A DEMAND MODEL WITH DEPARTURE TIME CHOICE FOR WITHIN-DAY DYNAMIC TRAFFIC ASSIGNMENT

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ABSTRACT

A within-day dynamic demand model is formulated, embodying, in addition to the classic generation, distribution and modal split stages, an actual demand model taking into account departure time choice. The work focuses on this last stage, represented through an extension of the discrete choice framework to a continuous choice set. The dynamic multimodal supply and equilibrium model based on implicit path enumeration, which have been developed in previous work are outlined here, to define within-day dynamic elastic demand stochastic multimodal equilibrium as a fixed point problem on users flows and transit line frequencies. A MSA algorithm capable, in the case of Logit route choice models, of supplying equilibrium flows and frequencies on real dimension networks, is presented, as well as the specific procedures implementing the departure time choice and actual demand models. Finally, the results obtained on a test network are presented and conclusions are drawn.

keywords: within-day dynamic traffic assignment, elastic demand, departure time choice

1 INTRODUCTION

The dynamic analysis of transportation networks gathered increasing attention through the past years. The static models, traditionally used in this field, are in fact unable to represent relevant phenomena, such as demand variations over time and temporary over saturation of network elements. On the other hand, dynamic models are more complex than static ones: on the supply side, they require to ensure temporal consistency, besides spatial consistency, among system variables (for instance, see Cascetta, 2001); on the demand side, choice of the time when to travel has to be modelled explicitly in order to achieve a correct representation of users' travel behaviour (Mahmassani and Chang, 1986; Van Vuren *et al.*, 1998; Mahmassani and Liu, 1999).
Different approaches to the latter problem can be found in the literature. One of them is to regard departure time choice as a discrete choice among temporal intervals and use static assignment to characterize the utility of each interval, as in Daly *et al*. (1990); models based on this approach yield a rough representation of travel demand during day time as a sequence of static equilibriums.

An alternative approach, relying on a more realistic representation of dynamic travel times, can be found in De Palma *et al*. (1983) and Ben-Akiva *et al*. (1986) where a within day dynamic stochastic equilibrium model and a doubly dynamic stochastic model, respectively, are presented. Travel, departure time and path choices are addressed through a mixed discrete/continuous nested Logit model and travel times are determined by means of a deterministic queuing model; however, both models are applied only to single origin destination pair idealized networks.
Another approach is based on the hypothesis that departure time and path choices are made jointly, so that, for each origin-destination pair, a discrete choice set is defined whose finite number of alternatives is equal to the number of departure intervals multiplied by the number of paths (Arnott *et al.*, 1990; Cascetta *et al.*, 1992); models based on this approach require explicit path enumeration and the introduction of a diachronic graph, as in Van der Zijpp and Lindveld (2000), so they can be hardly applied to congested urban networks.
An extensive analysis of departure time choice for shopping trips is presented in Bhat (1998) and Bhat and Steed (2002). In the first paper, a discrete choice model is proposed, able to represent correlation among adjacent departure time periods. In the second paper, a continuous-time model of departure time is proposed, accommodating time-varying coefficients. Both papers however doesn't investigate path choice, since travel times and cost are assumed exogenously.
In a recent paper by Bellei, Gentile, Papola (2003) the within-day road Dynamic Traffic Assignment (DTA) was regarded as a stochastic dynamic user equilibrium. A new fixed point formulation of the problem in terms of time-continuous real valued temporal profiles of arc flows and arc performances (travel times and generalized costs) was presented, where the concept of network loading map, yielding arc flows for given demand flows consistently with certain arc performances, was extended to the dynamic case, thus avoiding the introduction of both the dynamic network loading as a sub-problem, and the explicit path enumeration. OD flows temporal profiles were taken as given. An implicit path enumeration algorithm was thus proposed for the Logit case.
Based on the above modelling framework, two more papers, namely Gentile, Meschini, Papola (2002) and Gentile, Meschini, Papola (2003), focused on a new dynamic transit supply model relying on a frequency based approach; the introduction of a diachronic graph is thus avoided and implicit path enumeration is allowed. This model is able to represent both intra and inter modal congestion effects (i.e. interaction among cars and bus flows). In order to define a multimodal within-day DTA, a dynamic mode choice model was introduced in these papers, while demand flows were assumed to be rigid with respect to any other choice dimension.
In this paper a mixed discrete/continuous nested Logit dynamic demand model with five choice levels is presented, where, besides the usual to travel or not to travel (generation), destination, mode and path choices, the departure time choice is introduced. The model is conceived to extend previous work to the most general case of elastic demand multimodal

within-day DTA. With reference to departure time choice, the proposed demand model adopts a continuous approach, thus not requiring to enumerate explicitly the desired departure time intervals. The resulting within-day DTA model is then capable of representing both supply and demand dynamic phenomena concerning congested multimodal urban networks, and leads to a fixed point formulation that can be solved by an efficient implicit path MSA algorithm applicable to real networks.

2 THE CHOICE AND DEMAND MODELS

In modelling travel demand we follow the behavioural approach based on random utility theory, where it is assumed that each user is a rational decision-maker who, when making his travel choice: a) considers a positive, finite number of mutually exclusive *travel alternatives* constituting his *choice set* J; b) associates to each travel alternative j of his choice set a perceived utility, not known with certainty, and thus regarded by the analyst as a random variable U_j; and c) selects the travel alternative that maximises his utility. With these hypotheses the probability of alternative j is formally expressed as:

$$P_j = \text{Prob}[\cap_{k\in J}\ \varepsilon_k \geq V_j - V_k + \varepsilon_j\ ]\ , \tag{1}$$

where V_j and ε_j are respectively the systematic utility and the random residual of the generic alternative j. The expected value of the maximum perceived utility is called *satisfaction*:

$$W = \text{E}[\max_{j\in J}\ \{V_j + \varepsilon_j\ \}] \tag{2}$$

As usual, we assume that it is possible to divide the choice process into a hierarchic sequence of decisions; at each level the user has a specific choice set, dependent on the choices made at upper levels. The utility associated to each alternative available at a given level is the sum of a specific term and of the satisfaction that takes into account the alternatives available at the lower levels. The structure of the demand model parallels the structure of the choice model in that users' flows given as input, or having made a choice at an upper level, are split at the lower level accordingly to the corresponding choice model.

The multimodal network, where users departing during the time horizon $[0, \Theta]$ are represented, is defined as a graph $G(N, A)$ where N is the node set and A is the arc set. The generic node is denoted as x, while origin and destinations of trips, belonging to $C \subseteq N$, are denoted as o and d, respectively; the generic arc is denoted as a, $TL(a)$ and $HD(a)$ being, respectively, its initial and final nodes. Modal subgraphs $G_m(N_m, A_m)$ are implicitly defined associating to each arc a set $M_a \subseteq M$ of transport modes. Each travel alternative is biunivocally associated with an acyclic path k of graph G from o to d on mode m, belonging to an *efficient path* set K_m^{od}, defined shortly later.

The proposed demand model is a Nested Logit with five choice levels, specifically: *generation*, *distribution*, *modal split*, *departure time choice*, and *path choice*. It thus follows, apart from the departure time choice, the Oppenheim (1995) approach to travel demand modelling. Such an approach is extended to a dynamic equilibrium framework as follows:

i) the demand model takes as input the time rate $N^o(\sigma') \geq 0$, $\sigma' \in [0, \Theta]$, of potential users having σ' as their desired departure time, for each origin o;

ii) the travel behaviour, with regard to the choices of travelling, of destination d and of mode m, is represented by models having, for each desired departure time, the same functional form and parameters;

iii) the departure time choice is represented by a model supplying actual departure time probabilities in the interval $B(\sigma') = [\max\{\sigma' - ADV, 0\}, \min\{\sigma' + DEL, \Theta\}]$, in the form of a probability density function of actual departure time τ' for each o, d, m and σ', where ADV and DEL are respectively the maximum delay and advance time admitted by users;

iv) the travel behaviour, with regard to path choice, is represented by mode specific models having, for each o, d, and τ', the same functional form and parameters.

It is worth noting that the departure time choice model is defined over an infinite number of alternatives belonging to a continuous choice set in $\Re^1$, while the overall structure of the choice model is that of a discrete choice model. We will deal with this issue in section 2.3. Assumptions ii) and iii), are consistent with model specification, but could be easily relaxed if needed, for example allowing time varying parameters to better fit experimental data as suggested in Bhat (2002). On the contrary, dealing with path choice model, we need hypothesis iv) in order to adopt an implicit path enumeration approach where path probabilities must be consistent with the *arc conditional probabilities* of leaving a node $x = TL(a)$ through arc a. This is because on a same arc, at a given instant, we can have users coming from different origins and departing at different instants.
The choice model is of the type depicted in Figure 1. The hierarchical order proposed here keeps a widely accepted structure and it is consistent, with reference to the position of departure time choice, with previous experimental analysis both in De Palma *et al.* (1983) and in Bhat (1998).

