Inspections) [151, 153], shown in Figure 2.20 (a), (b). It is a robot that can be inserted


Figure 2.20: Singular robots with electromagnetic actuation corresponding to a robot developed to assemble and move with magnetic wheels once inserted into the region of interest (a) and (b) (images taken from [151]), a snake-robot capable of swimming developed in the LAMPETRA project (c) (image taken from [150], © 2012 IEEE), and an electroactive gripper (d) (image courtesy of Yoseph Bar-Cohen, JPL/Caltech/NASA, [152]).
through 15 mm diameter holes, and folds once inside using electric motors, becoming a vehicle with magnetic wheels that can move attached to ferromagnetic components to perform inspections. This robot concept, however, is not applicable to in situ inspections of machinery that includes few ferromagnetic components, such as a jet engine. In addition, it is not suitable for navigation in the environments considered in this work, which involve advancing in cluttered regions and performing turns in open spaces that require manoeuvring, where a gross-positioner and fine-positioner is a more appropriate concept.

The last type of singular devices are those using electro-active materials to generate actuation [152], as exemplified by the gripper shown in Figure 2.20 (d). These devices rely on the piezoelectric [154], dielectric [155, 156] , and electrostrictive [157] properties of certain polymers in order to generate actuation. These technologies, however, are still at early stages, and are focussed on the development of single actuators rather than the design of complete robotic devices. In addition, most of these technologies require high voltages to generate actuation with usable strokes, and these voltages cannot be used inside explosive environments like a jet engine [158]. As a result, devices with electro-active materials are not considered to be practical for the creation of a gross-positioner or fine-positioner for in in situ inspections.


Figure 2.21: Schematic of a segment of a typical device with SMA actuation (left) (image taken from [164], © 1998 IEEE), prototype of miniature manipulator with SMA actuation (centre) (image taken from [165]), and another example of miniature manipulator with SMA actuation (right) (image taken from [166]).

### 2.8 Shape memory alloy actuation devices

### 2.8.1 Description and review

Shape memory alloys (SMAs) are materials capable of recovering a predetermined geometry when subject to temperature variations [159]. This property can be exploited to generate actuation [160], and it has been implemented in manipulators and similar devices. The set of snake-robots and similar devices that rely on SMAs for actuation and control comprise this last category.

The force and capability of recovering a memorised shape provided by SMAs under temperature increases is significant [161]. Instead, the performance in the inverse process, i.e. shape recovery when cooled, is lower [162], and cannot be reliably applied to the actuation of robotic devices. As a consequence, SMAs are generally employed either antagonistically or with elastic restoring, so that the SMA only provides actuation in one direction. The former is the most common option in robotic devices since it allows greater controllability and higher force, as in [163]. Thus, the majority of devices in this section use antagonistic SMAs.

The design layout of snake-robots and similar devices actuated by SMAs generally consists on a slender backbone, which can be either flexible [165] or rigid [167], with three SMA elements distributed circumferentially around it, e.g. [164], as shown in Figure 2.21 (left). The SMA elements are typically helical, as also shown in Figure 2.21. This is because SMAs tend to have a relatively low usable strain [160], so to generate a sufficient stroke for actuation, a
helical geometry is required. The helical SMA elements are generally designed to contract when heated. Thus, they pull between the ends of the backbone when activated to generate bending. A segment with three or four SMA elements can thus be controlled to bend in any direction, providing 2 DOFs.

Multiple segments with 2 DOFs can be stacked serially to create a manipulator with 6 DOFs or more, as exemplified in Figure 2.22 (b). In manipulators, the heat is typically generated by Joule effect through the SMA and dissipated by convection. Thus, only a set of wires connected to each SMA element are required to actuate and control these devices. The layout of a manipulator made of multiple 2 DOF segments is similar to that of conventional tendondriven robots. However, in SMA, only wires transmitting electricity are required, instead of tendons transmitting mechanical work.

Multiple examples of these devices with SMAs relevant to create a manipulator exist. Pioneering work dates back to over two decades ago [168]. Since then, a various devices have been created with sizes ranging from a few millimetres in diameter, e.g. [164-166, 169, 170], some of which are shown in Figure 2.21 and in Figure 2.22 (a) and (c), to over one centimetre OD [168], as shown in Figure 2.22 (b). The design layout in all these devices is like the one described in the previous two paragraphs.

### 2.8.2 Analysis, discussion and applicability

Devices with SMAs as actuation can create manipulators with multiple DOFs and a miniature diameter. The segments of these devices are typically capable of 45 degrees bending or more, they offer significant force to support their own weight and significant payload, and they are modular since they only require a set of wires to provide electricity, so they can be easily coupled to other devices. In addition, these devices offer a large working channel, so payload and instrumentation can be easily incorporated, as shown in Figure 2.22 (c). Furthermore, the design of devices with SMAs can be easily adapted in terms of length and diameter, which makes them highly versatile.


Figure 2.22: Prototype of active catheter actuated by SMA with 0.9 mm OD (a) (image taken from [170]); snake-robot composed of multiple segments actuated by SMAs (b) (image taken from the article [168] authored by S. Hirose, K. Ikuta and M. Tsukamoto, entitled "Development of a shape memory alloy actuator. Measurement of material characteristics and development of active endoscopes", and published at Advanced Robotics, copyright © Taylor Francis and Robotics Society of Japan, reprinted by permission of Taylor Francis Ltd, www.tandfonline.com on behalf of Taylor Francis and Robotics Society of Japan); and schematic of integrated manipulator with payload and instrumentation (c) (image taken from [164], © 1998 IEEE).

The main drawbacks of devices with SMAs are the difficult manufacturing and control. Work on manufacturing and material selection is reported in the literature [164, 168, 170]. However, it generally requires specialised equipment and extensive experience, as confirmed in discussion with experts in the field. Similarly, some work on control of SMA is published, e.g. [163,171174], but generally the control is complex and sensitive to factors such as the environment, the operation history and the manufacturing, which makes the practical implementation difficult. Moreover, the heat dissipation of SMAs needs to be carefully considered when operating in confined environments, as it can affect any inspection operation.

Devices with SMAs are therefore well suited as fine-positioners. However, in practice, their fabrication and control can be very difficult, and can require significant time and expertise to achieve the required performance for a fine-positioner. Devices with SMAs are generally not appropriate as low-mobility gross-positioners that deform when in contact with the environment since the SMAs can be delicate. However, they could also be considered as integrated robots including both gross-positioner and fine-positioner. The length, diameter, force and accuracy achievable depend on their fabrication and control, which needs to be studied in detail.

In the refeence on-wing inspection case previously defined in subsection 1.1.3, devices with SMAs could be considered as fine-positoiners. Still, the difficult fabrication at miniature size, and the complex control in a confined environment with potential temperature variations would need to be carefully considered as they could complicate significantly the development of such a fine-positioner. The creation of an integrated robot comprising fine-positioner and grosspositioner fully actuated by SMAs is not considered viable, since it would require a 6 mm OD and a length over 1 m , an aspect ratio that is beyond existing prototypes.

### 2.9 Selection

Considering the review presented in the previous sections, two robot concepts were selected as the most promising to be explored for the insertion and deployment of probes on-wing. These are concentric tube robots, and soft robots with fluidic actuation, with a layout similar to that of the FMA, which can serve as initial reference design.

Soft robots with fluidic actuation can have diameters of a few millimetres, they can include a working channel, and they offer a relatively modular design capable of achieving 6 DOF. Their actuation and control only require three miniature, flexible conducts per robot segment, which makes them versatile and compatible with any gross-positioner, as opposed to tendon-driven devices. Manufacturing of the soft robots is relatively simple and does not require specialised equipment or extensive experience, unlike in robots relying on SMAs. Moreover, these devices can be easily scaled and miniaturised maintaining sufficient force to support their weight and additional payload, unlike devices employing electric motors.

CTRs offer a millimetric diameter, a working channel, a mobility that can reach 6 DOFs or more, and the capability of navigating in cluttered environments and advancing in follow-theleader motion. In addition, they are simple in design and manufacturing. Thus, they can be considered as gross-positioners and as fine-positioners.

It should be noted that vine-like robots are also a relevant robot concept with potential as gross-positioners. However, they were initially proposed during the third year of the work
reported in this thesis, so they were not considered in the initial literature review. In addition, they have limited accuracy and mobility, and they do not offer active DOFs. Thus, they are not explored further in this work.

### 2.10 Conclusions

Soft robots with fluidic actuation and CTRs are then the most promising concepts for this work. However, their capability of meeting the requirements described in Chapter 1 is unknown. Thus, research is necessary.

For CTRs, research on their kinematic capabilities is necessary to determine their full potential and applicability. This involves studying all possible design and control possibilities to achieve follow-the-leader motion. Research addressing this matter is presented in the next Chapter 3.

For soft robots with fluidic actuation, research on design and control is necessary to achieve devices with sufficient force and accuracy to deploy probes. The design of these devices is studied in Chapter 4 from a general perspective to find the most suitable design of a soft robotic manipulator. The work conducted on control of soft robotic manipulators with fluidic actuation is presented in Chapter 5.

## Chapter 3

## Concentric Tube Robots

The kinematic capabilities of CTRs are investigated in this chapter to find the complete follow-the-leader possibilities of CTRs. This allows for the determination of the general potential of these robots, and their suitability to on-wing operations. The research presented in this chapter is an edited version of that published in:

- A. Garriga-Casanovas and F. Rodriguez y Baena. Complete follow-the-leader kinematics using concentric tube robots. International Journal of Robotics Research, 37.1, pp. 197222, 2018.

The chapter is structured as follows. Initial considerations for the research are first introduced in section 3.1. The equations governing the behaviour of a general CTR are derived in section 3.2. The study of follow-the-leader motion is presented section 3.3, where a new, closed-form solution corresponding to the set of trajectories traceable in a follow-the-leader configuration is derived. In section 3.4, additional maneuvers of interest that can be drawn from the analysis of follow-the-leader motion are described. The effects of torsion of the tubes composing a CTR is considered in section 3.5, where a closed-form solution to the tubes' torsion in a two-tube configuration is derived. Lastly, a case study involving simulation and experiment is presented in section 3.6, together with the corresponding results. Conclusions from the research are drawn in section 3.7, where the suitability of CTRs for on-wing operations is also discussed.

### 3.1 Introduction

As previously mentioned in Chapter 2, CTRs are attractive to operate in cluttered environments, such as those found in MIS or on-wing inspections, since they offer the capability of moving in follow-the-leader motion, as well as moving in directions oblique to their centerline. This, combined with a small diameter similar to that of a surgical needle and a simple mechanical design requiring a small number of parts, makes CTRs a robot concept with high potential, as confirmed by the significant interest that CTRs are receiving in recent years.

Current work, however, is focused on the exploitation of robot designs composed of piecewise constant curvature tubes. This is predominantly due to the fact that the general follow-theleader capabilities of CTRs, and the corresponding robot behaviour in terms of torsion of the tubes, were unknown. A first study of other trajectories traceable in a follow-the-leader configuration was published in [175], at the same time that the research presented in this chapter was being conducted. However, [175] only offers solutions for some pre-determined and specific robot configurations, but it does not allow a general study, leaving the general follow-the-leader possibilities as an open question. A general study is therefore required to determine the complete follow-the-leader possibilities that CTRs can offer based on existing models, and thus establish the full potential of these devices.

In this chapter, the full follow-the-leader capabilities achievable with CTR are analysed, and a closed-form solution to the complete set of trajectories that can be followed in a follow-theleader configuration under the assumption of no axial torsion of the tubes is presented. The validity of such an assumption is subsequently considered in the set of trajectories discovered, which allows for the selection of a case study to showcase the work. The objective of this work is similar to that in [175], and therefore some parallels are inevitably present. However, the research presented here was conducted independently and prior to the publication of [175], which favored the formulation of a different approach that enables a general study and solution. The work presented here clarifies a currently open question, and broadens the potential of CTRs with a new set of trajectories that can be exploited in, for instance, MIS or on-wing inspections. A crucial part of the approach adopted here is a specific robot description, which allows for
a geometrical interpretation of the conditions for follow-the-leader motion. This enables the formulation of a treatable problem and the derivation of a general, closed-form solution under the assumption of no axial torsion of the tubes.

The formulation of the analysis developed in this work considers robots comprising any number of tubes with any desired pre-curvature and stiffness, and any possible control strategy in terms of rotation and insertion of the tubes. Discontinuities in robot curvature, which are inherent in telescopic robot deployment as well as in unconventional robot designs, are also considered in the study. Thus, the analysis of follow-the-leader motion reported here, together with the corresponding solutions, is completely general. In addition, the geometrical interpretation of follow-the-leader motion proposed in this work provides conceptual insight into these kinematics, which is useful for the future development of path planning and closed-loop control algorithms, and for the application of these robots to practical scenarios, where disturbances are present.

The strategy employed in this work to study the follow-the-leader possibilities, which involves first studying the problem assuming no torsion and then determining the validity of the assumption, is advantageous from both a theoretical and practical perspective. It establishes first the full capabilities under the assumption of no torsion, and then it enables selecting the admissible deviation in terms of torsion of the tubes. In this manner, useful trajectories with a small deviation away from an ideal follow-the-leader configuration are not discarded, which can be advantageous. Furthermore, since the admissible deviation in terms of torsion can be selected, it can be specified to be as close to zero as desired. Still, the design of CTRs accepting a relatively small deviation from follow-the-leader due to torsion is advisable, considering that it noticeably increases the number of feasible trajectories, and that in practice a certain degree of uncertainty generally exists in the predicted robot behavior. It should be noted that the focus here is on the deviation in terms local curvature from that corresponding to follow-the-leader motion, but this does not directly imply a specific deviation in task space. The relation between deviation in task space and local deviation due to torsion is illustrated with some simulations of relevant configurations, but the determination of the specific relation is a question beyond the scope of this present work. Interestingly, the analysis assuming no torsion is also applicable to
robot designs with non-annular cross sections, originally proposed in [80], by simply considering controls without relative rotation of the tubes.

In order to study the torsion of tubes and then conceive a case study to showcase this research, the general equilibrium of the robot is considered in the set of trajectories discovered. A closedform solution describing the torsion of the tubes along the arc length is obtained for two-tube robots with helical precurvatures, which represent the most relevant designs in the trajectories discovered. This solution then allows for the identification of the designs that guarantee that the torsion of the tubes is below a specified value. Interestingly, the torsional behavior is found to depend on two non-dimensional groups, which indicate that torsional deviation can be reduced by using helical tubes, the precurvatures of which have significantly different geometric torsion. These results are used to develop a case study involving simulation and experiment, where the tubes present a small torsional deformation and the robot maintains a near perfect follow-the-leader configuration, illustrating the capabilities described in this work.

The set of trajectories discovered in this work is non-trivial, and expands the currently known capabilities of CTRs. For robots composed of constant stiffness tubes, the corresponding robot designs required are found to be composed of tubes with precurvatures that are either helices or deformed helices with exponentially varying curvature magnitude. For robots with variable stiffness tubes, robot designs composed of tubes with more general geometries associated to the deformation of helices are found to be possible. Kinematic equivalences that can be exploited within the follow-the-leader set of trajectories are also extracted from the analysis. These include concatenation of segments of different trajectories, or the addition of idle tubes that become active once inserted.

Various maneuvers that combine follow-the-leader motion along a segment of the CTR with general displacements at its distal end, which do not correspond to follow-the-leader, are also distilled from the analysis. These maneuvers are aimed at applications where the robot endeffector is able work in a spacious cavity, which can only be accessed through a narrow path that requires follow-the-leader motion. Such situation is common in MIS and in on-wing inspections, where the kinematics identified here can offer a significant advantage. It should be noted that
some of these kinematic possibilities have been previously mentioned in the literature for robot designs composed of piecewise constant curvature tubes, e.g. [77], [74], [69]. In this work, these are generalised and integrated into the analysis developed here.

### 3.2 Governing equations

The relations that govern the behavior of a CTR are derived in this section. The analysis follows a similar approach to that in the established literature, and [69] is used as the main reference throughout the chapter to facilitate the reading. However, some variations on the analysis are introduced in order to adapt it to the aims of this work, with associated changes in nomenclature.

### 3.2.1 Problem Characterization

The problem description adopted in this work is crucial to allow the derivation of the solutions presented in the following sections. In this regard, a detailed characterization of the problem is presented in this subsection. The geometry of a tube, or a set of concentric tubes, is described by the curve corresponding to its centreline. Diameter variations are not expected, nor relevant to this study, and only the cross-sectional moment of inertia is necessary, as elucidated in the following subsection. Vectors, and in particular curvature, are expressed relying on Bishop reference frames ( [176]). In particular, a frame $W$ is defined as a Bishop frame corresponding to the final robot geometry, as initially proposed by [62], and a frame $F_{i}$ is defined as a frame materially attached to a tube $i$ that coincides with a Bishop frame associated to tube $i$ before undergoing structural deformation.

The following magnitudes are then used to characterise a concentric tube robot. The length of the relevant part of the robot, which generally corresponds to the inserted robot length, is denoted by $L$. The position along the arc length is represented by $s$, relative to the distal end and defined positive $s \in[0, L]$. An independent variable $t$, generally coinciding with time, is


Figure 3.1: Sketch of a general concentric tube robot composed of three tubes with relevant nomenclature definitions.
used to parametrise the evolution of the robotic system. The vector curvature of tube $i$ at cross section $s_{i}$ and instant $t$ is denoted by the first two components of $\mathbf{u}_{i}^{F_{i}\left(s_{i}\right)}\left(s_{i}, t\right)=\left[u_{i x}, u_{i y}, u_{i z}\right]^{T}$ , which is defined as the angular rate of increment of frame $F_{i}$ materially attached to tube $i$ respect to the arc length, and expressed in the same frame $F_{i}\left(s_{i}\right)$. The third component of $\mathbf{u}_{i}^{F_{i}\left(s_{i}\right)}\left(s_{i}, t\right)$ denotes the torsional deformation of tube $i$. Similarly, the first two components of $\mathbf{u}_{T}^{W(s)}(s, t)$ define as the curvature of the resulting robot in frame $W(s)$, while the third component of $\mathbf{u}_{T}^{W(s)}(s, t)$ is zero due to the definition of $W$. It should be noted that the vector curvature of a tube before and after applying external wrenches on it generally varies, so a circumflex is used to indicate the initial curvature $\hat{\mathbf{u}}_{i}^{F_{i}}\left(s_{i}\right)$. Since the initial geometry of a tube is described by the curve corresponding to its centreline, expressed in a Bishop frame, the third component of the initial curvature is zero by definition $\hat{u}_{i z}^{F_{i}}\left(s_{i}\right)=0$. The stiffness matrix corresponding to a tube $i$ is defined as

$$
\mathbf{k}_{i}=\left[\begin{array}{ccc}
E_{i x} I_{i x} & 0 & 0 \\
0 & E_{i y} I_{i y} & 0 \\
0 & 0 & J_{i} G_{i}
\end{array}\right]
$$

where $E$ is the Young modulus, and $I_{i x, y}$ is the cross-sectional moment of inertia in either direction $x$ or $y$. In this work, the tubes are assumed to have an annular cross section, since it allows relative rotation between the tubes, and it is therefore the most general case in terms
of follow-the-leader motion analysis. This implies $E_{i x} I_{i x}=E_{i y} I_{i y}$, and therefore the matrix $\mathbf{k}_{i}$ is independent of the Bishop frame used in the tube or robot description. The cross-sectional moment of inertia for tubes with annular cross section is

$$
\begin{equation*}
I_{i x}=\frac{\pi\left(d_{o}^{4}-d_{i}^{4}\right)}{64} \tag{3.1}
\end{equation*}
$$

where $d_{o}$ denotes the outer diameter of the tube and $d_{i}$ denotes the inner diameter. The length along the robot centreline between the distal end of tube $i$ and the robot's distal end is defined as $h_{i}$. At least one $h_{i}$ must be zero since the robot's distal end must comprise at least one tube, and here $h_{1}$ is chosen to be zero in a situation of ambiguity. The rotation angle between frame $F_{i}$ and frame $W$ is denoted by $\theta_{i}$. The scalar velocity at which the distal end of the robot advances through the workspace with respect to $t$ is represented by $v$. Finally, the internal moment vector associated with the resulting cross-sectional stress of tube $i$ is indicated as $\mathbf{m}_{i}^{F_{i}}$. A general CTR with some of the magnitudes here defined is illustrated in Figure 3.1.

From this problem description, the advantages of using Bishop frames ( [176]) are clear. First, Bishop frames are intrinsic reference frames with one component always parallel to the curve tangent vector, which is convenient considering that the vector curvature is orthogonal to the tube's centreline curve. In addition, they are defined in any curve that is sufficiently differentiable, even at points with zero curvature. Finally, for a tube with no axial torsion, the curvature along the tube can be transformed to another Bishop frame with a simple rotation that is constant along the entire tube.

### 3.2.2 Governing laws

The behavior of the robotic system is governed by three laws. First, an elastic constitutive law, which can be obtained following [69] as

$$
\begin{equation*}
\mathbf{m}_{i}^{F_{i}}=\mathbf{k}_{i}\left(\mathbf{u}_{i}^{F_{i}}-\hat{\mathbf{u}}_{i}^{F_{i}}\right) \tag{3.2}
\end{equation*}
$$

Second, a static equilibrium law (assuming a quasistatic operation of the robot), which can be written as

$$
\begin{equation*}
\sum_{i=1}^{n} \mathbf{m}_{i}^{W(s)}=0 \tag{3.3}
\end{equation*}
$$

Finally, a compatibility law (using a continuum mechanics description of matter), which translates into a condition that imposes a common final curvature to the tubes that compose a robot when arranged concentrically

$$
\begin{equation*}
\left.\mathbf{u}_{1}^{W(s)}\right|_{x, y}=\left.\mathbf{u}_{2}^{W(s)}\right|_{x, y}=\ldots=\left.\mathbf{u}_{T}^{W(s)}\right|_{x, y} \tag{3.4}
\end{equation*}
$$

which only applies to the $x, y$ components of $\mathbf{u}_{i}^{W(s)}$, as indicated by the subscripts $x, y$.

Assuming no external loads, and no axial torsion of the tubes, the combination of all three laws (3.2), (3.3), (3.4) determines the robot quasistatic model

$$
\begin{equation*}
\mathbf{u}_{T}^{W(s)}(s, t)=\left[\sum_{j=1}^{n} \mathbf{k}_{j}\right]^{-1} \sum_{i=1}^{n} \mathbf{R}\left(\theta_{i}(t)\right) \mathbf{k}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}\left(s-h_{i}(t)\right) \tag{3.5}
\end{equation*}
$$

where $h_{1}=0, n$ is the number of tubes comprising the robot, and

$$
\mathbf{R}\left(\theta_{i}(t)\right)=\left[\begin{array}{ccc}
\cos \left(\theta_{i}(t)\right) & -\sin \left(\theta_{i}(t)\right) & 0 \\
\sin \left(\theta_{i}(t)\right) & \cos \left(\theta_{i}(t)\right) & 0 \\
0 & 0 & 1
\end{array}\right]
$$

expressed in a Bishop frame corresponding to the final robot curvature with no axial torsion. The orientation of this final Bishop frame around the $z$ axis is defined by a desired arbitrary frame in a given cross section, e.g. the proximal end of the robot, and the corresponding extension to the entire curve of the robot centreline. As a consequence, rigid body rotations of the robot are represented by a simple rotation of all tubes with a common angular velocity. It should be noted that the composition $\left(s-h_{i}(t)\right)$ allows the evaluation of each tube's stiffness and initial curvature in a given cross section relative to the robot reference frame.

Equation (3.5) elucidates the fact that both the tubes and the robot's final curvature can be
expressed using a vector with only two components. However, in order to be consistent with literature, and to clarify the use of the assumption of no axial torsion, a three-dimensional vector is employed.

### 3.3 Follow-the-leader

Expression (3.5) describes all possible geometries that a concentric tube robot with design parameters $\mathbf{k}_{i}, \hat{\mathbf{u}}_{i}$ can achieve by relative rotation and insertion of the tubes that integrate it, and therefore the general movements it can perform. At each cross section, the possible robot curvature evolutions with time are given by the functions $\theta_{i}(t), h_{i}(t)$ for all tubes. And for a given instant in time, the shape of the continuum robot is determined by the curvature values along $s$.

In this section, the robot kinematics corresponding to follow-the-leader motion are studied. The condition for follow-the-leader motion is first elucidated in subsection 3.3.1. This condition is then imposed on the quasistatic model of a general CTR in subsection 3.3.2, yielding the vectorial equation that must be satisfied for a trajectory to be traceable in follow-the-leader motion. The complete solutions to this equation are then studied in subsection 3.3.3, leading to the complete set of trajectories where follow-the-leader is possible in subsection 3.3.4. It should be noted that the strategy of defining a kinematic condition for follow-the-leader motion and then imposing it on the robot model is similar to that proposed in [175]. However, the specific analysis is markedly different, which is a consequence of the fact that this research was conducted independently and prior to the publication of [175]. The different study in this thesis then leads to the new solutions derived in the following subsections.

### 3.3.1 General condition

Follow-the-leader motion requires the curve corresponding to the robot centreline to remain in a constant spatial curve, except for the differential segment that advances with a differential
of $t$. Thus, the curvature of the robot centreline must be constant for all spatial locations. Defining a magnitude $x$, which corresponds to spatial location in the workspace, the condition imposing curvature at each spatial location to remain constant can be expressed as

$$
\begin{equation*}
\mathbf{u}_{T}(x)=\text { constant } \forall x \in C \tag{3.6}
\end{equation*}
$$

where $C$ is the loci of the curve corresponding to the robot centreline. Considering that the robot curvature can be expressed as a function of $s$ and $t$, as described in the previous section, the expression of curvature at a spatial location can be differentiated. Since curvature must be constant at each spatial location as expressed in (3.6), the differentiation yields the condition for follow-the-leader in the robot segments with differentiable curvature as

$$
\begin{equation*}
-v \frac{\partial \mathbf{u}_{T}^{W(s)}}{\partial s}=\frac{\partial \mathbf{u}_{T}^{W(s)}}{\partial t} \forall s, t \tag{3.7}
\end{equation*}
$$

It should be noted that the time-dependant variables in $\mathbf{u}_{T}^{W(s)}$ are $\theta_{i}(t)$ and $h_{i}(t)$, and therefore the right hand side of (3.7) corresponds to $\frac{\partial \mathbf{u}_{T}^{W(s)}}{\partial \theta_{i}, h_{i} \ldots .} \frac{\partial \theta_{i}, h_{i} . . .}{\partial t}$ for all $i$. Condition (3.7) indicates that, in order to advance in a follow-the-leader configuration, the curvature of each cross section must pass to the immediate adjacent cross section towards the proximal end. In a reference frame positioned at the distal end of the robot, this motion resembles that of a wave without attenuation traveling towards the base of the CTR, as conceptually illustrated in Figure 3.2.

For robots with continuous $\frac{\partial \mathbf{u}_{T}^{W(s)}}{\partial s}$ and $\frac{\partial \mathbf{u}_{T}^{W(s)}}{\partial t}$, equation (3.7) is a necessary and sufficient condition for follow-the-leader motion. For robots with discontinuities in $\nabla \mathbf{u}_{T}^{W(s)}$, follow-the-leader motion is achieved if and only if the discontinuity step is finite, constant, and translating at velocity $v$ away from the distal end, and also equation (3.7) is satisfied in the segments of continuity. In other words, follow-the-leader requires the curvature discontinuity to remain constant in the given position relative to the workspace, and therefore it must distance away at rate $v$ from the robot distal end as the robot advances.

Interestingly, this condition for follow-the-leader motion is not only applicable to CTRs, but to any continuum robot. This condition indicates that the majority of continuum robots, such


Figure 3.2: Conceptual illustration of a curvature field corresponding to a follow-the-leader configuration. A vector of motion that satisfies follow-the-leader is indicated with a black arrow.
as standard tendon-driven robots or soft robotic manipulators, are not capable of follow-theleader motion. This is due to the fact that the majority of continuum robots are composed of a set of segments stacked serially, and the curvature along the arc length can only be selected by selecting of bending of each full robot segment. However, the follow-the-leader condition (3.7) requires the curvature at each cross section along the arc length to pass to the adjacent cross section towards the proximal end as the robot advances. In the majority of continuum robots, this is only possible by selecting all robot segments to have an equal curvature, which corresponds to a trivial case of a robot with a constant circumference arc geometry advancing in a circumferential trajectory. However, this configuration has little practical interest, and is equivalent to a rigid tool. Thus, follow-the-leader motion in standard continuum robots is not considered further.

### 3.3.2 Application to concentric tube robots

The imposition of condition (3.7) on the quasistatic model of the concentric tube robot (3.5) restricts the possible robot kinematics to those that correspond to perfect follow-the-leader motion (if any). This yields the condition that suffices for a trajectory to be traceable by a concentric tube robot in a follow-the-leader configuration

$$
\begin{equation*}
\sum_{i=1}^{n}\left[\mathbf{R}^{\prime}\left(\theta_{i}\right) \mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}\left(s-h_{i}\right) \dot{\theta}_{i}-\mathbf{R}\left(\theta_{i}\right) \frac{\partial}{\partial s}\left(\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}\left(s-h_{i}\right)\right) \dot{h}_{i}\right]=-v \sum_{i=1}^{n}\left[\mathbf{R}\left(\theta_{i}\right) \frac{\partial}{\partial s}\left(\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}\left(s-h_{i}\right)\right)\right] \forall s, t \tag{3.8}
\end{equation*}
$$

where

$$
\mathbf{R}^{\prime}\left(\theta_{i}(t)\right)=\left[\begin{array}{ccc}
-\sin \left(\theta_{i}(t)\right) & -\cos \left(\theta_{i}(t)\right) & 0 \\
\cos \left(\theta_{i}(t)\right) & -\sin \left(\theta_{i}(t)\right) & 0 \\
0 & 0 & 0
\end{array}\right]
$$

and

$$
\mathbf{P}_{i}=\left[\sum_{j=1}^{n} \mathbf{k}_{j}\right]^{-1} \mathbf{k}_{i}
$$

and both $\dot{\theta}_{i}$ and $\dot{h}_{i}$ are functions of time.

The variables $\dot{\theta}_{i}$ and $\dot{h}_{i}$ for all the tubes in a CTR are the control inputs available to control the robot. Mores specifically, $\dot{\theta}_{i}$ corresponds to the control of the rotation of each tube $i$ as a function of time, and $\dot{h}_{i}$ corresponds to the control of the insertion of each tube $i$. The dependence of $\dot{\theta}_{i}$ and $\dot{h}_{i}$ on $t$ is omitted in (3.8) and in the following equations for brevity, but both $\dot{\theta}_{i}$ and $\dot{h}_{i}$ should be considered to be functions of time in the entire presentation unless otherwise stated.

The rest of terms in (3.8) can be interpreted as follows. Matrix $\mathbf{P}_{i}$ is a diagonal matrix, and its three terms can be interpreted as the relative stiffness of tube $i$ in local directions $x, y, z$ with respect to the total stiffness of the robot in the same directions. The product $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}\left(s-h_{i}\right)$ corresponds to a vector that lies on the local $x, y$ plane in the robot cross section. Matrix $\mathbf{R}^{\prime}\left(\theta_{i}(t)\right)$ is the derivative with respect to $\theta_{i}$ of $\mathbf{R}\left(\theta_{i}(t)\right)$, which in turn corresponds to a rotation along the local tangential axis at each robot cross section. It should be noted that both $\mathbf{R}^{\prime}\left(\theta_{i}(t)\right)$
and $\mathbf{R}\left(\theta_{i}(t)\right)$ correspond to rotations along the local tangential axis, with a phase difference of $\frac{\pi}{2}$. Equation (3.8) then corresponds to a sum of vectors $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}\left(s-h_{i}\right)$ that lie on the local $x, y$ plane, and that are rotated along the local $z$ axis by matrices $\mathbf{R}^{\prime}\left(\theta_{i}(t)\right)$ and $\mathbf{R}\left(\theta_{i}(t)\right)$.

The curve describing trajectories where follow-the-leader is possible can be specified both by the corresponding $\mathbf{u}_{T}^{W}\left(s, t_{f}\right)$, which is parametrised by the arc length and evaluated at the time at the end of an insertion $t_{f}$, or by $\mathbf{u}_{T}^{W}(0, t)$, which parametrised by time and evaluated at the robot distal end $s=0$. Both expressions are equivalent in a follow-the-leader configuration. In this presentation, the expression $\mathbf{u}_{T}^{W}\left(s, t_{f}\right)$ is used for clarity of exposition.

The orientation at any robot cross section relative to the task space can be denoted by a rotation matrix $\mathbf{D}$, which is defined to have its $z$ component tangential to the centreline of the robot. The specific value of $\mathbf{D}$ at a given cross section $s$ and instant of time $t$ can be obtained by integrating the rotation determined by the local curvature $\mathbf{u}_{T}^{W}(s, t)$ along the robot arc length, which is a standard integration of a Bishop frame. The differential increment of $\mathbf{D}$ with respect to $s$ is determined by

$$
\frac{\partial \mathbf{D}}{\partial s}=-\left[\begin{array}{ccc}
-u_{T y} D_{13} & u_{T x} D_{13} & u_{T y} D_{11}-u_{T x} D_{12}  \tag{3.9}\\
-u_{T y} D_{23} & u_{T x} D_{23} & u_{T y} D_{21}-u_{T x} D_{22} \\
-u_{T y} D_{33} & u_{T x} D_{33} & u_{T y} D_{31}-u_{T x} D_{32}
\end{array}\right]
$$

where $D_{i j}$ denotes the individual components of $\mathbf{D}$ in row $i$ and column $j$. $\mathbf{D}$ can then be obtained from the integral equation

$$
\begin{equation*}
\mathbf{D}=\int_{L}^{L-s} \frac{\partial \mathbf{D}}{\partial s} d s \tag{3.10}
\end{equation*}
$$

It should be noted that integration of (3.10) generally needs to be solved numerically.

In concentric tube robots, $\frac{\partial \mathbf{u}_{T}^{W(s)}}{\partial s}$ and $\frac{\partial \mathbf{u}_{T}^{W(s)}}{\partial t}$ must be sectionally continuous since discontinuities can only be caused by either the end of one tube, or a locally non differentiable precurvature, both of which generate constant discontinuity steps. In this regard, the translation of discontinuity points towards the robot's proximal end at velocity $v$, and satisfaction of (3.8) in the
rest of the domain, are necessary and sufficient conditions for a trajectory to be traceable in a follow-the-leader configuration.

The complete solution to (3.8) therefore corresponds to the complete set of trajectories where follow-the-leader is possible under the assumption of no axial torsion. It should be noted that (3.8) is applicable to any robot design in terms of precurvatures, stiffness and number of tubes, for any possible control strategy. Thus, it represents a general condition for follow-the-leader motion.

The problem description employed in this work allows the derivation of a closed-form solution to (3.8). The key to such a solution is treating (3.8) from a vectorial perspective, rather than decoupling it into a system of individual differential equations. Considering that all terms in (3.8) either contain $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}$ or its derivative relative to $s$, and that $\mathbf{R}$ and $\mathbf{R}^{\prime}$ are closely related in terms of the rotations they represent, geometric relations simplify the study of (3.8). Such geometric interpretation also provides insight into the control inputs and geometries associated to follow-the-leader motion, and facilitates an intuitive interpretation of the follow-the-leader configuration. The rest of this section is dedicated to the solution of (3.8).

### 3.3.3 Solution cases

The approach adopted here to study the solution to (3.8) involves dividing the problem into cases of increasing complexity for clarity of exposition, as presented in this subsection. Cases with restrictions on the motions allowed with the tubes are considered first, serving as a foundation for the subsequent study of more general cases.

## Rotation only and different for each tube

Considering first a case where the rotation of the tubes is the only input allowed (equal insertion rate of all tubes), and considering that no groups of tubes are moving together, i.e. functions $\dot{\theta}_{i}(t)$ satisfy $\dot{\theta}_{i}(t) \neq \dot{\theta}_{j}(t) \forall i, j=1, \ldots, n$ over the course of an insertion, equation (3.8) simplifies
to

$$
\begin{equation*}
\sum_{i=1}^{n} \mathbf{R}^{\prime}\left(\theta_{i}\right) \mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}(s) \dot{\theta}_{i}=-v \sum_{i=1}^{n} \mathbf{R}\left(\theta_{i}\right) \frac{\partial}{\partial s}\left(\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}(s)\right) \tag{3.11}
\end{equation*}
$$

The possible solutions to (3.11) can be divided in two cases: the terms in (3.11) corresponding to each tube compensate so that their sum is null, which will be referred to as "compensating individually", or the terms in (3.11) from different tubes combine so that their sum is zero, which will be referred to as "compensating in conjunction".

In the case of compensating individually, (3.11) is particularised as

$$
\mathbf{R}^{\prime}\left(\theta_{i}\right) \mathbf{P}_{i}\left[\begin{array}{c}
\hat{u}_{i x}  \tag{3.12}\\
\hat{u}_{i y} \\
0
\end{array}\right](s) \dot{\theta}_{i}=-v \mathbf{R}\left(\theta_{i}\right) \frac{\partial}{\partial s}\left(\mathbf{P}_{i}\left[\begin{array}{c}
\hat{u}_{i x} \\
\hat{u}_{i y} \\
0
\end{array}\right](s)\right)
$$

which must be satisfied for all time. The only time-dependent terms are matrices $\mathbf{R}$ and $\mathbf{R}^{\prime}$ and $\dot{\theta}_{i}(t)$. Realizing that both $\mathbf{R}$ and $\mathbf{R}^{\prime}$ matrices represent a rotation of the $x, y$ components with a constant difference of $\frac{\pi}{2}$, and that component $z$ is not relevant here since the vector curvature always lies in the XY plane, equation (3.12) reduces to an ordinary differential equation (ODE) of the vector $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}$ with respect to $s$

$$
\mathbf{R}\left(\frac{\pi}{2}\right) \mathbf{P}_{i}\left[\begin{array}{c}
\hat{u}_{i x}  \tag{3.13}\\
\hat{u}_{i y} \\
0
\end{array}\right](s) \dot{\theta}_{i}=-v \frac{\partial}{\partial s}\left(\mathbf{P}_{i}\left[\begin{array}{c}
\hat{u}_{i x} \\
\hat{u}_{i y} \\
0
\end{array}\right](s)\right)
$$

The solution to (3.13) can be easily obtained realizing that it imposes $\frac{\partial}{\partial s}\left(\mathbf{P}_{i} \hat{\mathbf{u}}_{i}\right)$ to be orthogonal to $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}$. Specifically, the magnitude $\left\|\mathbf{P}_{i} \hat{\mathbf{u}}_{i}\right\|$ must be constant, and the direction of the vector $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}$ corresponding to tube $i$ in a Bishop frame must rotate in the local XY plane at a constant rate with respect to the arc length. In addition, $\dot{\theta}_{i}(t)$ must be constant and proportional to $v$ in order to satisfy (3.13) for all $t$. This applies to any individual tube, and therefore configurations corresponding to robots composed of individual tubes that satisfy (3.13) correspond to trajectories that satisfy follow-the-leader.

Follow-the-leader motion using only relative rotation of the tubes and compensating individually is therefore possible, and the resulting trajectories expressed as resulting geometry of the robot at the time corresponding to the end of an insertion are

$$
\mathbf{u}_{T}^{W}\left(s, t_{f}\right)=\sum_{i=1}^{n}\left[\begin{array}{c}
\left\|\mathbf{P}_{i} \hat{\mathbf{u}}_{i}\right\| \cos \left(w_{i} s+\phi_{i}\right)  \tag{3.14}\\
\left\|\mathbf{P}_{i} \hat{\mathbf{u}}_{i}\right\| \sin \left(w_{i} s+\phi_{i}\right) \\
0
\end{array}\right]
$$

where $w_{i}$ is a variable that can be selected in the robot design as desired and corresponds to the initial torsion of tube $i$, and $\phi_{i}$ is a parameter related to the relative rotation of the tubes at the proximal end of the trajectory, which can also be chosen freely. It should be noted that the trajectories (3.14) are parametrised by $s$ to elucidate that they correspond to a set of geometric curves, although the trajectories could also be parametrised by $t$, since both of these are equivalent in a follow-the-leader configuration.

In the case of compensating in conjunction, solutions to (3.11) can also be derived in specific configurations. Rewriting (3.11) relying on the fact that $\mathbf{R}^{\prime}\left(\theta_{i}\right)=\mathbf{R}\left(\theta_{i}\right) \mathbf{R}\left(\frac{\pi}{2}\right)$ yields

$$
\begin{equation*}
\sum_{i=1}^{n} \mathbf{R}\left(\theta_{i}\right)\left[\mathbf{R}\left(\frac{\pi}{2}\right) \mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}(s) \dot{\theta}_{i}+v \frac{\partial}{\partial s}\left(\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}(s)\right)\right]=0 \forall s, t \tag{3.15}
\end{equation*}
$$

The terms in (3.15) are a sum of planar vectors in each cross section, and thus vectors $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}$ and $\partial \mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}} / \partial s$ from a set of two or more tubes, defined as tubes $i \in l$, can be combined so that their sum is zero. For $\dot{\theta}_{i}(t) \neq \dot{\theta}_{j}(t)$, however, the relative orientation between vectors corresponding to different tubes changes with $t$. For follow-the-leader to be satisfied, these vectors need to compensate in conjunction at each instant of time and each cross section so that their sum is null, despite variations in their relative orientation from different evolutions of $\mathbf{R}\left(\theta_{i}(t)\right)$.

The magnitude of the vectors in (3.15) is either fixed, for $\partial \mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}} / \partial s$, or determined by $\dot{\theta}_{i}$, for $\dot{\theta}_{i} \mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}$. The $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W}$ of tubes $i \in l$ are generally not aligned and therefore the $\dot{\theta}_{i}$ determine the value of the sum of vectors corresponding to tubes $i \in l$ in (3.15) in each cross section. The $\dot{\theta}_{i}(t)$ can thus be selected so that the terms from a set of tubes $l$ compensate in conjunction
despite variations from different $\mathbf{R}\left(\theta_{i}(t)\right)$, with the values of $\dot{\theta}_{i}(t)$ chosen at each instant of time for each arrangement of vectors $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}$ and $\partial \mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}} / \partial s$. This enables a set of specific solutions, which are discussed in two further cases for clarity of exposition.

Considering first a case with $l=2$, two variables $\dot{\theta}_{1}$ and $\dot{\theta}_{2}$ are available to be selected at each instant of time. Specific $\dot{\theta}_{1}(t)$ and $\dot{\theta}_{2}(t)$ can thus be used to satisfy the two scalar equations implied by (3.15) for a given cross section, and all $t$. The functions $\dot{\theta}_{1}(t), \dot{\theta}_{2}(t)$ to satisfy (3.15) are unique for a given set of $\mathbf{P}_{1} \hat{\mathbf{u}}_{1}^{F_{1}}, \partial \mathbf{P}_{1} \hat{\mathbf{u}}_{1}^{F_{1}} / \partial s, \mathbf{P}_{2} \hat{\mathbf{u}}_{2}^{F_{2}}, \partial \mathbf{P}_{2} \hat{\mathbf{u}}_{2}^{F_{2}} / \partial s$ with a specific relative orientation and relative magnitude between these vectors, corresponding to a given cross section. The $\dot{\theta}_{i}(t)$, however, are common for all cross sections. Follow-the-leader is then satisfied if and only if the arrangement of vectors $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W_{i}}$ and $\partial \mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W_{i}} / \partial s$, in terms of relative orientation and relative magnitude of these vectors for tubes $i \in l$, is proportional in all cross sections along the arc length.

Two possible design solutions then arise: (i) the $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W}$ and $\partial \mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W} / \partial s$ of tubes $i \in l$ remain proportional along the arc length with an equal orientation, or (ii) the $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W}$ and $\partial \mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W} / \partial s$ remain proportional along the arc length, with an absolute orientation of all vectors corresponding to $i \in l$ rotating at a constant rate along the arc length when expressed in a Bishop frame. In solution (i), the $\left\|\mathbf{P}_{i} \hat{\mathbf{u}}_{i}\right\|$ of each tube must vary exponentially in order to maintain the proportionality between $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W}$ and $\partial \mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W} / \partial s$, and vector orientation must remain constant. In addition, the exponential increase rate must be equal for tubes $i \in l$ in order to maintain the proportionality between all vectors corresponding to tubes $i \in l$. In solution (ii), $\left\|\mathbf{P}_{i} \hat{\mathbf{u}}_{i}\right\|$ must also vary exponentially at an equal rate for all tubes $i \in l$, and in addition the direction of $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W}$ and $\partial \mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W} / \partial s$ must rotate along the arc length at an equal rate for tubes $i \in l$ in order to maintain proportionality.

Considering a general case with $l>2$, an equivalent analysis applies, although some specific differences are present. The number of variables $\dot{\theta}_{i}$ available in this case is $l$. This could suggest that condition (3.15) could be satisfied in $l / 2$ different cross sections (for even $l$ ) by selecting specific values of $\dot{\theta}_{i}$ at each instant of time. The design in terms of vectors $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W}$ and $\partial \mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W} / \partial s$ would then be freely selected at $l / 2$ cross sections, and designs with all other cross sections
proportional in terms of the $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W}$ and $\partial \mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W} / \partial s$ to any of the selected $l / 2$ cross sections, or linear combinations of them, would maintain follow-the-leader with the same common $\dot{\theta}_{i}$, as in the previous case for $l=2$. However, designs with vectors $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W}$ and $\partial \mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W} / \partial s$ proportional to the arrangement of these vectors in multiple cross sections are not possible. As described for the case $l=2$, proportionality in $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W}$ and $\partial \mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W} / \partial s$ implies an exponential variation in $\left\|\mathbf{P}_{i} \hat{\mathbf{u}}_{i}\right\|$. Thus, proportionality of vectors $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W}$ and $\partial \mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W} / \partial s$ for tubes $i \in l$ to any given cross section is propagated over all cross sections, and consequently all cross sections must be proportional to any given one. Therefore, also in the case $l>2$, the arrangement of $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W}$ and $\partial \mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W} / \partial s$ for tubes $i \in l$ in all cross sections must be proportional to a given cross section for follow-the-leader compensating in conjunction to be possible. The design of the tubes is then equivalent to that in the case $l=2$, with $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W}$ and $\partial \mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W} / \partial s$ for tubes $i \in l$ that must remain proportional along the arc length in terms of relative orientation and magnitude, and with an absolute orientation that must either be equal in all cross sections, or rotating at a constant rate along the arc length.

The trajectories that can be traced in a follow-the-leader configuration with robots comprising only a set of tubes $i \in l$ that compensate in conjunction must then correspond to a $\mathbf{u}_{T}^{W}\left(s, t_{f}\right)$ of either constant direction or constantly rotating direction, and magnitude varying exponentially. These trajectories are

$$
\mathbf{u}_{T}^{W}\left(s, t_{f}\right)=\sum_{i=1}^{l}\left[\begin{array}{c}
e^{\lambda s}\left\|\mathbf{P}_{i} \hat{\mathbf{u}}_{i}\right\| \cos \left(\rho s+\phi_{i}\right)  \tag{3.16}\\
e^{\lambda s}\left\|\mathbf{P}_{i} \hat{\mathbf{u}}_{i}\right\| \sin \left(\rho s+\phi_{i}\right) \\
0
\end{array}\right]
$$

where $\lambda$ is a parameter corresponding to the increase in curvature magnitude along the arc length, which can be selected with the tubes' design and is common for tubes $i \in l, \rho$ is a parameter corresponding to the geometric torsion of the tubes, also common for tubes $i \in l$, and $\phi_{i}$ is related to the tubes' orientation at the proximal end, as previously defined. Since $\rho$
and $\lambda$ are common for tubes $i \in l$, the trajectories (3.16) can also be expressed as

$$
\mathbf{u}_{T}^{W}\left(s, t_{f}\right)=\left[\begin{array}{c}
e^{\lambda s}\left\|\mathbf{u}_{R}\right\| \cos (\rho s+\nu)  \tag{3.17}\\
e^{\lambda s}\left\|\mathbf{u}_{R}\right\| \sin (\rho s+\nu) \\
0
\end{array}\right]
$$

where $\left\|\mathbf{u}_{R}\right\|$ is the curvature resulting from the interaction of tubes $i \in l$ at a given cross section $s=0$ and $t_{f}$, and $\nu$ is related to the robot orientation at the proximal end, and is analogous to $\phi_{i}$.

Compensating in conjunction requires at least two tubes in order to have two inputs $\dot{\theta}_{i}$ to satisfy the two components of (3.15). Configurations with additional tubes are also possible, and in these cases a degree of freedom appears for each additional tube. This does not expand the set of trajectories (3.17), but implies that a $\theta_{i}(t)$ can generally be freely selected for each additional tube, which can be exploited in additional maneuvers, described in section 3.4.

Follow-the-leader motion using only relative rotation of the tubes is thus possible both compensating individually and in conjunction. Condition (3.11) is a summation of terms corresponding to different tubes. Hence, any combination of solutions corresponding to a set of tubes compensating individually (3.14) and a set of tubes compensating in conjunction (3.17) must also satisfy (3.11). The resulting set of trajectories then is

$$
\mathbf{u}_{T}^{W}\left(s, t_{f}\right)=\sum_{i=1}^{n^{\prime}}\left[\begin{array}{c}
\left\|\mathbf{P}_{i} \hat{\mathbf{u}}_{i}\right\| \cos \left(w_{i} s+\phi_{i}\right)  \tag{3.18}\\
\left\|\mathbf{P}_{i} \hat{\mathbf{u}}_{i}\right\| \sin \left(w_{i} s+\phi_{i}\right) \\
0
\end{array}\right]+\sum_{j=1}^{g}\left[\begin{array}{c}
e^{\lambda_{j} s}\left\|\mathbf{u}_{R, j}\right\| \cos \left(\rho_{j} s+\nu_{j}\right) \\
e^{\lambda_{j} s}\left\|\mathbf{u}_{R, j}\right\| \sin \left(\rho_{j} s+\nu_{j}\right) \\
0
\end{array}\right]
$$

where $g$ is the number of sets of tubes that involve compensating in conjunction, and $n^{\prime}$ is the number of tubes that compensate individually.

As a particular solution in (3.18), the trajectory corresponding to a single tube being inserted is a helix relative to the workspace. In this case, the required tube precurvature is equal to the resulting trajectory, a configuration that corresponds to a common device, namely the corkscrew. It should be noted that the helix can be degenerated to a circumference arc, elucidating the
fact that this result is completely general.

## Different rotation and insertion for each tube

Considering now the case where any independent combination of insertion and rotation of the tubes as a function of time is allowed, but no groups of tubes move together, i.e. functions $\dot{\theta}_{i}(t)$ satisfy $\dot{\theta}_{i}(t) \neq \dot{\theta}_{j}(t) \forall i, j=1, \ldots, n$ over an insertion, the possible solutions to (3.8) can also be divided in two cases corresponding to the terms in (3.8) of each tube compensating individually or in conjunction.

In the case of compensating individually, condition (3.8) particularises to

$$
\begin{equation*}
\mathbf{R}^{\prime}\left(\theta_{i}\right) \mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}\left(s-h_{i}\right) \dot{\theta}_{i}-\mathbf{R}\left(\theta_{i}\right) \frac{\partial}{\partial s}\left(\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}\left(s-h_{i}\right)\right) \dot{h}_{i}=-v \mathbf{R}\left(\theta_{i}\right) \frac{\partial}{\partial s}\left(\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}\left(s-h_{i}\right)\right) \forall s, t, i \tag{3.19}
\end{equation*}
$$

Regrouping it, expression (3.19) can be rewritten as

$$
\begin{equation*}
\mathbf{R}^{\prime}\left(\theta_{i}\right) \mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}\left(s-h_{i}\right) \dot{\theta}_{i}=\left(\dot{h}_{i}-v\right) \mathbf{R}\left(\theta_{i}\right) \frac{\partial}{\partial s}\left(\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}\left(s-h_{i}\right)\right) \tag{3.20}
\end{equation*}
$$

which must also be satisfied for all $s, t, i$. Expression (3.20) simplifies the geometrical interpretation of the differential equation, elucidating the relation that must be satisfied between vector $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}$ and its derivative with respect to $s$.

Two different design possibilities in terms of precurvatures and stiffness of the tubes comprising the robot arise from equation (3.20), which depend on whether the magnitude of $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}=\mathbf{q}_{i}$ is designed to be constant or not.
(i) If $\left\|\mathbf{q}_{i}\right\|$ is constant, then the directions of $\mathbf{R} \frac{\partial}{\partial s} \mathbf{q}_{i}$ and $\mathbf{R}^{\prime} \cdot \mathbf{q}_{i}$ are parallel. This implies that there can be both $\dot{h}_{i}(t) \neq v, 0$ and $\dot{\theta}_{i}(t) \neq 0$ simultaneously. In this case, the solution of (3.20) has one degree of freedom to choose from, either $\dot{h}_{i}(t)$ or $\dot{\theta}_{i}(t)$. Regardless of the choice, provided that $\dot{h}_{i}(t) \neq v$, equation (3.20) represents an ODE analogous to that in the rotation only case,
since the difference between $\mathbf{R}_{i}$ and $\mathbf{R}_{i}^{\prime}$ is again constant and equal to $\frac{\pi}{2}$, yielding

$$
\begin{equation*}
\mathbf{R}\left(\frac{\pi}{2}\right) \mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}\left(s-h_{i}\right) \dot{\theta}_{i}=-\left(\dot{h}_{i}-v\right) \frac{\partial}{\partial s}\left(\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}\left(s-h_{i}\right)\right) \tag{3.21}
\end{equation*}
$$

The solution to (3.21) is, as in the previous case, a vector $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}$ of constant magnitude, and direction rotating in the intrinsic XY plane proportionally to the arc length. In equation (3.21), it is patent that the choice of $\dot{h}_{i}(t)$ is completely equivalent to the choice of $v$ and $\dot{\theta}_{i}(t)$, which determines the pace at which vector $\mathbf{q}_{i}$ rotates with the arc length. Hence, if $\left\|\mathbf{q}_{i}\right\|$ is constant, the follow-the-leader trajectories that can be obtained combining $\dot{h}_{i}(t)$ and $\dot{\theta}_{i}(t)$ are equivalent to those achievable using $\dot{\theta}(t)$ only. Naturally, this is only valid for the segment of the robot where the tube with $\dot{h}_{i}(t) \neq v, 0$ is present. The combination of $\dot{\theta}_{i}(t)$ and $\dot{h}_{i}(t)$ is only advantageous in a scenario where a variation of the relative insertion of a tube is desired. Such a maneuver does not increase the variety of single trajectories that can be traced in follow-theleader. However, it enables the linkage of some of these single trajectories, which can be useful in practical applications, as described in section 3.4. The satisfaction of (3.19) for a $t$ and any $s$ directly implies that (3.19) is satisfied for all $t$, since the vector $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}$ corresponding to each tube rotates along the arc length at a constant rate. Thus, the complete set of trajectories achievable for constant $\left\|\mathbf{q}_{i}\right\|$ are exactly equal as those in (3.18).
(ii) If $\left\|\mathbf{q}_{i}\right\|$ is not constant, then $\mathbf{R} \frac{\partial}{\partial s} \mathbf{q}_{i}$ generates a vector in a direction oblique to $\mathbf{R}^{\prime} \cdot \mathbf{q}_{i}$. Therefore, the only solution is $\dot{h}(t)=v, \dot{\theta}(t)=0$. This implies tube $i$ to be fixed with respect to the workspace, while the rest of the robot advances. Such a configuration may seem idle in terms of follow-the-leader kinematics as it does not contribute to the advancement of the robot. However, it shows that, once a tube has been inserted to some extent along the trajectory, it can be left fixed in that position while the rest of the robot continues forward, which is useful when linking trajectories composed of different numbers of tubes. It should be noted that a configuration with $\dot{h}_{i}(t)=v$ cannot be simultaneously adopted in all tubes since there must be at least one tube that advances the robot's distal end (functions $h(t)$ are defined nonnegative with respect to the robot's distal end). It is immediate to see that the solution identified for the design alternative (ii) holds for all times and cross sections.

In the case of compensating in conjunction, specific control inputs $\dot{\theta}_{i}$ and $\dot{h}_{i}$ together with specific designs can also satisfy (3.8). This can be elucidated by rewriting (3.8) using the definition $\mathbf{q}_{i}=\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}$ as

$$
\begin{equation*}
\sum_{i=1}^{n} \mathbf{R}\left(\theta_{i}\right)\left[\mathbf{R}\left(\frac{\pi}{2}\right) \mathbf{q}_{i}\left(s-h_{i}\right) \dot{\theta}_{i}+\left(v-\dot{h}_{i}\right) \frac{\partial}{\partial s}\left(\mathbf{q}_{i}\left(s-h_{i}\right)\right)\right]=0 \tag{3.22}
\end{equation*}
$$

which must hold for all $s, t$.

Equation (3.22) is a sum of planar vectors with a relative orientation that varies with $t$ due to the different $\mathbf{R}\left(\theta_{i}(t)\right)$ in different tubes. The magnitude of these vectors at each instant of time is determined by $\dot{\theta}_{i}$ for vectors $\dot{\theta}_{i} \mathbf{q}_{i}$, and by $\dot{h}_{i}$ for vectors $\dot{h}_{i} \partial \mathbf{q}_{i} / \partial s$. Thus, for a general design in a given cross section, $\dot{\theta}_{i}$ and $\dot{h}_{i}$ of a set of tubes $i \in l$ can be selected at each instant of time so that the sum of the corresponding terms in (3.22) is zero despite changes in relative orientation of the vectors.

The selection of $\dot{\theta}_{i}(t)$ and $\dot{h}_{i}(t)$ enables the satisfaction of (3.22) in a specific cross section. However, $\dot{\theta}_{i}(t)$ and $\dot{h}_{i}(t)$ affect all cross sections. For follow-the-leader to be satisfied in all cross sections, the arrangement of vectors $\mathbf{q}_{i}$ and $\partial \mathbf{q}_{i} / \partial s$ corresponding to tubes $i \in l$, in terms of relative orientation and relative magnitude of the vectors, must be proportional in all cross sections, in an equivalent manner as in the previous subsection. The corresponding design of the tubes is then equal to that in the previous subsection, with $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W}$ and $\partial \mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W} / \partial s$ for tubes $i \in l$ that must have a magnitude that varies exponentially along the arc length, and an absolute orientation either constant or rotating at a constant rate along the arc length. The trajectories that can be traced in a follow-the-leader configuration by compensating in conjunction using $\dot{\theta}_{i}$ and $\dot{h}_{i}$ are then equal to those in the previous subsection (3.17).

Compensating in conjunction involves two or more tubes. Configurations with two tubes lead to a robot with two degrees of freedom, as four inputs $\left(\dot{\theta}_{1}, \dot{\theta}_{2}, \dot{h}_{1}, \dot{h}_{2}\right)$ are available to satisfy the two equations implied by (3.22). Any additional tubes add two degrees of freedom per tube. Thus, even though the use of both $\dot{\theta}_{i}$ and $\dot{h}_{i}$ does not increase the follow-the-leader trajectories with respect to those traceable using $\dot{\theta}_{i}$ only, the use of both $\dot{\theta}_{i}$ and $\dot{h}_{i}$ provides additional
degrees of freedom. These degrees of freedom imply that either $\dot{\theta}_{i}, \dot{h}_{i}$, or a combination of them can be used to maintain follow-the-leader, as in the previous case involving tubes compensating individually with $\dot{\theta}_{i}$ and $\dot{h}_{i}$. As before, this applies to the region of robot that contains the tubes with $\dot{\theta}_{i}$ and $\dot{h}_{i}$. The exploitation of these kinematics combining $\dot{\theta}_{i}$ and $\dot{h}_{i}$ is described in section 3.4.

It should be noted that the trivial solution $\dot{h}(t)=v, \dot{\theta}(t)=0$ also satisfies (3.22) for any general design $\mathbf{q}_{i}$. As in the previous case, this solution does not contribute to the advancement of the robot in a follow-the-leader configuration, but it can be exploited in the additional kinematics described in section 3.4.

The discussion in the previous paragraphs for both configurations compensating individually or in conjunction shows that the use of the relative tube's insertion as control input $\dot{h}_{i}(t)$ does not contribute to the enhancement of the set of trajectories where follow-the-leader is possible. An alternative argument to discard relative tube insertion from contributing to follow-the-leader kinematics is that any positive $\dot{h}_{i}(t)$ motion prevents tube $i$ from remaining at the robot's distal end, and any negative $\dot{h}_{i}(t)$ implies a certain offset until the eventual instant of time when the tube becomes part of the distal end. Thus, a tube with $\dot{h}_{i}(t) \neq 0$ could only contribute to the distal ends kinematics during an instant of time. Nonetheless, the strategy of using $\dot{h}_{i}(t)=v$ remains useful for the linkage of trajectories achieved with different numbers of tubes, as previously mentioned.

In the case of compensating individually, the control input for each tube is also restricted by (3.20). In order to satisfy (3.20) at a given time instant, a specific tube geometry must be selected, as previously discussed. Once the geometry is specified, (3.20) imposes a constant relation between $\dot{\theta}(t), \dot{h}(t)$ and $v$ at each section for any time. Assuming constant stiffness of the tubes for simplicity, this relation can be written as

$$
\begin{equation*}
\dot{\theta}_{i}+\left(\dot{h}_{i}-v\right) w_{i}=0 \tag{3.23}
\end{equation*}
$$

where $w_{i}$ is the torsion of the tube expressed in $\left[\mathrm{m}^{-1}\right]$.

Relation (3.23) corresponds to the control input required in each individual tube to satisfy the follow-the-leader condition (3.20). Equation (3.23) elucidates the aforementioned freedom in the follow-the-leader control of each individual tube, where different combinations of $\dot{\theta}_{i}(t)$ and $\dot{h}_{i}(t)$ satisfy (3.21), and similarly (3.20). However, $\dot{h}_{i}(t)$ must be either zero or $v$ in the follow-the-leader configurations where the robot advances in order to satisfy the requirements on curvature discontinuities described in subsection 3.3.2. Thus, the relation between $\dot{\theta}_{i}(t)$ and $v$ is constant and determined by the geometry of the specific tube in the scenarios where the robot advances, with a specific rotation rate of each tube relative to the insertion rate. In particular, each advancing tube must rotate at a rate of $w_{i}$ relative to the arc length. A common example of such configuration is found in the insertion of a corkscrew, where the rotation rate relative to the insertion is determined by the helix geometry.

In the case of compensating in conjunction, the required control inputs $\dot{\theta}_{i}(t), \dot{h}_{i}(t)$ to maintain follow-the-leader motion can be determined from (3.22). In some cases, however, this can be complicated, and may lack insight into the mechanics of the robot. Alternatively, considering that the evolution of $\mathbf{u}_{T}^{W}\left(s_{0}, t\right)$ at any cross section $s_{0}$ is known for each trajectory (3.17), equation (3.5) can be used to determine $\theta_{i}(t), h_{i}(t)$ and thus $\dot{\theta}_{i}(t), \dot{h}_{i}(t)$. The $\theta_{i}(t)$ and $h_{i}(t)$ are the angles and insertions that satisfy that the sum of vectors $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W}(s)$ at a cross section is equal to the resulting curvature for all $t$. The control inputs determined for a cross section then apply to the entire robot, since the relative orientation and relative magnitude of $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W}$ and $\frac{\partial}{\partial s} \mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{W}$ in each cross section must remain proportional along the robots arc length for each set of tubes compensating in conjunction, as previously discussed.

The degrees of freedom of the control inputs can be seen in (3.5), where both $\dot{\theta}_{i}(t)$ or $\dot{h}_{i}(t)$ can be selected to achieve the desired evolution for the resulting curvature (3.17). As previously mentioned, the conditions on curvature discontinuities imply that the insertion inputs must be $\dot{h}_{i}=0$ or $\dot{h}_{i}=v$ for a CTR to maintain follow-the-leader motion over the entire robot. Then, only $\theta_{i}(t)$ can be used to follow the resulting curvature (3.17). Interestingly, in designs composed of two tubes, the $\theta_{i}(t)$ then involve the curvature vectors of both tubes monotonically tending towards an aligned or opposed configuration.

The condition for follow-the-leader (3.8) is a sum of terms corresponding to different tubes. Therefore, as in the previous subsection 3.3.3, combinations of configurations that involve compensating individually and compensating in conjunction also satisfy (3.8). The complete set of trajectories that can be traced in a follow-the-leader configuration under the assumptions of this second case is then equal to that in the previous subsection (3.18). The only extension in terms of follow-the-leader motion is the possibility of leaving tubes static relative to the workspace while the rest of the robot advances.

## General configuration including groups of tubes

Considering now the most general case, where any control inputs are allowed, the solutions to (3.8) are generally equivalent to those in the previous case, with the exception of configurations where groups of tubes move with a common $\dot{\theta}_{i}(t)$. These configurations are discussed in the following, both for groups of tubes compensating individually and in conjunction with other groups.

In the case of each group compensating individually, the terms of each group must then satisfy

$$
\begin{equation*}
\mathbf{R}^{\prime}\left(\theta_{j}\right) \sum_{i=1}^{m} \mathbf{q}_{i}\left(s-h_{i}\right) \dot{\theta}_{j}-\mathbf{R}\left(\theta_{j}\right) \sum_{i=1}^{m} \frac{\partial \mathbf{q}_{i}}{\partial s}\left(s-h_{i}\right) \dot{h}_{i}=-v \mathbf{R}\left(\theta_{j}\right) \sum_{i=1}^{m} \frac{\partial \mathbf{q}_{i}}{\partial s}\left(s-h_{i}\right) \forall s, t \tag{3.24}
\end{equation*}
$$

where $\theta_{j}(t)$ represents the common motion of the group of tubes, and $m$ is the number of tubes in the group. Equation (3.24) admits various solutions, which can be divided in different configurations.
(i) If $\dot{\theta}_{j}(t)=0$, then two possible solutions arise. First, (3.24) can be satisfied by selecting $\dot{h}_{i}(t)=v$ for all tubes, which an analogous situation to that discussed in 3.3.3 (ii).

Alternatively, by selecting specific $\dot{h}_{i}(t)$ for each tube, it is also possible to satisfy (3.24) at each instant of time in a given cross section. This solution requires at least two tubes, since two inputs $\dot{h}_{i}(t)$ are necessary to satisfy (3.24) for all $t$. In the case of the group of tubes coinciding with the robot's distal end, $\dot{h}_{i}(t)$ of one tube must always be zero by definition of $h_{i}(t)$, and then three tubes are necessary. The $\dot{h}_{i} \neq 0$ imply that the arguments of $\mathbf{q}_{i}$ vary with $t$. The
inputs $\dot{h}_{i}(t)$, however, apply to all cross sections. Thus, the configuration of vectors $\mathbf{q}_{i}$ and $\partial \mathbf{q}_{i} / \partial s$ must be proportional in all cross sections in order to satisfy $(3.24)$ for all $s, t$. The resulting trajectories are then equivalent to those described in the previous subsections 3.3.3, 3.3.3. It should be noted that this solution enables one to maintain follow-the-leader motion in the part of robot where the tubes with $\dot{h}_{i}(t) \neq 0$ are present, which cannot be all tubes of a robot for a sustained period of time.
(ii) If $\dot{h}_{j}(t)=0$, then it is necessary for $\dot{\theta}_{j}(t) \neq 0$, as well as $h_{j}(t)=0$, resulting in (3.24) transforming as

$$
\begin{equation*}
\mathbf{R}^{\prime}\left(\theta_{j}\right) \sum_{i=1}^{m} \mathbf{q}_{i}(s) \dot{\theta}_{i}=-v \mathbf{R}\left(\theta_{i}\right) \sum_{i=1}^{m} \frac{\partial \mathbf{q}_{i}}{\partial s}(s) \forall s, t \tag{3.25}
\end{equation*}
$$

which is equivalent to case (ii) of the previous subsection 3.3.3, so no new trajectories are added.
(iii) If $\dot{h}_{i}(t) \neq 0$ and $\dot{\theta}_{j}(t) \neq 0$, then two possible solutions arise. First, by selecting $h_{i}(t)=h_{j}(t)$ for all $i, j,(3.24)$ becomes analogous to (3.19). Then a solution exists where the group of tubes becomes equivalent to a single tube with the geometry and stiffness of the group in equilibrium, and thus the trajectories that can be traced in a follow-the-leader configuration are equivalent to those in subsection 3.3.3.

Alternatively, by selecting specific $\dot{h}_{i}(t)$ for each tube at each instant of time, (3.24) can be satisfied. This configuration is analogous to the previous case 3.3 .3 (i) for $\dot{h}_{i}(t) \neq v$, and therefore the trajectories that can be followed are equivalent to those in the previous case.

In the case of various groups of tubes compensating in conjunction, the groups must satisfy

$$
\begin{equation*}
\sum_{j=1}^{g^{\prime}}\left(\mathbf{R}\left(\theta_{j}\right) \sum_{i=1}^{l_{j}}\left[\mathbf{R}\left(\frac{\pi}{2}\right) \mathbf{q}_{i} \dot{\theta}_{j}+\left(v-\dot{h}_{i}\right) \frac{\partial}{\partial s}\left(\mathbf{q}_{i}\right)\right]\right)=0 \forall s, t \tag{3.26}
\end{equation*}
$$

where $g^{\prime}$ is the number of groups compensating in conjunction, $l_{j}$ denotes the number of tubes in group $j$, and arguments $\mathbf{q}_{i}\left(s-h_{i}\right)$ apply to the $\mathbf{q}_{i}$, although they are omitted for brevity. (3.26) is analogous to (3.22). The possible solutions can be divided in two further cases.
(iv) If the $\dot{h}_{i}(t)$ are common for all tubes in each group, then the groups act as single tubes with a geometry and stiffness equivalent to the combination of tubes in the group. The various groups
can then compensate in conjunction in an analogous manner as in the previous subsection for the case of single tubes compensating in conjunction. Hence, no trajectories are added.
(v) If the $\dot{h}_{i}(t)$ are different for the various tubes in each group, then the values of $\dot{h}_{i}(t)$ at each instant of time can be selected, either to achieve a desired evolution for the sum of terms in (3.26) corresponding to each tube so that the combination of tubes satisfies (3.26), or directly to satisfy (3.26) with the combination of terms from each individual tube. In either case, the arguments of vectors $\mathbf{q}_{i}$ and $\partial \mathbf{q}_{i} / \partial s$ at each cross section vary due to the different $\dot{h}_{i}(t)$. Specific control inputs are then required at each instant of time to satisfy (3.26) in a cross section, which represents a case analogous to that in subsection 3.3.3 when compensating in conjunction. Thus, the configuration of vectors $\mathbf{q}_{i}$ and $\partial \mathbf{q}_{i} / \partial s$ must be proportional along the arc length, and the resulting trajectories are equivalent to those in subsection 3.3.3.

From the discussion in this subsection, it can be concluded that the combination of a group of tubes with a common $\theta_{i}(t)$ does not expand the trajectories feasible in follow-the-leader configurations from those derived in the previous subsections. Nonetheless, the fact that groups of tubes moving in conjunction are equivalent to a single tube can be useful for the insertion of various tubes with singular precurvatures that cannot be inserted individually in a follow-the-leader configuration, but that in conjunction result in a geometry that can satisfy follow-the-leader. The exploitation of this configuration is considered and developed in the additional maneuvers described in section 3.4. It should be noted that the control input required for the insertion of a group of tubes is that corresponding to the single tube equivalent to the group, elucidated in (3.23).

## Curvature discontinuities

Up to this point, the study of trajectories where follow-the-leader is possible considered only continuous curves satisfying (3.7). However, trajectories with curvature discontinuities can also be traced in a follow-the-leader configuration, provided that the conditions described in subsection 3.3.1 are satisfied. An example are the well established trajectories composed of circumference arcs ([62]).

In general, the points of curvature discontinuity must remain in a constant position in the workspace, which implies that they must translate at velocity $v$ away from the robot's distal end as it advances. This requires the tubes causing the discontinuity to have $\dot{h}_{i}(t)=v$ from the point where the trajectory discontinuity is reached, onward. Considering that discontinuities appear due to either the end of a tube or a discontinuous precurvature of a tube, follow-theleader motion in trajectories with discontinuities is possible by leaving one or more tubes fixed at each point of curvature discontinuity while the rest of the robot proceeds forward. Each segment of trajectory between curvature discontinuities must satisfy (3.8). Thus, the complete trajectory must be a combination of segments of the trajectories identified in the previous subsections. These combined trajectories are discussed in more detail in subsection 3.4.1.

### 3.3.4 Set of trajectories summary

The trajectories found in the previous subsections 3.3.3-3.3.3, together with their combinations in subsection 3.3.3, constitute the set of trajectories that can be traced in a follow-the-leader configuration, since all possible cases solving (3.8) have been considered, in addition to curvature discontinuities. The trajectories, excluding combinations of them, can be synthesised in a single expression

$$
\mathbf{u}_{T}^{W}\left(s, t_{f}\right)=\sum_{i=1}^{n^{\prime}}\left[\begin{array}{c}
\left\|\mathbf{P}_{i} \hat{\mathbf{u}}_{i}\right\| \cos \left(w_{i} s+\phi_{i}\right)  \tag{3.27}\\
\left\|\mathbf{P}_{i} \hat{\mathbf{u}}_{i}\right\| \sin \left(w_{i} s+\phi_{i}\right) \\
0
\end{array}\right]+\sum_{j=1}^{g}\left[\begin{array}{c}
e^{\lambda_{j} s}\left\|\mathbf{u}_{R, j}\right\| \cos \left(\rho_{j} s+\nu_{j}\right) \\
e^{\lambda_{j} s}\left\|\mathbf{u}_{R, j}\right\| \sin \left(\rho_{j} s+\nu_{j}\right) \\
0
\end{array}\right]
$$

where $\left\|\mathbf{P}_{i} \hat{\mathbf{u}}_{i}\right\|, w_{i},\left\|\mathbf{u}_{R, j}\right\|, \lambda_{j}$, and $\rho_{j}$ are selected in the robot design, and $\phi_{i}$ and $\nu_{j}$ are determined by the rotational orientation of the tubes at the beginning of the trajectory. The magnitude of $\left\|\mathbf{P}_{i} \hat{\mathbf{u}}_{i}\right\|$ must be constant according to the previous discussion, but its value can be chosen as desired by selecting appropriate initial stiffness and curvature for each tube. Similarly, the $\left\|\mathbf{P}_{i} \hat{\mathbf{u}}_{i}\right\|$ of the tubes that compensate in conjunction to create $\left\|\mathbf{u}_{R, j}\right\|$ must vary exponentially, but the rate $\lambda_{j}$ and the magnitude of $\left\|\mathbf{u}_{R, j}\right\|$ can be selected as desired with the design of these tubes. The values of $w_{i}$ and $\rho_{j}$, which correspond to the initial torsion of either
tubes $i$ or $j$, and also can be freely selected provided that it is constant.

The initial designs of the individual tubes or groups of tubes comprising a CTR capable of follow-the-leader motion must satisfy $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}$ and $\frac{\partial}{\partial s} \mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}$ to remain proportional along the arc length, as discussed in the previous subsections. In the case of compensating in conjunction, the relative orientation and proportionality of the $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}$ and $\frac{\partial}{\partial s} \mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}$ must be equal for all tubes that compensate. Interestingly, for the common configuration of tubes with constant stiffness along the arc length, the initial geometry of the tubes that compensate individually is a helix, whereas that of tubes that compensate in conjunction is a deformed helix with continuously varying curvature magnitude. A particular case of degenerated helix is a circumference arc. Thus, (3.27) includes the well-established robot designs consisting of constant curvature tubes.

The robot designs corresponding to the set (3.27), however, are not limited to tubes with helical precurvatures. If tubes with variable stiffness are used, the precurvatures can present more general geometries that correspond to the deformation of helices, provided that the aforementioned relations on $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}^{F_{i}}$ are satisfied. These designs are equivalent to those of constant stiffness tubes in terms of follow-the-leader capabilities, but they can be exploited in additional maneuvers described in the next section, which combine follow-the-leader with other general kinematics in different parts of the robot, to increase the possibilities of motion and geometry in the parts of the robot that do not remain in a follow-the-leader configuration.

It should be noted that $\mathbf{P}_{i}$ is a non-dimensional stiffness determined by the stiffnesses of all tubes comprising a robot. In this regard, the design of the tubes in a robot is not decoupled, and instead a CTR must be designed considering all tubes that comprise it. In addition, in the case of configurations including tubes with $\lambda_{j} \neq 0$, which involve compensating in conjunction, two or more tubes are required for each term that involves a specific set of $\lambda_{j}$ and $\rho_{j}$ in the trajectories (3.27).

Expression (3.27), together with combinations of the trajectories linked as introduced in subsection 3.3.3, represent the complete set of trajectories that can be traced in follow-the-leader motion under the assumption of no axial torsion of the tubes. A broad variety of trajectories can therefore be followed. However, it should be noted that a generic robot design cannot be
used to follow any desired trajectory in the set (3.27), and instead a robot must be designed to follow a desired, small subset of the trajectories determined by variations in the $\phi_{i}, \nu_{j}$ and the insertion lengths of the tubes. In the particular case of using a robot with the minimum number of tubes necessary to follow a desired trajectory, the desired trajectory would require a specific robot design in terms of the initial $\mathbf{P}_{i} \hat{\mathbf{u}}_{i}$ of the tubes. It should also be noted that the length of trajectories involving terms with $\lambda \neq 0$ is typically limited as the curvature in these terms increases exponentially, rendering the trajectories prone to instability and of limited practical interest.

