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A systematic review of mathematics interventions for primary school students with intellectual disabilities

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ABSTRACT

This systematic review investigates the characteristics of effective mathematics interventions for students with intellectual disabilities (ID) age 5 to 12, focusing on early numeracy, arithmetic, and arithmetical problem solving skills. Twenty studies from 2008–2020 were reviewed: 17 had a single-subject design and three a group-comparison design. The studies included a total of 135 students with ID. Consistent with previous studies, the analysis showed that interventions with systematic and explicit instruction with feedback and the use of manipulatives are effective instructional approaches and strategies for students with ID. This study reveals that effective interventions are well-structured, high intensity learning sequences adapted to the students' achievement level. The intensity of an intervention requires careful consideration of the number of intervention sessions per week and their duration. Further studies should investigate which instructional strategy is most effective for each type of skill and the optimum intensity of interventions.

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KEYWORDS
Systematic review; mathematics instruction; intellectual disability; numeracy; intervention studies; intervention intensity

Introduction

Mathematical skills are important for academic learning and necessary for undertaking routine tasks in daily life, such as shopping, cooking, and managing time (Faragher and Brown 2005). Basic numerical and arithmetic skills are particularly important as facilitators of social participation and independence for children and adults with ID (Faragher and Brown 2005; Spooner et al. 2017). Good number sense in the early years is a strong predictor for the acquisition of further mathematical skills important in everyday life (Jordan et al. 2007).

Students with ID are able to acquire mathematical skills (Browder et al. 2008; Lemons et al. 2015) and their numerical development does not differ fundamentally from that of typically developing students (Baroody 1999; Brankaer, Ghesquière, and de Smedt 2011). However, their competence rarely progresses beyond basic numeracy and they require more time and repetition to learn mathematical concepts (Faragher and Clarke 2014). Most students with ID display significant limitations in both intellectual function and adaptive behaviour (American Psychiatric Association 2013). Differences in the

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Supplemental data for this article can be accessed here.

characteristics of their ID as well as their very heterogeneous achievement levels create challenges for teachers (Lee et al. 2016), who often feel ill-prepared for teaching mathematics to this group of students (Spooner et al. 2019). Therefore, teachers need effective instructional strategies for providing high quality mathematics instruction to students with ID (King, Lemons, and Davidson 2016) and the purpose of this systematic review is to identify the characteristics of effective mathematics interventions for 5–12 year old students with ID.

The review focuses on this age range because children generally acquire basic mathematical skills (i.e., early numeracy, arithmetic, and arithmetical problem solving skills) during primary school. Early numeracy comprises symbolic and non-symbolic number sense (approximate evaluation of magnitudes or symbols representing magnitudes), understanding mathematical relationships (early mathematical-logical principles, arithmetic principles, place-value and base-10 systems), counting skills, and basic skills in arithmetic (i.e., addition, subtraction, multiplication and division) (Aunio and Räsänen 2016). These skills are good predictors of later mathematics performance (Jordan et al. 2007). Arithmetic interventions for students with ID aim to improve pupils' knowledge of the four basic operations and mental arithmetic (Bowman et al. 2019; Hord and Bouck 2012). Arithmetical problem solving includes generalising and using these skills in different contexts and in the real world (Saunders et al. 2018). Arithmetical problem solving interventions for students with ID often emphasise steps in problem solving: problem translation, problem integration, solution planning, and solution execution (Root et al. 2017).

Instructional approaches and strategies for teaching mathematics to students with ID

To find the parameters for our systematic review, the most effective and often used instruction methods, we conducted a survey of the results of previous reviews and meta-analyses of research on teaching mathematics to students with ID. Eleven reviews and meta-analyses published over the past 20 years include intervention studies designed to improve mathematical skills (e.g. early numeracy, arithmetic, measurements, algebra, geometry, and data analysis) (see electronic supplement). Explicit instruction, direct instruction, and systematic instruction emerge as the three most frequently used methods for teaching mathematics to students with ID. They are all teacher directed and include similar instructional strategies.

Explicit instruction consists of research-supported instructional behaviour characterised by clarity of language and purpose and a reduction of the cognitive load. Providing feedback, modelling and think-alouds, breaking down complex tasks into smaller units of instruction, and offering students intensive practice opportunities with systematically faded support and prompts are all strategies for providing explicit instruction (Hughes et al. 2017).

Direct instruction has the same goals and strategies as explicit instruction, making it difficult to distinguish between them (Lemons et al. 2015). Direct instruction tends to include scripted lesson plans and curricula supplied with the same instructional strategies as explicit instruction.

Systematic instruction focuses on prompting procedures. When starting to teach a new skill, instructors fully control the learning by modelling and guiding practice using prompts. During the course of the lesson, the teacher gradually cedes control, encouraging students to work independently (Rosenshine 2008). Prompts (e.g. verbal, gestural or physical) are used to encourage correct responses. Common practices in the systematic instruction of students with ID are prompting procedures, constant time delay, and a system of least prompts (Bowman et al. 2019; Browder et al. 2008; Spooner et al. 2019).

According to Spooner et al. (2019) the most effective teacher directed mathematical interventions for students with ID include explicit and systematic instruction (see also Browder et al. 2008; Kroesbergen and Van Luit 2003; Goya, Ulloa, and Wells 2019). The use of task-analytic instruction, graphic organisers, and manipulatives have also been shown to be successful (Hudson, Rivera, and Grady 2018; Spooner et al. 2019; Peltier et al. 2019). Technology-aided instruction is another proven instructional strategy (e.g. Spooner et al. 2019).