[Figure 1 here]

The following part of this section is organized as follows. In subsection 2.1 further definitions and notations needed to place the demand model in a dynamic framework will be introduced. Then the choice models related to each level will be examined in ascending order, since lower level satisfactions are needed when an upper level model is defined. Subsection 2.2 and 2.3 will be specifically devoted respectively to implicit path choice and departure time choice, while we will deal with mode, destination and travelling choice together in subsection 2.4. The formal definition of the corresponding demand models, determining demand temporal profiles, will follow in the inverse order, since upper level profiles are input to lower level models. As for choice models, in subsection 2.5 we will deal with *desired demand* profiles

together, defined for each desired departure time and determined by generation, distribution and modal split models; sections 2.6 and 2.7 will be specifically devoted to actual demand model and to network flow propagation model, supplying *actual demand* profiles, defined for each actual departure time, at mode - *od* pair level and at arc level, respectively. The arc flow obtained are consistent with the dynamic loading to the network of implicit path flows, taking the arc performances as given.
We will denote by $g(\sigma)$ or $g(\tau)$ the generic temporal profile referred to desired or actual departure time, respectively; then, $g(\sigma')$ or $g(\tau')$ denote the value of the temporal profile at given instants. When compact form is utilized, vectors of temporal profiles, having the appropriate dimension, are identified by the use of bold symbols.

2.1 Definitions and notations

As already said, it is assumed that, when travelling from node x to destination d on mode m, users consider only the subset K_m^{xd} of *efficient paths* from x to d, which are defined on the basis of *node topological order* (Nguyen, Pallottino and Inaudi, 1996). With reference to a given destination d and mode m, we thus assume that the node topological order $TO_m^d(x)$ is monotone non-decreasing with some measure, independent of congestion and time, of the *distance* on graph G from x to d. An arc is efficient if its tail has a higher topological order than its head, while a path is efficient if all its arcs are efficient. The sets of the efficient arcs exiting and entering a given node x are referred to, respectively, as the *efficient forward star* $FSE(x)_m^d = \{a \in A: TL(a) = x, TO_m^d(x) > TO_m^d(HD(a))\}$ and the *efficient backward star* $BSE(x)_m^d = \{a \in A: HD(a) = x, TO_m^d(TL(a)) > TO_m^d(x)\}$.

At any time τ', each mode m is characterized by mode - specific arc entering flows and performances:

$f_a^m(\tau')$ mode m users flows entering at τ' into arc a,

$c_a^m(\tau')$ mode m generalized costs on arc a, for users entering at τ'.

$t_a^m(\tau')$ mode m exit time from arc a, when entering at τ'.

If function $t_a^m(\tau')$ is monotone increasing, its inverse, which also results to be monotone increasing, can be defined as:

$t_a^{m\text{-}1}(\tau')$ entering time into arc a when exiting at τ'.

The generalized arc cost for users entering at time τ is thus simply assumed to be:

$$c_a^m(\tau') = \eta \cdot (t_a^m(\tau') - \tau') + mc_a^m(\tau'), \tag{3}$$

where mc_a^m is the temporal profile of the monetary cost, while η is the Value of Time.

These variables, as all the arc and node variables defined in the following, are defined on a time horizon $[0, \Theta + \Delta\Theta]$, $\Delta\Theta$ being the time needed to complete all trips started at Θ.

Path performances are defined as a function of time consistently with arc performances by utilizing the following arc set notation:

A_k^{xdm} set of the arcs constituting path $k \in K_m^{xd}$;

$A_k^{xdm}{}_a$ set of the arcs constituting the sub-path of path $k \in K_m^{xd}$ between node x and the tail of arc $a \in A_k^{xdm}$;

together with the following path notation:

$C_k^{xdm}(\tau')$ generalized cost of path $k \in K_m^{xd}$ for users leaving node x at time τ';

$T_k^{xdm}(\tau')$ time when users following path $k \in K_m^{xd}$ and leaving node x at time τ' reach destination d;

$T_k^{xdm}{}_a(\tau')$ time when users following path $k \in K_m^{xd}$ and leaving node x at time τ' enter arc $a \in A_k^{xdm}$;

For each path $k \in K_m^{xd}$ and arc $a \in A_k^{xdm}$, the travel time of the sub-path between node x and node $TL(a)$ at time τ' is the sum of the travel times of its arcs $b \in A_k^{xdm}{}_a$, each of them referred to the time $T_k^{xdm}{}_b(\tau')$ when users leaving x at τ' reach $TL(b)$; that is:

$$T_k^{xdm}{}_a(\tau') = \tau' + \sum_{b \in A_k^{xdm}{}_a} [t_b^m(T_k^{xdm}{}_b(\tau')) - T_k^{xdm}{}_b(\tau')]. \tag{4}$$

Assuming additive costs, the generalized cost of path $k \in K_m^{xd}$ at time τ' is the sum of the costs of its arcs $a \in A_k^{xdm}$, each of them referred to the time $T_k^{xdm}{}_a(\tau')$ when users leaving node x at τ reach node $TL(a)$; that is:

$$C_k^{xdm}(\tau') = \sum_{a \in A_k^{xdm}} c_a(T_k^{xdm}{}_a(\tau')). \tag{5}$$

2.2 Implicit path choice model

The implicit path choice model presented in this paper is the same introduced and described in Bellei, Gentile, Papola (2003), and is founded on the concepts of *arc conditional probability* and *node satisfaction*, which are respectively defined as follows:

$p_a^{dm}(\tau')$ probability that users on mode m follow their trip to destination d with arc a, conditional on being at node $TL(a)$ at time τ';

$w_x^{dm}(\tau')$ expected value of the maximum perceived utility among all paths to d on mode m departing from x at time τ'.

It is worth noting that, if demand temporal profiles are defined, as in this case, with reference to the departure time from the origin, the implicit path choice model, as well as the network flow propagation model defined in section 2.7, have necessarily to be formalized with respect to destination. A formalization with respect to the origins is required, in fact, when demand temporal profiles are defined with respect to the arrival time to the destination, which in this case are unknown and are actually obtained from the network flow propagation model. Nevertheless, the node satisfaction temporal profiles yield path choice satisfactions for each o-d pair and mode m as the node satisfaction $w_o^{dm}(\tau')$ at o, while the choice probability of the generic path $k \in K_m^{od}$ from o to d on mode m at time τ' is equal to the product of the conditional probabilities of its arcs $a \in A_k^{odm}$, each of them referred to the time $T_k^{odm}{}_a(\tau)$ when users leaving o at τ' reach $TL(a)$; that is:

$P_k^{odm}(\tau') = \prod_{a\in A_k^{odm}} p_a^{dm}(T_k^{odm}{}_a(\tau'))$ (6)

With reference to the Logit case, node satisfaction can be expressed in a recursive form and used to calculate arc conditional probabilities:

$w_d^{md}(\tau') = 0$

$w_x^{md}(\tau') = \theta_R \cdot \ln(\sum_{a\in FSE_{md}(x)} \exp((-c_a^m(\tau') + w_{HD(a)}^{md}(t_a^m(\tau'))) / \theta_R))$; (7)

$p_a^{md}(\tau') = \exp((-c_a^m(\tau') + w_{HD(a)}^{md}(t_a^m(\tau')) - w_{TL(a)}^{md}(\tau')) / \theta_R)$; (8)

which, in compact form, can be formally expressed through the functionals:

$\boldsymbol{w}(\tau) = \mathrm{w}(\boldsymbol{c}(\tau), \boldsymbol{t}(\tau))$; (9)

$\boldsymbol{p}(\tau) = \mathrm{p}(\boldsymbol{w}(\tau), \boldsymbol{t}(\tau), \boldsymbol{c}(\tau))$. (10)

Moreover, it can be proven (Bellei, Gentile, Papola, 2003) that path probabilities defined by (6) are the same as the path probabilities derived by an explicit path choice model based on path costs (5):

$$P_k^{mod}(\tau') = \frac{\exp\left(-\dfrac{C_k^{mod}(\tau')}{\theta_R}\right)}{\sum_{j\in K_m^{od}} \exp\left(-\dfrac{C_j^{mod}(\tau')}{\theta_R}\right)} \quad . \tag{11}$$

2.3 Departure time choice model

With reference to a desired departure time σ', we assume, extending, to the continuous choice set $B(\sigma')$ the standard assumption of the Logit model, that the random residuals associated to the infinitesimal alternatives constituting a partition of $B(\sigma')$ are independently and identically distributed (i.i.d.) Gumbel variables as in De Palma *et al.* (1983).

Any consideration concerning correlation among alternatives is left to further investigations. We choose on purpose a simple and well known choice probability model in order to focus on our main objective, that is, to provide a modelling framework for the simulation of elastic demand in the context of within-day DTA. In any case, as stated by De Palma *et al.* (1983), there are experimental evidences that the logit model can serve as a reasonable model of departure time behaviour.