The control input required in each tube or group of tubes to maintain follow-the-leader motion over an entire CTR is (3.23) with $\dot{h}_{i}(t)=0$ for all tubes that are advancing and compensating individually in a possibly combined trajectory. The inputs required in tubes or groups of tubes compensating in conjunction is also $\dot{h}_{i}(t)=0$, and a $\theta_{i}(t)$ that can be determined from (3.5) so that the curvature resulting from the tubes compensating in conjunction follows the evolution of the corresponding term in (3.27). In both cases, the control input for tubes that remain stationary at the end of a segment of a combined trajectory is $\dot{h}_{i}(t)=v, \dot{\theta}_{i}(t)=0$.

The set of trajectories summarised in (3.27) is broad, and torsion can be expected to occur in some of the trajectories. This can render some of the trajectories partially inaccurate or completely unfeasible, as studied in section 3.5. Before the analysis of torsion, additional kinematics of interest are considered in the following section, completing the general study of motion related to follow-the-leader.

### 3.4 Additional maneuvers

The kinematic analysis presented up to this point focused on follow-the-leader motion. However, some potentially exploitable kinematic possibilities were also found in the discussion. The applicability of these kinematics, together with additional motions related to follow-the-leader, are described in this section. Some of these kinematics have been previously considered in the literature for robots comprising a set of piecewise constant curvature tubes. This work extends
some these kinematic possibilities to the new trajectories found here, and integrates them into the derivation in this thesis to complete the analysis.

### 3.4.1 Trajectory linking

The possibility of inserting one or multiple tubes that compose a robot with $\dot{h}_{i}=0, \dot{\theta}_{i} \neq 0$, and at a certain point switching the control of some of these tubes to $\dot{h}_{i}=v, \dot{\theta}_{i}=0$, was mentioned in subsection 3.3.3. This involves inserting one or multiple tubes to some extent together with the rest of the robot, and leaving these specific tubes fixed at a certain point while the rest of the robot proceeds forward, maintaining follow-the-leader motion throughout the entire robot (including the segment of the robot in which some tubes are left stationary).

This concept of telescopic deployment to enable follow-the-leader motion is known in the literature ( $[77],[74]$ ) and was originally introduced in [62] for tubes with piecewise constant curvature. In this work, the concept is extended to general trajectories composed of segments of trajectories from the set (3.27). More specifically, this deployment strategy can be exploited to follow trajectories in which the geometry of the first segment is determined by (3.27) for any desired number of tubes with selected precurvatures, and the geometry of the subsequent segments corresponds to (3.27) for equal precurvatures but a reduced number of tubes. In this manner, different trajectories from the set summarised in equation (3.27) can be linked and followed with a single robot, expanding the follow-the-leader kinematics. The telescopic insertion of tubes with piecewise constant curvature and no torsion is included as a particular case of linked trajectories. However, this deployment strategy is applicable to the broader set of trajectories discovered in this work (3.27).

The linkage of trajectories also enables extending the reachability of concentric tube robots. Tubes with significant precurvatures, which are generally prone to torsional instability, can be inserted a short length at the beginning of the trajectory, while tubes with shallower precurvatures can proceed forward. This is particularly relevant in keyhole surgery and on-wing operations, where reaching a desired location can require follow-the-leader motion in regions with clearly differentiated kinematic requirements. Examples in on-wing operations can be
inspections the access route involves entering into the engine in a radial direction, followed by a sharp turn to avoid a component, and then proceeding in a relatively straight passage to the location of interest. Typical MIS examples can be scenarios where entry into the body at a specific angle is a challenge, and the subsequent trajectory requires lower curvatures, as can be the case of interventional Magnetic Resonance (MR) procedures where access to the patient within the bore of the scanner is restricted. Specific examples of this can be focal ablation, brachytherapy, tissue sampling or drug delivery, performed under live MR imaging.

### 3.4.2 Combined follow-the-leader and general motion

One of the results drawn from the analysis in subsection 3.3.3 is that both $\dot{\theta}_{i}$ and $\dot{h}_{i}$ can be used to maintain follow-the-leader motion in the parts of the robot where the tubes with $\dot{\theta}_{i}$ and $\dot{h}_{i}$ are present. This applies both to configurations compensating individually, where it leads to one degree of freedom per tube, and configurations compensating in conjunction, where it leads to $2 l-2$ degrees of freedom. Once a robot has been inserted, it is then possible to operate individual tubes (or subsets of tubes in the case of compensating in conjunction) independently by using $\dot{h}_{i} \neq v, 0$ and the corresponding control input $\dot{\theta}_{i}$ determined from (3.23) for tubes compensating individually, or from (3.5) for tubes compensating in conjunction, as previously described. Follow-the-leader is then maintained throughout the part of the robot that contains the tubes controlled with $\dot{h}_{i} \neq v, 0$. Similarly, it is possible to independently operate some of the tubes composing a group that has been inserted with common $\dot{\theta}_{i}$ and $\dot{h}_{i}$, and maintain follow-the-leader provided that their individual design satisfies (3.20), or that the combination of designs of a subset of the tubes in the group satisfies (3.22). In the case of a tube or subset of tubes satisfying the design requirements for follow-the-leader in their proximal region only, their independent operation enables follow-the-leader motion in the part of the robot that contains the corresponding region of the tubes.

These kinematic equivalences enable general motion of the robot's distal part while maintaining a follow-the-leader configuration of the body of the robot once it has been inserted. In particular, there exist two main alternatives. The first involves varying the insertion of a tube
or a subset of tubes using follow-the-leader control in a configuration where the tubes being actuated present some offset $h_{i}>0$, i.e. the tubes are not at the robot's distal end. This leads to general, transversal motion of the robot's distal segment $s \in\left[0, h_{i}\right]$, while the rest of the robot, which contains the tube being actuated, remains in a follow-the-leader configuration. The second alternative involves using a group of tubes that satisfies the design requirements for follow-the-leader as a group, but is composed of tubes that, either individually or in conjunction for a subset of the tubes in the group, only satisfy the follow-the-leader requirements in the proximal part of the robot, presenting a general design in the distal part of the robot. In this configuration, the independent operation of the tubes using a follow-the-leader control corresponding to the proximal region of the tubes also enables follow-the-leader motion in the proximal part of the robot, combined with general motion of the distal region of the robot. The selection of the general curvature function in the distal part of the individual tubes determines the general motion generated at the robot.

It should be noted that the strategy of maintaining the proximal part of the robot in a steady configuration while the distal part is used as a manipulator had been previously introduced in [69]. In this regard, the contribution of this work is to expand the strategies to achieve this type of motion as well as the possible trajectories and kinematics under a common framework.

A relevant advantage of the kinematics proposed in this subsection, in particular the use of groups of tubes, is that, during the insertion, the group behaves as a single tube with $\left.\theta_{i}(t)=\theta_{j}(t)\right)$. Thus, it can contribute to the follow-the-leader kinematics during the robot insertion, reducing the number of tubes required, and then the group can be split for general maneuvers. Furthermore, the kinematics described in this subsection enable smooth variations of the robot's distal end configuration during insertion, which do not correspond to a follow-theleader configuration. These can be particularly useful in MIS applications, particularly during insertions through soft tissue where trajectory corrections are required.



Figure 3.3: Schematic illustration of the idle tubes concept corresponding to (a) two tubes (red and yellow) with opposite curvatures, resulting in a straight geometry (green) useful for insertion, and (b) the same tubes with aligned curvatures, which corresponds to an active configuration with bending in the segment near the distal end.

### 3.4.3 Idle tubes

The quasistatic model (3.5) shows that the combination of two tubes with opposite precurvatures results in a tube with zero curvature since the tubes' curvatures compensate at each cross section. Thus, a tube with a general desired curvature near the distal end and a straight geometry towards the proximal end can be integrated in a robot as a straight tube by combining it with its opposite, in an idle configuration shown in Figure 3.3(a). The incorporation of the resulting straight tube does not affect the possibility of follow-the-leader motion; it simply increases the robot stiffness.

Once the robot is inserted, the idle tubes can be activated by modifying their relative rotation or insertion, as shown in Figure 3.3(b). The active tubes only present curvature in the segment near the distal end, which is determined by their design. The result is the possibility of general motion at the robot's distal end once inserted, while maintaining a follow-the-leader configuration throughout the rest of the robot. The general motion achievable at the distal end is determined by the geometry of the idle tubes, which is selected by design. The idea of using idle tubes has been previously proposed in the literature. Here, the idea is generalised to the
precurvatures and trajectories discovered in this work, and the concept is extracted from the analysis in the previous sections of this chapter, leading to a more complete study.

The advantage of using idle tubes over the maneuvers described in the previous subsection is that idle tubes do not impose any restrictions on their control, since their proximal part is straight. On the other hand, idle tubes cannot contribute to the follow-the-leader kinematics, unlike groups of tubes described in the previous subsection. In this regard, idle tubes lead to a noticeable increase in robot stiffness, requiring higher precurvatures in the robot design to follow a specified trajectory. This results in devices prone to torsional instability, which is discussed in section 3.5. Thus, the practical applicability of the idle tubes concept is relatively limited.

### 3.5 Torsion

The analysis presented in the previous sections is predicated on the assumption of no axial torsion of the tubes composing the robot. Such an assumption can be used in the kinematic study of concentric tube robots, and it leads to the solutions described in previous sections. However, a certain degree of axial torsion is generally present in CTRs, and therefore a certain deviation from follow-the-leader can occur in the trajectories previously identified. When axial torsion is significant, CTRs can even become unstable in some of the previously identified trajectories due to the so-called snap-through instability described in [69]. Thus, even though some of the trajectories found under the assumption of no axial torsion can be tempting, as that shown in Figure 3.4, they may not be viable.

The axial torsion of CTRs is studied in this section in order to determine the validity of the assumption of no axial torsion, and therefore allow for the selection of robot configurations where such assumption is an acceptable approximation. It should be noted that, in this work, the term axial torsion refers to the torsional deformation of a tube along the axis locally tangential to its centreline. In the following derivation, both terms torsion and axial torsion are used to refer to axial torsion. These terms should not be confused with the geometric


Figure 3.4: Example of trajectory from the set (3.27), illustrating the fact that the assumption of no axial torsion can lead to intriguing predictions, but a study of torsion is required to determine feasibility.
torsion that can be associated to a curve in space, which can be used to describe the initial, undeformed geometry of a tube in space. In instances where it is necessary to refer to the geometric torsion of a curve in space, which can be describing the initial geometry of a tube, the term geometric torsion is used.

The study of torsion requires a general equilibrium analysis, which is derived in this section using special Cosserat rod equilibrium theory, following the approach in [69]. The study is then made specific to trajectories of interest in subsection 3.5.2, and a closed-form solution for a two-tube robot is presented. The implications of such a solution are subsequently discussed in subsection 3.5.3, and criteria to ensure that the torsion of the tubes is below a specified value are extracted. The relation between torsional deformation and deviation in task space is illustrated with some cases of interest in subsection 3.5.4, serving for the selection of a robot design for the case study described in section 3.6.

### 3.5.1 General formulation of torsional study

The derivation of the general differential equation governing torsion presented in this subsection is analogous to that in [69]. However, the main steps of the derivation are included for completeness, serving as a foundation for the subsequent analysis in this chapter. It should be noted that the derivation is for quasistatic operation of the robot, as in the previous sections of this chapter. In order to facilitate the integration of this work with existing literature, a new variable is defined $\zeta=L-s$, which corresponds to the arc length relative to the robot's proximal end. The study of torsion in the following is derived using $\zeta$ as the independent variable.

The equilibrium of a tube $i$ subjected to distributed external forces $\mathbf{f}$ and moments $\tau$ can be imposed as

$$
\left[\begin{array}{c}
\dot{\mathbf{m}}_{i}  \tag{3.28}\\
\dot{\mathbf{n}}_{i}
\end{array}\right]=\left[\begin{array}{c}
\tau \\
\mathbf{f}
\end{array}\right]-\left[\begin{array}{cc}
{\left[\mathbf{u}_{i}\right]} & {\left[\mathbf{v}_{i}\right]} \\
0 & {\left[\mathbf{u}_{i}\right]}
\end{array}\right]\left[\begin{array}{c}
\mathbf{m}_{i} \\
\mathbf{n}_{i}
\end{array}\right]
$$

where $\mathbf{u}_{i}$ and $\mathbf{v}_{i}$ represent the angular and linear deformations, respectively, $\mathbf{m}_{i}$ and $\mathbf{n}_{i}$ denote the internal moments and forces associated with the stress in the tube cross section, and the square brackets denote skew-symmetric matrix. The derivatives, indicated by a dot, are relative to the arc length of the curve describing the tube centreline, $\zeta$, and all the variables are a function of $\zeta$. The variables corresponding to a tube are expressed in the tube's frame, although the superscript indicating the frame is omitted for simplicity in the notation.

In this work, the focus is on the robot equilibrium resulting from the interaction between tubes. Thus, $\mathbf{f}$ and $\tau$ correspond to the forces and moments exerted on a tube by the adjacent tubes. Assuming the friction between the tubes comprising the robot to be negligible $\tau=0$, the equilibrium equation corresponding to the torques in (3.28) is

$$
\begin{equation*}
\dot{\mathbf{m}}_{i}=-\left[\mathbf{u}_{i}\right] \mathbf{m}_{i}-\left[\mathbf{v}_{i}\right] \mathbf{n}_{i} \tag{3.29}
\end{equation*}
$$

Considering the derivative of the constitutive relation (3.2) with respect to arc length

$$
\begin{equation*}
\dot{\mathbf{m}}_{i}=\mathbf{k}_{i} \frac{d \mathbf{u}_{i}}{d \zeta}+\frac{d \mathbf{k}_{i}}{d \zeta} \mathbf{u}_{i}-\frac{d\left(\mathbf{k}_{i} \hat{\mathbf{u}}_{i}\right)}{d \zeta} \tag{3.30}
\end{equation*}
$$

and combining (3.29) and (3.30) yields

$$
\begin{equation*}
\mathbf{k}_{i} \frac{d \mathbf{u}_{i}}{d \zeta}=-\left[u_{i}\right] \mathbf{m}_{i}-\left[v_{i}\right] \mathbf{n}_{i}-\frac{d \mathbf{k}_{i}}{d \zeta} \mathbf{u}_{i}+\frac{d\left(\mathbf{k}_{i} \hat{\mathbf{u}}_{i}\right)}{d \zeta} \tag{3.31}
\end{equation*}
$$

The angular strains can be assumed to be the prevailing deformation modes over linear strains, following [69], leading to

$$
\left[\mathbf{v}_{i}\right]=\left[\begin{array}{ccc}
0 & -1 & 0  \tag{3.32}\\
1 & 0 & 0 \\
0 & 0 & 0
\end{array}\right]
$$

Recalling that the initial curvature of a tube is defined in section 3.2 as the curvature of the curve corresponding to its centreline, the $z$ component of $\hat{\mathbf{u}}_{i}$ is zero, and therefore the $z$ component of $\frac{d\left(\mathbf{k}_{\mathbf{i}} \hat{\mathbf{u}}_{i}\right)}{d \zeta}$ is null. The tubes comprising the robot can be assumed to have an annular cross section with constant stiffness for convenience, which implies $\frac{\mathbf{k}_{i}}{d \zeta}=0$ and $k_{x}=k_{y}$. Using the constitutive relation (3.2), and after some manipulation, the $z$ component of (3.31) can be written as

$$
\begin{equation*}
\dot{u}_{i z}=\frac{k_{x}}{k_{z}}\left(u_{i x} \hat{u}_{i y}-u_{i y} \hat{u}_{i x}\right) \tag{3.33}
\end{equation*}
$$

which describes the torsional derivative of a tube with respect to $\zeta$ as a function of its initial and deformed bending curvatures. It should be noted that this expression is equivalent to that presented in [69], as it is applicable to any concentric tube robot design under the aforementioned assumptions.

Considering a robot composed of two tubes, the relative twist angle can be defined as

$$
\begin{equation*}
\alpha(\zeta)=\theta_{2}(\zeta)-\theta_{1}(\zeta) \tag{3.34}
\end{equation*}
$$

where $\theta_{i}$ represents the torsional displacement of tube $i$. Recalling the definition of $u_{i z}$ as the
torsional strain, the derivative of (3.34) with respect to the arc length relates the twist rate to the torsion of the tubes

$$
\begin{equation*}
\dot{\alpha}=u_{2 z}-u_{1 z} \tag{3.35}
\end{equation*}
$$

Combining the equilibrium of moments (3.3) in the $z$ direction and the constitutive law (3.2), the following relation in the $z$ direction can be obtained

$$
\begin{equation*}
k_{1 z} u_{1 z}+k_{2 z} u_{2 z}=0 \tag{3.36}
\end{equation*}
$$

Substituting (3.36) into (3.35), the twist rate between both tubes can be related to the torsion of one of the tubes

$$
\begin{equation*}
\dot{\alpha}=\left(1+\frac{k_{2 z}}{k_{1 z}}\right) u_{2 z} \tag{3.37}
\end{equation*}
$$

It should be noted that the twist rate can also be directly related to the torsion of the other tube using (3.36).

Combining the derivative of (3.37) with (3.33) , the second derivative of the twist can be related to the initial and final curvatures of tube 2 as

$$
\begin{equation*}
\ddot{\alpha}=\left(\frac{k_{2 x}}{k_{2 z}}+\frac{k_{2 x}}{k_{1 z}}\right)\left(u_{2 x} \hat{u}_{2 y}-u_{2 y} \hat{u}_{2 x}\right) \tag{3.38}
\end{equation*}
$$

The variables $u_{2 x}$ and $u_{2 y}$ can be expressed as functions of the initial curvatures of the tubes and the relative twist using the governing equations in section 3.2. Taking (3.5) in combination with (3.4), and expressing the relations in the Bishop frame associated to tube $2, F_{2}$, considering that in such case $\mathbf{R}\left(\theta_{1}\right)=\mathbf{R}(-\alpha)$ and $\mathbf{R}\left(\theta_{2}\right)=\mathbf{I}$, the following relations are obtained

$$
\begin{align*}
& u_{2 x}=\frac{1}{k_{1 x}+k_{2 x}}\left(k_{1 x} \hat{u}_{1 x} \cos \alpha+k_{1 y} \hat{u}_{1 y} \sin \alpha\right)  \tag{3.39}\\
& u_{2 y}=\frac{1}{k_{1 y}+k_{2 y}}\left(-k_{1 x} \hat{u}_{1 x} \sin \alpha+k_{1 y} \hat{u}_{1 y} \cos \alpha\right)
\end{align*}
$$

Substituting (3.39) into (3.38), and after some manipulation including the aforementioned assumption that $k_{x}=k_{y}$, the expression governing the relative twist of the tubes as a function
of their initial curvatures is obtained

$$
\begin{equation*}
\ddot{\alpha}=\left(\frac{k_{2 x}}{k_{1 z}}+\frac{k_{2 x}}{k_{2 z}}\right)\left(\frac{k_{1 x}}{k_{1 x}+k_{2 x}}\right)\left(\left(\hat{u}_{1 x} \hat{u}_{2 y}-\hat{u}_{2 x} \hat{u}_{1 y}\right) \cos \alpha+\left(\hat{u}_{1 x} \hat{u}_{2 x}+\hat{u}_{1 y} \hat{u}_{2 y}\right) \sin \alpha\right) \tag{3.40}
\end{equation*}
$$

A first boundary condition can correspond to the twist at the proximal end of the robot, i.e. at $\zeta=0$, which can generally be used as a control input,

$$
\begin{equation*}
\alpha(0)=\theta_{2}(0)-\theta_{1}(0) \tag{3.41}
\end{equation*}
$$

The second boundary condition can be obtained by considering that the torsional moment at the distal end of each tube must be zero, which implies no torsion of the tubes at $\zeta=L$ and therefore

$$
\begin{equation*}
\dot{\alpha}(L)=0 \tag{3.42}
\end{equation*}
$$

It should be noted that expression (3.40) together with boundary conditions (3.41) and (3.42) is general, and therefore valid for any two-tube robot design satisfying the assumptions used in the derivation.

### 3.5.2 Torsion in particular configurations

Expression (3.40) can be made specific to trajectories in the set (3.27) in order to determine the validity of the assumption of no axial torsion in practice. The most relevant trajectories in practice are those corresponding to tubes compensating individually since they only require one tube per component in (3.27), which enables following a wide variety of non-trivial trajectories with a low number of tubes, and they offer lengths and curvature values of typical practical interest. The following derivation is thus focused on robots composed of tubes with helical precurvatures. Substituting these helical precurvatures from (3.27) into (3.40), and after some manipulation,

$$
\begin{equation*}
\ddot{\alpha}=c \sin \left(\left(w_{2}-w_{1}\right) \zeta+\alpha(\zeta)+\phi_{d}\right) \tag{3.43}
\end{equation*}
$$

where $c=\left\|\hat{\mathbf{u}}_{1}\right\|\left\|\hat{\mathbf{u}}_{2}\right\|\left(\frac{k_{2 x}}{k_{1 z}}+\frac{k_{2 x}}{k_{2 z}}\right)\left(\frac{k_{1 x}}{k_{1 x}+k_{2 x}}\right)$ in which $\left\|\hat{\mathbf{u}}_{i}\right\|$ is constant considering tubes with constant stiffness, $\phi_{d}=\phi_{2}-\phi_{1}$, and the boundary conditions remain equal to those in (3.41) and (3.42). Defining a change of variable

$$
\begin{equation*}
\beta(\zeta)=\left(w_{2}-w_{1}\right) \zeta+\alpha(\zeta)+\phi_{d} \tag{3.44}
\end{equation*}
$$

equation (3.43) transforms into

$$
\begin{equation*}
\frac{d^{2} \beta(\zeta)}{d \zeta^{2}}=c \sin (\beta(\zeta)) \tag{3.45}
\end{equation*}
$$

with boundary conditions

$$
\begin{align*}
& \beta(0)=\alpha(0)+\phi_{d}  \tag{3.46}\\
& \dot{\beta}(L)=w_{2}-w_{1}
\end{align*}
$$

Differential equation (3.45), with boundary conditions (3.46), is similar to that obtained in [69], but differs in one of the boundary conditions, requiring a different solution. The rest of this subsection is dedicated to the solution of (3.45) with boundary conditions (3.46), and its application to solve (3.43).

The approach adopted in this work relies on the fact that equation (3.45) is analogous to the equation of a nonlinear pendulum. Thus, the solution to a nonlinear pendulum is adapted here for the specific boundary conditions (3.46). Considering that $\frac{d^{2} \beta(\zeta)}{d \zeta^{2}}=\ddot{\beta}=\dot{\beta} \frac{d \dot{\beta}}{d \beta}$, (3.45) can be integrated

$$
\begin{equation*}
\int_{\beta(0)}^{\dot{\beta}(\zeta)} \dot{\beta} d \dot{\beta}=c \int_{\beta(0)}^{\beta(\zeta)} \sin (\beta) d \beta \tag{3.47}
\end{equation*}
$$

and evaluated as

$$
\begin{equation*}
\dot{\beta}(\zeta)^{2}=\dot{\beta}(0)^{2}+2 c(\cos (\beta(0))-\cos (\beta(\zeta))) \tag{3.48}
\end{equation*}
$$

This expression can be evaluated at $\zeta=L$ considering the boundary conditions (3.46), and substituted in (3.48), resulting in the first order differential equation

$$
\begin{equation*}
\dot{\beta}(\zeta)^{2}=\left(w_{2}-w_{1}\right)^{2}+2 c(\cos (\beta(L))-\cos (\beta(\zeta))) \tag{3.49}
\end{equation*}
$$

Using separation of variables, the integral of (3.49) can be considered in the following interval

$$
\begin{equation*}
\zeta-L=\frac{1}{\sqrt{2 c}} \int_{\beta(L)}^{\beta(\zeta)} \frac{d \beta}{\sqrt{\frac{\left(w_{2}-w_{1}\right)^{2}}{2 c}+\cos (\beta(L))-\cos (\beta(\zeta))}} \tag{3.50}
\end{equation*}
$$

Defining $b=\frac{\left(w_{2}-w_{1}\right)^{2}}{2 c}+\cos (\beta(L))+1, K_{e}=\sqrt{\frac{2}{b}}$, and using the change of variable $h(\zeta)=\beta(\zeta)+\pi$, integral (3.50) can be rewritten as

$$
\begin{equation*}
\zeta-L=\frac{1}{\sqrt{2 c b}} \int_{h(L)}^{h(\zeta)} \frac{d h}{\sqrt{1-K_{e}^{2} \sin ^{2} \frac{h(\zeta)}{2}}} \tag{3.51}
\end{equation*}
$$

This integral corresponds to the incomplete elliptic integral of the first kind $F(x, K)$, which is defined for $0 \leq K \leq 1$. The closed-form solution to (3.51) can be obtained in two intervals of $K_{e}$.

If $0 \leq K_{e} \leq 1$, expression (3.51) can be directly integrated according to the definition of $F(x, K)$, yielding

$$
\begin{equation*}
(\zeta-L) \sqrt{\frac{c b}{2}}=F\left(\frac{h(\zeta)}{2}, K_{e}\right)-F\left(\frac{h(L)}{2}, K_{e}\right) \tag{3.52}
\end{equation*}
$$

Using the Jacobi elliptic functions $s n$ and $c n$, the incomplete elliptic integral of the first kind $F\left(\frac{h(\zeta)}{2}, K_{e}\right)$ can be inverted, which allows solving for $h(\zeta)$ as

$$
\begin{equation*}
h(\zeta)=2 \tan ^{-1}\left[\frac{s n\left((\zeta-L) \sqrt{\frac{c b}{2}}+F\left(\frac{h(L)}{2}, K_{e}\right), K_{e}\right)}{c n\left((\zeta-L) \sqrt{\frac{c b}{2}}+F\left(\frac{h(L)}{2}, K_{e}\right), K_{e}\right)}\right] \tag{3.53}
\end{equation*}
$$

If $K_{e}>1$, a change of variable can be defined using $\psi(\zeta)=\sin ^{-1}\left(K_{e} \sin \frac{h(\zeta)}{2}\right)$, which can be differentiated as $d \psi \cos \psi=K_{e} \sqrt{1-\frac{\sin ^{2} \psi}{K_{e}{ }^{2}}} \frac{d h}{2}$, and guarantees that the incomplete elliptic integral is well defined. Applying such change of variable to (3.51) yields

$$
\begin{equation*}
(\zeta-L) \sqrt{c}=\int_{\psi(L)}^{\psi(\zeta)} \frac{\cos \psi d \psi}{\sqrt{1-\frac{\sin ^{2} \psi}{K_{e}{ }^{2}}} \sqrt{1-\sin ^{2} \psi}} \tag{3.54}
\end{equation*}
$$

which can be integrated using the definition of the incomplete integral of the first kind, resulting
in

$$
\begin{equation*}
(\zeta-L) \sqrt{c}=F\left(\psi(\zeta), \frac{1}{K_{e}}\right)-F\left(\psi(L), \frac{1}{K_{e}}\right) \tag{3.55}
\end{equation*}
$$

Using Jacobi elliptic functions, and reversing the change of variables, (3.55) can be solved for $h(\zeta)$ as

$$
\begin{equation*}
h(\zeta)=2 \sin ^{-1}\left\{\frac{1}{K_{e}} \operatorname{sn}\left[(\zeta-L) \sqrt{c}+F\left(\sin ^{-1}\left[K_{e} \sin \left(\frac{h(L)}{2}\right)\right], \frac{1}{K_{e}}\right), \frac{1}{K_{e}}\right]\right\} \tag{3.56}
\end{equation*}
$$

The change of variable $h(\zeta)=\beta(\zeta)+\pi$ can be reversed to obtain the solution to (3.45) for $\beta(\zeta)$ from (3.53) and (3.56), which is immediate.

Finally, by reversing the change of variable (3.44), the closed-form solution to the relative twist $\alpha(\zeta)$ of two tubes in the trajectories where follow-the-leader motion is possible for given design parameters can be obtained in two intervals of $K_{e}$. For $0 \leq K_{e} \leq 1$,

$$
\begin{equation*}
\alpha(\zeta)=\left(w_{1}-w_{2}\right) \zeta-\pi-\phi_{d}+2 \tan ^{-1}\left[\frac{\operatorname{sn}\left((\zeta-L) \sqrt{\frac{c b}{2}}+F\left(\frac{\alpha(L)+\left(w_{2}-w_{1}\right) L+\phi_{d}+\pi}{2}, K_{e}\right), K_{e}\right)}{\operatorname{cn}\left((\zeta-L) \sqrt{\frac{c b}{2}}+F\left(\frac{\alpha(L)+\left(w_{2}-w_{1}\right) L+\phi_{d}+\pi}{2}, K_{e}\right), K_{e}\right)}\right] \tag{3.57}
\end{equation*}
$$

And for $K_{e}>1$,

$$
\begin{align*}
& \alpha(\zeta)=\left(w_{1}-w_{2}\right) \zeta-\pi-\phi_{d}+2 \sin ^{-1}\left\{\frac{1}{K_{e}} \operatorname{sn}[(\zeta-L) \sqrt{c}+\right. \\
& \left.\left.F\left(\sin ^{-1}\left[K_{e} \sin \left(\frac{\alpha(L)+\left(w_{2}-w_{1}\right) L+\phi_{d}+\pi}{2}\right)\right], \frac{1}{K_{e}}\right), \frac{1}{K_{e}}\right]\right\} \tag{3.58}
\end{align*}
$$

where the transition at $K_{e}=1$ is smooth.

It should be noted that the solution is expressed as a function of the relative twist at the distal end of the robot, instead of the proximal end as in the boundary condition (3.41). An equivalent result can be obtained using $\alpha(0)$ as independent variable instead of $\alpha(L)$ following an analogous derivation. However, $\alpha(L)$ is selected as the independent variable in this case since it facilitates the discussion on torsional stability, which is the final aim of this torsional study.

### 3.5.3 Torsion discussion

The implications of solution (3.57), (3.58) are analysed in this subsection. The focus is on the torsional magnitude in order to determine the validity of the assumption of no axial torsion employed in the previous sections of this chapter, and thus identify stable trajectories.

Expressions (3.57), (3.58) allow the determination of the relative twist at any cross section of a two-tube robot composed of helical tubes as a function of $\alpha(L)$ as well as the robot design parameters and $\phi_{d}$. The evaluation of expressions (3.57), (3.58) at $s=0$ provides the relation between $\alpha(0)$ and $\alpha(L)$ for a given robot design and $\phi_{d}$. The effect of $\phi_{d}$ on the relation between $\alpha(0)$ and $\alpha(L)$ is simply a translation of the origin about $\alpha(0)=\alpha(L)$, which is a consequence of the fact that $\phi_{d}$ corresponds to the relative rotation of the tubes at the proximal end. Since the torsional behavior of the tubes is cyclic with period $2 \pi$, the effect of $\phi_{d}$ is not relevant and is not considered further. On the other hand, the influence of the design parameters on the torsional behavior is through two non-dimensional groups: $L \sqrt{c},\left(w_{2}-w_{1}\right) L$. The relation between $\alpha(0)$ and $\alpha(L)$ can therefore be plotted for different values of the non-dimensional groups in order to study the tubes' torsional behavior.

Three illustrative examples of different relations between $\alpha(0)$ and $\alpha(L)$ are shown in Figure 3.5, which correspond to three different cases in terms of values of the non-dimensional groups. As can be seen, in two of the cases, the evolution of $\alpha(L)$ as a function of $\alpha(0)$ is stable, whereas in the third case the robot presents a torsional instability corresponding to a snap-through instability. The two stable examples, however, present markedly different evolutions of relative twist. The relation shown in blue is strongly nonlinear, which implies that the assumption of no axial torsion is not an accurate representation of the torsional behavior. Instead, the relation shown in green is closer to linear, and therefore can be approximated well by the assumption of no axial torsion of the robot.

Studying the evolution of $\alpha(L)$ as a function of $\alpha(0)$ for a range of values of the non-dimensional groups in combination with the solutions (3.57), (3.58), criteria to attain a desired torsional behavior can be extracted. The domain considered here is selected to include the configurations


Figure 3.5: Evolution of the relative twist between the distal and proximal ends of robots composed of two tubes with three different designs, which present a stable and approximately linear relation (green), a stable but nonlinear evolution (blue), and an unstable behavior in the interval $\alpha(0) \in[2.8,3.8]$.
of practical interest, with $L \sqrt{c} \in[0,3 \pi / 4]$ and $(w 2-w 1) L \in[0,24]$. In general, the robot is stable if $L \sqrt{c} \leq \frac{\pi}{2}$, although greater values can be reached in a stable manner by increasing $\left(w_{2}-w_{1}\right) L$. Similarly, it can be seen that greater values of $\left(w_{2}-w_{1}\right) L$ lead to a relation between $\alpha(L)$ and $\alpha(0)$ that is closer to $\alpha(L)=\alpha(0)$. The deviation from $\alpha(L)=\alpha(0)$, quantified as average deviation error squared, is plotted in Figure 3.6 as a function of the non-dimensional groups in the region of stable configurations of interest $L \sqrt{c} \in[0, \pi / 2]$ and $\left(w_{2}-w_{1}\right) L \in[0,24]$. The plot confirms the trends identified for $\left(w_{2}-w_{1}\right) L$ and shows that they are monotonic over the region considered. Interestingly, for the case of $w_{2}=w_{1}=0$, the results from (3.57), (3.58) converge with the results reported by [69]. In this regard, the solution (3.57), (3.58) represents a generalization of the work in [69] for two-tube robot designs with helical tubes, which correspond to the set (3.27).

A torsional deviation in the relation between $\alpha(0)$ and $\alpha(L)$ can therefore be selected to be below a specified value in order to ensure that the assumption of no axial torsion is an acceptable approximation. It should be noted, however, that a boundary on torsional deformation does not directly imply a specific boundary on the deviation with respect to follow-the-leader motion in


Figure 3.6: Plot of the average squared deviation from $\alpha(L)=\alpha(0)$ as a function of $L \sqrt{c}$, $\left(w_{2}-w_{1}\right) L$ in the domain of interest. The plot elucidates the trends identified, and confirms that they are monotonic in the domain of interest.
the resulting trajectory. The torsional deformation affects the local curvature values, whereas the deviation in the resulting trajectory is determined by the integration of the local curvature along the robot length. Thus, torsional deformation and resulting deviation in task space are related, but the relation depends on an integral.

The solution (3.57), (3.58) can be substituted into the well-known robot model including torsion, e.g. that described in [69], to determine the deviation in local curvature due to torsion in a two-tube robot. This can then be particularised to the robot designs and configurations found in this work to determine the local curvature deviation in the trajectories of interest (3.27). However, in order to determine the resulting position deviation due to torsion in task space, the local curvature needs to be integrated. A closed-form solution to such integral is not available. Thus, the specific deviation in task space due to torsion cannot be directly determined from the current analysis. The possibility of approximating this integral or finding boundaries on the deviations in task space from boundaries on local curvature deviations will be addressed in future work.

Nonetheless, in some practical cases, the typical deviation in task space due to torsional deformation can be considered to follow certain trends that can be approximated for a specific family of designs based on experience. In such cases, boundaries on torsional deformation can be used to identify the trajectories where follow-the-leader is possible within an admissible deviation. In order to exploit any trajectories of interest, however, these need to be subsequently verified to ensure that the deviation in task space is within the expected values. In more general cases, a hypothesis on the admissible torsional deformation in the specific scenario of interest can be formulated by exploring the effect of torsion on the resulting trajectory in some relevant configurations. The corresponding trajectories where approximate follow-the-leader is possible can then be identified, and trajectories of interest can be selected. However, any selected trajectory needs to be subsequently verified. This procedure can therefore require some iteration. In all cases, it should be noted that boundaries on torsional deformation typically involve using tubes with lower curvatures. In particular, in designs composed of tubes with planar precurvatures, this always applies as torsional deformation is determined by a single parameter $L \sqrt{c}$.

### 3.5.4 Illustration of torsion effects

A set of examples of torsional deformation and the corresponding deviation in task space are presented in this subsection. These are aimed at illustrating the relation between torsion and resulting deviation for some designs of interest.

Three simulated insertions are first used to show the behavior of three exemplary robot designs corresponding to the torsional relations shown in Figure 3.5, and then to quantify the follow-the-leader deviation in task space due to torsion. The simulations are implemented using the robot quasistatic model considering torsion (3.5) together with the solutions of torsion along the arc length (3.57), (3.58). The robot configuration is evaluated at ten regular intervals during an insertion. The three designs are all composed of two helical tubes with equal stiffness, a length of 20 cm , and $\left\|\hat{\mathbf{u}}_{1}\right\|=11 \mathrm{~m}^{-1},\left\|\hat{\mathbf{u}}_{2}\right\|=8 \mathrm{~m}^{-1}, w_{1}=8 \mathrm{~m}^{-1}, w_{2}=12 \mathrm{~m}^{-1}$ for the first design, $\left\|\hat{\mathbf{u}}_{1}\right\|=9 m^{-1},\left\|\hat{\mathbf{u}}_{2}\right\|=7 m^{-1}, w_{1}=-12 m^{-1}, w_{2}=9 m^{-1}$ for the second, and $\left\|\hat{\mathbf{u}}_{1}\right\|=6 m^{-1}$, $\left\|\hat{\mathbf{u}}_{2}\right\|=5 m^{-1}, w_{1}=-18 m^{-1}, w_{2}=9 m^{-1}$ for the third.

The resulting simulated insertions are shown in Figure 3.7 (a), (b) and (c), respectively. As can be seen, follow-the-leader is maintained in some parts of the trajectories, but significant deviations are present in both the first and second designs. In this work, the deviation, defined $\epsilon$, is quantified as the maximum of the minimum distances between any point on the robot centreline at any of the configurations during an insertion and the centreline at any other configuration. The maximum deviations for the insertions shown in Figure 3.7 (a), (b) and (c) then are $\epsilon_{1}=40.7 \mathrm{~mm}, \epsilon_{2}=16.0 \mathrm{~mm}$, and $\epsilon_{3}=2.8 \mathrm{~mm}$, respectively. The error in these three cases thus increases with the magnitude of torsion, as can also be observed in the plots. Interestingly, the snap-through instability appears in the first design at approximately $75 \%$ of the insertion, as can be seen in Figure 3.7 (a), where the geometry of the robot in the last three configurations is markedly different from that in the previous configurations.

Equivalent simulations can be conducted to explore the relation between torsion boundaries and deviation in task space in any set of designs. This is presented here for a relevant subset of designs corresponding to two-tube robots with helical tubes of equal stiffness, a length of 20 cm , and curvatures of each tube varied within $\left\|\hat{\mathbf{u}}_{i}\right\| \in[3,7]$, and geometric torsion varied within $w_{i} \in[-30,15]$, with $w_{1} \neq w_{2}$ for each design. This subset of designs is selected as it results in trajectories of potential practical interest, which present complex geometries with variations of curvature along the arc length, both in magnitude and direction. The maximum deviation from follow-the-leader is measured in each insertion as in the previous three cases.

The maximum deviation in task space is plotted in Figure 3.8 as a function of the maximum torsional deviation, defined as $\Delta \alpha_{M}=\max \|\alpha(L)-\alpha(0)\|$ over $\alpha(0) \in[0,2 \pi]$, for all designs in the subset. As can be seen in Figure 3.8, the maximum deviation in task space tends to increase with the maximum torsional deformation. Interestingly, the torsional deviation of some of the designs coincides, which is due to the fact that the non-dimensional groups coincide. In some specific cases, the deviation is low despite significant torsional deviation. These are designs where $w_{1}$ and $w_{2}$ are close, and therefore the robot behaves practically as a single helix with limited relative tube rotation. Still, in the cases explored, torsion boundaries translate as bounded deviation in task space.


Figure 3.7: Plot of three simulated insertions corresponding to three different designs, where (a) presents a significant deviation from follow-the-leader and includes a snap-through instability, (b) presents a noticeable deviation due to torsional deformation of the tubes, and (c) presents a low deviation.


Figure 3.8: Plot of maximum deviation from follow-the-leader in task space as a function of maximum torsional deviation for a wide variety of designs.

Torsion boundaries can then be defined in the subset of designs explored so that the assumption of no torsion is an admissible approximation, and thus the corresponding follow-the-leader trajectories can be followed within an acceptable deviation. This can be exemplified by considering admissible the relations between $\alpha(0)$ and $\alpha(L)$ that lie within two boundaries depicted as dashed lines in Figure 3.5, and without snap-through. These boundaries are arbitrarily set to be parallel to $\alpha(0)=\alpha(L)$ with an offset of $\pm 1 / 2 \mathrm{~m}^{-1}$, and correspond to maximum deviations in task space of near 4 mm . It should be noted, however, that these bounded deviations are only guaranteed in the specific configurations explored. The deviations on any other configuration, even if similar, must be verified.

The trajectories corresponding to the configurations explored within these bounds are plotted in Figure 3.9 for a common initial pose at the base. It should be noted that the trajectories shown in Figure 3.9 can also be rotated around the base $z$ axis while maintaining the base pose, increasing the follow-the-leader possibilities for that pose, although they are not plotted for clarity of illustration. It should also be noted that the solution in equations (3.57), (3.58) does not depend on the length units in the robot design variables, and therefore any isotropic scaling of the trajectories shown in Figure 3.9 results in a trajectory that can also be traced in an approximate follow-the-leader manner with a deviation that scales with $L$. Figure 3.9


Figure 3.9: Plot of the set of stable trajectories where follow-the-leader is possible using a robot composed of two tubes, with a common base pose.
illustrates the potential of the trajectories discovered in this work for surgical applications, showcasing the capability of following trajectories with a continuous variation of curvature, both in magnitude and direction, in an approximate follow-the-leader configuration to reach targets in different locations from a specified initial pose.

### 3.6 Case study: simulation and experiment

The results on torsional stability presented in the previous section allow for the selection of a robot design together with a trajectory to showcase the research reported in this chapter. The performance of the selected robot is presented in this section in the form of a case study involving simulation and experiment. This serves to illustrate both the capability of follow-the-leader

|  | $\kappa\left[\mathrm{m}^{-1}\right]$ | $w\left[\mathrm{~m}^{-1}\right]$ | OD $[\mathrm{mm}]$ | $\mathrm{ID}[\mathrm{mm}]$ |
| :--- | :--- | :--- | :--- | :--- |
| Tube 1 | 6.79 | -26 | 0.9652 | 0.8128 |
| Tube 2 | 6.22 | 4 | 1.1938 | 1.1176 |

Table 3.1: Characteristics of the tubes corresponding to the case study.
motion in a trajectory that is unique and representative of the research on follow-the-leader, as well as the validity of the assumption of no axial torsion in such trajectory.

### 3.6.1 Robot design and trajectory

The case study involves a two-tubes robot advancing in follow-the-leader motion along a trajectory with continuous variation of curvature, both in direction and magnitude, in the proximal part of trajectory, and helical geometry in the distal part. The trajectory selected is a combination of two trajectories in the set (3.27) linked as described in subsection 3.4.1, whereby one of the tubes remains static at the linkage between trajectories while the other proceeds forward. The case study therefore serves to demonstrate both the research reported in section 3.3, as well as some of the work on additional exploitable kinematics described in section 3.4. The behavior of the robot in the first and proximal part of trajectory is studied with simulations, whereas that in the second and distal part of trajectory is shown with an experiment.

The geometry of the complete selected trajectory can be described by the curvature $\kappa_{i}$, geometric torsion $w_{i}$ and stiffness of the two tubes comprising the robot, together with their respective insertion lengths. The tube's characteristics are summarised in Table 3.1, where OD denotes the outer diameter of the tube and ID represents the inner diameter. The total insertion of the outer tube, tube 2 , is 19 cm , whereas that of the inner tube is 26 cm . The complete trajectory is shown in Figure 3.10. As can be seen, it is a trajectory that cannot be followed using conventional, constant curvature tubes, as it presents continuous variation of curvature in the part corresponding to two tubes, and helical geometry in the part corresponding to a single tube.

The tube's characteristics are selected to minimise axial torsion. The evolution of $\alpha(L)$ as a function of $\alpha(0)$ can be predicted using (3.57), (3.58), as shown in Figure 3.13. In this case, the design parameters summarised in Table 3.1 result in the approximately linear relation between


Figure 3.10: Plot of the complete trajectory selected for the case study. It comprises both the first part of trajectory in the proximal region, where is presents continuous variation of curvature, both in direction and magnitude, and the second part of trajectory in the distal region, where it presents a helical geometry.
$\alpha(L)$ and $\alpha(0)$ shown in Figure 3.13. Thus, torsion is expected to be low in the entire trajectory.

### 3.6.2 Simulation

The first part of the trajectory corresponds to both tubes advancing with $\dot{h}_{1}=\dot{h}_{2}=0$ from the insertion point until full insertion of tube 2. The behavior of the robot in this part of the trajectory is studied by simulating it at a set of twenty configurations corresponding to insertion lengths between $L=9.5 \mathrm{~mm}$ and $L=19 \mathrm{~cm}$ at regular intervals. This enables evaluating the deviation from follow-the-leader and the magnitude of torsional deformation as the robot advances.

The geometry of the robot in each of these twenty configurations is simulated as in subsection 3.5.4, by combining (3.5) and (3.57), (3.58). The effects of friction between tubes and gravity are neglected, and the tubes are assumed to be made of nitinol with a Poisson ratio of $\nu=0.33$.

The desired control inputs at the insertion point for this part of trajectory are determined from (3.23) with $\dot{h}_{i}=0$. Thus, the rotation of each tube at the insertion point should be constant and at a rate corresponding to its geometric torsion. In practice, the tubes must be controlled by an actuation system, and therefore part of the tube will be inside this actuation system. The part of the tubes inside the actuation system may then undergo torsion as well, leading to a rotation at the insertion point different from that at the proximal ends where the tubes actuated. Considering an actuation box that constrains the tubes to remain straight inside it, the torsion in the part of the tubes inside the box is constant, according to the generalization of (3.33) for any number of tubes described in [69]. The specific torsion is then determined by the torsion at the cross section immediately after the insertion point, $u_{i z}(\zeta=0, t)$, which can be determined from (3.37) and (3.49). The desired constant rotation of $\theta_{i}$ at the insertion point can then be achieved with a rotation of $\gamma_{i}=\theta_{i}(\zeta=0)-u_{i z}(\zeta=0, t) d_{i}$ at the point where tube $i$ is actuated, where $d_{i}$ is the tube length between the point of actuation and the insertion point. The simulations then assume ideal actuation inputs, and thus a constant rotation at the insertion point at a rate corresponding to the geometric torsion of each tube.

The resulting simulated robot configurations are shown in Figure 3.11. As can be seen, an approximate follow-the-leader motion is maintained over this entire first part of trajectory, although a certain degree of deviation is present. The deviation from follow-the-leader is relatively low near the insertion point and increases towards the distal parts of the trajectory. The maximum deviation in task space, quantified as in the previous section, is 3.5 mm , and occurs between the configurations at $85 \%$ and $100 \%$ of the insertion, at an arc length of 163.9 mm of the final configuration.

The deviations shown in Figure 3.11 are due to torsion. The simulated torsional deviation along arc the length $\Delta \alpha(\zeta)=\alpha(\zeta)-\alpha(0)$ is shown in Figure 3.12 for the robot configurations corresponding to the twenty insertion lengths. As can be seen, the torsional behavior varies as the insertion of the robot increases, which results in changes in the local curvature along the arc length, and ultimately leads to the deviations from follow-the-leader in task space. The relation between deviations in local curvature and follow-the-leader error in task space is determined by the integration of curvature along the arc length, and therefore the effect of local curvature deviations is amplified with the arc length, which results in the larger errors in the distal parts of the trajectory shown in Figure 3.11.

### 3.6.3 Experiment

The second part of the trajectory is the continuation of the first one. It begins with both tubes inserted as described in the previous subsection. One of the tubes is then advanced to trace this second part of trajectory while the other tube remains stationary relative to the task space. The robot behavior in this second part of trajectory is demonstrated with an experiment in order to illustrate follow-the-leader motion in practice.

The experiment implementation starts with the distal end of both tubes coinciding, which corresponds to the end of the first part of trajectory. Tube 1 is subsequently advanced, which involves a combination of insertion and rotation of the tube at a rate of $w_{1} \mathrm{~m}^{-1}$, while tube 2 remains stationary. The geometry of the complete device is measured as tube 1 advances in order to evaluate the satisfaction of follow-the-leader motion over the entire device. The


Figure 3.11: Simulated insertion of the robot in the first part of trajectory.


Figure 3.12: Simulated torsional deviation as a function of arc length for twenty robot configurations during an insertion.
experiment proceeds until full insertion of tube 1 , which corresponds to the end of the complete trajectory shown in Figure 3.10.

The design of the tubes used in the experiment matches the description in subsection 3.6.1, summarised in Table 3.1. Both tubes are made of nitinol, supplied by Nitinol Devices and Components Inc., with part numbers TSE0380X0320GS and TSE0470X0440GS, respectively. It should be noted that the stiffness of both tubes is practically equal, which requires the result in (3.27) to be correct for follow-the-leader motion to occur throughout the entire robot.

Starting the experiment from the point of linkage between the two parts of the complete trajectory enables achieving follow-the-leader motion without the need for an actuation system. Tube 1 can be simply advanced with free rotation, relying on the elastic equilibrium of the system to naturally rotate it at the required rate $w_{1}$.

This rotational behavior is necessary in this configuration corresponding to follow-the-leader, where the curvature at each point in the workspace must be constant. For tubes with constant stiffness as in this experiment, follow-the-leader requires the curvature vector of each tube to remain constant at each point in the workspace. Since the tubes are in a minimum energy


Figure 3.13: Predicted evolution of the relative twist at the distal end as a function of the proximal end of the robot design selected for the experiment.
equilibrium at the beginning of the experiment, tube 1 is expected to rotate to remain in the minimum energy equilibrium as it is being inserted. Considering that the tubes have a helical geometry, remaining at a minimum energy configuration implies maintaining a constant curvature vector at each point in the workspace, and therefore rotating at the follow-the-leader rate $w_{1}$. This structural behavior can therefore be exploited to design a simpler experiment that suffices for the illustration of the research on follow-the-leader, which is the strategy adopted in this work for the implementation.

The experimental set up used in the implementation is shown in Figure 3.14. The shape of the device is measured at regular intervals during advancement using a 3D laser scanner (PICZA LPX-250, manufactured by Roland). The desired initial geometry of the tubes was achieved by means of a shape setting process. Since the tubes' stiffness is constant, their precurvatures are helical, and the shape setting process simply involved constraining each tube to a cylindrical fixture of the specified diameter, heating the assembly in air up to 550 degrees Celsius under free convection for 10 minutes, and quenching it in water. The assembled device with both tubes arranged concentrically was held vertically to minimise the deformation due to gravitational
forces. In this work, the set-up was placed inside the 3D laser scanner, and tube 1 was advanced manually while tube 2 remained fixed relative to the scanner workspace.

A total of six robot shape measurements were recorded using the 3D laser scanner as tube 1 was advanced. Each measurement consists of a set of points describing the device shape, as shown in Figure 3.15 for the third measurement, with the corresponding projections on the XZ and YZ planes, shown in Figures 3.16 and 3.17, respectively. A curve is fitted to determine the geometry of the curve corresponding to the device centreline, which is also shown in Figures 3.15, 3.16 and 3.17, for the same measurement. As can be seen, the measurement presents a certain degree of noise, which can be attributed to vibrations induced on the device by the rotation of the 3D scanner, and to reflections from the surface of the tubes, which is more reflective than the desired surface for typical operation of the 3D laser scanner. The noise is zero mean, and the fitted curve allows for the reliable extraction of the geometry of the device. The fitted curves of the different measurements are subsequently used to assess the follow-the-leader motion.

The result of the experiment is an accurate follow-the-leader configuration throughout the entire device. The 3D points from the different measurements recorded during device advancement, together with their corresponding fitted curves, are shown in Figure 3.18 using specific colors for each measurement. The projections of the fitted curves on the XZ and YZ planes are shown in Figures 3.19 and 3.20 , respectively. As can be seen, the motion in both parts of the trajectory corresponding to two tubes and one tube remains within a follow-the-leader configuration. The maximum deviation estimated from the fitted curves in each measurement is 4 mm . This can be partially attributed to the limited accuracy of the experimental set-up, 3D scanner and shape setting process, as well as small discrepancies between the idealised robot behavior and the practical implementation, mainly in terms of external forces or friction between tubes.

The trajectory displayed by the device in the experiment presents the same approximate characteristics as the planned trajectory, as shown in Figure 3.21, although there are some discrepancies. The discrepancies are considered to be related to imperfections in the experimental implementation, as well as small inaccuracies in the assumptions used in the derivation. In-


Figure 3.14: Experimental set-up with device held vertically inside the 3D laser scanner.


Figure 3.15: Exemplary measurement of the 3D device geometry as a cloud of orange points, with a fitted 3D curve in blue.


Figure 3.16: Projection on the XZ plane of the recorded points describing the geometry of the device in one exemplary measurent, with the corresponding fitted curve.


Figure 3.17: Projection on the YZ plane of the measured points corresponding to the device shape in a specific configuration during the experiment, and fitted curve.


Figure 3.18: Experimental measurements of the device geometry during the advancement of one of the tubes, plotted as a point cloud with a different color for each recorded configuration. The different measurements overlap, confirming follow-the-leader motion throughout the entire device. The curves fitted to each measurement are also displayed.


Figure 3.19: Projection on the XZ plane of the curves fitted to the experimental measurements during advancement of one of the tubes.
terestingly, in the experimental implementation, tube 1 presented an estimated rotation at the expected rate as it was being inserted according to visual observation of the rotation at the base of the tube aided by markers. The apparent torsion of the tubes, also estimated from visual observations at $\alpha(0)$ and $\alpha\left(L-h_{2}\right)$ aided by markers, was minimal, as predicted. Overall, and despite practical imperfections, the experiment satisfactorily illustrates the research on follow-the-leader kinematics and on torsion of the tubes.

### 3.7 Conclusions

### 3.7.1 General conclusions of research on CTRs

The complete set of trajectories where follow-the-leader motion is possible under the assumption of no axial torsion within the robot was discovered in this work. A closed-form expression summarising the set of trajectories was derived. The solution obtained showed that the majority of trajectories in the set present a continuous variation of curvature along the arc length, both


Figure 3.20: Projection on the YZ plane of the curves fitted to the experimental measurements during advancement of one of the tubes.
in direction and magnitude; still, the solution includes all currently known piecewise constant curvature trajectories as a particular case. The analysis presented in this chapter also elucidated the control required for a robot to advance in a follow-the-leader configuration, where the individual tubes must be either static or advancing as part of the robot's distal end.

Additional maneuvers of interest were also extracted from the study of follow-the-leader kinematics. These include the possibility of combining follow-the-leader motion in the proximal part of the robot with general motion at the distal end, or the linkage of trajectories that can be traced in follow-the-leader configuration.

The general analysis of follow-the-leader motion was developed under the assumption of no axial torsion of the tubes. In order to determine the validity of such an assumption, and then be able to select stable robot configurations for follow-the-leader motion in practice, the torsion of the tubes was considered in the trajectories of interest. A closed-form solution describing the torsion of the tubes in the most relevant trajectories where follow-the-leader is possible using two-tube robots was derived. Criteria for the structural stability of the robot were then extracted from such a solution, and a relevant subset of designs was explored. This allowed for


Figure 3.21: Plot of the planned and measured trajectories.
the identification of stable trajectories that can be traced in follow-the-leader motion within an admissible deviation value, which can be specified as desired.

In order to illustrate the work, a suitable stable trajectory was selected for a case study of a prototypical, two-tube CTR. The case study was developed with simulations and an experiment, showcasing the capability of follow-the-leader motion in a trajectory with continuous curvature variation, both in direction and magnitude. This capability in the wider set of trajectories found in this work expands the potential of CTRs.

### 3.7.2 Applicability of CTRs

The complete set of trajectories discovered is non-trivial and significantly increases the known capabilities of CTRs. However, the research conducted on the effects of torsion also suggests that the trajectories found are affected by torsion in a similar manner as existing trajectories traceable with robots composed of piecewise constant curvature tubes. Thus, in the trajectories discovered, the robot also suffers from torsional deviations that introduce deviations from follow-the-leader motion, and it can experience snap-through instabilities in some configurations. The research indicates that the deviations due to torsion and the snap-through instabilities tend to increase with the robot length and the magnitude of curvature in the design and, even though the use of tube designs with significantly different geometrical torsion can reduce the effects of torsion to some extent, the issues caused by torsion remain prominent in the new trajectories found.

The practical applicability of the new trajectories is thus generally limited to trajectories that are relatively short and with shallow curvatures for the requirements of on-wing operations. The possible values of magnitude of curvature in a trajectory depends on the acceptable deviations due to torsion, the desired length, and the environmental constraints that define the overall trajectory geometry. Thus, the viability of CTRs needs to be considered in each individual case. However, in general, for trajectories of interest in on-wing operations where the required length is over 30 cm , the typical magnitude of the curvature radius should not be significantly lower than a 20 cm radius to enable standard operation of a CTR.

This generally does not have a wide applicability in on-wing operations. CTRs can only be useful in specific scenarios that require the insertion of tools along a shallow trajectory, or in scenarios where significant curvature magnitude is only required at the first part of the trajectory, or where trajectory linking can be used. However, CTRs are generally not useful for fine manipulation of an end-effector in on-wing operations. In the particular, reference case previously defined in subsection 1.1.3, CTRs are not useful in general since the required robot length is over 1 m , and torsion makes the robot practically inviable.

CTRs with non-annular cross section can be used to prevent the issues with torsion, although it should be noted that this reduces one DOF per tube in the robot. The same analysis developed in this chapter applies. However, as previously noted, the analysis shows that the new trajectories discovered are not viable in designs with non-annular cross section since they require the relative rotation of the tubes for robot advancement. Thus, the only viable trajectories in CTRs with non-annular cross section are those composed of piecewise constant curvature arcs, with telescopic deployment of the tubes.

This is generally not useful for the fine-positioner in the reference on-wing inspection case defined in subsection 1.1.3. However, it can be used to create a gross-positioner that can follow the access route, performing the 90 degree turn in open space in the turbine chamber, and advancing over the entire route to insert a fine-positioner. The use of a non-annular cross section implies that the gross-positioner has no length limit and, even though it limits the mobility of the device, a few DOFs can suffice for the insertion. A gross-positioner consisting of a CTR with non-annular cross section is selected for robotic system in Chapter 6. Instead, for the fine-positioner, soft robots with fluidic actuation are the main robot concept explored, as presented in the next Chapter 4.

Even though generally not relevant to on-wing operations, the new trajectories for CTRs discovered in this work show promise in MIS. In medical applications, the required trajectory length in general is significantly shorter than in on-wing operations, since the distance between the locations of interest in the human body and the entry points is shorter. Thus, CTRs with annular cross section can be used to exploit some of the new trajectories discovered, and relevant
curvature magnitudes can be used thanks to the shorter trajectory length. CTRs exploiting the new trajectories found could be used, for example, in neurosurgery or heart surgery, where CTRs have the potential to enable new procedures and improve existing ones.

## Chapter 4

## Design of Soft Robotic Manipulators with Fluidic Actuation

The work presented in the previous chapter indicates that CTRs cannot be used for the finepositioner. Thus, soft robots with fluidic actuation represent the main robot concept explored in the present and following chapters to create a fine-positioner. Existing soft robots are generally designed by intuition, and offer a low force. The design of soft robotic manipulators with fluidic actuation is investigated in this chapter by taking a novel, general approach, which is intended as a common framework for the design of soft robotic manipulators. This is then applied to determine the most suitable design of the fine-positioner. It should be noted that this chapter is entirely dedicated to the design of soft robotic manipulators, which are generally composed of a set of segments that need to be capable of bending. The term device is used in this chapter to refer to each of these segments, which need to be designed.

The research presented in this chapter is an edited version of that published in:

- A. Garriga-Casanovas, I. Collison, F. Rodriguez y Baena. Towards a Common Framework for the Design of Soft Robotic Manipulators with Fluidic Actuation. Soft Robotics, 5.5, pp. 622-649, 2018.

In addition, the design method developed in this chapter and the new soft robotic manipulator
designs found are the material of the following patent application:

- A. Garriga-Casanovas, F. Rodriguez y Baena. Manipulator. Patent Application 1812408.1, 2018.

The chapter is structured as follows. The research on design is introduced in section 4.1. The general design problem is formulated in section 4.2 , where the study of design is divided into two parts: design of soft robotic manipulators for a given maximum pressure, and design of soft robotic manipulators for any pressure. The design of soft robotic manipulators for given maximum pressure is addressed first. The outline of the corresponding designs of interest is justified in section 4.3, and the layouts of interest are classified into two categories, which define the possible types of the segments of a robotic manipulator. These correspond to extending and contracting devices. The study of the design of extending devices is presented in section 4.4. A similar derivation for the design of contracting devices is reported in section 4.5. The main design principles derived for extending and contracting devices with a given maximum pressure are summarised in section 4.6. In section 4.7, these design principles are applied to the design of a bending device in a prototypical scenario that showcases the work. Finite element (FE) simulations to determine two parameters of the bending device in the prototypical scenario and verify the work are also reported in section 4.7. The extension of the design study to devices where any desired value of maximum pressure can be used is described in section 4.8. A non-dimensional analysis of the designs of interest is outlined in section 4.9. Finally, the most suitable design of the fine-positioner is presented in section 4.10, and conclusions on the overall design study and the final fine-positioner design found are summarised in section 4.11.

It should be noted that nomenclature is redefined in each chapter of this thesis, including this one. This is due to the fact that the work presented in the various chapters addresses very different aspects of robotics, and requires a significant number of different variables in each chapter.

### 4.1 Introduction

As previously mentioned in chapter 2 , a myriad of soft robots with fluidic actuation exist, which are aimed at very different applications including manipulation, locomotion, or gripping. The majority of these applications require the soft robot to provide a controlled motion between two points of interest in a solid structure while supporting external wrenches. Soft robots with fluidic actuation offering this type of operation represent the technology required to create a manipulator that can be used as fine-positioner, and thus are the focus of this work. Specifically, in this chapter we concentrate on the design of the individual elements providing the controlled motion between two points, which can be part of a system comprising multiple similar elements, such as a manipulator composed of a set of serially stacked segments. The design of each of these elements can be studied relatively independently, and therefore in the rest of this chapter the focus is on the design of the individual elements. These individual elements, or segments of robotic manipulator, are treated as individual devices, such as that shown in Figure 4.1, and the term device is used to refer to them.

These devices can be divided according to the motion they provide between the two points of interest when pressurized. This results in three preliminary categories: devices that provide elongation, contraction, and bending. The design of elongating devices is relatively straightforward, as the elongation is directly created by the pressure applied to the chamber walls in the elongation direction, and the structure generally opposes to it. Thus, the design simply involves a structure that facilitates elongation while containing the pressurized fluid and preventing radial expansion. Furthermore, piston-cylinder devices provide efficient solutions to elongation needs [177], hence elongating devices are not considered further. Contracting devices are equivalent to pneumatic artificial muscles (PAMs). The design and mechanical properties of PAMs are extensively studied in the literature [123,124], and therefore are not analyzed further in this work.

The design of devices that provide bending, on the other hand, is challenging, and a general rationale for their design is not available. The canonical application for bending devices is manipulation, which is also the application of this work. In this regard, bending devices are
generally interpreted as segments of soft robotic manipulators. As previously mentioned in chapter 2, a profusion of bending devices have been proposed in recent decades, mostly aimed at manipulation, including the FMA [98, 101], the OctArm [178, 179], or Stiff-Flop [112, 113], among others. However, in spite of the wide range of designs now available, bending devices are still designed mostly by intuition.

A first study of the design of bending devices was recently published [83]. However, it only offers a specific analysis of a set of predefined designs, but it is not applicable for a generalized design study. In addition, the derivation in [83] relies on equilibrium conditions that may not always be justified, and the paper only considers the effects of external forces at zero deflection configurations. A comprehensive set of tools for the design of soft robots are available at the soft robotics toolkit [180]. However, these tools are predominantly based on finite element (FE) methods and experiments centred on a set of predefined designs, which are suitable for the analysis and optimisation of specified classes of designs, but are not applicable to address the design problem in general. In this regard, to the best of this author's knowledge, there is no general framework for the design of bending devices, which hinders the identification of the best existing designs, complicates the development of novel and improved devices (such as that required for the fine-positioner), and ultimately hampers the advancement of the field.

In this chapter, a general study of the design of soft robots with fluidic actuation that provide bending is presented. In the study, the design layouts of interest are first justified, and a set of design principles is then derived. These principles can be used to outline the design of devices for each application, and to define subsequent design optimization, which is also developed in this chapter. The work is a applied in a medical case study to illustrate the new design methodology proposed and verify the design principles. Finally, the work is applied to determine the most suitable design of the fine-positioner. The study is developed first for devices with a given maximum pressure, which represents the most relevant design problem, and is then extended to cases where any pressure can be used.

The foundation for the study is a novel approach that considers the equilibrium of devices isolated in arbitrary cross sections to provide insight into their mechanical behavior. Such an


Figure 4.1: Schematic diagram of a bending device with a completely general design.
approach is adapted from existing work on tendon-driven, continuum manipulators [181], with parallelisms that are apparent in the analysis. The approach serves both to study the design of soft robotic manipulators with fluidic actuation, and to mechanically model them for accurate control. In this chapter, the focus is on design; mechanical modelling and control are presented in chapter 5 .

The approach proposed in this chapter is applicable to any design, and therefore the study developed here is general. The findings in terms of design principles coincide with some design trends in existing literature, elucidating the relevance of this work. In this regard, this work aims to contribute towards the development of a common framework for the design of bending devices, serving as a reference to compare existing designs, and providing an analytical instrument, together with a set of principles for the design of soft robotic manipulators. It should be noted that the nature of the analysis in this work is generally qualitative, although mathematical elements are employed to facilitate the derivation.

### 4.2 Problem formulation

The purpose of the segments of soft robotic manipulator considered here is to provide a desired motion between two points on the device, which in this case is associated to bending, together with a certain force. In soft robots with fluidic actuation, the motion is achieved by pressurizing
a set of chambers in the device to produce structural deformation. The most common scenario of interest is that where the robot must generate work to produce the motion, overcoming external wrenches, and this is the case of interest for the design of a fine-positioner. However, the study presented in this chapter is completely general, without limitations on the possible designs or on the operational scenario.

The design problem is to select the geometry and structural properties of the soft robot to achieve the desired motion and maximise a specified performance. In this work, the design problem considered is completely general, without predefined design variables. Solving this problem generally requires determining the solution to a non-linear structural problem with large deformations, for which analytical solutions are not available in general. Thus, an innovative approach is required, as presented in the following sections.

The exact maximum pressure that a soft robot design can withstand can be very complex to determine. In general, however, this can be considered to be primarily determined by outer wall design, and by the sealing points in the chambers. In addition, in the common case of medical applications, the maximum pressure can also be limited by the operation environment, e.g. to guarantee the safety of a patient in the case of a malfunction during a medical procedure. It should be noted that in cases where the maximum pressure is limited by the outer wall, this refers to the wall in the device that separates any chamber from the outside. Any partition walls defining chambers inside the device generally do not limit the maximum pressure since the rubbers typically used in soft robots can reach significant strains, so partition walls may deform when pressurised, but remain internal and generally do not burst.

The effect that the choice of outer wall has on the maximum pressure that a device can withstand is a relatively simple and independent design factor to consider. In general, an outer wall that can withstand the maximum possible pressure is desirable to maximise the force of the device (typically by making it with fibres and silicone). And generally, a thicker and stronger wall can withstand higher pressure, but also introduces more bending stiffness on the device and takes more space, which limits performance. In this regard, it is a relatively independent factor to consider in the design. The effect that the strength of the sealing points in the chambers has
on the maximum pressure is also a relatively independent factor, and is usually related to the fabrication method and assembly of parts rather than design of the soft robot.

In this regard, the first design problem of interest is typically the design of a device for a given maximum pressure, since this maximum pressure is generally either dictated by the application such as in MIS, or by factors that are relatively independent of the rest of the design. In the particular case of this work, the limiting factor on maximum pressure initially was the strength of the sealing points. Thus, the design of soft robotic devices is first studied for given maximum pressure. The study is then extended to a case where any value of pressure can be used.

The performance criteria for the design must generally be related to the purpose of these devices, i.e. to provide a bending motion while supporting external wrenches. Typically, soft robotic manipulators are required to be capable of reaching a specific deflection determined by the desired workspace. The wrenches that can be supported at that deflection tend to be their main limitation. In this regard, the design objective selected in this work is to maximise the wrenches that can be supported while achieving a desired deflection.

The design of devices to maximise the wrenches that can be supported with a given maximum pressure is thus studied first in sections 4.3, 4.4, 4.5, 4.6, 4.7. The work is then extended to the case of any possible pressure in sections 4.8, 4.9, and 4.10.

### 4.3 General design layouts

The wide diversity of possibilities for the design of a soft robotic manipulator makes it difficult to directly address the general design problem and determine the design. It is therefore appropriate to first outline the design space, and then use a more detailed study to derive the design principles. In this work, a preliminary analysis is first used to bound the design space and discretise the design options, as described in this section. This enables a subsequent detailed study of the two layouts of interest, which is derived in the following sections. It should be noted that the analysis in this section is general and independent of the desired performance
criteria; the study is then particularised in subsequent sections to the performance criteria selected for this work.

Any potential design must consist of a general structure linking the two points of interest, as illustrated in Figure 4.1. In soft robotic manipulators with fluidic actuation, the structure is passive, and therefore the design must contain a set of chambers that can be pressurised to generate the desired motion by deforming the structure. This set of chambers must generally cover the region between the two points of interest in a nearly continuous manner, as otherwise parts of the device would act as structures that simply transmit loads, which are not the focus of this work.

The set of chambers, together with the direction of bending, which is approximately perpendicular to the vector between the two points of interest, define two sides of the device, which can be considered as two walls. Kinematic considerations show that, in order to achieve bending, a differential deformation in the structure at either side of the device is required. This involves either one wall extending more than the other, or one wall contracting more than the other. A segment of soft robotic manipulator can therefore generate bending in two elementary ways, and the designs can be classified accordingly, leading to two general categories: extending-type devices, and contracting-type devices, as illustrated in Figure 4.2.

The equilibrium of a system corresponding to the general design isolated at an arbitrary cross section perpendicular to the vector between the two points of interest can then be considered, as shown in Figure 4.3. This exposes the reaction forces as well as the pressure applied by the fluid. The system equilibrium can thus be used to extract insights into the mechanical behaviour and to study the design, and it represents a cornerstone of the analysis presented in this chapter. Before a detailed study, the equilibrium can first be applied to the two categories of a segment of soft robotic manipulator, extending and contracting devices, to outline the design layouts, as described in the following two paragraphs.

Considering the equilibrium in extending devices, this indicates that the pressure in the chambers generally creates tensioning reactions on the structure. The reactions associated to each side of the structure depend on the design. These reactions translate into deformations, with


Figure 4.2: Conceptual illustration of the general layouts corresponding to the two possible types of soft robotic manipulators: extending devices (a) and contracting devices (b).


Figure 4.3: Equilibrium diagram of a general bending device isolated at an arbitrary cross section, exposing the pressure applied by the fluid as well as the structural reactions.
the elongation of each side depending on the stiffness in the longitudinal direction. The differential elongation necessary for bending can therefore be achieved with either an asymmetric pressure loading or an asymmetric longitudinal stiffness. It should be noted that the reactions can also produce lateral expansion, but this generally does not contribute to elongation, rather the opposite, so it is undesirable in extending devices. Thus, the layout of extending devices must consist of an elongated structure that cannot expand radially and has a combination of asymmetric geometry and asymmetric longitudinal stiffness so that one side extends more than the other. The specific combination of geometry and stiffness affects the performance, and requires a detailed study, presented in section 4.4.

Considering the equilibrium in contracting devices, this also shows that the pressure tends to generate tensioning reactions. Contraction cannot be achieved with a compression of the structure at one side of the device that substantially reduces its length, and instead one side of the structure must either protrude outwards or buckle inwards to generate the contraction. The layout of contracting devices must then consist of a structure with one side that either protrudes
or buckles to produce a contraction while the other side maintains the original length, resulting in bending. The principle of operation is similar to that of PAMs, e.g. see [124], and some of the analysis can be adapted from there. Still, the equilibrium analysis indicates that both the design geometry and the longitudinal and bending stiffnesses affect the reaction forces, the protrusion geometry, and ultimately the performance, requiring a detailed examination. The study of the design of contracting devices is reported in section 4.5.

Considering that extending and contracting devices are the only alternatives to produce bending, the study of these two layouts represents a complete study of the design of soft robotic manipulators with fluidic actuation. Devices combining extension and contraction are also possible, and their design is a combination of the design principles for both types of operation. The design of a device combining both extending and contracting operation is presented in section 4.7.

### 4.4 Design of extending devices

Extending devices achieve bending thanks to a differential extension in their structure when pressurised, which is created by a design asymmetry in terms of geometry and stiffness. The design of extending devices is studied in detail in this section in order to derive a set of design principles and determine the design that maximises the design objective. It should be noted that, previously mentioned in the previous sections, the term device is used here to refer to a segment of soft robotic manipulator.

Considering that the design objective is to achieve a desired deflection and maximise the force for a given maximum pressure, the study is divided into two parts. First, the study is focused on the design to maximise the forces and moments that can be supported at a given deflection with a constrained pressure, as described in subsections 4.4.1, 4.4.2, 4.4.3. Then, the analysis considers the design objective of reaching the desired deflection with a minimum pressure, as presented in subsection 4.4.4. The results of both analyses are combined to extract design principles and determine the most suitable design, summarised in subsection 4.4.5, while the
overall analysis is finally generalised to 3D in subsection 4.4.6.

### 4.4.1 Equilibrium approach

## Equilibrium formulation

The equilibrium of an extending device isolated at an arbitrary cross section can be considered, as shown in Figure 4.4, exposing the reactions as well as the pressure applied by the fluid. The equilibrium of moments and forces in the direction perpendicular to the cross section can thus be imposed as

$$
\begin{align*}
& T_{1}+T_{2}=p x-F_{n}  \tag{4.1}\\
& T_{1}\left(c_{1} d+x\left(1-c_{1}\right)+b\left(c_{2}-c_{1}\right)\right)-p \frac{x^{2}}{2}-p x c_{2} b+F_{n}\left(h-b\left(1-c_{2}\right)\right)=M
\end{align*}
$$

where $d$ denotes the total region of the cross section, $x$ represents the region of the cross section corresponding to the pressurised fluid, and $b$ is the region of the cross section corresponding to wall 2. The external forces are decomposed into two directions, parallel and perpendicular to the cross section. The perpendicular forces are aggregated into a resulting normal force, denoted by $F_{n}$, and the parallel forces are aggregated into a resulting tangential force $F_{t}$. M corresponds to the sum of external moments together with the moment created by $F_{t}$ with respect to the cross section.

The distributed normal stresses corresponding to wall 1 and 2 are aggregated into two equivalent forces, denoted by $T_{1}$ and $T_{2}$, respectively, while the distributed tangential stresses are aggregated into $T_{t 1}$ and $T_{t 2}$, respectively. The location of the equivalent line of application of $T_{1}$ and $T_{2}$ is defined by the non-dimensional parameters $c_{1}$ and $c_{2}$, respectively. The specific equivalent line of application of these two forces may not be constant and can be difficult to determine as it depends on the specific stress distribution, which is determined by a complex structural behavior. However, considering that, in soft robots with fluidic actuation, and particularly in extending devices, the walls are in tension, the equivalent point of application of $T_{1}$ and $T_{2}$ must be within the respective walls. Thus, the variables $c_{1}$ and $c_{2}$ are bounded


Figure 4.4: Equilibrium diagram of extending device isolated at an arbitrary cross section, exposing the reaction forces, aggregated into $T_{1}$ and $T_{2}$, and the pressure applied by the fluid.
$c_{1}, c_{2} \in[0,1]$. As will be seen in the following, the walls should be thin, and therefore the stress distribution can generally be considered to be relatively uniform, leading to values of $c_{1}$ and $c_{2}$ near $1 / 2$. However, the specific point of application does not affect the subsequent derivation, and therefore need not be considered further.

The description of the cross section with $d, x$, and $b$ is convenient, as $d$ is generally a parameter determined by constraints from the environment, and then the design study involves selecting the variables $x$ and $b$. It should be noted that the variables $x, b$ and $d$ are then geometrically bounded. In particular, $x>0, b>0, d>x+b$. Thus, some of the constraints are coupled. It should also be noted that for extending devices to operate, $F_{n}<p x$.

The external wrenches applied in the device can be any combination of forces and moments. The point of application of $F_{n}$ is determined by the specific external forces in each scenario. The contribution of $F_{n}$ to the moments equation in (4.1) depends on the distance between the line of application of $F_{n}$ and the line of application of $T_{2}$. The $F_{n}$ applied may thus influence the $M$ that can be supported, and vice versa. However, maintaining the contribution of $F_{n}$ to the moments as a separate force with a certain point of application is desirable as it shows the separate wrenches that can be supported by a design, and the effect of $F_{n}$ on $M$.

## Equilibrium discussion

Equation (4.1) indicates that $b$ affects the contribution of $F_{n}$ to $M$ through the term $F_{n} b\left(1-c_{2}\right)$, which has an effect on the device's performance. However, this is due to the fact that changes in $b$ involve displacing the point of application of $T_{2}$. Equivalent alternatives for displacing the point of application of $T_{2}$ relative to the point of application of $F_{n}$ include displacing the entire wall 2 , or displacing the entire device. However, any possible offset of the external forces relative to the device to improve performance is considered to be already applied in practice. The problem of interest in terms of design is to maximise performance for a given external loading. In this regard, the effect of varying $b$ on $F_{n} b\left(1-c_{2}\right)$ is not relevant from a design perspective as it is equivalent to offsetting the device, and it is therefore disregarded in the design derivation.

