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Opening the DICE black-box

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1 | Motivation

Integrated Assessment Models (IAMs) attempt to capture and describe the interactions of (i) human behaviour, (ii) economic activity, and (iii) and climate dynamics and impacts. However, IAMs are often treated as some sort of black-box when calculating solutions.

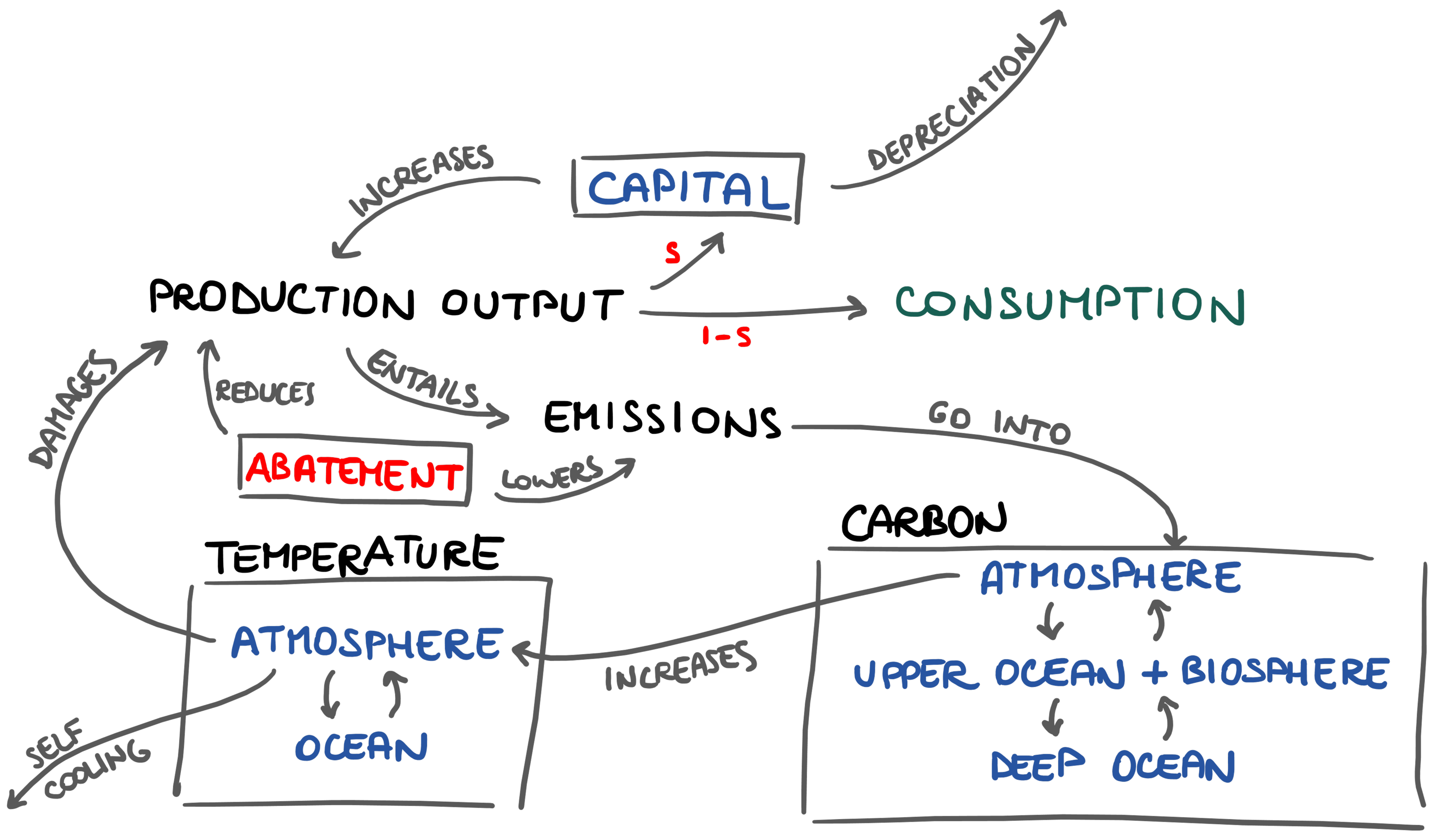
Enter system dynamics $\Longrightarrow$ Press button $\Longrightarrow$ Get numeric results $\Longrightarrow$
$\Longrightarrow$ Obtain SCC as the optimal carbon price

This raises several questions:

- Do we really understand, what is happening in DICE (or other IAMs) and what the SCC actually is?
- Are we aware what DICE can and cannot do / what it should and shouldn't be used for?
- Is the SCC actually the optimal carbon price/tax?