Although we refer to users travelling from origin *o* to destination *d* by mode *m*, the corresponding indices will be dropped in order to improve readability. Let's then define:

$p(\tau'/\sigma')$ probability density of leaving at time τ', conditional to desired departure time σ';

$V(\tau',\sigma')$ specific utility of leaving at time τ', when σ' is the desired departure time;

$w(\tau')$ path choice satisfaction, given, for each *o*-*d* pair and mode *m*, by (7)

The Logit formula can be used to calculate the probability of choosing the generic departure interval $[\tau'-\mathrm{d}\tau/2,\ \tau'+\mathrm{d}\tau/2]$ as follows:

$$p\left(\tau'/\sigma'\right)\cdot \mathrm{d}\tau = \frac{\exp\left(\dfrac{\mathrm{V}\left(\tau',\sigma'\right)+w\left(\tau'\right)}{\theta_{DT}}\right)}{\int\limits_{\sigma'-ADV}^{\sigma'+DEL}\exp\left(\dfrac{\mathrm{V}\left(x,\sigma'\right)+w\left(x\right)}{\theta_{DT}}\right)\cdot \mathrm{d}x}\cdot \mathrm{d}\tau \quad , \tag{12}$$

where, clearly, the summation is replaced by an integration. On the analogy of the discrete case, the denominator in equation (12) is directly related to the departure time satisfaction:

$$W\left(\sigma'\right)=\theta_{DT}\cdot \ln\left[\int\limits_{\sigma'-ADV}^{\sigma'+DEL}\exp\left(\frac{\mathrm{V}\left(x,\sigma'\right)+w\left(x\right)}{\theta_{DT}}\right)\cdot \mathrm{d}x\right] \tag{13}$$

Finally we obtain, by assuming that the specific utility is proportional to the departure advance or delay:

$$V\left(\tau',\sigma'\right)=-\max\left\{b_{ADV}\cdot\left(\sigma'-\tau'\right),b_{DEL}\cdot\left(\tau'-\sigma'\right)\right\} \quad , \tag{14}$$

an explicit expression of the actual departure time τ' probability density function, conditional to desired departure time σ' and dependent on path choice satisfaction at τ':

$$p\left(\tau'/\sigma'\right)=\frac{\exp\left(\dfrac{w\left(\tau'\right)-\max\left\{b_{ADV}\cdot\left(\sigma'-\tau'\right),b_{DEL}\cdot\left(\tau'-\sigma'\right)\right\}}{\theta_{DT}}\right)}{\int\limits_{\sigma'-ADV}^{\sigma'+DEL}\exp\left(\dfrac{w\left(x\right)-\max\left\{b_{ADV}\cdot\left(\sigma'-x\right),b_{DEL}\cdot\left(x-\sigma'\right)\right\}}{\theta_{DT}}\right)\cdot \mathrm{d}x} \tag{15}$$

The departure time choice model is expressed in compact form through the functionals:

$$\boldsymbol{p_{DT}}(\tau/\sigma) = \mathrm{p}(\boldsymbol{V}(\tau, \sigma), \boldsymbol{w}(\tau)) \tag{16}$$

$$\boldsymbol{W_{DT}}(\sigma) = \mathrm{W}(\boldsymbol{V}(\tau, \sigma), \boldsymbol{w}(\tau)) \tag{17}$$

2.4 Mode, destination and travelling choice models

Referring to users travelling from origin *o* toward destination *d* and to a desired departure time σ', let's define:

$P_m^{od}(\sigma')$ choice probability of modal alternative *m*;
$V_m^{od}(\sigma')$ specific utility of modal alternative *m*;
$W_m^{od}(\sigma')$ departure time choice satisfaction, given by (13).

In the Logit case, the mode choice probabilities are:

$$P_m^{od}\left(\sigma'\right)=\frac{\exp\left(\dfrac{V_m^{od}\left(\sigma'\right)+W_m^{od}\left(\sigma'\right)}{\theta_M}\right)}{\sum\limits_{m'\in M}\exp\left(\dfrac{V_{m'}^{od}\left(\sigma'\right)+W_{m'}^{od}\left(\sigma'\right)}{\theta_M}\right)} \quad , \tag{18}$$

in compact form: $\boldsymbol{P_M}(\sigma) = \mathrm{P}(\boldsymbol{V_M}(\sigma), \boldsymbol{W_{DT}}(\sigma))$

and the resulting mode choice satisfaction is:

$$W^{od}\left(\sigma'\right)=\theta_M\cdot\ln\left[\sum_{m\in M}\exp\left(\frac{V_m^{od}\left(\sigma'\right)+W_m^{od}\left(\sigma'\right)}{\theta_M}\right)\right] \tag{19}$$

in compact form: $\boldsymbol{W_M}(\sigma) = \mathrm{W}(\boldsymbol{V_M}(\sigma), \boldsymbol{W_{DT}}(\sigma))$

We assume $V_m^{od}(\sigma') = \boldsymbol{\beta_M}^{\mathrm{T}}\cdot\boldsymbol{X_m}^{od}(\sigma')$, where $\boldsymbol{X_m}^{od}(\sigma')$ is a vector of time-dependent mode m attributes for users travelling from o to d and $\boldsymbol{\beta_M}$ is the corresponding vector of coefficients.

Referring to users departing from origin o and to a desired departure time σ', let's define:

$P_d{}^o(\sigma')$ choice probability of destination alternative d;

$V^{od}(\sigma')$ specific utility of destination alternative d,

while the mode choice satisfaction $W^{od}(\sigma')$ is given by (19).

In the Logit case, the destination choice probabilities are::

$$P_d^o\left(\sigma'\right)=\frac{\exp\left(\dfrac{V^{od}\left(\sigma'\right)+W^{od}\left(\sigma'\right)}{\theta_D}\right)}{\sum_{d'\in C}\exp\left(\dfrac{V^{od'}\left(\sigma'\right)+W^{od}\left(\sigma'\right)}{\theta_D}\right)}\quad, \tag{20}$$

in compact form: $\boldsymbol{P_D}(\sigma) = \mathrm{P}(\boldsymbol{V_D}(\sigma), \boldsymbol{W_M}(\sigma))$

and the resulting destination choice satisfaction is:

$$W^{o}\left(\tau'\right)=\theta_D\cdot\ln\left[\sum_{d\in C}\exp\left(\frac{V_d^{o}\left(\tau'\right)+W^{od}\left(\tau'\right)}{\theta_D}\right)\right] \tag{21}$$

in compact form: $\boldsymbol{W_D}(\sigma) = \mathrm{W}(\boldsymbol{V_D}(\sigma), \boldsymbol{W_M}(\sigma))$

We assume: $V^{od}(\sigma') = \boldsymbol{\beta_D}^{\mathrm{T}}\cdot\boldsymbol{X}^{od}(\sigma')$, where $\boldsymbol{X}^{od}(\sigma')$ is a vector of time-dependent destination d attributes for users travelling from o and $\boldsymbol{\beta_D}$ is the corresponding vector of coefficients.

Referring to users potentially departing from origin o and to a desired departure time σ', let's define:

$P^o(\sigma')$ choice probability of travelling

$V^o(\sigma')$ specific utility of travelling

while the destination choice satisfaction $W^o(\sigma')$ is given by (21).

In the Logit case, the travelling choice probabilities are:

$$P^{o}\left(\sigma'\right)=\frac{\exp\left(\dfrac{V^{o}\left(\sigma'\right)+W^{o}\left(\sigma'\right)}{\theta_E}\right)}{1+\exp\left(\dfrac{V^{o}\left(\sigma'\right)+W^{o}\left(\sigma'\right)}{\theta_E}\right)}\quad, \tag{22}$$

in compact form: $\boldsymbol{P_E}(\sigma) = \mathrm{P}(\boldsymbol{V_E}(\sigma),\boldsymbol{W_D}(\sigma))$,

where utility of not travelling is set to 0 because the model is additive and the resulting travelling choice satisfaction, which is the perceived utility of potential users is:

$$S^o(\sigma') = \theta_E \cdot \ln\left[1+\exp\left(\frac{V^o(\sigma')+W^o(\sigma')}{\theta_E}\right)\right] \quad , \tag{23}$$

in compact form: $\boldsymbol{S}(\sigma) = \mathrm{S}(\boldsymbol{V_E}(\sigma), \boldsymbol{W_D}(\sigma))$.

We assume $V^o(\sigma') = \boldsymbol{\beta_E}^{\mathrm{T}} \cdot \boldsymbol{X}^o(\sigma')$, where $\boldsymbol{X}^o(\sigma')$ is a vector of time-dependent choice of travelling attributes for potential users in o and $\boldsymbol{\beta_E}$ is the corresponding vector of coefficients.

2.5 Desired demand models

We remember that the flow $N^o(\sigma')$ of users potentially travelling from origin o at time σ' is assumed to be known. Let's denote the desired demand flows by:

$q^o(\sigma')$ flow from origin o with desired departure time σ'

$q^{od}(\sigma')$ flow from origin o to destination d with desired departure time σ'

$q_m^{od}(\sigma')$ flow from origin o to destination d on mode m with desired departure time σ'

Thus, the generation model is expressed by:

$$q^o(\sigma') = N^o(\sigma') \cdot P^o(\sigma') \quad , \tag{24}$$

where $P^o(\tau')$ has the expression (22), the distribution model by:

$$q^{od}(\sigma') = q^o(\sigma') \cdot P_d{}^o(\sigma') \quad , \tag{25}$$

where $P^{od}(\sigma')$ has the expression (20), and the modal split model by:

$$q_m^{od}(\sigma') = q^{od}(\sigma') \cdot P_m^{od}(\sigma') \tag{26}$$

where $P_m^{od}(\sigma')$ has the expression (18).