The cross section where equilibrium is considered is arbitrary, and therefore the analysis can be applied to any cross section on the device. This provides insight into the mechanical behavior of the entire device, and therefore it serves to study the design.

The equilibrium of forces also shows that external forces parallel to the cross section must be supported at the boundary where the device is isolated. Considering the definition of fluid, the direction of the pressure force is always normal to the boundary. Thus, the lateral forces must be supported by the structure in any design, particularly by $T_{t 1}$ and $T_{t 2}$. The contribution of this shear stress to the deflection, however, is considered to be relatively small, following the standard study of structures. In this regard, the equilibrium in the direction parallel to the cross section is not considered further.

The system of equations (4.1) provides the reactions $T_{1}$ and $T_{2}$ for any $M$ and $p$ given a design. These solutions, however, correspond to different structural deformations and therefore different displacements. Thus, the equilibrium alone cannot be used to determine the design to maximise $M$, as a combination of $T_{1}$ and $T_{2}$ to increase $M$ always exists, but it may correspond to an undesirable deflection. In order to study the design for a given deflection of interest, a condition imposing a desired deflection to be maintained is required.

### 4.4.2 Deflection condition

The purpose of the deflection condition is to define the relation between $T_{1}$ and $T_{2}$ that must be satisfied for a desired deflection to remain constant. In particular, the deflection must remain constant despite variations in the external wrenches and pressure applied.

Deflection depends on the differential wall extension. Thus, deflection can be maintained even at different pressures provided that both walls extend. The deflection condition can therefore not be determined from a specified extension value at each wall, but rather must be derived from a ratio between the extensions of both walls.

In order to attain a desired deflection, even without external wrenches, a certain extension at each wall is necessary, which corresponds to the initial extension of the walls. Once the initial
deflection is achieved, it can be maintained even for variable external wrenches by compensating with pressure. More specifically, deflection can be maintained at variable values of wall extension provided that any increase in length in a wall is accompanied by a certain increase in length at the other wall. A condition to maintain a deflection can therefore be obtained by imposing the increase in length at both walls to be related through a certain ratio $R$ as

$$
\begin{equation*}
\delta_{1}=R \delta_{2} \tag{4.2}
\end{equation*}
$$

where $\delta_{i}$ denotes the increase in length in wall $i$ with respect to the length necessary to attain the initial deflection. The value of the ratio $R$ is generally close to 1 , but it can depend on the desired deflection. However, the derivation in this work does not require the exact value of $R$, and it is therefore not specified. It should be noted that any variation in extension must be associated with a variation in both external wrenches and pressure.

The extension in a wall depends both on the stress applied and the wall stiffness. In addition, the initial extension required in each wall to reach the initial deflection involves a certain initial tension $T_{i 0}$ for $i=1,2$. In this regard, the deflection condition cannot simply impose a relation between $T_{1}$ and $T_{2}$, but it must include the stiffnesses of the walls as well as the initial tension of the walls. The increase in extension $\delta_{i}$ in a wall $i$ can be related to the increase in tension in that wall $T_{i}-T_{i 0}$ through a variable stiffness $s_{i}$ as

$$
\begin{equation*}
T_{i}-T_{i 0}=s_{i} \delta_{i} \tag{4.3}
\end{equation*}
$$

The value of $s_{i}$ can be difficult to determine, and it is not necessarily constant. In general, $s_{i}$ can depend on the material, the design, and the deformation. However, the specific $s_{i}$ is not calculated here since it is not necessary for the derivation.

Substituting the relation between extension and tension (4.3) into (4.2), the condition that must be satisfied for a deflection to be maintained is obtained as

$$
\begin{equation*}
T_{2}=\frac{T_{1} s_{2}}{R s_{1}}+T_{20}-\frac{T_{10} s_{2}}{R s_{1}} \tag{4.4}
\end{equation*}
$$

The deflection condition is thus expressed as a relation between $T_{1}$ and $T_{2}$, as well as as set of parameters.

This condition (4.4) is applicable to any scenario with any desired deflection and external wrenches. The two terms on the right depend on the conditions to achieve initial deflection, and thus the desired deflection in each scenario is imposed by these terms. These two terms are constant, and are analysed in subsection 4.4.4. The value of $R$ may also vary to some extent for some of these different scenarios, although in some instances the value of $R$ can be equal for different deflections. Still, all these parameters are specified for a given scenario. Thus, (4.4) defines the relation between $T_{1}$ and $T_{2}$ that guarantees the deflection to be maintained in any scenario.

Interestingly, in the case of infinite stiffness at wall 1, the deflection condition (4.4) simply imposes $T_{2}$ to be constant. This is a typical situation as will be seen in the following, where designs with infinite wall 1 stiffness are particularly relevant. However, a constant $T_{2}$ is not a valid condition to maintain deflection in general, since, in extending devices, wall 2 may need to extend to a certain degree as pressure increases in order to compensate the extension in wall 1.

### 4.4.3 Design derivation

The equilibrium and the deflection condition can be combined to analyse the design problem and derive a set of design principles, as described in the following.

## Preliminary qualitative considerations

The equilibrium analysis, illustrated in Figure 4.4, indicates that the moment at the cross section necessary to support the external wrenches is created between the pressure and the reactions. In particular, since pressure can only act in one direction, and the structure generally only acts in the opposite direction, the moment is created between the pressure pushing and the structure pulling.

The main challenge is supporting wrenches that tend to reduce the deflection, i.e. wrenches that contribute as positive values of $M$. Opposite wrenches increase the deflection, and supporting them is thus trivial.

The pressure is always acting between the two walls in tension. Thus, the moment must be created between $T_{1}$ pulling and $p$ pushing. $T_{2}$, on the other hand, opposes to this moment, and is therefore undesirable in general. The only purpose of wall 2 is to contain the pressurised fluid.

This qualitative analysis indicates that maximum $T_{1}$ and minimum $T_{2}$ are desirable. This could lead to the impression that concentrating the pressure application near wall 1 , for example using thick or even hollow structure in wall 2 , maximises performance, as it maximises $T_{1}$ and minimises $T_{2}$. However, this arrangement also promotes an undesirable deflection. In the extreme case, a design with $T_{2}=0$ and thus $T_{1}=p x$ would be possible, but it would yield zero or negative deflection, which is undesirable as deflection must be maintained. Conversely, concentrating the pressure application close to wall 2 would minimise the increase in $T_{1}$ and thus the reduction in deflection when pressure is increased, enabling $p$ to compensate generate the majority of the moment without loss of deflection. However, this also results in low $T_{1}$ and thus low wrenches that can be supported. The analysis combining the equilibrium (4.1) and deflection condition (4.4) is derived in the following to resolve these design questions.

## Detailed analysis and derivation

Imposing the condition requiring a deflection to be maintained (4.4) into the equilibrium of forces in (4.1) yields

$$
\begin{equation*}
T_{1}=\frac{p x-F_{n}-\kappa}{1+\frac{s_{2}}{R s_{1}}} \tag{4.5}
\end{equation*}
$$

where $\kappa=T_{20}-T_{10} s_{2} / R s_{1}$ which corresponds to the initial deflection conditions. Substituting (4.5) into the equilibrium of moments in (4.1), the $M$ that can be supported for a given design
and a certain deflection is obtained as

$$
\begin{equation*}
\frac{p x-F_{n}-\kappa}{1+\frac{s_{2}}{R s_{1}}}\left[c_{1} d+x\left(1-c_{1}\right)+b\left(c_{2}-c_{1}\right)\right]-p \frac{x^{2}}{2}-p x c_{2} b+F_{n}\left(h-b\left(1-c_{2}\right)\right)=M \tag{4.6}
\end{equation*}
$$

It should be noted that, as previously mentioned, $T_{1}>0$ for operation to be possible since otherwise the structure would be in compression, and the device would not act as an extending device but rather as a passive structure. Thus, from (4.5), this implies a bound $p x-F_{n}-\kappa>0$.

Expression (4.6) enables determining the design to maximise the desired performance, which in this case involves maximizing $M$. Expression (4.6) is applicable to any deflection, and therefore it can be used to address the design problem in any scenario. It should be noted that the effect of the terms corresponding to initial deflection, aggregated in $\kappa$, is studied separately in subsection 4.4.4.

The design principles can be extracted by considering the contribution of the design variables to $M$ in (4.6). The stiffnesses $s_{1}$ and $s_{2}$ appear only as a ratio $s_{2} / s_{1}$. The ratio only contributes to the denominator of a term that should be maximised for the case of interest $p x-F_{n}-\kappa>0$, and therefore $s_{2} / s_{1}$ should be minimised. As previously mentioned, the values of $s_{1}$ and $s_{2}$ may depend on the design as well as the material. However, in soft robotic devices, the material can generally be chosen to provide any desired stiffness, particularly including low values. In this regard, the material can be used to select $s_{1}$ and $s_{2}$, compensating for any variation in stiffness associated to the geometry. Thus, the minimization of $s_{2} / s_{1}$ is considered to be attainable with the material choice, independently of the rest of the design.

The variables $s_{1}$ and $s_{2}$ represent the overall stiffness of a wall, but the local stiffness within the wall needs not be constant. The specific stiffness distribution affects the line of application of $T_{1}$ and $T_{2}$, and therefore can be used to modify $c_{1}$ and $c_{2}$. The line of application is determined by the location where the moment generated by the distributed stress within a wall is equal to that created by $T_{1}$ or $T_{2}$. For a given wall in extension, corresponding to a deflection, the normal stress within the wall can be considered to be strongly dependent on the local stiffness, especially if the stiffness distribution over the cross section presents significant differences.

Thus, the wall layers with markedly higher stiffness generally involve higher local stress, and the line of application of the equivalent force can be considered to tend to these layers.

The stiffness distribution can therefore be used to modify $c_{1}$ and $c_{2}$. However, it should only be used for $c_{1}$. Considering that $s_{2}$ should be minimised, and that low stiffness is difficult to attain, any stiffness variation typically involves an increase in $s_{2}$, reducing the performance. Instead, a high $s_{1}$ can generally be maintained since local stiffness can typically be increased to compensate local reductions. Equation (4.6) indicates that a high $c_{1}$ is desirable, and therefore wall 1 should have a high stiffness in the outer layers and lower stiffness in the inner layers. Still, this is only relevant in designs where wall thickness is substantial, which are typically not the designs of interest, as shown in the following.

For a $s_{2} / R s_{1}$ that is minimised, $\kappa$ is typically negligible, as can be seen from the analysis in the next subsection. The derivation of the rest of design principles can then be divided in two cases for clarity of exposition.

Case with $F_{n}=0$ and negligible $\kappa$

A case with $F_{n}=0$ and $\kappa$ negligible can be considered first as it represents a common scenario of interest where the robotic manipulator must support external forces in the direction of bending, as in a nearly horizontal robot segment supporting and moving a payload against gravity, or a nearly horizontal segment moving a set of additional segments stacked serially at its distal end, which generate an external lateral force and moment. In addition, the case with and negligible provides a first intuitive understanding of the design principles. In this case, and for $s_{2} / R s_{1}$ negligible relative to 1 , each of the variables $b, x, d$ only affects one or a small number of terms in (4.6), and thus can be easily determined. In addition, $p$ can be factorised, so the desired value of these variables is independent of pressure.

The variable $b$ affects three terms, the combination of which always reduces $M$ since $s_{2} / R s_{1}>=$ 0 , and $x, c_{1}$ and $c_{2}$ are non-negative. Hence, $b$ should be minimised, which can be written as $b=0$. Then, for $b=0$, the value of $x$ to maximise $M$ depends on $c_{1}, s_{2} / R s_{1}$ and $d$. If $1-2 c_{1}-s 2 / R s 1>0$ then $x$ should be maximised, and therefore $x=d$. If $1-2 c_{1}-s 2 / R s 1<0$
then the value of $x$ to maximise $M$ is

$$
\begin{equation*}
x=\frac{2 c_{1} d}{2 c_{1}+s 2 / R s 1-1} \tag{4.7}
\end{equation*}
$$

Considering that $s_{2} / R s_{1}$ should be minimised, this implies $x=d$. Thus, in a design where $s_{2} / R s_{1}$ is minimal, the most suitable cross section is $x=d$ regardless of the value of the parameters $c_{1}$ and $c_{2}$. The design of the cross section with maximal $x$ and minimal $b$, so that the cross-sectional region corresponding to the pressurised fluid is maximal, in designs were $s_{2} / R s_{1}$ is minimised, represents another relevant design principle. It should be noted that in some practical cases it may not be possible to minimise $s_{2} / R s_{1}$ due to manufacturing or material constraints. In these cases, $x$ is determined by (4.7), which may be lower than $x=d$.

Finally, the parameter $d$ is determined by the practical application, but expression (4.6) highlights that increasing $d$ results in higher force. Thus, $d$ should be maximised to occupy all available room in each scenario.

Case with $F_{n}, \kappa \neq 0$

Considering a general scenario including $F_{n}$ and $\kappa$, a similar analysis can be applied to determine the design principles. In this case, the contribution of $F_{n} b\left(1-c_{2}\right)$ to $M$ in (4.6) should be disregarded, as discussed in subsection 4.4.1. Then, for a $s_{2} / R s_{1}$ negligible relative to 1 , the variables $x$ and $b$ can be analysed in conjunction. The analysis is divided in two further cases depending on the sign of $F_{n}+\kappa$.

For $F_{n}+\kappa<0$, the terms in (4.6) containing either of these variables can be aggregated into three groups: terms containing $x b$, terms containing sums of $x$ and $b$ and terms containing only $x$, as

$$
\begin{align*}
\tau_{1} & =-p x b c_{1} \\
\tau_{2} & =\left(-F_{n}-\kappa\right)\left[c_{1} d+x\left(1-c_{1}\right)+b\left(c_{2}-c_{1}\right)\right]  \tag{4.8}\\
\tau_{3} & =p x \frac{x\left(1-2 c_{1}\right)+2 c_{1} d}{2}
\end{align*}
$$

The terms corresponding to $\tau_{1}$ reduce $M$ and should therefore be minimised, which entails that either $x$ or $b$ should be minimised. The term corresponding to $\tau_{2}$ should be maximised which,
considering that $b+x<d$, implies that combinations of $x$ and $b$ that yield $b+x=d$ are desirable. In particular, if a trade-off between $x$ and $b$ is possible, combinations with higher values of $x$ are preferable since the contribution of $x$ to $\tau_{2}$ is higher. Finally, the terms corresponding to $\tau_{3}$ contribute to $M$, and should therefore be maximised.

The value of $x$ to maximise $\tau_{3}$ deserves consideration as the relation between $\tau_{3}$ and $x$ is parabolic. If $c_{1}>1 / 2$, then $\tau_{3}$ is maximised with the specific value of $x$

$$
\begin{equation*}
x_{m}=-\frac{c_{1} d}{1-2 c_{1}} \tag{4.9}
\end{equation*}
$$

which is always $x_{m}>=d$. Considering the constraint $x<d$, the value of $x$ should then be $x=d$. If $c_{1}<1 / 2$, then $\tau_{3}$ as a function of $x$ is a parabola that tends to infinity and intersects the $x$ axis at 0 and at a negative value. Hence, $x$ should also be maximised. Thus, for any $c_{1}$ within the possible values, if $s_{2} / R s_{1}$ can be minimised as previously discussed, then the $x$ to maximise $T_{3}$ should be $x=d$.

The desirable values of $x$ and $b$ can thus be determined. $\tau_{1}$ requires either $x$ or $b$ to be minimised, $\tau_{2}$ indicates that a trade-off between $x$ and $b$ be achieved, prioritizing $x$, and $\tau_{3}$ requires $x$ to be maximised. Hence, $b$ should be minimised, which can be expressed as $b=0$, and $x$ should be maximised, yielding $x=d$.

For $F_{n}+\kappa>0$, a similar derivation can be used. Defining a change of variable $y=p x-F_{n}-\kappa$, the terms in (4.6) containing either $y$ or $b$ can be aggregated into three groups: terms containing $y b$, terms containing only $b$ and terms containing only $y$, as

$$
\begin{align*}
& \tau_{1}^{\prime}=-y c_{1} b \\
& \tau_{2}^{\prime}=-\left(F_{n}+\kappa\right) c_{2} b  \tag{4.10}\\
& \tau_{3}^{\prime}=y c_{1} d+\frac{y^{2}}{p}\left(\frac{1-2 c_{1}}{2}\right)-\frac{y\left(F_{n}+\kappa\right) c_{1}}{p}
\end{align*}
$$

The terms corresponding to both $\tau_{1}^{\prime}$ and $\tau_{2}^{\prime}$ reduce $M$, and thus should be minimised. Instead, the terms corresponding to $\tau_{3}^{\prime}$ increase $M$, and should be maximised.

As in the previous case, the maximization of $\tau_{3}^{\prime}$ requires some consideration. If $c_{1}<1 / 2$, the
relation between $y$ and $\tau_{3}^{\prime}$ is a positive parabola that intersects the $y$ axis at 0 and at a negative value, since $F_{n}+\kappa<p x$. Thus, $y$ should be maximised. If $c_{1}>1 / 2, \tau_{3}^{\prime}$ as a function of $y$ is a negative parabola that is maximised at

$$
\begin{equation*}
y_{m}=-\frac{p c_{1} d-\left(F_{n}+\kappa\right) c_{1}}{1-c_{1}} \tag{4.11}
\end{equation*}
$$

which is always $y_{m}>p d-F_{n}-\kappa$. The value of $y$, however, is bounded $0<y<p d-F_{n}-\kappa$ since $p x>F_{n}+\kappa$ and $x<d$. Thus, for $c_{1}>1 / 2, y$ should also be maximised.

The desirable values of $y$ and $b$ to maximise $\tau_{3}^{\prime}$ and minimise $\tau_{1}^{\prime}, \tau_{2}^{\prime}$ are then maximum $y$ and minimum $b$. Reversing the change of variable $y=p x-F_{n}-\kappa$, this implies $b=0$ and $x=d$.

The design principles for all admissible values of $F_{n}+\kappa$ are therefore equal to those in the case where $F_{n}=0$. Hence, these constitute general principles to maximise the wrenches that can be supported at a given deflection.

## Final derivation considerations

This analysis was derived considering the equilibrium in an arbitrary cross section, and therefore it is applicable to any cross section on the device. In addition, it also applies to any deflection, pressure, and external wrenches. Thus, the design principles can generally be used to determine the design of a device to maximise the wrenches that can be supported.

For a given design and deflection, both the reactions at the cross section and $p$ vary with the external wrenches applied to create the moment that maintains equilibrium. Specifically, for an increase in $M$, both $p$ and $T_{1}$ must increase. If $s_{1}$ is not negligible, then the increase in $T_{1}$ is accompanied by an increase in $T_{2}$ that maintains deflection, with a ratio that depends on $s_{2} / R s_{1}$. However, as discussed in the previous subsections, $s_{2} / R s_{1}$ should always be minimised, and therefore the increase in $T_{2}$ is generally low.

The moment created by the external wrenches can vary in different cross sections. However, the design to maximise performance remains equal in all cross sections regardless of the equivalent
wrenches, as argued in the previous paragraphs. The variable moment in different cross sections can result in uneven deformation along the device, but that simply implies a small variation in $R$, which does not affect the derivation. Thus, a constant cross-sectional design throughout the device, with a design determined by the design principles derived in previous paragraphs, is the most suitable design solution in general.

## Derivation discussion

It should be noted that, in designs determined by the design principles derived here, bending is mainly achieved with a differential stiffness in the two sides of the structure, rather than an asymmetric geometry. The values of $T_{1}$ and $T_{2}$ can therefore be equal, but the different longitudinal stiffness in both walls produces the deflection. In addition, when external wrenches are supported, $T_{2}$ can be lower than $T_{1}$, but the deflection can be maintained thanks to the different stiffness in both walls.

The designs principles derived here are valid for $T_{1}$ and $T_{2}$ with a line of application anywhere within the wall thickness. Thus, even singular designs with a hollow wall structures to create separation are considered, but these are undesirable according to the design principles, which is a consequence of the fact that maximizing the cross-sectional region corresponding to the pressurised fluid is always desirable. In this regard, the results of the design analysis are general in terms of maximizing the force of the device at a given deflection.

The analysis indicates that the wrenches that can be supported depend on the maximum pressure. Thus, if the pressure limit was infinite, the device would be capable of supporting practically any wrenches, which illustrates the potential of soft robots with fluidic actuation. Still, elongation of the device would occur for finite $s_{1}$, complicating the practical implementation.

It should also be noted that the design principles only require the ratio $s_{2} / s_{1}$ to be minimised, but the absolute value is not imposed. This could lead to the false impression that the absolute stiffness is not relevant to the device performance. However, the absolute stiffness affects the pressure required to reach the desired deflection, as described in the following.

### 4.4.4 Initial deflection

A similar approach as that described in previous subsections is applied here in order to study the most suitable design to attain a desired deflection with minimum pressure. The same equilibrium of the device isolated at an arbitrary cross section can be considered, as illustrated in Figure 4.4. This provides the reactions for a given cross sectional design.

Deflection is achieved with a differential extension of the walls. This can be attained with either a difference between $T_{1}$ and $T_{2}$, a difference in stiffness of the walls, or a combination. The absolute extension in a wall $i$, denoted by $\Delta_{i}$, can be related to the tension using a similar expression as (4.3), but here in absolute terms

$$
\begin{equation*}
T_{i}=s_{i} \Delta_{i} \tag{4.12}
\end{equation*}
$$

As in subsection 4.4.4, $s_{i}$ can be difficult to determine, but the specific value is not necessary for the derivation, and is therefore not considered further. The use of (4.12) is advantageous as it elucidates the two methods to achieve deflection.

In order to attain the desired deflection with minimum pressure it is necessary to facilitate achieving the desired difference between $\Delta_{1}$ and $\Delta_{2}$. Using (4.12), the desired differential extension of the walls can be expressed as

$$
\begin{equation*}
\Delta_{2}-\Delta_{1}=T_{2} / s_{2}-T_{1} / s_{1} \tag{4.13}
\end{equation*}
$$

Thus, in terms of stiffness, the difference between $s_{1}$ and $s_{2}$ should be maximised. It should be noted that maximizing the difference between $s_{1}$ and $s_{2}$ facilitates attaining the desired deflection regardless of the tensions in the walls. In this regard, it represents a general principle in terms of attaining the desired deflection at minimum pressure.

In terms of tensions, (4.13) elucidates that difference between $T_{1}$ and $T_{2}$ should also be maximised for a given $p$. Since $s_{1}$ should be maximised and $s_{2}$ minimised, the determining factor in (4.13) to maximise deflection is $T_{2}$, which should be maximised. Considering the equilibrium
(4.1), and after some manipulation, it can be seen that the tensions depend on the cross section design as

$$
\begin{align*}
& T_{1}=\frac{p \frac{x^{2}}{2}+p x c_{2} b+M+F_{n}\left(h-c_{2} b\right)}{x\left(1-c_{1}+d c_{1}+b\left(c_{2}-c_{1}\right)\right)} \\
& T_{2}=\frac{\frac{p x^{2}}{2}\left(1-c_{1}\right)+p x c_{1}(d-b)-M+F_{n}\left(h-b c_{2}\right)}{x\left(1-c_{1}\right)+d c_{1}+b\left(c_{2}-c_{1}\right)} \tag{4.14}
\end{align*}
$$

As discussed in subsection 4.4.1, the contribution of the term $F_{n} b\left(1-c_{1}\right)$ is disregarded since it is equivalent to offsetting the device. Then, from (4.14), it can be seen that for $M>0$, which are the external wrenches of interest as previously discussed, reducing $b$ to increase $x$ is always desirable since $\partial T_{2} / \partial b<0$ and $\partial T_{2} / \partial x>0$. Thus, in order to maximise $T_{2}$ and therefore deflection, $b$ should be minimised and $x$ should be maximised, which can be written as $b=0$ and $x=d$. For $x=d$, (4.14) also elucidates that a maximum $d$ is desirable, hence $d$ should be selected to occupy all space available.

Interestingly, the performance in terms of initial deflection depends on the absolute stiffness of the walls, as elucidated in (4.13). Hence, for a given difference between $s_{1}$ and $s_{2}$ that cannot be increased, the absolute stiffness should be minimised to achieve deflection at minimum pressure.

The analysis in this subsection therefore indicates that the design principles to attain a desired deflection with minimum pressure are maximum $s_{1}$, minimum $s_{2}, b=0, x=d$, maximum $d$, and minimum absolute stiffness when possible. It should be noted that these equalities in practice denote that the variables should tend to the desired values, i.e. minimum wall thickness and maximum region corresponding to the pressurised fluid. In the optimal design $x=d,(4.14)$ indicates that $T_{2}$ increases with $d$, which should therefore be maximised to occupy all available room in each scenario. The derivation of these results is independent of the desired deflection or the pressure, and therefore they represent general principles.

### 4.4.5 Complete design

The designs to maximise the wrenches that can be supported at a given deflection and maximum pressure, and to achieve a deflection at minimum pressure were elucidated in the two previous subsections. The design objective in this work involves attaining a desired deflection and
maximizing the wrenches that can be supported with a given maximum pressure, which couples both analyses.

Fortunately, there is an agreement in the design principles to achieve both objectives, as summarised in the following. The ratio $s_{1} / s_{2}$ should be maximised in both cases, which can be attained, for example, with a pleated structure in wall 2 . Then, for a high $s_{1} / s_{2}, x$ in both cases should be maximised, $d$ should be maximised, and $b$ should be minimised. The only difference is that the absolute values of $s_{1}$ and $s_{2}$ are not relevant in terms of maximizing force at a given deflection and maximum pressure, but they are relevant to attain the desired deflection at minimum pressure. Thus, absolute stiffness should generally be minimised.

This applies to any cross section on the device, and to any deflection and pressure value. Thus, these design principles summarised in the previous paragraph can be used to determine the most suitable design. Since the design principles are independent of the maximum pressure and the deflection, the most suitable design is relatively independent of the desired application.

### 4.4.6 Generalization to 3D

The study up to this point considered a planar scenario. The generalization to 3D is presented in this subsection. The analysis in 3D is mostly analogous; it involves considering the equilibrium of a device isolated in a cross section, aggregating the distributed reactions onto two tensioning force variables, distilling a condition to maintain deflection, and combining them in order to determine the design. However, the generalization of elements such as the aggregation of forces and deflection condition requires a careful examination.

In the 3D scenario, the soft robotic manipulator is considered to bend in a desired plane. External forces are considered to act in the plane of bending, as it represents the most relevant case for the design study. This scenario lends itself to the analysis of symmetric designs, but this symmetry is not used in the derivation in order to maintain generality of the study. The study can then be directly extrapolated to the design of devices capable of supporting out of plane forces.

The 3D device isolated in an arbitrary cross section can be considered, as in 2D. Here, the force associated to the pressure is $p A$, where $A$ is the area of the cross section corresponding to the chamber, and $p$ is pressure as before. The force $p A$ is applied at the centre of pressures, which depends on the chamber geometry.

## Aggregation of forces $T_{1}$ and $T_{2}$

The distributed normal stresses at the cross section can also be aggregated into two forces $T_{1}$ and $T_{2}$ as in the planar case. However, the specific division of the cross section into two regions, the stresses of which correspond to $T_{1}$ and $T_{2}$, affects the analysis, and therefore must be considered. The moment at the cross section that produces bending and supports external wrenches is created between the pressure and distributed reaction stresses at one side of the structure, with the reactions at the other side opposing to it. Thus, a suitable dividing line is that passing through the centre of pressures and perpendicular to the bending plane, as it yields a $T_{1}$ aggregating all distributed stresses that contribute to the moment, and a $T_{2}$ aggregating all stresses that oppose to it, as in the planar scenario.

A dividing line passing through the centre of pressures implies that the relative location of this line can vary with the cross-sectional design. However, this is desirable, as the crosssectional stresses that contribute to the moment also depend on the design. Thus, the dividing line proposed here ensures that the stresses are appropriately aggregated, since the stresses associated to each force always share a common objective in terms of contribution to the device performance.

It should be noted that, as in the planar case, the equivalent line of application of $T_{1}$ and $T_{2}$ can be assumed to be within the region of the cross section they correspond to. Indeed, considering that extending devices achieve deflection thanks to a differential extension of the walls, and that this is produced with a pressurised fluid, it can generally be assumed that the normal stresses at the cross section are predominantly tensioning stresses, and therefore $T_{1}$ and $T_{2}$ are applied within the cross section.


Figure 4.5: General cross section of a 3D device with variable stiffness, with the regions in dark and light gray indicating to higher and lower stiffness, respectively. The approximate lines of application of $T_{1}, T_{2}$ and the centre of pressures $c_{p}$ are also indicated.

## Effect of stiffness distribution on $T_{1}$ and $T_{2}$

The specific line of application of $T_{1}$ or $T_{2}$ is affected by the stiffness distribution in the region they correspond to, as illustrated in Figure 4.5. As in the planar case, the stiffness in a region needs not be constant, and specific stiffness distributions can be used to displace $T_{1}$ and $T_{2} . T_{1}$ and $T_{2}$ are applied at the point where the moment they create is equivalent that generated by the normal stress in their corresponding region. In designs with a constant cross section and at a certain deflection, the local stress in the cross section can be considered to be higher at the sub-regions with markedly higher stiffness, particularly when the variations in the stiffness distribution are significant. Thus, the line of application of $T_{1}$ and $T_{2}$ can be considered to tend to the location of higher stiffness within their regions.

As in the planar case, the desired stiffness is considered to be selectable with the material choice, compensating for any effects from the design geometry. Thus, a typical configuration of interest with $T_{1}$ applied at an edge of the cross section can be attained with a high-stiffness material in the desired sub-region, and a lower-stiffness material over the rest of cross section,
as shown in Figure 4.5. In this case, the line of application of $T_{1}$ can be considered to be relatively independent of the cross section geometry.

## Generalization of deflection condition to 3D

The condition to maintain deflection can also be generalised to 3D. In order to maintain deflection, the overall normal strain distribution in the cross section should be approximately preserved, which implies that any increase in extension should be relatively homogeneous over the cross section. Considering that the stiffnesses at the cross section regions corresponding to $T_{1}$ and $T_{2}$ can be anticipated to be markedly different, a stress distribution with two distinct values corresponding to two regions in terms of stiffness can be expected.

The specific relation between $T_{1}$ and $T_{2}$ to maintain deflection is difficult to determine, as these average values may correspond to different stress and strain distributions. However, a ratio between $T_{1}$ and $T_{2}$ that guarantees that the deflection is maintained must always exist. Indeed, an increase in $M$ while $p$ and the external forces remain constant results in a decrease in deflection, whereas an increase in $p$ while all external wrenches are constant leads to an increase in deflection. Thus, a configuration where deflection is maintained exists, and this corresponds to a certain ratio between $T_{1}$ and $T_{2}$. In particular, following a similar structure as in 2D, at each configuration of equilibrium in each cross section, a relation of the type

$$
\begin{equation*}
T_{2}=\frac{T_{1} S_{2}}{R S_{1}}+T_{20}-\frac{T_{10} S_{2}}{R S_{1}} \tag{4.15}
\end{equation*}
$$

exists, which guarantees that the deflection is maintained with a certain value of $R$. It should be noted that the variables $S_{1}$ and $S_{2}$ denote the longitudinal stiffnesses of the cross section regions corresponding to $T_{1}$ and $T_{2}$, respectively, and are analogous to $s_{1}$ and $s_{2}$ in 2D.

The specific value of $R$ can be difficult to determine, and may depend on the cross section. In general, considering the discussion in the previous paragraph, it can be bounded to be positive. Provided that it is positive, the specific value of $R$ is not relevant to the design derivation in general, as in the planar case, and it is therefore not considered further.

It should be noted that that the existence of the condition (4.15) with a certain $R$ is independent of the deformation distribution over the cross section. In some cross sections, it can occur that maintaining the deflection with different external wrenches leads to a somewhat different strain distribution, resulting in a variation in the bending mode of the overall device. However, this only implies a somewhat different $R$ in the cross sections, but the overall deflection is maintained. In addition, $R$ remains positive in general, which is the main requisite for the derivation of the design principles.

## Generalization of design derivation to 3D

With these concepts generalised to 3D, the equilibrium of the device isolated at an arbitrary cross section can also be considered in 3D. As in the planar case, the equilibrium indicates that $T_{1}$ and $p$ generate the moment, and are desirable, whereas $T_{2}$ opposes to it. However, for a deflection to be maintained, relation (4.15) between $T_{1}$ and $T_{2}$ must be satisfied. The equilibrium of forces

$$
\begin{equation*}
T_{1}+T_{2}=\iint_{A} p d A-F_{n} \tag{4.16}
\end{equation*}
$$

can therefore be combined with (4.15), and substituted into the equilibrium of moments, yielding

$$
\begin{equation*}
-\left(F_{n}+K\right) \frac{D}{1+S_{2} / R S_{1}}+\iint_{A} \frac{p D}{1+S_{2} / R S_{1}}-p \chi d A+F_{n} H=M \tag{4.17}
\end{equation*}
$$

where $D$ is the distance between the line of application of $T_{1}$ and $T_{2}, \chi$ is the distance in the direction of bending between a point in the cross section and the line of application of $T_{2}$, generalizing $x$ in $2 \mathrm{D}, K$ is a constant associated to the initial deflection of the device, which generalises $\kappa$ and is also typically low and positive, $H$ is the distance between the line of application of $F_{n}$ and $T_{2}$, generalizing $h-b\left(1-c_{2}\right)$, and the rest of variables are a direct generalization of those in 2D. Both $D$ and $\chi$ depend on the design geometry and stiffness distribution. However, $\chi$ does not depend on the line of application of $T_{1}$, and therefore is not affected by the stiffness in the region corresponding to $T_{1}$. The value of $H$ can also vary with the design but, as in the planar case, this variation is disregarded since it is equivalent to offsetting the device. Expression (4.17) is equivalent to (4.6), and can be used to derive the
design principles in 3D.

Case with $F_{n}+K=0$ Considering a case with $F_{n}+K=0$ first, expression (4.17) indicates that $S_{2} / R S_{1}$ should be minimised. Thus, maximal $S_{1}$ and minimal $S_{2}$ are desirable. As previously discussed, the overall stiffness in a region, $S_{1}$ or $S_{2}$, can be composed of different stiffnesses in different sub-regions, which can be used to displace the line of application of $T_{1}$ and $T_{2}$ towards the sub-regions of higher stiffness. Equation (4.17) indicates that $D$ should be maximised while maintaining the values of $\chi$, i.e. by displacing the line of application of $T_{1}$. Hence, the stiffness distribution in the region corresponding to $T_{1}$ should be analogous to that in 2 D , and consist of a high-stiffness sub-region near the edge in the direction of bending and a lower stiffness over the rest, as previously introduced and illustrated in Figure 4.5. This preserves a minimal $S_{2} / R S_{1}$, and maintains $T_{1}$ applied near the edge despite variations in the cross-sectional geometry.

The integrand in (4.17) can then be considered to be always positive. Its local value is the distance between $T_{1}$ and a differential element of chamber area, $D-\chi$, which is not affected by variations in the line of application of $T_{2}$. Hence, the integrand is relatively independent of design geometry, since the line of application of $T_{1}$ is relatively constant. Then, the area of the integral in (4.17) should be maximised in order to maximise $M$. This implies that the design should have minimum wall thickness, maximum chamber area, and a cross section that occupies all the available room.

As previously mentioned, a minimal $S_{2}$ is desirable. It should be noted that in very specific cases where the reduction of $S_{2}$ through material choice has reached the possible minimum, a cross section outline to some degree smaller than the room available may result in a noticeably lower $S_{2}$, and therefore improved performance despite the reduction in $p A$. However, these cases are generally unusual, and the performance improvement is typically low as the reduction in $S_{2} / R S_{1}$ is marginal. Hence, the design of a cross section to occupy all available room can be considered a general design principle.

Case with $F_{n}+K \neq 0$ Considering a case with $F_{n}+K \neq 0$, a similar analysis can be applied. Here, for operation to be viable, $F_{n}+K<p A$. Thus, (4.17) indicates that $S_{2} / R S_{1}$ should be minimised. As in the case with $F_{n}=0$, (4.17) indicates that $D$ should be maximised
while maintaining the values of $\chi$, and therefore the same stiffness distribution in the region corresponding to $T_{1}$ applies. The point of application of $T_{2}$, however, is relevant in this case since an equal variation in $D$ and $\chi$ can modify $M$. If $F_{n}+K>0$, a high $D$ is desirable despite an equal increase in $\chi$. However, a large $A$ is also desirable, which can involve a reduction in $D$. Conversely, if $F_{n}+K<0$, a low $D$ is desirable provided that $\chi$ reduces equally. Still, an extensive $A$ is also desirable with $F_{n}<0$ to maximise the contribution of the integral in (4.17), which can increase $D$ and thereby reduce the performance. In this regard, the most suitable design depends on $F_{n}, K$, as well as the variation of $D$ and $\chi$ with $A$ and the geometry. This design problem in the case $F_{n}+K \neq 0$ in 3D is analogous to that in 2D, but in 3D an ad hoc analysis is required to determine the 3 D equivalents of $c_{1}$ and $c_{2}$ for a given geometry as well as $K$, and then generalise the design principles. This involves a numerical study that is beyond the scope of this work; thus, the specific geometry for each configuration under $F_{n}+K \neq 0$ remains as an open question.

## Final derivation considerations

The derivation with both $F_{n}=0$ and $F_{n}+K \neq 0$ considered the equilibrium at an arbitrary cross section of the device. As in the planar case, the moment created by the external wrenches depends on the cross section, and therefore can vary along the device. However, the design study is equal despite variations in the external moments, and therefore applicable to all cross sections. Thus, the design principles can be applied to all cross sections, defining the most suitable design of the device.

It should be noted that the derivation of the design involves first establishing that the ratio between the stiffness of wall 2 and that of wall 1 needs to be minimised, and then determining the geometry. However, in the case that the ratio of stiffnesses could not be minimised, and for $F_{n}=0$, the design would then need to have an area of the cross section corresponding to the pressurised fluid not occupying all the cross section, which in 2D can be determined from (4.7). In 3D this can involve using structures to prevent cross section deformation, which can justify the introduction of braided chambers in some of the existing designs [113].

It should also be noted that, as previously discussed, the structure of extending devices is considered to extend only longitudinally, without expanding radially. The introduction of radial expansion would lead to contraction, which is undesirable in extending devices as it reduces the extension. Devices employing contraction are discussed in the following section. Extending devices should therefore maintain a constant cross section occupying all available space, which can be achieved by incorporating a set of braces or transversal fibers on the structure of the device.

### 4.5 Design of contracting devices

The deflection in contracting devices is generated by a protruding wall, which forces one side of the device to contract, causing bending of the device. Thus, in contrast to extending devices, the pressure in contracting devices primarily serves to force a wall to protrude, and the moment for bending and supporting external wrenches is created to some extent between the tension in the protruding wall and the compression of another wall. The performance of the device depends on the design geometry and stiffness, which requires a detailed examination. It should be noted that, as introduced at the beginning of this chapter, the term device is used here to refer to a segment of soft robotic manipulator.

The design of contracting devices is studied in this section using the same framework as in extending devices. First, the equilibrium of the device is formulated in subsection 4.5.1. Energy considerations are then presented in subsection 4.5.2, justifying a set of design principles in terms of the stiffnesses of the device's structure. In subsection 4.5.3, a condition to impose a constant deflection is determined. The equilibrium, energy considerations, and deflection condition are combined in subsection 4.5.4 to study the design and derive design principles. The design principles to attain a desired deflection with minimum pressure are presented in 4.5.5, leading to the complete design principles for contracting devices, summarised in 4.5.6. The generalization of the analysis to a 3D scenario is finally described in subsection 4.5.7.

### 4.5.1 Equilibrium

## Equilibrium formulation

The equilibrium of a general contracting device isolated at an arbitrary cross section can be considered, as illustrated in Figure 4.6. Imposing equilibrium of forces in the direction orthogonal to the cross section and equilibrium of moments with respect to the point where $T_{2}$ is applied, two equations are obtained

$$
\begin{align*}
& T_{1}+T_{2}=p x-F_{n} \\
& T_{1}\left[c_{1} d+x\left(1-c_{1}\right)+b\left(c_{2}-c_{1}\right)\right]-\frac{p x^{2}}{2}-p x c_{2} b-m_{2}+F_{n}\left(h-b\left(1-c_{2}\right)\right)=M \tag{4.18}
\end{align*}
$$

where $b, x, d, c_{1}, c_{2}, T_{1}, T_{2}, T_{t 1}, T_{t 2}, F_{n}, M$ and $p$ are equivalent to those of extending devices. Equations (4.18) are analogous to those in extending devices, including the comments on the aggregation of external wrenches into $F_{n}, F_{t}$ and $M$, as well as the inequalities relating $x, b$ and $d$ based on geometric constraints.

In contracting devices, wall 1 must protrude and generate a contraction by pulling between its ends, whereas wall 2 must approximately maintain the initial length and bend. Wall 2 therefore serves as a backbone, which may undergo compression stresses. In particular, when $T_{1}>p x-F_{n}$, wall 2 must be in compression, which typically occurs at low deflections. Hence, the structure of wall 2 typically needs to be capable of supporting compressive stress, and the stress distribution in wall 2 may combine tensioning and compressive stresses. The aggregation of these stresses is decoupled here into a moment associated to bending of the wall, defined as $m_{2}$, which can be generally considered to be negative and to reduce further with wall thickness, and the tensioning force $T_{2}$. This aggregation of the distributed stresses into $m_{2}$ and a normal force $T_{2}$ is generally admissible since, as will be seen in the subsequent presentation, wall 2 can generally be considered to act as a rod. The equivalent point of application of $T_{2}$ can thus be considered to lay within the wall thickness, $0<c_{2}<1$, and typically near the centre, $c_{2}=1 / 2$.


Figure 4.6: Equilibrium diagram of a 2D contracting device isolated at an arbitrary cross section, exposing reaction forces as well as pressure.

## Equilibrium discussion

As in extending devices, the equilibrium can be considered on any cross section of the device, and therefore the analysis derived from this equilibrium can be used to study the design of the entire device. Similarly, the equilibrium in the lateral direction also indicates that the structure of the device must support any lateral reactions in a passive manner. However, the effect of shear stresses on deflection is generally negligible, and and therefore not considered further.

The equilibrium equations (4.18) indicate that, in order to maximise the moment that can be supported, $T_{1}$ should be maximised and $T_{2}$ should be minimised, working in compression. Equations (4.18) also highlight that the pressure in contracting devices serves two separate purposes. First, and most importantly, it presses on wall 1 to create a protrusion, indirectly contributing to the equilibrium of moments through $T_{1}$. Second, it acts on the cross section, directly contributing to the equilibrium of moments as in extending devices. In this regard, contracting devices with equal diameter but different $x$ can present different performance and the contribution $p x$ can be exploited. The direct contribution of $p x$, however, also implies a higher tension at the walls, tending to reduce the protrusion, or equivalently limiting $M$, which couples both purposes of pressure.

The design in terms of geometry, including $x$ and $b$, and stiffness, predominantly in terms of the protruding wall, must therefore be determined to maximise the wrenches that can be supported. Considering (4.18), configurations that attain high values of $M$ in equilibrium with low or even zero $p$ can be found. However, each of these equilibrium configurations may correspond to a different deflection. A condition imposing a desired deflection is therefore required in order to study the effect of design on performance, as in extending devices.

An important difference with respect to extending devices, however, is that in contracting devices the protruding wall is not perpendicular to the cross section along most of the device. The geometry of the protrusion therefore affects the device's performance, and must be first considered, as described in the next subsection.

### 4.5.2 Energy considerations

## General energetic analysis

The similarities between PAMs and contracting devices imply that some of the existing energetic approaches used in PAMs [123] can be adapted for the study of contracting devices, and thereby extract insight into the behavior of contracting devices. In particular, energetic considerations can be used to elucidate the effect of some aspects of the design, such as structural stiffness, on the performance. Thus, specific aspects of the design, such as stiffness of the protruding wall, can be determined, defining specific protrusion geometries.

Energy conservation must be satisfied in a system corresponding to a general device with a given deflection and supporting general wrenches. Following [123], virtual works can be considered for a structural deformation caused by a virtual element of fluid $d V$ entering the device, with associated virtual increment of displacement $d l$ at the point of application and in the direction of the resulting external force $F$, and associated virtual increment of rotation $\theta$ where $M$ is applied. Considering an incompressible fluid, this yields

$$
\begin{equation*}
p d V=F d l+M d \theta+d W_{s} \tag{4.19}
\end{equation*}
$$

where $d W_{s}$ is the work required to deform the structure, which is $d W_{s}>0$. Equation (4.19) elucidates that any $d W_{s}$ tends to reduce the wrenches that can be supported, and therefore should generally be minimised.

In order to minimise $d W_{s}$ while maintaining operational capability, the longitudinal stiffness of the protruding wall should tend to infinity. The bending stiffness of the protruding wall, defined $s_{b}$, should either be $s_{b}=0$ over the entire protruding wall, or a combination of $s_{b}=0$ and $s_{b}=\infty$ in different parts of the protruding wall. It should be noted that $s_{b}$ must be $s_{b}=0$ at least at some parts of the protruding wall to enable operation. Finally, the bending stiffness of wall 2 should be minimal to minimise $d W_{s}$, or equivalently to reduce the effect of $m_{2}$ in (4.18), but the wall should be capable of supporting compression stresses with minimal contraction.

The energy dedicated to deform the structure can be considered to be practically zero both in designs with only $s_{b}=0$ in wall 1 and in designs with a combination of $s_{b}=0$ and $s_{b}=\infty$ in wall 1 , which renders both configurations equivalent in this regard. However, equation (4.19) also indicates that the wrenches that can be supported with a given pressure are maximised when the $d V$ that corresponds to a pair of $d l, d \theta$ is maximised. Thus, the geometry of the protrusion is relevant as it can increase $d V$ for a given deflection. The protrusion geometry of designs with only $s_{b}=0$ in wall 1 is completely determined by the structural behavior. Instead, the protrusion geometry in designs combining $s_{b}=0$ and $s_{b}=\infty$ depends on the distribution of $s_{b}=0$ and $s_{b}=\infty$, and can therefore be selected. Considering that a maximum $d V$ associated to a $d l, d \theta$ at the deflection of operation is desirable, the specific distribution of $s_{b}=0$ and $s_{b}=\infty$ should be selected to maximise the $d V$ associated to an increment of contraction of the protruding wall at the desired deflection, using all available room. This is generally determined geometrically considering that the wall geometry is composed of parts with predetermined geometry corresponding to $s_{b}=\infty$, and parts with a specific geometry corresponding to $s_{b}=0$, which is a circumference arc as shown in the next subsection. The constraints from the environment and the desired deflection, however, depend on each scenario, and therefore the distribution of $s_{b}=0$ and $s_{b}=\infty$ is specific for each application.

## Braces and braids

A set of braces can be used as an alternative design option to reduce the protrusion and adapt it to the environmental constraints. The braces need not involve any additional $d W_{s}$ provided that wall 1 is only comprised of parts with $s_{b}=0$ and $s_{b}=\infty$ and infinite longitudinal stiffness, although the braces should enable wall 1 to protrude to reach the desired deflection. The effect of these braces is thus analogous to that of a wall combining $s_{b}=0$ and $s_{b}=\infty$, and therefore they represent an equivalent alternative to select the desired protrusion geometry.

Another design option for wall 1 in 3D scenarios is to include a braided structure such as those used in PAMs [124], which may also minimise $d W_{s}$. In particular, a braid that couples longitudinal and transversal tension (and therefore stiffness) through a certain ratio determined
by the braid angle may also offer a performance equivalent to that of designs with infinite longitudinal stiffness provided that it requires minimal work to deform it. The braid then simply acts as a mechanism to transform transversal deformation into longitudinal deformation. This provides the capability of increasing contraction for a given protrusion, but it requires in-plane deformation in two directions, and thus a 3D structure. Considering that in-plane extension in the transversal direction is generally not desirable nor practical in soft robotic manipulators with contracting operation, and that braids generally involve a certain degree of $d W_{s}$, the use of braids in wall 1 is considered disadvantageous over infinite longitudinal stiffness, and therefore not the main focus of this study.

## Final energy discussion

The design in terms of stiffness can therefore be determined using energetic considerations, as described in previous paragraphs. The energetic considerations, however, do not directly imply a specific design in terms of $x$ or $b$, since the relation between these and the maximization of $d V$, for a $d l, d \theta$ is difficult to determine a priori. The equilibrium approach introduced in the previous subsection can be used to determine the rest of design, and also to develop the study of contracting devices under the same framework as extending devices, but first a deflection condition is required.

### 4.5.3 Deflection condition

An incompressible wall 2 is desirable in contracting devices, as argued in previous and following sections. Then, a deflection condition imposing the distance between the ends of wall 1 to remain constant suffices to ensure that deflection is maintained.

## General deflection condition

The distance between the ends of wall 1 depends on the protrusion geometry and any extension of wall 1. As argued in the previous subsection, a maximal longitudinal stiffness is desirable
for wall 1 , and therefore wall 1 can be considered to be inextensible. In this case, the distance between the ends of wall 1 only depends on the protrusion geometry, which is generally a function of $p, T_{1}$, and $s_{b}$. For a given $s_{b}$, the distance between the ends of the protruding wall can thus be expressed as $\zeta$, which a function of $p$ and $T_{1}$.

The deflection is then determined by $\zeta$. The specific $\zeta\left(T_{1}, p\right)$ can be difficult to determine in general as it involves solving a nonlinear structural problem with general boundary conditions. However, considering that $\zeta$, and therefore deflection, depend on the protrusion geometry, insight into the structural behavior of the protrusion can be used in order to obtain a condition to impose a desired deflection.

In a general protruding wall, an increase in $p$ for constant $T_{1}$ given $s_{b}$ leads to a greater protrusion and more contraction, so $d \zeta / d p<0$. Conversely, an increase in $T_{1}$ for constant $p$ and $s_{b}$ tends to reduce the protrusion, hence $d \zeta / d T_{1}>0$. Thus, a relation between $T_{1}, p$ and $\zeta$ generally exists as well for a given $s_{b}$, which can be defined as $f(p, \zeta)$. Even though $f(p, \zeta)$ is also difficult to determine in general, the function $f(p, \zeta)$ can be either bounded or determined in specific designs of interest, which can suffice to obtain a deflection condition that enables a subsequent design study.

In particular, in designs with $s_{b}=0$, the equilibrium of a differential element of wall can be considered, as shown in Figure 4.7, yielding

$$
\begin{align*}
& d m_{1} / d h=V_{1} \\
& d V_{1} / d h=p-T_{1} d \psi / d h  \tag{4.20}\\
& d T_{1} / d h=V_{1} d \psi / d h
\end{align*}
$$

where $m_{1}$ is the resulting moment at the cross section of the wall, $V_{1}$ is the resulting vertical force at the cross section of the wall, and $\psi$ is an angle corresponding to the orientation of the cross section. For $s_{b}=0, m_{1}=0$. Thus, $V_{1}=0$, and therefore $T_{1}$ is constant over the wall region where $s_{b}=0$. Finally, the relation between $T_{1}, p$ and the curvature radius of the wall,


Figure 4.7: Equilibrium of a differential wall element, where $m, V$ and $T$ denote the resulting moment, vertical force and tensioning force at the wall cross section, respectively, and $p$ is the pressure that the wall is withstanding.
which can be defined as $R=1 / d \psi / d h$, is

$$
\begin{equation*}
T_{1}=p R \tag{4.21}
\end{equation*}
$$

The curvature of a wall or part of it over the region where $s_{b}=0$ is therefore constant. In this regard, the protrusion geometry in designs with purely $s_{b}=0$ is a circumference arc, whereas the protrusion geometry in designs combining $s_{b}=0$ and $s_{b}=\infty$ is a combination of circumference arcs and the preselected geometry for the parts with $s_{b}=\infty$. A bijective relation then exists between the geometry of wall 1 and the distance between its ends $\zeta$ in a given design, which is determined geometrically. In particular, in the case of a wall 1 with only $s_{b}=0$, a certain $\zeta$ implies a specific $R$. In the case of a wall 1 combining $s_{b}=0$ and $s_{b}=\infty$, a given $\zeta$ also implies a certain $R$ that is common in all regions where $s_{b}=0$, and is generally lower than the $R$ in designs with only $s_{b}=0$ for an equal $\zeta$.

The wall curvature is directly related to $T_{1}$ and $p$ according to (4.21). Hence, a pair of $T_{1}$ and $p$ imply a protrusion geometry, which in turn entails a certain $\zeta$, and can therefore be used as a condition to impose a desired deflection. Equivalently, using the relation between $R$ and $\zeta$ described in the previous paragraph for each particular design, (4.21) can be transformed into the condition

$$
\begin{equation*}
f(p, \zeta)=p R(\zeta) \tag{4.22}
\end{equation*}
$$

where $R(\zeta)$ is determined geometrically. Thus, for a given design in terms of distribution of $s_{b}=0$ and $s_{b}=\inf$, the tension in wall 1 to maintain a desired deflection is proportional to $p$, and determined by (4.22) with the $R$ corresponding to the regions where $s_{b}=0$. The condition applies to the entire wall. This includes regions where $s_{b}=\infty$ since the moment at the ends of these regions is zero, and therefore the tension and its line of application within these regions is constant and equal to the tension at the ends.

The result that the wall geometry is specific for a certain distance between the ends of a protruding wall in wall designs with $s_{b}=0$ and infinite longitudinal stiffness is coherent with the energetic considerations. Indeed, if deformation energy cannot be stored in the structure, an increase in $p$ and $T_{1}$ that maintains the distance between the ends of the wall and thus involves no motion of the device cannot result in any change in geometry in order to satisfy energy conservation.

## Particular designs with braces or braids

In designs including a set of braces, the deflection condition is similar. However, the specific design of the braces can lead to different values of tension in each segment of wall between two braces, particularly if the braces are not perpendicular to the cross section, resulting in different curvatures at each segment of the protrusion. The effect of the braces on the resulting wall tension must therefore be considered in order to then use condition (4.22) with the corresponding $R$. This effect can be determined by considering equilibrium at the point of attachment of the braces. However, it is not developed in this work since braces simply represent an alternative to modify the protrusion geometry equivalent to designs combining $s_{b}=0$ and $s_{b}=\infty$, but do not provide specific performance advantages, as discussed in the previous subsection. It should be noted, however, that braces typically involve a reduction in $R$, which entails lower $T_{1}$ and therefore lower force for a given $p$ and deflection, but also lower protrusion magnitude.

A deflection condition similar to (4.22) can also be obtained in designs with braids provided that $s_{b}=0$ is a valid assumption for the braid. However, condition (4.22) is derived considering
a planar case, and a direct generalization to 3D only applies to protrusions with bending in a plane. Braids, on the other hand, couple transversal and longitudinal deformation, and therefore are intrinsically 3D. Considering that the use of braids is generally disadvantageous as discussed in the previous subsection, and that the generalization of this study to 3D is discussed in subsection 4.5.7, the deflection condition for designs with braids is not considered further in this subsection.

### 4.5.4 Design derivation

The equilibrium equations (4.18) and the deflection condition (4.22), together with the energetic considerations, can be combined to study the design, and derive design principles to maximise the wrenches that can be supported.

## Detailed analysis and derivation

First, the energetic considerations described in subsection 4.5.2 can be used to determine the wall stiffnesses of the design. In particular, the design should generally have an inextensible protruding wall with either $s_{b}=0$ or a combination of $s_{b}=0$ and $s_{b}=\infty$ to maximise the $d V$ associated to a $d l, d \theta$ at the desired operation deflection, using all available room. The specific combination of $s_{b}=0$ and $s_{b}=\infty$ depends on each specific application, but typically larger regions of $s_{b}=\infty$ provide higher $d V$ and thus higher performance at low deflections, whereas larger wall regions with $s_{b}=0$ enable reaching and providing some support of external wrenches at larger deflections. With these stiffnesses, the tension of the protruding wall (4.21) can be combined with the equilibrium of forces (4.18) in order to show that wall 2 must be designed to be capable of supporting compressive stress to allow operation at low deflections where $R$ tends to infinity.

In these designs of interest, the equilibrium equations (4.18) can then be considered, and a
desired deflection can be imposed by substituting (4.22), yielding

$$
\begin{equation*}
p R(\zeta) \cos \alpha\left[c_{1} d+x\left(1-c_{1}\right)+b\left(c_{2}-c_{1}\right)\right]-p \frac{x^{2}}{2}-p x c_{2} b-m_{2}+F_{n}\left(h-b\left(1-c_{2}\right)\right)=M \tag{4.23}
\end{equation*}
$$

where $\alpha$ is the angle between the direction of the resulting tensioning force in wall 1 and the direction normal to the plane of the cross section. (4.23) elucidates the fact that the design to maximise $M$ depends on $\alpha$, and therefore on the protrusion geometry. This is determined by the aforementioned stiffnesses, which maximise performance as discussed in subsection 4.5.2, and thus $\alpha$ is a specified value at each cross section.

Expression (4.23) provides the relation between the wrenches that can be supported at a desired deflection and the design. (4.23) is analogous to (4.6) in extending devices, and can therefore be used to derive the additional design principles to meet the objective of maximizing $M$. Expression (4.23) is valid in general, and thus enables the determination of the design in a general scenario.
$\underline{\text { Case with } F_{n}=0}$

A case with $F_{n}=0$ can be studied first, as it represents a common scenario of interest in practice, and is illustrative of the design principles. The effect of $x, b$ and $d$ on (4.23) is relatively decoupled in the majority of terms. However, their contributions to $M$ depend on the values of $c_{1}$ and $c_{2}$, especially in terms of $\operatorname{sgn}(c 2-c 1)$, and therefore these two parameters must be first considered.

Specific values of $c_{1}$ and $c_{2}$ can be difficult to select with the design, but general tendencies for the desired values of the parameters can be considered, which can suffice for the design study. The value of $c_{1}$ affects $M$ through three terms: a positive and two negative ones. However, considering that the variables $x, b$ and $d$ are related through $x+b<d$, it can be seen that the total contribution of $c_{1}$ to $M$ is always positive, and therefore $c_{1}$ should be maximised to the extent it is possible with the design. The contribution of $c_{2}$ to $M$ is more complex to analyse, and therefore its desired tendency is difficult to determine. However, considering that $c_{1}$ and $c_{2}$ are equivalent design parameters, their maximum values can be considered similar. Thus, in
a design where $c 1$ is maximised, it can be assumed that $c_{2}<=c_{1}$.
The $\operatorname{sgn}\left(c_{2}-c_{1}\right)$ can then be considered to be negative. This implies that $b$ tends to reduce $M$ in (4.23). In addition, $b$ also tends to reduce $m_{2}$, leading to more negative values, which reduces further $M$. Thus, $b$ should generally be minimised, which can be expressed as $b=0$.

The contribution of $x$ to $M$ in (4.23) is then only through two terms, with a quadratic relation. Thus, the value of $x$ to maximise $M$ can be directly determined as $x=\left(1-c_{1}\right) R(\zeta)$. Considering the constraint $x<d$, the value of $x$ should tend to $d$ at low deflections where $R \rightarrow \infty$, even for a $c_{1}$ that is maximised, which can be expressed as $x=d$. At larger deflections, the required value of $x$ may be lower than $d$. A design where $x$ reduces with deflection can be considered to be inviable in practice unless pressures below atmospheric pressure are used, which is typically impractical as it limits the maximum pressure difference. Thus, at larger deflections, the design in terms of $x$ should be selected for a specific operation according to $x=R(\zeta)\left(1-c_{1}\right)$.

Finally, the value of $d$ is determined by the environment in each application. (4.23) shows that a high $d$ is desirable, and therefore it should be the selected so that the device reaches the constraints from the environment at the maximum protrusion. This agrees with the aforementioned design of wall 1 to use all available room, although the specific wall geometry, determined by the regions with $s_{b}=0$ and $s_{b}=\infty$, should be selected to maximise the $d V$ associated to $d l, d \theta$, as previously described.
$\underline{\text { Case with } F_{n} \neq 0}$
The design in the general case $F_{n} \neq 0$ can be studied in a similar manner. In contracting devices, the deflection condition (4.22) only imposes a constraint on $T_{1}$, but does not involve $T_{2}$. As a consequence, the contribution of $F_{n}$ to $M$ in (4.23) is through a constant term $F_{n} h$ and a term depending on the design $F_{n} b\left(1-c_{2}\right)$. As in extending devices, the contribution of the term $F_{n} b\left(1-c_{2}\right)$ to $M$ can be disregarded since it is equivalent to offsetting the device with respect to the external forces. The contribution $F_{n} h$ is fixed and equivalent to an additional external moment to be supported.

The design derivation in the case $F_{n} /=0$ is therefore analogous to that in the case $F_{n}=0$, and
the principles for contracting devices both with and without external wrenches are equivalent. These, together with the aforementioned principles corresponding to the stiffnesses of the walls, constitute the design principles to maximise the $M$ that can be supported at a given deflection with contracting devices.

## Final derivation considerations

It should be noted that, in designs combining $s_{b}=0$ and $s_{b}=\infty$, the derivation also applies to the regions where $s_{b}=\infty$. However, in these regions, both the line of application of $T_{1}$ and $R(\zeta)$ correspond to those at the boundaries with the adjacent regions, at the side where $s_{b}=0$. Thus, the line of application of $T_{1}$ needs not necessarily be within the wall in designs with curved, rigid wall regions. The design principles, however, indicate that $d$ should be maximised within the room available, and $x$ should generally occupy the entire cross section. Hence, the rigid parts in wall 1 should be straight, and the same principles derived in previous paragraphs apply.

Interestingly, the geometric principles indicate that the thickness of wall 1 and 2 should be minimised in the majority of cases. This is coherent with the principles in terms of stiffness indicating that the bending stiffness of wall 2 should be minimal while supporting compression stress, and the bending stiffness of wall 1 should be minimal in the desired regions. Thus, the resulting designs can be produced in practice.

## Derivation discussion

This derivation confirms that wall 2 in standard contracting devices must undergo compressive stress when $R(\zeta)>=x-F_{n} / p$, which typically occurs at low deflections. High values of $x$ can aid in reducing the compressive stress but, in general, designs without the capability of supporting compressive stress in wall 2 cannot operate at low deflections. This is due to the fact that a protrusion generally involves a $T_{1}$.

The need for a wall 2 capable of supporting compressive stress can only be prevented by reducing
the $T_{1}$ associated to a protrusion and $p$, which requires exceptional solutions. One of such solutions is to include an elastic sheet that acts as a continuous set of elastic braces opposing to the protrusion, thereby reducing $T_{1}$ and thus leading to a contracting device without the need for a wall 2 capable of supporting compressive stress. Such a design solution is relevant in the application described in section 4.7. However, in general such a solution also involves a reduction in the wrenches that can be supported.

### 4.5.5 Initial deflection

The design principles to attain a desired initial deflection with minimum pressure can be determined by following a similar derivation as that to derive the principles to maximise the wrenches that can be supported.

First, the energetic considerations of subsection 4.5.2 indicate that the structure should store minimum energy. This implies an inextensible protruding wall with either $s_{b}=0$ or a combination of $s_{b}=0, s_{b}=\infty$. Wall 2 should then be incompressible, and with minimum bending stiffness.

The study of the protrusion in subsection 4.5.3 then indicates that the protrusion geometry is directly related to the deflection. Thus, the desired initial deflection can be imposed by selecting a protrusion geometry with a desired $R$, and using the deflection condition (4.22), where the specific $R(\zeta)$ is determined geometrically.

Equilibrium of moments can also be considered in a device at the desired initial deflection and with no external wrenches. This is equivalent to the equilibrium in (4.18), shown in Figure 4.6, particularised to $F_{n}=0, M=0$. The imposition of the desired initial deflection (4.22) to (4.18)b yields

$$
\begin{equation*}
p R_{i}\left(c_{1} d+x\left(1-c_{1}\right)+b\left(c_{2}-c_{1}\right)\right)-\frac{p x^{2}}{2}-p x c_{2} b-m_{2}=0 \tag{4.24}
\end{equation*}
$$

This equation can be used to determine the design to attain the desired initial deflection with minimum pressure. First, (4.24) indicates that, in order to minimise $p, m_{2}$ should be minimised, which agrees with the energy considerations. Then, factorizing $p$, it can be seen that the design
to minimise $p$ in (4.24) is equivalent to the design to maximise $M$ in (4.23). Hence, the design geometry and stiffness should be equal to those derived in the previous subsection.

### 4.5.6 Complete design

The design principles to attain a desired deflection at minimum pressure are equal to those to maximise the wrenches that can be supported at a desired deflection, both with and without $F_{n}$. Thus, these represent the general design principles for contracting devices, and are summarised in the following.

The protruding wall should have infinite longitudinal stiffness, and either $s_{b}=0$ or a combination of $s_{b}=0$ and $s_{b}=\infty$ to maximise the $d V$ associated to an increment in the contracting wall, using all available space. Wall 2 should be incompressible with minimum bending stiffness. The parameter $c_{1}$ should be maximised to the extent possible. The total width $d$ should be selected so that wall 1 reaches the constraints from the environment at maximum protrusion. And finally, the design geometry should be $b=0$, and $x=R(\zeta)\left(1-c_{1}\right)$, which is typically $x=d$.

### 4.5.7 Generalization to 3D

The design derivation presented up to this point can be generalised to 3D. The study in 3D is mostly equivalent: it involves using energy considerations to outline the device's stiffness, and then combining it with an equilibrium analysis to derive the design principles. However, some aspects of the generalization require a detailed analysis.

## Generalization of design derivation to 3D

The energy considerations in subsection 4.5 .2 can be applied to 3D, showing that the structure of a 3D device should store minimum energy to maximise the wrenches that can be supported. Thus, the structure of a 3D contracting device must be composed of two regions: a first region
corresponding to a protruding wall, which should be inextensible and with either $s_{b}=0$ or a combination of $s_{b}=0$ and $s_{b}=\infty$, and a second region of the device acting as a backbone, which should be incompressible and with minimum bending stiffness, equivalent to wall 2 in the planar case.

The cross section of the device must then be divided, with parts corresponding to these two structural regions. Unlike in extending devices where the role of the cross-sectional stress in the cross section is dictated by the position relative to the centre of pressures, in contracting devices the purpose of the local stress in each element of area over the cross section is not clear a priori.

A cross section divided along an arbitrary curve can be considered. This defines the two regions in terms of stiffness, where one region corresponds to the protruding, inextensible wall, and the other region corresponds to the incompressible wall. The equilibrium of the 3D device isolated in this general cross section divided along an arbitrary curve can then be considered in an analogous manner as in subsection 4.5.1, with $T_{1}$ corresponding to the aggregated normal stresses in the region of the protruding wall, and $T_{2}$ corresponding to the aggregated stresses in the other region. The equilibrium indicates that, in order to maximise the wrenches that can be supported, the separation between $T_{1}$ and $T_{2}$ should be maximised. Thus, the curve dividing the cross section must be selected to maximise the distance between $T_{1}$ and $T_{2}$ in the direction perpendicular to these forces and in the plane of bending. This specifies the purpose of each region of the cross section, and defines the stiffnesses of the device.

Equilibrium of the 3D device isolated in an arbitrary cross section with $T_{1}$ and $T_{2}$ defined by this dividing curve can be used to determine the rest of the design in an equivalent manner as in the planar case. The design involves minimizing the thickness of the region corresponding to $T_{2}$, maximizing the area of the cross section corresponding to the pressurised chamber for typical operation deflections, and maximizing the increment of volume in the device for an increment in contraction of the protruding wall at the operation deflection, using all available space.

The specific division of the cross section along a curve, or equivalently the allocation of the
different parts of the cross section to the different regions, in order to maximise the distance between $T_{1}$ and $T_{2}$ depends on each scenario. In typical scenarios where the spatial constraints in a cross section are defined by a rectangle, wall 1 should correspond to one side of the rectangle, and wall 2 to the opposite side, as shown in Figure 4.8 (a). In more general scenarios with any spatial constraints, wall 1 should correspond to the entire frontal region of the device when observed from the direction in which it bends, as illustrated in the example in Figure 4.8 (b), creating a frontal protrusion, while wall 2 should correspond to the opposite side.

These designs oppose to designs with a wall 1 that extends to the lateral regions, such as that shown in Figure 4.8 (c). Protrusion in the lateral direction, or in any direction different from a frontal protrusion, is generally undesirable. This can be elucidated using the equilibrium, as it generally involves increasing the region corresponding to $T_{1}$ to the laterals, which modifies the line of application of $T_{1}$, reducing the distance between $T_{1}$ and $T_{2}$. The undesirable lateral protrusions can also be explained using energetic considerations. The wrenches that can be supported depend on the volume increase of the device (4.19) for a contraction increment. However, the geometry of the protrusion generally cannot be selected to adapt exactly to the volume available from the spatial constraints, which are commonly prismatic, leaving some volume unused. In designs with lateral protrusions, the unexploited volume is typically larger than in designs with only frontal protrusion, as unused volume appears at both sides or near vertices of the available room, leading to lower performance. Thus, both equilibrium and energetic considerations confirm that the protrusion should generally be only frontal.

Designs in 3D such as those in Figures 4.8 (a), (b) typically include lateral walls. However, these should not contribute to the protruding wall nor to the opposite wall in order to maintain the distance between $T_{1}$ and $T_{2}$ to a maximum. These lateral walls only serve to contain the pressurised fluid and enable protrusion of wall 1, but should not affect the structural behaviour of the device. Thus, these walls should generally be designed to minimise any resistance to deformation while containing the fluid without protruding laterally, e.g. using a pleated structure with tendons connecting both laterals. In specific cases, however, these lateral walls can be used to reduce the $T_{1}$ associated to a deflection and pressure, reducing the compression on wall 2. This is equivalent to the use of an elastic sheet introduced in subsection


Figure 4.8: Diagrams of a typical cross-sectional design in a scenario with rectangular constraints (a), typical cross-sectional design in a general scenario with curved constraints (b), and undesirable cross-sectional design in general scenario. In all diagrams, wall 1 is depicted in red, wall 2 in blue, lateral walls in black, and the available room in the scenario in dashed green lines.
4.5.4 for planar designs, and is a relevant solution in the design presented in the following section.

## Discussion of 3D derivation

The design of contracting devices in 3D presented in this subsection elucidates that contracting devices are similar to a segment of continuum robot actuated by PAMs and with an elastic backbone, such as [182]. However, contracting devices integrate the different parts, and can be designed with the principles elucidated in this work to improve performance. Still, both contracting devices and devices including PAMs present the disadvantage of involving a protruding wall, which typically protrudes outwards, requiring additional room to operate.

### 4.6 Summary of design principles for given maximum pressure

The main design principles derived in the previous sections are summarised in the following, first for extending devices in subsection 4.6.1, and then for contracting devices in subsection 4.6.2. The overall procedure to design a soft robotic manipulator using the design principles is then outlined in subsection 4.6.3. It should be noted that this section is intended as a
summary of the main principles, and the reader is referred to the previous sections for details and clarifications on the principles and their derivation.

### 4.6.1 Extending devices

The design principles for extending devices in both 2D and 3D can be summarised as follows. The longitudinal stiffness in the region corresponding to wall 1 should be maximised, which can be expressed as maximal $s_{1}$ in 2D and equivalently maximal $S_{1}$ in 3 D . The stiffness distribution should be selected so that the maximum stiffness is concentrated near the edge of the cross section in the direction of bending in order to displace the line of application of $T_{1}$ towards the cross section contour. The stiffness in the region corresponding to wall 2 should be minimised, which can be expressed as minimal $s_{2}$ in 2 D , and minimal $S_{2}$ in 3 D . This minimal stiffness can be achieved, for example, with a pleated structure. The total cross section of the device should be maximised to occupy all available room. The thickness of the walls should be minimised. Finally, the chamber area should be maximised, in general case were $S_{2} / R S_{1}$ is minimised, to ensure that the region of the cross section corresponding to the pressurised fluid is maximal. It should be noted that these last three principles apply to any 2 D case and to the 3D case $F_{n}+K=0$. However, in the 3D case $F_{n}+K \neq 0$, the specific geometry of the cross section must be determined using numerical methods.

The performance of extending devices is related to their operation. In extending devices, the combination of $T_{1}$ and the direct contribution of pressure in the cross section create the moment that supports external moments and equivalent moments generated by external forces. Thus, the performance of extending devices tends to be relatively low at low pressures, but remains relatively constant as deflection and pressure increase. As a result, extending devices are relatively well suited to operate at large deflections and corresponding higher pressures.

### 4.6.2 Contracting devices

The design principles for contracting devices in both 2D and 3D can be summarised as follows. The longitudinal stiffness of the protruding wall should be maximal. Its bending stiffness should generally be a combination of parts with infinite and minimal bending stiffness, selected to maximise the $d V$ corresponding to an increase in wall contraction at the operation deflection, although in specific cases braids or braces can be used to maximise the $d V$ associated to a contraction increase. Wall 2 should be capable of bending with minimum resistance while generally being capable of supporting compression forces. The distance between $T_{1}$ and $T_{2}$ should be maximised by selecting appropriate regions for walls 1 and 2, as illustrated in Figure 4.8. This implies that in some cases lateral walls may be included, typically in the form of pleated structures with braces to prevent lateral expansion. However, these lateral walls should only serve to contain the pressure and not affect the structural behavior of the device. The total cross section should be maximised so that the device occupies all available room at the operation deflection, where the protrusion should be maximal. The thickness of the walls should be minimised. Finally, the region of the cross section with pressurised fluid should generally be selected to be maximal at the operation deflection.

The performance of contracting devices is also related to their operation. The support of external moments and equivalent moments generated by external forces is primarily achieved between wall 1 , which is in tension thanks to the pressure forcing wall 1 to protrude, and wall 2 in compression. The direct contribution of pressure to the moment at the cross section is then secondary. As a result, their performance is relatively high at low deflections, where low pressures produce significant $T_{1}$, but tends to reduce at higher deflections, where the $T_{1}$ created by a given pressure is lower. This behavior is analogous to that of PAMs [123].

### 4.6.3 Outline of design principles application

The design principles can be used in the process of determining the most suitable design in each scenario. The design depends on multiple factors in terms of requirements and constraints


Figure 4.9: Flow chart outlining the overall procedure to design a soft robotic manipulator in a given scenario. The chart summarises the main design steps, which are implemented using the design principles derived in this work.
of the scenario, so each case needs to be considered individually. Nonetheless, an overall design procedure exists, which is generally common. This is schematised in Figure 4.9, and outlined in the following.

First, the spatial constraints and the scenario requirements (typically desired deflection) are considered, and the category of device is selected accordingly. If the desired deflection is relatively low and some space is available for a protrusion, a contracting device is selected. Conversely, if the desired deflection is high, or the maximum diameter is very constricted, an extending device is selected. If the desired deflection presents a broad range of values of interest, a device combining extending and contracting actuation can be selected. Finally, if the desired deflection is intermediate, both an extending and a contracting device need to be explored, and the most suitable design needs to be selected by comparing the performance of the final designs of both types of device.

Once the type of device is chosen, the total cross section is selected to occupy all room available at the desired deflection, as indicated in Figure 4.9. In some cases, braids or braces may be introduced to adapt to the total cross section to the spatial constraints. A preliminary crosssectional geometry is then designed, following the design principles and defining a preliminary estimate of the regions corresponding to each wall.

The stiffness distribution is then selected, following the design principles. In most contracting devices, this can affect the design of the total cross section to use all available room and any braids or braces associated with it, and thus they need to be designed in conjunction. Once the stiffness distribution and total cross section are established, the cross-sectional geometry is adjusted according to the design principles. Iteration can then be conducted to satisfy all design principles to the best possible extent, as shown in Figure 4.9.

The design procedure up to this point provides the most suitable design layout. In some cases, the design principles can show that a compromise is necessary, as not all principles can be concurrently satisfied. In addition, the value of specific design parameters may need to be optimised, which is also generally identified by the design principles. FE simulations can be used to optimise the parameters, and resolve the compromises, yielding the final design. The FE simulations can also be used to compare final performance of designs in the case that both an extending and a contracting device are explored, and thus select the best. An example of design application is presented in the next section, which showcases this design procedure in a problem that illustrates the different steps described in this subsection.

### 4.7 Case study: design of a manipulator for MIS

The design principles distilled in the previous subsections are applied in this section to derive the design of a segment of soft robotic manipulator in a prototypical scenario.

### 4.7.1 Scenario definition

A minimally invasive surgery (MIS) scenario requiring a segment of soft robotic manipulator is selected as the prototypical scenario in this work. Soft robotic manipulators are well suited to MIS, offering compliance, modularity, compatibility with magnetic resonance imaging, and miniaturization possibilities that are particularly desirable in keyhole surgery. The recent interest in the subject [112] illustrates the relevance of these devices in medical applications.

The specific requirements for the segment of soft robotic manipulator in the selected scenario are for it to be able to bend laterally in any direction, providing 2 degrees of freedom (DOFs), and to maximise the lateral force that can be supported at deflections near 20 degrees. This deflection is measured as the angle between the centres of the manipulators ends in undeformed and deformed configurations, and is selected arbitrarily to illustrate the determination of the design in a representative case. The outer diameter of the device is constrained to 6 mm , and the operation pressure is limited to 6 psi. These are typical values in MIS where a small diameter is required for entry into the body, and the maximum pressure is limited due to the relatively weak sealing at miniature size and to prevent damage in case of bursting. These values are also similar to the pressures and deflections considered in the literature for devices with similar characteristics $[116,183]$.