2 | The DICE framework

The DICE-model combines an macro-economic growth model with the geophysical development of carbon stocks and different temperatures.



We propose a continuous-time version of the standard DICE-model. It thereby takes a social-planer perspective and the solution of the model is the socially optimal outcome.

$$\max_{s\in[0,1],\mu\in[0,\bar{\mu}]} \int_0^T e^{-\rho t} L(t) u(c(t)) dt$$

$\dot{K} = sQ - \delta_K K$	$K(0) = K_0$	Production capital
$\dot{M} = \Phi M + \alpha E(t) \cdot [1,0,0]^T$	$M(0) = M_0$	Carbon stocks
$\dot{T} = \xi T + \zeta_1 F(M) \cdot [1,0]^T$	$T(0) = T_0$	Temperatures
$Y(t) = A(t) K(t)^\gamma L(t)^{1-\gamma}$		Gross-production
$Q(t) = \left[1 - \Lambda(\mu(t),t) - \Omega(T^{AT}(t),t)\right] Y(t)$		Net-production
$E(t) = [1-\mu(t)]\,\sigma(t) Y(t) + E_{land}(t)$		Carbon emissions
$c(t) = [1 - s(t)] \dfrac{Q(t)}{L(t)}$		Per-capita consumption

3 | Disentangling the SCC

The social cost of carbon (SCC) captures the marginal rate of substitution between consumption and emissions along the socially optimal path:

$$\text{SCC} = \frac{dC}{dE} = -\frac{\partial_E V}{\partial_C V} = \alpha \frac{-\lambda^{M^{AT}}}{\lambda^K}$$

The last equation relates the SCC to shadow-prices λ^X (and holds for interior solutions of s). We can analytically derive the growth rate of the SCC as

$$\frac{\frac{d}{dt}\text{SCC}}{\text{SCC}} = -\delta_K + \gamma \frac{Q}{K}\left(1 + \frac{\mu \Lambda_\mu - \text{SCC}\cdot\sigma}{(1-\Lambda-\Omega)}\right) - \phi_{11} - \phi_{21}\frac{\lambda^{M^{UP}}}{\lambda^{M^{AT}}} - \phi_{31}\frac{\lambda^{M^{LO}}}{\lambda^{M^{AT}}} - \frac{\lambda^{T^{AT}}}{\lambda^{M^{AT}}}\xi_1 F'(M^{AT})$$

We find that

- the SCC discounts with the capital depreciation rate δ_K;
- the SCC growth is related to the capital ratio of production;
- $-\phi_{11} - \phi_{21}\frac{\lambda^{M^{UP}}}{\lambda^{M^{AT}}} - \phi_{31}\frac{\lambda^{M^{LO}}}{\lambda^{M^{AT}}}$ captures the impact of the carbon levels;
- $-\frac{\lambda^{T^{AT}}}{\lambda^{M^{AT}}}\xi_1 F'(M^{AT})$ accounts for the long-turn damages through the current atmospheric carbon stock and its impact on the temperature development.

4 | SMAC vs. SCC

In contrast to the SCC, the social marginal abatement cost (SMAC) lack a precise definition. Considering the SMAC only from a static perspective, we find

$$\text{SMAC}_1(t) := \frac{dQ(t)}{dE(t)} = \frac{dQ(t)/d\mu(t)}{dE(t)/d\mu(t)} = \frac{1}{\sigma(t)}\frac{\partial\Lambda(t)}{\partial\mu(t)}.$$