On the basis of relations (24), (25) and (26), it is possible to express the three desired demand models together as $q_m^{od}(\sigma') = N^o(\sigma') \cdot P^o(\sigma') \cdot P^{od}(\sigma') \cdot P_m^{od}(\sigma')$, or, in compact form, through the functional:

$$\boldsymbol{q}(\sigma) = \mathrm{q}(\boldsymbol{N}(\sigma), \boldsymbol{P_E}(\sigma), \boldsymbol{P_D}(\sigma), \boldsymbol{P_M}(\sigma)) \tag{27}$$

2.6 Actual demand model

The role of the actual demand model is to transform the desired demand temporal profile of any *o*-*d* pair and mode *m* into the corresponding actual demand temporal profile, on the basis of the departure time probabilities produced by the departure time choice model. As in section 2.3 the *o*, *d* and *m* indices will be dropped. The desired demand profile provided by (24), (25) and (26) is thus denoted as $q(\sigma)$.

To transform $q(\sigma)$ into the *actual demand* temporal profile $d(\tau)$ the contributes coming from departure time choices corresponding to different desired departure times have to be considered. The number of trips started within the infinitesimal departure interval $[\tau'\text{-d}\tau/2, \tau'\text{+d}\tau/2]$ is thus given by the integral, for each σ': $\tau' \in B(\sigma')$, of the flow $q(\sigma')$ of trips with desired departure time within the infinitesimal interval $[\sigma'\text{-d}\sigma/2, \sigma'\text{+d}\sigma/2]$, multiplied by the probability $\mathrm{p}(\tau'/\sigma')$ that their actual departure time is in $[\tau'\text{-d}\tau/2, \tau'\text{+d}\tau/2]$; that is:

$$d(\tau')\cdot \mathrm{d}\tau = \int_{\tau'-DEL}^{\tau'+ADV} q(\sigma)\cdot \mathrm{d}\sigma \cdot \mathrm{p}(\tau'/\sigma)\cdot \mathrm{d}\tau \tag{28}$$

On the basis of (15) and (28) the actual demand profile may be expressed as:

$$d(\tau') = \int_{\tau'-DEL}^{\tau'+ADV} q(\sigma)\cdot \frac{\exp\left(\dfrac{w(\tau') - \max\{b_{ADV}\cdot(\sigma-\tau'), b_{DEL}\cdot(\tau'-\sigma)\}}{\theta_{DT}}\right)}{\int_{\sigma-ADV}^{\sigma+DEL} \exp\left(\dfrac{w(x) - \max\{b_{ADV}\cdot(\sigma-x), b_{DEL}\cdot(\tau-x)\}}{\theta_{DT}}\right)\cdot \mathrm{d}x}\cdot \mathrm{d}\sigma \tag{29}$$

As in general the profiles $q(\sigma)$, and $w(\tau)$ can assume any form, the integrals in equations (15) and (29) cannot be solved in closed form. An ad-hoc procedure, based on the assumption that the profile $q(\sigma)$ is piece-wise constant, while the profile $w(\tau)$ is piece-wise linear, is presented in section 4.2.

The actual demand model is thus expressed in compact form through the functional:

$$\boldsymbol{d}(\tau) = \mathrm{d}(\boldsymbol{q}(\sigma), \boldsymbol{p_{DT}}(\tau/\sigma)) = \mathrm{d}(\boldsymbol{q}(\sigma), \mathrm{p}(\boldsymbol{V}(\tau, \sigma), \boldsymbol{w}(\tau))) \tag{30}$$

in fact, when solving the integral in (29) σ' disappears, while profile $q(\sigma)$ does not.

2.7 Network flow propagation model

The path flows could be easily derived from the actual demand profiles $d(\tau)$, provided by (29) for each o, d and m, and by the path choice probabilities (5). This however would lead, to ensure consistency with travel times when they are loaded to the network, to the formulation of Dynamic Network Loading (DNL) problem. As shown in Bellei, Gentile, Papola (2003), an implicit path enumeration approach allows to define an equivalent fixed point DTA formulation, not requiring DNL formulation, which is applied to the problem at hand in the following section.

Within this approach, an arc network loading consistent with travel times, actual demand profiles and arc conditional probabilities is obtained, trough what we call a network flow propagation model, since it describes how arc flow profiles by mode and destination propagate from upstream to downstream arcs:

$$f_a^{md}(\tau') = p_a^{md}(\tau')\cdot[d_m^{TL(a)d}(\tau') + \textstyle\sum_{b\in BSE_{md}(TL(a))} [f_b^{md}(t_b^{m\,-1}(\tau'))\cdot\partial t_b^{m\,-1}(\tau')/\partial\tau]] \tag{31}$$

where $d_m^{TL(a)d}(\tau') = 0\ \forall\, \tau' \in [0, \Theta]$ if $TL(a) \notin \mathrm{C}$. Arc flows by mode are then obtained by simply summing over destinations:

$$f_a^{m}(\tau') = \textstyle\sum_{d\in C} f_a^{md}(\tau') \tag{32}$$

The network flow propagation model is expressed in compact form through the functional:

$$\boldsymbol{f}(\tau) = \omega(\boldsymbol{d}(\tau), \boldsymbol{p}(\tau), \boldsymbol{t}(\tau)) \tag{33}$$

3 SUPPLY AND EQUILIBRIUM MODELS

In analogy with the static case, the within-day DTA, regarded as a dynamic user equilibrium, can be consistently formalized through a fixed point problem expressed in terms of the temporal profiles of arc flows and transit frequencies, by combining the arc performance function with the Network Loading Map (NLM) and thus avoiding to introduce the DNL. To this purpose, the NLM is here extended to the case of elastic demand with departure time choice, as depicted in Figure 2.

[Figure 2 here]

In the following subsections, the arc performance function and the equilibrium model will be briefly described only in a qualitative manner, providing references where this matters are widely investigated and discussed.

3.1 Arc performance models

The concept of *equivalent flow* $v_a(\tau)$ is introduced first, in order to represent the congestion phenomena considered. With reference to the arcs of the transit network, the equivalent flows coincide with the user flows, while, with reference to each arc *a* of the road network, we assume that the equivalent flow is given by a linear combination of flows for modes $m \in M_a$, including transit vehicles. Then, equivalent flows are expressed in compact form as:

$$\boldsymbol{v}(\tau) = \mathrm{v}(\boldsymbol{f}(\tau), \boldsymbol{\phi}(\tau)) \tag{34}$$

With reference to the road network, the dynamic arc performance function is specified through the macroscopic and non-stationary *link-node* model presented in Bellei, Gentile, Papola (2003), where the vehicular flow, considered as a monodimensional partially compressible fluid, behave accordingly with the simplified kinematic wave theory and with a triangular shaped fundamental diagram. The arc is modelled by considering two serial phases: a *running phase*, modelled through a *bottleneck*, which simulates the link travel time including the delay due to the *over-saturated* queue; a *waiting phase*, which represents the average delay due to the *under-saturated* queue proper to an intermittent service (such a traffic light).

With reference to the transit network, the dynamic performance function is specified through the transit supply model presented in Gentile, Meschini, Papola (2002) and (2003). With reference to the generic line *l*, the characterizing element of this model is the non separable *stop model*, depicted in Figure 3.

[Figure 3 here]

The *line arc* represents both the dwelling time at the stop corresponding to its tail, and the running time between two successive stops. The dwelling time is dependent on the flows that are boarding, alighting, and on board the line at the corresponding stop; the running time is dependent on the congestion level along the road arcs employed by the line.
The *waiting arc* represents the average waiting time for the transit vehicle at the stop, and it is assumed to be proportional to the inverse of the line frequency.
The *queue arc*, modelled through a *bottleneck* with variable outgoing capacity, represents the additional travel time due to an over saturation of the line, which occurs whenever boarding and onboard flows overwhelm the line capacity.
The *stop* and *waiting arcs*, both with no travel time and cost allow expressing the non separable dynamic stop model as a function of arc entering flow temporal profiles only.
The *stop access arc* allows distinguishing between different stop nodes corresponding to the same walking node.
The *alighting* and *walking arcs* are assumed uncongested.
On the basis of the above outlined models, congestion phenomena are represented assuming that, in general, the temporal profiles of exit times depend on the temporal profiles of both the arc equivalent flows and the line frequencies:

$$\boldsymbol{t}(\tau) = \mathrm{t}(\boldsymbol{v}(\tau), \boldsymbol{\phi}(\tau)) \tag{35}$$

The arc cost is assumed to be the sum of a term proportional to the travel time, and a term taking into account the monetary cost. Then, the temporal profiles of arc cost are expressed in compact form by the functional:

$$\boldsymbol{c}(\tau) = \mathrm{c}(\boldsymbol{v}(\tau), \boldsymbol{\phi}(\tau)) \tag{36}$$

In order to complete the dynamic representation of the supply we need to define a *line model*, which, for given transit frequencies at terminals, yields the frequency pattern along each line. Contrary to the static case in fact, the line frequency temporal profiles are generally not constant along the line: on one side, there is a translation of the terminal frequency in space and time; on the other side, the variation in time of the line travel times makes vehicles spread and jam (see for example Figure 4).