The minimum wall thickness is considered to be limited by manufacturing constraints and associated resilience to puncture, leakage, and withstanding the maximum pressure. The manufacturing of soft robots commonly involves casting the hyperelastic structure of the device, adding fibers, sheets or other inextensible elements, and finally affixing all the elements typically with additional layers of hyperelastic material. Considering the typical tolerances associated to these processes, a minimum wall thickness of $400 \mu \mathrm{~m}$ is selected for the prototypical scenario, which is considered to be a thickness value that could be achieved reliably with sufficient investment in manufacturing. The suitability of this thickness to withstand the maximum pressure with a safety margin to cope with manufacturing tolerances while providing a certain degree of resilience is confirmed in the simulations in subsection 4.7.6.

### 4.7.2 Design derivation

## Primary design derivation

The principles of operation of extending and contracting devices are different, which makes the devices suitable for operation at different deflections. Contracting devices predominantly support external wrenches thanks to the pressure forcing the protruding wall to be in tension and thus the opposite wall in compression, and the direct contribution of pressure to support external moments is secondary. As a consequence, they generally offer higher performance at lower deflections where even low pressures create significant tension in the protruding wall. However, as deflection increases, the relation between $T_{1}$ and $p$ reduces, and $T_{2}$ becomes a tension force, leading to lower performance. Conversely, extending devices support external wrenches thanks to the direct contribution of pressure to generate a moment when considering equilibrium of a device isolated in a general cross section, in combination with $T_{1}$. Thus, they typically offer lower performance at low deflections and low pressure, but their performance remains relatively constant as deflection and pressure increase, offering higher performance at higher deflections. A design combining extending and contracting operation would therefore be advantageous in this application that requires operation at various deflections.

The design principles for extending and contracting devices share many similarities. The wall thickness should generally be minimised, and the area of the cross section corresponding to the pressurised fluid should be maximised; the devices should use all available room; the region corresponding to wall 1 should present a maximal longitudinal stiffness, and this should be concentrated near the edge to maximise the distance between the line of application of $T_{1}$ and $T_{2}$. In addition, these principles are generally independent of the desired deflection and pressure. Thus, a design combining both types of operation can be conceived for this scenario.

The main design difference is that, in extending devices, a wall 2 with minimum longitudinal stiffness is desirable, as elucidated in section 4.4, which can be attained with a pleated structure. Instead, in contracting devices, wall 2 must typically support compressive stresses, as shown in section 4.5, and therefore a pleated structure is not viable. Thus, a certain degree of compromise
is necessary.

In this prototypical scenario, any protrusion over 6 mm diameter is undesirable. Thus, the outer structure should be cylindrical with 6 mm diameter. In order to provide bending in any direction, the design must be 3D, and should then include at least three chambers in the cross section along the device. Since chambers involve partition walls that increase bending stiffness, the number of chambers should be minimised, leading to three chambers being selected. The design principles indicate that cross section deformation is desirable from an extending device perspective in order to maximise the area of the cross section corresponding to the pressurised chambers, and displace the line of application of $T_{1}$ towards the outer contour, with maximum concentration of stiffness at the region corresponding to $T_{1}$. Such cross section deformation leads to a protruding central rod. This can be exploited as the protruding wall in contracting devices. Thus, the central rod should have an infinite longitudinal stiffness, which is desirable for it to act as the protruding wall of extending devices, and as wall 1 of extending devices. This results in a device combining extending and contracting operation, with a design that is desirable for both types of operation as it maximises area of the cross section corresponding to the pressurised fluid, and presents a desirable stiffness at the equivalent of wall 1.

Since the design includes contracting operation, a structure capable of supporting compressive stress is necessary to act as wall 2 . The device must be capable of bending in any direction, hence the line of application of the equivalent of $T_{2}$ must be near the centre of the device. The most suitable solution is then the incorporation of an outer cylindrical structure made of superelastic material such as nitinol, with notches in alternating perpendicular directions to enable bending with minimum resistance while supporting compression forces.

The most suitable design of the soft robotic manipulator is therefore a cylinder with a constant cross section that consists of three equal chambers that can deform and present a maximum area, and an outer metallic structure, as conceptually illustrated in Figure 4.10 (left). The ratio $S_{1} / S_{2}$ should be maximised according to the design principles, which implies a minimal stiffness at the outer wall, and maximal longitudinal stiffness at the central rod. This can be obtained by designing an outer wall made of minimal stiffness material and with minimum thickness, which


Figure 4.10: Conceptual illustrations of the most suitable design (left), and alternative design (right). The design on the left includes three partition walls with minimal stiffness to facilitate cross-sectional deformation, an inextensible central rod, a minimal outer wall thickness of 400 $\mu m$ made of low stiffness material, and a notched outer structure to support compression force while minimizing resistance to bending. The design on the right also has an outer wall with minimal thickness and minimal resistance to bending, but it does not include an outer structure, and instead is has outer fibers to prevent radial expansion. In addition, its three partition walls are also deformable but present some stiffness, which combines with a central rod that can extend to some degree to prevent compression of the outer wall.
in this scenario corresponds to $400 \mu m$ as described in the previous subsection, and including an inextensible thread at the central rod. The stiffness of the partition walls should be minimal to facilitate cross section deformation, and wall thickness should be minimal to maximise the area of the chambers in the cross section. Since the maximum cross-sectional deformation is limited by the outer wall, the partition walls are always below the maximum strain of typical hyperelastic materials, and thus the minimum wall thickness in this scenario, $400 \mu \mathrm{~m}$, can be selected. It should be noted that the outer structure serves to prevent radial expansion of the outer wall. The maximum protrusion of the central rod is also limited by the outer diameter, and therefore the device respects the diameter constraints while offering contracting operation.

## Discussion of primary design found

This design layout resembles that of the FMA, but the principles of operation and the specific geometry and stiffnesses are different. This layout combines extending and contracting operation, in contrast to the FMA that only involves extending operation. In addition, the wall thickness in this layout is lower than in the FMA to maximise the chamber area in the cross section, the central rod is inextensible to maximise force, and the design includes an outer structure to support compression forces. Finally, the partition walls in the proposed layout contrast with those in the FMA, as they are designed to facilitate cross section deformation, which maximises the area corresponding to the pressurised fluid in the cross section and leads to contracting operation.

The manufacturing of the proposed outer structure capable of supporting compression forces is challenging, particularly at the miniature size of this prototypical scenario. In addition, it can introduce bending resistance, limiting performance. Furthermore, the structure can limit extending-type operation at relatively high pressures.

## Alternative design

An alternative design without the outer structure can be conceived, which is easier to manufacture and more illustrative of the research presented in this chapter, enabling the verification of some of the design principles. The need for a structure to support compressive forces stems from the significant tension at the central rod associated to a protrusion, and mainly occurs at low deflections. As mentioned in subsections 4.5.4 and 4.5.7, this tension can be reduced by introducing some resistance to the protrusion. In this 3D design, the resistance can be introduced by using partition walls with some stiffness. Thus, by combining partition wall stiffnesses together with some extension of the central rod, a design without compression force on the outer structure can be achieved. It should be noted that, in such design, the central rod still serves to increase performance by introducing contracting actuation and by maximizing $S_{1} / S_{2}$, hence a high central rod stiffness in the longitudinal direction is desirable. The main purpose of the partition walls is to compensate any excessive effect of the protrusion for a given pressure and deflection, and therefore the partition wall stiffness (PWS) depends on the longitudinal central rod stiffness (LCRS), with higher LCRS requiring higher PWS.

The alternative design is therefore similar to the previous design, as conceptually illustrated in Figure 4.10 (right), but with different values of PWS and LCRS. This leads to a design without compression at the outer wall, which eliminates the need for complex structures while enabling operation at low deflection. Consequently, it represents the design selected for this prototypical scenario.

The outer wall can then be made of soft material, and according to the design principles should have a minimal wall thickness to maximise the area of the cross section corresponding to the pressurised fluid, and a minimal bending stiffness to maximise $S_{1} / S_{2}$. A pleated structure with circumferential fibers could be used to minimise bending stiffness, but manufacturing at millimetric scale can be challenging. Instead, a cylindrical outer wall made of soft material with circumferential fibers to prevent radial expansion while allowing longitudinal deformation is practically equivalent and easier to manufacture, hence is the solution selected. The material of the outer wall should be hyperelastic, with low stiffness, and capable of withstanding pressure
when combined with fibers. In order to consider a realistic material that is readily available, DragonSkin 10 (Smooth-On, USA) is selected for this prototypical scenario. This is a common material in soft robotics and it has been previously characterised in the literature [184]. The wall thickness should be the minimum possible, which corresponds to $400 \mu m$ in this scenario, as described in the previous subsection. This wall thickness can withstand $p_{\max }$ with only minor bulging of the rubber between the fibers, which corresponds to a maximum strain in the rubber below the failure limit of the material, as confirmed in the simulations in subsection 4.7.6. The cross section area corresponding to the pressurised fluid should also be maximised according to the design principles, which implies a minimum partition wall thickness of 400 $\mu m$. This principle also implies that cross section deformation is also desirable, which however can be limited by PWS. Thus, the contributions of PWS and LCRS need to be matched to achieve the desired performance.

## Compromise in optimal design parameters

The optimal values of LCRS and PWS depend on the maximum pressure, denoted by $p_{\max }$, as well as the outer wall characteristics. Increasing the LCRS improves $S_{1} / S_{2}$, and thus the design principles indicate that it increases performance, as qualitatively shown in Figure 4.11 (left). However, it requires a high PWS to prevent buckling, and therefore cross section deformation can be compromised, which can reduce performance. Conversely, lower PWS facilitates cross section deformation, which according to the design principles is desirable, leading to higher initial deflections and higher performance at lower pressures, as qualitatively shown in Figure 4.11 (left). However, the maximum LCRS is then limited, which can reduce performance at higher pressures. A compromise is therefore necessary, which depends on $p_{\max }$. The performance of designs optimised for different $p_{\max }$ is qualitatively illustrated in Figure 4.11 (right), elucidating the fact that the optimal values of the parameters must be selected for the operating pressure in each scenario.

Since cross-sectional deformation is limited by the outer wall, all designs become equivalent in terms of cross section once full cross-sectional deformation is reached. On the other hand,


Figure 4.11: (left) Qualitative graph illustrating the design trends corresponding to the variation of LCRS in a design where the rest of the design remains equal, shown in blue, and the variation of PWS in a design where the rest remains equal, shown in magenta. (right) Qualitative graph illustrating the performance of designs optimised for different $p_{\max }$ in terms of their PWS and LCRS, elucidating the fact that the design parameters PWS and LCRS must be optimised for the specific pressure of operation in each scenario.
maximal PWS enables higher LCRS and therefore higher performance. In addition, high PWS also contributes to the longitudinal stiffness of wall 1 , increasing and thereby leading to better performance. Thus, the optimization of the design involves selecting the maximum PWS that enables reaching full cross section deformation at, and then the maximum LCRS to minimise tension at the outer wall during operation of the device while avoiding buckling. These parameters need to be optimised for each specific scenario. FE simulations were developed in this work for the optimization in the prototypical scenario. The specific simulations, the optimization process, and results obtained are reported in the next four subsections.

It should be noted that, in the design selected, the partition wall stiffness serves to prevent buckling before full cross section deformation. After reaching full cross section deformation, this cross section remains practically constant despite further increases in pressure, and buckling does not occur since the contribution of the contracting effect is practically completed. Thus, designs with different PWS become equivalent once they reach full cross section deformation provided that the rest of the design is equal and that the contribution of the partition walls to the longitudinal stiffness is relatively low.


Figure 4.12: Configuration of the simulations with soft robotic manipulator and rigid block. The distances and angles are specified in the diagram.

### 4.7.3 FE simulations: evaluation criteria

FE simulations were developed in order to optimise the design parameters for the device in the prototypical scenario, and verify the design principles extracted in the previous subsections. The criteria to evaluate the performance of the soft robotic manipulator deserve consideration. These are first considered in this subsection.

The design objective in this prototypical scenario is to maximise the lateral force at a deflection near 20 degrees for a given $p_{\max }$. Thus, the performance is evaluated by measuring the normal force applied onto a prismatic block positioned as shown in Figure 4.12, with frictionless contact. This corresponds to an approximate deflection near 20 degrees of the manipulator at initial contact, and an interaction that is normal to the rigid block and approximately lateral on the soft robotic manipulator. It should be noted that the deflection at initial contact is somewhat lower than 20 degrees. This is intentional since the relative rotation between the ends of the manipulator varies with pressure even after contact, which implies that the distance between the centre of the distal end of the device and the block changes even after contact. Since
deflection is measured based on the position of the centres of the manipulator's ends, this varies at different pressures during contact. Thus, the rigid block is specifically positioned so that deflection is near 20 degrees for the range of pressures of interest.

This configuration selected for the simulations is also a representative of the typical operation of soft robotic manipulators. The design principles were shown to be independent of maximum pressure and deflection. Thus, the FE simulations conducted in this configuration also serve to verify some of the design principles derived in the previous sections.

### 4.7.4 FE simulations: parameter optimization

The objective of the optimization of the LCRS and PWS is to obtain both maximum PWS while reaching full cross section deformation at $p_{\text {max }}$, and minimal outer wall tension in the operation range of the device, while preventing buckling due to compression of the outer wall. This maximises the wrenches that can be supported at $p_{\max }$ and enables operation at low deflection. The procedure to determine the optimal values of LCRS and PWS is as follows.

First, PWS is selected to obtain full cross-sectional deformation at $p_{\max }$ for a generic LCRS. This is achieved by conducting quasistatic simulations for a set of values of PWS with regular stiffness increments while maintaining constant material properties elsewhere. The simulations are executed using a gradual increase in pressure until a practically full cross-sectional deformation, which here is specified by the central rod reaching approximately $70 \%$ of the radius, and the corresponding pressure is recorded. The PWS of the design that achieves practically full crosssectional deformation at a pressure closest to $p_{\max }$ is selected. Then, for the optimal PWS, the LCRS to achieve minimal outer wall tension is determined. This is done by conducting simulations with the optimal PWS and gradually increasing value for LCRS, starting with a stiffness corresponding to that of DragonSkin 10, until the outer wall stiffness is minimal. The LCRS that reaches minimal outer wall stiffness without buckling, together with the PWS to provide practically full cross section deformation at $p_{\text {max }}$, constitute the optimal design. It should be noted that this optimization process of determining the PWS first independently of


Figure 4.13: Undeformed cross section geometry of the design selected viewed with a perspective projection (left), and deformed cross section of the same design with partition walls made of a material with $C_{10}=127500 \mathrm{~Pa}$ when pressurised at 6 psi in two chambers (right).
the LCRS is possible since cross-sectional deformation is relatively independent of the value of LCRS.

It should also be noted that rupture of the partition walls due to excessive strain is not considered since the maximum cross-sectional deformation is limited by the outer wall. The maximum possible extension of the partition walls is approximately double their initial length, which is significantly below the failure limit of typical rubbers. Similarly, extension of the central rod is typically lower than double the initial length, which is also below the failure limit. Thus, the PWS and LCRS are varied freely.

### 4.7.5 FE simulation implementation

The simulations were implemented using Abaqus/Standard - Simulia ${ }^{\mathrm{TM}}$, Dassaut Systemes ${ }^{\circledR}$ (Velizy-Villacoublay, France). The simulation set up involves the soft robotic manipulator and a rigid block situated as shown in Figure 4.12. The geometry of the soft robotic manipulator is a 6 mm diameter cylinder, with a constant cross section as shown in Figure 4.13 (left), a solid end cap of 1 mm thickness, and a total length of 31 mm .

The material of the outer wall was modeled as an incompressible, hyperelastic material with a Neo-Hookean constitutive law with $c_{10}=42500 P a$ and $D=0$, following [184]. The constitutive behavior of the material of the partition walls was also approximated with an incompressible Neo-Hookean law, and the different values of the PWS were selected by varying the parameter $c_{10}$, with values ranging between $c_{10}=42500 \mathrm{~Pa}$ and $c_{10}=425000 \mathrm{~Pa}$ at regular increments of 42500Pa. Similarly, the material of the central rod was also approximated with an incompressible Neo-Hookean law, with a $c_{10}$ that was modified to vary LCRS. The bending stiffness of the central rod was not relevant at the stiffness values of interest since this was sufficiently thin. Finally, the fibers were modeled as circular beams of $10 \mu \mathrm{~m}$ diameter made of a material with a Young's modulus of 51 GPa , and a Poisson ratio of 0.36 , which is representative of Kevlar.

The deformation of the device resulting from the pressurisation of two chambers was simulated by applying a pressure in the chambers that increased linearly, from zero to the maximum applied pressure. This linear load increment approach enabled solving the geometrically nonlinear problem. An encastre boundary condition was imposed at one end of the manipulator, and another encastre was defined at one point of the rigid block. The contact between the manipulator and the rigid block was modeled as frictionless. The contact force was measured as the force applied by the manipulator on the rigid block.

The force corresponding to wall $2, T_{2}$, was measured as the aggregated tension force over the outer wall of the device in a free body cut corresponding to the cross section indicated in Figure 4.14. This is due to the fact that the outer wall in this 3D design provides the equivalent function as wall 2 in the analytical derivation. The mesh was maintained constant when varying material properties in the different simulations, and mesh convergence testing was conducted to ensure that the analysis was not affected by the characteristics of the mesh.

### 4.7.6 FE simulation results

The results of the simulations provide the deformation of the device and the force it applies on the rigid block as a function of pressure, as illustrated in Figure 4.14 for a representative simulation. These results serve both to determine the optimal design parameters of the device


Figure 4.14: Simulation of deformed geometry in design with partition walls made of a material with $c_{10}=127500 \mathrm{~Pa}$ and central rod made of a material with $c_{10}=425 \mathrm{MPa}$, pressurised at 6 psi in two chambers. The bulging at the outer wall between the fibers is shown, as well as the deformation of the cap caused by the tension of the central rod.
in the prototypical scenario, and to verify some of the design principles, as described in the following.

## Design optimization results

In terms of optimal parameters, the results of varying PWS for constant LCRS show that partition walls made of a material with $c_{10}=127500 P a$ yield practically full cross-sectional deformation at $p_{\max }$, as shown in Figure 4.13 (right). This cross section corresponds to the section marked in orange in Figure 4.14, which is representative of the cross-sectional deformation along the device. Thus, the optimal PWS in this scenario corresponds to $c_{10}=127500 \mathrm{~Pa}$ since it is the highest PWS that reaches practically full cross section deformation at $p_{\max }$.

The results of increasing LCRS for the optimal PWS are shown in Figure 4.15. As can be seen, the performance improves with increasing values of LCRS. At an LCRS of $c_{10}=425 \mathrm{MPa}$, the tension at the outer wall becomes zero and even slightly negative during operation, as can


Figure 4.15: Plot of lateral force as a function of pressure for different designs. The performance of designs with a PWS of $c_{10}=127500 \mathrm{~Pa}$ and a hyperelastic LCRS, the value of which is indicated in the legend in terms of the $c_{10}$ parameter, are shown with continuous lines. The performance of designs with a PWS of $c_{10}=127500 \mathrm{~Pa}$ and an elastic LCRS, with the stiffness indicated in the legend in terms of the Youngs modulus, are shown in dashed lines. The performance of the FMA design in terms of partition wall stiffness and LCRS, but with an outer wall thickness of $400 \mu \mathrm{~m}$, is shown with an orange line combining dots and dashes.
be seen in Figure 4.16 (left), where the tension at the outer wall is plotted as a function of pressure for the different LCRS. Thus, the optimal LCRS corresponds to $c_{10}=425 M P a$ since higher values of LCRS would involve compression stress at the outer wall, which could lead to structural instabilities such as buckling. This was confirmed by executing simulations at higher LCRS, which presented converge issues due to structural instabilities.

A design with partition walls made of a material with $c_{10}=127500 P a$ and central rod with $c_{10}=425 \mathrm{MPa}$ therefore represents optimal design in this scenario, together with the aforementioned geometry and the outer wall made of DragonSkin 10 with $c_{10}=42500$ Pa . The higher performance of the optimal design is predominantly due to two factors. First, it presents practically full cross section deformation at $p_{\max }$, and therefore it provides a high performance in terms of cross section as the area corresponding to the pressurised fluid is maximised and the majority of the stiffness is concentrated near the cross section contour corresponding to wall

1. Second, it has the highest LCRS, and therefore the force spent stretching the structure is minimised, particularly at the central rod which corresponds to wall 1 , leading to a maximal contribution of pressure to support external wrenches. Interestingly, a relation can be observed between the reduction in tension at the outer wall, shown in Figure 4.16 (left), and the improvement in performance due to the increase in LCRS, shown in Figure 4.15, where the magnitude of the improvement in performance between two designs is directly related to the magnitude of the reduction in outer wall tension.

The results of the simulations show that buckling of the outer wall does not occur, as can be seen in Figure 4.14. The results also indicate that any bulging of the outer wall between the circumferential fibers is minor, as can also be seen in Figure 4.14, which corresponds to a strain at the outer wall that is maintained significantly below the failure limit of the rubber. Thus, the wall performs as desired, withstanding the pressure applied without excessive wall thickness.

This outer wall behaves similarly to a pleated structure, presenting longitudinal extension with only minimal radial expansion between the fibers. Thus, this design with the soft outer wall and circumferential fibers is mostly equivalent to a previously mentioned design with a pleated structure and circumferential fibers, and the study developed here can be generally extrapolated due to the similar structural behavior.

The results of the simulations also confirm that the tension at the inner rod is relevant and significant. Observing the end cap, as shown in Figure 4.14, a depression can be noted, which is caused by the inner rod in tension.

## Performance comparison

The performance of the design obtained in this work with optimal parameters was also compared with that of the FMA design, in terms of material stiffnes of the partition walls and LCRS, since it is as well-established design, and is representative of some of the highest performing soft robotic manipulator designs that can meet the requirements of the scenario defined in this work. It should be noted that, in this FMA design, only the partition walls and central rod


Figure 4.16: (left) Plot of outer wall tension as a function of pressure (left) for designs with a PWS of $c_{10}=127500 \mathrm{~Pa}$ and various LCRS, indicated in the legend in terms of the $c_{10}$ coefficient. (right) Plot of lateral force as a function of pressure for designs with varying PWS and equal material properties in the rest of design, including LCRS. The PWSs are indicated in the legend in terms of the $c_{10}$ parameter of the Neo-Hookean constitutive law used to model these hyperelastic, incompressible materials.
of the FMA were used. The outer wall thickness used was the same as in the rest of this case study, $400 \mu m$, to compare performance in equivalent conditions, removing the potential effect of a different outer wall on bending stiffness. DragonSkin 10 was selected as the material for the FMA, also to compare both designs in equivalent conditions. The results of lateral force as a function of pressure are shown in Figure 4.15. As can be seen, the design obtained here provides a higher force at $p_{\max }$. The results also show that, at lower pressures, the FMA design presents a somewhat higher performance, primarily due to the softer partition walls that enable larger cross-sectional deformation at lower pressure, confirming that a design must be optimised for a specific pressure.

The performance of the design obtained in this work was also compared with that of the standard FMA with an outer wall thickness of 0.75 mm , which matches exactly the standard FMA design. DragonSkin 10 was also used as the material for the rubber. The results of lateral force obtained, however, could not be included in Figure 4.15, since the standard FMA makes contact with the rigid block at 6.39 psi , which is beyond the interval of pressures in the plot. The performance of the standard FMA is plotted later in section 4.10 Figure 4.22 where a suitable interval of pressures is shown. The fact that the standard FMA does not
even make contact with the rigid block at the range of pressures considered here shows that the performance of the standard FMA is significantly below that of the design obtained here for the range of pressures in this case study, and is lower than that of the FMA with an outer wall thickness of $400 \mu \mathrm{~m}$. This is due to the thicker outer wall in the standard FMA, which adds bending stiffness, and thus reduces performance. It should be noted, however, that the comparison with the standard FMA is not entirely valid since this is designed to withstand higher pressures than $p_{\text {max }}$ here, and to operate in a range of pressures that exceeds that of Figure 4.15. The design for any desired pressure, which includes the operating conditions of the standard FMA, is considered in the next sections 4.8, 4.9, 4.10.

## Alternative materials

The design obtained in this work can be fabricated using readily available silicones for the outer wall, partition walls, and fibers. However, the hyperelastic material selected for the central rod can be difficult to obtain in practice as it presents a stiffness significantly higher than that of standard rubbers. In order to consider more realistic materials, equivalent simulations were conducted using elastic material properties for the central rod, with Young moduli between $E=10^{8} \mathrm{~Pa}$ and $E=10^{10} \mathrm{~Pa}$, which are representative of cotton or wool threads. The results are shown in Figure 4.15, together with the previous results for hyperelastic central rod. As can be seen, the performance of designs with central rods made of stiff, elastic materials are equivalent to those with hyperelastic materials. Thus, the design can be fabricated by using readily available materials such as textile threads as the central rod.

## Principles and operation verification results

The results of the simulations also serve to verify two of the most relevant design principles. In addition, they can be used to confirm that the operation of the device is as predicted.

The performance of different designs with varying PWS and constant LCRS and material properties elsewhere is plotted in Figure 4.16 (right) as a function of pressure. The plots
indicate that lower PWS increases the lateral force that the device can apply, and reduces the pressure required to attain an initial deflection. These results agree with the behaviour predicted based on this analysis in this work, shown in Figure 4.11 (left). Thus, the results confirm that cross-sectional deformation is desirable to improve performance. Equivalently, the results verify that maximizing the area of the cross section corresponding to the pressurised fluid is desirable to maximise the force of soft robotic manipulators. This contrasts with some of the designs in the literature [113], and shows that, unless additional constraints are present, such as those exposed in subsection 4.4.6, the exploitation of cross-sectional deformation can yield designs with improved performance.

The results of increasing values of LCRS with a constant design elsewhere, shown in Figure 4.15 for both hyperelastic and elastic central rods, confirm that increasing LCRS leads to higher force in general. These results also agree with the predicted trends, shown in Figure 4.11 (left). This verifies another of the design principles, namely that high LCRS is desirable to maximise the performance or, equivalently, that maximal $S_{1} / S_{2}$ is desirable to maximise the force that can be applied, provided that it does not lead to buckling of wall 2 . It should be noted that the result of the simulation with a central rod stiffness of $c_{10}=4.25 M P a$ does not reach the full pressure. This is due to the fact that the simulation did not converge at pressures above 5.7 psi since some mesh elements presented excessive distortion. Nonetheless, the plot elucidates the trends of interest.

Finally, the results of tension at the outer wall for different LCRS, shown in Figure 4.16 (left), confirm that increasing LCRS leads to lower values of overall tension at the outer wall. Thus, these results confirm that the performance improves as less force is spent stretching the outer wall. In particular, for the optimal LCRS, the results in Figure 4.16 (left) show that the tension at the outer wall becomes zero and even to a slight extent negative, which indicates that the objective of the optimization in terms of minimizing outer wall tension is achieved. Moreover, the results on outer wall tension confirm that the contracting operation is effective, particularly at low deflections, where the tension at the equivalent of wall 2 becomes practically zero. At larger deflections, the contribution of the contracting operation is significantly reduced, since the protrusion is limited by the outer wall and cannot increase further. Then, the extending
operation becomes relevant, which involves some inevitable tension at the outer wall, but provides a high overall performance.

### 4.8 Design for unconstrained pressure

The design study presented up to this point was developed for a given maximum pressure. The study can be extended to the case where any maximum pressure can be used. This is presented in this section.

The design study in the case of unconstrained maximum pressure is mostly equivalent to that for a given maximum pressure. The main difference is that here the maximum pressure is not given, but depends on the design.

### 4.8.1 Effect of design on pressure

As previously noted in section 4.2 , the pressure that a device can withstand primarily depends on its sealing points and its outer walls. The resistance of the sealing points can be increased with improved fabriation techniques, and it is not considered an issue in the design problem. This is discussed further in the fabrication section in chapter 6. The resistance of the outer walls, on the other hand, depends on their design, and this affects the performance of the device. Thus, it must be added to the design problem.

The strength of the outer walls chiefly depends on the type of wall used and on its thickness. The outer wall generally needs to be able to extend in order to enable bending of the device, while withstanding the pressure without expanding radially beyond the operation constraints. Thus, it needs to be anisotropic in stiffness. This can generally be achieved with two types of wall design: either a pleated structure with circumferential fibres, or with a straight structure (without pleats) made of a soft material such as rubber and circumferential fibres.

In both types of outer wall, a thicker wall can generally withstand higher pressures. And the application of higher pressures in a device generally allows for the support of greater wrenches,
as can be seen by formulating the equilibrium of the device isolated in a general cross section in analogous manner as in sections 4.4 and 4.5. However, a thicker wall introduces bending resistance, and occupies room in the cross section, both of which are undesirable since they affect the design by reducing the wrenches that can be supported.

The effect of the outer wall on the design can be then summarised with two additional principles. First, a thicker outer wall can withstand higher pressures, which lead to higher wrenches that can be supported. Second, a thicker outer wall increases the bending stiffness and reduces the area in the cross section corresponding to the pressurised fluid, which reduces the wrenches that the design can support.

### 4.8.2 Extension of design to unconstrained pressure

The design of devices with unconstrained maximum pressure is then equivalent to the design with a given maximum pressure, with the added factor that here the $p_{\max }$ depends on the outer wall, particularly in terms of its thickness. Each outer wall implies a $p_{\max }$, and for this $p_{\max }$ the principles in the previous sections apply. However, the fact that here the $p_{\max }$ depends on the outer wall implies that the factors summarised in the two additional principles in the previous paragraph must also be considered in the selection of the outer wall.

These two additional principles generally introduce a compromise in the design of the outer wall, which in turns affects the $p_{\max }$ and thus the specific parameters in the rest of design. The design process in the case with unconstrained maximum pressure is then mostly equivalent to the case with a given maximum pressure, and can be conducted using the principles summarised in section 4.6 together with the two additional principles in the previous subsection 4.8.1. However, the two additional principles mean that an extended optimisation process is generally required. This involves considering various outer walls with corresponding $p_{\max }$, optimising the design for each of them, and finally comparing the performance of the designs for different $p_{\max }$ to select the best. This is presented by way of example in section 4.10.

It should be noted that analytical solutions to the maximum pressure that an outer wall can
withstand are not available, so numerical solutions are required. This implies that the design process must involve a numerical part to consider each outer wall. In this regard, the approach adopted in this work to study the design for a given maximum pressure, and then extend it to consider any maximum pressure, which depends on the outer wall, is well suited to the requirements of the design process, since it enables outlining the design first for various possible $p_{\max }$ and then performing the numerical simulations to determine the most suitable outer wall and thus $p_{\max }$.

### 4.9 Non-dimensional analysis

The material selection is important in both the design of the fine-positioner, and in the more general design of soft robotic manipulators. In addition, in the development of soft robotic manipulators, it can be relevant to extrapolate between different scales in order to reduce the number of simulations and experiments required for development.

A non-dimensional analysis was thus developed for soft robotic manipulators with a design outline similar to that of the FMA or the design shown in Figure 4.10 (right), which are similar to the design of the fine-positioner selected in the next section. This provides valuable insight for the material selection for the fine-positioner, and allows for the extrapolation of the work between different scales. In this regard, it enables conducting experimental studies at convenient sizes, and reducing the number of simulations and experiments required.

### 4.9.1 Development of non-dimensional analysis

The deformation of geometrically equivalent soft robotic manipulators similar to those of interest under equivalent external load distribution, and pressurisation in one chamber or multiple chambers with a given ratio of pressures between them, depends on a set of non-dimensional groups. These can be selected to be $\frac{p}{E_{r}}, \frac{F}{E_{r} d^{2}}, \frac{E_{f}}{E_{r}}, \frac{E_{c}}{E_{r}}, \frac{E_{p}}{E_{r}}$, as well as a parameters $\nu_{r}, \nu_{f}, \nu_{c}$, and $\nu_{p}$, where $p$ is the reference magnitude of the pressure applied, $F$ is the magnitude of the
external wrenches applied for a given wrench distribution that generally excludes gravitational forces since they are typically negligible, $d$ is a reference length corresponding to the diameter of the device, $E_{r}$ is a coefficient related to the stiffness of the rubber (which can correspond to the Young's modulus or the $c_{10}$ coefficient in a Neo-Hookean constitutive law), $E_{f}$ is the stiffness of the outer fibres, $E_{c}$ is the stiffness of the central rod, $E_{p}$ is the stiffness of the partition walls in designs where these are made of a different material than the outer wall, and $\nu_{r}, \nu_{f}, \nu_{c}$, and $\nu_{p}$ are the Poisson ratios of the rubber, outer fibres, central rod and partition walls, respectively. It should be noted that here the use of the magnitude $p$ in the non-dimensional groups imposes the stress at the surfaces where pressure is applied, and is equivalent to the use of a reference magnitude of stress in the structure of the device.

In the deformation of the soft robotic manipulator, it can be assumed that the outer fibres are inextensible. In addition, the effect of $\nu_{f}, \nu_{c}$, and $\nu_{p}$ on the behaviour of the complete device can be assumed to be negligible in general. Moreover, rubbers used as soft material are generally incompressible, so $\nu_{r}$ is generally constant. Thus, the non-dimensional groups of $\frac{E_{f}}{E_{r}}$, $\nu_{f}, \nu_{c}, \nu_{r}$ and $\nu_{p}$ are considered to have a negligible effect or be irrelevant, and the important non-dimensional groups are reduced to $\frac{p}{E_{r}}, \frac{F}{E_{r} d^{2}}, \frac{E_{c}}{E_{r}}$, and $\frac{E_{p}}{E_{r}}$. Also, in some designs such as that selected in section 4.7, the central rod can be assumed to be practically inextensible, and in other devices such as the FMA the central rod is not present. In these cases, the group $\frac{E_{c}}{E_{r}}$ is not relevant. Finally, in designs where the partition walls must also be made of the same material as the outer wall due to fabrication constraints, the non-dimensional groups are reduced to $\frac{p}{E_{r}}$, $\frac{F}{E_{r} d^{2}}$.

Using this non-dimensional analysis, the behaviour of geometrically equivalent devices with either different size, different material, different magnitude of external wrenches, or different applied pressure can be guaranteed to be equivalent if the non-dimensional groups are equal. In addition, the number of variables influencing the behaviour of a soft robotic manipulator in the non-dimensional case is reduced by two relative to the dimensional case, which simplifies the study of these devices.This analysis can also be generalised to designs with additional regions or fibres with different stiffness by adding non-dimensional groups corresponding to the ratio of stiffnesses in the different regions.

### 4.9.2 Applications of non-dimensional analysis

This non-dimensional analysis enables the extrapolation of the behaviour of a soft robotic manipulator to different materials. It indicates that an equal deformation is achieved in designs made of materials with different stiffness if the pressure and external wrenches are varied proportionally to maintain the non-dimensional groups constant. This suggests that stiffer materials lead to devices capable of supporting higher wrenches with an equal deformation, which is generally desirable, provided that pressure is also increased.

The stress in a stiffer material also increases proportionally in the case of equivalent deformation. Thus, the use of stiff materials with a relatively low ultimate stress can be undesirable. However, for materials with similar ultimate strain, which is the case of some of the hyperelastic rubbers used as soft materials in manipulators with fluidic actuation, the non-dimensional analysis indicates that the use of stiffer materials is desirable as it leads to devices that can support higher wrenches, provided that the assumptions in the previous subsection are satisfied. This is further elucidated in the material selection for the fine-positioner in the next section 4.10.

The non-dimensional analysis provides additional insights on the mechanical behaviour of soft robotic manipulators. One relevant insight is that, for devices with equal material properties and no external loads, the relation between applied pressure and deformation is constant regardless of the device scale. This implies that the pressure required for operation is determined by the design and desired deflection, but not by the scale of the device, which is particularly relevant when considering medical applications.

Another insight of interest is related to the non-dimensional group $\frac{F}{E_{r} d^{2}}$. This elucidates that the external wrenches that can be supported by a device with specified material properties and a given deformation varies with the square of the diameter of the device. This is particularly relevant when considering the use of a given device, such as a fine-positioner, in alternative applications with different diameter constraints.

### 4.10 Design of fine-positioner

The requirements for the fine-positioner are similar to those in the case study in section 4.7, with the main difference that in this application the maximum pressure is not limited by the scenario. The design procedure is then generally equivalent to that in the previous case study, which serves as foundation. However, here the maximum pressure depends on the selected outer wall of the design, which must be added to the design problem.

### 4.10.1 Design derivation

The design derivation is then analogous to that in subsection 4.7.2 for the previous case study, and involves satisfying the design principles to the extent possible, and using the principles to identify compromises that need to be resolved through numerical optimisation where necessary. The main difference is that in the case of the fine-positioner, the maximum pressure depends on outer wall thickness, which must be selected. Therefore, here, the complete principles used in the derivation are those summarised in section 4.6 together with those in subsection 4.8.1.

The two additional principles from subsection 4.8.1 do not affect significantly the derivation of the layouts of interest, which remains equivalent to that in subsection 4.7.2. This involves first considering that each segment of fine-positioner should be capable of bending to achieve deflections of 30 degrees or more in order to provide a significant workspace. The design of this robot segment should be selected to maximise the wrenches that it can support at the operation deflections. As in 4.7.2, this implies that the best design should be a combination of extending device and contracting device. Considering that each robot segment needs to be capable of bending in any direction in 3D space, the design needs to have at least three partition walls in the cross section that define three chambers. The number of partition walls is then selected to be three considering the design principles in section 4.6 stating that the region in the cross section corresponding to pressurised fluid should be maximised, and that the bending stiffness of the design should be minimised.

The rest of design layout can then be determined by applying the design principles in section
4.6 together with the two principles in subsection 4.8.1. The aim in the application of the principles is to determine a design layout that satisfies as many principles as possible, and to identify compromises that need to be optimised numerically where necessary. The application of the principles here is analogous to that in subsection 4.7.2, with the main difference that the outer wall thickness in this case needs to be selected in a trade-off defined by the principles described in subsection 4.8.1. A given outer wall then determines the maximum pressure, which in turn implies a specific design as in the previous case study in subsection 4.7.2.

The specific design layout of interest for each outer wall thickness is determined by design principles that are simply those in section 4.6 , since the two principles in subsection 4.8 .1 only relate to the outer wall. Thus, the layouts of interest for the design of the fine-positioner for any selected outer wall are equivalent those in the previous case study shown in Figure 4.10, but with specific parameters in terms of partition wall stiffness and central rod stiffness that depend on the maximum pressure dictated by the outer wall thickness.

In this regard, the resulting possible design layouts for the fine-positioner are equivalent to those in Figure 4.10. The main difference in the design of the fine-positioner is that here the outer wall thickness also needs to be optimised, together with the longitudinal stiffness of the central rod and the stiffness of the partition walls. Another minor difference is that, in the case of the fine-positioner, the fabrication of rubber structures combining multiple soft materials is not considered viable, as described further in Chapter 6. Thus, the material of the partition walls must be the same as that of the outer wall. The stiffness of the partition walls must then be selected through their thickness. The thickness of the partition walls affects the region of cross section that corresponds to the pressurised fluid, and contributes to the bending stiffness of the device, and to the longitudinal stiffness of the central rod. Therefore, it needs to be determined in conjunction with the other parameters by following an optimisation procedure, which is presented in subsection 4.10.3.

The fabrication of any outer structure with notches such as that shown in Figure 4.10 (let) is considered difficult in the miniature size required for the fine-positioner, and this can introduce bending stiffness that reduces performance, in a similar manner as in the previous case study.

Thus, as in subsection 4.7.2, the layout in Figure 4.10 (left) is discarded.

The design layout selected as the most suitable for a segment of the fine-positioner is then that shown in Figure 4.10 (right), with an outer wall thickness, partition wall thickness and longitudinal central rod stiffness that need to be selected. This selection requires numerical optimisation, which is described in subsection 4.10.3. Before the optimisation, the most suitable soft material for the rubber structure in the device is considered and chosen, as presented in the following subsection 4.10.2.

### 4.10.2 Hyperelastic material selection

The hyperelastic material of which the structure of the device should be made, including outer wall and partition walls, affects the performance of the device and needs to be selected carefully. After surveying suppliers and literature for hyperelastic rubbers, a set of relevant materials available was identified, which are summarised in Table 4.1. These materials present significant differences in their stiffness and ultimate strain. The selection of material can be made with aid from the non-dimensional analysis presented in the previous section 4.9.

As can be seen in Table 4.1, the ultimate strain of Elastosil M4601, by Wacker Chemie AG (Munich, Germany), is higher than that of DragonSkin 10, 20, and 30, by Smooth-On Inc. (Macungie, US). In addition, the stiffness of Elastosil M4601 is also higher or comparable to that of these other materials. Hence, according to the non-dimensional analysis in the previous section 4.9, Elastosil M4601 is preferable over DragonSkin 10, 20 or 30, since it can reach equivalent deformations to DragonSkin 10 or 20 while supporting higher wrenches, and at the deformation where DragonSkin 30 reaches its ultimate strain, Elastosil M4601 can still increase its strain by further increasing pressure, leading to higher support of wrenches.

When comparing the ultimate strain of Elastosil M4601 with that of Ecoflex OO-30 and OO-50, by Smooth-On Inc., it can be seen that the ultimate strain of the former is somewhat lower. This means that a device made of Ecoflex OO-30 or OO-50 can reach an equal deformation as a device made of Elastosil M4601 at the point where the latter reaches its ultimate strain, and
then keep increasing deformation and thus support of wrenches by increasing pressure. However, the stiffness of Ecoflex OO-30 or OO-50 is at least three times lower that of Elastosil M4601. According to the non-dimensional analysis in section 4.9, this implies that at the maximum strain of Elastosil M4601, with an equivalent deformation, the wrenches that a device made of Elastosil M4601 can support are at least three times those that a design made of Ecoflex OO-30 or OO-50 can support. In this regard, even though devices made of Ecoflex OO-30 or OO-50 can keep increasing the strain from $700 \%$ to $900 \%$, with an associated increase in the support of wrenches, it is expected that this additional increase in wrenches is not sufficient to triple the wrenches supported at $700 \%$ strain. Thus, Elastosil M4601 is expected to yield devices capable of supporting higher wrenches.

The stiffness of Smooth-Sil 950, by Smooth-On Inc., is higher than that of Elastosil M4601, which according to the non-dimensional analysis in section 4.9 implies that at an equal deformation, a device made of Smooth-Sil 950 can support higher wrenches. Using a coarse approximation to estimate the stiffness of these two materials based on their Shore hardness, the wrenches that a device made of Smooth-Sil 950 can support can be estimated to be nearly $80 \%$ higher than those of a device made of Elastosil M4601 for an equal deformation. However, the ultimate strain of Elastosil M4601 is more than double that of Smooth-Sil 950. This implies that at the maximum deformation of a device made of Smooth-Sil 950, a device made of Elastosil M4601 can keep deforming by increasing pressure to double the strain. Thus, even though a device made of Smooth-Sil 950 can support wrenches approximately $80 \%$ higher at its maximum deformation, the strain in a device made of Elastosil M4601 can be increased to more than double the strain at the maximum of Smooth-Sil 950, with an associated increase in pressure and thus in the wrenches it can support. The increase in wrenches that a device can support as a function of strain is not necessarily linear, and can require numerical solutions to determine. However, in a first linear approximation, the increase in wrenches that a device made of Elastosil M4601 from the maximum strain of Smooth-Sil 950 to the maximum strain of Elastosil M4601 can be expected to exceed $80 \%$. Thus, Elastosil M4601 is expected to yield a device capable of supporting higher wrenches than the other materials, and is the material selected for the rubber of the fine-positioner.

| Material | Ultimate stress [MPa] | Ultimate strain | Shore hardness | $c_{10}$ coeff. [kPa] |
| ---: | :--- | :--- | :--- | :--- |
| Smooth-Sil 950 | 5 | $320 \%$ | 50 A | 340 |
| DragonSkin 10 | 2.75 | $663 \%$ | 10 A | 42.5 |
| DragonSkin 20 | 3.8 | $620 \%$ | 20 A | $\mathrm{n} / \mathrm{a}$ |
| DragonSkin 30 | 3.45 | $384 \%$ | 30 A | $\mathrm{n} / \mathrm{a}$ |
| Elastosil M4601 | 6.5 | $700 \%$ | 28 A | 110 |
| Ecoflex OO-30 | 1.38 | $900 \%$ | OO30 | 12.662 |
| Ecoflex OO-50 | 2.17 | $980 \%$ | OO50 | near 25 |

Table 4.1: Material properties of available hyperelastic rubbers relevant to create deformable structures in soft robotic manipulators.

### 4.10.3 Design optimisation

An optimisation needs to be conducted to select the outer wall thickness (OWT), partition wall thickness (PWT), and longitudinal central rod stiffness (LCRS) in the design layout shown in Figure 4.10 (right), and thus determine the final design of the fine-positioner. The objective of the optimisation is to find the combination of parameters that maximises the wrenches that can be supported.

## Initial optimisation procedure

The procedure to determine the design parameters deserves consideration. As summarised in subsection 4.8.1 and mentioned in subsection 4.10.1, a larger OWT increases $p_{\max }$, which is desirable, but also introduces bending stiffness and reduces the area of the chambers in the cross section, which is undesirable. Thus, a range of OWT must be explored. Each OWT implies a $p_{\max }$. Then for each $p_{\max }$, the selection of LCRS and PWT is similar to that described in the case study in section 4.7. In terms of the LCRS, higher LCRS generally leads to improved performance, and thus the aim is to use a high LCRS without compression on the outer wall. PWT can be introduced to limit the protrusion associated to the central rod initially, and thus help prevent buckling. It should be noted, however, that in this instance the stiffness of the partition walls is selected through their thickness and not their material. The choice of partition wall stiffness then affects the region of the cross section corresponding to the chambers. Thus, an optimal combination of LCRS and PWT must be found for each $p_{\max }$. The overall optimal design can then be obtained by comparing the performance of the designs with pairs
of LCRS and PWT optimised for each OWT, which corresponds to a $p_{\text {max }}$, and then selecting the combination of OWT, LCRS, and PWT, that maximises wrenches that can be supported.

A specific optimisation procedure can be used to explore the combinations of parameters of interest without conducting unnecessary simulations. As described in the previous paragraph, the most suitable pair of LCRS and PWT depends on $p_{\max }$, which is determined by OWT. In general, LCRS and PWT are chosen so that there is practically full cross-sectional deformation at $p_{\max }$, and LCRS is maximum without causing buckling. A higher $p_{\max }$ allows for larger PWT while achieving full cross-sectional deformation at this $p_{\max }$. This in turn can generally enable higher LCRS. Thus, LCRS and PWT are typically higher with higher $p_{\max }$ that corresponds to a larger OWT.

This implies that a pair of LCRS and PWT selected for a OWT can also be suitable for a larger OWT. And typically, for the larger OWT, it is possible to increase the pair of LCRS and PWT to some extent, leading to increase performance. The drawback is that a greater OWT can also reduce performance. However, if the performance in a design with a given pair of LCRS and PWT increases when increasing OWT, then the greater OWT is generally desirable as typically it will also be possible to keep equal or greater LCRS and PWT, leading to even better performance.

In this regard, it is possible to explore first a range of OWT with a constant pair of LCRS and PWT. Then, if performance increases monotonically with OWT for a set of OWT, those OWT can be initially discarded. The starting point for the optimisation then is the OWT at which performance presents a first peak for constant LCRS and PWT, together with the higher values OWT. These represent an initial shortlist of OWT of interest.

The optimisation process must then explore combinations of LCRS and PWT for the shortlisted OWT to find the most suitable pair of LCRS and PWT for each OWT. The exploration of pairs of LCRS and PWT can also be economised relying on the discussion in the previous paragraphs of this subsection. Since the pair of LCRS and PWT suitable for an OWT is generally also applicable for larger OWT, it is possible to first find the optimal pair of LCRS and PWT for the smallest shortlisted OWT, and then only explore equal or higher combinations of LCRS
and PWT for each subsequent larger OWT. This optimisation procedure, which involves first a shortlisting a set of OWT, and then exploring pairs of LCRS and PWT that only increase, is the procedure initially adopted in this work.

The range of OWT to be explored initially is selected to be between 0.4 mm and 1.4 mm . The lowest value is selected to be smaller than the minimum OWT that can be manufactured, which is approximately 0.8 mm in the work conducted for this thesis as discussed in Chapter 6, in order to clearly elucidate the trends. The highest value is initially selected by intuition, as the optimal design is expected to have an OWT less than half the radius of the device, but it can be extended if the simulation results suggest it. The discretisation interval for OWT is selected to be 0.2 mm considering the fabrication accuracy in practice, and the personal experience that suggests that the performance variations corresponding to variations of OWT less than 0.2 mm are relatively low.

The results of the simulations are presented in the subsection 4.10.3. But first, the simulation set-up is summarised in the next subsection.

## Simulation set-up

The simulation set-up used for this design optimisation is equivalent to that in the case study described in subsections 4.7.3 and 4.7.5, and shown in Figure 4.12. This is because the objective of maximising the loads that can be supported at typical operation deflections is similar.

The simulation of the central rod, however, is somewhat different from that in the previous simulations in subsection 4.7.5. Here, the central rod is simulated as a beam using beam elements in Abaqus Standard, with a radius of 0.025 mm . This is to represent more accurately the behaviour of a design with a central fibre (made of a material such as a thread) embedded in rubber, and ensure that the bending stiffness of this fibre is negligible, as in a standard fibre. In addition, it helps with the meshing as it does not require transitions between fine mesh and coarse mesh that occurred in the previous case of simulating the central rod as a solid tubular region. The material properties for the central rod are selected to be similar to those of typical
fibres available in practice, with a linear constitutive law with a Young's modulus that is varied between $10^{5} \mathrm{~Pa}$ and $10^{10} \mathrm{~Pa}$, and a Poisson ratio that is maintained constant at 0.35 . It should also be noted that in the simulations in this section, the thickness of the end cap was increased to 2.5 mm to support the tension of the central rod applied to the centre of the cap without excessive stress in the cap.

The simulations are executed until the equivalent stress in the rubber material (Elastosil M4601), determined using the Von Mises criterion, reaches a value of $\sigma_{e}=5.2 \mathrm{MPa}$. This is approximately $80 \%$ of the ultimate stress of Elastosil M4601, and is selected to leave a safety margin of $20 \%$, and to ensure that the simulations are performed in a range of stress where the Neo-Hookean constitutive law used for the rubber is an acceptable approximation. This criterion for maximum stress also leads to simulation results where the outer wall presents relatively small protrusions on the outer wall between the fibres at $p_{\max }$, which correspond to typical conditions of maximum pressurisation.

## Initial optimisation results

The results of the simulation of designs with a range of OWT and a constant pair of PWT $=0.8 \mathrm{~mm}$ and LCRS $=10^{5} \mathrm{~Pa}$ are shown in Figure 4.17. As can be seen, the designs with lower OWT contact the rigid block at lower pressures, which matches the expected behaviour. However, at the maximum equivalent stress of $\sigma_{e}=5.2 \mathrm{MPa}$, the performance increases from the lowest $\mathrm{OWT}=0.4 \mathrm{~mm}$ up to $\mathrm{OWT}=1 \mathrm{~mm}$, where it peaks. After OWT $=1 \mathrm{~mm}$, the performance decreases. These results suggest that the shortlist of OWT of interest are OWT $=1 \mathrm{~mm}$ and larger values of OWT.

The subsequent optimisation is performed first for OWT $=1 \mathrm{~mm}$. This is due to the fact that, for the given pair of LCRS and PWT used initially to explore OWT, performance with $\mathrm{OWT}=1.2 \mathrm{~mm}$ or $\mathrm{OWT}=1.4 \mathrm{~mm}$ is lower than with $\mathrm{OWT}=1 \mathrm{~mm} . \mathrm{OWT}=1.2 \mathrm{~mm}$ and $\mathrm{OWT}=1.4 \mathrm{~mm}$ enable higher pressures which, as discussed in the previous subsection 4.10.3, would generally imply that larger LCRS and PWT could be used, potentially leading to higher performance. However, the pressures achieved at OWT $=1 \mathrm{~mm}, \mathrm{OWT}=1.2 \mathrm{~mm}$ and OWT


Figure 4.17: Plot of lateral force as a function of pressure for a set of designs with OWT varying between $\mathrm{OWT}=0.4 \mathrm{~mm}$ and $\mathrm{OWT}=1.4 \mathrm{~mm}$ at intervals of 0.2 mm , and constant parameters $\mathrm{PWT}=0.8 \mathrm{~mm}$ and LCRS $=10^{5} \mathrm{~Pa}$.
$=1.4 \mathrm{~mm}$ are already significantly higher than those in the previous case study in section 4.7, where the optimal LCRS was already nearly inextensible. This suggests that the optimal LCRS at $\mathrm{OWT}=1 \mathrm{~mm}$ may be already inextensible, and it may be difficult to achieve any noticeable improvement with OWT $=1.2 \mathrm{~mm}$ or $\mathrm{OWT}=1.4 \mathrm{~mm}$. Thus, $\mathrm{OWT}=1 \mathrm{~mm}$ was selected as the initial OWT of interest.

The next step of the optimisation, which involves determining the optimal LCRS and PWT for the OWT of interest, was then implemented first for OWT $=1 \mathrm{~mm}$. Since this OWT is larger than in the previous case study in section 4.7, and corresponds to higher pressure, the optimal LCRS and PWT can then be expected to be equal or higher than those in section 4.7, according to the discussion in the previous subsection 4.10.3. The central rod in this section is simulated as a beam, which is half the diameter (a fourth of the area) of the solid region defined as central rod previous case study in section 4.7. In addition, the rubber selected for the fine-positioner is stiffer than that in the previous case study, which requires scaling the material properties of the entire design by a factor of $110 / 42.5=2.588$ maintain equivalent behaviour. In order to consider central rods with longitudinal stiffnesses similar to those in the final design in section 4.7 (where the central rod materials that yield the highest performance
have stiffnesses of $10^{9}-10^{10} \mathrm{~Pa}$ ), the initial Young's modulus for the central beam in this optimisation should then be approximately $10^{9} * 4 * 2.588=10^{10} \mathrm{~Pa}$ or $10^{10} * 4 * 2.588=10^{11}$ Pa. Considering that these values of stiffness are nearly inextensible in practice, especially in relation to the stiffness of the rubber, the stiffness values initially used in this optimisation are in the range $2.5 * 10^{7} \mathrm{~Pa}$ to $2.5 * 10^{10} \mathrm{~Pa}$. The partition wall stiffness in the fine-positioner design is selected through PWT. In order to approximately match the partition wall stiffness in the final design in section 4.7, the PWT should be PWT $=1.2 \mathrm{~mm}$. However, as previously noted in subsection 4.10.3, the effect of PWT is somewhat different in the fine-positioner due to the fact that the thickness varies in PWT and occupies space in the cross section. Thus, lower PWT are also considered, with corresponding lower LCRS.

The starting set of values for the optimisation of the fine-positioner are then between LCRS $=$ $2.5 * 10^{8} \mathrm{~Pa}$ and $\mathrm{LCRS}=2.5 * 10^{10} \mathrm{~Pa}$, and $\mathrm{PWT}=0.8 \mathrm{~mm}, \mathrm{PWT}=1.0 \mathrm{~mm}, \mathrm{PWT}=1.2$ mm . It should be noted that, for PWT $=0.8 \mathrm{~mm}$, the maximum LCRS initially considered is $\mathrm{LCRS}=2.5 * 10^{9} \mathrm{~Pa}$ since the this PWT is not expected to be sufficient to prevent buckling for the higher LCRS. For PWT $=1.0 \mathrm{~mm}$, the LCRS initially considered are $2.5 * 10^{9} \mathrm{~Pa}$ and $2.5 * 10^{10} \mathrm{~Pa}$, since lower values are not expected to be relevant. Finally, for PWT $=1.2 \mathrm{~mm}$, the LCRS considered is $2.5 * 10^{10} \mathrm{~Pa}$, as lower values are not expected to be relevant.

The results of force for this initial set of parameters of interest are shown in Figure 4.18. As can be seen, the design that presents the highest performance is the design with OWT $=1$ $\mathrm{mm}, \mathrm{PWT}=1 \mathrm{~mm}, \mathrm{LCRS}=2.5 * 10^{10} \mathrm{~Pa}$. It should be noted that some of the results do not reach the maximum pressure, which is due to convergence issues in the simulations at higher pressures. However, the results reach a sufficient pressure to show the trends of interest. Figure 4.18 also shows that, for a given PWT, performance increases with higher LCRS, as expected. The results also show that, for a given LCRS, lower PWT is desirable. Lower PWT, however, can be associated with risk of buckling in the outer wall.

The results of tension at the outer wall for this initial set of parameters of interest is shown in Figure 4.19. It should be noted that some of the results in Figure 4.19, and particularly the result for the design with OWT $=1 \mathrm{~mm}, \mathrm{PWT}=0.8 \mathrm{~mm}, \mathrm{LCRS}=2.5 * 10^{9} \mathrm{~Pa}$, show


Figure 4.18: Plot of lateral force as a function of pressure for designs with OWT $=1 \mathrm{~mm}$, and various combinations of PWT and LCRS as indicated in the legend, which are the combinations of parameters considered in the initial optimisation process.
an abrupt variation in the trends near the values corresponding to highest pressures. This is due to the fact that, at higher pressures, the deformation in the device increases, and some of the mesh elements tend to distort excessively, causing numerical instabilities. Still, these instabilities are limited to isolated values that can be clearly distinguished, and do not affect the results in terms of extracting the trends of interest.

The results in Figure 4.19 suggest that high values of PWT tend to increase tension at the outer wall for a given LCRS and OWT. This is attributed to the fact that PWT contributes to the tension of the central rod for high values of PWT. Thus, reducing PWT can help prevent buckling. This contrasts with some of the discussion used to economise the optimisation procedure in 4.10.3. Thus, the optimisation procedure needs to be revisited to include all relevant combinations of design parameters considering this new information.

## Additional optimisation process

The new information extracted from the results in the previous subsection suggests that the parameters PWT and LCRS are coupled, as PWT contributes to the longitudinal stiffness of the


Figure 4.19: Plot of tension at the outer wall as a function of pressure for designs with OWT $=1 \mathrm{~mm}$, and various combinations of PWT and LCRS as indicated in the legend.
central rod. In addition, PWT affects the chamber area in the cross section. Thus, optimisation must consider both parameters together, exploring all combinations in the interval of values interest.

The exploration of various combinations of PWT and LCRS is already conducted in part in the results in the previous subsection. Thus, the results obtained up to this point remain relevant. The optimisation process only needs to be extended to include a broader interval of parameters LCRS and PWT. The minimum PWT can be manufactured with reasonable accuracy in this work is considered to be 0.6 mm , as discussed in Chapter 6. PWT higher than 1.2 mm are not considered relevant according to the results in the previous subsection 4.10.3. Thus, the range of PWT is extended to be between 0.6 mm and 1.2 mm . As discussed in the previous subsection 4.10.3, an LCRS $=2.5 * 10^{10} \mathrm{~Pa}$ is considered practically inextensible, especially relative to the stiffness of the rubber, and thus it represents the maximum LCRS of interest. LCRS lower than $2.5 * 10^{9} \mathrm{~Pa}$ is not expected to be relevant considering the results in the previous subsection that show that buckling does not occur for LCRS $=2.5 * 10^{9} \mathrm{~Pa}$, and that the additional PWT explored in this subsection are expected to further reduce the risk of buckling. However, an LCRS $=2.5 * 10^{10}$ is considered possible for a PWT $=0.8 \mathrm{~mm}$ considering the results in the previous subsection. The additional optimisation process is then an extension of the previous


Figure 4.20: Plot of lateral force as a function of pressure for designs with $\mathrm{OWT}=1 \mathrm{~mm}$, various combinations of PWT and LCRS, as indicated in the legend, which represent the designs of interest in the extended optimisation process.
one which include all combinations of LCRS and PWT in these extended intervals of interest.

It should be noted that, as in the previous subsection, some of the simulations are not executed up to the maximum pressure, but only up to a pressure where the trends of interest are visible. This is due to the fact that convergence at high pressures can be very sensitive to the mesh selected, and achieving this convergence can be time-consuming. Thus, the simulations are developed up to a point where they show the relevant trends.

The results of force for these additional combinations of values of the parameters, together with the previous ones, are shown in Figure 4.20. The corresponding results of tension at the outer wall are shown in Figure 4.21. As can be seen in Figure 4.20, the trends from the previous Figure 4.18 are maintained, and performance increases with higher LCRS for given OWT and PWT. Figure 4.20 also shows that, for given OWT and LCRS, performance increases with lower PWT. The design with the highest force is with an $\mathrm{OWT}=1 \mathrm{~mm}$, a $\mathrm{PWT}=0.6 \mathrm{~mm}$ and an LCRS $=2.5 * 10^{10} \mathrm{~Pa}$.

Figure 4.21 confirms that tension at the outer wall is generally positive, so the designs explored are viable. The tension at the outer wall for some designs in Figure 4.21 reaches locally negative


Figure 4.21: Plot of tension at the outer wall as a function of pressure for designs with OWT = 1 mm , various combinations of PWT and LCRS, as indicated in the legend, which correspond to the extended optimisation process.
values. However, these are very close to zero, and do not lead to buckling in the simulations. Still, the fact that the tension at the outer wall begins to reach negative values in some of the designs, as shown in Figure 4.21, indicates that the range of designs explored is near the viable limit. Therefore, the results in Figure 4.20 indicate that the most suitable design for an OWT $=1 \mathrm{~mm}$ is with a $\mathrm{PWT}=0.6 \mathrm{~mm}$ and an $\operatorname{LCRS}=2.5 * 10^{10} \mathrm{~Pa}$. This does not buckle.

These results show that an LCRS $=2.5 * 10^{10} \mathrm{~Pa}$ is viable and suitable for $\mathrm{OWT}=1 \mathrm{~mm}$. This LCRS is practically inextensible, and higher values need not be considered. In this regard, designs with $\mathrm{OWT}=1.2 \mathrm{~mm}$ need not be considered since higher LCRS achievable at higher pressures are not expected to have a relevant impact in the performance of the device, and on the other hand, a thicker outer wall reduces performance for an equivalent pair of LCRS and PWT, as previously shown in Figure 4.17.

## Final optimisation steps

The optimisation process up to this point was developed considering that the limitation that a larger OWT can impose on the possible protrusion of the central rod is not relevant. However,
a constraint on the protrusion caused by the OWT could hinder the contracting effect, and thus reduce the performance of the device. Thus, to complete the optimisation process and determine the final fine-positioner design, it is necessary to consider whether a reduction in OWT can lead to a performance improvement.

As previously noted, in this work, the minimum OWT that can be manufactured reliably is estimated to be near 0.8 mm . This is 0.2 mm larger than the minimum PWT that can be manufactured, which is due to the fact that the outer wall includes fibres and the manufacturing process is different, as described in Chapter 6. Thus, the designs with a reduced OWT explored have a $\mathrm{OWT}=0.8 \mathrm{~mm}$. The exploration is equivalent to that for $\mathrm{OWT}=1 \mathrm{~mm}$ presented in the previous subsections, and involves considering all relevant combinations of LCRS and PWT.

The results of force for designs with OWT $=0.8 \mathrm{~mm}$ and a set of LCRS and PWT of interest are shown in Figure 4.22, together with the most relevant of the previous results. The corresponding results of tension at the outer wall are shown in Figure 4.23. The results in Figure 4.22 indicate that, for a OWT $=0.8 \mathrm{~mm}$, the most suitable design is with a PWT $=0.6$ and $\mathrm{LCRS}=$ $2.5 * 10^{10} \mathrm{~Pa}$. The results in Figure 4.23 show that, for all designs of interest, the tension at the outer wall is practically non negative and therefore do not lead to buckling, so they are viable designs.

The results in Figure 4.22 also indicate that devices with OWT $=0.8 \mathrm{~mm}$ contact the rigid block at lower pressures than equivalent devices with OWT $=1 \mathrm{~mm}$. This is expected considering that the bending stiffness of devices with OWT $=0.8 \mathrm{~mm}$ is lower than with $\mathrm{OWT}=1 \mathrm{~mm}$, and that the former allows for a larger chamber area in the cross section. For a given maximum equivalent stress of $\sigma_{e}=5.2 \mathrm{MPa}$, the performance of the design with $\mathrm{OWT}=1 \mathrm{~mm}, \mathrm{PWT}=$ 0.8 mm and LCRS $=2.5 * 10^{10} \mathrm{~Pa}$ is very similar to that of the design with $\mathrm{OWT}=0.8 \mathrm{~mm}$, $\mathrm{PWT}=0.6 \mathrm{~mm}$ and LCRS $=2.5 * 10^{10} \mathrm{~Pa}$, although the performance of the latter is near $5 \%$ higher. The similar performance is attributed to the fact that, even though designs with OWT $=0.8 \mathrm{~mm}$ have lower bending stiffness and larger area of the chambers in the cross section, they also offer lower $p_{\max }$ than designs with OWT $=1 \mathrm{~mm}$. The fact that the performance of


Figure 4.22: Plot of lateral force as a function of pressure for designs with both OWT $=0.8$ mm and $\mathrm{OWT}=1 \mathrm{~mm}$, and a set of combinations of PWT and LCRS of interest, as indicated in the legend. The results show the highest performing designs with both OWT $=0.8 \mathrm{~mm}$ and $\mathrm{OWT}=1 \mathrm{~mm}$. The results for the standard FMA design are also included for comparison of performance.


Figure 4.23: Plot of tension at the outer wall as a function of pressure for designs with both $\mathrm{OWT}=0.8 \mathrm{~mm}$ and $\mathrm{OWT}=1 \mathrm{~mm}$, and a set of combinations of PWT and LCRS of interest, as indicated in the legend.
the best designs with OWT $=0.8 \mathrm{~mm}$ and $\mathrm{OWT}=1 \mathrm{~mm}$ is similar at maximum equivalent stress indicates that lower OWT are not relevant.

Comparing the force for all the designs of interest for equivalent stress levels, the design with the highest performance is the design with OWT $=0.8 \mathrm{~mm}, \mathrm{PWT}=0.6 \mathrm{~mm}$ and $\mathrm{LCRS}=$ $2.5 * 10^{10} \mathrm{~Pa}$. It should be noted that the performance is very similar to that of the design with $\mathrm{OWT}=1 \mathrm{~mm}, \mathrm{PWT}=0.8 \mathrm{~mm}$ and $\operatorname{LCRS}=2.5 * 10^{10} \mathrm{~Pa}$, so the selection of the former over the latter does not have a significant impact, as at this stage of the optimisation both designs are near the optimal. In this regard, the latter design could also be used if in the future it was advantageous for manufacturing or other practical reasons.

### 4.10.4 Final selected design and discussion

The final design selected for the fine-positioner in this work then is the design with OWT $=0.8$ $\mathrm{mm}, \mathrm{PWT}=0.6 \mathrm{~mm}$ and $\mathrm{LCRS}=2.5 * 10^{10} \mathrm{~Pa}$. This has the highest performance, as shown in Figure 4.22.

The OWT and PWT of the design found in this work are similar to those of the FMA. However, the design derived in this work is different, and provides a higher performance. Unlike the FMA, which is designed as an extending device only, the design proposed in this work is conceived to combine extending and contracting operation, which enables it to support higher wrenches at a range of deflections. In addition, as an extending device, the FMA design is not optimal since the central part is designed to be soft and extensible, and thus provide one DOF corresponding to extension. This lowers its performance in terms of bending and supporting external wrenches, and is considered a design aspect with significant potential for improvement, which attributed to the fact that the FMA was originally designed mostly by intuition. Conversely, in this work, the design is derived using a set of design principles, which lead to a design that includes a nearly inextensible central rod, which improves its performance. Moreover, the design proposed in this work is designed exploit any cross-sectional deformation, which contrasts with the FMA, which is designed for a constant cross section. Finally, the fact that the FMA is mostly designed by intuition implies that it is not necessarily optimal even as an extending device, which can
be seen in the partition walls that are excessively thick and are not designed to deform, which leads to chambers in the cross section that are somewhat smaller than they could be, and in the fact that the FMA lacks an inextensible central rod.

The higher performance of the design found in this work is confirmed in Figure 4.22, where it is compared with that of the standard FMA. As can be seen, the force of the design proposed here is higher than that of the standard FMA for all pressures, and at an equal pressure of approximately 1.5 bar, which is the highest common pressure simulated for both designs, the performance of the design proposed in this work is nearly double that of the standard FMA. In addition, the trends of force as a function of pressure indicate that at higher pressures, the performance of the design proposed here can increase further at a significantly higher rate than that of the standard FMA. This is attributed to the fact that the design proposed in this work has a practically inextensible central rod that prevents it from wasting the increase in pressure in extension of the device. Lastly, the design found in this work has an OWT that is 0.05 mm larger than in the standard FMA, which enables it to withstand slightly higher pressures with an equivalent maximum stress, and thus support higher forces.

It should be noted that the final design selected in this work was obtained by following an optimisation process where the discretisation step for both OWT and PWT is 0.2 mm . This was initially justified considering the manufacturing accuracy that can be reliably achieved in practice, and that the variation in performance associated to this discretisation step was expected to be low. The results shown in Figures 4.22 and 4.23 confirm that the variation in performance between designs near the optimal is relatively low, and therefore confirm the suitability of the discretisation step used.

The discretisation step for LCRS is in increments of one order of magnitude. This is considered reasonable taking into account that in practice the fibres available to create this central rod are limited, so not any stiffness value can be manufactured, and that the variation in performance between designs with different LCRS is relatively low near the final design selected.

It should also be noted that the design optimisation was conducted without including significant wires or other elements required for the payload, which may need to be routed through the fine-
positioner. In the case that wires or other elements from the payload need to be incorporated, these should be passed through the central rod, and can potentially be used as the element providing the longitudinal stiffening. Adding wires or other elements through the central rod is expected to have a negligible effect on performance, and they can integrate well with the inextensile central rod, provided that these have a relatively small diameter. These wires or other elements should generally not be passed through the chambers, since this could reduce performance due to the fact that the cables can occupy room in the cross section, reducing chamber area.

Lastly, the final design selected here has an unpleated outer wall made of rubber with circumferential fibres. A pleated outer wall could improve performance to some extend by reducing the bending stiffness of this outer wall. However, this is considered very difficult to manufacture in miniature size, and is left for future work. Similarly, the design derivation in this work also identified a possible design variation with an outer wall made of a tubular structure with notches. However, the fabrication of such a structure in miniature size and ensuring that it does not introduce significant bending stiffness is considered very difficult, and is left for future work.

### 4.11 Conclusions on design

A novel approach to study the design of soft robotic manipulators with fluidic actuation was proposed in this chapter. This can serve as the foundation towards a common framework for the design of these devices. This approach can be first applied to justify the two main design layouts, which correspond to extending and contracting devices. Design principles for each of the two layouts can be subsequently extracted. These are summarised in section 4.6 for designs with a given maximum pressure. The study can be extended to designs where the maximum pressure is not constrained by the application, leading to two additional principles summarised in subsection 4.8.1.

In addition to the main design study, a non-dimensional analysis of soft robotic manipulators
similar to the FMA and to the designs of interest in this work was also developed. This primarily helps in the selection of soft robotic manipulators. Moreover, it provides insights on the scaling of these devices, and the force and pressures they can support at different sizes.

The design principles for extending and contracting devices can be applied to determine the design of a soft robotic manipulator in each desired scenario. To showcase this, a prototypical scenario in MIS was defined in this work, which serves as foundation for the design of the finepositioner. The application of the design principles led to the determination of the design of a segment of soft robotic manipulator that combines the extending and contracting operation, which represents the most suitable design in the scenario defined. Optimal values for the stiffness of the partition walls and central rod in the design selected were found to require a numerical analysis of the deformation of the device. FE simulations were developed to determine these optimal stiffness values, yielding the optimised design. The FE simulations also served to confirm some of the main trends predicted by the design study, thereby verifying some of the main research results.

The design principles were then applied to outline the most suitable design of the segments of the fine-positioner. The design derivation was similar to that used for the previous case study, but generalising it to the jet engine inspection application, where pressure is not constrained. The non-dimensional analysis was used to aid in the material selection for this fine-positioner design. Finally, a set of compromises in this design outline were identified using the design principles, and an optimisation process was defined. The results of the implementation of this optimisation yielded the final design for each of the segments of the fine-positioner.

This final design selected for the segments of the fine-positioner consists of the layout shown in Figure 4.10 (right), with an outer wall thickness of 0.8 mm , a partition wall thickness of 0.6 mm , and a longitudinal central rod stiffness of $2.5 * 10^{10} \mathrm{~Pa}$. The outer wall and partition walls should be made of Elastosil M4601. This final design found in this work shows a performance that is higher than that of the FMA, which in turn is one of the highest performing soft robotic manipulators developed to date, as presented in 2. This can be attributed to the design improvements and differences summarised in the previous section. The fabrication of
this final design is described is Chapter 6. But first the research conducted on control of the fine-positioner, which should be made of three segments of this final design, and more generally of other similar soft robotic manipulators, is presented in the next chapter.

## Chapter 5

## Control of a Soft Robotic Manipulator

The robotic system comprising the fine-positioner and gross-positioner needs to be controlled in order to reach the engine region of interest and accurately position the end-effector to deploy probes. The insertion into the overall engine region of interest is performed mostly by the gross-positioner, and needs not be accurate. It only requires a path planning capability, which is discussed in Chapter 6. The positioning of the end-effector is performed using the finepositioner, and this must be accurate for a correct probe deployment, which requires accurate control.

The control of soft robotic manipulators similar to the design selected for the fine-positioner is considered in this chapter. The kinematics of continuum robots with bending and extension capabilities are first studied in sections 5.1 to 5.6 , and closed-form solutions are derived. Work on the development of closed-loop control laws for soft robotic manipulators such as the finepositioner is then presented in the subsequent sections 5.7 and 5.8. This includes a mechanical model of the fine-positioner, which is described in section 5.7, and the development of closedloop control laws for a segment of the fine-positioner operating in a plane, which presented in section 5.8.

The work presented in this Chapter is in part an edited version of the work published in:

- A. Garriga-Casanovas, F. Rodriguez y Baena. Kinematics of Continuum Robots with

Constant Curvature Bending and Extension Capabilities. Journal of Mechanisms and Robotics, 11.1, 011010, 2018.

### 5.1 Introduction to kinematics of continuum robots with bending and extension capabilities

As previously mentioned in Chapter 2, continuum robots have received significant attention in recent years. This is not least because of the advantages they offer in manipulation, dexterity and even locomotion inside cluttered environments. A relevant part of continuum robots are actuated by means of a pressurised fluid and can be considered soft robots. Prominent examples of this are the FMA [98, 100], the OctArm robot [178], a manipulator similar to the OctArm [185], the AirOctor [43], or the Stiff-Flop [112]. The fine-positioner selected in this work also belongs to the categories of soft and continuum robots.

The capability of bending and extending is common in soft, continuum robots actuated by a pressurised fluid. This provides these robots with dexterity that, in specific applications, can surpass that of traditional serial manipulators. However, solutions to the kinematics problems, and particularly the inverse kinematics, are generally not available, which obscures their potential and can hamper the application of these robots.

Finding solutions to the full kinematics would help in establishing the way in which these robots manoeuvre, the DOFs they can offer, and even their workspace, which in turn would help in clarifying and exploiting these robots' full potential. In addition, having solutions to the full kinematics would help in the development of closed-loop control laws and path planning algorithms. The possibility of clarifying the potential of soft, continuum robots, particularly in terms of applicability to cluttered environments such as a jet engine, and the possibility of having a foundation for the development of closed-loop control laws and path planning algorithms, originally motivated the research on kinematics presented here.

The kinematics can be decoupled into a robot-specific mapping, between actuator space and


Figure 5.1: Illustration of a robot configuration corresponding to the inverse kinematics solution for a specified end-effector pose, in a robot composed of two segments with a total of six actuation degrees of freedom.
configuration space, and a robot-independent mapping, between configuration space and task space, as proposed in [15]. The kinematics work presented here in sections 5.1 to 5.6 focuses on the robot-independent mapping for robots composed of sections that can both elongate and bend with constant curvature, such as the device illustrated in Fig. 5.1. The kinematics problem considering the capability of both elongating and bending represents a general and relevant kinematics problem in continuum robots, which applies to a variety of robots including FMA-type robots [98], the OctArm and a kinematically similar master device to control it [178, 186], more recent robots similar to the OctArm [128], or tendon-driven devices with extensible backbone [187], and [44].

Various studies of the kinematics of continuum and soft robots exist in the literature [15], although the inverse kinematics for a specified end-effector pose remains an open problem. A relatively complete formulation of the kinematics is presented in [188], although it does not provide a closed-form solution to the inverse kinematics. A modal approach that allows numerical calculation of the inverse kinematics is proposed in [189,190], and is extended in [191]. However, these approaches rely on approximations of the robot geometry that do not match the common constant curvature bending kinematics. An algorithm to calculate the inverse kinematics of the distal end position is introduced in [192], but it does not account for the tip orientation and does not provide closed-form solutions. Various approaches to solving the inverse kinematic control problem have been developed using the robot Jacobian, where [93] and [81] are recent examples. However, these require some computational time that can vary depending on the end-effector pose, especially when redundancies exist, they do not directly yield the reachable end-effector poses, and they present issues with singularities. Furthermore, these approaches based on the Jacobian lack insight into the kinematics, which complicates subsequent path planning and control. Formulations of both the robot-specific and robotindependent mappings are presented in [193]. However, closed-form solutions to the inverse kinematics are not available, and a numerical approximation is used. In [194], the self-motion of 2D continuum manipulators is analysed, but closed-form solutions to the inverse kinematics are not derived, and the research cannot be extrapolated to a 3D scenario. An adaptation of the Denavit-Hartenberg parameters is described in [195], but it does not yield a closed-form
solution to the inverse kinematics. An analytical kinematic formulation is proposed in [17] for a 2D application, although it cannot be extrapolated to 3D. In [196], a closed-form solution to the inverse kinematics for a specified end-effector position in 3D is presented, but the approach is not applicable to solve the problem of a specified end-effector pose, hence it cannot be used in general.