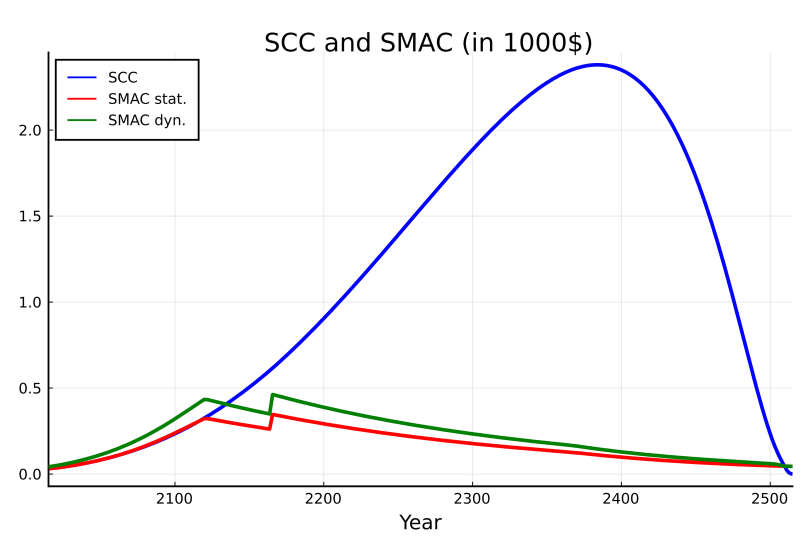
It can be shown that $\text{SMAC}_1 = \text{SCC}$ for interior solutions of abatement rate μ. Taking the dynamic impacts of changes in μ into account, we obtain:

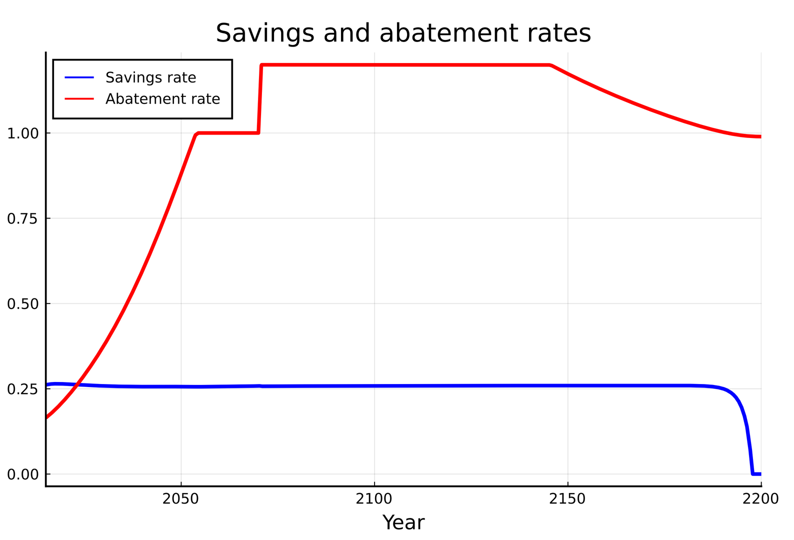
$$\text{SMAC}_2(t) = \text{SMAC}_1(t) \cdot \frac{1+\gamma\cdot\text{GKR}}{1+\gamma\cdot\text{GKR}\cdot\varepsilon(\Lambda,\mu)}$$

GKR . . . Gross capital investment ratio $s\gamma Q/K$

$\varepsilon(\Lambda,\mu)$ . . . Elasticity of the factor productivity $(1-\Omega-\Lambda)$ with respect to the share of not abated emissions $(1-\mu)$

Evaluating SMAC_1, SMAC_2 and the SCC numerically along the optimal solution we obtain significant differences in the profiles.





5 | SCC as a carbon tax

To analyse whether the SCC is the best choice for the carbon-tax we need to answer the question:

Does setting carbon-price = SCC in a decentralised setting lead to the first-best solution of the social planer setting?

A decentralised setting consistent with the standard DICE model consists of 2 sectors:

Firms maximize their profits Π at every point in time.

$$\Pi(t,\mu,K,L) = \left[1 - \Omega(T^{AT}(t)) - \Lambda(\mu,t)\right] A(t) K^\gamma L^{1-\gamma} - R\cdot K - w\cdot L - p\cdot E$$

R . . . interest rate w . . . labour income p . . . carbon price

Households maximize utility accounting for the asset dynamics.

$$\max \int_0^T e^{-\rho t} U(c(s,a,L),L) dt$$

$$\dot{a} = s\cdot(R\cdot a + w\cdot L + p\cdot E) - \delta_K a \quad , \quad a(T) \geq 0$$

$$c(s,a,L) := \frac{(1-s)\cdot(R\cdot a + w\cdot L + p\cdot E)}{L}$$

We further assume that (i) tax-income from carbon-pricing is redistributed to the households and (ii) firms and households assume they have no impact on the development of the geophysical system. We can then mathematically prove that setting $p(t) = \text{SCC}(t)$ results in the firms choosing the socially optimal abatement level and households choosing the socially optimal savings rate.

$\Longrightarrow$ In this specific setting, the **SCC is the optimal carbon price/tax.**