[Figure 4 here]

As explained in Gentile, Meschini, Papola (2002) and (2003), an effective representation of this phenomenon can be achieved regarding transit frequencies as flows on the network. In this way, in analogy with the network flow propagation, the temporal profiles of the frequency at each stop of the generic line l is expressed as:

$$\phi_a(\tau') = \phi_{FA(l)}(T_a^{l\,-1}(\tau')) \cdot (\partial T_a^{l\,-1}(\tau') / \partial \tau) \tag{37}$$

where: $\phi_{FA(l)}(\tau)$ is the temporal profile of the line frequency at terminal (assumed given); $T_a^l(\tau')$ express the time when a vehicle running line l and departed from terminal at time τ'

reaches $TL(a)$; $T_a^{l\,-1}(\tau)$ is the inverse of the temporal profile just defined, and express the departure time from the terminal for a vehicle running line l and reaching $TL(a)$. Expressing profile $T_a^l(\tau)$ as a function of arc exit time profiles through equation (4), we have in compact form:

$$\boldsymbol{\phi}(\tau) = \phi(\boldsymbol{t}(\tau)) \tag{38}$$

3.2 Formulation of the dynamic assignment model

Relations (35), (36), and (38) determine a circular dependence between transit frequencies and arc performances, in addition to the classical one between user flows and arc performances. Then, the DTA is formulated as a fixed-point problem expressed in terms of the temporal profiles of arc inflows and transit frequencies on the time horizon $[0, \Theta + \Delta\Theta]$. Formally, combining (9), (10), (16), (17), (18), (19), (20), (21), (22), (27), (30), (33), (34), (35), (36) and (38) it is:

$$[\boldsymbol{f}(\tau), \boldsymbol{\phi}(\tau)] = \Phi[\boldsymbol{f}(\tau), \boldsymbol{\phi}(\tau)] \ , \tag{39}$$

where the functional relation denoted by (39), whose expression is quite complex, is explained through the flow-diagram depicted in Figure 5.

[Figure 5 here]

To be notice that, from the point of view of fixed point definition, σ is an auxiliary variable, since it just serves to take into account the effects on the demand of the difference between actual and desired departure times.

4 ALGORITHM

In order to implement the proposed DTA model, the period of analysis is divided into I time intervals identified by the sequence of instants $(\tau^0, \dots, \tau^i, \dots, \tau^I)$. In the following we assume to approximate the generic temporal profile x through either a piece-wise constant or a piece-wise linear function defined by the values taken at such instants, so that for the two cases we have respectively:

$$x(\tau^0) = x^0 \ , \ x(\tau) = x^i \ , \ \tau \in (\tau^{i-1}, \tau^i] \ , i = 1, \dots, I \tag{40a}$$

$$x(\tau^0) = x^0 \ , \ x(\tau) = x^{i-1} + (\tau - \tau^{i-1}) \cdot (x^i - x^{i-1}) / (\tau^i - \tau^{i-1}) \ , \ \tau \in (\tau^{i-1}, \tau^i] \ , i = 1, \dots, I \tag{40b}$$

Specifically, the temporal profiles of the flows are assumed piece-wise constant, while the temporal profiles of performances, satisfactions and choice probabilities are assumed to be piece-wise linear.

With these assumptions, it is possible to devise an efficient elastic demand DTA solution algorithm, outlined in the first section, that requires only some more trivial calculus than a rigid demand DTA with regard to the inclusion of generation, distribution and modal split models. Since taking into account the choice of departure time is somewhat more demanding, the corresponding procedures for the calculation of choice probabilities and of actual demand are illustrated in more detail in the second section.

4.1 Elastic Demand Dynamic Traffic Assignment

The resulting fixed point problem formalizing the elastic demand DTA is solved through the Method of Successive Averages (MSA). It can be thus defined by the following macro steps:

0)	$k = 0$, $\boldsymbol{f}^{k+1} = \{\mathbf{0} \mid \boldsymbol{f}_{\text{iniz}}\}$	initialization
1)	$k = k + 1$	updates the iteration counter
2)	$\boldsymbol{v}^k = \mathrm{v}(\boldsymbol{f}^k, \boldsymbol{\phi}^k)$	calculates equivalent flows
3)	$\boldsymbol{t}^k = \mathrm{t}(\boldsymbol{v}^k, \boldsymbol{\phi}^k)$, $\boldsymbol{c}^k = \mathrm{c}(\boldsymbol{v}^k, \boldsymbol{\phi}^k)$	calculates the arc performances
4)	$\boldsymbol{w}^k = \mathrm{w}(\boldsymbol{c}^k, \boldsymbol{t}^k)$, $\boldsymbol{p}^k = \mathrm{p}(\boldsymbol{w}^k, \boldsymbol{c}^k, \boldsymbol{t}^k)$	calculates arc conditional probabilities
5)	$\boldsymbol{P_{DT}}^k = \mathrm{P}(\boldsymbol{V_{DT}}, \boldsymbol{w}^k)$, $\boldsymbol{W_{DT}}^k = \mathrm{W}(\boldsymbol{V_{DT}}, \boldsymbol{w}^k)$	calculates departure time probabilities
6)	$\boldsymbol{P_M}^k = \mathrm{P}(\boldsymbol{V_M}, \boldsymbol{W_{DT}}^k)$, $\boldsymbol{W_M}^k = \mathrm{W}(\boldsymbol{V_M}, \boldsymbol{W_{DT}}^k)$	calculates modal choice probabilities
7)	$\boldsymbol{P_D}^k = \mathrm{P}(\boldsymbol{V_D}, \boldsymbol{W_M}^k)$, $\boldsymbol{W_D}^k = \mathrm{W}(\boldsymbol{V_D}, \boldsymbol{W_M}^k)$	calculates distribution probabilities
8)	$\boldsymbol{P_E}^k = \mathrm{P}(\boldsymbol{V_E}, \boldsymbol{W_D}^k)$, $\boldsymbol{S}^k = \mathrm{S}(\boldsymbol{V_E}, \boldsymbol{W_D}^k)$	calculates generation probabilities
9)	$\boldsymbol{q}^k = \mathrm{q}(\boldsymbol{N}, \boldsymbol{P_G}^k, \boldsymbol{P_D}^{\mathrm{k}}, \boldsymbol{P_M}^k)$	calculates the desired demand flows
10)	$\boldsymbol{d}^k = \mathrm{d}(\boldsymbol{q}^k, \boldsymbol{W_{DT}}^k, \boldsymbol{w}^k)$	calculates the actual demand flows
11)	$\boldsymbol{f_{NLM}}^k = \omega(\boldsymbol{p}^k, \boldsymbol{t}^k, \boldsymbol{d}^k)$	performs network flow propagation
12)	$\boldsymbol{\phi_{NLM}}^k = \phi(\boldsymbol{t}^k)$	performs line frequency propagation
13)	$\boldsymbol{f}^{k+1} = \boldsymbol{f}^k + 1/k \cdot (\boldsymbol{f_{NLM}}^k - \boldsymbol{f}^k)$	updates the arc flows
14)	$\boldsymbol{\phi}^{k+1} = \boldsymbol{\phi}^k + 1/k \cdot (\boldsymbol{\phi_{NLM}}^k - \boldsymbol{\phi}^k)$	updates the line frequencies
15)	if $\max_{a\in A\ i\in I} \lvert y_a^{i\,k} - f_a^{i\,k} \rvert > \varepsilon$ and $k < k_{\max}$ then goto 2	stop criterion

Steps 3), 4), 11) and 12) present algorithmic procedures quite specific for the dynamic context, and are detailed in Bellei, Gentile, Papola (2003) and in Gentile, Meschini, Papola (2003).