In sections 5.1 to 5.6 of this Chapter, the kinematics of soft, continuum robots composed of segments with piece-wise constant curvature bending and extending capabilities are studied, and analytical, closed-form solutions to the direct and inverse kinematics are presented. The analysis is focused on devices composed of serially stacked segments operating in 3D space since they represent the most relevant type of robots. The solution to the inverse kinematics is derived in closed-form thanks to a novel approach that relies on quaternions to describe the rotations associated to the robot's segments. This, combined with a strategy inspired by the Paden-Kahan sub-problems [197] that involves dividing the problem into parts of reduced complexity, yields a particularly simple formulation of the inverse kinematics, which can be treated analytically, leading to explicit solutions. It should be noted that quaternions have been previously used to study different aspects of continuum robots. In [198, 199], quaternions are used for the mechanical modeling of elastic rods, and a similar approach is applied in [200] to study the dynamics of soft robotic manipulators. Quaternions are also used in [201] to develop efficient finite element methods applicable to continuum rods that can also expand radially. In addition, quaternions can be used to reliably integrate orientation along the arc length of continuum robots [202,203], and they are used in [204] to develop efficient numerical solutions to the kinematics of continuum robots. However, to the best of this author's knowledge, the work presented here is the first instance where quaternions are used to derive closed-form solutions to the full robot kinematics.

A set of relevant considerations that arise from the central study of kinematics are also discussed in this work. The number of degrees of freedom (DOFs) at the distal end of the robot is analysed using the direct kinematics Jacobian, and redundancies are identified. The solution to the inverse kinematics is then shown to be a curve that corresponds to such redundancy, and is also obtained in closed-form. A condition on the reachable end-effector poses with a six
actuation DOFs robot is distilled from the derivation, and it is related to the discussion on the robot's DOFs. This discussion also shows that a robot with nine actuation DOFs is required to achieve six end-effector DOFs, and therefore the kinematics of robots with nine actuation DOFs are also analysed.

It should be noted that this work on kinematics cannot be directly applied to the final design selected for the fine-positioner since this cannot extend. This is due to the fact that the kinematics research presented here was conducted before the majority of work on design, and with the information available at the time, the fine-positioner design was expected to be closer to the FMA, and thus be capable of extending. Nonetheless, the kinematics research presented here serves as foundation for the kinematics of the fine-positioner and subsequent development of control laws, and it is applicable to all other continuum robots with constant curvature bending and extension capabilities.

The rest of the work on kinematics is structured as follows. The kinematic problem is outlined in section 5.2, where nomenclature is also defined. The direct kinematics are presented in section 5.3, together with a discussion on the end-effector DOFs corresponding to robots with six and nine actuation DOFs. The analysis of the inverse kinematics is presented in section 5.4, leading to the derivation of closed-form solutions. In addition, the implications of such solutions are discussed in the same section, including the redundancies of the solution, the condition on reachable poses, and the analysis of robots with nine actuation DOFs. Finally, simulations of the robot configuration corresponding to the kinematic solutions are plotted in section 5.5, leading to the conclusions on the kinematics work in section 5.6.

### 5.2 Problem formulation

The kinematics of a robot concern the study of the relation between the configuration of the robot end-effector, which can be described by $g_{t} \in S E(3)$ when operating in a 3D workspace, and the robot joint configuration, which can be described by $\theta \in Q \subset R^{n}$, where $n$ denotes the dimensions of the configuration space. The direct kinematics correspond to the study of the
function $g: Q \rightarrow S E(3)$. The inverse kinematics concern the study of the solution to

$$
\begin{equation*}
g(\theta)=g_{t} \tag{5.1}
\end{equation*}
$$

for $\theta \in Q$, where $g_{t}$ is a specified end-effector configuration inside the workspace.

The kinematic study presented in this work considers a continuum robot composed of a set of serially stacked segments, each of which can be individually controlled to bend in any direction in 3D space and also extend, providing 3 DOFs. The deformation modes of the segments represent the foundation for the kinematic study of any continuum robot. Here, the segments are assumed to bend as constant curvature arcs, and the extension DOF is assumed to be independent of the bending, following the same circumference arc of the selected bending. It is also assumed that attachments between any two segments present negligible length, and that adjacent arcs are tangential.

The geometry of the robots considered here can therefore be described by a set of circumference arcs stacked serially, which correspond to the robot's segments. Each segment can be characterised by three independent variables. The kinematic mapping $g(\theta)$ thus corresponds to $n / 3$ subsequent transformations associated to constant curvature arcs.

This robot layout together with these of assumptions on bending modes satisfactorily model FMA-type robots [98], which originally motivated this work. However, the kinematic study reported here is not only limited to an FMA-type robot; it applies to all robots that can be approximated by the aforementioned bending and extension modes, which can correspond to a variety of devices, such as $[44,186,187]$. It should be noted that the deformation modes considered in this work are selected according to their relevance. Robots composed of 3-DOF segments that bend as circumference arcs and also extend represent a relevant part of the soft, continuum robots introduced in the previous section. In addition, the kinematics considered here provide a foundation for the kinematics of devices with other deformation modes. The kinematics, however, are not simplified by the deformation modes considered in this work, and they differ from the kinematics of traditional multi-linkage robots, calling for a novel approach.

The primary aim in the operation of the serial robots considered in this work is to control the robot's end-effector pose, commonly for manipulation purposes. Operation in $S E(3)$ generally requires near 6 DOFs at the end-effector. Considering that the devices studied here offer 3 actuation DOFs per segment, the kinematics of robots composed of 2 segments represent the most relevant problem, and are the focus of this work. The main objective in the study reported here are the kinematics to attain a desired end-effector pose. The solution to the inverse kinematics of a robot with $n=6$ involves determining the two tangential arcs required to reach a desired $g_{t}$. The solution to such a problem is not simple, as will be seen in the following sections, requiring an innovative derivation. The analysis of the direct and inverse kinematics also shows that $g(\theta)$ is neither injective nor surjective, hence configuration spaces with dimension $n>6$ are also considered.

### 5.3 Direct kinematics

Various general derivations of the direct kinematics of a continuum robot exist in the literature, e.g. see [15]. However, the specific variables used to describe the robot $\theta \in Q$ strongly influence the complexity of the mapping $g(\theta)$.

The most suitable description of the robot configuration is discussed in the following subsection 5.3.1. The direct kinematics are then derived in subsection 5.3.2, and the corresponding Jacobian is studied in subsection 5.3.3 to determine the DOFs of different robot layouts.

### 5.3.1 Robot Description

The configuration of the continuum robot is completely determined when the configuration of all sections that comprise it is specified. A suitable description of the segments is crucial in order to obtain a simple formulation for the kinematics, which subsequently allows for the derivation of closed-form solutions.

Each robot segment corresponds to a transformation from the pose at its proximal end to a


Figure 5.2: Diagram of one segment of the robot (yellow), with the different variables corresponding to the first segment description $\left(\sigma_{i}, \zeta_{i}, \phi_{i}\right)$, and the second segment description $\left(x_{i}^{F}, y_{i}^{F}, z_{i}^{F}\right)$, as well as the reference frame at the base of the segment $\{F\}$, the rotation vector $\mathrm{w}_{i}$, and rotation angle $\rho_{i}$.
new pose at its distal end, as illustrated in Fig. 5.2. The main challenge in the kinematics formulation resides in the conceptually distant transformations associated with translation and rotation. Descriptions of the robot segments that simplify the translation transformation associated with the segment generally complicate the rotation transformation, and vice versa.

There are two main segment descriptions that are used in this work, which complement in different parts of the analysis. Both of them are relative to a reference frame, defined as $\{F\}$, situated at the segment's base, as shown in Fig. 5.2.

The first description employs $\sigma_{i}$, which is a scalar corresponding to the Euclidean distance between segment $i$ 's base and tip, $\zeta_{i}$, which is the angle between the vector of the segment tip position and the $\mathbf{k}^{F}$ axis of $\{F\}$, and $\phi_{i}$, which is the angle between the projection of the segment on the $\mathbf{i}^{F}, \mathbf{j}^{F}$ plane and the $\mathbf{i}^{F}$ axis of $\{F\}$, as shown in Fig. 5.2. It should be noted that the definitions of segment base and tip are arbitrary, and interchangeable. This segment description represents a compromise in the complexity of the transformations corresponding to translation and rotation, and is used for the derivation of the direct kinematics.

The second description employs the Cartesian coordinates of the tip of a segment, defined as $x_{i}^{F}, y_{i}^{F}, z_{i}^{F}$, relative to a reference frame at its base $\{F\}$, as shown in Fig. 5.2, where the subscripts in $x_{i}^{F}, y_{i}^{F}, z_{i}^{F}$ indicate the segment index, $i$, and the superscripts the reference frame, $\{F\}$. As in the previous description, the segment base and tip are selected arbitrarily, and can be interchanged in each analysis, as applied in the inverse kinematics derivation in section 5.4. It should be noted that the position of the reference frame used in the definition of the variables $x_{i}^{F}, y_{i}^{F}, z_{i}^{F}$ determines the side of the segment corresponding to the base. This second description simplifies the translation transformation, but generally complicates the rotation transformation. This description is used in the inverse kinematics derivation in section 5.4, where its advantages become apparent.

It should be noted that both segment descriptions are directly related. For example, $\sigma_{i}, \zeta_{i}, \phi_{i}$
can be obtained as a function of $x_{i}^{F}, y_{i}^{F}, z_{i}^{F}$ using

$$
\begin{align*}
& \sigma_{i}=\sqrt{\left(x_{i}^{F}\right)^{2}+\left(y_{i}^{F}\right)^{2}+\left(z_{i}^{F}\right)^{2}} \\
& \zeta_{i}=\arccos \frac{z_{i}^{F}}{\sqrt{\left(x_{i}^{F}\right)^{2}+\left(y_{i}^{F}\right)^{2}+\left(z_{i}^{F}\right)^{2}}}  \tag{5.2}\\
& \phi_{i}=\arctan \frac{y^{F}}{x_{i}^{F}}
\end{align*}
$$

The bending and extension of a segment are coupled in both of these descriptions. A given set of values of $\sigma_{i}, \zeta_{i}, \phi_{i}$ generally implies both bending and extension of the segment. Equally, a set of values of $x_{i}^{F}, y_{i}^{F}, z_{i}^{F}$ generally involves both bending and extension of segment $i$. Furthermore, segment motions that involve variations in only one of the variables $\sigma_{i}, \zeta_{i}$ or $x_{i}^{F}, y_{i}^{F}, z_{i}^{F}$ generally lead to variations in both bending and extension. Similarly, variations in only bending or extension generally involve coupled variations in $\sigma_{i}, \zeta_{i}, \phi_{i}$ or $x_{i}^{F}, y_{i}^{F}, z_{i}^{F}$.

The decoupled bending and extension of a segment can be determined from $\sigma_{i}, \zeta_{i}, \phi_{i}$ using the fact that the triangle shown in blue in Fig. 5.2 is isosceles, together with trigonometric relations. The resulting expression is

$$
\begin{align*}
& b_{i}=\frac{2 \sin \zeta_{i}}{\sigma_{i}}  \tag{5.3}\\
& l_{i}=\frac{\zeta_{i} \sigma_{i}}{\sin \zeta_{i}}
\end{align*}
$$

where the bending curvature of the segment is $b_{i}$, the arc length of the extended segment is $l_{i}$, and the direction of bending is simply determined by $\phi_{i}$. Similarly, for a set of $x_{i}^{F}, y_{i}^{F}, z_{i}^{F}$, the bending and extension of a segment are determined by

$$
\begin{align*}
& b_{i}=\frac{2 \sqrt{\left(x_{i}^{F}\right)^{2}+\left(y_{i}^{F}\right)^{2}}}{\left.\left.\left(x_{i}^{F}\right)^{( }+y_{i}^{F}\right)^{2}+z_{i}^{F}\right)^{2}} \\
& l_{i}=\arcsin \left(\frac{\sqrt{\left(x_{i}^{F}\right)^{2}+\left(y_{i}^{F}\right)^{2}}}{\sqrt{\left(x_{i}^{F}\right)^{2}+\left(y_{i}^{F}\right)^{2}+\left(z_{i}^{F}\right)^{2}}}\right) \frac{\left(x_{i}^{F}\right)^{2}+\left(y_{i}^{F}\right)^{2}+\left(z_{i}^{F}\right)^{2}}{\sqrt{\left(x_{i}^{F}\right)^{2}+\left(y_{i}^{F}\right)^{2}}}  \tag{5.4}\\
& \phi_{i}=\arctan \frac{y_{i}^{F}}{x_{i}^{F}}
\end{align*}
$$

As can be seen from (5.3), the segment description $\sigma_{i}, \zeta_{i}, \phi_{i}$ yields a relatively simple decoupling of bending and extension, whereas the decoupling in (5.4) involves additional complexity. Equations (5.3) and (5.4) also elucidate the specific variations in bending and extension of a segment for variations in $\sigma_{i}, \zeta_{i}, \phi_{i}$ or $x_{i}^{F}, y_{i}^{F}, z_{i}^{F}$. In addition, the equations show that, for fixed
bending or extension, the possible values of $\sigma_{i}, \zeta_{i}, \phi_{i}$ or $x_{i}^{F}, y_{i}^{F}, z_{i}^{F}$ are determined by nonlinear relations with a certain degree of complexity.

Robot segment descriptions where bending and extension are directly decoupled in different variables are also possible. For example, using $b_{i}, l_{i}, \phi_{i}$, bending is directly determined by $b_{i}$ and $\phi_{i}$, and extension by the total length $l_{i}$. However, these descriptions complicate the formulation of the kinematics, rendering the subsequent study of the direct kinematics impractical, and the derivation of the inverse kinematics practically inviable. In addition, the use of these descriptions does not provide specific advantages in the study of the kinematics, and the specific bending and extension of segments can be obtained from the results obtained with the other segment descriptions using (5.3) and (5.4). Hence, the segment descriptions used in this work are either $\sigma_{i}, \zeta_{i}, \phi_{i}$ or $x_{i}^{F}, y_{i}^{F}, z_{i}^{F}$.

The complete robot configuration is determined by the multiple individual segments described using either of the descriptions above.

### 5.3.2 Direct Kinematics Derivation

The direct kinematics mapping of the continuum robot can be obtained by subsequently applying the transformations corresponding to its serially stacked segments, each of which involves a translation and a rotation. Here, the segments are described using $\sigma_{i}, \zeta_{i}, \phi_{i}$. The orientation of the end-effector is described using ZYZ Euler angles, as introduced at the latter part of this subsection, since it yields a simpler formulation of the direct kinematics that facilitates the subsequent Jacobian-based analysis of DOFs.

The position of the distal end of a segment $i$ relative to reference frame $\{F\}$ is defined as $\mathbf{p}_{i}^{F}$. This position $\mathbf{p}_{i}^{F}$ corresponds to the translation associated to segment $i$, and can be determined as a function of $\sigma_{i}, \zeta_{i}, \phi_{i}$ as

$$
\begin{equation*}
\mathbf{p}_{i}^{F}=\left[\sigma_{i} \sin \zeta_{i} \cos \phi_{i}, \sigma_{i} \sin \zeta_{i} \sin \phi_{i}, \sigma_{i} \cos \zeta_{i}\right] \tag{5.5}
\end{equation*}
$$

It should be noted that $\left\|\mathbf{p}_{i}^{F}\right\|=\sigma_{i}$, which is simply a consequence of the definition of $\sigma_{i}$. Still, the variable $\sigma_{i}$ is generally used here for clarity.

The formulation of the rotation corresponding to the orientation at the tip of segment $i$ relative to $\{F\}$, defined as $\mathbf{R}_{i}$, requires some preliminary consideration. The rotation axis corresponding to $\mathbf{R}_{i}$ is perpendicular to the segment's bending plane, and therefore always lies in plane $\mathbf{i}^{F}, \mathbf{j}^{F}$ in Fig. 5.2. The distal end of a segment can therefore reach any position inside the reachable $3 D$ space, but only the subspace of $S O(3)$ subtended between two orientation variables can be reached.

The rotation axis can thus be expressed in $\{F\}$ as

$$
\begin{equation*}
\mathbf{w}_{i}=\left[-\sin \phi_{i}, \cos \phi_{i}, 0\right] \tag{5.6}
\end{equation*}
$$

The rotation angle associated to segment $i$, defined as $\rho_{i}$, can be obtained as a function of $\zeta_{i}$ considering trigonometric relations. Since the triangle shown in blue in Fig. 5.2 is isosceles, then

$$
\begin{equation*}
\rho_{i}=2 \zeta_{i} \tag{5.7}
\end{equation*}
$$

Using Rodrigues' formula [197], $\mathbf{R}_{i}$ can then be directly obtained as a function of $\zeta_{i}$ and $\phi_{i}$ using (5.6) and (5.7). Thus, the homogeneous transformation associated to a segment $\mathbf{T}_{i}$ can be obtained as a function of $\sigma_{i}, \zeta_{i}, \phi_{i}$ from $\mathbf{R}_{i}$ and $\mathbf{p}_{i}^{F}$, as

$$
\mathbf{T}_{i}=\left[\begin{array}{cccc}
\left(s_{\phi_{i}}\right)^{2}\left(1-c_{2 \zeta_{i}}\right)+c_{2 \zeta_{i}} & s_{\phi_{i}} c_{\phi_{i}}\left(c_{2 \zeta_{i}}-1\right) & c_{\phi_{i}} s_{2 \zeta_{i}} & \sigma_{i} s_{\zeta_{i}} c_{\phi_{i}}  \tag{5.8}\\
s_{\phi_{i}} c_{\phi_{i}}\left(c_{2 \zeta_{i}}-1\right) & \left(c_{\phi_{i}}\right)^{2}\left(1-c_{2 \zeta_{i}}\right)+c_{2 \zeta_{i}} & s_{\phi_{i}} s_{2 \zeta_{i}} & \sigma_{i} s_{\zeta_{i}} s_{\phi_{i}} \\
-c_{\phi_{i}} s_{2 \zeta_{i}} & -s_{\phi_{i}} s_{2 \zeta_{i}} & c_{2 \zeta_{i}} & \sigma_{i} c_{\zeta_{i}} \\
0 & 0 & 0 & 1
\end{array}\right]
$$

where $c_{\omega}$ and $s_{\omega}$ denote $\cos \omega$ and $\sin \omega$, respectively. The total transformation of a robot
composed of $n / 3$ segments between its distal and proximal ends then is

$$
\begin{equation*}
\mathbf{T}_{t}=\prod_{i=1}^{n / 3} \mathbf{T}_{i} \tag{5.9}
\end{equation*}
$$

which is a function of $\sigma_{i}, \zeta_{i}, \phi_{i}$ for $i=1, \ldots, n / 3$.

The orientation of the robot's distal end can also be described using the ZYZ Euler angles $\alpha, \beta, \gamma$. The corresponding rotation matrix can be obtained, e.g. see [197], as

$$
\mathbf{R}_{t}=\left[\begin{array}{ccc}
c_{\alpha} c_{\beta} c_{\gamma}-s_{\alpha} s_{\gamma} & -c_{\alpha} c_{\beta} s_{\gamma}-s_{\alpha} c_{\gamma} & c_{\alpha} s_{\beta}  \tag{5.10}\\
s_{\alpha} c_{\beta} c_{\gamma}+c_{\alpha} s_{\gamma} & -s_{\alpha} c_{\beta} s_{\gamma}+c_{\alpha} c_{\gamma} & s_{\alpha} s_{\beta} \\
-s_{\beta} c_{\gamma} & s_{\beta} s_{\gamma} & c_{\beta}
\end{array}\right]
$$

Comparing (5.10) and the rotational component of (5.9), the ZYZ Euler angles of the robot distal end as a function of the robot configuration can be extracted for $\sin \beta \neq 0$ as

$$
\begin{align*}
& \alpha=\operatorname{atan} 2\left(\frac{T_{t 23}}{\sin \beta}, \frac{T_{t 13}}{\sin \beta}\right) \\
& \beta=\operatorname{atan} 2\left(\sqrt{T_{t_{31}}^{2}+T_{t_{32}}^{2}}, T_{t_{33}}\right)  \tag{5.11}\\
& \gamma=\operatorname{atan} 2\left(\frac{T_{t 32}}{\sin \beta},-\frac{T_{t 31}}{\sin \beta}\right)
\end{align*}
$$

where $T_{t_{i j}}$ denotes the components of $\mathbf{T}_{t}$. In particular, $\alpha, \beta, \gamma$ can be directly obtained as a function of $\sigma_{i}, \zeta_{i}, \phi_{i}$ for $i=1, \ldots, n / 3$ from (5.11) with the $T_{t_{i j}}$ determined from (5.9) combined with (5.8).

Defining a reference frame at the robot's proximal end as $\{G\}$, which coincides with reference frame $\{F\}$ of the first robot segment, the position of the robot's distal end relative to $\{G\}$ can be denoted by $\mathbf{p}_{t}^{G}$. The expression of $\mathbf{p}_{t}^{G}$ as a function of the robot configuration can also be directly obtained from $\mathbf{T}_{t}$ (determined using (5.9) combined with (5.8)). It corresponds to the first three terms in the fourth column of $\mathbf{T}_{t}$.

The direct kinematics can thus be determined by the distal end pose, defined by $\alpha, \beta, \gamma$ and $\mathbf{p}_{t}^{G}$, obtained as a function of the robot configuration $\sigma_{i}, \zeta_{i}, \phi_{i}$ for $i=1, \ldots, n / 3$, as described in
the last two paragraphs.

### 5.3.3 Degrees of Freedom Analysis

As previously mentioned in section 5.2 , the most common requirement for operation of robotic manipulators in 3 D space is to provide near 6 DOFs at the end-effector, which refer to the ability to control the end-effector pose. In this regard, the DOFs of robots composed of two and three segments, which offer six and nine actuation DOFs from their segments, respectively, are considered in this subsection. It should be noted that in this analysis the DOFs refer to the end-effector pose, and not to the possibility of continuous deformation of the robot segments in infinitely different ways. The robot segments are considered to provide 3 actuation DOFs each, as previously described in section 5.2.

The DOFs at the end-effector of a robot can be determined by studying the Jacobian $\mathbf{J}$ corresponding to the differentiation of the direct kinematics, i.e. differentiation of the end-effector pose, $\alpha, \beta, \gamma$ and $\mathbf{p}_{t}^{G}$, with respect to the actuation DOFs $\sigma_{i}, \zeta_{i}, \phi_{i}$ for $i=1, \ldots, n / 3$. However, some of the trigonometric functions in (5.11) complicate such study. A Jacobian $\mathbf{J}^{\prime}$ can be defined, which corresponds to the differentiation of $\tan \alpha, \tan \beta, \tan \gamma$ and $\mathbf{p}_{t}^{G}$ with respect to $\sigma_{i}, \zeta_{i}, \phi_{i}$ for $i=1, \ldots, n / 3$. Since

$$
\begin{align*}
\delta \tan \alpha & =\frac{\delta \alpha}{\cos ^{2} \alpha} \\
\delta \tan \beta & =\frac{\delta \beta}{\cos ^{2} \beta}  \tag{5.12}\\
\delta \tan \gamma & =\frac{\delta \gamma}{\cos ^{2} \gamma}
\end{align*}
$$

the rank of $\mathbf{J}$ is equal ${ }^{1}$ to the rank of $\mathbf{J}^{\prime}$.

The expression of $\mathbf{J}^{\prime}$ is not reproduced here since it has a significant extension, which makes it impractical to write explicitly. However, it can be calculated using a symbolic toolbox, such as the Symbolic Math Toolbox ${ }^{T M}$ of Matlab®(Mathworks Inc.), as implemented in this work, by

[^1]entering the expressions of $\tan \alpha, \tan \beta, \tan \gamma$ and $\mathbf{p}_{t}^{G}$ as a function of $\sigma_{i}, \zeta_{i}, \phi_{i}$ for $i=1, \ldots, n / 3$ (obtained as described in the previous subsection), and deriving with respect to $\sigma_{i}, \zeta_{i}, \phi_{i}$ for $i=1, \ldots, n / 3$.

By studying $\mathbf{J}^{\prime}$ for a robot with $n=6$, the rank is found to be 5 since

$$
\begin{equation*}
\operatorname{det}\left|\mathbf{J}^{\prime}\right|=0 \quad \forall \sigma_{1}, \zeta_{1}, \phi_{1}, \sigma_{2}, \zeta_{2}, \phi_{2} \tag{5.13}
\end{equation*}
$$

and at least one minor exists with $\operatorname{det}\left|\mathbf{J}^{\prime}\right| \neq 0$. One degree of redundancy therefore exists. This result is also obtained in section 5.4 using a different derivation, where the redundancy is also elucidated.

The redundancy, however, differs from those in traditional multi-link robots since the kinematics are fundamentally different, and therefore a geometric analogy is not available. The fact that a robot with $n=6$ provides 5 DOFs at the end-effector also implies a constraint on the reachable end-effector poses. The study of the inverse kinematics provides a simple derivation of the condition on the end-effector poses that can be reached, as presented in subsection 5.4.4.

The study of $\mathbf{J}^{\prime}$ also indicates that at least a minor with rank 3 exists in the rows corresponding to $\tan \alpha, \tan \beta, \tan \gamma$. This implies that any end-effector orientation can be reached, with an associated constraint on position. Similarly, the study of $\mathbf{J}$ also shows that any end-effector position can be reached since a minor with rank 3 exists in the rows corresponding to $\mathbf{p}_{t}^{G}$. A constraint on the reachable end-effector orientations then applies. These results on the reachable end-effector poses are confirmed and elucidated in the derivation in subsection 5.4.4.

Interestingly, the end-effector orientation is the concatenation of the rotations associated to the robot segments, as expressed in 5.9. The rotation associated to a robot segment with a given bending and extension can also be achieved with zero extension and a different, specific bending of the segment. This bending can be directly determined from (5.3) by imposing the segment rotation angle $\rho_{i}=2 \zeta_{i}$ and the $l_{i}$ corresponding to zero extension, and determining the $\sigma_{i}$ and corresponding $b_{i}$. Therefore, an end-effector orientation reached using both bending and extension can also be reached using only bending of the segments, with zero extension,
which enables decoupling both types of actuation in this instance. Conversely, any end-effector orientation cannot be reached by only selecting the extension of the segments for a given bending, as a robot with $n=6$ only has 2 DOFs corresponding to extension.

For a specified, desired end-effector orientation, the inverse kinematics solution derived in section 5.4 can be used with an arbitrary end-effector position to determine the required segment rotation using both bending and extension. Then, the equivalent bending of the segments for a zero extension configuration can be calculated as described here to reach the desired end-effector orientation without extension.

A robot with $n=9$ provides 6 DOFs at the distal end. This result can be obtained by studying the rank of the corresponding Jacobian, following an analogous procedure to that described for a two-segment robot. A three-segment robot therefore provides the ability to reach any pose in 3D space, as well as three degrees of redundancy that can be used, for instance, to avoid an obstacle.

### 5.4 Inverse kinematics

The closed-form solution to the inverse kinematics problem is presented in this section. This involves determining the configuration of the two arcs composing a robot with $n=6$ to reach a specified end-effector pose. Despite the apparent simplicity of the problem, its solution is not trivial. Attempts to solve (5.1) with $g(\theta)$ formulated as in the previous section do not yield closed-form solutions. Instead, an alternative approach is required.

The approach proposed here is conceptually illustrated in Fig. 5.3. It involves considering the orientation at the point of junction between the two segments, which can be defined as $\mathbf{p}_{m}^{G}$ relative to the robot's proximal end, as a result of the transformations associated to the segments from the robot's proximal and distal ends. For an arbitrary position of $\mathbf{p}_{m}^{G}$, the approaches from both ends generally lead to different orientations. By imposing that both orientations coincide, a set of conditions emerge, which constitute the inverse kinematics problem.


Figure 5.3: Conceptual approach to the inverse kinematics solution. The rotations associated to a robot composed of two segments, which are defined by quaternions, are illustrated. The point of junction $\mathbf{p}_{m}^{G}$, and the reference frames $\{G\}$ and $\{T\}$ are also included.

### 5.4.1 Inverse Kinematics Formulation

Simplicity in the conditions constituting the inverse kinematics problem is crucial to enable the derivation of a closed-form solution. The use of Euler angles to describe the end-effector orientation is not suitable in the case of the inverse kinematics, as it complicates significantly the problem formulation, rendering it practically intractable. Instead, in this instance, orientation is described using quaternions, which are better suited to address the inverse kinematics problem. In addition, the robot segments are described using the second description introduced in subsection 5.3.1, which employs $x_{i}^{F}, y_{i}^{F}, z_{i}^{F}$ (Fig. 5.2). The combination of quaternions and this segment description enables the derivation of the closed-form solutions to the inverse kinematics reported in the following subsections. A key challenge is finding the relative orientation between the ends of a segment as a function of $x_{i}^{F}, y_{i}^{F}, z_{i}^{F}$.

The rotation associated to a general segment $i$ is determined by an axis $\mathbf{w}_{i}$ and an angle $\rho_{i}$, as discussed in subsection 5.3.2. The orientation at the segment tip can be expressed by a unit quaternion $\mathbf{q}_{i}$ relative to a reference frame at the base of the segment $\{F\}$, which is

$$
\begin{equation*}
\mathbf{q}_{i}=\cos \frac{\rho_{i}}{2}+w_{i i} \sin \frac{\rho_{i}}{2} \mathbf{i}^{F}+w_{i j} \sin \frac{\rho_{i}}{2} \mathbf{j}^{F}+w_{i k} \sin \frac{\rho_{i}}{2} \mathbf{k}^{F} \tag{5.14}
\end{equation*}
$$

where $\mathbf{i}^{F}, \mathbf{j}^{F}, \mathbf{k}^{F}$ are the unit vectors of the $\{F\}$ frame, and $w_{i i}, w_{i j}, w_{i k}$ denote the three components of $\mathbf{w}_{i}$. It should be noted that $w_{i k}$ is zero, as previously introduced in (5.6), and therefore the orientation of the segment tip corresponds to a rotation of $\{F\}$ about an axis that lies in the $\mathbf{i}^{F}, \mathbf{j}^{F}$ plane. The rotation axis $\mathbf{w}_{i}$ is perpendicular to the plane of bending of segment $i$. Thus, the orientation at the tip of segment $i$ described by quaternion $\mathbf{q}_{i}$ in (5.14) corresponds to a zero twist configuration of segment $i$ from a continuum body perspective. Quaternion $\mathbf{q}_{i}$ then correctly represents the full orientation at the tip of segment $i$ relative to $\{F\}$ in an actual continuum robot.

It should be noted that in this work, quaternion $\mathbf{q}_{i}$ in (5.14) is directly obtained as the total rotation from frame $\{F\}$ to the orientation at the tip of segment $i$. This approach differs from the three successive rotations commonly used in the literature [15] to find the orientation at the tip of segment $i$. Still, our approach leads to an equal resulting orientation at the tip of segment $i$, and is more straightforward when using quaternions.

Obtaining $\mathbf{q}_{i}$ as a simple function of $x_{i}^{F}, y_{i}^{F}, z_{i}^{F}$ requires some consideration. First, by using the identity in the scalar product between the vector corresponding to the position of the segment's $\operatorname{tip}\left[x_{i}^{F}, y_{i}^{F}, z_{i}^{F}\right]$ and the unit vector $\mathbf{k}^{F}$,

$$
\begin{equation*}
\sqrt{\left(x_{i}^{F}\right)^{2}+\left(y_{i}^{F}\right)^{2}+\left(z_{i}^{F}\right)^{2}}\left\|\mathbf{k}^{F}\right\| \cos \frac{\rho_{i}}{2}=[0,0,1] \cdot\left[x_{i}^{F}, y_{i}^{F}, z_{i}^{F}\right] \tag{5.15}
\end{equation*}
$$

the $\cos \frac{\rho_{i}}{2}$ can be obtained as a simple function of $x_{i}^{F}, y_{i}^{F}, z_{i}^{F}$.
Then, by using the vector product identity for the same vectors $\left[x_{i}^{F}, y_{i}^{F}, z_{i}^{F}\right]$ and $\mathbf{k}_{F}$

$$
\begin{equation*}
\sqrt{\left(x_{i}^{F}\right)^{2}+\left(y_{i}^{F}\right)^{2}+\left(z_{i}^{F}\right)^{2}}\left\|\mathbf{k}^{F}\right\| \sin \frac{\rho_{i}}{2}=\left\|\left[x_{i}^{F}, y_{i}^{F}, z_{i}^{F}\right] \times \mathbf{k}^{F}\right\| \tag{5.16}
\end{equation*}
$$

the $\sin \frac{\rho_{i}}{2}$ can be obtained as a function of $x_{i}^{F}, y_{i}^{F}, z_{i}^{F}$.

The normalised $\mathbf{w}_{i}$ as a function of $x_{i}^{F}, y_{i}^{F}, z_{i}^{F}$ can be obtained as

$$
\begin{equation*}
\mathbf{w}_{i}=\frac{\mathbf{k}^{F} \times\left[x_{i}^{F}, y_{i}^{F}, z_{i}^{F}\right]}{\left\|\mathbf{k}^{F} \times\left[x_{i}^{F}, y_{i}^{F}, z_{i}^{F}\right]\right\|}=\frac{\left[-y_{i}^{F}, x_{i}^{F}, 0\right]}{\sqrt{\left(x_{i}^{F}\right)^{2}+\left(y_{i}^{F}\right)^{2}}} \tag{5.17}
\end{equation*}
$$

Finally, by combining (5.15), (5.16) and (5.17), $\mathbf{q}_{i}$ can be obtained as a function of $x_{i}^{F}, y_{i}^{F}, z_{i}^{F}$ as

$$
\begin{equation*}
\mathbf{q}_{i}=\frac{z_{i}^{F}-y_{i}^{F} \mathbf{i}^{F}+x_{i}^{F} \mathbf{j}^{F}}{\sqrt{\left(x_{i}^{F}\right)^{2}+\left(y_{i}^{F}\right)^{2}+\left(z_{i}^{F}\right)^{2}}} \tag{5.18}
\end{equation*}
$$

The simplicity of (5.18) enables the subsequent derivation of a closed-form solution to the inverse kinematics.

Considering a robot with $n=6$, as illustrated in Fig. 5.4 by plotting the centreline of the robot's segments, the reference frame at the robot's proximal end is $\{G\}$. Another reference frame at the robot's distal end can be denoted by $\{T\}$. The orientation of $\{T\}$ is defined so that it coincides with $\{G\}$ when the robot is in a straight configuration. The orientation of the robot's end-effector relative to $\{G\}$ can be defined as

$$
\begin{equation*}
\mathbf{q}_{t}=\kappa+\lambda \mathbf{i}^{G}+\mu \mathbf{j}^{G}+\nu \mathbf{k}^{G} \tag{5.19}
\end{equation*}
$$

and the corresponding rotation matrix is denoted by $\mathbf{R}_{t}$.

The configuration of the proximal segment (segment 1) can be described by the position of its distal end $x_{1}^{G}, y_{1}^{G}, z_{1}^{G}$ relative to $\{G\}$. This distal end of segment 1 is the same as the point of junction between both segments $\mathbf{p}_{m}^{G}$, and thus the Cartesian coordinates $x_{1}^{G}, y_{1}^{G}, z_{1}^{G}$ correspond to the three components of $\mathbf{p}_{m}^{G}$. The orientation at the distal end of segment 1 can then be determined using (5.18) as

$$
\begin{equation*}
\mathbf{q}_{1}=\frac{z_{1}^{G}-y_{1}^{G} \mathbf{i}^{G}+x_{1}^{G} \mathbf{j}^{G}}{\sqrt{\left(x_{1}^{G}\right)^{2}+\left(y_{1}^{G}\right)^{2}+\left(z_{1}^{G}\right)^{2}}} \tag{5.20}
\end{equation*}
$$

The configuration of the distal segment (segment 2) can be described by the position of its proximal end $x_{2}^{T}, y_{2}^{T}, z_{2}^{T}$, relative to $\{T\}$. The proximal end of segment 2 is $\mathbf{p}_{m}^{T}$, which is the same point in space as $\mathbf{p}_{m}^{G}$, but here it is expressed relative to $\{T\}$. Thus, in this case the proximal end of segment 2 acts as the tip of the segment, and the base of segment 2 lies at the origin of $\{T\}$ (Fig. 5.4). The rotation $\mathbf{q}_{2}^{-1}$ corresponding to the second segment, which is relative to the robot's distal end reference frame, can therefore be expressed as a function of
$x_{2}^{T}, y_{2}^{T}, z_{2}^{T}$ as

$$
\begin{equation*}
\mathbf{q}_{2}^{-1}=\frac{z_{2}^{T}-y_{2}^{T} \mathbf{i}^{T}+x_{2}^{T} \mathbf{j}^{T}}{\sqrt{\left(x_{2}^{T}\right)^{2}+\left(y_{2}^{T}\right)^{2}+\left(z_{2}^{T}\right)^{2}}} \tag{5.21}
\end{equation*}
$$

It should be noted that the rotation $\mathbf{q}_{2}^{-1}$ corresponds to a segment that begins at the robot's distal end in a direction opposite to the $\mathbf{k}^{T}$ axis of $\{T\}$. Still, expression (5.21) remains valid due to geometric symmetry.

The vectors $\left[x_{1}^{G}, y_{1}^{G}, z_{1}^{G}\right]$ and $\left[x_{2}^{T}, y_{2}^{T}, z_{2}^{T}\right]$ both indicate the position of the point of junction between the two robot segments relative to $\{G\}$ and $\{T\}$, respectively. Reference frames $\{T\}$ and $\{G\}$ are related through a translation $\mathbf{p}_{t}^{G}$ and a rotation $\mathbf{R}_{t}^{-1}$. The components of $\mathbf{R}_{t}^{-1}$ can be denoted by $R_{t, i j}^{-1}$, which correspond to row $i$ and column $j$. These components of $\mathbf{R}_{t}^{-1}$ are given by the specified end-effector pose. Thus, vectors $\left[x_{1}^{G}, y_{1}^{G}, z_{1}^{G}\right]$ and $\left[x_{2}^{T}, y_{2}^{T}, z_{2}^{T}\right]$ are also directly related for a specified end-effector pose. The relation can be expressed as

$$
\left[\begin{array}{c}
x_{2}^{T}  \tag{5.22}\\
y_{2}^{T} \\
z_{2}^{T}
\end{array}\right]=\left[\begin{array}{ccc}
R_{t, 11}^{-1} & R_{t, 12}^{-1} & R_{t, 13}^{-1} \\
R_{t, 21}^{-1} & R_{t, 22}^{-1} & R_{t, 23}^{-1} \\
R_{t, 31}^{-1} & R_{t, 32}^{-1} & R_{t, 33}^{-1}
\end{array}\right]\left[\begin{array}{c}
p_{t i}^{G}-x_{1}^{G} \\
p_{t j}^{G}-y_{1}^{G} \\
p_{t k}^{G}-z_{1}^{G}
\end{array}\right]
$$

where $p_{t i}^{G}, p_{t j}^{G}, p_{t k}^{G}$ denote the three components of $\mathbf{p}_{t}^{G}$.

The rotation $\mathbf{q}_{2}^{-1}$ in (5.21) can then be expressed as a function of $x_{1}^{G}, y_{1}^{G}, z_{1}^{G}$ using (5.22). Thus, for any position of $\mathbf{p}_{m}^{G}$, the resulting orientation when approached from the robot's proximal and distal ends can be expressed by $\mathbf{q}_{1}\left(x_{1}^{G}, y_{1}^{G}, z_{1}^{G}\right)$ and $\mathbf{q}_{2}^{-1}\left(x_{1}^{G}, y_{1}^{G}, z_{1}^{G}\right)$, respectively.

In the robot configuration corresponding to the inverse kinematics solution, (5.9) must be satisfied. Hence, the concatenation of rotations must satisfy

$$
\begin{equation*}
\mathbf{q}_{t}=\mathbf{q}_{1} \mathbf{q}_{2} \tag{5.23}
\end{equation*}
$$

Defining $\mathbf{q}_{t}^{-1}$ as the inverse of $\mathbf{q}_{t}$, equation (5.23) can be reordered as

$$
\begin{equation*}
\mathbf{q}_{2}^{-1}=\mathbf{q}_{t}^{-1} \mathbf{q}_{1} \tag{5.24}
\end{equation*}
$$



Figure 5.4: Reference frames in inverse kinematics solution for a $n=6$ robot, with endeffector position at $\mathbf{p}_{t}^{G}=[2.64,0.92,-0.26]\left[\right.$ a.u.] and orientation $\mathbf{q}_{t}=0.87+0.13 \mathbf{i}^{G}-0.27 \mathbf{j}^{G}+$ $0.40 \mathbf{k}^{G}$. The centreline of the first segment is plotted in cyan, and the centreline of the second segment in magenta, and four lines following the outer surface of both segments of continuum body separated circumferentially at 90 degrees are plotted in red, green, blue and yellow. Reference frame $\{G\}$ at the robot's proximal end is depicted in turquoise, reference frame $\{T\}$ at the specified end-effector pose is depicted in purple, and the pose resulting from the robot configuration is shown in dashed green, with an exact overlap.
which is a function of $x_{1}^{G}, y_{1}^{G}, z_{1}^{G}$, as well as the end-effector pose, from (5.20) and (5.21) combined with (5.22). The quaternion components of (5.24) define the inverse kinematics problem.

### 5.4.2 Inverse Kinematics Solution

The solution to (5.24) is the solution to the inverse kinematics. In the subsequent presentation, $x, y, z$ is used to indicate $x_{1}^{G}, y_{1}^{G}, z_{1}^{G}$. Substituting (5.20) and (5.21) into (5.24) and using the change of variable (5.22), the following conditions emerge

$$
\begin{align*}
\lambda x+\mu y+\nu z & =0  \tag{5.25a}\\
-\frac{-\mu x+\lambda y+\kappa z}{\mathbf{h}_{3} \cdot\left(\mathbf{d}-\mathbf{p}_{t}^{G}\right)} & =\frac{\|\mathbf{d}\|}{\left\|\mathbf{d}-\mathbf{p}_{t}^{G}\right\|}  \tag{5.25b}\\
-\frac{\nu x+\kappa y-\lambda z}{\mathbf{h}_{2} \cdot\left(\mathbf{d}-\mathbf{p}_{t}^{G}\right)} & =\frac{\|\mathbf{d}\|}{\left\|\mathbf{d}-\mathbf{p}_{t}^{G}\right\|}  \tag{5.25c}\\
-\frac{\kappa x-\nu y+\mu z}{\mathbf{h}_{1} \cdot\left(\mathbf{d}-\mathbf{p}_{t}^{G}\right)} & =\frac{\|\mathbf{d}\|}{\left\|\mathbf{d}-\mathbf{p}_{t}^{G}\right\|} \tag{5.25d}
\end{align*}
$$

where $\mathbf{d}=[x, y, z]$, and $\mathbf{h}_{1}=\left[R_{t, 11}^{-1}, R_{t, 12}^{-1}, R_{t, 13}^{-1}\right], \mathbf{h}_{2}=\left[R_{t, 21}^{-1}, R_{t, 22}^{-1}, R_{t, 23}^{-1}\right], \mathbf{h}_{3}=\left[R_{t, 31}^{-1}, R_{t, 32}^{-1}, R_{t, 33}^{-1}\right]$, which correspond to the rows of $\mathbf{R}_{t}^{-1}$. The components of $\mathbf{R}_{t}^{-1}$ are determined by the specified end-effector orientation, and are thus directly related to $\mathbf{q}_{t}$. It should be noted that the main nonlinearities in (5.25) arise from the exponentials related to the moduli on the right hand side.

The equations in the system (5.25) are not independent. Different approaches to solving it are possible. This work proposes that (5.25a) be used, as well as the difference between (5.25b) and (5.25c). From (5.25a),

$$
\begin{equation*}
y=-\frac{\lambda x+\nu z}{\mu} \tag{5.26}
\end{equation*}
$$

Substituting (5.26) into the difference between (5.25b) and (5.25c), a second order polynomial equation relating $x$ and $z$ is obtained

$$
\begin{equation*}
c_{4} x^{2}+c_{3} z^{2}+c_{2} x z+c_{1} x+c_{0} z=0 \tag{5.27}
\end{equation*}
$$

where

$$
\begin{align*}
& c_{4}=-\mu R_{t, 21}^{-1}-\nu R_{t, 31}^{-1}+\frac{\left(\lambda R_{t, 22}^{-1}-\kappa R_{t, 32}^{-1}\right) \lambda^{2}}{\mu^{2}}-\frac{\left(-\mu R_{t, 22}^{-1}+\lambda R_{t, 21}^{-1}-\nu R_{t, 32}^{-1}-\kappa R_{t, 31}^{-1}\right) \lambda}{\mu} \\
& c_{3}=\frac{\left(\lambda R_{t, 22}^{-1}-\kappa R_{t, 32}^{-1}\right) \nu^{2}}{\mu^{2}}+\kappa R_{t, 23}^{-1}+\lambda R_{t, 33}^{-1}-\frac{\left(\lambda R_{t, 23}^{-1}+\kappa R_{t, 22}^{-1}-\kappa R_{t, 33}^{-1}+\lambda R_{t, 32}^{-1}\right) \nu}{\mu} \\
& c_{2}=\frac{2 \nu \lambda\left(\lambda R_{t, 22}^{-1}-\kappa R_{t, 32}^{-1}\right)}{\mu^{2}}-\frac{\left(-\mu R_{t, 22}^{-1}+\lambda R_{t, 21}^{-1}-\nu R_{t, 32}^{-1}-\kappa R_{t, 31}^{-1}\right) \nu}{\mu}-\mu R_{t, 23}^{-1}+ \\
& \kappa R_{t, 21}^{-1}-\nu R_{t, 33}^{-1}+\lambda R_{t, 31}^{-1}-\frac{\lambda\left(\lambda R_{t, 23}^{-1}+\kappa R_{t, 22}^{-1}-\kappa R_{t, 33}^{-1}+\lambda R_{t, 32}^{-1}\right)}{\mu}  \tag{5.28}\\
& c_{1}=\left(\mu R_{t, 21}^{-1}+\nu R_{t, 31}^{-1}\right) p_{t i}^{G}+\left(\mu R_{t, 22}^{-1}+\nu R_{t, 32}^{-1}\right) p_{t j}^{G}+\left(\mu R_{t, 23}^{-1}+\nu R_{t, 33}^{-1}\right) p_{t k}^{G}- \\
& \frac{\left.\left.\lambda\left(\left(\kappa R_{t, 31}^{-1}-\lambda R_{t, 21}^{-1}\right) p_{t i}^{G}+\kappa R_{t, 32}^{-1}-\lambda R_{t, 22}^{-1}\right) p_{t j}^{G}+\kappa R_{t, 33}^{-1}-\lambda R_{t, 23}^{-1}\right) p_{t k}^{G}\right)}{\mu} \\
& c_{0}=\frac{\nu\left(\kappa R_{t, 31}^{-1} p_{t i}^{G}+\kappa R_{t, 32}^{-1} p_{t j}^{G}+\kappa R_{t, 33}^{-1} p_{t k}^{G}-\lambda R_{t, 21}^{-1} p_{t i}^{G}-\lambda R_{t, 22}^{-1} p_{t j}^{G}-\lambda R_{t, 23}^{-1} p_{t k}^{G}\right)}{\mu}- \\
& \kappa R_{t, 21}^{-1} p_{t i}^{G}-\kappa R_{t, 22}^{-1} p_{t j}^{G}-\kappa R_{t, 33}^{-1} p_{t k}^{G}-\lambda R_{t, 31}^{-1} p_{t i}^{G}-\lambda R_{t, 32}^{-1} p_{t j}^{G}-\lambda R_{t, 33}^{-1} p_{t k}^{G}
\end{align*}
$$

The analytical, closed-form solution to (5.25) can then be obtained for $x$

$$
\begin{equation*}
x=\frac{-\left(c_{2} z+c_{1}\right) \pm \sqrt{\left(c_{2} z+c_{1}\right)^{2}-4 c_{4}\left(c_{3} z^{2}+c_{0} z\right)}}{2\left(c_{4}\right)} \tag{5.29}
\end{equation*}
$$

which is the solution to the inverse kinematics problem in combination with (5.26), as a function of $z$, which acts as a parameter. The point $x, y, z$ corresponds to the point of junction between the two segments, $\mathbf{p}_{m}^{G}$, and completely defines the configuration of each of the two robot segments. This solution can also be expressed with the more conventional variables $\sigma_{i}, \zeta_{i}, \phi_{i}$ using the change of variable (5.5) for the proximal segment, and by using an analogous relation with the change of variables (5.22) for the distal segment.

The solution to the inverse kinematics is therefore a curve in $3 D$ space of the possible positions of the point of junction $\mathbf{p}_{m}^{G}$. This solution can be expressed as

$$
\begin{align*}
& x=f_{1}\left(\mathbf{p}_{t}^{G}, \mathbf{q}_{t}, z\right)  \tag{5.30}\\
& y=f_{2}\left(\mathbf{p}_{t}^{G}, \mathbf{q}_{t}, z\right)
\end{align*}
$$

where the curve is parametrised by $z$ as in (5.29). This corresponds to a degree of redundancy
in the robot space, which is discussed in the following subsection.
The solution to the inverse kinematics derived here always exists for any $g_{t}$ inside the robot's workspace, and is not affected by singularities. The solution is expressed in closed-form by (5.29) and (5.26) for the general case $\mu \neq 0$. For the particular case $\mu=0$, the solution is determined by substituting the relation between $x$ and $z$ determined by (5.25a) with $\mu=0$ into the difference between (5.25b) and (5.25c), in an analogous manner as previously described in this subsection, but for the simpler case $\mu=0$. The resulting expression is equivalent to (5.29).

The fact that the solution is derived in closed-form implies that it is straightforward to implement in practice, requiring a negligible computational time. In addition, the solution applies to any reachable $g_{t}$ without any additional complexity. The closed-form solution can then be used in the design of control laws and path planning algorithms. The derivation of the inverse kinematics solution in closed-form also elucidates a kinematic redundancy, which enables one to select the most desirable robot configuration for each $g_{t}$, as described in the next subsection.

### 5.4.3 Redundancy in Inverse Kinematics

The direct kinematics analysis of subsection 5.3.3 indicates a degree of redundancy in a robot with six actuation DOFs operating in $S E(3)$. This redundancy corresponds to the fact that a robot with six actuation DOFs can reach a given end-effector pose in multiple configurations. This redundancy is verified and elucidated by the solution to the inverse kinematics system (5.25). For a specified $g_{t}$ inside the workspace, there exists an infinite number of solutions for the point of junction between the two robot segments $[x, y, z]$ that allow $g_{t}$ to be reached, which determine the robot's self-motion.

These solutions define on a curve, determined by (5.29) and (5.26) as a function of the parameter $z$. This curve lies on a plane determined by $\lambda, \mu, \nu$, and is elliptical in geometry.

An example of such an ellipse is plotted in orange in Fig. 5.5 for a $g_{t}$ at $\mathbf{p}_{t}^{G}=[-0.14,5.28,1.02]$ [a.u.] position, and $\mathbf{q}_{t}=0.1+0.36 \mathbf{i}^{G}-0.17 \mathbf{j}^{G}+0.91 \mathbf{k}^{G}$ orientation. The different points on the orange curve are possible positions of the point of junction $\mathbf{p}_{m}^{G}$, and thus correspond to


Figure 5.5: Curve corresponding to the loci of the distal end of the first segment, for an $n=6$ robot with end-effector position at $\mathbf{p}_{t}^{G}=[-0.14,5.28,1.02]\left[\right.$ a.u.] and orientation $\mathbf{q}_{t}=$ $0.1+0.36 \mathbf{i}^{G}-0.17 \mathbf{j}^{G}+0.91 \mathbf{k}^{G}$. Two of the possible robot configurations to reach this specified end-effector pose are also shown, with the distal end of the first segment at two of the possible locations on the curve.
different extension and bending of the robot segments. Two robot configurations corresponding to the inverse kinematics solution for the same specified $g_{t}$ and different positions of $\mathbf{p}_{m}^{G}$ on the orange curve of possible solutions are also plotted in Fig. 5.5 to help illustrate the kinematic redundancy. The two configurations correspond to different extension and bending of the segments, but reach the same $g_{t}$. The most desirable robot configuration to reach a $g_{t}$ can therefore be selected, which enables avoiding collisions between the robot and obstacles in the environment, and respecting the physical constraints on extension and bending of the segments.

### 5.4.4 Condition on End-Effector Configuration

An alternative, relevant reordering of (5.23) is

$$
\begin{equation*}
\mathbf{q}_{1}=\mathbf{q}_{t} \mathbf{q}_{2}^{-1} \tag{5.31}
\end{equation*}
$$

Expressing the terms in (5.31) as explicit functions of $x, y, z$, and the end-effector pose by using (5.20), (5.21) and (5.22), a set of equations equivalent to (5.25) is obtained as

$$
\begin{align*}
\lambda \mathbf{h}_{1} \cdot\left(\mathbf{d}-\mathbf{p}_{t}^{G}\right)+\mu \mathbf{h}_{2} \cdot\left(\mathbf{d}-\mathbf{p}_{t}^{G}\right)+\nu \mathbf{h}_{3} \cdot\left(\mathbf{d}-\mathbf{p}_{t}^{G}\right) & =0  \tag{5.32a}\\
\mu \mathbf{h}_{1} \cdot\left(\mathbf{d}-\mathbf{p}_{t}^{G}\right)-\lambda \mathbf{h}_{2} \cdot\left(\mathbf{d}-\mathbf{p}_{t}^{G}\right)+\kappa \mathbf{h}_{3} \cdot\left(\mathbf{d}-\mathbf{p}_{t}^{G}\right) & =-z \frac{\left\|\mathbf{d}-\mathbf{p}_{t}^{G}\right\|}{\|\mathbf{d}\|}  \tag{5.32b}\\
\nu \mathbf{h}_{1} \cdot\left(\mathbf{d}-\mathbf{p}_{t}^{G}\right)-\kappa \mathbf{h}_{2} \cdot\left(\mathbf{d}-\mathbf{p}_{t}^{G}\right)-\lambda \mathbf{h}_{3} \cdot\left(\mathbf{d}-\mathbf{p}_{t}^{G}\right) & =y \frac{\left\|\mathbf{d}-\mathbf{p}_{t}^{G}\right\|}{\|\mathbf{d}\|}  \tag{5.32c}\\
\kappa \mathbf{h}_{1} \cdot\left(\mathbf{d}-\mathbf{p}_{t}^{G}\right)+\nu \mathbf{h}_{2} \cdot\left(\mathbf{d}-\mathbf{p}_{t}^{G}\right)-\mu \mathbf{h}_{3} \cdot\left(\mathbf{d}-\mathbf{p}_{t}^{G}\right) & =-x \frac{\left\|\mathbf{d}-\mathbf{p}_{t}^{G}\right\|}{\|\mathbf{d}\|} \tag{5.32~d}
\end{align*}
$$

It should be noted that the left hand side of the system of equations (5.32) is linear.

Since systems (5.25) and (5.32) are equivalent, the constituting equations must be concurrently satisfied. Equations (5.32a) and (5.25a) correspond to two parallel planes. However, they are not necessarily coincident, as this depends on the desired end-effector pose. Thus, the poses $g_{t}$ that simultaneously satisfy (5.25a) and (5.32a) constitute the reachable end-effector configurations.

Comparing (5.25a) and (5.32a), and after manipulation, the condition determining the reachable end-effector configurations can be distilled as

$$
\begin{equation*}
\lambda p_{t i}^{G}+\mu p_{t j}^{G}+\nu p_{t k}^{G}=0 \tag{5.33}
\end{equation*}
$$

Equation (5.33) indicates that the position of the robot's end-effector must be on a plane determined by $\lambda, \mu, \nu$, which is the same plane where the distal end of the proximal segment, $\mathbf{p}_{m}^{G}$, must be. Interestingly, condition (5.33) does not constrain $\kappa$. The condition on the reachable end-effector configurations can also be expressed in terms of the ZYZ Euler angles by transforming $\lambda, \mu, \nu$ into $\alpha, \beta, \gamma$, e.g. as in [205].

Thus, by selecting five variables to specify the desired end-effector pose, one of which must correspond to $\kappa$ or its equivalent in Euler angles, condition (5.33) can be then used to obtain the 6th variable, thereby completely defining the robot's end-effector pose. The inverse kinematics solution can be subsequently determined, as described in the previous subsection.

### 5.4.5 Higher Dimensional Robot Configurations

The discussion in the previous subsections shows that a robot with $n=6$ provides 5 DOFs at the end-effector. In order to achieve 6 DOFs at the end-effector, an additional robot segment is required, as justified in (5.3.3), resulting in a robot with $n=9$. The generalization of the work to robots with $n=9$ is outlined in this subsection.

Considering a robot composed of three segments, a reference frame $\{B\}$ can be defined, which coincides with the robot's proximal end. The configuration of the proximal segment can be described by $x_{0}^{B}, y_{0}^{B}, z_{0}^{B}$, which correspond to the position of the proximal segment's distal end relative to $\{B\}$. The orientation of the proximal segment's distal end relative to $\{B\}$ can be expressed by a quaternion using (5.18) as

$$
\begin{equation*}
\mathbf{q}_{0}=\frac{z_{0}^{B}-y_{0}^{B} \mathbf{i}^{B}+x_{0}^{B} \mathbf{j}^{B}}{\sqrt{\left(x_{0}^{B}\right)^{2}+\left(y_{0}^{B}\right)^{2}+\left(z_{0}^{B}\right)^{2}}} \tag{5.34}
\end{equation*}
$$

A reference frame can then be defined at the distal end of the proximal segment $\left\{G^{\prime}\right\}$, the position and orientation of which are a function of $x_{0}^{B}, y_{0}^{B}, z_{0}^{B}$.

The pose of the robot's end-effector relative to $\{B\}$ can be denoted by $\mathbf{p}_{\tau}^{B}$ and $\mathbf{q}_{\tau}$. The orientation of the robot's end-effector relative to $\left\{G^{\prime}\right\}$, which can be defined as $\mathbf{q}_{\tau}^{\prime}$, can then be obtained as a function of $x_{0}^{B}, y_{0}^{B}, z_{0}^{B}$ and $\mathbf{q}_{\tau}$ as

$$
\begin{equation*}
\mathbf{q}_{\tau}^{\prime}=\mathbf{q}_{0}^{-1} \mathbf{q}_{\tau} \tag{5.35}
\end{equation*}
$$

The robot's end-effector position relative to $\left\{G^{\prime}\right\}$, which can be denoted by $\mathbf{p}_{\tau}^{G^{\prime}}$, can also be obtained as a function of the proximal segment's configuration and $\mathbf{p}_{\tau}^{B}$ by using the translation $\left[x_{0}^{B}, y_{0}^{B}, z_{0}^{B}\right]$ and the rotation associated with $\mathbf{q}_{0}^{-1}$, see [197], yielding

$$
\mathbf{p}_{\tau}^{G^{\prime}}=\left[\begin{array}{c}
p_{\tau i}^{B}\left(\left(z_{0}^{B}\right)^{2}+\left(x_{0}^{B}\right)^{2}\right)+x_{0}^{B}\left(p_{\tau j}^{B} y_{0}^{B}-2 p_{\tau k}^{B} z_{0}^{B}+\left(z_{0}^{B}\right)^{2}-\left(y_{0}^{B}\right)^{2}+\left(x_{0}^{B}\right)^{2}\right)  \tag{5.36}\\
p_{\tau j}^{B}\left(\left(z_{0}^{B}\right)^{2}-\left(y_{0}^{B}\right)^{2}\right)+y_{0}^{B}\left(2 p_{\tau k}^{B} z_{0}^{B}+p_{\tau i}^{B} x_{0}^{B}-3\left(z_{0}^{B}\right)^{2}-\left(x_{0}^{B}\right)^{2}-\left(y_{0}^{B}\right)^{2}\right) \\
p_{\tau k}^{B}\left(\left(z_{0}^{B}\right)^{2}-\left(y_{0}^{B}\right)^{2}-\left(x_{0}^{B}\right)^{2}\right)+z_{0}^{B}\left(2 p_{\tau i}^{B} x_{0}^{B}-2 p_{\tau j}^{B} y_{0}^{B}+3\left(y_{0}^{B}\right)^{2}-\left(z_{0}^{B}\right)^{2}\right)
\end{array}\right]
$$

where $p_{\tau i}^{B}, p_{\tau j}^{B}, p_{\tau k}^{B}$ are the three components of $\mathbf{p}_{\tau}^{B}$.

The kinematics subproblem corresponding to the two distal segments of the robot implies a condition on the reachable $\mathbf{p}_{\tau}^{G^{\prime}}, \mathbf{q}_{\tau}^{\prime}$, elucidated in (5.33). Instead, the three-segment robot allows 6 DOFs at the end-effector. Using (5.35) and (5.36), condition (5.33) corresponding to the two distal segments can be translated into a condition on $x_{0}^{B}, y_{0}^{B}, z_{0}^{B}$ for a given $\mathbf{p}_{\tau}^{B}$ and $\mathbf{q}_{\tau}$.

The inverse kinematics subproblem for the two distal segments can then be solved using (5.30), for a pose specified by $\mathbf{p}_{\tau}^{G^{\prime}}$ and $\mathbf{q}_{\tau}^{\prime}$, which now satisfies (5.33). Substitution of expressions (5.35) and (5.36) into the $\mathbf{p}_{\tau}^{G^{\prime}}$ and $\mathbf{q}_{\tau}^{\prime}$ of such solution (5.30) provides the general solution to the inverse kinematics of the complete robot as a function of $x_{0}^{B}, y_{0}^{B}, z_{0}^{B}$, which in turn are related by the aforementioned condition.

Thus, the three-segment robot allows for the complete control of the end-effector pose inside the workspace, and three degrees of redundancy. In a typical scenario, one of them can correspond to the two distal segments, and the other two may correspond to the proximal segment.

### 5.5 Simulations

The robot configurations corresponding to the inverse kinematics solution in different scenarios are simulated in this section for robots with $n=6$ in order to help illustrate the results obtained. The simulations also provide a verification of the work presented in the previous sections, and show the behavior of continuum robots with bending and extension capabilities in some representative cases.

The configuration of a robot with a specified end-effector pose $\mathbf{p}_{t}^{G}=[2.64,0.92,-0.26][$ a.u.] and $\mathbf{q}_{t}=0.87+0.13 \mathbf{i}^{G}-0.27 \mathbf{j}^{G}+0.40 \mathbf{k}^{G}$ is illustrated in Fig. 5.4 with a plot of the centreline of the robot's segments, together with four lines that follow the outer contour of the continuum robot, showing that this does not undergo any twist and that its torsional alignment is correct. The end-effector pose is selected to satisfy (5.33). The solution is calculated using (5.29), (5.26), with an arbitrary value of $z=-3$. The coordinates of the point of junction between the two


Figure 5.6: Set of inverse kinematics solutions corresponding to a robot with $n=6$, for a specified end-effector at $\mathbf{p}_{t}^{G}=[2.64,0.92,-0.26][a . u$.$] and \mathbf{q}_{t}=0.87+0.13 \mathbf{i}^{G}-0.27 \mathbf{j}^{G}+0.40 \mathbf{k}^{G}$.
segments $\mathbf{p}_{m}^{G}$ are found to be $x=1.40, y=-3.80, z=-3$ [a.u.]. Using (5.2) and (5.22), the variables directly describing the two segments can be obtained as $\sigma_{1}=5.04, \zeta_{1}=2.21, \phi_{1}=$ $-1.22, \sigma_{2}=5.60, \zeta_{2}=1.00, \phi_{2}=-0.58$. As can be seen in Fig. 5.4, the tangency of the arcs is respected, and the resulting robot end-effector pose matches the specified pose exactly.

The robot configuration shown in Fig. 5.4 is a solution to the inverse kinematics, but it requires significant room to maneuver, which may not be available when operating in confined environments. In this regard, different possible robot configurations for the same end-effector pose, which correspond to the redundancy presented in subsection 5.4.3, are plotted in Fig 5.6. These highlight the capability provided by the inverse kinematics solution to select the most suitable robot configuration to reach a desired end-effector pose.

Finally, four robot configurations corresponding to the robot moving vertically and with an end-effector orientation changing gradually are plotted in Fig. 5.7, with pose values specified
in the figure caption. All four end-effector poses satisfy (5.33), and the corresponding robot configurations are determined using the inverse kinematics solution (5.29), (5.26), with appropriate $z$ values to prevent excessive bending or extension of the segments. As can be seen in Fig. 5.7, these robot configurations result in a smooth motion of the robot, which illustrates the suitability of the inverse kinematics solution in determining appropriate robot configurations to execute a desired motion.

### 5.6 Conclusions on kinematics

The direct and inverse kinematics of continuum robots with constant curvature bending and extending capabilities can be solved in closed-form using the approach proposed in this work. The problem description is decisive in the complexity of the kinematic mappings. The use of quaternions enables the derivation of the closed-from solution to the inverse kinematics presented here.

The kinematic analysis required to obtain these solutions also produces additional results, which are of interest. Among the most prominent of these is the fact that a manipulator with six actuation DOFs is only capable of five DOFs at the end-effector. This redundancy is translated as a curve corresponding to the inverse kinematics solution, which can be expressed in closedform as described in this work. A condition on the reachable end-effector poses using a robot with six actuation DOFs therefore exists, which is also drawn from the analysis presented in the previous sections. The kinematic solutions derived for a robot with six actuation DOFs can also be used to determine the solution to the inverse kinematics of a higher order system necessary to reach six DOFs at the end-effector, as outlined in this work. Finally, the simulated solutions presented here show a variety of robot configurations available to reach a desired end-effector pose, illustrating the possibility of selecting suitable configurations for different scenarios.

As previously noted, the kinematic solutions derived in the previous sections cannot be directly applied to the final fine-positioner design, since this is composed of segments that cannot extend. Thus, the work presented in the previous sections cannot be directly used for the development


Figure 5.7: Four inverse kinematics solutions corresponding to the motion of an $n=6$ robot with end-effector poses at $\mathbf{p}_{t 1}^{G}=[-2,2,2.97][a . u$.$] and \mathbf{q}_{t 1}=0.73+0.31 \mathbf{i}^{G}-0.39 \mathbf{j}^{G}+0.47 \mathbf{k}^{G}$, $\mathbf{p}_{t 2}^{G}=[-2,2,3.30][$ a.u. $]$ and $\mathbf{q}_{t 2}=0.73+0.29 \mathbf{i}^{G}-0.44 \mathbf{j}^{G}+0.44 \mathbf{k}^{G}, \mathbf{p}_{t 3}^{G}=[-2,2,3.64][a . u$.$] and$ $\mathbf{q}_{t 3}=0.73+0.27 \mathbf{i}^{G}-0.48 \mathbf{j}^{G}+0.41 \mathbf{k}^{G}$, and $\mathbf{p}_{t 4}^{G}=[-2,2,3.97]\left[\right.$ a.u.] and $\mathbf{q}_{t 4}=0.73+0.24 \mathbf{i}^{G}-$ $0.51 \mathrm{j}^{G}+0.38 \mathbf{k}^{G}$.
of control laws.

The generalisation of this work on kinematics to continuum robots composed of segments that cannot extend, such as the fine-positioner, was considered, and some efforts were spent on it by this author. However, even though this work can serve as foundation, and important elements such as the use of quaternions to formulate the inverse kinematics can be adapted, the generalisation is non-trivial, and is considered to require a significant amount time.

At the same period of time when the issues on compatibility of this work on kinematics with the final fine-positioner design were arising, the possibility of starting a collaboration between this author and Dr Enrico Franco, a post-doctoral researcher working on control theory at Imperial College London, arose. This enabled the possibility of adopting an alternative approach to the development of control laws for the fine-positioner, which involved the use of energy shaping methods to derive closed-loop control laws. Such an alternative approach presented a higher probability of success, and thus was adopted. The work on this alternative approach is presented in the next sections.

### 5.7 Mechanical modelling

The development of closed-loop control laws for the fine-positioner using energy shaping methods first requires an analytical model of the mechanical behaviour of the fine-positioner. The development of such an analytical model is presented in this section. It should be noted that notation is redefined in this section and in the next section 5.8 , since they correspond to a new approach to the control presented in [206], and the new notation used in this and next sections generally follows that in [206] to simplify comprehension for the reader.

### 5.7.1 Initial modelling exploration

An initial approach to the mechanical modelling was first explored using beam theory. A segment of the fine-positioner was considered as a beam, subjected to external loads that
corresponded to pressure and external wrenches. A hypothesis on the deformation modes of the fine-positioner was then formulated based on a coarse approximation of the deformation of the outer wall observed in the similations. This assumed that the cross sections that were planar in the undeformed segment remained planar in the deformed segment, although they needed not remain perpendicular to the centreline.

Imposing equilibrium on this beam model, and developing it, an approximate mechanical model was obtained. This yielded closed-form solutions to the curvature of the centreline of a segment of fine-positioner for the case of a pressurised segment without external wrenches. This predicted a constant curvature bending, which approximately matches the behaviour in simulations and experimental observations. However, it was not possible to obtain closed-form solutions to the deformation for the case of a segment undergoing external wrenches.

### 5.7.2 Mechanical model selected

In this regard, an alternative approach was then adopted, which is the approach selected for the mechanical model used in this work for the derivation of closed-loop control laws. In this approach, a segment of the fine-positioner is approximated as a set of four rigid links articulated at three pin joints, as illustrated in Figure 5.8. The joints are initially considered to have an elastic stiffness, defined $k_{i}$ for joint $i$, that needs to be determined using either simulations or experiments, with a resting configuration of equal orientation between adjacent links. The angle in the joints is defined $q_{i}$, and the resting configuration corresponds to $q_{i}=0$. Nonlinear stiffnesses can also be considered in future work to improve the model. The bending moment created by the pressurisation of the chambers in the fine-positioner is applied as a moment at the distal end of the model, or equivalently as an equal moment at each of the joints. The specific value of moment created by a given pressure needs to be determined using simulations or experiments.


Figure 5.8: Schematic of the model made of four links with three pin joints, which corresponds to a segment of the fine-positioner in planar operation. The joint angles, $q_{i}$, are indicated, as well as the joint stiffnesses, $k_{i}$, links lengths, $L_{i}$, and control input $u$ that corresponds to the moment created by pressure and is equal for all three joints in the model.

## Link lengths

This model is initially developed for planar operation of the fine-positioner segment. The model has the advantage that it does not result in redundancies, since the imposition of a pose at the distal end of the segment generally implies a specific set of joint angles. The lengths of the links in the model affect the joint angles for each distal end pose, and must be determined so that the bending behaviour in the model is representative of the bending behaviour of a segment of the fine-positioner.

The lengths of the four links in this work are determined so that the potential elastic energy associated to a deformed segment is similar to that of the continuous segment. It should be noted that an exact match is not possible for all segment deformations or distal end poses due to the discretisation used in the model to approximate a continuum robot segment. The lengths are selected to yield a similar behaviour between model and device for the most relevant configurations.

In order to match the behaviour in the model and in the fine-positioner in the common configuration of a straight segment with zero external loads and zero pressure, the links at the two ends of the robot segment should be tangential to the segment. In addition, the sum of the length of the four links should be equal to the length of a segment of fine-positioner, $L_{T}$. Then, for a common case of a robot segment bending as constant curvature (CC) arc, symmetry in the model is important so that it is representative of the behaviour of the fine-positioner. This implies that the length of the rigid link near the proximal end, $L_{1}$, and that of the link near the distal end, $L_{4}$, should be equal. In addition, the length of the second link, $L_{2}$, and the third, $L_{3}$, should also be equal.

Finally, in robot segments bending as CC arcs, in a first, coarse approximation assuming that the elastic energy associated to each slide of fine-positioner is constant and that the bending stiffness is constant with curvature, the elastic potential energy should be proportional to the square of curvature. Then, in the model, in order to obtain an elastic potential energy proportional to the square of the deflection angle at the distal end, the rotation in all three joints should be equal for tip poses corresponding to a segment bending as CC arc. Considering the kinematics of a model made of four rigid links, it is not possible to find a set of link lengths such that the rotation in all joints is equal for distal end poses corresponding to CC arcs of the fine-positioner. It is only possible to select a reference deflection corresponding to CC arc bending where this is satisfied, and then ensure that the deviations from equal rotation angles in all joints is relatively low for other deflections. In this work, the reference deflection selected is a 60 degree deflection at the distal end, since it is representative of the desired operation of the fine-positioner where control laws are expected to be particularly relevant.

Given these considerations, to achieve symmetry, it is necessary for $L_{1}=L_{4}$ and $L_{2}=L_{3}$. In addition, $L_{1}+L_{2}+L_{3}+L_{4}=L_{T}$, to match the total length at zero deflection. Then, link lengths of $L_{1}=L_{4}=0.125 L_{T}$ and $L_{2}=L_{3}=0.375 L_{T}$ lead to rotations in the segment joints $q_{i} 1=q_{2}=q_{3}=20$ degrees for a distal end deflection of 60 degrees. In addition, at other bending deflections corresponding to CC arcs, a model with these link lengths also leads to rotations in the joints that are relatively equal, which results in elastic potential energy proportional to the square of deflection.

Finally, for other general deformations of the segment that are not CC, a model with $L_{1}=L_{4}=$ $0.125 L_{T}$ and $L_{2}=L_{3}=0.375 L_{T}$ is also considered to be suitable. This is predominantly due to the fact that for typical deformation modes of the robot observed in the simulations in the previous Chapter 4, this model results in joint rotations that are approximately representative of the curvature at the cross sections of the segment near the equivalent location of the joint. Thus, the model selected consists of four links with lengths $L_{1}=L_{4}=0.125 L_{T}$ and $L_{2}=L_{3}=$ $0.375 L_{T}$.

## Mass distribution

A mass is associated to each of the four links. The objective is to obtain dynamics in the model similar to those of the fine-positioner segment. In this work, the total mass in the model is selected to be equal to the mass of a segment of the fine-positioner, and this is distributed between the four links, assigning mass proportional to the length of the links. This implies $12.5 \%$ of the total mass at first link, $m_{1}, 37.5 \%$ at second link, $m_{2}, 37.5 \%$ at third link, $m_{3}$, and $12.5 \%$ at fourth link, $m_{4}$. For all links, the mass is concentrated at the midpoint of the link.

This mass distribution is expected to result in an acceptable model in a first approximation that suffices to derive control laws for the following reasons. First, a mass distribution proportional to the link lengths implies that the inertia of each segment relative to the base can be similar to that in the fine-positioner. In addition, the mass distribution is symmetric like the link lengths, and the more significant parts of the total mass are in the longer links of the model, that generally present displacements that are similar to those of the larger parts of the finepositioner segment. And finally, the mass in the first link of the proximal end of the robot, which considered fixed relatively to the environment for control purposes, does not affect the kinetic energy, which can be acceptable considering that it is only $12.5 \%$ of the total mass, and can be considered to be representative of the fact that the proximal $12.5 \%$ of the first segment of fine-positioner does not move significantly relatively to the environment.

## Moment generated by pressure

The bending moment generated by a given pressure is determined from the simulation of the selected design of the fine-positioner. This moment is determined by the magnitude of pressure, and by the distance between the centre of pressures and the stiffness centre, which is analogous to the centre of mass of the cross section but using the distribution of stiffness in the cross section instead of the distribution of density. On a first instance, this moment is estimated to be proportional to the pressure, which neglects the effects of cross-sectional deformation. This moment generated by pressure is measured at a given pressure, and is then interpolated linearly to any other pressure. It should be noted that this is a coarse approximation that tends to increase in error as the cross section varies from that used to extract the values of the moment. Thus, the values of moment should be extracted for a representative state.

The moment was measured by using a free body cut in a simulation of a segment of the finepositioner without external loads and with a deflection near 60 degrees. Considering the fact that the outer wall remains circular in the deformed cross section, the fact that the stiffness of the central rod is orders of magnitude higher than that of the rubber, and the geometry of the deformed cross section in the simulations, the stiffness centre was estimated to be near the central rod. From this, the bending moment generated by pressure was estimated to be $1.1 \mathrm{mNm} / \mathrm{bar}$. It should be noted that the determination of this bending moment generated by pressure is only a first, coarse approximation, and can be refined with experiments in case the control laws do not perform correctly.

## Joint stiffness

The stiffness of the joints in the model was then determined so that the deflection of the model matched that of the segment of fine-positioner in the simulation without external loads, for a given pressure, and for the moment generated by pressure obtained in the previous subsection. In the case without external loads, the rotation in all joints of the model is equal. Thus, this simply involved extracting a deflection from the simulation, dividing it by three to determine
the rotation in each joint, and then finding the joint stiffness that corresponds to this joint rotation for the moment generated by pressure calculated using the value of $1.1 \mathrm{mNm} / \mathrm{bar}$. The resulting joint stiffness is $5 \mathrm{mNm} / \mathrm{rad}$.

These values of joint stiffness and bending moment generated by pressure imply that, at a pressure of 1 bar, each joint rotates 0.22 rad , and thus the deflection at the distal end of the model is near 38 degrees. This resulting model is relatively similar to the observed behaviour of a segment of the fine-positioner. As noted in the previous subsection, this represents a first, coarse model, and its parameters can be improved based on the first results in the practical implementation of the control laws. In particular, the values of the moment generated by pressure and joint stiffness can be tuned based on the results obtained in practice.