4.2 Departure time choice and actual demand

Since we approximate flow temporal profiles through piece-wise constant functions over predefined time intervals, we are only concerned in determining the time interval in which each user actually depart, and not the exact user departure time. Hence, we only need to evaluate the probability $P((\tau^{i-1}, \tau^i]/\sigma')$ of departing during interval $(\tau^{i-1}, \tau^i]$, when σ' is the desired departure time. This can be done integrating equation (15) over the interval $(\tau^{i-1}, \tau^i]$:

$$P\left(\left(\tau^{i-1},\tau^{i}\right]/\sigma'\right)=\int_{\tau^{i-1}}^{\tau^{i}}\frac{\exp\left(\dfrac{w(\tau)-\max\left\{b_{ADV}\cdot(\sigma'-\tau),b_{DEL}\cdot(\tau-\sigma')\right\}}{\theta_{DT}}\right)}{\int_{\sigma'-ADV}^{\sigma'+DEL}\exp\left(\dfrac{w(x)-\max\left\{b_{ADV}\cdot(\sigma'-x),b_{DEL}\cdot(x-\sigma')\right\}}{\theta_{DT}}\right)\cdot\mathrm{d}x}\cdot\mathrm{d}\tau \qquad (41)$$

Since profile $w(\tau)$ is expressed through relation (40b), the arguments of the integrals in equation (41) are piece-wise linear. The points $P_i^{\,j} = P((\tau^{i-1}, \tau^i]/\sigma^j)$ and W^j, $i = 1, \ldots, I$, $j = 1, \ldots, I$, which respectively define, coherently with the relation (40b), profiles $P((\tau^{i-1}, \tau^i]/\sigma)$ and $W(\sigma)$, can be calculated through the following procedure:

Function *departure_time_choice*

$A = 0$; $D = 0$

For $j = 1$ **To** I

1) **For** $i = 1$ **To** I

$p_i^{\,j} = 0$

Next i

2) **Do While** $\tau^A \le \sigma^j - ADV$ **And** $A < I$

$A = A + 1$

Loop

3) **Do While** $\tau^D < \sigma^j + DEL$ **And** $D < I$

$D = D + 1$

Loop

4) $$\alpha=\left(w^{i-1}-\tau^{i-1}\cdot\frac{w^{i}-w^{i-1}}{\tau^{i}-\tau^{i-1}}-\sigma^{A}\cdot b_{ADV}\right)\Big/\theta_{DT}$$

$$\beta=\left(\frac{w^{i}-w^{i-1}}{\tau^{i}-\tau^{i-1}}+b_{ADV}\right)\Big/\theta_{DT}$$

$$P_A^{\,j}=\exp(\alpha)/\beta\cdot\left[\exp\left(\tau^{A}\cdot\beta\right)-\exp\left(\left(\sigma^{j}-ADV\right)\cdot\beta\right)\right]$$

$W^j = p_{ADV(j)}^{\,j}$

5) **For** $i = A + 1$ **To** j

$$\alpha=\left(w^{i-1}-\tau^{i-1}\cdot\frac{w^{i}-w^{i-1}}{\tau^{i}-\tau^{i-1}}-\sigma^{j}\cdot b_{ADV}\right)\Big/\theta_{DT}$$

$$P_i^{\,j}=\exp(\alpha)/\beta\cdot\left[\exp\left(\tau^{i}\cdot\beta\right)-\exp\left(\tau^{i-1}\cdot\beta\right)\right]$$

$W^j = W^j + p_i^{\,j}$

Next i

6) **For** $i = j + 1$ **To** $D - 1$

$$\beta=\left(\frac{w^{i}-w^{i-1}}{\tau^{i}-\tau^{i-1}}-b_{DEL}\right)\Big/\theta_{DT}$$

$P_i{}^j = \exp(\alpha)/\beta \cdot \left[\exp(\tau^i \cdot \beta) - \exp(\tau^{i-1} \cdot \beta)\right]$

$W^j = W^j + p_i{}^j$

Next *i*

7) $\alpha = \left(w^{i-1} - \tau^{i-1} \cdot \dfrac{w^i - w^{i-1}}{\tau^i - \tau^{i-1}} + \sigma^D \cdot b_{DEL} \right) \Big/ \theta_{DT}$

$P_D{}^j = \exp(\alpha)/\beta \cdot \left[\exp\left((\sigma^j + DEL) \cdot \beta\right) - \exp(\tau^{D-1} \cdot \beta)\right]$

$W^j = W^j + p_D{}^i$

8) **For** $i = ADV(j) + 1$ **To** $DEL(j) - 1$

$p_i{}^j = p_i{}^j / W^j$

Next *i*

$W^j = \theta_{DT} \cdot \ln(W^j)$

Next *j*

For each desired departure time σ^j, step 1) resets the choice probabilities; steps 2) and 3) find τ^A and τ^D, which are, respectively, the first predefined instants after $\sigma^j - ADV$ and $\sigma^j + DEL$; steps 4) through 7) evaluate the numerator of equation (41) for each interval $(\tau^{i-1}, \tau^i]$, $i = A, \ldots, D$; steps 8) finally calculate the probability for each interval $(\tau^{i-1}, \tau^i]$, $i = A, \ldots, D$.
The number D^i of users actually departed during the generic interval $(\tau^{i-1}, \tau^i]$ is then given by the integral, for each instant σ' such that $\sigma' \in [\tau^{i-1} - DEL,\ \tau^i + ADV]$, of the desired demand flow $q(\sigma')$ multiplied by the related probability $P((\tau^{i-1}, \tau^i]/\sigma')$:

$$D^i = \int_{\tau^{i-1} - DEL}^{\tau^i + ADV} q(\sigma) \cdot P\left((\tau^{i-1}, \tau^i]/\sigma\right) \cdot d\sigma \qquad (42)$$

Then, coherently with the relation (40a), profile $d(\tau)$ is simply given by:

$$d(\tau') = d^i = D^i / (\tau^i - \tau^{i-1}) \ , \quad \tau' \in (\tau^{i-1}, \tau^i] \ , \ i = 1, \ldots, I \qquad (43)$$

Since profile $P((\tau^{i-1}, \tau^i]/\sigma)$ is expressed through relations (40b), the points d^i can be calculated through the following procedure:

Function *actual_demand*

$A = 0$; $D = 0$

For $i = 1$ **To** I

1) **Do While** $\sigma^D \leq \tau^j - DEL$ **And** $D < I$

$D = D + 1$

Loop

2) **Do While** $\sigma^A < \tau^j + ADV$ **And** $A < I$

$A = A + 1$

Loop

3) $d^i = q^D \cdot 0{,}5 \cdot P_i^D \cdot \left(\tau^D - \left(\tau^i - DEL\right)\right) / \left(\tau^i - \tau^{i-1}\right)$

4) **For** $j = D + 1$ **To** $A - 1$

$d^i = d^i + q^j \cdot 0{,}5 \cdot \left(P_i^j + P_i^{j-1}\right) \cdot \left(\sigma^j - \sigma^{j-1}\right) / \left(\tau^i - \tau^{i-1}\right)$

Next j

5) $d^i = d^i + q^A \cdot 0{,}5 \cdot P_j^{A-1} \cdot \left(\left(\tau^i + ADV\right) - \tau^{A-1}\right) / \left(\tau^i - \tau^{i-1}\right)$

Next i

For each actual departure interval (τ^{i-1}, τ^i], steps 1) and 2) find σ^D and σ^A, which are respectively the first periodization instant after τ^{i-1}–DEL and τ^i+ADV; steps 3) to 5) evaluate equation (42) for each interval (σ^{j-1}, σ^j], $j = D, \ldots, A$.

5 NUMERICAL APPLICATION

The network of Sioux Falls, consisting of 76 directed arcs and 24 centroids, has been considered for a numerical application. In order to investigate the effectiveness of the proposed departure time choice model and algorithm, we compare the results of two DTA on this network, performed considering both rigid and elastic departure time choice.

The period of analysis, 6 hours long, was subdivided in 36 time intervals 10 minutes long; the known daily demand has been distributed consistently with an arbitrary temporal profile simulating a morning peak; suitable values were assigned arbitrarily to all the input parameters previously discussed throughout the paper.

Results obtained appear to be plausible. Figure 6, referring to one of the most congested arc of the network, shows flow spreading from congested time intervals toward uncongested ones; local increase of flow in congested time intervals, although counterintuitive, does not clash with the effectiveness of the model, since travel times (and thus travel costs) are non-increasing both locally (Figure 7), and globally (Figure 9), when the elasticity with respect to the departure time is considered. These results confirm that the departure time elasticity yields a more satisfactory flow pattern on the network, thus allowing higher loads on the most attractive arcs (Figure 8). Finally, Figure 10 shows a comparison in terms of the calculation time and number of iterations needed to obtain equilibrium, with and without departure time choice, on a PC with a 1.9 Ghz CPU. The same convergence criterion, defined above, was adopted in both cases:

$$\frac{\sum_{a \in A} \sum_{i \in I} \left| f_{a,NLM}^i - f_{a,EQ}^i \right| \cdot \left(\tau^i - \tau^{i-1}\right) \cdot c_{a,EQ}^i}{\sum_{a \in A} \sum_{i \in I} f_{a,EQ}^i \cdot \left(\tau^i - \tau^{i-1}\right) \cdot c_{a,EQ}^i} \leq \varepsilon = 0.01$$

As expected, the case with departure time choice is more resource demanding, although calculation times remains quite acceptable for real applications.

[Figure 6 here]

[Figure 7 here]

[Figure 8 here]

[Figure 9 here]

[Figure 10 here]

6 CONCLUSIONS

The purpose of this paper was to provide a modelling framework for the simulation of elastic demand in the context of within-day dynamic traffic assignment. To this end, a multimodal within-day dynamic traffic assignment model is presented with a Nested Logit demand model considering combined travel, destination, mode, departure time, and route choices. With specific reference to departure time choice, a continuous version of the logit model is adopted, so that enumerating explicitly the desired departure time intervals is not needed.

The resulting dynamic traffic assignment model, which allows us to represent demand and supply dynamic phenomena concerning multimodal urban networks under congested conditions, is formulated through a fixed-point problem, which can be solved through an efficient implicit path MSA algorithm capable of dealing with real networks.

Both, the model and the devised algorithm, can be easily extended to the multi-class case, so overcoming the restrictions due to the hypotheses we assume of constancy over time of the functional form as well as of the parameters of the choice models.

Any question concerning existence and uniqueness of solutions has not been tackled in this paper. Yet, we can say that the devised algorithm, applied to a test network, converges to plausible solutions.

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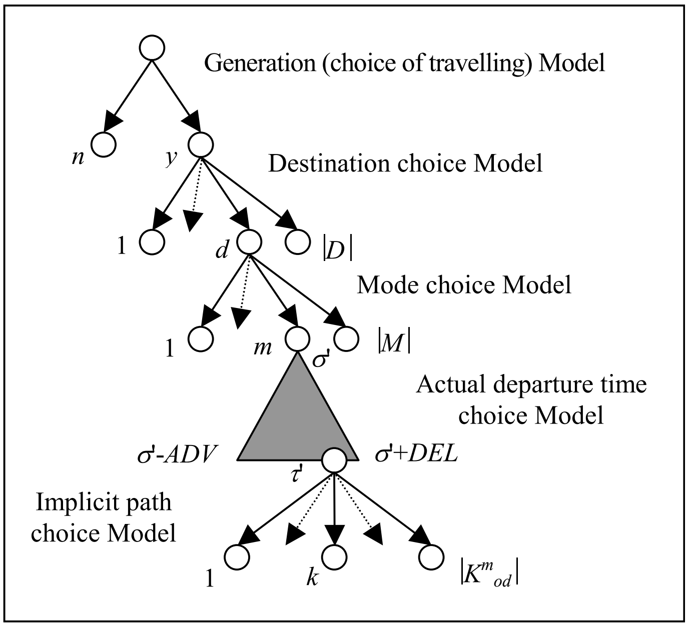

Figure 1 – Choice Tree for potential users from origin o with desired departure time τ'