### 5.8 Closed-loop control laws

Work on the development of control laws for the fine-positioner is presented in this section. The control laws are based on the mechanical model presented in the previous section 5.7. It should be noted that the derivation of the control laws for this work was predominantly done by Dr Enrico Franco. The contribution of this author to the derivation of these laws was only in an advisory capacity, through discussions on the suitability of the laws for the mechanical model, suggestions on potential solutions adapted from existing literature, and insights on the behaviour of the device relevant for the derivation of the laws. In this regard, the derivation of the control laws is only briefly outlined, and the resulting control laws for planar operation of a segment of the fine-positioner are then presented.

### 5.8.1 Control objective

The aim of the control laws initially developed here is to reach and track a desired configuration in a segment of the fine-positioner by using feedback information regarding the state of this segment. In this case, the control input is the pressure applied in two chambers of this fine-
positioner, which is equal for both chambers and makes the segment bend in a plane. This pressure input in the model, which generates the bending moment applied to all joints, is defined as $u$. The segment is considered to be subjected to external wrenches that act as external disturbances. These cause the equivalent of external moments acting as disturbances on the model joints, which are defined as $\delta_{i}$. The effect of all external disturbances is incorporated as these $\delta_{i}$. The control law should then generally be a function of the state of the segment, and potentially of an estimate of the disturbances, that determines the pressure to be applied at each instant of time to track the desired configuration, compensating for the external disturbances.

The desired configuration to be tracked is generally a function of the distal end of the segment. The control law developed in the following is to track the deflection at the distal end of a segment. From this, equivalent control laws can be developed in a relatively straightforward manner to track other desired variables, e.g. position at the distal end.

### 5.8.2 Concept of approach adopted

The approach adopted in this work to derive the control laws is generally referred to as energy shaping [207,208]. The concept for energy shaping can be interpreted considering the fact that, in general, systems without a control input present a set of minimum energy states that are generally stable configurations. In energy shaping, a control law is defined such that the closedloop behaviour of the system, resulting from the control input, presents an energy minimum at the desired system configuration. This is achieved by defining a control law such that the control input is a function of the system state that results in a closed-loop system behaviour with an equivalent minimum energy point at the desired configuration.

One of the main formulations of energy shaping is the so-called interconnection-and-dampingassignment passivity-based-control (IDA-PBC) [208], which was successfully developed over a decade ago. In IDA-PBC, the control law consists of two separate terms that correspond to the energy shaping and the so-called injection-damping control, and neglects dissipative forces. Recent work, however, indicates that using a control based on IDA-PBC but with less rigid structures can lead to control laws suitable to a wider range of systems [209, 210].

Following these recent advances, Dr Enrico Franco proposed a new control design that relies on the framework of IDA-PBC, and improves it to include adaptive estimation of friction forces, and a nonlinear dissipative term in the closed-loop system dynamics [206], which builds upon published work from the same author [211,212].

This adaptive, energy shaping control proposed in [206] is considered to be well-suited for the control of the fine-positioner. It can cope with the nonlinearities of the system, it is robust and suitable for operation in unstructured environments, it considers the system dynamics, and it is adaptive so it can compensate for disturbances and tolerate model inaccuracies.

It should be noted that alternative approaches to the control of soft robotic manipulators have also been recently proposed in the literature, although the problem remains largely open in general. A recent, relevant publication is [93], where a dynamic controller is proposed for operation of a soft robotic manipulator in a plane, which is based on a model of the robot consisting of a set of rigid links and elastic joints. The controller, however, is based on partial feedback linearisation, and is not adaptive. This can lead to issues with convergence in the case of external disturbances and significant nonlinearities in the system behaviour, and in general the linear control can be relatively slow to converge for typical gains required to avoid significant instabilities and overshooting. Another recent, relevant approach is presented in [213], where a model-free control for continuum robots is proposed, which is based on an adaptive Kalman filter. The resulting control in this case is adaptive so it can cope with external disturbances and does not require a robot model. This control, however, does not consider dynamic effects, and it can be relatively slow to adapt to changes in the external disturbances and in the system dynamics, especially if these are constantly varying. Furthermore, it relies on a linear approximation of nonlinear system behaviour, and its stability can be difficult to prove in a general case.

### 5.8.3 Control laws

The control laws for a segment of the fine-positioner were derived by Dr Enrico Franco by applying [206]. The derivation is based on the model presented in the previous section 5.7. In
this case, the system state is $\mathbf{q} \in \mathbb{R}^{3}$, the control input is $u \in \mathbb{R}$, and the external disturbances are $\delta \in \mathbb{R}^{3}$. The control problem therefore corresponds to an underactuated system, since only one control input is available to control the bending of multiple degrees of freedom, which approximate to the continuous bending of a segment of robot in practice.

In the derivation, it was assumed that the potential energy in the segment is primarily due to elastic potential energy, and thus gravitational potential energy was neglected, following [130]. It was also assumed that kinetic energy is primarily translational energy, and thus rotation could be neglected, also following [130]. Finally, any effects of hysteresis or damping were also neglected considering the behaviour observed in practice.

The potential energy then is

$$
\begin{equation*}
V_{k}=\frac{1}{2} \sum_{i=1}^{3} k_{i}\left(q_{i}\right)^{2} \tag{5.37}
\end{equation*}
$$

The kinetic energy is

$$
T=\frac{1}{2}\left[\begin{array}{lll}
\dot{q}_{1} & \dot{q}_{2} & \dot{q}_{3}
\end{array}\right] \mathbf{M}\left[\begin{array}{c}
\dot{q}_{1}  \tag{5.38}\\
\dot{q}_{2} \\
\dot{q}_{3}
\end{array}\right]
$$

where $M$ is the inertia matrix

$$
\mathbf{M}=\left[\begin{array}{ccc}
c_{1}+c_{2} \cos q_{2}+c_{3} \cos q_{3}+c_{4} \cos \left(q_{2}+q_{3}\right) & * & *  \tag{5.39}\\
c_{8}+c_{9} \cos q_{2}+c_{10} \cos q_{3}+c_{11} \cos \left(q_{2}+q_{3}\right) & c_{5}+c_{6} \cos q_{3} & * \\
c_{12}+c_{13} \cos q_{3}+c_{14} \cos \left(q_{2}+q_{3}\right) & c_{15}+c_{16} \cos q_{3} & c_{7}
\end{array}\right]
$$

where $c_{1}=L_{2}^{2}\left(m_{2} / 4+m_{3}+m_{4}\right)+L_{3}^{2}\left(m_{3} / 4+m_{4}\right)+L_{4}^{2} m_{4} / 4, c_{2}=L_{2} L_{3}\left(m_{3}+2 m_{4}\right), c_{3}=L_{3} L_{4} m_{4}$, $c_{4}=L_{2} L_{4} m_{4}, c_{5}=L_{3}^{2} m_{3} / 4+L_{3}^{2} m_{4}+L_{4}^{2} m_{4} / 4, c_{6}=L_{3} L_{4} m_{4}, c_{7}=L_{4}^{2} m_{4} / 4, c_{8}=L_{3}^{2} m_{3} / 4+$ $L_{3}^{2} m_{4}+L_{4}^{2} m_{4} / 4, c_{9}=L_{2} L_{3}\left(m_{3} / 2+m_{4}\right), c_{10}=L_{3} L_{4} m_{4}, c_{11}=L_{2} L_{4} m_{4} / 2, c_{12}=L_{4}^{2} m_{4} / 4$, $c_{13}=L_{3} L_{4} m_{4} / 2, c_{14}=L_{2} L_{4} m_{4} / 2, c_{15}=m_{4} L_{4}^{2} / 4, c_{16}=L_{3} L_{4} m_{4} / 2$.

The equations governing the behaviour of the system can then be formulated as

$$
\left[\begin{array}{c}
\dot{\mathbf{q}}  \tag{5.40}\\
\dot{\mathbf{p}}
\end{array}\right]=\left[\begin{array}{cc}
0 & \mathbf{I}^{3} \\
-\mathbf{I}^{3} & -\mathbf{R}
\end{array}\right]\left[\begin{array}{c}
\nabla_{q} H \\
\nabla_{p} H
\end{array}\right]+\left[\begin{array}{c}
0 \\
\mathbf{G}
\end{array}\right] u-\left[\begin{array}{l}
0 \\
\delta
\end{array}\right]
$$

where $\mathbf{q}$ are the three angles, as previously defined, $\mathbf{p}=\mathbf{M} \dot{\mathbf{q}}$ which are the momenta, $\mathbf{G}^{T}=$ $\left[\begin{array}{lll}1 & 1 & 1\end{array}\right]$ is the input matrix, $\mathbf{I}^{3}$ is the identity matrix of rank 3 , and $H=\frac{1}{2} \mathbf{p}^{T} \mathbf{M}^{-1} \mathbf{p}+V_{k}$ which is the open-loop Hamiltonian. The equations (5.40) elucidate the fact that the control problem corresponds to an underactuated system.

The control laws were initially developed to track a desired distal end deflection in the system (5.40). For a segment without external loads, a desired distal end deflection implies equal rotations in all three joints. This rotation is denoted by $q *$, which is a third of the tracked deflection. The derivation of the control laws was conducted by Dr Enrico Franco by applying [206], which resulted in the control law

$$
\begin{equation*}
u=\frac{k}{3}\left(q_{1}+q_{2}+q_{3}-q^{*}\right)-K_{p}\left(q_{1}+q_{2}+q_{3}-3 q^{*}\right)-K_{v}\left(\dot{q}_{1}+\dot{q}_{2}+\dot{q}_{3}\right)+\frac{1}{3}\left(\hat{\delta_{1}}+\hat{\delta_{2}}+\hat{\delta}_{3}\right) \tag{5.41}
\end{equation*}
$$

where $K_{p}$ and $K_{v}$ are control parameters that can be tuned, in an equivalent manner as the parameters in proportional-derivative control.

The control law (5.41) is composed of four terms. The first two terms are equivalent to the energy shaping terms to assign a desired closed-loop equilibrium in IDA-PBC; the third term is equivalent to the damping-injection control term in IDA-PBC; and the fourth term is the disturbance compensation term from [206]. In this work, the external disturbances can be estimated adaptively by using either time-delay-control (TDC) approach [214], or the immersion and invariance (I\&I) approach [215]. Using the TDC approach, the adaptive estimation is

$$
\begin{equation*}
\hat{\delta}=-\nabla_{q} H(t-\tau)+\mathbf{G} u(t-\tau)-\mathbf{M} \ddot{q}(t-\tau)-\dot{\mathbf{M}} \dot{q}(t-\tau) \tag{5.42}
\end{equation*}
$$

where $\tau$ is a time delay corresponding to the instant in the past $t-\tau$ when the system state and inputs are observed for the estimation of $\hat{\delta}$, and should generally be selected to be as small as possible while allowing sufficient time for the available feedback system to provide information regarding the system state.

Using the I\&I approach, the adaptive estimation can be obtained for various cases in terms of
the expected disturbances. For a case with constant disturbances $\delta$, the adaptive estimation is

$$
\begin{equation*}
\hat{\delta}=-\alpha \mathbf{p}+\int \alpha\left(-\nabla_{q} H+\mathbf{G} u-\hat{\delta}\right) d t \tag{5.43}
\end{equation*}
$$

where $\alpha$ is a parameter that can be tuned based on the application.

For a more general case where the disturbances correspond to a vertical force $F$ of unknown value acting on the distal end of the segment, this force $F$ generates disturbances that act as moments on the model joints, which are a nonlinear function of the state as

$$
\begin{equation*}
\delta=F L_{T} \mathbf{f}(q) \tag{5.44}
\end{equation*}
$$

where

$$
\mathbf{f}(q)=\frac{1}{L_{T}}\left[\begin{array}{c}
L_{4} \sin \left(q_{1}+q_{2}+q_{3}\right)+L_{3} \sin \left(q_{1}+q_{2}\right)+L_{2} \sin q_{1}  \tag{5.45}\\
L_{4} \sin \left(q_{1}+q_{2}+q_{3}\right)+L_{3} \sin \left(q_{1}+q_{2}\right) \\
L_{4} \sin \left(q_{1}+q_{2}+q_{3}\right)
\end{array}\right]
$$

The adaptive estimation of this force using I\&I then is

$$
\begin{equation*}
\hat{F}=-\frac{1}{L_{T}} \alpha \mathbf{f}^{T}(q) \mathbf{p}+\frac{1}{L_{T}} \int \alpha \mathbf{f}(q)^{T}\left(-\nabla_{q} H+\mathbf{G} u-\hat{F} L_{T} \mathbf{f}(q)\right) d t \tag{5.46}
\end{equation*}
$$

and thus the adaptive estimation of $\hat{\delta}$ is

$$
\begin{equation*}
\hat{\delta}=\hat{F} L_{T} \mathbf{f}(q) \tag{5.47}
\end{equation*}
$$

As in the previous case, $\alpha$ in (5.46) is a parameter that can be tuned. In general, increasing $\alpha$ leads to faster convergence of the adaptive estimation of the disturbances. However, the use of higher values of $\alpha$ can lead to the excitation of unmodelled dynamics in the system in practice. Thus, the selection of $\alpha$ depends on the accuracy in the model and the desired convergence rate, and is generally determined based on experimental exploration of possible values. It should be noted that in the control law proposed in this work, it is only necessary for $\alpha>0$ to ensure stability.

It should also be noted that the function $\mathbf{f}(q)$ in (5.46) corresponds to the type of external disturbances that are expected to act on the system. In the case of a vertical force acting on the distal end of a segment, $\mathbf{f}(q)$ is proportional to the moment generated by this force on the model joints. However, other general functions $\mathbf{f}(q)$ can be defined to adaptively estimate more general disturbances using I\&I.

Comparing both estimation approaches, the TDC approach is simpler and more general than I\&I since it does not involve any structural assumption on the disturbance except for the assumption that the variation of the disturbance over the period of time $\tau$ is bounded. However, it is more susceptible to noise since the update depends on acceleration. Conversely, I\&I tends to present less noise since the update is determined by velocity, and the parameter $\alpha$ offers more versatility for tuning to the desired system in practice. In addition, using I\&I it is possible to show local stability in general, and global stability in some cases where the disturbance is matched or where the system state is bounded, which applies to a significant number of systems such as the one in this work. Instead, using TDC it is only possible to show that the system behaviour is ultimately bounded.

It should be noted that the adaptive estimation of disturbances also applies to adaptively estimate uncertainties in the bending stiffness in the model. The effect of the disturbances $\hat{\delta}$ is a moment in the joints in the model, which acts on the system in an equivalent manner as a discrepancy between in the joint stiffness model and bending stiffness of the robot in practice. Thus, the adaptive control proposed in this section also applies to adapt and compensate for model discrepancies in terms of bending stiffness, which further increases the robustness of this control.

### 5.8.4 Results of performance

The performance of the control laws presented in the previous subsection was evaluated using simulations. These involved simulating the dynamics of the model of the fine-positioner segment described in subsection 5.7.2 when applying the control law (5.41). The objective of the control in the simulations was set to be that of reaching and holding a desired distal end deflection of
$\pi / 2 \mathrm{rad}$, which in a case without external loads corresponds to a rotation of $\pi / 6 \mathrm{rad}$ in all three model joints. The initial configuration of the model in the simulations was set to be horizontal, with zero deflection in all joints. The behaviour of the system was recorded by saving the rotations of the three joints as a function of time in the simulations.

The simulations were initially implemented both using a fixed time step in the integration of the system dynamics, and using a variable time step in the integration, which was determined by the solver. The fixed time step enables measuring the state of the system at a set of specific instants of time that can correspond to the sampling rate of typical sensing systems used in practice, to then input this system state to the control law at the specific instants of time. This closely matches practical scenarios where some limitations can exist in terms of sampling rate of the sensing used to measure the robot state. However, this results in relatively long simulation times. The simulations with a variable time step, on the other hand, enable faster simulations, but in these simulations the system state is always made available to the control law, so the specific sampling rate of the sensing used in practice cannot be exactly reproduced in the simulations. The results of system behaviour obtained in the initial simulations conducted with both methods were practically equal. The simulations with variable time step were then adopted, since they require significantly shorter simulation time. The results presented in this section were predominantly produced using variable time step simulations. The estimation of the external disturbances in the simulations presented in this section was performed using the I\&I approach since it is more robust to noise associated to acceleration, which is desirable, especially for the fixed time step simulations.

The simulations were first performed for a case without external disturbances, which implies that the model used in the simulation of the dynamics of the system matches the model used in the derivation of the control laws. The results of the simulations in this first case without external disturbances are shown in Figure 5.9, where the rotation in all three joint angles is plotted. The desired rotation angle in each joint, which corresponds to a third of the desired distal end deflection, is also plotted as reference. As can be seen in Figure 5.9, the system tends to the desired configuration without overshooting or oscillations, which confirms the correct performance of the control law derived in this work.


Figure 5.9: Results of performance of the control law in simulations corresponding to a case without external loads. The rotation in all three joints $\left(q_{1}, q_{2}, q_{3}\right)$ is plotted in different colours, as indicated in the legend. The desired joint configuration corresponding to a third of the desired distal end deflection is also plotted for reference.

Simulations with external disturbances corresponding to a vertical external force acting on the distal end of the fine-positioner segment were then implemented, for a control without disturbance compensation. In this case, the external force creates different disturbances in terms of moments in the three joints, and these disturbances are a nonlinear function of the system configuration. This is commonly referred to as a problem with unmatched disturbances, and is a particularly challenging problem in the control of underactuated systems such as the one considered in this work.

The results of the simulations in this second case with external force and unmatched disturbances are shown in Figure 5.10 (left). The results indicate that the behaviour of the system presents a significant deviation from the desired configuration, and the distal end deflection does not converge to the desired value. This is due to the external disturbances on the system that are not matched with the control.

Finally, simulations were performed for a case with an external vertical force applied to the


Figure 5.10: Results of simulations corresponding to a case with external disturbances in terms of an external force at the distal end, performed with a control law without disturbance compensation (left), and with the control law with disturbance compensation (right). The rotation in all three joints $\left(q_{1}, q_{2}, q_{3}\right)$ is plotted in different colours, as indicated in the legend. A reference joint rotation corresponding to a third of the desired distal end deflection is also plotted.
distal end of the segment, acting as a disturbance, and with disturbance compensation on the control. The results of these simulations with external disturbances and compensation on the control are shown in Figure 5.10 (right). As can be seen, the system converges to the desired configuration, confirming the correct performance of the control law. It should be noted that in this case, the rotations in the three joints do not tend to the same value. This is due to the fact that in the desired distal end deflection, the assignable equilibrium of the system with the external disturbances corresponds to different rotations in each joint, since the external force creates a different moment in each joint, and it is thus not possible to reach an equilibrium with equal rotations in all three joints. In other words, the configuration corresponding to constant curvature bending is not an assignable equilibrium as a consequence of the unmatched disturbances in the system. Still, the distal end deflection, which is the sum of the rotations in the three joints, tends to the selected, desired deflection thanks to the control law that compensates for the external disturbances, which is the control aim.

It should be noted that in all simulations shown in Figures 5.9, 5.10 (left), 5.10 (right) the transient time to reach an equilibrium configuration in the system is between 8 and 12 s . This is primarily determined by the parameters $K_{p}$ and $K_{v}$ used in the control. These parameters
can be adjusted to reduce the settling time in the future, although it should be noted that combinations of parameters that lead to markedly shorter settling time can also lead to overshooting. In addition, in robust control, the parameters $\alpha$ and $1 / \tau$ can also be increased to reduce the settling time, but as discussed in the previous subsection, higher values of these parameters can be unsuitable in practice, and could lead to oscillations due to the excitation of unmodelled dynamics, or due to noise generated in the estimation of the disturbances.

The time to reach the equilibrium configuration in Figures 5.9, 5.10 (left), 5.10 (right) also presents some differences between these figures. This can be attributed to the fact that in the cases with external disturbances, these correspond to a vertical force at the distal end that contributes to the bending of the segment of fine-positioner, and thus helps reach the desired deflection in a shorter time.

### 5.8.5 Discussion

The control law presented in this work can be used to reach and track a desired configuration in terms of distal end deflection, and performs correctly both in cases without external disturbances and with external disturbances. The control law (5.41) is relatively simple, and can resemble control laws that can be obtained with simpler approaches such as proportional derivative and integral controls. However, the adaptive energy shaping approach adopted in this work is relevant since it is a more robust approach that can cope with additional complexity. For example, it can be extend in a relatively straightforward manner to track a desired distal end position, and it can be generalised to more complex models or alternative models of other types of continuum robots.

Similarly, it is expected to be possible to generalise it to derive control laws for the full finepositioner. The generalisation of the work on control to the full fine-positioner operating in 3D space is expected to be performed in future work. Lastly, the control approach adopted in this work is also advantageous over simpler control approaches since it offers an adaptive feature, which is attractive considering that some of the parameters in the model such as the joint stiffness present some uncertainty.

### 5.9 Conclusions on control

The kinematics of continuum robots with bending and extension capabilities were studied, and closed-form solutions were derived. These solutions elucidate a redundancy in the kinematics of a manipulator with 6 actuation DOFs, and a condition on the reachable end-effector poses was also obtained. These kinematics studied apply to soft robotic manipulators similar to the FMA and to other continuum robots that can extend and bend describing piece-wise constant curvature arcs. However, these kinematics do not exactly match the final design obtained for the fine-positioner.

An alternative approach to the control of the fine-positioner using adaptive energy shaping control was then proposed in this work. A mechanical model was developed for it, and control laws were subsequently derived for operation of a segment of fine-positioner in 2D. The successful performance of these control laws was shown in simulations, which considered practical factors such as a limited sampling frequency in the sensing used to measure the system state.

Experiments are planned to be conducted in future work to evaluate the performance of the control law in practice, where discrepancies in the model are expected, as well as potential issues with noise, and limited sampling frequency in sensing. These experiments can also be used to select between the TDC and I\&I adaptive estimation based on performance in practice. In addition, the experiments are expected to serve to illustrate the work. The experiments are not included in this thesis due to time constraints, and due to difficulties in the fabrication of segments of the fine-positioner that are sufficiently durable in practice. The issues with reliability in the fabricated devices, and the systems complementing the fine-positioner in the overall inspection system to perform on-wing operations, are described in the next chapter.

## Chapter 6

## Robotic System for On-wing

## Inspections

The most important and challenging part in the development of the robotic system for onwing inspections is the development of the fine-positioner, which is presented in the previous chapters. This fine-positioner needs to be complemented by a set of other components to insert it into the engine region of interest, ensure a correct probe deployment, and navigate inside the engine. In this chapter, the selection of concepts for these complementary components is presented, together with a justification of the design process. In addition, the fabrication and intended operation of these different components of the robotic system are also described, and practicalities of their application to on-wing inspections are discussed.

The selection of a solution concept for the gross-positioner is presented in section 6.1, together with its intended operation; the electronics for fine-positioner are outlined in section 6.2; a path planner is described in section 6.4; deployment mechanism solutions are summarised in section 6.5; and possible feedback system are introduced in section 6.6. In addition, the fabrication of both gross-positioner and fine-positioner is described in section 6.3; and the assembly of the different systems is considered in section 6.7.

The work on path planning presented in this chapter is an edited version of the work published in:

- F. Liu, A. Garriga-Casanovas, R. Secoli, and F. Rodriguez y Baena. Fast and Adaptive Fractal Tree-Based Path Planning for Programmable Bevel Tip Steerable Needles. Robotics and Automation Letters, 1.2, pp. 601-608, 2016. © 2016 IEEE.

It should also be noted that part of the details regarding the fabrication of soft robots described in this Chapter were learned during a placement of this author at the Suzumori Endo Robotics laboratory of Tokyo Institute of Technology. Thus, the fabrication method was not developed entirely by this author. Additional details about the fabrication of soft robots learned during the placement are presented in the conference paper:

- A. Garriga-Casanovas, A. A. M. Faudzi, T. Hiramitsu, F. Rodriguez y Baena, K. Suzumori. Multifilament Pneumatic Artificial Muscles to Mimic the Human Neck. IEEE International Conference on Robotics and Biomimetics, 2017.

However, the work reported in this paper is mostly relevant to fabricate devices including PAMs, which is not central in this work. Thus, the work reported in this paper is not presented in this thesis.

### 6.1 Gross-positioner

As previously noted in Chapter 1, the gross-positioner must perform the insertion into the region of interest, which in the reference case defined in subsection 1.1.3 involves negotiating the entry route shown in Figure 1.3. Considering the literature review in Chapter 2, the study of CTRs in Chapter 3, and the work on design and control of soft robotic manipulators in Chapters 4, 5, the robot concept selected for the gross-positioner is a non-annular CTR composed of three tubes, which carries the fine-positioner attached at its distal end, as illustrated in Figure 6.1. This robot concept can achieve lengths over 1 m with diameters of 6 mm or less, as previously noted in Chapter 2, and it does not present the issues with torsion of the tubes of annular CTRs, that limit their lengths and maximum curvatures. In addition, using telescopic deployment of the tubes, non-annular CTRs can negotiate the obstacles shown in Figure 1.3, and reach the


Figure 6.1: Concept of gross-positioner as a non-annular CTR composed of three tubes, together with the fine-positioner attached at its distal end. The assembly of both devices is illustrated in a configuration similar to that of the resulting system after insertion into the chamber with the HPC discs.
engine region of interest. Furthermore, a non-annular CTRs can provide a working channel of a few millimetres in diameter to accommodate any wires and elements for the payload, as well as tubes to pressurise the fine-positioner, and any elements from a feedback system.

The non-annular CTR gross-positioner is intended to work together with the fine-positioner to reach the engine region of interest. In the reference case previously defined in subsection 1.1.3, the proposed operation mode to advance inside the engine involves a combination of the mobility of both devices so that they can successfully reach chamber with the HPC discs. The insertion into the chamber with the HPC discs consists of four parts, described in the following.

The first part of the entry route simply involves entering through a narrow conduct with constant curvature, which can for example be the conduct of a temperature probe for the turbine, and this guides both fine-positioner and gross-positioner via contact forces. Thus, the gross-positioner only needs to act as a passive rod with some flexibility that advances the fine-positioner.

The second part of the route is the turn in the turbine chamber, which is illustrated in Figure 6.2. This intended to be performed with telescopic deployment of two pre-curved tubes of the non-annular CTR, such that the device initially curves towards the rear end of the engine, and then it curves in the opposite direction, defining an $S$ shape that brings the base of the fine-positioner to the correct pose for insertion into the gap between the shafts. The first turn in this telescopic deployment is intended to be performed by advancing the entire CTR,
designed with an outer tube that has a stiffness higher than the other two tubes, and thus dominates the geometry of the device, as shown in Figure 6.2 (top). Then, this outer tube is expected to remain static, and the other two tubes of the CTR are expected to advance to perform the second turn, shown in Figure 6.2 (middle), and thus point the distal end of the gross-positioner (the proximal end of the fine-positioner) near the desired pose. For this, the middle tube of the CTR should be pre-curved, whereas the inner tube should be straight, so that the combination of the two create the desired curvature of the CTR in this second turn. Then, using the dexterity of the fine-positioner, its distal end is expected to be pointed into the gap between shafts, potentially with aid from the 3 DOFs provided by the non-annular CTR to compensate for deviations. Finally, the inner tube of the CTR is intended to advance while the other tubes remain static, as shown in Figure 6.2 (bottom), in order to advance the fine-positioner into the gap between shafts, while the fine-positioner and remaining 2 DOFs of the gross-positioner are used to correct for any deviations. In this regard, the fine-positioner is only expected to be attached to the inner tube.

Once the fine-positioner is at the beginning of the gap between shafts, the third part of the route starts, which is the advancement through the gap between the shafts to reach the chamber of interest. In this part, the gross-positioner and fine-positioner are intended to work together to advance. The fine-positioner is intended to perform a 'snaking' motion shown in Figure 6.3, and originally proposed in [104]. This motion can be achieved by applying control inputs that impose a sequential bending of two or more segments of devices such as the fine-positioner or the FMA, following the sequence shown in Figure 6.3. The result of this motion, which resembles that of a wave, is an advancement of the device in narrow spaces thanks to the contact with the surrounding structures. Then, by combining this 'snaking' motion of the fine-positioner together with the advancement of the inner tube from the gross-positioner, the combined inspection system is intended to advance through the gap between shafts and overcome any frictional forces. Dynamic frictional forces are expected to be low, since the 'snaking' motion of the fine-positioner avoids dynamic friction, and thus the only dominant dynamic frictional forces are those associated to the inner tube of the gross-positioner advancing. The fine-positioner is expected to be composed of three segments, so the sequential bending


Figure 6.2: Insertion procedure for gross-positioner, corresponding to initial insertion of device with three tubes (top), advancement of middle and inner tube while leading the outer tube static (middle), and advancement of inner tube only (bottom).
should generally be applied to all three segments to achieve the 'snaking' motion. However, it can also be applied to two segments while another segment is used for other purposes such as steering. Lateral deviations from the desired, straight trajectory can occur while advancing through the gap between shafts. Using the capability of the segments of the fine-positioner to bend in any direction, including the lateral direction, is expected to be possible to correct for these deviations, and steer the robotic system in the desired direction.

After advancing through the gap between shafts, the robotic system needs to enter the chamber with HPC discs through a hole, which is the fourth part of the access route. For this manoeuvre, the dexterity of the fine-positioner is expected to be used to steer the distal end into the hole. Then, a combined motion of the gross-positioner advancing, and the proximal segments of the fine-positioner performing the 'snaking' motion in Figure 6.3, is intended to be used to advance the fine-positioner into the chamber with the HPC discs. This fourth part of the access route ends once the entire fine-positioner is inside the chamber, and the distal end of the grosspositioner is beginning to enter the chamber. It should be noted that the intended method to access the chamber with the HPC discs described in this subsection needs to be tested in practice to determine its viability.

Once the robotic system is inside the chamber, the fine-positioner is intended to be used to deploy the probe on the first disc encountered when accessing the chamber through the hole in the shaft. After inspecting this first disc, the robotic system must pass through the gap between the first disc and the shaft to reach the second disc. This manoeuvre can be performed by first directing the distal part of the fine-positioner into the gap, using its dexterity. Then, a combination of advancement of the gross-positioner together with a 'snaking' motion of the fine-positioner are intended to be used to advance the robotic system through the gap between the disc and shaft until the fine-positioner is passed the first disc in the direction towards the front of the engine. At this point, the fine-positioner is intended to be used to deploy the probe on the second disc, relying on its dexterity. After inspecting this second disc, the operation to reach and inspect the following disc is intended to be repeated, until all subsequent discs are inspected. As in the previous case, this manoeuvre to reach all discs needs to be tested in practice to determine its viability.


Figure 6.3: Schematic of snaking motion to advance in a narrow space between two walls using a device composed of two FMA-type devices. The schematic illustrates the sequence of bending (from left to right) in a robot composed of two segments to advance upwards, although an analogous sequence applies to advance in a robot composed of additional segments. Image courtesy of [104].

The retraction of the robotic system is intended to be equivalent to the insertion, but performed in reverse. Retraction is simpler since the gross-positioner is already inserted through the route, so deviation from the desired path is not an issue. In this regard, the 'snaking' motion may not be required if friction is not significant, but can be used in case friction is an issue.

The design of the gross-positioner is thus simple. The device is a non-annular CTR composed of three tubes, which needs to be designed for each specific application. For the reference case defined in subsectino 1.1.3 and addressed in the previous paragraphs, only the outer and middle tubes are curved to achieve the desired resulting $S$ shape. The curvature of the tubes to achieve this resulting geometry can be determined using the analysis in Chapter 3, and this is relatively simple case. The inner tube is straight, so it only adds stiffness to the robot, and then the curvatures and stiffnesses of the other two tubes need to be selected to achieve the resulting geometry in equilibrium, and not exceed the maximum strain of nitinol. The process to fabricate this gross-positioner is described in section 6.3.

The control of the insertion of the gross-positioner can be performed with linear actuators for high accuracy. However, since this application is relatively simple, the insertion of the tubes
could also be performed by hand, by simply attaching grippers to each tube. It should be noted, however, that the turn in the turbine chamber to enter into the gap between the shafts can be difficult. The dexterity of the fine-positioner can help in this insertion into the gap between shafts. In addition, the three DOFs of the gross-positioner can also be used to aid in directing the fine-positioner into the gap between shafts. In the case that the DOFs of the gross-positioner are used to aid in the manoeuvre, the theoretical framework in Chapter 3 can be used to determine the geometry of the gross-positioner for each control input in terms of tube insertion. It should also be noted that, even though relative rotation of the tubes of the gross-positioner is not possible due to the non-annular cross section, the rotation of the entire gross-positioner could also be used to help align it.

### 6.2 Fine-positioner

The design selected for the fine-positioner is that presented in Chapter 4. This fine-positioner is controlled by pressure inputs to its chambers, which are determined by a computer that uses the control introduced in Chapter 5, in combination with possible path planning algorithms described in section 6.4, and with information from a feedback system introduced in 6.6. The systems to communicate with the computer determining the desired pressure values, and regulate the pressure inputted to the fine-positioner, are described in this section.

Proportional pressure regulators were used in this work to impose the desired pressure values to the chambers of the fine-positioner via tubes, which are described in section 6.3. The pressure regulators selected here are the PRE1-U08 supplied by AirCom Pneumatic GmbH (Ratingen, Germany). These operate at a pressure between 0 and 8 bar. The desired pressure values are imposed on the pressure regulators via analog voltage inputs.

Electronic hardware to interface between these pressure regulators and the computer was developed with advice from Dr Enrico Franco. The electronics consist of a mictrocontroller that receives commands from the computer, converts them to digital signals with the appropriate protocol, and outputs them to a set of digital to analogue converters (DAC) that then transform


Figure 6.4: Design of the printed circuit board developed to integrate the electronics.
the signals into analogue voltages, which are finally inputted to the pressure regulators. The microcontroller selected for this application is the mbed LPC 1768 manufactured by ARM ltd (Cambridge, UK), and the DACs are MCP4922 manufactured by Microchip Technology Inc. (Chandler, US). In order to simplify the systems, the communication between computer and microcontroller was implemented through the USB connector that powers the mbed using serial communication, and more specifically using remote procedure calls (RPC) as the communication protocol. The commands are converted at the mictrocontroller and outputted using serial peripheral interface (SPI) to the DACs.

A printed circuit board ( PCB ) was developed to integrate all the electronics and thereby simplify the practical use of the electronics. The design of the PCB is shown in Figure 6.4, and includes the microcontroller, six DACs, and a set of ports where the pressure regulators and power supplies can be connected.

The PCB was ordered from Newbury Electronics ltd (Newbury, UK), and the electronic components from RS Components ltd (Corby, UK). The hardware was subsequently assembled and tested. The code to interface in RPC between computer and microcontroller was implemented


Figure 6.5: Electronic systems assembled on the PCB and connected to the computer to confirm their correct performance.
in $\mathrm{C}++$ on the mbed, and in Matlab on the computer in order to facilitate a possible future integration using the robotics operating system (ROS) via the Matlab Robotics Toolbox. The code to output SPI commands from the microcontroller to the DACs was also implemented on the mbed after consulting the relevant datasheets. The complete electronic system with hardware and software, illustrated in Figure 6.5, was finally tested, verifying its correct performance.

### 6.3 Fabrication

The fabrication of soft robots, and in particular the fine-positioner, is presented in the next subsection 6.3.1. The fabrication of the gross-positioner is presented in subsection 6.3.2.


Figure 6.6: Fabrication process for general soft robots, corresponding to cast silicone structure (a), structure with added outer fibres (b), and structure with supply tubes and separate end caps before sealing (c).

### 6.3.1 Fine-positioner

Soft robots with fluidic actuation, such as the fine-positioner, are generally fabricated in three main steps. First, the rubber structure is cast. Second, the outer fibres are winded on the outer surface. And third, the ends of the segments are sealed while including pressurisation tubes, and the outer fibres are affixed to the rubber using an additional layer of rubber. This process is illustrated in Figure 6.6, and details about these three steps are presented in the following subsections. First, however, the moulds used for casting are introduced in subsection 6.3.1.

## Moulds

Moulds for soft robots can be manufactured using a variety of methods depending on the desired geometry. In the case of the fine-positioner, the primary mould used was fabricated using additive manufacturing. More specifically, the mould was fabricated using laser sintering of a titanium alloy powder. The final mould is shown in Figure 6.7 (a) and (b). As can be seen, the mould is composed of two parts corresponding to the inner chambers and the outer surface, which facilitates demoulding. The accuracy in this mould was estimated to be near $50 \mu \mathrm{~m}$.

The fabrication of the same mould using additive manufacturing of plastic was also explored. However, the lower accuracy of the 3D printer used, a FormLabs 2 (Somerville, USA), and the issues encountered with warping of the thin rods and surface roughness, resulted in moulds that were generally unusable due to the low accuracy and defects they created. Thus, plastic


Figure 6.7: Primary mould used for casting composed of two parts made with additive manufacturing, shown in (a) and (b), and alternative mould composed of a set of machine parts that are partially assembled in (c), where the three metallic rods are assembled to an end piece, and fully assembled in (d).
moulds were discarded.

An alternative mould made of machined parts that were subsequently assembled was also created. This is shown in Figure 6.7 (c) in a partly assembled configuration, and in Figure 6.7 (d) in a fully assembled configuration. As can be seen, this mould is made of three metallic rods corresponding to the chambers, which are held in position by two pieces at the ends, which are cut using wire electrical discharge machining. An outer tube, also held in position by the end pieces, defines the outer wall, which is shown separately in Figure 6.7 (c). This mould is simpler to manufacture and is a viable alternative to the metallic mould created by additive manufacturing. However, it presented some issues with leaking of the rubber during casting, and the accuracy in this mould was similar to that of the one created by additive manufacturing, so the mould created by additive manufacturing was generally preferred.

## Casting

The fabrication of soft robots generally begins by casting the rubber in the mould, to create a rubber structure such as that shown in Figure 6.6 (a). In the fabrication of the fine-positioner, the fibre for the central rod was also added at the centre of the mould before adding the liquid rubber, and was held in place to embed it.

The rubber was prepared by first mixing the two components that constitute it in the specified proportions, then degassing it by placing it into a vacuum chamber for 8 minutes, and finally pouring it into the mould. The mould with the cast rubber was then left at room temperature for 8 hours until it cured. The mould was then separated into its two parts, and the rubber structure was removed.

The process of removing the rubber was found to be delicate, and in multiple occasions the rubber corresponding to the partition walls or outer wall broke. In addition, despite the degassing process, micro bubbles were found to remain in some cases. Considering these factors, and the accuracy in the mould, it was considered to be difficult to cast wall thicknesses lower than 0.6 mm in a reliable manner.

## Outer fibres

Once the rubber structure is created, the outer fibres can be added, as shown in Figure 6.6 (b). In the fabrication of the fine-positioner, the fibres were manually wound, creating a double helix arrangement with an approximate pitch between 1-2 mm.

## Pressurisation tubes and sealing

The final step of the manufacturing is adding the tubes supplying the pressure to the chambers of the soft robot, sealing these chambers, and securing the outer fibres in their position. In the fine-positioner, the tubes used were EXLON PFA Micro-Fluoro Resin Tubing, manufactured by IWASE Co. (Kanagawa, Japan), and distributed by Elematec Czech s.r.o. (Prague, Czech Republic). Two tube sizes were used: the first with an OD of 0.3 mm and an ID of 0.1 mm , and the second with an OD of 0.6 mm and an ID of 0.4 mm . The thinner tubes were used to deliver the pressure to the chambers of the distal segments of the fine-positioner, passing through the chambers of proximal segments to reach the proximal end of the fine-positioner and extend out of it a few centimetres. The wider tubes were used to pressurise the chambers of the proximal segment. Wider tubes were also attached to the thinner tubes at the proximal end of the


Figure 6.8: Example of preliminary fabricated segment (shown on the right) with the tubes to pressurise it attached to its proximal end, and the tubes to pressurise another distal segment passing through it. The distal segment is also shown on the left, which represents the segment where the tubes passing through the proximal segment should be attached. The composition aims to illustrate the concept of stacking the segments serially, and passing the tubes to pressurise the distal segments through the proximal ones.
fine-positioner to supply the pressure through the length corresponding to the gross-positioner. This helped reduce the pressure drop by providing a wider conduct to supply the pressurised air throughout most of the length of the supply tubes, which corresponds to the length of the tubes passing through the gross-positioner.

The micro tubes were placed inside the corresponding chambers, and rubber was applied to the ends of the segments to seal them. The proximal segment was fabricated first, with the tubes corresponding to the distal segments passing through it. At the same time, a layer of rubber was also applied to the outer wall to affix the outer fibres, with an additional thickness of approximately $100 \mu \mathrm{~m}$ to $200 \mu \mathrm{~m}$. The result was that the minimum wall thickness that could be manufactured reliably was near 0.8 mm . An example of fabricated segment with the tubes to pressurise it attached at is proximal end, and the tubes to pressurise another distal segment passing through it, is shown in Figure 6.8.

Once the additional rubber of the sealing points cured, Loctite ${ }^{\circledR} 401^{\mathrm{TM}}$ (Dusseldorf, Germany) was applied at the region of contact between the micro tubes and the rubber to improve the bonding, and prevent leakage. In some cases, the device was then pressurised to test it, and if


Figure 6.9: Example of preliminary fabricated segment without pressure (left), and pressurised (right). The excess material from the casting is attached at the distal end of the segment, which needs to be removed.
leakage occurred, additional Loctite ${ }^{\circledR} 401^{\mathrm{TM}}$ was used to improve the sealing. An example of a single fabricated segment being tested to confirm that it can withstand the pressure without leakage is shown in Figure 6.9.

## Discussion

This fabrication process described in the previous subsections can be successfully used for the fabrication of the fine-positioner developed in this work. The resulting bonding of the central fibre with the silicone, however, was found to be relatively weak, and in some cases this central fibre tended to slide relative to the silicone. This can create friction forces that tend to cut the silicone, eventually breaking the inner structure of the device and rendering this unusable. As a consequence, the device fabricated using the method described in the previous subsections presents a low reliability, and in the experience by this author, it can only work for a few minutes before it breaks.

The reliability needs to be improved in future work. Possible solutions to be explored in the future are the use of fibre materials and adhesives that improve the bonding between the central fibre and the rubber; widening the region of rubber around the central rod, which can contribute to the LCRS while having a minimal impact on the area of the chambers, and thus have a minimal impact on performance; and using a miniature chain as central rod that is mechanically locked to the silicone.

### 6.3.2 Gross-positioner

The gross-positioner can be fabricated by following an equivalent method as that described in Chapter 3 for annular CTRs, but by initially creating a non-annular cross section. Circular tubes can be first purchased in the desired sizes from a supplier such as Nitinol Devices and Components Inc. The non-annular cross section can then be created by inserting a mandrel into the tubes, heating them to $550 \mathrm{deg} C$ as in Chapter 3, and mechanically deforming them into the mandrel, for example using pliers, to create an elliptical cross section. An elliptical cross section is an effective design of non-annular cross section that prevents relative torsion of the tubes and is relatively simple to manufacture.

Once the tubes are elliptical, the curvature of the two tubes that need to be pre-curved can be created by following a process of heating and quenching, as described in Chapter 3. The three tubes can then be arranged concentrically to create the gross-positioner. Finally, linear actuators can be attached to the tubes for high accuracy insertion control, or the tubes can be inserted manually into the jet engine. The fabrication of the gross-positioner using this method, and its practical application, are expected to be performed in future work, as described in Chapter 8. Major technical obstacles are not expected in the fabrication. The main expected issues are practicalities such as fabricating fixtures of the required size that can be used to set the shape of the tubes and withstand 550 degrees Celsius, and obtaining access to furnaces of the required size for the tubes which are over 1 m long. The performance of the resulting gross-positioner in terms of accuracy and viability is expected to depend on the accuracy of the fabrication, particularly on the process of creating an elliptical tube cross section. However, this is expected to be relatively accurate, and it can be improved by iteration.

### 6.4 Path planning

The path planning capability can be helpful when navigating inside cluttered environments such as a jet engine. In the reference on-wing inspection case defined in subsection 1.1.3, this is particularly relevant for the insertion of the inspection system into the gap between shafts, for
the advancement through the gap to the chamber with the HPC discs, and for the operation of the fine-positioner with the end-effector in the vicinity of the components that need to be inspected without collisions.

The path planning problem was therefore considered in this work. The path planning problem to navigate inside a jet engine presents similarities with the path planning problem for MIS, and particularly for steerable needles navigating in cluttered environments such as brain or liver. Considering these similarities, a collaboration with a post doctoral researcher investigating path planning algorithms for nonholonomic steerable needles, Dr Fangde Liu, was started to develop path planning algorithms for nonholonomic systems.

### 6.4.1 Concept for path planner

The path planner developed in this work aims to suit the requirements of navigating inside jet engines, and of MIS, which present similar challenges. The most prominent of these challenges are the complex and numerous obstacles in the environment; the confined spaces where the robot needs to operate; and the need for a path planner that can replan a path in real-time to compensate for unexpected variations in the robot configuration, or in the case of MIS, in the environment.

The concept for the path planner proposed here, in collaboration with Dr Liu, is to parallelise the path planning problem, and harness the power of the graphics processing unit (GPU) to solve it in real time. The path planner proposed first generates a significant number of paths covering the domain of interest without considering the obstacles in the environment, and then checks their viability in parallel, to finally select the path that minimises a given cost function, which can be freely selected for each application. This path planner relies on fractal theory to generate the paths in an efficient manner, and create a data structure that enables efficient parallel path planning on the GPU. The resulting data structure has a recursive structure, is adaptable in size, is constructed procedurally, is invariant, and allows for a dense coverage of the entire domain. This is conceptually illustrated in Figure 6.10, where a tree of paths generated with the proposed approach is shown.


Figure 6.10: Generic tree illustrating the concept for the path planner. The density of paths corresponds to the space coverage that can be achieved in real-time with modern GPUs. © 2016 IEEE.

The generated cache of paths can then be analyzed in parallel using the GPU to determine the most suitable path in a fraction of a second. The ability to cope with nonholonomic constraints, as well as constraints in the space of states of any complexity or number, is intrinsic to the path planner proposed, rendering it highly versatile. Details about the proposed path planner are presented in Appendix A.

The proposed path planner has three main advantages with respect to imaged-based algorithms: (i) it works directly with voxels, optimizing computational performance; (ii) it is capable of real-time replanning with a bounded computational time; (iii) it can be used regardless of the number or complexity of the obstacles, rendering it robust and versatile, with a high success rate compared to other path planning algorithms. It should be noted that jet engines have a known geometry, but the obstacles can be complex and numerous, and the robot being inserted has a certain degree of compliance so it can deform in a different manner from the predicted one, requiring path replanning in real-time. In this regard, these features of the path planner are attractive for jet engine navigation.

It should be noted that the original idea for this path planner was proposed by Dr Fangde

Liu. The contributions of this author to this path planning work are: the formulation of the theoretical framework for the work, the review of literature, help outlining and clarifying the final algorithm, help in the selection and generation of the final results, and writing the paper.

### 6.4.2 Results and illustration of path planning capability

The result of the path planning work is a fast and robust path planner for nonholonomic systems that can work in real-time, which is detailed in Appendix A. The capability of the path planner is also showcased in Appendix A, in a scenario requiring real-time path planning for a steerable needle that must navigate through a complex liver structure to reach a given target. An illustrative result from this work is shown in Figure 6.11, which corresponds to a particular case where the steerable needle is required to reach a target that moves, replanning the path as it advances. As can be seen in Figure 6.11, the path planner initially finds a first path and, as the target moves, it repeats the path planning process to find an alternative path online.

This application is illustrative of the speed of the path planning algorithm proposed here. The implementation used in this work employed Matlab 2014b (Mathworks Inc.), Linux Ubuntu 64bit, and was executed on an Intel CORE i7 CPU @ 3.2Ghz with a GTX TITANX from NVIDIA corp., with 3072 threads, a 1 GHz base-clock and 12 GB of memory, which has an approximate computing power of 7 TFLOP and supports CUDA 7.5 API [216]. With this implementation, it was possible to explore 300 million paths per second. More details can also be found in Appendix A.

### 6.4.3 Conclusions on path planning

A path planner for nonholonomic systems was developed in this work. The algorithm was applied to a nonholonomic steerable needle, which showcases its performance. The path planner is versatile, and represents a capability that can be used for navigation inside jet engines using the inspection system. The application of the path planner, together with other control laws,


Figure 6.11: Simulation results of online path replanning during needle insertion into liver, with target moving along the red arrow direction. The best initial path is shown in green, and and subsequent best paths in red, blue, cyan, and magenta. © 2016 IEEE.
to a jet engine is expected to be performed in future work, together with implementation of the complete inspection system in jet engines.

### 6.5 Deployment mechanism

The purpose of the deployment mechanism is to ensure a correct contact between the probe and the inspected component, passively compensating for any misalignments, and allowing for a correct inspection. In addition, in the case of flexible probes, it should facilitate the probe insertion into the engine. The deployment problem was considered for both flexible and rigid probes, and suitable solutions were identified. These are summarised in subsection 6.5.1 for flexible probes, and in subsection 6.5.2 for rigid probes.

### 6.5.1 Deployment mechanism for flexible probes

The deployment mechanism together with any flexible probe used must fit through the entry port, which in the reference case previously defined in subsection 1.1 .3 is 6 mm ID, and must be compatible with the fine-positioner and the rest of the inspection system. If the flexible probe is larger than the entry port, it must be furled or redesigned so that it can be inserted. The flexible probe must then be deployed on the component compensating for misalignments, compensating for any shape mismatching, ensuring a correct contact, and in the case that a couplant medium is used, removing any air bubbles in it.

The design of the deployment mechanism for flexible probes can therefore be divided in three parts: furling and unfurling, conforming while removing air bubbles, and orientation corrections. The analysis of each part and solutions proposed are summarised in the following three subsections.


Figure 6.12: Concepts of solutions for furling and unfurling corresponding to (a) a tube with a slit, and (b) tapered probe.

## Furling and Unfurling

Four main options were identified for furling and unfurling. The first option involves placing the probe inside a tube with a slit so that the probe can extend and retract when rotated, as illustrated in Figure 6.12 (a). The second alternative is to design a tapered probe towards the base and place it inside a tube, so that it elastically unfurls when pushed outside, and is furled by the reaction forces from the tube walls when pulled inside, as shown in Figure 6.12 (b). The third option is to attach a backing on the probe that elastically furls it, and to employ a balloon to unfurl it. The last alternative is to employ SMA antagonistically or with elastic restoring to generate the furling and unfurling. However, these last two options are considered secondary due to implementation difficulties they entail.

## Conforming on component

In order to force any flexible probe to conform on the surface of the component, forces and moments need to be applied on it. The surface of the component can be used to compensate for shape mismatching, and in general the application of a distributed load at the back of the probe can help make it conform. Considering the possible forces available, four potential solutions were identified.

The first alternative is to employ a balloon to apply a pressure at the back of the probe, forcing it to conform, as shown in Figure 6.13 (a). The second option is similar, but placing a foam at the back of the probe, as illustrated in Figure 6.13 (b). In this manner, by pressing at a single point the back of the foam, this can help distribute the force on the probe. A third option is to


Figure 6.13: Concepts of solutions to force the probe to conform on the component corresponding to (a) a balloon at the back of the probe, (b) a foam at the back of the probe, (c) pressing at a single point at the back of the probe, and (d) only the probe held with significant base bending.
press at a discrete number of points at the back of the probe by means of a simple structure, as depicted in Figure 6.13 (c). Considering the main deformation modes of the probe, it can be possible to force it to conform on a suitable component. A last option is to simply place the probe on the component and force it to bend near the base by holding it at specific orientations, as in Figure 6.13 (d), in order to create a moment that aids in conforming on concave surfaces. This option is inspired by [217], which tackles a similar problem and reports successful results.

Bubbles are expected to be removed by the load applied at the back of the probe in the solutions employing a balloon, foam or pressing at discrete points. In the design that simply involves placing the probe on the component, sliding the probe while feeding additional couplant as it slides is a potential strategy to remove air bubbles. However, this needs to be tested for each application, probe, and final inspection system.

## Orientation corrections

Forces and moments together with some degree of mobility between the probe and the finepositioner are required to correct for orientation misalignments. These misalignments refer to
differences between the direction of a vector perpendicular to the surface of the component and a vector perpendicular to the surface of the probe, which should be aligned in an ideal deployment. Reaction forces from the component at misaligned probe angles can generate the stabilising moment necessary to correct for orientation misalignments. In this regard, two main solutions were identified.

The first solution is to place a pivoting point between the fine-positioner and the probe that enables the required degree of mobility. This pivoting point can be mechanical, or simply an elastic element such as rubber or a spring. The range of motion from the pivoting point, and its stiffness, should be selected based on the expected misalignments for each application, which depend on the final fine-positioner performance.

The second solution is to exploit the elasticity of the probe in the region close to the base to provide the required degree of mobility. The reaction forces can then provide the stabilising moment to make the probe align. In this manner, the probe deployment is expected to intrinsically correct for small orientation misalignments provided that some mobility is allowed by the flexible probes proximal part.

The majority of deployment mechanism designs require a structure to transmit force from the fine-positioner to the back of the probe. A rigid structure as in Figure 6.13 (b)-(d) provides sufficient force, but presents a limited tolerance to longitudinal misalignments if a backing such as a balloon or foam is used. Considering Figure 6.14 (a), it can be seen that a deployment mechanism without backing and with a rigid structure can pivot around the tip of the structure to tolerate orientation variations that correspond to a rotation along an axis perpendicular to the plane of symmetry of the probe, although it leads to lift-off at the proximal part of the probe. Designs with backing and a rigid structure as in Figure 6.14 (b) cannot pivot, and instead the end of the structure must slide relative to the backing if orientation variations occur, which also leads to lift-off and limits the admissible misalignments.

A flexible structure presents a higher tolerance to misalignments since it does not require pivoting in the presence of orientation variations. The deformation of the structure also leads to some variations in the position of the structure end relative to the probe, although the sliding


Figure 6.14: Schematic diagram of probe deployment with a rigid structure in (a) a mechanism with no backing on the probe, and (b) a mechanism with backing such as a balloon or foam. The probe is depicted in orange, its base in black, the structure in dark gray, any probe backing in blue, and the component in light gray.
is considered lower than in a rigid structure. Thus, a flexible bar from the fine-positioner to the back of the probe (or foam) together with a plaque at the end to distribute the force was concluded the most suitable solution. The degree of misalignment tolerable for each deployment mechanism design needs to be investigated, as reported in the next subsection.

### 6.5.2 Deployment mechanism for rigid probes

The deployment mechanism for rigid probes is simpler than that for flexible probes. It only needs to compensate for misalignments, and ensure that any couplant medium used between the probe and the component has no bubbles. Thus, the solutions in this case are focussed on orientation corrections, and removal of bubbles.

As in the previous subsection, reaction forces from the component can also be used to create the moment that corrects for probe misalignments. In this case, the probe does not have any flexibility that can be exploited. Thus, the main solution identified here is to use a pivoting point between the probe and the fine-positioner that provides the required degree of mobility, as schematised in Figure 6.15, in an equivalent manner as in the previous subsection. Then, in the case of misalignment in the approximation by the fine-positioner, the contact forces from the component make the probe realign, for moderate misalignments. As before, this pivoting point can be mechanical or simply an elastic element. In the realignment due to contact forces, the probe can be expected to slide to some extent relative to the component until it reaches


Figure 6.15: Schematic of solution involving a pivoting point between the probe and the finepositioner to provide a degree of mobility that tolerates misalignments together with reaction forces from the component.
the correct orientation. The resulting position can present small errors, which are ultimately due to errors in the initial approximation from the fine-positioner, and these are expected to be compensated by the fine-positioner.

It should be noted that in the case of a rigid probe there is no need for a structure to transmit forces from the fine-positioner to the probe. The pivoting point can already transmit the required forces. The removal of bubbles in this case of a rigid probe is expected to be performed by sliding the probe on the component. However, the assessment of this solution needs to be considered for each application, probe, and inspection system.

### 6.6 Feedback system

A feedback system is required to obtain information about the robot state, and then control it accurately to deploy the probes in the correct location. In general, feedback systems can be regarded as a chain of physical phenomena relating a set of magnitudes of interest (in this case the robot state), with a magnitude that can be measured. The surrounding engine in this project obstructs the use of conventional exteroceptive systems such as external cameras or


Figure 6.16: Examples of potential cameras from (a) Medigus (Tel Aviv, Israel), (b) AWAIBA (Yverdon-les-Bains, Switzerland), (c) Fujikura (Tokyo, Japan), (d) Omnivision (California, USA). Images courtesy of the respective companies.
electromagnetic trackers, requiring the use of a proprioceptive system.

Considering this feedback problem, a suitable solution was identified in this work. This is a combination of an optical fibre shape sensing technology, together with a camera mounted at the distal end of the robot with an algorithm that combines the camera images with information about the engine in order to infer its position. These two technologies complement appropriately. The former provides feedback of the continuous robot shape with high precision, but presents errors proportional to the fibre length, and is blind to unexpected issues. The latter only informs about the tip pose and can have issues with localisation in the case of limited features in the image, but its accuracy is independent of the cable length, and provides information regarding unexpected issues, as well as a record of the inspection.

In terms of shape sensing, technologies based on Rayleigh backscattering [218] are considered preferrable over those relying on Fibre Bragg Gratings [219]. In particular, shape sensing based on phase interference employing coherent optical frequency domain reflectometry is advantageous since it offers quasi-continuous shape sensing, it provides high spatial resolution, it is commercially available, and the fibres required are inexpensive; only the interrogator system external to the robot is costly. Luna Innovations Incorporated (Virginia, USA) was identified as the most suitable supplier considering the technology they can provide.

Regarding the camera with a localisation algorithm, possible technologies were identified. Examples of miniature cameras available are shown in Figure 6.16 (a)-(d). In terms of localisation algorithms, monoSLAM [220] or patch descriptors [221] are potential solutions.

The selection of a specific imaging algorithm and camera is considered dependent on each
application. The selection of a specific camera and algorithm is expected to be performed in future work. The selected camera and shape sensing technologies are then expected to be incorporated on the complete inspection system also in future work.

### 6.7 Integrated robotic system

The integrated robotic system consists on the gross-positioner, fine-positioner, deployment mechanism, feedback system, and any potential probe, assembled into a robotic system, as illustrated in Figure 1.4, and working together. The weight of the probe that the system can carry depends on the dimensions of the cabling associated to it, on the dimensions of the feedback system, and on the desired workspace for the fine-positioner carrying the payload. However, in a first estimate, the system could be capable of carrying a probe weight of up to 10 g .

The proposed, integrated robotic system must also include a central computer that processes information from the feedback system, and determines the control inputs to the pressure regulators for the fine-positioner, and potentially to the linear actuators for the gross-positioner. This central computer that acts as the brain of the system, and all components of the system should communicate to it. The communication between components is envisaged to be implemented using ROS, where each component part is expected to be a node, and the computer to be the master node. The computer is then expected to receive information from the feedback system, and potentially process it to generate an estimate of the robot state. Then it is expected to use the control laws, and possibly path planning algorithm, to determine the control inputs to the fine-positioner, and if applicable, to the gross-positioner as well. The implementation of this communication in ROS, and the subsequent assembly of the different components into an integrated robotic system is expected to be performed in future work, as described in Chapter 8.

### 6.8 Conclusions on inspection system

A robotic system capable of performing on-wing inspections was proposed, which is composed of a gross-positioner, a fine-positioner, a deployment mechanism, a feedback system, and any potential probe. Research was conducted on the fine-positioner, as presented in the previous chapters, leading to advancements and new solutions to design and control. A concept for the gross-positioner was selected, as presented in this chapter, and a method to navigate inside the engine combining the gross-positioner and fine-positioner was proposed to reach the regions of interest. Methods to fabricate the fine-positioner and gross-positioner were also proposed, as described in this chapter, which in the case of the fine-positioner were used to fabricate segments of it. Moreover, the electronics and pressure regulators required to actuate and control the fine-positioner were also developed.

Solutions to the deployment mechanism and feedback system were also proposed. In addition, a path planner for nonholonomic systems was also developed, which can be used in the inspection system to navigate inside jet engines. In order to create the inspection system, the solutions found to its different parts need to be assembled. This is expected to be conducted in future work, as described in Chapter 8.

## Chapter 7

## Study of Illustrative Inspection Case

An example of application of the work presented in this thesis is the reference on-wing inspection case previously defined in subsection 1.1.3. In that reference case, and more generally in on-wing inspections, the inspection procedure used for the detection of any potential defects may differ from those used off-wing, and can be challenging to determine. In this regard, the reference on-wing inspection case is extended in this chapter both to consider the viability of a typical inspection performed with the robotic system developed in this project, and more generally to illustrate the selection of the most suitable technique, the optimization of the inspection strategy, and the quantification of the corresponding SNR in a reference case.

The inspection requirements in terms of characteristics of the reference defects and components to be inspected are first described in section 7.1, together with the access constraints. The suitability of the main existing NDE techniques is then discussed in section 7.2, leading to the selection of a specific NDE technique and probe. A set of possible inspection strategies for the selected technique are also identified. 2D simulations are reported in section 7.3, which are used to select the most suitable of these inspection strategies, together with the optimal implementation parameters. The impact of 3D effects and typical noise levels in practice is considered in section 7.4, where the expected SNR of the selected inspection strategies is quantified. Lastly, the final conclusions of the inspection study are presented in section 7.5, together with a discussion of the results.

### 7.1 Inspection requirements

The components to be inspected in the reference on-wing inspection case are selected to be the HPC discs. As previously mentioned in Chapter 1, these discs are located in a chamber near the centre of the engine in the arrangement shown in Figure 7.1. The discs have all their surfaces available for the deployment of any desired probe, as can be seen in Figure 7.1, and the surface roughness is practically negligible for standard NDE inspection techniques. However, any inspection device must respect the surrounding structural obstacles shown in Figure 7.1. In addition, and as previously mentioned, it needs to be possible to enter any inspection device into the chamber with the HPC discs through the access route shown in Figure 1.3, which requires any inserted device to fit through 6 mm ID holes. Moreover, any probe or inspection device must be compatible with the robotic system described in previous chapters. It should be noted that this robotic system is expected to be capable of reaching the chamber with the HPC discs by relying on a combined motion of the gross-positioner and fine-positioner, as described in Chapters 6 and 1.

The HPC discs are solid and made of a titanium superalloy. The discs can be considered to present isotropic characteristics, distributed uniformly. The reference structural properties for the discs are a Young's modulus of approximately 120 GPa , a Poisson ratio of near 0.31 , and an approximate density of $4450 \mathrm{~kg} / \mathrm{m}^{3}$. The discs are conductive but not ferromagnetic.

The potential defects to be inspected for are considered to be cracks, as these are typical defects of interest in NDE. These potential cracks are considered to present relatively uniform characteristics. They are considered to be located in the subsurface, near the centre of the HPC disc bores, which is typically more than 10 mm below any available surface of the discs. The cracks are also considered to be flat with negligible width, smooth, and elliptical in shape, generally presenting a low eccentricity. The size of the potential cracks to be detected is selected to be 0.7 mm in their major axis, which is also representative of typical inspection requirements in the aerospace sector. Lastly, the potential cracks are considered to be oriented nearly parallel to the radial-axial plane, with a potential variation of $\pm 20^{\circ}$. This represents a challenging inspection case, which aims to be illustrative of a difficult on-wing inspection in the


Figure 7.1: Sketch of a cross section of the engine chamber with HPC discs, and surrounding structural elements. The blue arrows indicate the access route, which passes through the gap between the shafts, enters the chamber with the HPC discs via a hole in the shaft, and finally reaches the HPC disc bores. A combined motion of the gross-positioner and fine-positioner is expected to be used to follow the access route and reach the disc bores, as described in previous chapters.
aerospace sector.

### 7.2 Technique selection

A technique capable of detecting the potential cracks described in the previous section while respecting the access and manoeuvrability constraints is necessary. The technique should also be executable using a payload, typically a probe, that can be deployed with a robotic system and imposes a minimal restrictions on it. These requirements are stringent, and constrain significantly the NDE techniques applicable. Techniques are reviewed in the following subsection 7.2.1, and a technique and probe are selected in subsection 7.2.2. Possible strategies for the implementation of the technique are then identified in subsection 7.2.3.

### 7.2.1 NDE techniques

A wide variety of NDE techniques exist [1]. These generally involve exploiting the effect that the presence of a defect causes on a physical phenomenon by relating it to a variable that can be measured through a subsequent chain of physical phenomena. The phenomena exploited in NDE techniques are broad, including static and dynamic electromagnetic fields [4], high frequency electromagnetic waves [222], elastic waves [2], and thermal conductivity [223]. The suitability of the main NDE techniques for the inspection considered here is discussed in the following.

## Electromagnetic

Electromagnetic techniques involve the use of either constant or variable electromagnetic fields for the detection of defects or characterisation of materials [224]. One of the most common electromagnetic techniques is eddy current testing [5]. In general, this involves using a coil to generate a varying magnetic field which, when interacting with the inspected component, induces eddy currents. The induced currents generate subsequent magnetic fields, which interact with the excitation coil, affecting its impedance. The impedance of the coil is thus determined by the electromagnetic characteristics of the inspected component. The presence of a defect disturbs the behaviour of eddy currents in the component, which creates a change in the impedance of the excitation coil, and can therefore be used for inspection. Eddy currents, however, generally offer a low penetration depth due to the so-called skin effect [225]. In practice, this translates into a maximum inspection depth of only a few millimetres, which renders eddy current testing unsuitable for the detection of the defects described in the previous section 7.1.

Another type of electromagnetic techniques are potential drop techniques, either alternating current potential drop (ACPD) [226-228] or direct current potential drop (DCPD) [229]. Both techniques involve placing electrodes on the inspected component and injecting a current. This current has a potential drop associated to it, which is affected by the presence of defects between
the electrodes, as these force the current to divert around it. Thus, by measuring the value of the potential drop, defects can be detected and sized [230]. Both techniques, however, share the main disadvantage of not being applicable to detect subsurface defects at significant depth. As a result, potential drop methods are not applicable for the reference inspection case defined in the previous section 7.1.

Alternating current field measurement (ACFM) is a technique similar to ACPD but relying on eddy currents for excitation. ACFM offers a sizing capability [231], as in ACPD, but without requiring contact with the inspected component or calibration, as in the case of eddy currents. In ACFM, a uniform current is induced with a coil, and the deflections caused on it by any potential defect, which causes changes in the magnetic field above the component, are detected $[231,232]$. As in the case of potential drop techniques, however, ACFM is not suitable for the detection of subsurface defects such as the defects of interest defined in the previous section 7.1.

Finally, magnetic flux leakage (MFL) is a technique that relies on steady magnetic fields to detect defects [233]. In MFL, a steady magnetic field is induced on a component, which deflects around potential defects [234]. At high levels of induction, this deflected field leaks out from the component, and can be detected using either Hall sensors, coils or magnetic particles, indicating the presence of a defect, [235]. In practice, however, MFL is not suitable for the in situ detection of defects significantly below the surface such as the potential cracks of interest defined in the previous section 7.1 , since it requires very voluminous equipment.

## Ultrasonic

Ultrasonic techniques involve the use of elastic waves for either the detection of defects or the measurement of structural properties [236]. In general, in ultrasonic testing, elastic waves are excited, which propagate through a structure and, in the presence of a defect, scatter [237]. The scattering then creates additional waves that also travel through the structure and can be measured. This enables both detection of defects and in some cases sizing of these.

Multiple embodiments of ultrasonic testing exist. In general, these can be divided into techniques that rely on surface waves, and techniques that rely on bulk waves. The potential cracks of interest defined in the previous subsection 7.1 are considered to be located in the subsurface, near the centre of disc bores. The techniques of interest are then those employing bulk waves that propagate through the bulk of the material to produce a crack scattering.

In traditional bulk wave techniques, a piezoelectric element, either placed on the surface of the inspected component or coupled to it through a fluid medium, is excited with an electric impulse to force the element to vibrate, and thereby generate a bulk wave [238]. This wave propagates through the component, and any scattering from potential defects is then recorded either with the same element, or with another piezoelectric element also placed on the inspected component. The use of a single element enables the detection of cracks, but presents a low sensitivity to subsurface cracks that are perpendicular to the surface of the component. The use of two elements placed on the same surface of the component and separated a certain distance enables the detection of cracks perpendicular to the surface of the component, as well as the location and sizing of these. This is achieved by measuring the time at which the scattering from the two crack tips arrive at the receiving element relative to the excitation time, as well as the time it takes for a surface wave to travel between both elements, and using trigonometry. This corresponds to a technique known as time of flight diffraction (TOFD) [3,239]. In practice, however, TOFD requires a distance between both elements in the same order of magnitude as the defect depth.

An array of individual piezoelectric elements arranged together in a single transducer can also be used. These are commonly referred to as phased array ultrasonic transducers (PAUT) [240], and have become increasingly popular in the recent decade. The use of an array of elements generally enables a higher detectability than the use of single elements since the array can capture the information corresponding to the excitation and recording in each element, as well as the information corresponding to the excitation and recording in different elements, for all pairs of elements in the array. This is referred to as the full matrix capture [241], and can be combined with post-processing algorithms to produce images of the inspected region [242].

Both PAUT, TOFD, and single-element ultrasound are techniques that could be applicable for the detection of the cracks in the reference on-wing inspection case defined in the previous section 7.1. They are all suitable for the inspection of subsurface defects, and can be used in solid discs such as HPC discs. TOFD offers the highest detectability for cracks perpendicular to the surface of the component [239], but it requires the deployment of two probes separated a few centimetres, which is not practical in situ. Thus, either single-element ultrasound or PAUT are the most appropriate techniques. The only disadvantage of these techniques is that the SNR can be low for cracks perpendicular to the surface, and therefore an inspection study is required.

Alternative methods to generate ultrasonic, bulk waves exist. One of them are electromagnetic acoustic transducers (EMATs), where either a coil or a magnet are used to generate ultrasonic waves relying on magnetostriction or Lorenz forces [243-245]. This enables the excitation of acoustic waves without requiring contact in conductive and ferromagnetic materials, and therefore EMATs can be used to perform inspections in components with coatings or rough surfaces, in harsh environments, and in scenarios where accurate deployment of ultrasonic probes is challenging. Another alternative to generate ultrasonic, bulk waves are lasers [246]. The localised heating created by a laser on a material can produce a rapid expansion which generates elastic waves. The waves then propagate and scatter, and this scattering can be measured at the surface of the component using a laser vibrometer [247]. As a result, laser ultrasound techniques are also non-contact and can be used in any material provided that the surface is polished. The main advantage of both EMATs and laser ultrasound over piezoelectric transducers is that they are non-contact techniques. However, these techniques generally lead to lower SNR than ultrasonic techniques using piezoelectric elements [243], and involve larger equipment to generate and detect waves. In the reference on-wing inspection case defined in the previous section 7.1, contact is not an issue, and instead the inspection SNR and miniature dimensions of the equipment required are important. Thus, piezoelectric elements are the most suitable method to generate ultrasonic, bulk waves.

## Radiographic

Radiographic techniques rely on a beam of high energy radiation and a detector to find potential defects [222]. In general, the beam is directed to the inspected component and attenuates as it traverses it. The presence of a defect reduces the beam attenuation due to the void associated with the defect, and therefore defects can be imaged using a film or detector plate placed at the opposite site of the component in the direction of the beam.

Radiographic techniques can be implemented either by taking a single radiograph obtained from a specific orientation [248], or by combining multiple radiographs obtained from different orientations, which is referred to as computed tomography (CT) [249]. In general, single radiographs are only able to detect cracks oriented such that the incident beam lies in the same plane as the crack. CT, on the other hand, is able to detect cracks in any general orientation. CT, however, requires scanning the component, which is time-consuming, costly, and can generally not be performed in situ as the scanning system needs to surround the inspected component.

The cracks of interest defined in the previous section 7.1 are located near the centre of jet engines. Any in situ radiographic inspection would therefore require either entering a radioactive source and detector film near the HPC discs, or performing the inspection from the outside of the engine with a beam traversing the entire engine structure. The first option is difficult to implement, as the source and film would need to be deployed at opposite sides of the discs, which is very challenging in practice since only one robotic manipulator can be entered through the access route. The second option involves a beam path intersecting multiple structures in the engine in addition to the HPC discs, which adds a significant noise level to the inspection, masking any indication from potential cracks. In addition, any radiographic technique requires stringent health and safety precautions, which typically include a shielded bay that is not practical in in situ inspections.

## Visual

Visual techniques involve visually examining a component to find defects [6]. These techniques include inspections with the naked eye [250], inspections using optical devices such as lenses, mirrors and borescopes [251], and inspections aided by elements to improve the visibility of defects, which are referred to as enhanced visual techniques [6].

The main enhanced visual technique is liquid penetrant inspection (LPI) [7]. In LPI, a highly visible fluid is applied on the surface of the inspected component, and is let to flow into any potential defect. The surface of the component is subsequently cleaned, and any potential defects, typically cracks, are revealed as the highly visible fluid emerges from the defect [252]. A developer can be applied to help draw the fluid from the defects [253].

Visual techniques are generally limited to the inspection for surface-breaking defects. In this regard, they are not applicable for the detection of the cracks of the reference on-wing inspection case.

## Thermographic

Thermographic techniques generally involve heating the inspected component and measuring its surface temperature over time. The temperature of the surface of the component depends on the conductivity of heat within the component. The presence of a defect generally obstructs heat conductivity, which causes a localised change in the surface temperature, and can therefore be used for inspection purposes [254, 255].

Thermography is generally used for the inspection for delaminations, cracks and similar planar defects that lie in a plane parallel to the surface of the component [256-258], as these act as insulators of the heat applied to the surface. However, it is not suitable for the detection of subsurface cracks perpendicular to the surface of the component, such as the cracks defined in the previous section 7.1, since the presence of these cracks has a negligible effect on the conductivity of heat applied to the surface. In addition, thermographic techniques require a heat source and an infrared camera to increase and measure the temperature of the component,
respectively, which are typically large and cannot be inserted into an engine through bore holes [259]. Thus, thermography is not applicable for the detection of the cracks in the reference on-wing inspection case.

### 7.2.2 Selected technique and probe

Considering the discussion in the previous subsection, the technique selected for the reference on-wing inspection case is ultrasound. Ultrasound has the potential of detecting the cracks defined in the previous section 7.1, as opposed to electromagnetic, visual, and thermographic techniques, and it offers the possibility of being deployed in situ using a single robotic manipulator with a single probe, unlike radiographic techniques.

Ultrasound is a common technique for the inspection of discs in the aerospace sector. The established, standard frequency for the inspection of discs in the sector is 10 MHz .

The inspection of defects similar to those defined in the previous section 7.1 using ultrasound is tackled in [260], where a possible inspection procedure using a single probe is reported. However, that study considers defect sizes and depths somewhat different from the inspection requirements here, and it only considers a single, predefined inspection strategy relying on direct scattering from the defect. In addition, the study in [260] focusses on immersion testing, which is not practical in situ, and only considers probes made of a single element. Inspections using single-element probes can be considered as a simple and specific case of inspections using PAUT, and typically an inspection study for PAUT includes the beams and strategies that can be achieved with single-element probes.

The use of PAUT for the in situ detection of the cracks defined in the previous section 7.1 is thus explored in this project. The inspection frequency is selected to be 10 MHz , as it is the established standard in the inspection of these discs in the aerospace sector. The velocity of longitudinal, bulk waves in the material is $6046 \mathrm{~m} / \mathrm{s}$. A PAUT with an element pitch corresponding to half of the longitudinal wavelength is selected as the probe design for the inspection since it yields the highest performance, which corresponds to a pitch of approximately
0.3 mm . A probe with these characteristics can also be miniaturised in size, rendering it well suited for an in situ inspection.

PAUTs with an operating frequency of 10 MHz and a 0.3 mm pitch are commercially available. They generally have an element width of 0.2 mm , and a total number of elements that can be 16, 32, or 64 . A larger number of elements leads to higher SNR in the inspection, but it also entails a probe wire with a larger diameter, which is undesirable as it needs to pass through the fine-positioner and gross-positioner. Hence, the most suitable number of elements is a compromise and needs to be determined.

A particularly relevant PAUT is the BFAP probe, manufactured by JP-Probes (Yokohama, Japan) and supplied by Phoenix ISL (Warrington, UK). In particular, the design shown in Figure 7.2 presents desirable features. It is a linear, flexible array composed of 32 elements, with an operation frequency of 10 MHz , a 0.3 mm pitch, and a 0.2 mm element size, which matches the specifications previously outlined. In addition, since it is flexible and can bend up to a curvature radius of 3 mm , it can conform to a wide range of surfaces not only limited to HPC discs, which agrees with the desired versatility of the inspection system mentioned in Chapter 1. One possible disadvantage of using this flexible probe is that deployment can be difficult, as discussed in section 6 . Thus, the possibility of using a rigid probe with an equivalent layout, and connectors to the elements routed through the back of the probe so that the probe can fit through a 6 mm ID hole cannot be discarded a priori.

The selected, reference probe for the inspection is therefore the small version of the BFAP, or rigid version of it, with a number of elements initially considered to be 32 . The current design of this probe is 10 mm wide and its rigid casing is 12 mm wide, as shown in Figure 7.2 , which is 6 mm wider than the holes in the access route that it must fit through. Potential redesigns to reduce the probe width are possible, as confirmed in correspondence with the supplier, although those would need to be tested to ensure a correct probe performance. In order to obtain a probe that can fit through 6 mm ID holes, the distance between connectors to the individual piezoelectric elements would need to be reduced, and the lateral margins at the size of the connectors as well; the dimensions of the rigid casing where these connectors are
wired to the cables would also need to be reduced; and the total width of the cable would also need to be reduced. The length of the piezoelectric elements, which is currently 4 mm , could also be reduced if necessary, although this entails a reduction in performance, so it should be avoided if possible.