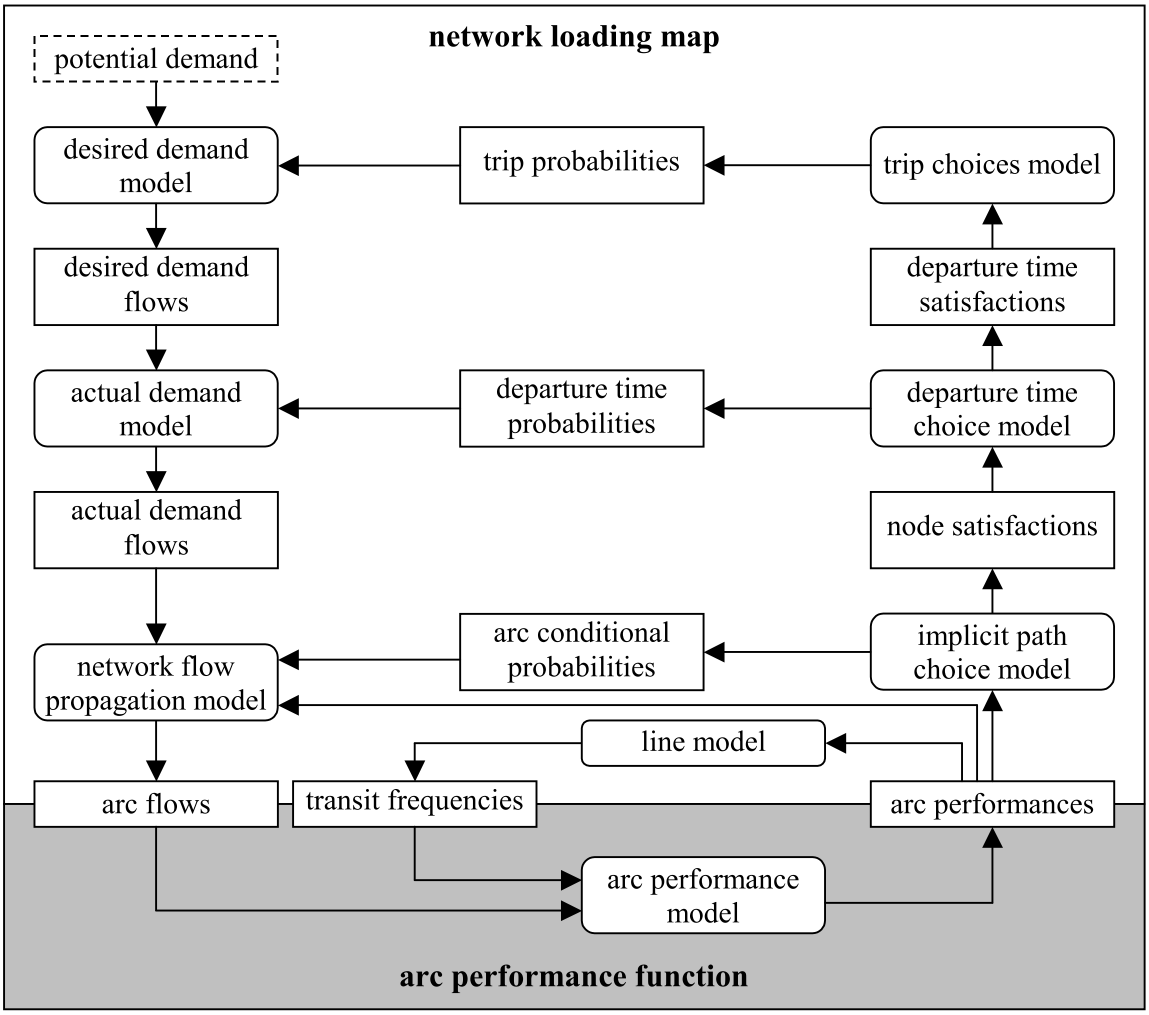

Figure 2 – Flow Diagram of Elastic Demand DTA *Fixed Point Problem*

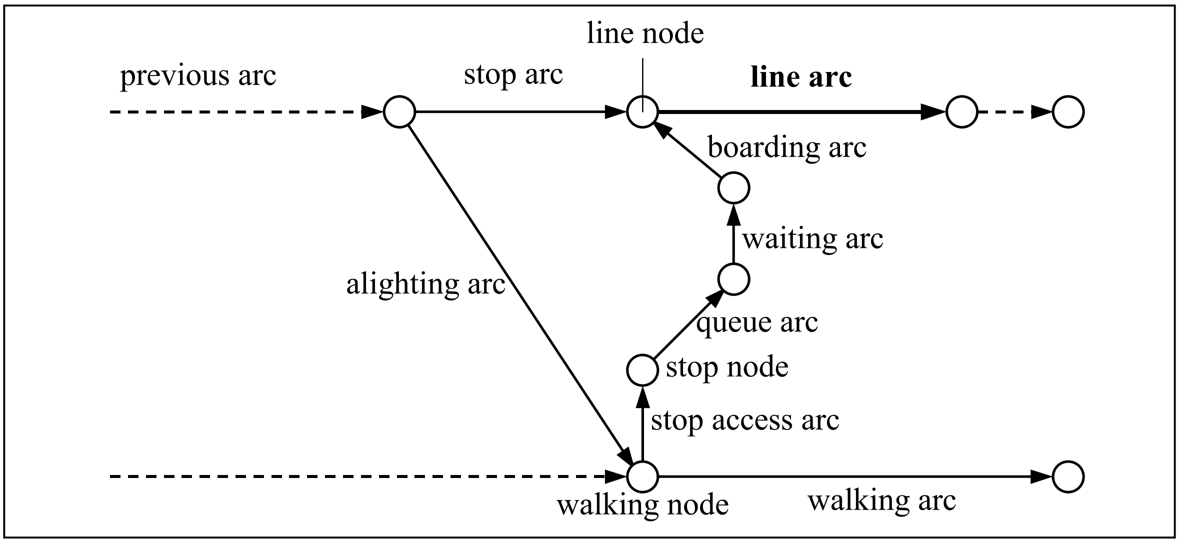

Figure 3 – Transit stop model

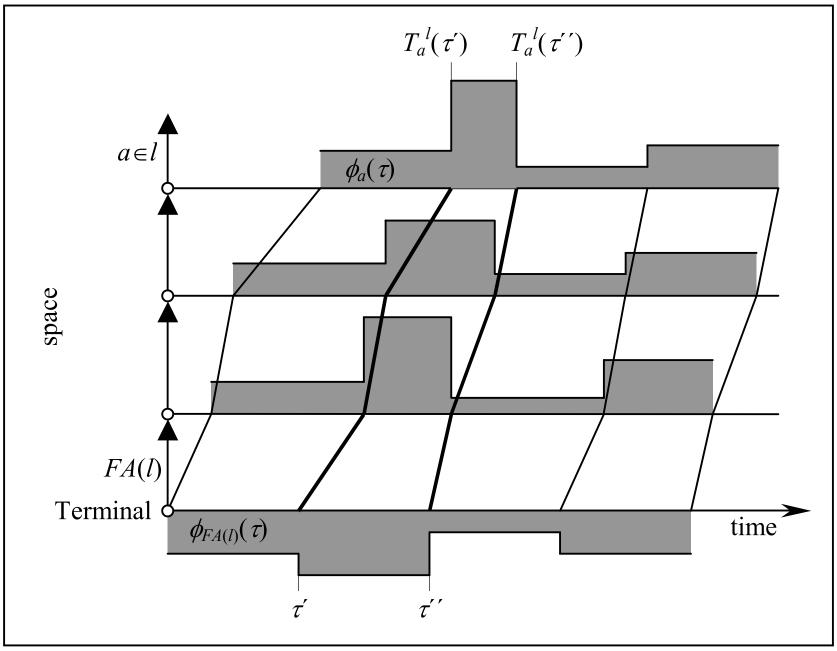

Figure 4 – Frequency temporal profiles along the transit line

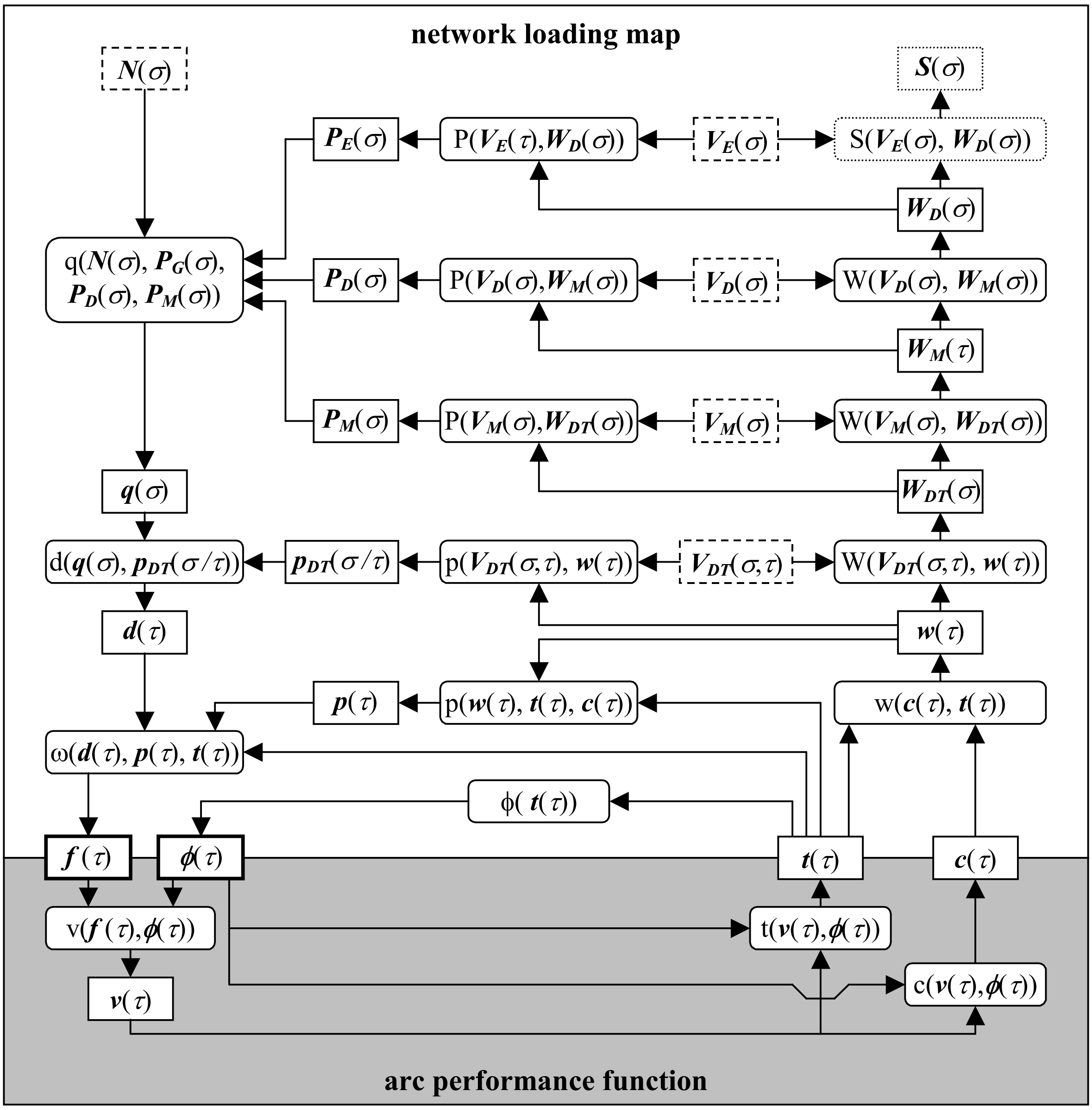

Figure 5 – Functional of the fixed point problem

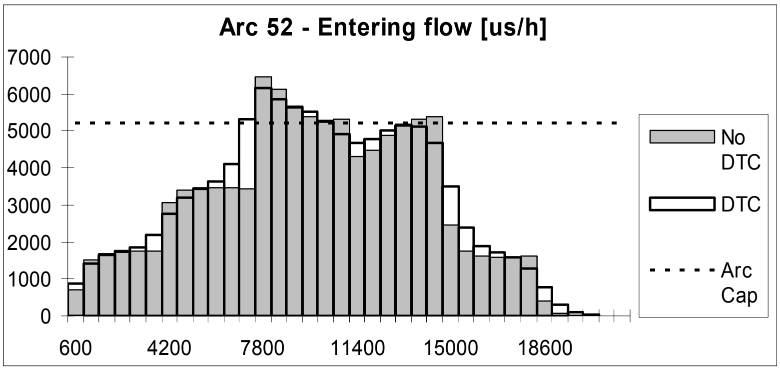

Figure 6 – Inflow temporal profile for a congested arc with and without departure time choice

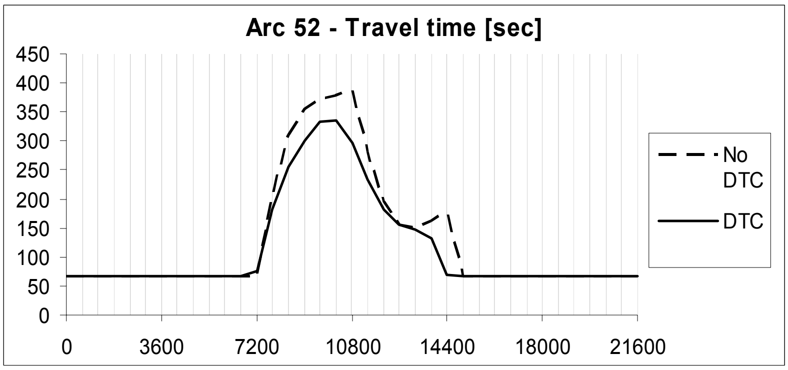

Figure 7 – Travel time temporal profile for a congested arc with and without departure time choice

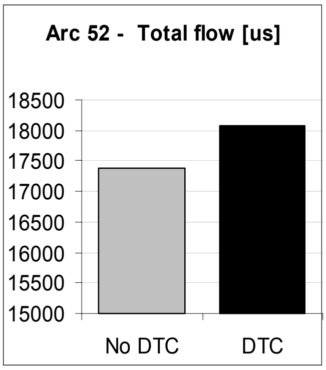

Figure 8 – Total flow for a congested arc with and without departure time choice

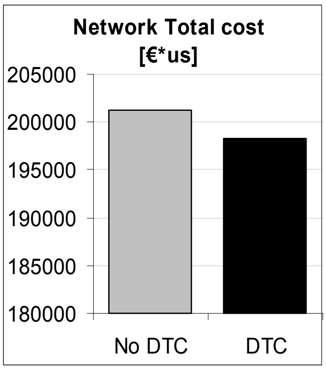

Figure 9 – Total travel cost on the network with and without departure time choice

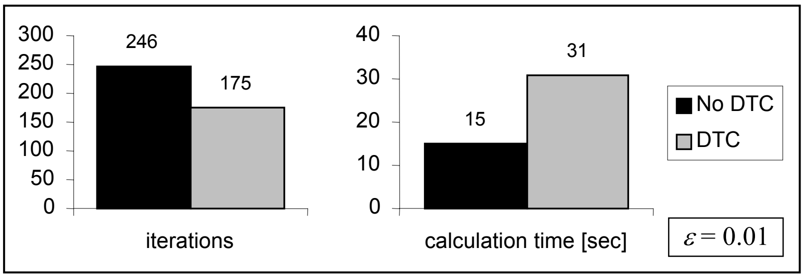

Figure 10 – Performances of the algorithm for the two DTA