These reductions are considered to be possible but challenging. Reducing the distance between the connectors is considered to be possible with high accuracy manufacturing, but it can lead to a more fragile probe. Alternatively, the connectors could be routed through a layer above the piezoelectric elements, but the capability of bending of the probe would need to be tested to ensure that the connectors do not break. Reducing the lateral margins at the side of the connectors is considered to be simpler, although the probe robustness would also need to be tested. Reducing the size of the rigid casing where the connectors are wired to the cable is also considered to be viable, and major technical problems are not expected. Lastly, reducing the width of the cable while maintaining the number of elements in the array is considered to be possible, especially given the fact that electric shielding may not be required for inspections inside a jet engine that acts as a Faraday cage, and that cable protection may not be necessary when incorporating the cable on the robotic system.

It should also be noted that the existing probe shown in Figure 7.2 has 32 elements. This leads to an existing probe cable of 4.2 mm diameter, which is near the 6 mm diameter constraint of the access route, and is excessive for the fine-positioner. The potential cable reduction mentioned in the previous paragraphs could be used to reduce this diameter so that it is more suitable for the fine-positioner. However, a reduction in the number of elements in the probe can also be considered to reduce the cable diameter. The inspection study presented in the following subsections is initially developed considering a probe with 32 elements, since it is a viability study in the most desirable conditions, but if the SNR is found to be sufficient, a reduction in the number of elements can be considered.


Figure 7.2: Drawing of the BFAP probe selected as reference probe for the inspection. All units are in millimetres. Drawing courtesy of JP-Probes.

### 7.2.3 Possible inspection strategies

Despite the advantages of ultrasound relative to other NDE techniques, the orientation and location of the defects of interest defined in section 7.1 makes them challenging to detect. In particular, the fact that the cracks are considered to lie near the radial-axial plane of the disc implies that they are nearly perpendicular from any available surface. In this regard, the inspection strategy must be carefully considered to select the strategy and implementation parameters that yield the highest possible SNR.

Single-probe inspection strategies are the focus of this work since only one robotic manipulator can be entered through the access route, and this can only deploy a probe in one location at a given time. The position of the cracks is nearly equidistant from both the lateral and inner surfaces of the disc bores. The lateral walls of the bores are parallel, and therefore one of them can be used as a reflecting wall for the inspection. Thus, both inspection strategies using direct scattering from the crack and strategies involving reflections on the opposite wall are possible a priori. Any reflection on the wall opposing the surface with the probe requires the wave to travel a relatively long path, which implies a significant beam spread associated with it. Given the disc dimensions and crack orientations of interest, the inspection strategies with a single
probe considered relevant in this reference on-wing inspection case are those using either direct scattering signals, or using signals that scatter on the crack, reflect at the opposite wall, and return to the probe.

These inspection strategies of interest can be executed using either longitudinal waves, shear waves, or combinations of longitudinal and shear waves that mode convert at the scattering events. The strategies using direct scattering signals have one event where the wave mode can change, whereas the tandem strategies include two events where the wave mode can change. This leads to twelve, single-probe inspection strategies to be explored, which are described in the following two paragraphs.

The first four strategies rely on direct scattering. These involve positioning the probe either at the lateral or inner part of the disc, as shown in Figures 7.3 (a) and (b), respectively, and generating a beam that interacts with the crack and produces either a lateral or crack tip scattering that returns to the same probe where it is measured, as schematically shown in Figure 7.4 (a). The transmitted wave can be either a longitudinal or shear wave, and the returning wave can be either the same mode of the transmitted wave or mode-converted. Thus, the four possible strategies with direct scattering are: to use a longitudinal transmitted wave and a longitudinal returning wave, which is denoted by LL; to use a longitudinal transmitted wave and a shear returning wave, denoted by LS; to use a shear transmitted wave and a longitudinal returning wave, denoted by SL; and to use a shear transmitted wave and a shear returning wave, denoted by SS. The SS strategy is equivalent to the strategy proposed in [260] for a similar scenario.

The next eight inspection strategies are tandem strategies. These involve positioning the probe at the lateral side of the disc, as shown in Figure 7.3 (a), and generating a beam that strikes the crack laterally to produce a reflection that proceeds towards the opposite wall, where it is reflected a second time, and then returns to the same probe where it is captured, as shown in Figure 7.4 (b). The use of this same path but in opposite direction is possible, and is equivalent using the imaging technique described in subsection 7.3.3, so it is included in these tandem strategies. The wave path in these tandem strategies is composed of three segments:
the segment between the probe and the crack; the segment between the crack and the opposite wall; and the segment between the opposite wall and the probe. Each of these segments can be completed by a longitudinal or shear wave, leading to eight possible strategies: LLL, LLS, LSL, LSS, SLL, SLS, SSL, SSS, where the first letter refers to the wave mode in the segment between the probe and the crack ( L referring to longitudinal wave and S to shear wave), the second letter refers to the wave mode in the segment between the crack and the opposite wall, and the third letter refers to the wave mode in the segment between the opposite wall and the probe, following the same notation as that used in the direct scattering strategies.

An inspection strategy using two probes is also available. This involves placing the two transducers at opposite sides of the disc, as shown in Figure 7.3 (c), and generating a skewed beam with one of the transducers, which strikes the crack laterally, and produces a reflection that is received by other second transducer, as depicted in Figure 7.4 (c). This last alternative requires positioning two probes at opposite sides of the disc (these can be 40 mm in thickness), which is practically inviable since only one robotic manipulator can be entered through the single access route available. Thus, the other strategies employing a single probe represent the main inspection strategies considered in this work.

The dimensions of the probe are small relative to the size of the disc bores, so the entire region of interest in a given disc section cannot be inspected from a single probe position. In addition, the discs need to be inspected in all sections circumferentially. In this regard, for any of the strategies, the inspection procedure involves deploying the probe in a selected position (one of the positions shown in Figure 7.3), performing the inspection with the selected strategy, and then rotating the disc as indicated in Figure 7.3 (a), (b), (c) while keeping the probe in a fixed position relative to the ground to repeat the inspection at constant intervals over the disc circumference, and thus scan it circumferentially. The probe is then displaced to a neighbouring position with an equal orientation, and the circumferential inspection scan is repeated. This is executed for probe positions over the entire lateral or inner side of the disc to achieve full coverage.

For the inspection strategies of interest, the probe position relative to the region with potential


Figure 7.3: Sketches of possible probe positions at the side of the disc (a), at the inner part of the disc (b), and two probes at both sides of the disc (c). The dimensions are not to scale.


Figure 7.4: Sketches of inspection strategies corresponding to direct scattering strategies (a); tandem strategies (b); and strategy using two probes (c). The potential crack is represented by a red bar, the beam propagation is indicated using green arrows, and the probe is represented in yellow.
cracks affects the inspection signal intensity. The optimal probe position relative to the imaged region must therefore be determined for each inspection strategy and for the crack orientations of interest. The maximum signal intensity achievable with each strategy can then be compared, and the optimal strategy and corresponding probe positioning can be selected. The SNR can then be evaluated to determine the viability of the inspection. Analytical solutions to the signals expected from the inspection in each configuration are not available. Thus, either numerical solutions or experiments are required, as described in the following section.

### 7.3 Study in planar case

The selection of the most suitable inspection strategy and corresponding probe positioning requires determining the expected signal intensity for the different configurations of interest. Considering the difficulty of producing representative subsurface defects in practice, and the various factors affecting the inspection, the approach adopted in this work is a combination of finite element (FE) simulations of ultrasound propagation and scattering from the crack, together with experiments to take into account the noise in practice. The FE simulations are initially developed in a planar case to have a first study of the inspection strategies, and select a shortlist of strategies of interest together with the corresponding optimal probe positioning. This simulation study in the planar case is reported in this section.

### 7.3.1 Set up of 2 D simulations

The aim of the FE simulations is to determine the magnitude of the signals received by the probe using either of the previously identified inspection strategies for all relevant probe positions and crack orientations, enabling the subsequent selection of the most suitable strategy. The simulations were developed using Abaqus - Simulia ${ }^{\mathrm{TM}}$, Dassaut Systemes ${ }^{\circledR}$ (Velizy-Villacoublay, France). Considering that the probe is a linear array of elements, and that the ultrasound propagation and scattering relevant to the inspection occurs in a plane, a two-dimensional domain was selected for the FE simulations in a first instance, as shown in Figure 7.5. This corresponds to an idealised situation in terms of measured signal amplitude since it is equivalent to an infinitely long crack and an infinite element length in a three-dimensional scenario. However, it is considered sufficiently representative of the behaviour of ultrasound in a first approximation, and allows for the formulation of a numerical problem of an addressable size. The probe width does not affect these simulations, which can therefore be used to provide a first study of the inspection independently of any probe modifications done in the future to reduce the probe width.

In order to consider all relevant probe positions and crack orientations, a significant number


Figure 7.5: Illustrative snapshot of FE simulation corresponding to a single array element exciting a 5-cycle tone-burst at 10 MHz with a 1 N force normal to the component, which generates ultrasound that propagates and scatters. The different colours in the resulting waves indicate the magnitude of node displacement, although the absolute magnitude is arbitrary considering a linear behaviour of the waves.
of simulations need to be studied. Instead of simulating each configuration independently, the FMC [241] corresponding to elements placed over the entire surface of the relevant domain and spaced 0.3 mm (a distance equal to the probe pitch) is simulated in this work. This then enables reproducing any beam generated with the probe positioned anywhere in the relevant domain within a 0.3 mm discretisation interval. The width of the domain selected here, shown in Figure 7.5, is 100 mm since it is considered to include all relevant probe positions. The height is 39 mm , which corresponds to the standard thickness of the discs.

Reflections from the lateral boundaries of the domain are undesirable since these boundaries do not correspond to walls in the discs. Absorbing layers using increasing damping (ALIDs) can be added to prevent reflections [261,262], but they complicate the preparation of the simulation and increase the simulation size to some extent. In this work, signal subtraction was employed. This involves conducting the simulations exciting one element and monitoring in all elements of the FMC both with and without the crack, and then subtracting the resulting signals to isolate only the signals associated with scattering from the crack. In this manner, the undesirable boundary reflections are effectively eliminated from the simulated signals. This is also useful
when reflections such as that from the back wall need to be suppressed to isolate the signals associated with crack scattering only.

In the simulations, the crack was approximated as a slit of 0.7 mm height and negligible width, as highlighted in Figure 7.5, which matches the characteristics of the cracks of interest. This was created in Abaqus CAE, by untying the nodes corresponding to each side of the crack using the command Assign Seam. Since the orientation of the cracks of interest can vary between $\pm 20$ degrees relative to the vertical direction, crack angles at $0^{\circ}, 5^{\circ}, 10^{\circ}$, and $20^{\circ}$ were all simulated in order to determine the evolution of the received signals with crack orientation. In this case, positive crack angles denote clockwise rotations of the crack in Figure 7.5. The symmetry of the domain implies that the response from negative crack angles can be inferred from the positive crack angles studied.

The presence of a crack affects the mesh generated by Abaqus to discretise the domain, particularly if it is an angled crack. Variations in the mesh lead to small variations in the signal, which translates as noise in the final signals obtained after signal subtraction. In order to prevent numerical noise due to mesh differences, simulations without crack were repeated for all the different meshes used in the simulations with crack.

The material properties of the disc were assumed isotropic and homogeneous, without grain scattering. The frequency employed in the simulations is 10 MHz . The global element size of the mesh is 0.0198 mm to allow 16 elements per wavelength of the slower wave, the shear wave. The simulation time step is $2.5 \cdot 10^{-9} \mathrm{~s}$ to ensure stability and therefore convergence according to Lax's equivalence theorem [263]. The total simulation time is $5.4 \cdot 10^{-5} \mathrm{~s}$ to allow the signals corresponding to the shear wave to complete the longest paths, which correspond to tandem strategies.

The simulated excitation of an element was applied to specific nodes selected based on the geometric position corresponding to the element, and the received signals were also monitored in specific nodes based on position of the receiving elements. A five-cycle toneburst was used as a representative approximation of the excitation signal, with a sampling period equal to the simulation step time.

### 7.3.2 Implementation of 2D simulations

The FE simulations were initially implemented in Abaqus Explicit to ensure valid results. Pogo [264] was subsequently employed to repeat the simulations exciting each of the nodes over the top surface of the domain in order to simulate the entire FMC. The use of Pogo enabled a reduction of the computational time by near two orders of magnitude relative to Abaqus.

The total number of simulations required was significant. The full FMC corresponding to 333 elements had to be simulated for each crack angle and also for each mesh without crack, resulting in a total of over 3000 simulations. In order to fully automate the process of executing the simulations in Pogo, a Matlab code was developed. Using this code, the full FMC simulations corresponding to all relevant crack angles, as well as the absence of crack were completed.

The resulting, simulated time-traces corresponding to a pair of elements in the FMC present a low numerical noise, as can be seen in Figure 7.6, which corresponds to a time-trace without signal subtraction. The resulting time-traces also show that in general the signals associated with crack scattering are practically indistinguishable in amplitude compared to the back wall reflection or surface waves, which suggests that an in situ inspection using single-element probes is unlikely to be viable, and instead PAUT is preferrable. An imaging technique that combines the signals corresponding to all pairs of transmitter-receiver elements in a probe is therefore required to generate an image of the crack with a higher signal amplitude.

### 7.3.3 Imaging

A signal received at a given instant of time for a pair of transmitter-receiver elements is generally the result of a scattering event that can have occurred at any point such that the wave propagation time from the transmitter to that point plus the propagation time from that point to the receiver is equal to the time at which the signal is received in the time-trace. For each inspection strategy and each time instant in the time-trace, the possible origin of the scattering event can be on a one-dimensional set of points in the imaged region, provided that the received signal is the result of primary scattering, and that the inspection strategy in terms of number


Figure 7.6: Representative time-trace associated to a pair of transmitter-receiver of the FMC without signal subtraction.
of reflections of the wave on boundaries is specified. For a direct scattering inspection strategy using longitudinal waves (an LL strategy), this one-dimensional set of points defines half of an elliptical curve. The foci of the ellipse are at the positions of the transmitter and receiver, and the major axis of the ellipse is equal to the product of the time at which the signal is received and the longitudinal wave propagation velocity. For other inspection strategies such as direct scattering strategies combining two wave modes or tandem strategies, the points define a more general curve.

The behaviour of ultrasonic waves in a bulk medium can be considered to be linear provided that the wave amplitude is relatively low. The signals received in each pair of transmitter-receiver elements can then be combined using superposition to form an image. The one-dimensional curves of points associated to a scattering event for different pairs of transmitter-receiver generally coincide at the image points with a scatterer, and differ in the rest of the image. In this regard, a suitable imaging method is to assign a value to each pixel equal to the sum of the signals received in all pairs of transmitter-receiver, evaluated at the instant of time corresponding to the propagation time from the transmitter to the pixel of interest and to the receiver, with the selected inspection strategy. This method uses all the information available from any scattering occurring at any point in the image, for the inspection strategy selected.

In addition, the sum of signals in a point with a scatterer is constructive, whereas the sum of signals in points that do not correspond to a scatterer tends to cancel out. This imaging method is equivalent to the standard total focussing method (TFM) [241] when used with an LL inspection strategy. In this regard, TFM is the reference imaging method selected in this work.

For the direct scattering inspection strategies, TFM with the wave propagation velocity of either longitudinal or shear waves corresponding to each strategy was used to generate simulated images corresponding to the crack inspected with a 32 -element probe of the same proportions as the BFAP, from all relevant probe positions introduced in subsection 7.3.1, and for the different crack angles. The value in each image pixel was thus calculated as

$$
\begin{equation*}
I(x, z)=\left|\sum_{i=1}^{n^{2}} h\left(\frac{\sqrt{\left(x_{t i}-x\right)^{2}+z^{2}}}{c_{1}}+\frac{\sqrt{\left(x_{r i}-x\right)^{2}+z^{2}}}{c_{2}}\right)\right| \tag{7.1}
\end{equation*}
$$

where $I$ is the value of the pixel positioned at coordinates $x, z$ in the domain shown in Figure $7.5, x_{t i}, x_{r i}$ are the horizontal coordinates of the centres of the transmitter and receiver elements, respectively, for element pair $i$ which are positioned over the top surface of the domain, $c_{1}$ is the wave velocity corresponding to the outgoing wave travelling from the excited element to the imaged pixel, $c_{2}$ is the wave velocity corresponding to the returning wave, $n$ is the number of array elements (32 in this case), and $h(t)$ is the Hilbert transform of the simulated time-traces, which is used to smooth the resulting TFM image, following [241].

The time-traces employed to generate the images included signal subtraction. This involved simulating the time-traces in a set-up with the crack, then simulating them in an equivalent set-up without the crack, and subtracting both signals to remove reflections from the lateral boundaries of the domain that do not correspond to any boundaries in practice, as previously described in subsection 7.3.1. It should be noted that ALIDs are an alternative to remove undesirable reflections from the boundaries of the domain, as previously discussed in subsection 7.3.1, but in this work signal subtraction was employed.

An example of an image produced with (7.1) for an LL strategy is shown in Figure 7.7 (left),
which corresponds to a $0^{\circ}$ crack imaged with a BFAP probe with its centre positioned 15 mm to the left of the crack. The magnitude in this example image is normalised. As can be seen, the indication corresponding to the crack is clear, although some artefacts appear in the image. The artefacts generally correspond to either direct scattering from the crack in wave modes that do not match the modes used in the strategy selected to produce image (such as mode-converted waves in the LL strategy used for the image in Figure 7.7 (left)), to scattering from the crack that reflects on the domain boundaries, or to shadowing created by the crack on the back wall for probe positions near the centre of the domain. The artefacts, however, do not interfere with the relevant part of the image, which is the scattering from the crack. Therefore, the image can be used to determine the magnitude of the indication corresponding to the crack, which is the desired information.

For the tandem inspection strategies, an adaptation of TFM was used as the imaging method. In this case, the value in each pixel is the sum of the signals received for all pairs of transmitterreceiver in the probe, evaluated at the time corresponding to a wave propagating from the transmitter to the pixel through a specular back wall reflection, plus the time to propagate from the pixel to the receiver, all with the propagation velocity of the modes corresponding to each segment in the inspection strategy selected. This can be expressed as

$$
\begin{equation*}
I(x, z)=\left|\sum_{i=1}^{n^{2}} h\left(\frac{\sqrt{d^{2}\left(x-x_{t i}\right)^{2}+4 d^{2}(d-z)^{2}}}{2(d-z) c_{3}}+\frac{\sqrt{\left(x-x_{t i}\right)^{2}\left(4(d-z)^{2}-4(d-z) d+d^{2}\right)+4(d-z)^{4}}}{2(d-z) c_{4}}+\frac{\sqrt{\left(x_{r i}-x\right)^{2}+z^{2}}}{c_{5}}\right)\right| \tag{7.2}
\end{equation*}
$$

where $d$ is the height of the domain, which corresponds to the disc thickness and is taken as 39 $\mathrm{mm}, c_{3}$ is the velocity of the wave travelling between the excited element and the opposite wall, $c_{4}$ is the velocity of the wave travelling between the opposite wall and the pixel of interest, $c_{5}$ is the velocity of the wave travelling between the pixel and the receiving element, and the rest of variables are equal to those in the previous case. Simulated images were then obtained with this imaging method for a BFAP probe is all relevant positions and for all crack angles, with the different tandem strategies, using the simulated FMC data with signal subtraction.

An example of image corresponding to an LLL tandem inspection strategy is shown in Figure 7.7 (right). As in the previous case, the indication from the crack is clear, but some artefacts


Figure 7.7: Simulated TFM images for an LL direct scattering inspection (left) and an LLL tandem inspection (right). Both images correspond to a BFAP probe positioned with its centre offset 15 mm to the left of the crack, which is 19.5 mm below the surface, and are obtained from the simulated FMC using signal subtraction. The colour scale in the images is expressed in decibels relative to the maximum magnitude in the image.
are present. The artefacts are generally caused by signals resulting from interactions with the crack in wave modes that do not match those of the strategy used in the imaging, and by the shadowing created by the crack over the domain boundaries. The shadowing artefacts only affect the region of the image with the crack when part of the probe is positioned above the crack. In tandem inspections, however, the imaged region exactly below the probe is not relevant, since the back wall signal masks any crack signal in that region. In practice, this appears as a large region with back wall signal only, as confirmed in Figure 7.14 (right) of subsection 7.4.3. Hence, the relevant information, which is the magnitude of the crack indication, can be extracted from the simulated images for the probe positions of interest.

In all simulated images, the axes used were selected to match the XZ axes of the simulation set-up shown in Figure 7.5. Using these axes, the position of the crack in all images was 50 mm in the horizontal direction ( X direction in Figure 7.5), and 19.5 mm in the vertical direction ( Z direction in Figure 7.5). The position of the probe was also defined in these same axes when reporting results of crack indication intensity as a function of probe position. The imaging region was defined such that the indications from the crack and from the back wall were clearly
visible, with a significant margin. This generally led to images that were 48 mm in the vertical direction, and 40 mm in the horizontal direction ( 20 mm at each side of the crack), such as those shown in Figure 7.7. In specific configurations where the position of the probe was near the lateral boundaries of the domain in Figure 7.5, the imaging region was increased in the horizontal direction to ensure that the regions corresponding to the probe position and back wall indication were included in the image, with at least a 5 mm margin.

### 7.3.4 Results of 2 D simulations

The quality of the images obtained with each inspection strategy and configuration is assessed based on the maximum amplitude from the indication corresponding to the crack in the TFM image, since the objective of the inspection is the detection of the cracks of interest. This was implemented by defining a square region surrounding the crack, 2.5 mm from the crack centre in the vertical direction and 3 mm in the horizontal direction, for all simulated images, and recording the value of maximum amplitude.

The value of amplitude of the crack indication was extracted relative to the amplitude of the back wall reflection in order to have a reference. This was implemented by recording the maximum amplitude of the crack indication in the images obtained using the simulated FMC with signal subtraction, and then recording the maximum amplitude from the back wall indication in images generated using a simulated FMC without signal subtraction. The ratio of amplitudes in decibels was then saved. It should be noted that signal subtraction only removes signals corresponding to reflections from boundaries, but it does not affect the amplitude of the signals scattering from the crack, so the desired ratios can be reliably obtained. The probe is composed of 32 elements with a 0.3 mm pitch, as previously described in section 7.2.2.

The results of ratios between crack indication and back wall indication corresponding to direct scattering inspection strategies LL, LS, SL, and SS are plotted in Figure 7.8 as a function of probe position, and for a $0^{\circ}$ crack. As described in the previous subsection, the probe position here is defined as the position of the centre of the probe in the domain and axes shown in Figure 7.5 , and the position of the crack is always at 0.05 m . Thus, a probe position of 0.05 m in

Figure 7.8 corresponds to a configuration with the probe directly above the crack. A $0^{\circ}$ crack is used to initially evaluate the strategies since it is the most critical orientation, so it can be used to select the strategies that can be viable and need to be explored further. The results of the simulations in Figure 7.8 show that the intensity of the crack indication can reach values of approximately -30 dB relative to the back wall indication, and that these are achieved with the LL and SS strategies. With the LS and SL strategies, the crack indication can reach maximum values of approximately -34 dB relative to the back wall indication.

The results in Figure 7.8 also show that the crack intensity varies significantly as a function of probe position. Interestingly, the maximum amplitude when inspecting a $0^{\circ}$ crack is not achieved with the probe centred above the crack, but with the probe centre offset 12 mm with respect to the crack. This result agrees with some of the results in the literature, e.g. [265], [266], which suggest that the maximum scattering amplitude is achieved when striking a crack at angles of approximately $30^{\circ}$ for similar cracks. Performing the inspection with a probe position relative to the inspected region near the configuration with maximum amplitude is important, since the significant variations in amplitude shown in Figure 7.8 for different probe positions can have an important impact on detectability.

The results of ratios between crack indication and back wall indication corresponding to tandem inspection strategies LLL, LLS, LSL, LSS, SLL, SLS, SSL, SSS are plotted in Figure 7.9 as a function of the probe position, for the same $0^{\circ}$ crack. As in the previous case, the probe position is defined as the position of the centre of the probe in the domain and axes shown in Figure 7.5, and the position of the crack is always at 0.05 m . In the case of tandem inspection strategies, probe positions near the region above the crack are not valid since, in these configurations, the indications from the crack and the back wall overlap and cannot be distinguished, as previously mentioned in subsection 7.3.3. The invalid range of probe positions was determined by imaging various experimental FMC acquisitions of a disc with a back wall signal, as described in section 7.4.3 and shown in Figure 7.14 (right) for an LLL strategy, and visually identifying the image region covered by the back wall signals. The approximate invalid interval of probe positions is marked with a red band in Figure 7.9. Interestingly, some of the results obtained in these invalid probe positions follow a different trend from those in the rest of positions for some


Figure 7.8: Curves of ratio between crack indication and back wall indication for direct scattering strategies LL, LS, SL, SS, and vertical crack, as a function of position of array centre. The position of the array centre is relative to left domain boundary in Figure 7.5. The crack is positioned at 0.05 m relative to the same left domain boundary, so an array position of 0.05 m corresponds to the array positioned directly above the crack. The probe is composed of 32 elements with a 0.3 mm pitch.


Figure 7.9: Curves of ratio between crack indication and back wall indication for the tandem strategies LLL, LLS, LSL, LSS, SLL, SLS, SSL, SSS, and vertical crack, as a function of position of array centre relative. The position of the array centre is relative to left domain boundary in Figure 7.5, so it is expressed in the same X axis in Figure 7.5. The crack is positioned at 0.05 m relative to the same left domain boundary. The interval marked with a red band corresponds to the invalid probe positions, where the reflections from the back wall mask any crack indications. The probe is composed of 32 elements with a 0.3 mm pitch.
of the strategies. This is a consequence of the fact that back wall shadowing from the crack occurs in these configurations, and this back wall shadowing appears in the images generated when performing signal subtraction, overlapping with the region corresponding to the crack indications, and affecting the magnitude of the indications.

### 7.3.5 Selection of strategies of interest

Considering the results shown in Figures 7.8 and 7.9, the strategy that yields the highest crack indications is the tandem strategy LLL. The magnitude of the indications of the SLL strategy can be near that of the LLL strategy. However, the SLL strategy involves exciting shear waves, which can be difficult to achieve in an on-wing inspection since the BFAP probe is designed to generate longitudinal waves. Thus, considering that both LLL and SLL are tandem strategies that are expected to yield similar crack indications, and that the SLL strategy can be difficult


Figure 7.10: Curves of ratios between crack indication and back wall indication for direct scattering strategy LL, and crack orientations of $0^{\circ}, 5^{\circ}, 10^{\circ}$, and $20^{\circ}$, as a function of position of array centre. The position of the array centre is relative to left domain boundary in Figure 7.5. The crack is positioned at 0.05 m relative to the same left domain boundary.
to implement in practice, the SLL is disregarded.

The LLL strategy is then considered the most suitable strategy based on the 2D results. The LLL, however, involves the use of waves travelling a relatively long path, which, when considering 3D effects, can lead to a significant reduction in signal intensity due to beam spread. Direct scattering strategies involve shorter paths, and therefore a less significant reduction in signal due to beam spread, particularly in the 3D case. The direct scattering strategies that yield the highest signals are LL and SS. SS is not considered suitable in practice since the shear wave signals from the crack arrive at approximately the same time as the longitudinal wave reflections from the back wall, which mask any crack indication. Therefore, the selected strategies are LL and LLL, which need to be explored further.


Figure 7.11: Curves of ratio between crack indication and back wall indication for the tandem strategy LLL, and crack orientations of $0^{\circ}, 5^{\circ}, 10^{\circ}$, and $20^{\circ}$, as a function of position of array centre relative. The position of the array centre is relative to left domain boundary in Figure 7.5. The crack is positioned at 0.05 m relative to the same left domain boundary. The invalid interval of probe positions is marked with a red band.

### 7.3.6 Optimal probe positioning

The intensity of the crack indications depends on the probe positioning and on the crack orientation. In order to determine the optimal probe positioning for the selected strategies LL and LLL, the ratios between crack indication and back wall indication as a function of probe positioning were also generated for crack orientations of $0^{\circ}, 5^{\circ}, 10^{\circ}$, and $20^{\circ}$. The results corresponding to the LL strategy are plotted in Figure 7.10, and the results corresponding to the LLL strategy are plotted in Figure 7.11. As can be seen in the plots, in both cases the curves corresponding to a $0^{\circ}$ crack are symmetrical as expected, whereas the other curves present an asymmetry that increases as the crack angle increases. The plots also show that the maximum intensity of the crack indication varies for the different crack orientations, with lower values as the crack orientation is closer to vertical, as expected.

The optimal probe positioning for the LL strategy is relatively similar for all crack orientations, as can be seen in Figure 7.10. The probe centre needs to be offset between 8 mm and 12 mm
from the position above the potential crack. Since the variation in intensity in this interval is relatively low, and the most critical crack orientation is the vertical crack, the optimal probe positioning for the LL strategy is taken to be with a 12 mm offset relative to the crack. Or, in other words, imaging in a region offset 12 mm from the probe centre yields the highest detectability. This corresponds to an inspection at an angle of approximately $31.6^{\circ}$.

The optimal probe positioning for the LLL strategy is also similar for all crack orientations, as can be seen in Figure 7.11. The probe centre relative to the imaged region needs to be offset the minimum distance from the position above the crack, provided that it does not involve entering into the invalid interval marked in red in Figure 7.11. This corresponds to an offset of approximately 12 mm , which coincides with the optimal probe positioning for the LL strategy. Thus, only one probe positioning needs to be considered when exploring further the LL and LLL strategies to determine their viability, which is with an offset of 12 mm and approximately corresponds to the probe positioning shown in Figure 7.3 (a) for a crack lying on the plane cutting the disc in the same image.

### 7.4 Study considering 3D effects

The simulations presented in the previous section were developed in 2D. The planar case provides a valid first approximation, but 3D effects are expected in practice, which can reduce the intensity of the signals from the crack. The impact of 3D effects on the resulting signal intensity, as well as the noise levels in practice, need to be quantified. 3D simulations and experiments were therefore conducted for the inspection strategies selected and the optimal probe positioning in order to estimate the expected inspection performance in the reference on-wing inspection case. These are presented in this section.

### 7.4.1 Simulations in 3D

The commercial simulation software CIVA [267] was used to develop 3D simulations. The development of the 3D simulations using Pogo was considered, but it was disregarded since the required number of nodes to simulate the entire domain in 3D was excessively large for the computational resources available. The use of a hybrid model such as [268], which combines FE simulations of complex parts of the model such as defect scattering together with analytical solutions of simple parts such as beam propagation, was also considered, but it was not used since the technology readiness level (TRL) of the software available was not sufficient to conduct all the simulations required in this work.

The 3D simulations in CIVA involved simulating the FMC for the selected strategies LL and LLL and for the optimal probe positioning, which corresponds to the probe centre offset 12 mm relative to the crack. The simulations were implemented for a probe with the characteristics of the BFAP, described in subsection 7.2.2. The crack was selected to be circular with 0.7 mm diameter and vertical orientation, as it is the most critical case. The probe was positioned on the surface of a prismatic domain. The distance between the probe and the opposite wall was selected to be 39 mm , to match the disc thickness, and only reflections on this opposite wall were considered (the lateral dimensions of the domain were significantly larger and reflections from these were not included). The material properties, crack positioning, inspection frequency, and excitation used in the 3D simulations were the same as those in the 2D case.

The main result of the simulations in 3D was an FMC dataset that was used to produce simulated TFM images corresponding to the selected inspection strategies LL and LLL. The images were generated using the same imaging techniques described in subsection 7.3.3. Two TFM images for the LL and LLL inspection strategies considering 3D effects are shown in Figure 7.12. As can be seen, the simulated images show the crack indication for both LL and LLL strategies, as well as the back wall signals.

The images in Figure 7.12 also present artefacts. In particular, Figure 7.12 (left) presents artefacts that are due to the signals from the back wall reflection extending towards the laterals,
which are typical in TFM images, and additional artefacts near the horizontal position of 55 mm and vertical position of 30 mm , which are due to mode-converted waves scattering from the crack.

Figure 7.12 (right) also presents artefacts, which are more significant. The main artefact in Figure 7.12 (right) is vertical and covers a significant part of the image, approximately between horizontal positions of 30 mm and 47 mm . This is caused by the signals reflecting at the back wall, which arrive at a similar time at the probe as any signals corresponding to a tandem inspection for any potential defects in the region directly below the probe, and thus are imaged as an artefact. The similar arrival time is due to the fact that any signals corresponding to a tandem inspection for any potential defects directly below the probe would travel from the probe to the back wall, then to the crack, and back to the probe, following a very similar path to signals simply travelling from the probe to the back wall, and back to the probe. The artefact disappears in the regions at the sides of the probe location, where the path of any signals of a tandem inspection for any potential defects differ significantly in length with respect to the path of signals reflecting at the back wall only. Figure 7.12 (right) also presents additional, smaller artefacts, which are due to mode-converted waves.

The crack signal was determined for both the LL and LLL strategies. In both cases, the crack signal was recorded relative to the back wall reflection in order to have a reference. The crack signal relative to the back wall reflection is -43.54 dB for the LL strategy, and -47.58 dB for the LLL strategy, both of them for the optimal probe positioning with an offset of 12 mm relative to the crack.

### 7.4.2 Convergence of 3 D simulations to 2 D

The results of intensity of the crack indication in 3D present a significant reduction from those in the 2D case shown in Figures 7.8 and 7.9 , which is a consequence of the 3D effects. In order to elucidate the impact of the 3D effects, and determine whether the 3D results tend towards matching the 2 D results as the crack length increases in the passive direction of the probe, a set of 3D simulations with different crack lengths were conducted in CIVA. It should be noted


Figure 7.12: TFM images generated with the 3D simulated FMC for a 0.7 mm vertical crack with a 12 mm offset from the probe, and for the inspection strategy LL (left) and LLL (right). Both images are normalised. (left) presents artefacts due to mode-converted waves, and due to the back wall signals extending laterally. (right) presents a significant artefact, which is due to the back wall reflection arriving at the same time as any potential defects in the region directly below the probe, as well as additional, smaller artefacts due to mode-converted waves.
that the signals simulated with CIVA are more approximate than those of the FE simulations, and this is more prominent for smaller defects.

The FMC was simulated using CIVA for a set of vertical cracks with 0.7 mm height and lengths of $100 \mathrm{~mm}, 5 \mathrm{~mm}, 2 \mathrm{~mm}$ and 0.7 mm . The inspection strategy selected here was LL, as it is considered to be representative of the trends in the convergence of 3D simulations to a planar case. The inspection parameters and probe used in the simulations were the same as in those described previous subsection.

The results of ratios between crack indication intensity and back wall intensity for different crack lengths are shown in Figure 7.13. The results show that the intensity of the crack indication varies significantly with crack length. It increases rapidly as crack length increases for lengths of a few millimetres, and then presents an asymptotic trend. The results also show that, in the case of a 100 mm long crack, the intensity of the crack signal relative to the back wall signal in 3D converges to the 2D results in Figure 7.8, confirming a good agreement between the planar and 3D cases. It should be noted, however, that the results obtained with CIVA


Figure 7.13: Ratio between intensity of crack indication and back wall indication for vertical cracks of 0.7 mm height and various crack lengths, showing the impact of 3D effects on crack signals. The ratio between intensity of crack indication and back wall from the planar simulations is also included with a dashed line as a reference, elucidating the convergence of the 3D simulations to a planar case. A logarithmic scale is used in the horizontal axis.
present a certain degree of inaccuracy, especially for the smaller defects, which is associated to the approximate methods employed by the software.

### 7.4.3 Experimental noise incorporation

The signals and images generated with the simulations correspond to an ideal scenario. In order to consider the noise levels in practice, 50 FMC acquisitions with an averaging of 10 were conducted in a disc sample. The probe was positioned at the lateral side of the disc in all 50 acquisitions, but it was repositioned to a different location for each of the 50 acquisitions in order to obtain a different measurement of any potential coherent noise originating from the material.

The experimental measurements confirmed a certain level of coherent noise, which is attributed to grain scattering. A set of ten equivalent FMC acquisitions were also conducted on two aluminium blocks with a similar thickness to the disc and a fine grain in order to determine whether the noise changed. The TFM images from the aluminium blocks showed a marked



Figure 7.14: Illustrative TFM images resulting from the combination of the simulated scattering from the crack in 3D with experimental noise. The images correspond to an LL direct scattering strategy (left), and an LLL tandem strategy (right), with a 12 mm offset between probe centre and crack position, and both images are normalised.
reduction in coherent noise levels, confirming that the noise in the disc measurements can be attributed to grain scattering.

The 50 experimental FMC acquisitions on the disc containing the noise were combined with the 3D simulated signals described in subsection 7.4.1, using the amplitude of the back wall indication as common reference for amplitude. The direct reflection from the back wall was removed from the simulated signals before combining them with the experimental measurements in order to retain only the simulated signals associated to crack scattering. TFM images were subsequently generated in the same manner as described in subsection 7.3.3. Two of these images are shown in Figure 7.14 for the two inspection strategies LL and LLL. These resulting images reproduce the images that would be obtained in practice when inspecting for the cracks of interest in the reference on-wing inspection case, since they correspond to the superposition of the experimental measurements of noise and back wall reflection in a disc with the 3D simulated signals of scattering from the crack.

The resulting images indicate that the crack is not visible, neither with the LL strategy nor with the LLL strategy, as can be seen in Figures 7.14 (left) and (right), respectively. This is due to the fact that the noise magnitude is higher than the magnitude of the crack signals. The
noise is particularly localised in the region below the probe, and tends to reduce in the lateral direction. The results in the previous section indicate that probe positioning that yields the highest intensity of the crack indication is with the probe centre offset 12 mm from the imaged region, both for the LL and LLL strategies. In addition, as previously mentioned in section 7.1, any potential cracks are considered to be near the centre of the discs. In this regard, a region of interest can be defined, such that the inspection only considers indications in that region. The magnitude of the crack indications relative to the noise can then be evaluated in that region. The region of interest used in this work is depicted in Figure 7.14, and corresponds to a rectangular region that is 7 mm from the disc surfaces, with a lateral edge 10 mm from the probe centre, and the other lateral edge 30 mm from the probe centre.

The SNR in the region of interest for each inspection strategy can then be evaluated in two steps. First, the maximum amplitude corresponding to the noise in the region of interest can be measured relative to the back wall amplitude for 50 images generated with the 50 experimental noise measurements obtained moving the probe to different positions, without any crack signal. Second, the maximum amplitude of the crack indication relative to the back wall can be measured in an image generated using the 3D simulations for a vertical crack (the most challenging orientation), as previously described in subsection 7.4.1. Then, using the back wall indication as a common reference, the SNR can then be evaluated as the difference between the maximum amplitude of the crack indication expressed in decibels and the maximum amplitude corresponding to the noise also expressed in decibels. It should be noted that this evaluation of the SNR is based on 50 FMC measurements with the probe in different positions on a given disc to consider variations in the grain noise. The grain noise in different discs is assumed to be similar to the grain noise in the disc used for the experiments, which is taken as a representative reference.

The evaluation of the SNR was implemented for both the LL and LLL inspection strategies. The maximum amplitude of noise in the region of interest, relative to the back wall, for the 50 experimental acquisitions is plotted in Figure 7.15 (left) for the LL strategy, and in Figure 7.15 (right) for the LLL strategy. The plots show some variation in the noise levels, which is due to the variation in the grain scattering noise in each measurement. However, in general the plots


Figure 7.15: Maximum noise amplitude relative to maximum back wall amplitude in the region of interest for 50 images produced using experimental data and the LL strategy (left), and the LLL strategy (right).
show that the maximum noise indication is between -31.5 dB and -36 dB relative to the back wall for the LL strategy, and between -28.5 dB and -33 dB for the LLL strategy. The values of crack indications obtained from the simulated signals are -43.54 dB for the LL strategy and -47.58 dB for the LLL strategy, which are lower than the noise levels. Therefore, the resulting SNR is negative, which implies that the crack is not detectable with either inspection strategy.

### 7.5 Results of inspection viability and discussion

The results of the inspection study described in the previous sections indicate that, in the reference on-wing inspection case, the inspection is not viable using a single probe. The maximum intensity of the signals corresponding to the crack is more than 10 dB lower than the maximum intensity of the noise in the region of interest for both the LL and LLL strategies, which are the most suitable inspection strategies. It should be noted that the SNR was evaluated for a vertical crack, which is the most challenging orientation to inspect. However, given that the crack orientation is considered to vary between $\pm 20$ degrees with an unspecified probability distribution of crack orientation, it is necessary to consider the most critical case.

The fusion of the data corresponding to different inspection strategies, such as LL and LLL, could be considered to improve the intensity of the crack signals relative to the noise, following
the work reported in [269]. However, given the low intensity of the signals corresponding to the cracks of interest, which are over 10 dB lower than the noise, the potential improvements from the fusion of data is not expected to be sufficient to make the inspection viable.

The inspection study was developed for a 32 element probe, which is 9.6 mm long. The use of a longer probe could lead to higher signals corresponding to the crack indication. However, the improvement is not expected to be significant. In addition, the increase in probe length is limited by the access requirements, and any larger probe with more elements would require a greater number of wires to pass through the robot, which would lower its force and could easily fill the limited space available.

The inspection study suggests that a crack of approximately 2.5 mm in size would be detectable, based on the simulation results shown in Figure 7.13, and the noise levels measured experimentally. This crack size is significantly larger than the 0.7 mm crack size specified in the reference on-wing inspection case, which is representative of typical crack sizes that are currently desirable to detect in the aerospace sector. However, the availability of a robotic system for on-wing inspections could enable a higher frequency of inspections in the future, making larger defects admissible. Thus, it can be relevant to revisit the defect size to be detected in the future, when a system capable of on-wing inspections is available.

The study was conducted using an inspection frequency of 10 MHz since it is the typical frequency used for the inspection of discs in the aerospace sector. A lower frequency could be considered to reduce the noise due to grain scattering in future work. However, the reduction in frequency cannot be significant as the wavelength of longitudinal waves at 10 MHz is already near the size of the crack of interest. In order to achieve a viable inspection, it would be necessary to improve the intensity of the crack signal relative to the noise by over 10 dB , which is considered to be very difficult with a moderate frequency reduction.

The most promising inspection solution for the reference on-wing inspection case is considered to be the use of two probes deployed at opposite sides of the disc, adopting the alternative inspection strategy mentioned in section 7.2 .3 and schematised in Figure 7.4 (d) and (g). However, the simultaneous deployment of two probes at opposite sides of the disc is not considered
to be viable using a single robotic manipulator such as those investigated in this work, and instead would require a different robot concept.

## Chapter 8

## Conclusions

### 8.1 Concluding remarks

### 8.1.1 General robotic system for on-wing inspections

There is a need for new technology to perform on-wing operations such as inspection and repair of jet engines, as described in Chapter 1. This need was addressed in this work, focussing on on-wing inspections in difficult to access locations.

The on-wing inspection problem was analysed, literature was reviewed, and a novel robotic system was proposed to perform on-wing inspections, and potentially repairs. This robotic system resembles a 'snake-robot', and is conceived to be capable of inserting an end-effector relying on reaction forces from the engine, and then deploy it in a desired location. Its development can be divided into five parts: a fine-positioner, a gross-positioner, an inspection strategy, a deployment mechanism, and a feedback system. The key part of the work is the development of the fine-positioner, which was the centre of the research presented in this thesis.

A soft robotic manipulator with fluidic actuation was concluded the most suitable concept for the fine-positioner. The FMA was identified as a relevant reference device for the fine-positioner. However, the FMA is designed mostly by intuition, and as a result it has a relatively low force.

In addition, neither kinematic solutions nor control laws are available for it. Research on design and control was therefore conducted.

### 8.1.2 Design

The design of soft robotic manipulators with fluidic actuation was studied from a general perspective, as presented in Chapter 4. This led to the development of a general framework for the design of soft robotic manipulators. A set of design principles were derived, which can be used to determine the most suitable design in each application. These were first applied in a MIS case study to illustrate the work, verify the study, and provide a foundation for the design of the fine-positioner. The application of the principles in the case study led to two layouts of interest. The design principles also served to identify compromises in a set of design parameters that needed to be optimised. An optimisation procedure was outlined and implemented using FE simulations. These produced successful results, verifying the work, and leading to the most suitable design for the MIS case study.

In order to apply the design study to determine the most suitable fine-positioner design, this was extended to consider the possibility of using any maximum pressure. In addition, a nondimensional analysis was developed. The extended design principles, together with the nondimensional analysis, were then applied to the design of the fine-positioner. Two main layouts of interest were found, in a similar manner as in the MIS case study, and compromises were identified. An optimisation procedure with FE simulations was implemented to resolve these compromises. This yielded the most suitable design of the fine-positioner, which shows an important performance improvement with respect to the FMA, which is representative of the highest performing existing designs.

### 8.1.3 Control

In terms of control, research on kinematics of continuum robots, on mechanical modelling of the fine-positioner, and on preliminary control laws was conducted, as described in Chapter 5. The
kinematics of continuum robots composed of segments with constant curvature bending and extension capabilities were investigated, which correspond to FMA-type devices. Closed-form solutions to the full kinematics were derived, which closes a currently open problem. However, the final design selected for the fine-positioner does not match the deformation modes of this work, so the control efforts then focussed on the derivation of control laws relying on mechanical modelling.

In terms of mechanical modelling, after exploring various approaches, a model was developed, which approximates the robot segments as a set of rigid links with articulated joints. The specific parameters of the model were extracted from FE simulations and experimental measurements. The development of control laws was started in collaboration with Dr Enrico Franco. Control laws were obtained for planar operation of a robot segment. These are based on the mechanical model developed. The performance of these new control laws was evaluated in simulations and successful results were obtained. These control laws in the planar case represent the basis for the development of control laws for the fine-positioner in 3D, which is expected to be the next main step of the control work.

In addition to the work on control laws, path planning algorithms were also explored in another collaboration with Dr Fangde Liu. This resulted in a fast and robust path planning algorithm for nonholonomic systems, which can be used for navigation inside cluttered environments like a jet engine.

### 8.1.4 Concentric tube robots

As part of the exploration of continuum robots for on-wing inspections, research on CTRs was also conducted, presented in Chapter 3. This led to the discovery of the complete set of trajectories where follow-the-leader motion is possible in the case of no torsion of the tubes, closing an open question. The effects of torsion were subsequently studied, and a closed-form solution to torsion for the case of two tubes was derived. Simulations and an experiment were finally developed to discuss the effects of torsion, and illustrate the work with a case study. The results of this research indicate that CTRs are not suitable for on-wing inspections of jet
engines. However, the trajectories discovered show promise in MIS. In addition, it was found that the use of CTRs with a non-annular cross section would be a suitable solution for the gross-positioner.

### 8.1.5 Fabrication and complementary systems

The fabrication of the fine-positioner was also explored, which was aided by a placement at Tokyo Institute of Technology. A method to fabricate the fine-positioner was selected and implemented in practice, as summarised in Chapter 6. This method, however, leads to devices with a low reliability, and needs to be improved.

The development of the other parts of the robotic system for on-wing inspections was also analysed, and appropriate solutions were identified, as also outlined in Chapter 6. For the gross-positioner, the solution proposed is to use a non-annular CTR composed of three tubes. The work completed on CTRs in Chapter 3 is applicable to develop this device. For the deployment mechanism, various solutions were identified, both for rigid and flexible probes. These generally rely on the use of a pivoting point between the probe and the fine-positioner, or the probe elasticity, together with reaction forces from the component to passively compensate for small misalignments. For the feedback system, shape sensing based on optical fibres together with a camera and a localisation algorithm were identified as the most promising solutions.

### 8.1.6 Inspection study

A reference on-wing inspection case was defined and studied using analytical discussion, simulations, and experimental measurements, as presented in Chapter 7. The most suitable inspection strategy was identified. However, it was found that the inspection is not viable using a single probe, even with the optimal strategy, since the SNR is excessively low.

Nonetheless, the robotic system proposed is still expected to be useful in general for the on-wing insertion and deployment of probes inside jet engines. This can be applied to other on-wing
inspections that can be conducted with a single probe, which include current inspections. In addition, the fine-positioner developed, and the work on CTRs, are also applicable to MIS, where there is a significant need for miniature robotic manipulators and similar devices.

### 8.2 Future Work

The work presented in this thesis addresses the primary challenges for the development of the inspection system proposed to conduct on-wing inspections of jet engines. In order to create a working prototype of the complete inspection system that can be translated to industry, the research conducted on control needs to be extended, and the different parts of the complete system need to be further developed and assembled, as summarised in this section.

### 8.2.1 Control of fine-positioner

In terms of control, experiments need to be conducted to evaluate the performance of the control law derived in Chapter 5 for a segment of the fine-positioner operating in a plane, and determine whether it performs as expected in practice, despite the various discrepancies in the model developed in this work. Once these validation experiments are successfully completed, this work on control needs to be generalised to obtain control laws for operation of a full fine-positioner with 6 DOFs in 3D space.

This is expected to be possible by applying the adaptive energy shaping approach used in this thesis to a full manipulator. However, the generalisation to 3D requires modifications to the model of a segment of fine-positioner in Chapter 5 to consider deformations out of plane. The modified, 3D model is expected to be composed of a set of rigid links and elastic joints, in an equivalent manner as in the model proposed in this work for 2D operation, although research to select the specific model in 3D and confirm its suitability may be necessary. This future work on control is expected to lead to a relatively robust fine-positioner capable of accurate positioning in 3D.

### 8.2.2 Fabrication of fine-positioner

The main limitation of the current fine-positioner is its short durability. The method for fabricating the fine-positioner described in Chapter 6 thus needs to be improved in future work to achieve longer durability and reliability. This future work on fabrication should lead to a method to fabricate a fine-positioner composed of three segments that can operate reliably for tens or hundreds of hours without rupture.

### 8.2.3 Complete fine-positioner

The combination of the future work on control laws and on fabrication are expected to lead to a working prototype of the fine-positioner, which represents the key part of the inspection system. Testing of the prototype of the fine-positioner inside a cluttered region of a jet engine may also be conducted in the future to confirm its reliable operation when in contact with representative environments.

This prototype of fine-positioner is not expected to include any specific payload initially, since the inspection system is developed to be versatile. However, the prototype should include a working channel to be able to accommodate payload. In this regard, once a first prototype is available, it can also be relevant to explore minor variations of the design of the fine-positioner to include working channels of different diameters. This can be conducted by applying the same design procedure presented in Chapter 4 but adding working channels of different diameters along the central rod.

Another independent, potential future avenue for research in the fine-positioner is to explore the alternative designs, mostly in terms of the outer wall, derived in Chapter 4. These correspond to a design with a notched outer structure, and a design with a pleated outer wall, schematised in Figure 4.10. The main challenge in this potential work is the fabrication of these outer wall structures in miniature size. If the outer walls can be fabricated, the design framework and procedure in Chapter 4 can then be applied to determine the optimal design in each case. It should be noted that the derivation in Chapter 4 relies on a set of assumptions, which are
noted throughout the chapter, to enable the derivation of a general design framework. Another independent future avenue for research then is to explore the limitations of these assumptions, and investigate design layouts outside the boundaries of the design framework defined by these assumptions.

### 8.2.4 Further development of other system parts

The gross-positioner described in Chapter 6 is also expected to be fabricated and tested in future work. This was not conducted as part of the work reported in this thesis since the reference on-wing inspetion case was found to be inviable using a single probe, and a specific application for the inspection system was not immediately available. In addition, the fabrication and testing of a gross-positioner requires equipment that is not readily available, and from a research perspective is not considered to represent a significant challenge. Instead, the efforts were focussed on developing the main research required to be able to create an inspection system for each given application, which primarily corresponds to research on the fine-positioner, and on finding solutions to the different parts of the inspection system.

The fabrication and testing of the gross-positioner is thus expected to be implemented in the future, once an application of inspection that can be conducted using a single probe is selected. A certain degree of redesign of the geometry of the tubes comprising the gross-positioner is likely to be necessary to adapt it to each application. This can be performed by following an equivalent procedure as that described in Chapters 3 and 6.

The fabrication and testing of the deployment mechanism is also expected to be conducted in future work, once a specific application and probe are selected, and a working prototype of the fine-positioner is available. Similarly, the selection of a specific feedback system, and its incorporation on the fine-positioner to then test it and improve it iteratively is also expected to be performed in future work, once a specific application is selected, and a working prototype of the fine-positioner is available.

### 8.2.5 Assembly of complete inspection system

The assembly of the different parts of the inspection system represents the last main task to create a working prototype of the inspection system for on-wing inspections. This assembly can be conducted to some extent for a general gross-positioner, fine-positioner, probe, deployment mechanism, and feedback system. However, it is expected to require refinement for each specific application, since all these parts depend in part on the application, so the final assembly of the inspection system is expected to be conducted once an application that is viable using a single probe is selected.

The assembly of the different parts is also expected to require some iteration and testing in a jet engine to tend to an optimal integration of all parts, and thus create a working prototype of the inspection system. Once a working prototype is available, entry manoeuvres such as those described in Chapter 6 can be tested to confirm their viability. The path planner for navigation inside an engine can also be implemented onto the inspection system once a prototype of this that can be tested in a jet engine is available. Finally, the technology readiness level of the working prototype can be advanced, preparing the robotic system for implementation in industry.

### 8.2.6 Medical applications

The set of trajectories discovered in Chapter 3 for follow-the-leader motion using CTRs show promise in MIS. The exploitation of these trajectories to perform new or improved medical procedures using CTRs may also be explored in future work.

The fine-positioner design found in this work, and the design framework developed in Chapter 4, also show significant promise in MIS. A soft robotic manipulator is well-suited for MIS since it is compliant and inherently safe, it is MRI compatible, it can be miniaturised, it is low-cost so it can be disposable, and it is relatively modular and can be easily coupled to other tools. In addition, the design framework and design procedure developed in this work enable the determination of the most suitable design for each application. In this regard, applications of
the fine-positioner to MIS are also an interesting avenue to explore in future work.

## Bibliography

[1] R. Halmshaw, Non-Destructive Testing. Edward Arnold, London, 1987.
[2] J. Blitz and G. Simpson, Ultrasonic methods of non-destructive testing. Springer, 1996.
[3] T. Kundu, Ultrasonic Nondestructive Evaluation. CDC Press, 2003.
[4] R. C. McMaster, P. McIntire, and M. L. Mester, Electromagnetic Testing, volume 4 of Nondestructive Testing Handbook. American Society for Nondestructive testing, Columbus, OH,, 2nd edition ed., 1986.
[5] J. Garcia-Martin, J. Gomez-Gil, and E. Vazquez-Sanchez, "Non-destructive techniques based on eddy current testing," Sensors, vol. 11, no. 3, pp. 2525-2565, 2011.
[6] W. McGonnagle, Nondestructive Testing. Gordon and Breach Science Publishers, 1969.
[7] N. Tracy and P. Moore, "Liquid Penetrant Testing," in Nondestructive Testing Handbook, ch. Volume 2, American Society for Nondestructive Testing, Columbus, OH, 1999.
[8] M. P. Kummer, J. J. Abbott, B. E. Kratochvil, R. Borer, A. Sengul, and B. J. Nelson, "Octomag: An electromagnetic system for 5-DOF wireless micromanipulation," IEEE Transactions on Robotics, vol. 26, no. 6, pp. 1006-1017, 2010.
[9] H. Choset and W. Henning, "A Follow-the-leader Approach to Serpentine Robot Motion Planning," 1999.
[10] W. S. AB, "Sword tool. www.wesdyne.se/index.php/ct-menu-item-15/ct-menu-item-29," (Accessed June 2018).
[11] E. W. Hawkes, L. H. Blumenschein, J. D. Greer, and A. M. Okamura, "A soft robot that navigates its environment through growth," Science Robotics, vol. 2, no. 8, p. eaan3028, 2017.
[12] S. I. plc, "PretzelFlex ${ }^{T M}$ Instrument. www.surginno.com/pretzelflex/," (Accessed May 2018).
[13] J. Mohd Jani, M. Leary, A. Subic, and M. a. Gibson, "A review of shape memory alloy research, applications and opportunities," Materials and Design, vol. 56, pp. 1078-1113, 2014.
[14] I. A. Gravagne, C. D. Rahn, and I. D. Walker, "Large deflection dynamics and control for planar continuum robots," IEEE/ASME Transactions on Mechatronics, vol. 8, no. 2, pp. 299-307, 2003.
[15] R. J. Webster III and B. A. Jones, "Design and Kinematic Modeling of Constant Curvature Continuum Robots: A Review," The International Journal of Robotics Research, 2010.
[16] V. Anderson and R. Horn, "Tensor Arm Manipulator Design," ASME transactions, 1967.
[17] S. Hirose, Biologically Inspired Robots, Snake-like Locomotiors and Manipulators. 1993.
[18] D. Caleb Rucker and R. J. Webster, "Mechanics of continuum robots with external loading and general tendon routing," IEEE Transactions on Robotics, pp. 645-654, 2011.
[19] I. D. Walker, "Continuous Backbone Continuum Robot Manipulators," ISRN Robotics, vol. 2013, pp. 1-19, 2013.
[20] A. Ataollahi, R. Karim, A. Soleiman Fallah, K. Rhode, R. Razavi, L. Seneviratne, T. Schaeffter, and K. Althoefer, "3-DOF MR-Compatible Multi-Segment Cardiac Catheter Steering Mechanism.," IEEE transactions on bio-medical engineering, no. c, 2013.
[21] I. D. Walker and M. Hannan, "A novel 'elephant's trunk' robot," 1999 IEEE/ASME International Conference on Advanced Intelligent Mechatronics (Cat. No.99TH8399), pp. 0-5, 1999.
[22] W. Lee, A. Chamorro III, and B. Weitzner, "Robotically Controlled Medical Instrument with a Flexible Section," 2003.
[23] N. Simaan, Kai Xu, Wei Wei, a. Kapoor, P. Kazanzides, R. Taylor, and P. Flint, "Design and Integration of a Telerobotic System for Minimally Invasive Surgery of the Throat," The International Journal of Robotics Research, vol. 28, no. 9, pp. 1134-1153, 2009.
[24] N. Simaan, "Snake-like units using flexible backbones and actuation redundancy for enhanced miniaturization," in Proceedings - IEEE International Conference on Robotics and Automation, vol. 2005, pp. 3012-3017, 2005.
[25] N. Simaan, R. Taylor, and P. Flint, "A dexterous system for laryngeal surgery," IEEE International Conference on Robotics and Automation, 2004. Proceedings. ICRA '04. 2004, vol. 1, 2004.
[26] X. Kai and N. Simaan, "Actuation compensation for flexible surgical snake-like robots with redundant remote actuation," Proceedings - IEEE International Conference on Robotics and Automation, vol. 2006, no. May, pp. 4148-4154, 2006.
[27] K. Xu and N. Simaan, "Analytic Formulation for Kinematics, Statics, and Shape Restoration of Multibackbone Continuum Robots Via Elliptic Integrals," Journal of Mechanisms and Robotics, vol. 2, no. February 2010, p. 011006, 2010.
[28] W. Wei, X. Kai, and N. Simaan, "A compact two-armed slave manipulator for minimally invasive surgery of the throat," Proceedings of the First IEEE/RAS-EMBS International Conference on Biomedical Robotics and Biomechatronics, 2006, BioRob 2006, vol. 2006, pp. 769-774, 2006.
[29] N. Simaan, Kai Xu, Wei Wei, a. Kapoor, P. Kazanzides, R. Taylor, and P. Flint, "Design and Integration of a Telerobotic System for Minimally Invasive Surgery of the Throat," The International Journal of Robotics Research, vol. 28, no. 9, pp. 1134-1153, 2009.
[30] N. Simaan, a. Bajo, a. Reiter, L. Wang, P. Allen, and D. Fowler, "Lessons learned using the insertable robotic effector platform (IREP) for single port access surgery," Journal of Robotic Surgery, vol. 7, no. 3, pp. 235-240, 2013.
[31] D. B. Camarillo, C. F. Milne, C. R. Carlson, M. R. Zinn, and J. K. Salisbury, "Mechanics modeling of tendon-driven continuum manipulators," IEEE Transactions on Robotics, vol. 24, no. 6, pp. 1262-1273, 2008.
[32] D. B. Camarillo, C. R. Carlson, and J. K. Salisbury, "Task-Space Control of Continuum Manipulators with Coupled Tendon Drive," Springer Tracts in Advanced Robotics, vol. 54, no. 4, pp. 271-280, 2009.
[33] M. D. M. Kutzer, S. M. Segreti, C. Y. Brown, R. H. Taylor, S. C. Mears, and M. Armand, "Design of a new cable-driven manipulator with a large open lumen: Preliminary applications in the minimally-invasive removal of osteolysis," in Proceedings - IEEE International Conference on Robotics and Automation, pp. 2913-2920, 2011.
[34] S. M. Segreti, M. D. M. Kutzer, R. J. Murphy, and M. Armand, "Cable length estimation for a compliant surgical manipulator," in Proceedings - IEEE International Conference on Robotics and Automation, pp. 701-708, 2012.
[35] R. J. Murphy, Y. Otake, R. H. Taylor, and M. Armand, "Predicting Kinematic Configuration from String Length for a Snake-like Manipulator Not Exhibiting Constant Curvature Bending," IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS 2014), no. Iros, pp. 3515-3521, 2014.
[36] Z. Du, W. Yang, and W. Dong, "Kinematics Modeling of a Notched Continuum Manipulator," Journal of Mechanisms and Robotics, vol. 7, no. 4, p. 041017, 2015.
[37] Y. Wenlong, D. Wei, and D. Zhijiang, "Mechanics-based kinematic modeling of a continuum manipulator," in IEEE International Conference on Intelligent Robots and Systems, pp. 5052-5058, 2013.
[38] X. Dong, M. Raffles, S. C. Guzman, D. Axinte, and J. Kell, "Design and analysis of a family of snake arm robots connected by compliant joints," Mechanism and Machine Theory, vol. 77, pp. 73-91, 2014.
[39] X. Dong, M. Raffles, S. Cobos-Guzman, D. Axinte, and J. Kell, "A Novel Continuum Robot Using Twin-Pivot Compliant Joints: Design, Modeling, and Validation," Journal of Mechanisms and Robotics, vol. 8, no. 2, p. 021010, 2015.
[40] E. Gomez-acedo, K. Txoperena, K. Pfeiffer, F. Messmer, and M. Gruhler, "MiRoR Miniaturized Robotic Systems for Holistic," vol. 23, no. 2, pp. 978-981, 2018.
[41] X. Dong, D. Axinte, D. Palmer, S. Cobos, M. Raffles, A. Rabani, and J. Kell, "Development of a slender continuum robotic system for on-wing inspection/repair of gas turbine engines," Robotics and Computer-Integrated Manufacturing, vol. 44, pp. 218-229, 2017.
[42] D. Palmer, S. Cobos-Guzman, and D. Axinte, "Real-time method for tip following navigation of continuum snake arm robots," Robotics and Autonomous Systems, vol. 62, no. 10, pp. 1478-1485, 2014.
[43] W. McMahan, B. a. Jones, and I. D. Walker, "Design and implementation of a multisection continuum robot: Air-octor," in 2005 IEEE/RSJ International Conference on Intelligent Robots and Systems, IROS, pp. 3345-3352, 2005.
[44] T. D. Nguyen and J. Burgner-Kahrs, "A tendon-driven continuum robot with extensible sections," IEEE International Conference on Intelligent Robots and Systems, vol. 2015December, pp. 2130-2135, 2015.
[45] N. G. Cheng, M. B. Lobovsky, S. J. Keating, A. M. Setapen, K. I. Gero, A. E. Hosoi, and K. D. Iagnemma, "Design and analysis of a robust, low-cost, highly articulated manipulator enabled by jamming of granular media," Proceedings - IEEE International Conference on Robotics and Automation, pp. 4328-4333, 2012.
[46] G. Guthart and J. Salisbury, "The Intuitive telesurgery system: overview and application," Proceedings 2000 ICRA. Millennium Conference. IEEE International Conference on Robotics and Automation. Symposia Proceedings (Cat. No.00CH37065), vol. 1, no. April, pp. 618-621, 2000.
[47] D. J. Abbott, C. Becke, R. I. Rothstein, and W. J. Peine, "Design of an endoluminal NOTES robotic system," IEEE International Conference on Intelligent Robots and Systems, pp. 410-416, 2007.
[48] IntuitiveSurgical, "Endowrist / Single-site Catalog," 2014.
[49] D. Q. Larkin, T. G. Cooper, and C. Mohr, "Minimally Invasive Surgical System," 2012.
[50] O. Robotics, "Snake-robots. http://www.ocrobotics.com/," (Accessed June 2018).
[51] R. O. Buckingham and a. C. Graham, "Dexterous manipulators for nuclear inspection and maintenance - Case study," 2010 1st International Conference on Applied Robotics for the Power Industry, CARPI 2010, pp. 1-6, 2010.
[52] R. Anscombe, P. Brandrick, A. Bryant, R. Buckingham, G. Ferguson, A. Graham, B. Green, M. Lichon, N. Parry, M. Redman, and M. Summers, "Snake-arm robots: a new approach to aircraft assembly," SAE International, 2006.
[53] M. C. Yip and D. B. Camarillo, "Model-Less Feedback Control of Continuum Manipulators in Constrained Environments," IEEE Transactions on Robotics, 2014.
[54] A. Degani, H. Choset, A. Wolf, T. Ota, and M. A. Zenati, "Percutaneous intrapericardial interventions using a highly articulated robotic probe," in Proceedings of the First IEEE/RAS-EMBS International Conference on Biomedical Robotics and Biomechatronics, 2006, BioRob 2006, vol. 2006, pp. 7-12, 2006.
[55] M. A. Zenati, "Medrobotics FLEX System,"
[56] T. Ota, A. Degani, D. Schwartzman, B. Zubiate, J. McGarvey, H. Choset, and M. A. Zenati, "A Highly Articulated Robotic Surgical System for Minimally Invasive Surgery," Annals of Thoracic Surgery, vol. 87, pp. 1253-1256, 2009.
[57] T. Ota, A. Degani, D. Schwartzman, B. Zubiate, J. Mcgarvey, H. Choset, and M. A. Zenati, "A Novel Highly Articulated Robotic Surgical System For Epicardial Ablation," pp. 250-253, 2008.
[58] T. Ota, A. Degani, B. Zubiate, A. Wolf, H. Choset, D. Schwartzman, and M. a. Zenati, "Epicardial Atrial Ablation Using a Novel Articulated Robotic Medical Probe Via a Percutaneous Subxiphoid Approach," Innovations: Technology and Techniques in Cardiothoracic and Vascular Surgery, vol. 1, no. 6, pp. 335-340, 2006.
[59] B. Kang, R. Kojcev, and E. Sinibaldi, "The first interlaced continuum robot, devised to intrinsically follow the leader," PLoS ONE, vol. 11, no. 2, pp. 1-16, 2016.
[60] L. A. Lyons, R. J. Webster III, and R. Alterovitz, "Motion planning for active cannulas," in 2009 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), (St. Louis, USA, October 11-15, 2009), pp. 801-806, 2009.
[61] R. J. Webster III, A. M. Okamura, and N. J. Cowan, "Toward active cannulas: Miniature snake-like surgical robots," in 2006 IEEE International Conference on Intelligent Robots and Systems (IROS), (Beijing, China, October 9-15, 2006), pp. 2857-2863, 2006.
[62] P. Sears and P. E. Dupont, "A steerable needle technology using curved concentric tubes," in 2006 IEEE/RSJ International Conference on Intelligent Robots and Systems, (Beijing, China, October 9-15, 2006), pp. 2850-2856, 2006.
[63] P. E. Dupont, J. Lock, and E. Butler, "Torsional kinematic model for concentric-tube robots," in 2009 IEEE International Conference on Robtics and Automation (ICRA), (Kobe, Japan, May 12-17, 2009), pp. 3851-3858, 2009.
[64] R. J. Webster III, J. M. Romano, and N. J. Cowan, "Mechanics of precurved-tube continuum robots," IEEE Transactions on Robotics, vol. 25, no. 1, pp. 67-78, 2009.
[65] M. Mahvash and P. E. Dupont, "Stiffness control of surgical continuum manipulators," IEEE Transactions on Robotics, vol. 27, no. 2, pp. 334-345, 2011.
[66] D. C. Rucker, B. a. Jones, and R. J. Webster III, "A geometrically exact model for externally loaded concentric-tube continuum robots," IEEE Transactions on Robotics, vol. 26, no. 5, pp. 769-780, 2010.
[67] J. Lock and P. E. Dupont, "Friction modeling in concentric tube robots," in 2011 IEEE International Conference on Robotics and Automation (ICRA), (Shanghai, China, May 9-13, 2011), pp. 1139-1146, 2011.
[68] P. E. Dupont, J. Lock, and B. Itkowitz, "Real-time position control of concentric tube robots," in 2010 IEEE International Conference on Robotics and Automation (ICRA), (Anchorage, Alaska, USA, May 3-8, 2010), pp. 562-568, 2010.
[69] P. E. Dupont, J. Lock, B. Itkowitz, and E. Butler, "Design and Control of ConcentricTube Robots," IEEE Transactions on Robotics, vol. 26, no. 2, pp. 209-225, 2010.
[70] L. a. Lyons, R. J. Webster III, and R. Alterovitz, "Planning active cannula configurations through tubular anatomy," in 2010 IEEE International Conference on Robotics and Automation, no. 1, (Anchorage, Alaska, USA, May 3-8, 2010), pp. 2082-2087, 2010.
[71] C. Bergeles and P. E. Dupont, "Planning stable paths for concentric tube robots," in 2013 IEEE International Conference on Intelligent Robots and Systems (IROS), (Tokyo, Japan, November 3-7, 2013), pp. 3077-3082, 2013.
[72] S. C. Ryu and P. E. Dupont, "FBG-based Shape Sensing Tubes for Continuum Robots," in 2014 IEEE International Conference on Robotics and Automation, (Hong Kong, China, May 31 - June 7, 2014), pp. 3531-3537, 2014.
[73] J. Burgner, P. J. Swaney, D. C. Rucker, H. B. Gilbert, S. T. Nill, P. T. Russell, K. D. Weaver, and R. J. Webster III, "A bimanual teleoperated system for endonasal skull base surgery," in 2011 IEEE/RSJ International Conference on Intelligent Robots and Systems, (San Francisco, CA, USA, 25-30 September, 2011), pp. 2517-2523, 2011.
[74] A. H. Gosline, N. V. Vasilyev, E. J. Butler, C. Folk, A. Cohen, R. Chen, N. Lang, P. J. Nido, and P. E. Dupont, "Percutaneous intracardiac beating-heart surgery using metal MEMS tissue," International Journal of Robotics Research, vol. 31, no. 9, pp. 1081-1093, 2012.
[75] N. V. Vasilyev, A. H. Gosline, E. Butler, N. Lang, P. J. Codd, H. Yamauchi, E. N. Feins, C. R. Folk, A. L. Cohen, R. Chen, D. Zurakowski, P. J. Del Nido, and P. E. Dupont,
"Percutaneous steerable robotic tool delivery platform and metal microelectromechanical systems device for tissue manipulation and approximation: Closure of patent foramen ovale in an animal model," Circulation: Cardiovascular Interventions, vol. 6, pp. 468475, 2013.
[76] E. J. Butler, R. Hammond-Oakley, S. Chawarski, A. H. Gosline, P. Codd, T. Anor, J. R. Madsen, P. E. Dupont, and J. Lock, "Robotic neuro-endoscope with concentric tube augmentation," in 2012 IEEE International Conference on Intelligent Robots and Systems (IROS), (Vilamoura, Algarve, Portugal, October 7-12, 2012), pp. 2941-2946, 2012.
[77] P. E. Dupont, A. Gosline, N. Vasilyev, J. Lock, E. Butler, C. Folk, A. Cohen, R. Chen, G. Schmitz, H. Ren, and P. del Nido, "Concentric Tube Robots for Minimally Invasive Surgery," in Hamlyn Symposium on Medical Robotics, pp. 3-5, 2012.
[78] P. E. Dupont, J. Lock, and E. Butler, "Torsional kinematic model for concentric-tube robots," IEEE International Conference on Robtics and Automation, pp. 3851-3858, 2009.
[79] P. J. Swaney, H. B. Gilbert, R. J. Hendrick, O. Commichau, R. Alterovitz, and R. J. W. Iii, "Transoral Steerable Needles in The Lung : How Non - Annular Concentric Tube Robots Can Improve Targeting," in Hamlyn Symposium on Medical Robotics, 2016.
[80] E. Greenblatt, T. Trovato, A. Popovic, and D. Stanton, "Interlocking nested cannula," 2011.
[81] T. Mahl, A. Hildebrandt, and O. Sawodny, "A variable curvature continuum kinematics for kinematic control of the bionic handling assistant," IEEE Transactions on Robotics and Automation, 2014.
[82] G. Chen, M. T. Pham, T. Maalej, H. Fourati, R. Moreau, and S. Sesmat, "A Biomimetic steering robot for Minimally invasive surgery application," IN-TECH, 2009.
[83] P. Polygerinos, Z. Wang, J. T. B. Overvelde, K. C. Galloway, R. J. Wood, K. Bertoldi, and C. J. Walsh, "Modeling of Soft Fiber-Reinforced Bending Actuators," IEEE Transactions on Robotics, vol. 31, no. 3, pp. 778-789, 2015.
[84] A. D. Marchese and D. Rus, "Design, kinematics, and control of a soft spatial fluidic elastomer manipulator," International Journal of Robotics Research, 2016.
[85] A. De Greef, P. Lambert, and A. Delchambre, "Towards flexible medical instruments: Review of flexible fluidic actuators," Precision Engineering, vol. 33, no. 4, pp. 311-321, 2009.
[86] D. Rus and M. T. Tolley, "Design, fabrication and control of soft robots," Nature, vol. 521, no. 7553, pp. 467-475, 2015.
[87] D. Trivedi, C. D. Rahn, W. M. Kierb, and I. D.Walkerc, "Soft robotics: Biological inspiration, state of the art, and future research," Applied Bionics and Biomechanics, 2008.
[88] B. Mosadegh, P. Polygerinos, C. Keplinger, S. Wennstedt, R. F. Shepherd, U. Gupta, J. Shim, K. Bertoldi, C. J. Walsh, and G. M. Whitesides, "Pneumatic networks for soft robotics that actuate rapidly," Advanced Functional Materials, vol. 24, no. 15, pp. 21632170, 2014.
[89] F. Ilievski, A. D. Mazzeo, R. F. Shepherd, X. Chen, and G. M. Whitesides, "Soft robotics for chemists," Angewandte Chemie - International Edition, vol. 50, no. 8, pp. 1890-1895, 2011.
[90] B. Chang, A. Chew, N. Naghshineh, and C. Menon, "A spatial bending fluidic actuator: fabrication and quasi-static characteristics," Smart Materials and Structures, vol. 21, no. 4, p. 045008, 2012.
[91] K. Ikuta, H. Ichikawa, K. Suzuki, and D. Yajima, "Multi-degree of freedom hydraulic pressure driven safety active catheter," Proceedings - IEEE International Conference on Robotics and Automation, vol. 2006, no. May, pp. 4161-4166, 2006.
[92] G. Agarwal, N. Besuchet, B. Audergon, and J. Paik, "Stretchable Materials for Robust Soft Actuators towards Assistive Wearable Devices," Scientific Reports, 2016.
[93] C. D. Santina, U. Pisa, R. K. Katzschmann, C. D. Santina, R. K. Katzschmann, A. Bicchi, and D. Rus, "Dynamic Control of Soft Robots Interacting with the Environment Dynamic Control of Soft Robots Interacting with the Environment," no. April, 2018.
[94] A. D. Marchese, R. K. Katzschmann, and D. Rus, "A Recipe for Soft Fluidic Elastomer Robots," Soft Robotics, vol. 2, no. 1, pp. 7-25, 2015.
[95] N. Correll, C. Onal, H. Liang, E. Schoenfeld, and D. Rus, "Soft Autonomous Materials - Using Programmed Elasticity and Embedded Distributed Computation," International Symposium on Experimental Robotics (ISER). Springer Tracts in Advanced Robotics., pp. 1-14, 2010.
[96] K. Korane, "Robot imitates nature," Machine Design, vol. 82, no. 18, pp. 68-70, 2010.
[97] K. Ogura, S. Wakimoto, K. Suzumori, and Y. Nishioka, "Micro pneumatic curling actuator - Nematode actuator -," in 2009 IEEE International Conference on Robotics and Biomimetics, pp. 462-467, 2009.
[98] K. Suzumori, S. Iikura, and H. Tanaka, "Applying a Flexible Microactuator to Robotic Mechanisms," IEEE International Conference on Robotics and Automation (ICRA), 1992.
[99] K. Suzumori, "Elastic Materials Producing Compliant Robots," Robotics and Autonomous Systems, 1996.
[100] K. Suzumori, S. Iikura, and H. Tanaka, "Flexible microactuator for miniature robots," [1991] Proceedings. IEEE Micro Electro Mechanical Systems, pp. 204-209, 1991.
[101] K. Suzumori, S. Iikura, and H. Tanaka, "Development of flexible microactuator and its applications to robotic mechanisms," IEEE International Conference on Robotics and Automation (ICRA), 1991.
[102] H. Taniguchi and H. Tanaka, "Applying Flexible-MicroActuators to Multi-Fingered Robot-Hand," 1996.
[103] Y. Nobumoto and H. Tanaka, "Enhanced Fma Legs for Quadruped Walking," Proceedings of the JFPS International Symposium on Fluid Power, vol. 1996, no. 3, pp. 115-120, 1996.
[104] K. Suzumori and A. Abe, "Applying Flexible Microactuators to Pipeline Inspection Robots," Robotics, Mechatronics and Manufacturing Systems, 1993.
[105] K. Suzumori, T. Miyagawa, M. Kimura, and Y. Hasegawa, "Micro inspection robot for 1-in pipes," IEEE/ASME Transactions on Mechatronics, vol. 4, no. 3, pp. 286-292, 1999.
[106] S. Wakimoto, I. Kumagai, and K. Suzumori, "Development of variable stiffness colonoscope consisting of pneumatic drive devices," International Journal of Automation Technology, vol. 5, no. 4, pp. 551-558, 2011.
[107] G. Chen, M. T. Pham, T. Maalej, H. Fourati, R. Moreau, and S. Sesmat, "A Biomimetic steering robot for Minimally invasive surgery application," IN-TECH, 2009.
[108] C. Ferraresi, A. M. Bertetto, and L. Mazza, "Design and realisation of a flexible pneumatic actuator for robotics," 1997.
[109] R. Abe, K. Takemura, K. Edamura, and S. Yokota, "Concept of a micro finger using electro-conjugate fluid and fabrication of a large model prototype," Sensors and Actuators, A: Physical, 2007.
[110] L. Yu, G. She, Z. Hu, and J. Cheng, "High speed switch electromagnetic valve control for a miniature pneumatic robot locomotion," Proceedings - 3rd International Conference on Measuring Technology and Mechatronics Automation, ICMTMA 2011, vol. 2, pp. 324327, 2011.
[111] G. Thomann, G. Chen, and T. Redarce, "Design and control of an autonomous bendable tip for colonoscopy," Journal of Micro-Nano Mechatronics, vol. 4, no. 3, pp. 103-114, 2008.
[112] M. Cianchetti, T. Nanayakkara, T. Ranzani, G. Gerboni, K. Althoefer, P. Dasgupta, and A. Menciassi, "Soft Robotics Technologies to Address Shortcomings in Today's Minimally

Invasive Surgery: The STIFF-FLOP Approach," Soft Robotics, vol. 1, no. 2, pp. 122-131, 2014.
[113] J. Czarnowski, M. Macia, J. Główka, M. Cianchetti, and A. Menciassi, "New STIFFFLOP module construction idea for improved actuation and sensing," in IEEE International Conference on Robotics and Automation (ICRA), pp. 2901-2906, 2015.
[114] H. Abidi, G. Gerboni, M. Brancadoro, J. Fras, A. Diodato, M. Cianchetti, H. Wurdemann, K. Althoefer, and A. Menciassi, "Highly dexterous 2-module soft robot for intra-organ navigation in minimally invasive surgery," International Journal of Medical Robotics and Computer Assisted Surgery, vol. 14, no. 1, pp. 1-9, 2018.
[115] A. Arezzo, Y. Mintz, M. E. Allaix, S. Arolfo, M. Bonino, G. Gerboni, M. Brancadoro, M. Cianchetti, A. Menciassi, H. Wurdemann, Y. Noh, K. Althoefer, J. Fras, J. Glowka, Z. Nawrat, G. Cassidy, R. Walker, and M. Morino, "Total mesorectal excision using a soft and flexible robotic arm: a feasibility study in cadaver models," Surgical Endoscopy and Other Interventional Techniques, vol. 31, no. 1, pp. 264-273, 2017.
[116] G. Gerboni, T. Ranzani, A. Diodato, G. Ciuti, M. Cianchetti, and A. Menciassi, "Modular soft mechatronic manipulator for minimally invasive surgery (MIS): overall architecture and development of a fully integrated soft module," Meccanica, vol. 50, no. 11, pp. 28652878, 2015.
[117] D. Lane, J. Davies, G. Casalino, G. Bartolini, G. Cannata, G. Veruggio, M. Canals, and C. Smith, "AMADEUS: Advanced MAnipulation for DEep Underwater Sampling," Robotics, no. December, 1997.
[118] D. O'Brien and D. Lane, "3D force control system design for a hydraulic parallel bellows continuum actuator," Proceedings 2001 ICRA. IEEE International Conference on Robotics and Automation (Cat. No.01CH37164), vol. 3, pp. 2375-2380, 2001.
[119] S. Hirai, T. Masui, and S. Kawamura, "Prototyping Pneumatic Group Actuators Composed of Multiple Single-motion Elastic Tubes," vol. 20, no. 3, pp. 299-306, 2001.
[120] Q. Y. Q. Yang, L. Z. L. Zhang, G. B. G. Bao, S. X. S. Xu, and J. R. J. Ruan, "Research on novel flexible pneumatic actuator FPA," IEEE Conference on Robotics, Automation and Mechatronics, 2004., vol. 1, no. c, pp. 1-3, 2004.
[121] J. Bishop-Moser, G. Krishnan, C. Kim, and S. Kota, "Design of soft robotic actuators using fluid-filled fiber-reinforced elastomeric enclosures in parallel combinations," in IEEE International Conference on Intelligent Robots and Systems, pp. 4264-4269, 2012.
[122] R. Kang, Y. Guo, L. Chen, D. T. Branson, and J. S. Dai, "Design of a Pneumatic Muscle Based Continuum Robot with Embedded Tendons," IEEE/ASME Transactions on Mechatronics, vol. 22, no. 2, pp. 751-761, 2017.
[123] C.-P. Chou and B. Hannaford, "Measurement and modeling of McKibben pneumatic artificial muscles," IEEE Transactions on Robotics and Automation, vol. 12, no. 1, pp. 90102, 1996.
[124] F. Daerden and D. Lefeber, "Pneumatic artificial muscles: actuators for robotics and automation," European Journal of Mechanical and Environmental Engineering, vol. 47, no. 1, pp. 11-21, 2002.
[125] W. McMahan, V. Chitrakaran, M. Csencsits, D. Dawson, I. D. Walker, B. Jones, M. Pritts, D. Dienno, M. Grissom, and C. D. Rahn, "Field trials and testing of the OcotArm continuum manipulator," IEEE international conference on robotics and automation (ICRA), 2006.
[126] I. D. Walker, D. M. Dawson, T. Flash, F. W. Grasso, R. T. Hanlon, B. Hochner, W. M. Kier, C. C. Pagano, C. D. Rahn, and Q. M. Zhang, "Continuum robot arms inspired by cephalopods," SPIE Conference on Unmanned Ground Vehicle Technology, vol. 5804, pp. 303-314, 2005.
[127] N. Giri and I. D. Walker, "Three module lumped element model of a continuum arm section," in IEEE International Conference on Intelligent Robots and Systems (IROS), (San Francisco, CA, USA, September 25-30), pp. 4060-4065, 2011.
[128] I. S. Godage, G. A. Medrano-Cerda, D. T. Branson, E. Guglielmino, and D. G. Caldwell, "Dynamics for variable length multisection continuum arms," International Journal of Robotics Research, vol. 35, no. 6, pp. 695-722, 2015.
[129] I. S. Godage and I. D. Walker, "Dual Quaternion based modal kinematics for multisection continuum arms," Proceedings - IEEE International Conference on Robotics and Automation, vol. 2015-June, no. June, pp. 1416-1422, 2015.
[130] I. S. Godage, R. Wirz, I. D. Walker, and R. J. Webster, "Accurate and Efficient Dynamics for Variable-Length Continuum Arms: A Center of Gravity Approach," Soft Robotics, vol. 2, no. 3, pp. 96-106, 2015.
[131] A. Devreker, B. Rosa, A. Desjardins, E. J. Alles, L. C. Garcia-Peraza, E. Maneas, D. Stoyanov, A. L. David, T. Vercauteren, J. Deprest, S. Ourselin, D. Reynaerts, and E. Vander Poorten, "Fluidic actuation for intra-operative in situ imaging," in IEEE International Conference on Intelligent Robots and Systems, vol. 2015-Decem, pp. 1415-1421, 2015.
[132] G. Smoljkic, G. Borghesan, D. Reynaerts, J. D. Schutter, J. V. Sloten, and E. V. Poorten, "Hybrid robotic system for applications in robotic surgery," Proceedings of the 5th Joint Workshop on New Technologies for Computer/Robot Assisted Surgery, pp. 1-3, 2015.
[133] G. Smoljkic, G. Borghesan, A. Devreker, E. V. Poorten, B. Rosa, H. De Praetere, J. De Schutter, D. Reynaerts, and J. V. Sloten, "Control of a hybrid robotic system for computer-assisted interventions in dynamic environments," International Journal of Computer Assisted Radiology and Surgery, vol. 11, no. 7, pp. 1371-1383, 2016.
[134] Y. Lu and C.-J. Kim, "Characterisation of Balloon-joined micro-fingers," pp. 1-6, 2003.
[135] K. Suzumori, A. Koga, F. Kondo, and R. Haneda, "Integrated flexible microactuator systems," Robotica, vol. 14, no. 05, p. 493, 1996.
[136] K. Suzumori, T. Maeda, H. Watanabe, and T. Hisada, "Fiberless flexible microactuator designed by finite-element method," IEEE/ASME Transactions on Mechatronics, vol. 2, no. 4, pp. 281-286, 1997.
[137] K. Suzumori, S. Endo, T. Kanda, N. Kato, and H. Suzuki, "A bending pneumatic rubber actuator realizing soft-bodied manta swimming robot," Proceedings - IEEE International Conference on Robotics and Automation, no. April, pp. 4975-4980, 2007.
[138] H. S. H. Shin, K.-M. J. K.-M. Jeong, and J.-J. K. J.-J. Kwon, "Development of a snake robot moving in a small diameter pipe," Control Automation and Systems (ICCAS), 2010 International Conference on, no. 1, pp. 1826-1829, 2010.
[139] O. Salomon and A. Wolf, "Inclined Links Hyper-Redundant Elephant Trunk-Like Robot," Journal of Mechanisms and Robotics, vol. 4, p. 045001, 2012.
[140] A. Johnson, C. Wright, M. Tesch, K. Lipkin, and H. Choset, "A Novel Architecture for Modular Snake Robots," Ricmuedu, pp. 1-7, 2011.
[141] J. Shang, D. P. Noonan, C. Payne, J. Clark, M. H. Sodergren, A. Darzi, and G. Z. Yang, "An articulated universal joint based flexible access robot for minimally invasive surgery," Proceedings - IEEE International Conference on Robotics and Automation, vol. 1, pp. 1147-1152, 2011.
[142] P. Vartholomeos, C. Bergeles, L. Qin, and P. E. Dupont, "An MRI-powered and controlled actuator technology for tetherless robotic interventions," The International Journal of Robotics Research, vol. 32, pp. 1536-1552, 2013.
[143] P. Liljebäck, Ø. Stavdahl, K. Y. Pettersen, and J. T. Gravdahl, "A modular and waterproof snake robot joint mechanism with a novel force/torque sensor," IEEE International Conference on Intelligent Robots and Systems, pp. 4898-4905, 2012.
[144] C. Wright, A. Buchan, B. Brown, J. Geist, M. Schwerin, D. Rollinson, M. Tesch, and H. Choset, "Design and architecture of the unified modular snake robot," Proceedings IEEE International Conference on Robotics and Automation, pp. 4347-4354, 2012.
[145] E. Shammas, A. Wolf, J. Brown, H.B., and H. Choset, "New joint design for threedimensional hyper redundant robots," Proceedings 2003 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS 2003) (Cat. No.03CH37453), vol. 4, no. October, pp. 3594-3599, 2003.
[146] J. Shang, C. J. Payne, J. Clark, D. P. Noonan, K. W. Kwok, A. Darzi, and G. Z. Yang, "Design of a multitasking robotic platform with flexible arms and articulated head for Minimally Invasive Surgery," IEEE International Conference on Intelligent Robots and Systems, pp. 1988-1993, 2012.
[147] D. P. Noonan, V. Vitiello, J. Shang, C. J. Payne, and G. Z. Yang, "A modular, mechatronic joint design for a flexible access platform for MIS," IEEE International Conference on Intelligent Robots and Systems, pp. 949-954, 2011.
[148] Z. Zhang, J. Shang, C. Seneci, and G. Z. Yang, "Snake robot shape sensing using microinertial sensors," IEEE International Conference on Intelligent Robots and Systems, pp. 831-836, 2013.
[149] K. W. Kwok, K. Hung Tsoi, V. Vitiello, J. Clark, G. C. T. Chow, W. Luk, and G. Z. Yang, "Dimensionality reduction in controlling articulated snake robot for endoscopy under dynamic active constraints," IEEE Transactions on Robotics, vol. 29, no. 1, pp. 15-31, 2013.
[150] C. Stefanini, S. Orofino, L. Manfredi, S. Mintchev, S. Marrazza, T. Assaf, L. Capantini, E. Sinibaldi, S. Grillner, P. Wallen, and P. Dario, "A compliant bioinspired swimming robot with neuro-inspired control and autonomous behavior," Proceedings - IEEE International Conference on Robotics and Automation, pp. 5094-5098, 2012.
[151] W. Fischer, G. Caprari, R. Siegwart, I. Thommen, W. Zesch, and R. Moser, "Foldable magnetic wheeled climbing robot for the inspection of gas turbines and similar environments with very narrow access holes," Industrial Robot: An International Journal, vol. 37, no. 3, pp. 244-249, 2010.
[152] Y. Bar-cohen, "Electro-active polymers : current capabilities and challenges," 2002.
[153] G. Caprari, A. Breitenmoser, W. Fischer, C. Hurzeler, F. Tache, and R. Siegwart, "Highly Compact Robots for Inspection of Power Plants," Journal of Field Robotics, 2011.
[154] Y. Fu, E. C. Harvey, M. K. Ghantasala, and G. M. Spinks, "Design, fabrication and testing of piezoelectric polymer PVDF microactuators," Smart Materials and Structures, vol. 15, no. 1, pp. S141-S146, 2005.
[155] C. Keplinger, M. Kaltenbrunner, N. Arnold, and S. Bauer, "Rontgen's electrode-free elastomer actuators without electromechanical pull-in instability.," Proceedings of the National Academy of Sciences of the United States of America, vol. 107, no. 10, pp. 45054510, 2010.
[156] R. Pelrine, "High-Speed Electrically Actuated Elastomers with Strain Greater Than $100 \%$," Science, vol. 287, no. 5454, pp. 836-839, 2000.
[157] a. W. Richards and G. M. Odegard, "Constitutive Modeling of Electrostrictive Polymers Using a Hyperelasticity-Based Approach," Journal of Applied Mechanics, 2010.
[158] E. legislation), "Directive 94/9/EC of the European Parliament and the Council," 1994.
[159] H. Funakubo, Shape Memory Alloys. 1987.
[160] J. Mohd Jani, M. Leary, A. Subic, and M. a. Gibson, "A review of shape memory alloy research, applications and opportunities," Materials and Design, vol. 56, pp. 1078-1113, 2014.
[161] S. Lagoudas, Shape Memory Alloys: modelling and engineering applications. Springer, 2010.
[162] W. Huang and W. Toh, "Training two-way shape memory alloy by reheat treatment," Journal of Materials Science Letters, vol. 19, no. 17, pp. 1549-1550, 2000.
[163] K. Ikuta, M. Tsukamoto, and S. Hirose, "Shape memory alloy servo actuator system with electric resistance feedback and application for active endoscope," Proceedings. 1988 IEEE International Conference on Robotics and Automation, pp. 427-430, 1988.
[164] Y. Haga and M. Esashi, "Small Diameter Active Catheter Using Shape Memory Alloy Coils," IEEJ Transactions on Sensors and Micromachines, 1998.
[165] Y. Haga, M. Mizushima, T. Matsunaga, and M. Esashi, "Medical and welfare applications of shape memory alloy microcoil actuators," Smart Materials and Structures, vol. 14, no. 5, pp. S266-S272, 2005.
[166] G. Lim, K. Park, M. Sugihara, K. Minami, and M. Esashi, "Future of active catheters," Sensors and Actuators A: Physical, vol. 56, no. 1-2, pp. 113-121, 1996.
[167] M. Ho and J. P. Desai, "Characterization of SMA actuator for applications in robotic neurosurgery," Proceedings of the 31st Annual International Conference of the IEEE Engineering in Medicine and Biology Society: Engineering the Future of Biomedicine, EMBC 2009, pp. 6856-6859, 2009.
[168] S. Hirose, K. Ikuta, and M. Tsukamoto, "Development of a shape memory alloy actuator. Measurement of material characteristics and development of active endoscopes," Advanced Robotics, vol. 4, no. 1, pp. 3-27, 1989.
[169] T. Fukuda, S. G. S. Guo, K. Kosuge, F. Arai, M. Negoro, and K. Nakabayashi, "Micro active catheter system with multi degrees of freedom," Proceedings of the 1994 IEEE International Conference on Robotics and Automation, pp. 2290-2295, 1994.
[170] T. Mineta, T. Mitsui, Y. Watanabe, S. Kobayashi, Y. Haga, and M. Esashi, "Batch fabricated flat meandering shape memory alloy actuator for active catheter," Sensors and Actuators, A: Physical, vol. 88, no. 2, pp. 112-120, 2001.
[171] J. Jayender, R. Patel, S. Nikumb, and M. Ostojic, "Modelling and gain scheduled control of shape memory alloy actuators," Proceedings of 2005 IEEE Conference on Control Applications, 2005. CCA 2005., pp. 767-772, 2005.
[172] J. Jayender, R. Patel, S. Nikumb, and M. Ostojic, "H_inf Loop Shaping Controller for Shape Memory Alloy Actuators," Proceedings of the 44th IEEE Conference on Decision and Control, pp. 653-658, 2005.
[173] a. Veeramani, G. Buckner, S. Owen, and R. Cook, "Modeling the dynamic behavior of a shape memory alloy actuated catheter," Smart Mater. Struct., vol. 17, 2008.
[174] J. Jayender, R. Patel, S. Nikumb, and M. Ostojic, "Modeling and Control of Shape Memory Alloy Actuators," IEEE Transactions on Control Systems Technology, 2008.
[175] H. B. Gilbert, J. Neimat, and R. J. Webster III, "Concentric Tube Robots as Steerable Needles : Achieving Follow-the-Leader Deployment," IEEE Transactions on Robotics, vol. 31, no. 2, pp. 246-258, 2015.
[176] R. L. Bishop, "There is more than one way to frame a curve," American Mathematical Monthly, vol. 82, pp. 246-251, 1975.
[177] M. De Volder and D. Reynaerts, "Pneumatic and hydraulic microactuators: a review," Journal of Micromechanics and Microengineering, vol. 20, no. 4, p. 043001, 2010.
[178] W. McMahan, V. Chitrakaran, M. Csencsits, D. Dawson, I. D. Walker, B. Jones, M. Pritts, D. Dienno, M. Grissom, and C. D. Rahn, "Field trials and testing of the OcotArm continuum manipulator," IEEE international conference on robotics and automation (ICRA), 2006.
[179] N. Giri and I. D. Walker, "Three module lumped element model of a continuum arm section," in IEEE International Conference on Intelligent Robots and Systems (IROS), (San Francisco, CA, USA, September 25-30), pp. 4060-4065, 2011.
[180] "Soft Robotics Toolkit. https://softroboticstoolkit.com," (Accessed March 2018).
[181] D. B. Camarillo, C. F. Milne, C. R. Carlson, M. R. Zinn, and J. K. Salisbury, "Mechanics modeling of tendon-driven continuum manipulators," IEEE Transactions on Robotics, vol. 24, no. 6, pp. 1262-1273, 2008.
[182] A. Devreker, B. Rosa, A. Desjardins, E. J. Alles, L. C. Garcia-Peraza, E. Maneas, D. Stoyanov, A. L. David, T. Vercauteren, J. Deprest, S. Ourselin, D. Reynaerts, and E. Vander Poorten, "Fluidic actuation for intra-operative in situ imaging," in IEEE International Conference on Intelligent Robots and Systems, vol. 2015-Decem, pp. 1415-1421, 2015.
[183] M. Cianchetti, T. Ranzani, G. Gerboni, I. De Falco, C. Laschi, and A. Menciassi, "STIFFFLOP surgical manipulator: Mechanical design and experimental characterization of the
single module," in IEEE International Conference on Intelligent Robots and Systems, pp. 3576-3581, 2013.
[184] F. Connolly, C. J. Walsh, and K. Bertoldi, "Automatic design of fiber-reinforced soft actuators for trajectory matching," Proceedings of the National Academy of Sciences, vol. 114, no. 1, p. 201615140, 2016.
[185] J. Bishop-Moser, G. Krishnan, C. Kim, and S. Kota, "Design of soft robotic actuators using fluid-filled fiber-reinforced elastomeric enclosures in parallel combinations," IEEE International Conference on Intelligent Robots and Systems (IROS), 2012.
[186] C. G. Frazelle, A. Kapadia, and I. Walker, "Developing a Kinematically Similar Master Device for Extensible Continuum Robot Manipulators," Journal of Mechanisms and Robotics, vol. 10, no. 2, p. 025005, 2018.
[187] B. A. Jones, W. Mcmahan, and I. Walker, "Design and Analysis of a Novel Pneumatic Manipulator," IFAC Proceedings Volumes, 2004.
[188] B. A. Jones and I. D. Walker, "Kinematics for Multisection Continuum Robots," IEEE Transactions on Robotics, vol. 22, pp. 43-55, 2006.
[189] G. S. Chirikjian and J. W. Burdick, "A Modal approach to hyper-redundant manipulator kinematics," IEEE Transactions on Robotics and Automation, vol. 10, no. 3, pp. 343-354, 1994.
[190] G. S. Chirikjian, "Hyper-redundant manipulator dynamics: a continuum approximation," Advanced Robotics, vol. 9, no. 3, 1995.
[191] F. Fahimi, H. Ashrafiuon, and C. Nataraj, "An improved inverse kinematic and velocity solution for spatial hyper-redundant robots," IEEE Transactions on Robotics and Automation, vol. 18, 2002.
[192] B. A. Jones and I. D. Walker, "Practical kinematics for real-time implementation of continuum robots," IEEE Transactions on Robotics, 2006.
[193] K. Xu and N. Simaan, "Analytic Formulation for Kinematics, Statics, and Shape Restoration of Multibackbone Continuum Robots Via Elliptic Integrals," Journal of Mechanisms and Robotics, 2010.
[194] A. D. Kapadia and I. D. Walker, "Self-Motion Analysis of Extensible Continuum Manipulators," in IEEE International Conference on Robotics and Automation (ICRA), pp. 1988-1994, 2013.
[195] M. W. Hannan and I. D. Walker, "Kinematics and the Implementation of an Elephant's Trunk Manipulator and Other Continuum Style Robots," Journal of Robotic Systems, vol. 20, no. 2, pp. 45-63, 2003.
[196] S. Neppalli, M. A. Csencsits, B. A. Jones, and I. D. Walker, "Closed-Form Inverse Kinematics for Continuum Manipulators," Advanced Robotics, vol. 23, pp. 2077-2091, 2009.
[197] R. M. Murray, Z. Li, and S. S. Sastry, A Mathematical Introduction to Robotic Manipulation. CRC Press, 1994.
[198] S. Kehrbaum and J. Maddocks, "Elastic rods, rigid bodies, quaternions and the last quadrature," Phil. Trans. R. Soc. Lond. A, pp. 2117-2136, 1997.
[199] E. Celledoni and N. Säfström, "A Hamiltonian and multi-Hamiltonian formulation of a rod model using quaternions," Computer Methods in Applied Mechanics and Engineering, vol. 199, no. 45-48, pp. 2813-2819, 2010.
[200] D. Trivedi, A. Lotfi, and C. D. Rahn, "Geometrically exact dynamic models for soft robotic manipulators," 2007 IEEE/RSJ International Conference on Intelligent Robots and Systems, vol. 24, no. 4, pp. 1497-1502, 2007.
[201] I. Tunay, "Spatial continuum models of rods undergoing large deformation and inflation," IEEE Transactions on Robotics, vol. 29, no. 2, pp. 297-307, 2013.
[202] J. Burgner-Kahrs, D. C. Rucker, and H. Choset, "Continuum Robots for Medical Applications: A Survey," IEEE Transactions on Robotics, vol. 31, no. 6, pp. 1261-1280, 2015.
[203] J. Park and W. K. Chung, "Geometric integration on Euclidean group with application to articulated multibody systems," IEEE Transactions on Robotics, vol. 21, no. 5, pp. 850863, 2005.
[204] I. S. Godage and I. D. Walker, "Dual Quaternion based modal kinematics for multisection continuum arms," Proceedings - IEEE International Conference on Robotics and Automation, vol. 2015-June, no. June, pp. 1416-1422, 2015.
[205] NASA, "Euler angles, quaternions, and rotation matrices," Working Relationships, Mission Planning and Analysis Division, 1977.
[206] E. Franco, "IDA-PBC with Adaptive Friction Compensation for Underactuated Mechanical Systems," International Journal of Control, vol. (in press), 2018.
[207] R. Ortega, A. J. V. D. Schaft, I. Mareels, and B. Maschke, "Putting energy back in control," IEEE Control Systems Magazine, vol. 21, pp. 18-33, April 2001.
[208] R. Ortega, M. W. Spong, F. Gomez-Estern, and G. Blankestein, "Stabilization of a class of underactuated mechanical systems via interconnection and damping assignment," IEEE Transactions on Automatic Control, vol. 47, no. 8, pp. 1218-1233, 2002.
[209] N. Crasta, R. Ortega, and H. K. Pillai, "On the matching equations of energy shaping controllers for mechanical systems," International Journal of Control, vol. 88, no. 9, pp. 1757-1765, 2015.
[210] A. Donaire, R. Ortega, and J. G. Romero, "Simultaneous interconnection and damping assignment passivity-based control of mechanical systems using dissipative forces," Proceedings of the American Control Conference, vol. 2016-July, pp. 6610-6615, 2016.
[211] E. Franco, "Discrete-time IDA-PBC for underactuated mechanical systems with inputdelay and matched disturbances," 2018 26th Mediterranean Conference on Control and Automation (MED), no. 1, pp. 1-9, 2018.
[212] E. Franco, A. Astolfi, and F. Rodriguez y Baena, "Robust balancing control of flexible inverted-pendulum systems," Mechanism and Machine Theory, vol. In press, pp. 1-19, 2018.
[213] M. Li, R. Kang, D. T. Branson, and J. S. Dai, "Model-free Control for Continuum Robots Based on an Adaptive Kalman Filter," IEEE/ASME Transactions on Mechatronics, vol. 23, no. 1, pp. 286-297, 2018.
[214] K. Youcef-Toumi and O. Ito, "A Time Delay Controller for Systems With Unknown Dynamics," Journal of Dynamic Systems, Measurement, and Control, vol. 112, no. 1, p. 133, 1990.
[215] A. Astolfi and R. Ortega, "Immersion and invariance: A new tool for stabilization and adaptive control of nonlinear systems," IEEE Transactions on Automatic Control, vol. 48, no. 4, pp. 590-606, 2003.
[216] J. Nickolls and W. J. Dally, "The gpu computing era," IEEE Micro, vol. 30, pp. 56-69, 2010.
[217] K. a. Daltorio, T. E. Wei, a. D. Horchler, L. Southard, G. D. Wile, R. D. Quinn, S. N. Gorb, and R. E. Ritzmann, "Mini-Whegs TM Climbs Steep Surfaces Using Insect-inspired Attachment Mechanisms," The International Journal of Robotics Research, vol. 28, no. 2, pp. 285-302, 2009.
[218] L. Palmieri and L. Schenato, "Distributed Optical Fiber Sensing Based on Rayleigh Scattering," The Open Optics Journal, vol. 7, pp. 104-127, 2013.
[219] K. Thyagarajan and A. Ghatak, "Fiber optic essentials," John Wiley \& Sons, Inc., pp. 168-185, 2007.
[220] A. J. Davison, I. D. Reid, N. D. Molton, and O. Stasse, "MonoSLAM: Real-time single camera SLAM," IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 29, no. 6, pp. 1052-1067, 2007.
[221] J. Rivera-Rubio, I. Alexiou, and A. a. Bharath, "Appearance-based indoor localization: A comparison of patch descriptor performance," 2015.
[222] R. Halmshaw, Industrial Radiology: Theory and Practice. Non-Destructive Evaluation Series. Springer Netherlands, 1995.
[223] X. P. Malgaue, Theory and Practice of Infrared Technology for Nondestructive Testing. Wiley New York, 2001.
[224] H. L. Libby, Introduction to Electromagnetic Nondestructive Testing Methods. WileyInterscience, 1971.
[225] R. C. McMaster, "Introduction to electromagnetic testing," in R. C. McMaster, P. McIntire, and M. L. Mester, editors, Electromagnetic Testing, volume 4 of Nondestructive Testing Handbook, pp. 1-23., American Society for Non-destructive Testing, Columbus, $\mathrm{OH}, 2$ nd edition ed., 1986.
[226] I. S. Hwang and R. G. Ballinger, "A multi-frequency AC potential drop technique for the detection of small cracks," Measurement Science and Technology, vol. 3, no. 1, pp. 62-74, 1992.
[227] N. Bowler and Y. Huang, "Model-based characterization of homogeneous metal plates by four-point alternating current potential drop measurements," IEEE Transactions on Magnetics, vol. 41, no. 6, pp. 2102-2110, 2005.
[228] M. C. Lugg, "Data interpretation in ACPD crack inspection," NDT International, vol. 22, no. 3, pp. 149-154, 1989.
[229] M. D. H. Beevers and C. J., The DC electrical potential method for crack length measurement. Engineering Materials Advisory Services, Warley, UK, 1980.
[230] G. Sposito, Advances in potential drop techniques for non-destructive testing. PhD thesis, 2009.
[231] R. LeTessier, R. Coade, and B. Geneve, "Sizing of cracks using the alternating current field measurement technique," International Journal of Pressure Vessels and Piping, vol. 79, no. 8-10, pp. 549-554, 2002.
[232] C. Raine and L. A., "Additional applications with the Alternating Current Field Measurement (ACFM) technique," Insight, 1998.
[233] D. L. Atherton, "From high-resolution MFL signals to accurate defect sizing," Pipes and Pipelines International, 1995.
[234] S. S. Udpa and R. K. Stanley, "Magnetic flux leakage testing," in S. S. Udpa and P. O. Moore, editors, Electromagnetic Testing, volume 5 of Non-destructive Testing Handbook, pp. 227-245, American Society of Nondestructive Testing, Columbus, OH, 2nd editio ed., 2004.
[235] J. T. Schmidt, K. Skeie, and P. McIntire, "Magnetic Particle Testing," in Nondestructive Testing Handbook, ch. Volume 6, American Society for Nonde- structive Testing, Columbus, OH, 2nd editio ed., 1989.
[236] J. Blitz and S. Geoff, Ultrasonic methods of non-destructive testing. Chapman \& Hall, 1996.
[237] K. F. Graff, Wave Motion in Elastic Solids. Dover Publications, 1991.
[238] L. Lynnworth, Ultrasonic measurements for process control: theory, tech- niques, applications. Academic Press, 1989.
[239] J. P. Charlesworth and J. A. G. Temple, Engineering Applications of Ultrasonic Time-of-Flight Diffraction Second Edition. Research Studies Press LTD, 2001.
[240] B. W. Drinkwater and P. D. Wilcox, "Ultrasonic arrays for non-destructive evaluation: A review," NDT and E International, vol. 39, no. 7, pp. 525-541, 2006.
[241] C. Holmes, B. W. Drinkwater, and P. D. Wilcox, "Post-processing of the full matrix of ultrasonic transmit-receive array data for non-destructive evaluation," NDT\&E International, 2005.
[242] A. Velichko and P. D. Wilcox, "An analytical comparison of ultrasonic array imaging algorithms.," The Journal of the Acoustical Society of America, vol. 127, no. March 2009, pp. 2377-2384, 2010.
[243] M. Hirao and H. Ogi, Emats for Science and Industry: Noncontacting Ultrasonic Measurements. Kluwer Academic Publishers, 2003.
[244] R. Ribichini, F. Cegla, P. B. Nagy, and P. Cawley, "Evaluation of electromagnetic acoustic transducer performance on steel materials," NDT\&E International, 2012.
[245] J. Isla, M. Seher, R. Challis, and F. Cegla, "Optimal impedance on transmission of Lorentz force EMATs," in Review of Progress in Quantitative Nondestructive Evaluation, 2016.
[246] C. Scruby and L. Drain, Laser ultrasonics: techniques and applications. CRC Press, 1990.
[247] J. P. Monchalin, "Optical Detection of Ultrasound," IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control, vol. 33, no. 5, pp. 485-499, 1986.
[248] K. Ltd, Radiography in Modern Industry. Kodak Ltd, Rochester, New York, USA, 4th edition ed., 1980.
[249] P. Reimers, W. Gilboy, and J. Goebbels, "Recent developments in the industrial application of computerized tomography with ionizing radiation," NDT International, vol. 17, no. 4, pp. 197-207, 1984.
[250] M. G. Silk, A. M. Stoneham, and J. A. G. Temple, The reliability of non-destructive inspection: assessing the assessment of structures under stress. Arthur Hilger, 1987.
[251] D. E. Bray and D. McBride, Nondestructive testing techniques. Wiley-Interscience, 1992.
[252] B. F. Larson, "Study of the factors affecting the sensitivity of liquid penetrant inspections: Review of literature published from 1970 to 1998," FAA Technical Report Number DOT/FAA/AR-01/95, 2002.
[253] H. B. and J. V., "Liquid Penetrant Inspection," in Non-Destructive Testing, pp. 7 - 17, Palgrave, London, 1988.
[254] X. P. Malgaue, Theory and Practice of Infrared Technology for Nondestructive Testing. Wiley New York, 2001.
[255] D. P. Almond, B. Weekes, T. Li, S. G. Pickering, E. Kostson, J. Wilson, G. Y. Tian, S. Dixon, and S. Burrows, "Thermographic techniques for the detection of cracks in metallic components," Insight: Non-Destructive Testing and Condition Monitoring, vol. 53, no. 11, pp. 614-620, 2011.
[256] D. P. Almond and S. G. Pickering, "Analysis of the delamination detection capabilities of pulse stimulated thermographic non-destructive testing techniques," Materials Evaluation, 2014.
[257] D. P. Almond and S. G. Pickering, "An analytical study of the pulsed thermography defect detection limit," Journal of Applied Physics, vol. 111, no. 9, 2012.
[258] S. Quek and D. P. Almond, "Defect detection capability of pulsed transient thermography," Insight-Non-Destructive Testing and Condition Monitoring, vol. 47, no. 4, pp. 212215, 2005.
[259] A. P. Mouritz, "Nondestructive inspection and structural health monitoring of aerospace materials," in Introduction to Aerospace Materials, pp. 534 - 557, Woodhead Publishing, 2012.
[260] J. Bamberg and M. Spies, "Optimal probe arrangement for ultrasonic inspection of spin test disks," in AIP Conference Proceedings, 2008.
[261] M. Drozdz, L. Moreau, M. Castaings, M. J. S. Lowe, and P. Cawley, "Efficient numerical modelling of absorbing regions for boundaries of guided waves problems," AIP Conference Proceedings, vol. 820 I, no. 2006, pp. 126-133, 2006.
[262] P. Rajagopal, M. Drozdz, E. A. Skelton, M. J. S. Lowe, and R. V. Craster, "On the use of absorbing layers to simulate the propagation of elastic waves in unbounded isotropic
media using commercially available Finite Element packages," NDT and E International, vol. 51, pp. 30-40, 2012.
[263] G. D. Smith, Numerical Solution of Partial Differential Equations: Finite Difference Methods. Oxford University Press, 1985.
[264] P. Huthwaite, "Accelerated finite element elastodynamic simulations using the GPU," Journal of Computational Physics, vol. 257, pp. 687-707, 2014.
[265] J. A. Ogilvy and J. A. G. Temple, "Diffraction of elastic waves by cracks: application to time-of-flight inspection," Ultrasonics, vol. 21, no. 6, pp. 259-269, 1983.
[266] J. Zhang, B. Drinkwater, and P. D. Wilcox, "Defect characterization using an ultrasonic array Defect Characterization Using an Ultrasonic Array to Measure the Scattering Coefficient Matrix," IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control, vol. 55, no. October, pp. 2254-2265, 2008.
[267] EXTENDE, "CIVA software. www.extende.com/civa-in-a-few-words," (Accessed April 2018).
[268] R. Phillips, D. Duxbury, P. Huthwaite, and M. Lowe, "Simulating the ultrasonic scattering from complex surface-breaking defects with a three-dimensional hybrid model," NDT and E International, vol. 97, no. March, pp. 32-41, 2018.
[269] N. Brierley, The computational enhancement of automated non-destructive inspection. PhD thesis, 2014.
[270] D. Glozman and M. Shoham, "Image-guided robotic flexible needle steering," Robotics, IEEE Transactions on, vol. 23, no. 3, pp. 459-467, 2007.
[271] R. J. Webster, J. S. Kim, N. J. Cowan, G. S. Chirikjian, and A. M. Okamura, "Nonholonomic modeling of needle steering," The International Journal of Robotics Research, vol. 25, no. 5-6, pp. 509-525, 2006.
[272] N. van de Berg, D. van Gerwen, J. Dankelman, and J. van den Dobbelsteen, "Design choices in needle steering 2014;a review," Mechatronics, IEEE/ASME Transactions on, vol. PP, no. 99, pp. 1-12, 2014.
[273] K. Reed, A. Majewicz, V. Kallem, R. Alterovitz, K. Goldberg, N. Cowan, and A. Okamura, "Robot-assisted needle steering," Robotics Automation Magazine, IEEE, vol. 18, pp. 35-46, Dec 2011.
[274] K. Reed, V. Kallem, R. Alterovitz, K. Goldberg, A. Okamura, and N. Cowan, "Integrated planning and image-guided control for planar needle steering," in Biomedical Robotics and Biomechatronics, 2008. BioRob 2008. 2nd IEEE RAS EMBS Int. Conf. on, pp. 819 -824 , oct. 2008.
[275] V. Kallem and N. Cowan, "Image guidance of flexible tip-steerable needles," Robotics, IEEE Trans. on, vol. 25, pp. 191 -196, feb. 2009.
[276] J. A. Engh, D. S. Minhas, D. Kondziolka, and C. N. Riviere, "Percutaneous intracerebral navigation by duty-cycled spinning of flexible bevel-tipped needles," Neurosurgery, vol. 67, pp. 1117-1122, Oct 2010.
[277] P. Swaney, J. Burgner, H. Gilbert, and R. Webster, "A flexure-based steerable needle: High curvature with reduced tissue damage," Biomedical Engineering, IEEE Transactions on, vol. 60, pp. 906-909, April 2013.
[278] P. Dupont, J. Lock, B. Itkowitz, and E. Butler, "Design and control of concentric-tube robots," Robotics, IEEE Transactions on, vol. 26, pp. 209-225, April 2010.
[279] D. Rucker, B. Jones, and R. Webster, "A model for concentric tube continuum robots under applied wrenches," in Robotics and Automation (ICRA), 2010 IEEE Int. Conference on, pp. $1047-1052$, may 2010.
[280] S. C. Ryu, Z. F. Quek, J.-S. Koh, P. Renaud, R. Black, B. Moslehi, B. Daniel, K.-J. Cho, and M. Cutkosky, "Design of an optically controlled MR-Compatible active needle," Robotics, IEEE Transactions on, vol. 31, pp. 1-11, Feb 2015.
[281] P. Qi, H. Liu, L. Seneviratne, and K. Althoefer, "Towards kinematic modeling of a multiDOF tendon driven robotic catheter," in Engineering in Medicine and Biology Society (EMBC), 2014 36th Annual International Conference of the IEEE, pp. 3009-3012, Aug 2014.
[282] L. Frasson, T. Parittotokkaporn, B. Davies, and F. Rodriguez y Baena, "Early developments of a novel smart actuator inspired by nature," in Mechatronics and Machine Vision in Practice, 2008. M2VIP 2008. 15th Int. Conference on, pp. 163 -168, dec. 2008.
[283] L. Frasson, S. Ko, A. Turner, T. Parittotokkaporn, J. F. Vincent, and F. Rodriguez y Baena, "Sting: a soft-tissue intervention and neurosurgical guide to access deep brain lesions through curved trajectories," Proceedings of the Institution of Mechanical Engineers, Part H: Journal of Engineering in Medicine, vol. 224, no. 6, pp. 775-788, 2010.
[284] S. Y. Ko, L. Frasson, and F. Rodriguez y Baena, "Closed-loop planar motion control of a steerable probe with a "'programmable bevel"' inspired by nature," Robotics, IEEE Trans. on, vol. 27, pp. 970-983, oct 2011.
[285] S. Y. Ko and F. Rodriguez y Baena, "Trajectory following for a flexible probe with state/input constraints: An approach based on model predictive control," Robotics and autonomous systems, vol. 60, no. 4, pp. 509-521, 2012.
[286] S. Patil, J. Burgner, R. J. Webster, and R. Alterovitz, "Needle steering in 3-d via rapid replanning," Robotics, IEEE Transactions on, vol. 30, no. 4, pp. 853-864, 2014.
[287] S. Lazard, J. Reif, and H. Wang, "The complexity of the two dimensional curvatureconstrained shortest-path problem," in Proceedings of the Third International Workshop on the Algorithmic Foundations of Robotics,(Houston, Texas, USA), pp. 49-57, 1998.
[288] D. R. Smith, "Variational methods in optimization," Mineola, N.Y.: Dover Publications, Inc., 1998.
[289] O. Junge, J. E. Marsden, and S. Ober-Blöbaum, "Discrete mechanics and optimal control," in Proceedings of the 16th IFAC World Congress, vol. 16, pp. 00310-1, 2005.
[290] B. Fornberg, "A practical guide to pseudospectral methods," Cambridge University Press, 1998.
[291] C. I. Connolly, J. Burns, and R. Weiss, "Path planning using laplace's equation," in Robotics and Automation, 1990. Proceedings., 1990 IEEE International Conference on, pp. 2102-2106, IEEE, 1990.
[292] R. M. Murray and S. S. Sastry, "Steering nonholonomic systems in chained form," in Decision and Control, Proceedings of the 30th IEEE Conference on. IEEE, pp. 11211126, 1991.
[293] W. Park, J. S. Kim, Y. Zhou, N. J. Cowan, A. M. Okamura, and G. S. Chirikjian, "Diffusion-based motion planning for a nonholonomic flexible needle model," in Robotics and Automation, 2005. ICRA 2005. Proceedings of the 2005 IEEE International Conference on, pp. 4600-4605, IEEE, 2005.
[294] V. Duindam, J. Xu, R. Alterovitz, S. Sastry, and K. Goldberg, "Three-dimensional motion planning algorithms for steerable needles using inverse kinematics," The International Journal of Robotics Research, vol. 29, no. 7, pp. 789-800, 2010.
[295] R. A. Knepper, S. Srinivasa, and M. T. Mason, "Toward a deeper understanding of motion alternatives via an equivalence relation on local paths," International Journal of Robotics Research, vol. 31, pp. 168-187, February 2012.
[296] K. M. Seiler, S. P. Singh, S. Sukkarieh, and H. Durrant-Whyte, "Using lie group symmetries for fast corrective motion planning," The International Journal of Robotics Research, vol. 31, no. 2, pp. 151-166, 2012.
[297] R. Secoli and F. Rodriguez y Baena, "Closed-loop 3d motion modeling and control of a steerable needle for soft tissue surgery," in Robotics and Automation (ICRA), 2013 IEEE International Conference on, pp. 5831-5836, May 2013.
[298] S. M. LaValle, "Planning algorithms," Cambridge University Press, 2006.
[299] M. Likhachev, D. Ferguson, G. Gordon, A. T. Stentz, and S. Thrun, "Anytime dynamic a*: An anytime, replanning algorithm," in Proceedings of the International Conference on Automated Planning and Scheduling (ICAPS), pp. 262-271, June 2005.
[300] S. M. LaValle and J. J. Kuffner, "Randomized kinodynamic planning," The International Journal of Robotics Research, vol. 20, no. 5, pp. 378-400, 2001.
[301] J. Xu, V. Duindam, R. Alterovitz, and K. Goldberg, "Motion planning for steerable needles in 3d environments with obstacles using rapidly- exploring random trees and backchaining," in Automation Science and Engineering, 2008. CASE 2008. IEEE International Conference on, pp. 41-46, IEEE, 2008.
[302] S. Patil and R. Alterovitz, "Interactive motion planning for steerable needles in 3d environments with obstacles," Proceedings of the 2010 3rd IEEE RAS \& EMBS International Conference on Biomedical Robotics and Biomechatronics, 2010.
[303] S. Patil, J. Burgner, R. Webster, and R. Alterovitz, "Needle steering in 3-d via rapid replanning," Robotics, IEEE Transactions on, vol. 30, pp. 853-864, Aug 2014.
[304] S. Patil et al., "Motion planning under uncertainty in highly deformable environments," Robotics science and systems: online proceedings, 2011.
[305] C. Caborni, S. Y. Ko, E. De Momi, and G. Ferrigno, "Risk-based path planning for a steerable flexible probe for neurosurgical intervention," in Biomedical Robotics and Biomechatronics (BioRob), 2012 4th IEEE RAS \& EMBS International Conference on, pp. 866-871, IEEE, 2012.
[306] J. Kider, M. Henderson, M. Likhachev, and A. Safonova, "High-dimensional planning on the gpu," in Robotics and Automation (ICRA), 2010 IEEE International Conference on, pp. 2515-2522, May 2010.
[307] C. Park, J. Pan, and D. Manocha, "Real-time optimization-based planning in dynamic environments using gpus," in Robotics and Automation (ICRA), 2013 IEEE International Conference on, pp. 4090-4097, May 2013.
[308] J. Ichnowski and R. Alterovitz, "Parallel sampling-based motion planning with superlinear speedup," in Intelligent Robots and Systems (IROS), 2012 IEEE/RSJ International Conference on, pp. 1206-1212, Oct 2012.
[309] H. Rivaz, S. J.-S. Chen, and D. L. Collins, "Automatic deformable mr-ultrasound registration for image-guided neurosurgery," Medical Imaging, IEEE Transactions on, vol. 34, no. 2, pp. 366-380, 2015.
[310] R. M. Murray, Z. Li, and S. S. Sastry, "A mathematical introduction to robotic manipulation," CRC Press, 1994.
[311] 3D-Slicer, "Liver segmentation tutorial," 2009.
[312] A. de Brebisson and G. Montana, "Deep neural networks for anatomical brain segmentation," arXiv preprint arXiv:1502.02445, 2015.
[313] P. Chatelain, A. Krupa, and N. Navab, "3d ultrasound-guided robotic steering of a flexible needle via visual servoing," in Robotics and Automation (ICRA), 2015 IEEE International Conference on, pp. 2250-2255, May 2015.

## Appendix A

## Path planning

The work on path planning completed in collaboration with Dr Fangde Liu is presented in this appendix.

The work presented in this appendix is an edited version of the work published in:

- F. Liu, A. Garriga-Casanovas, R. Secoli, and F. Rodriguez y Baena. Fast and Adaptive Fractal Tree-Based Path Planning for Programmable Bevel Tip Steerable Needles. Robotics and Automation Letters, 1.2, pp. 601-608, 2016. © 2016 IEEE.
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## A. 1 Introduction

MIS is becoming the standard of care for a range of medical procedures, including biopsies, targeted drug delivery, and brachytherapy cancer treatment. The advantages of MIS include less trauma for the patient, lower risk of complications, and a shorter full-recovery time. Current standard medical practice uses rigid tools, which enable good accuracy, but are not capable of accessing locations behind delicate regions. Steerable needles [270], [271] have the potential
to overcome these limitations, and to improve reliability through automation, resulting in a significant advancement in keyhole surgery.

Existing steerable needle concepts can be classified in seven different groups, as outlined in [272]: base manipulation [273], bevel tip (with and without a "kinked tip") [274-276], pre-curved stylet [277], active cannula [278,279], optically controlled needle [280], tendon actuated tip [281] and programmable bevel tip [282-284]. The design developed by this author's colleagues, codenamed Soft Tissue Intervention and Neurosurgical Guide (STING) [285]), has a bio-inspired design that reproduces the multi-segment ovipositor of certain parasitic wasps, is made of flexible plastic and is fully Magnetic Resonance Imaging (MRI) compatible. It has the ability to steer along three-dimensional paths without duty cycle spinning along the insertion axis, as shown in Figure A.1, and thus offers an ideal target system for the path planning technique described in this work.

In most of these applications, the uncertainties arising from tissue deformation during insertion and consequent need of frequent path replanning to track the motion of one or several targets, warrants a real-time path planning algorithm, with a high update frequency [286]. The design of real-time path planning algorithms capable of online updates, however, is challenging, especially when differential constraints are present. The problem is NP-hard [287]. General methods from variational optimization [288] [289], or approaches from optimal control such as the Gauss pseudospectral method [290], are capable of accurately finding the optimal solution; however, they require significant computational time. Potential fields based methods [291] and most other probabilistic methods are unable to handle nonholonomic constraints. Linear path planners for chained-form systems [292] have been applied for some steerable needle designs, but cannot cope with control saturation associated with large tissue deformation. The limited robustness of probability maps [293] or inverse kinematics based approaches [294] prevents their use in safety-critical applications, such as in surgery. Path planners based on homotopy groups [295] or Lie groups [296] exploit symmetries in the path to accelerate the processing speed. However, these solutions must be computed iteratively due to their nonlinear nature, leading to an unbounded computational time.


Figure A.1: a) Rendering of the STING distal end [297] b) STING cross-section with interlocking mechanism. © 2016 IEEE.

Sampling-based methods are the dominant trend [298] in problems with differential constraints. Standard approaches such as the Dijkstra method, or the improved, heuristics based version, A* [298], are able to effectively find paths with obstacle avoidance, but the search is excessively time consuming. Even algorithms that improve on $A^{*}$ by reusing previous search information [299] require significant computational time, and can only be scaled to multiple CPUs. RapidlyExploring Random Trees (RRTs) [300] [301], and specifically Reachability-Guided RRTs (RGRRTs) [302], are becoming increasingly popular due to their ability to quickly explore the entire domain and cope with curvature constraints for needle steering. RRTs perform well in environments with relatively simple obstacles, presenting short computational times that allow online path replanning during insertion [303]. However, in congested environments, with complex obstacles, even purpose-developed heuristically accelerated RRTs present computation times that are relatively long and unbounded [304] [305].

A common issue in the majority of existing approaches is that they perform the search sequentially, relying on serial CPU computing, the speedup potential of which is limited. Instead, by exploiting the power of the Graphics Processing Unit (GPU) for general purpose processing, the computation time can be reduced by over one order of magnitude. Some early approaches
to path planning on the GPU are reported in [306] [307], highlighting their potential advantages over CPU-based algorithms. However, the performance improvement of these algorithms is limited to ten times that of CPU based implementations, a result which can be improved. Parallelization of RRTs is also reported in the literature [308], although the algorithm is only scalable to multiple CPUs, and presents a limited speed improvement. This is a consequence of the search procedure in RRTs, which leads to a variable computational load due to an iterative growth, potentially causing the system to stall when multiple threads require the same tree to update simultaneously, and may not meet the "single instruction multiple data operations" requirement, which the GPU is designed for.

This paper proposes a novel approach to path planning, which is tailored for a GPU-based implementation. The strategy introduced in this work employs fractal theory to create a data structure that enables efficient parallel path planning. The resulting parallelized problem has a recursive structure, is adaptable in size, is constructed procedurally, and allows a dense coverage of the entire domain, as illustrated in Fig. 6.10. For this, the method has been termed Adaptive Fractal Trees (AFT). The approach adopted in this work presents three main advantages with respect to existing imaged-based algorithms. First, it works directly with voxels, optimizing computational performance. Second, it is capable of real-time replanning with a bounded computational time. Third, it can be used regardless of the number or complexity of the obstacles, rendering it robust and versatile, with a high success rate compared to other path planning algorithms.

The paper is structured as follows. The path planning problem for a steerable needle is formally stated in Section II. Section III provides a description of the AFT approach, together with an analysis of its specific properties for parallelization. Simulated results, together with the corresponding discussion, are presented in Section IV, leading to the conclusion of this paper in Section V.

## A. 2 Problem formulation

## A.2.1 Path Planning for Programmable Bevel Tip Needles

For the purpose of path planning, only a description of the STING's distal end is necessary, since it can be assumed that the body will follow the path dictated by the tip [283]. The robot configurations form a subspace of the special Euclidian group, with $q(t) \in S E(2)$ for 2D [284] and $q(t) \in S E(3)$ for 3D [297]. The initial and target configurations are indicated by $q_{i}$ and $q_{f}$, respectively. The interaction between needle and tissue, together with the robot design, lead to a set of nonholonomic constraints, valid at least locally in an infinitesimal neighborhood of time and space. Defining a direction $x$ tangent to the insertion path, two first constraints arise from a no-slip condition, $V_{y}=V_{z}=0$, which are the linear velocities along the $y$ and the $z$ axes respectively. The STING is designed to steer in 3D without duty-cycling along the insertion axis, $x$, hence a kinematic constraint on the rotational velocity along the insertion axis arises, $w_{x}=0$. The curvatures of the resulting path along the $y$ and $z$ directions, defined as $k_{y, z}=\frac{w_{y, z}}{V_{x}}$, are determined by the bevel tip geometry. This is specifically calculated to prevent excessive stress on the needle, leading to a bounded curvature between a minimum $L_{y, z}$ and a maximum $U_{y, z}: L_{y, z} \leq \frac{w_{y, z}}{V_{x}} \leq U_{y, z}$. Considering these premises, along with a needle design that suffers from negligible torsional effects, we employed the Bishop frame [176] as the most suitable frame to describe the needle motion.

The obstacles in the configuration space correspond to either physical obstacles or virtual constraints. Due to tissue deformation, the spatial position of the obstacles may vary [309]. It is assumed that feedback from their position, as well as from the current and target configurations of the needle tip, is available from an appropriate source (e.g. an intraoperative imaging or tracking system).

The aim of a path planner is to find a feasible path from $q_{i}$ to $q_{f}$ that respects all of the constraints, and optimizes a cost function. In general, the cost function to minimize is defined as a risk-based function, possibly with additional components, such as the minimization of the insertion length, as to reduce tissue damage.

## A.2.2 General Path Planning Problem

More generally, we are considering a system described in implicit form by $q \in \mathbb{R}^{n}$, with a set of $k \leq n$ smooth linearly independent ${ }^{1}$ nonholonomic Pfaffian constraints

$$
\begin{equation*}
w_{i}(q) \dot{q}=0 i=1, \ldots, k \tag{A.1}
\end{equation*}
$$

which may also include any number of obstacles of any complexity, denoted in the configuration space by $Q_{\text {obs }}$. The path planning problem for this system can be equivalently formulated as a steering control problem [310].

The corresponding system can be expressed as

$$
\begin{equation*}
\dot{q}=\sum_{i=1}^{m} g_{i}(q) u_{i} \tag{A.2}
\end{equation*}
$$

where $m=n-k$ and $u \in U \subset \mathbb{R}^{m}$ are the control inputs, with

$$
\begin{equation*}
\operatorname{span}\left\{g_{1}, \ldots, g_{m}\right\}=\operatorname{span}\left\{w_{1}, \ldots, w_{k}\right\}^{\perp} \tag{A.3}
\end{equation*}
$$

Considering the obstacles $Q_{o b s}$ to be static, the path planning problem is then to find the input functions $u_{1, \ldots, k}$ that steer the system from an initial $q_{i}$ to a target configuration $q_{f}$, while optimizing a cost function and avoiding $Q_{o b s}$.

## A. 3 Adaptive Fractal Trees Algorithm

The recursive nature of motion in nonholonomic systems closely resembles the topological structure of a tree. The possible motion at each step depends on the previous one, a process that reverses recursively to the initial point, or the tree origin. Despite the advantages of

[^2]the parametric form (A.2), path planning for systems with differential constraints remains challenging; the majority of existing numerical solutions are sampling-based and rely on serial iterative computing processes, requiring often excessive, and unbounded computational time. Their parallelization to suit GPU specifications is either difficult or impossible.

By uniformly discretizing the control space, the path adopts a fractal structure. Such fractal space can be divided into sub-spaces, in a coarse to fine manner, as

$$
\begin{equation*}
T_{s}=T_{s 1}+T_{s 12}+T_{s 13}+\ldots+T_{s i} \tag{A.4}
\end{equation*}
$$

Each subspace $T_{s i}$ can be parallel processed by the GPU. This results in a novel method for massively parallel path planning, with an efficient search. The resolution increases exponentially with each subspace, leading to fast convergence.

## A.3.1 Motion Fractal Tree

Relying on the parametric form of a nonholonomic system (A.2), all possible paths can be mapped to an L-tree, as shown in Fig. 6.10. Beginning at $q_{i}$, the first set of tree ramifications corresponds to the action vectors $g_{i}$ of the system, advancing by an increment that can be symbolized by $\delta_{u}$ in each of the $s$ directions. Then, each branch is divided and subsequently given the motion action inputs, generating a fractal structure.

The number of required increments is determined by the needle insertion distance, and the computational time is bounded by the limited needle length. A path is then determined by a string of configurations $q_{T}=\left[i_{1}, i_{2}, \ldots, i_{N}\right]$, where N is the total number of increments required. The entire domain of possible motions is discretized exhaustively using a fractal tree, as illustrated in Fig. 6.10. A differential increment between ramifications would lead to an exact approximation of all possible paths. However, the number of paths increases exponentially with the number of ramifications and, as the discretization step decreases, the size of the path space grows, becoming infinite for a differential increment. Hence, for any given application, a specific incremental step must be selected.


Figure A.2: Illustration of the adaptive search concept in a two-stage approach. The coarse tree (green) first explores the entire domain. The fine tree (red) is concentrated around the most promising region, providing higher resolution. The blue line highlights the most suitable path. © 2016 IEEE.

This structured construction of the tree is implemented efficiently by the GPU, as explained in the following subsections. This property is in contrast with the random construction of RRTs, and it represents one of the distinctive advantages of AFT for fast computation.

## A.3.2 Adaptive Discretization

By exploiting the tree property, as in (A.4), it is possible to break down the search into subspaces. This division has the particular property that all subspaces share the same number of motion segments and topology.

Each tree can be parametrized by three elements: $l$, which corresponds to the segment's length,
$\delta_{k}$, which describes the branch's aperture, and $C$, the tree's central path. The latter is either provided by a previous coarse search, or taken as a straight line for the first generated tree. The size of a tree is therefore adaptable, depending on the construction parameters.

In this way, the path search can be executed in a coarse to fine manner, reducing the problem's complexity exponentially, and achieving high accuracy in the fine search. First, the path planner creates and searches a coarse tree $T_{s 1}$. Then, the path that minimizes a cost function within $T_{s 1}$ is used to build a second, finer tree around it, the density of which is increased exponentially with respect to the previous one.

The adaptable search concept is illustrated in Fig. A.2, where a two-stage approach is depicted. First, a coarse tree is generated covering the entire domain, in order to determine the most promising region. Then, a second tree is constructed to perform the fine search, focusing the computational resources around the region identified by the coarse tree, with a higher density of paths that minimizes the error. In general, after two or three stages, the desired resolution is reached.

## A.3.3 Parallel Path Planning Algorithm

The AFT path planning algorithm is composed of three parts: (1) motion segment reconstruction, (2) collision detection and distance to target calculation, and (3) back-tracking and pooling.

A cost function is defined in order to evaluate the paths and determine the most suitable one. In this case, the cost function is composed of three parts, as

$$
\begin{equation*}
C\left(q_{T}\right)=w_{1} R\left(q_{T}\right)+w_{2} T\left(q_{T}\right)+w_{3} D\left(q_{T}\right) \tag{A.5}
\end{equation*}
$$

where $w_{i}$ represents a weighting parameter, $R$ is a risk-based function, $T$ is a function associated to trauma, and $D$ represents the distance between the needle tip and the target configuration. The distance to target is defined here as the Euclidean distance.

The database of paths is generated at any time using the aforementioned tree parameters. The cache does not need to be stored in memory, which suits the GPU architecture. The initial and target configurations, as well as the obstacles, are assumed to be available from an appropriate feedback source. The tree is constructed starting from the initial point. The limited path length of steerable needles allows a fast computation of the action list.


Figure A.3: Diagram of the enumeration and allocation of tree segments to the GPU threads. Each tree segment is assigned to one GPU thread. The most suitable path and corresponding threads are highlighted in cyan. © 2016 IEEE.

Collision detection is then applied to the cache of paths. Medical applications require high accuracy, and the anatomical obstacles tend to present complex/irregular boundaries. Here, it is assumed that some image processing has been applied on the raw feedback data, and the voxels representing the obstacles have been identified. The path planner proposed in this work then checks each voxel on the tree for possible collision, marking the path segments where this occurs. The distance to target is also computed and stored for each segment.

Back-tracing is then performed. It begins with checking whether the segments are collisionfree. Then it proceeds towards the tree root, assessing possible collisions within the paths. If all segments of a path are collision-free, then it is marked as viable.

Finally, a parallel maximum pooling is executed, selecting, among the collision-free paths, the one that minimizes the cost function. This path then becomes the central line for the next stage, around which the path planner then refines the search.

```
Algorithm 1 AFT Basic Algorithm
Input: \(q_{i}, q_{f}, Q_{\text {obs }}\)
Output: \(q_{c_{\text {min }}}\)
    Initialization
    \(N, B, J\)
    \(q_{T}=\emptyset\)
    Recursion Loop
    for \(j=1\) to \(J\) do
        RefineTreeAround \(\left(q_{T}\right)\)
        for all ID: \(i \leq N\) do
            \(q_{i} \leftarrow \operatorname{MotionPlan}(i)\)
            \(c_{i} \leftarrow \operatorname{Cost}\left(q_{i}\right)\)
        end for
        \(T \leftarrow \operatorname{Index} \operatorname{OfMin}\left(c_{1}, c_{2}, \ldots, c_{N}\right)\)
    end for
    return \(q_{T}\)
```

As a result, the method described here combines the robustness of RRTs with the parallelization possibilities of path caches, leading to an algorithm that is advantageous with respect to both. This algorithm is reported as Algorithm 1. The initial and target configurations, as well as the obstacles, are first inputted. The parameters for the tree construction, $l, \delta_{k}$ and $C$, are then determined according to the number of segments, $N$, and branches, $B$. The recursion depth, $J$, is also introduced, which represents the number of tree refinements (typically two). A recursion loop is then executed to generate and evaluate a tree at each step. The tree is adapted in the function RefineTreeAround $\left(q_{T}\right)$ around path $q_{T}$, which is taken to be straight for the first iteration. All processes are executed in parallel to construct the tree and compute the cost of each path, as defined in equ.(A.5), which represents the function $\operatorname{Cost}\left(q_{T}\right)$. Subsequently, the minimum cost path is identified in the function IndexOfMin, which is executed by parallel reduction. This minimum cost path is used in the next iteration of the "for loop". When
iteration $J$ is reached, the path that minimizes the cost function, $q_{T}$, is determined, which is the output of the algorithm.

Fractal trees can be easily parallelized. Each tree segment can be allocated to a GPU thread, as shown in Figure A.3, optimizing the use of computational resources. The cost evaluation and motion plan reconstruction are the kernel for parallel computing, which consumes the majority of computational time and space. Due to the GPU architecture, the kernel (line 5-8 of Algorithm 1) is optimized, as described in Algorithm 2.

```
Algorithm 2 AFT Optimized Kernel
Input: ID, \(q_{i}, q_{f}\)
Output: \(c_{I D}\)
    Initialization
    \(q \leftarrow q_{i}\)
    \(i \leftarrow 0\)
    parent \(_{I D}\) is root \({ }_{I D}\)
    child \(_{I D}\) is root \({ }_{I D}\)
    \(\operatorname{Cost}\left(q_{\text {parent }}^{I D}() \leftarrow 0\right.\)
    while child \(_{I D} \neq I D\) do
        child \(_{I D}\), parent \(_{I D} \leftarrow \mathbf{C h i l d}\left(\right.\) parent \(\left._{I D}, I D\right)\)
        if child \(d_{I D} \neq \emptyset\) then
            parent \(_{I D \text { cur }} \leftarrow\) parent \(_{I D}\)
            \(q \leftarrow q+\mathbf{A c t i o n}\left(\right.\) child \(\left._{I D}\right)\)
            continue
        end if \(\{\) Calculating the last segment cost \(\}\)
        \(S_{\text {last }} \leftarrow\) BuildSegment parent \(_{\text {IDcur }}\), parent \(_{I D}\) )
        \(p_{1}, p_{2}, p_{3}, \ldots, p_{w} \leftarrow \operatorname{Dice}\left(S_{\text {last }}\right)\).
        \(c_{I D}=\sum_{i=1}^{N} \operatorname{Cost}\left(p_{i}\right)\)
    end while
    SYNCHRONIZE GPU THREADS
18: while parent \({ }_{I D}\) is not root \(_{I D}\) do
        parent \(_{I D} \leftarrow \operatorname{Parent}\left(\right.\) parent \(\left._{I D}\right)\)
20: \(\quad c_{I D} \leftarrow c_{I D}+\operatorname{Cost}\left(q_{\text {parent }_{I D}}\right)\)
    end while
```

The inputs of Algorithm 2 are the ID of each segment, and the initial and target configurations. The algorithm initializes by establishing the maximum path length $L$ and variable $i$, as well as as creating an array of costs $\operatorname{Cost}\left(q_{I D}\right)$. To maximize efficiency, the cost for each segment is only computed once. As the ID of each segment is allocated, the cost is then calculated and stored into an array for all segments. The BuildSegment function builds the segment $S_{\text {last }}$ between the parent $I_{\text {IDcur }}$ and parent $t_{I D} . S_{\text {last }}$ is then sampled into $p_{1}, p_{2}, \ldots, p_{w}$ sub-segments,
using the Dice function. The corresponding cost of each sample is calculated and accumulated to define the total cost $c_{I D}$ of the whole segment $S_{\text {last }}$. After synchronizing the parallel threads, back tracking is then applied to calculate the cost of each path. This is executed using the function Parent, which determines the parent segment corresponding to each segment. In this manner, the system tracks back each node to its parent, summing the contribution of each segment to the aggregate cost, and thus obtaining the total cost associated with each path. The array of costs for each path is the output of the algorithm.

An important factor for efficient parallelization is the enumeration of each segment with an $I D$. Exploiting the fractal structure of the tree, parent and child $I D \mathrm{~s}$ have a regular pattern. By travelling up and down the tree, an enumeration maps each path $I D$ to a series of control actions.

## A. 4 Simulation Setup

An application of AFT to minimally invasive surgery is presented in this section, in order to validate the algorithm in a statistically significant manner. In particular, simulations corresponding to 3D liver navigation are reported, as they showcase the capability of AFT to plan a path in real-time in a highly congested and complex environment.

Tissue deformation during needle insertion can lead to displacements of a few centimeters. As a consequence, target migration and variations in the spatial position of the obstacles can be significant, requiring path replanning with a high update frequency, as the needle is being inserted. In this work, it is assumed that the initial and target configurations of the steerable needle, as well as the obstacles, are available from a suitable intra-operative imaging modality, e.g. Interventional Magnetic Resonance Imaging or Ultrasound.

AFT is specifically designed to recalculate a path as the environment varies during needle insertion. The short and fixed computational time associated with AFTs allows replanning with an update frequency that can match the feedback imaging system, eliminating the need for complex low level control.

The simulations reported here include a representative set of 100 different 3D path planning problems encountered during needle insertion into liver, and simulated online replanning during needle insertion, with target motion. The 100 problems correspond to different initial configurations randomly generated within a bounded domain, and three fixed targets, as shown in Figures A.5,A.6. The online replanning is simulated in a particularly complicated needle insertion, with a target moving continuously during the insertion, with a total displacement of 2 cm . The simulations are in a common 3D environment, which represents a segmented CT scan of a liver (Liver Dataset [311]) in voxel format, with a resolution of 256 x 256 x 256 . This CT scan image volume was selected as it includes a high number of vessels that define a challenging obstacle map, where existing algorithms such as RRTs experience difficulties in finding a solution. It is assumed that an image processing algorithm is available to label the obstacles in the intra-operative images [312]. Similarly, it is assumed that the needle configuration can be estimated from the images [313].

In the simulations, the Cost function is defined to favor the shortest path that arrives closest to the target, without intersecting any obstacle. The Euclidean distance was used to measure the proximity to the target. The parameters for the simulations were as follows: maximum needle curvature $=0.014 \mathrm{~mm}^{-1} ;$ search space $=100 \times 100 \times 200 \mathrm{~mm}^{3}$; discretization AFT step $=2 \mathrm{~mm}$; maximum insertion length $=160 \mathrm{~mm}$; entry points randomly generated in a bounding box of $-30 \leq x \leq 0 \mathrm{~mm},-5 \leq y \leq 5[\mathrm{~mm}]$ and $-83 \leq z \leq 100[\mathrm{~mm}]$ in position, and a variation of 10 degrees with respect to vector $[0,1,0]$ in orientation; target positions $[x, y, z]=$ $[57,157,58],[-57,157,58],[57,-157,-5] m m$; cost function parameters: $w_{1}=w_{2}=w_{3}=1$; number of search paths $N=1024 * 1024 * 17, B=17$ and $\delta_{k}=1 / 280$.

In the setup used in this work, the code is implemented in Matlab 2014b (Mathworks Inc.), Linux Ubuntu 64bit, and executed on an Intel CORE i7 CPU @ 3.2Ghz with a GTX TITANX from NVIDIA corp., with 3072 threads, a 1 GHz base-clock and 12GB of memory. This GPU has an approximate computing power of 7 TFLOP and supports CUDA 7.5 API [216].

The simulation of RRTs in the same path planning problems is also reported in order to compare the performance of AFT with one of the most widely used algorithms in MIS. RRTs
are implemented in the same setup, with a maximum number of iterations of 16,000 . An RGRRT implementation is adopted, as it provided faster convergence in the tests performed in this work. A goal bias sampling strategy was used, with $50 \%$ of the samples on the target and the remaining $50 \%$ randomly distributed.

The performance tests conducted indicate that, with the setup used in this work, 300 million paths per second can be evaluated. Consequently, considering a typical surgical application, where a 20 Hz update frequency is required, the algorithm would be capable of assessing 15 million paths per second. This computational power translates into a resolution that approaches the limits of the imaging device.

## A. 5 Results and Discussion

The results of an illustrative AFT path planning problem are shown in Fig. A.4. As can be seen, a high density of paths (red) are surveyed, and the path that minimizes the cost function is selected (green), with a total computation time of just 5.2 ms .

The results of the simulation of 100 different AFT path planning problems in a prototypical scenario are shown in Figure A.5. In this case, only the selected paths are displayed for clarity, showing the ability of the AFT algorithm to negotiate complex obstacles. The average final error in the feasible paths identified is 1.45 mm , with a standard deviation of 1.19 mm . The average computation time is 5.15 ms , with a corresponding standard deviation of 0.048 ms . The small variation in computation time between the different simulations is a result of the automatic speed adjustment of the GPU. However, the computation time is fixed and independent of the complexity of the obstacles. This represents a significant advantage of AFT in surgical applications.

In comparison, a standard RRTs implementation, which is taken here to be one of the best competing algorithms in the literature, performs considerably worse than the algorithm proposed in this work. The RRTs simulation performed on the same data set and with the same setup indicates that, after 16000 iterations (corresponding to an approximate computation time
of 30 s in the non-optimized implementation in this work), a path is found in $42 \%$ less cases than in AFT. The preliminary paths found using RRTs after 16,000 iterations are shown in Figure A.6. As can be seen, only the simpler cases are solved, whereas the remaining cases would require a significantly higher number of iterations to reach a solution.

The relatively low success rate of RRTs in an environment with complex obstacles is a consequence of the search strategy employed by the algorithm. In RRTs, the space is sampled, and then paths linking to the tree are searched. While this strategy is successful in many environments, links to the tree can be difficult to find in the presence of complex obstacles, requiring high sampling resolution. The computational cost increases exponentially with the number of samples, hindering the use of RRTs in highly congested environments.

AFTs, on the other hand, provide a higher success rate in real-time, regardless of the number and complexity of the obstacles. Such robustness is a result of the algorithm construction and implementation, which exploits the GPU architecture to survey a high number of paths in parallel. In this regard, the AFT algorithm is particularly suited to surgical applications, where the ability to update a plan in real-time in the presence of any number of complex obstacles, would be advantageous.

The result of a simulated online replanning using AFT during needle insertion is shown in Fig. 6.11. As can be seen, the algorithm initially calculates a path (green). However, as the target moves during insertion, the online replanning finds a more suitable path (red), which is re-calculated and improved to account for target motion.

## A. 6 Conclusion on path planning

Efficient three-dimensional path planning in complex environments remains challenging, especially in scenarios requiring a real-time implementation. In this work, the path planning problem can be solved in real-time, even for systems with nonholonomic constraints and complex environments, with a novel algorithm which we named Adaptive Fractal Trees (AFT). The application of AFT enables the parallelization of the path planning problem, which in
turn unlocks the massive computational speedup potential of the GPU, leading to ms long path searches, regardless of the complexity of the surgical scenario. The use of AFT enables the search to be conducted in a coarse to fine manner, with a database of paths that can be procedurally produced. In this way, a perfect match between the algorithm and the hardware capabilities is achieved. In addition, the fractal tree that is generated translates into a dense, invariant and organized exploration of the entire domain. This represents an advancement with respect to existing algorithms in terms of robustness and success rate of path planning in highly constrained and complex environments. As a result, the approach described in this paper allows the path planning problem to be computed in real-time, with the resolution and update frequency necessary for many surgical applications.


Figure A.4: Simulation of a coarse 3D search through the segmented vasculature of liver. The full tree of paths is colored red. The best path is shown in green. © 2016 IEEE.


Figure A.5: Simulation results of liver path planning with the AFT algorithm, showing all of the best paths which intersect three random targets (blue, green and magenta), varying the entry position (black dots) and insertion direction. © 2016 IEEE.


Figure A.6: Simulation results of liver path planning with the RG-RRT algorithm, showing all of the best paths that intersect three random targets (blue, green and magenta), varying the entry position (black dots) and insertion direction. For some entry points, the algorithm fails to provide suitable paths to reach the targets. © 2016 IEEE.

## Appendix B

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Robotic Manipulators for In Situ Inspections of Jet Engines

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Subject: Re: Form Submission: Technical Support

Hi Tahzeeb,

As George mentioned, I'm finishing my doctorate at the Mechanical Engineering Department, supervised by Prof Ferdinando Rodriguez y Baena, so not related to Dan or George.

I did my doctorate on miniature robots, and wanted to include an image of the smallest Omnivision camera in my thesis to illustrate existing technology.

Best wishes,

Arnau

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Country

United Kingdom (GB)

## Phone

Email<br>a.garriga-casanovas14@imperial.ac.uk

## Application

Medical Imaging

Form Factor<br>Module

## Output Data Format n/a

## Resolution

Digital

## Additional Requests or Comments

Hello, Iâ $\epsilon^{\mathrm{TM}} \mathrm{m}$ finishing my doctoral thesis at Imperial College on miniature robots, and I would like to include an image of the smallest Omnivision camera as an example of existing technology. I found some examples in the catalogue online, and I wanted to use one of them. I would like to ask for your permission to include such an image in my thesis, which will be added to Spiral, Imperial's institutional repository
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From: Yaron Silberman
To: Garriga Casanovas, Arnau
Attachments: image003.jpg

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Organisation: Medigus
Job title: VP Sales \& Marketing

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From: Bar-Cohen, Yoseph (355N)
To: Garriga Casanovas, Arnau
Attachments: image001.png

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Yosi


FIGURE 5: 4-finger EAP gripper lifting a rock.

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Sent: Friday, January 4, 2019 7:03 AM
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To: Garriga Casanovas, Arnau

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From：Anthony Rickatson
To：Garriga Casanovas，Arnau
CC：香里 松村，Ishii，Shingo，Robert Walker
Attachments：B－15D5004B－160k CIS Brochure（UK）．pdf，B－17D5003D＿40k CIS Brochure（for UK）．pdf

Hi Arnau，
You are welcome to use these images in our Brochures（attached）．
Best Regards

Anthony Rickatson
Technical Sales Executive
Electronics Division

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Sent： 08 January 2019 10：30
To：Anthony Rickatson＜arickatson＠fujikura．co．uk＞
Cc：香里 松村＜kaori．matsumura＠jp．fujikura．com＞；Ishii，Shingo＜Shingo＠fujikura．com＞；Robert Walker ＜rwalker＠fujikura．co．uk＞
Subject：Re：FEL／／ICL／include an image of the Fujikura miniature camera

Hi Anthony，

Thank you for your response．

The image I would like to include in my thesis is the attached one．

Best wishes，

Arnau

On 08／01／2019，09：36，＂Anthony Rickatson＂＜arickatson＠fujikura．co．uk＞wrote：

Hi Arnau Garriga Casanovas，
Happy new year．
I am in charge of your request．
Please can you resend the image to me for review？

Best Regards

Anthony Rickatson
Technical Sales Executive

## Electronics Division

## Fujikura Europe Ltd

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To: Fujikura Web contents Manager [medical@jp.fujikura.com](mailto:medical@jp.fujikura.com)
Subject: Inquiries from Fujikura Global Web site

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company: Imperial College London
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address:Dept. of Mechanical Engineering, Imperial College London, London, SW7 2AZ name:Arnau Garriga Casanovas
[Contact information]
PHONE:7835146371
FAX:
e-mail:a.garriga-casanovas14@imperial.ac.uk
[Inquiry details]
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[^3]
[^0]:    376

[^1]:    ${ }^{1}$ Singularities can be locally present at $\cos \beta=0, \cos \gamma=0, \cos \alpha=0$, which require a separate analysis. However, these singular regions are not relevant to the general study of the DOFs achievable by the robot.

[^2]:    ${ }^{1}$ A subset of the constraints may be locally linearly dependent. In such case, the rank of the distribution associated to the action vectors $g_{i}$ increases locally, without further consequences on the path planning presented in this paper.

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