Some studies have tried to determine which method is best suited for acquiring and improving each mathematical skill. Butler et al. (2001) found that while numeracy skills were best taught using direct instruction, arithmetical skills were mainly learned through prompts, and mathematical problem solving was fostered using self-regulation and strategy instruction. According to Kroesbergen and Van Luit (2003), repetitive direct instruction is the most effective instructional approach for developing basic arithmetic skills.

Intensity and duration

Students with ID need more repetition and time to acquire mathematical skills than those without (Baroody 1999), and it is also important to have an evidence-based determination of optimal intervention intensity so that interventions can be planned and resources distributed effectively in special education support systems (Lemons et al. 2018). However, how intensive any intervention needs to be in order to be effective is not known since intervention intensity is rarely reported. In their review, King, Lemons, and Davidson (2016) report that most studies have a duration of between one week and one month but only half of the 14 studies included in their review had provided information about session duration. According to Kroesbergen and Van Luit (2003) a longer intervention had a smaller effect size. They suggested that the greater number of skills targeted in long-term interventions led to small achievement gains in multiple areas, resulting in a small overall effect size. Because comparing intervention intensity between studies is important for assessing effectiveness, Warren, Fey, and Yoder (2007) suggest calculating a *cumulative intervention intensity* (CII), the product of the number of intervention sessions per week, their duration and the total duration (number of weeks) of the intervention.

Present study

While previous reviews and meta-analyses confirm that mathematics interventions for students with ID are effective, they do not reveal the specific characteristics of those interventions that best enable primary school age students with ID to acquire early numeracy, arithmetic, and arithmetical problem solving skills. They consider the impact of interventions on pupils aged 4 to 22 and investigate the responses of a number of subgroups of students with ID, such as students with mild, moderate or severe ID, or with

Down syndrome or autism. This study helps to fill this gap in the current body of research by answering the following question:

What are the characteristics of effective interventions for improving early numeracy, arithmetic and arithmetical problem solving skills for students with ID age 5 to 12? Specifically:

- What instructional approaches and strategies have been used for interventions?
- Which maths skills have been targeted?
- How important is the intensity of the intervention?

As a corollary, we also consider whether effect size, as defined by percentage of non-overlapping data (PND), is a valid measure of an intervention's success for all of the studies reviewed. Because IQ assessments of students with ID are not very reliable, this review does not differentiate between degrees of ID (Jenni et al. 2015).

Methods

Literature search procedures

First, relevant sources were searched in the Eric, PsychInfo, Jstor, Researchgate and Web of Science databases. The following syntax with keywords was used: ("intellectual disabilit*" OR "developmental disability*" OR "cognitive disability*" OR "special education") AND ("intervention" OR "instruction" OR "training") AND ("numeracy" OR "maths*" OR "arithmetic*" OR "problem solving" OR "math fluency" OR "number sense"). Different keywords for disability, setting, content and intervention were combined. The search was limited to academic journals. Second, a manual search of journals publishing studies in the field of intellectual disability was performed. Third, we searched in the reference lists of the studies we had already selected.

Inclusion criteria

Studies had to fulfil the following criteria in order to be selected for the current review. First, only studies with students with ID age 5 to 12 were included. Second, the students with ID had to have a diagnosed ID or an IQ below 75. Third, the intervention had to be evaluated by a group comparison or a single-subject design. Fourth, the intervention had to be described in detail and the focus had to be on early numeracy, arithmetic, or problem solving with numeracy or arithmetic. Finally, only studies written in English and published between 2008 to 2020 were selected.

Coding procedure

Both authors independently read and coded the studies for the following variables: participants' age, IQ, sample size, study design, content, duration, group setting, leader, instructional strategies, measures, results/effect size, and maintenance/ follow-up. When information in single-case studies about the percentage of non-overlapping data (PND) was missing, the authors calculated the PND on the basis of the baseline and intervention graphs. For group design studies, the effect size, Hedge's *g*, was calculated by dividing the difference between the treatment and control group by the pooled standard deviation corrected for small sample sizes (Institute of Education Sciences 2020). CII was calculated using Warren, Fey, and Yoder (2007) formula: The product of intervention length in weeks, minutes or trials per session and number of sessions per week (weeks x minutes or trials per session x sessions per week).

Included studies

The database search yielded 762 records, 20 of which met the inclusion criteria (Figure 1). All students had a diagnosed ID. Some students had specific diagnoses, for example, autism spectrum disorder (about 28) or specific syndromes. Two studies with single-case designs included one student older than 12. Both students were excluded from the analyses. Seventeen of the 20 selected studies had a single-case design with a total of 67 students with ID ($M = 3.6$, $SD = 1.7$). Three studies had a group comparison design with a total of 107 students ($M = 35.7$, $SD = 15.3$), but of these students, only 68 students had an ID ($M = 22.7$, $SD = 8.1$). The most common design in single-case studies was a (staggered) multiple-probe-across-participants design. One study combined this design with an alternating treatment design. Only one of the three group studies had a follow-up, 8 months later. Eight of the 17 single-case studies had a maintenance phase with one to five measurement points. Maintenance phases started between 1 day and 10 weeks

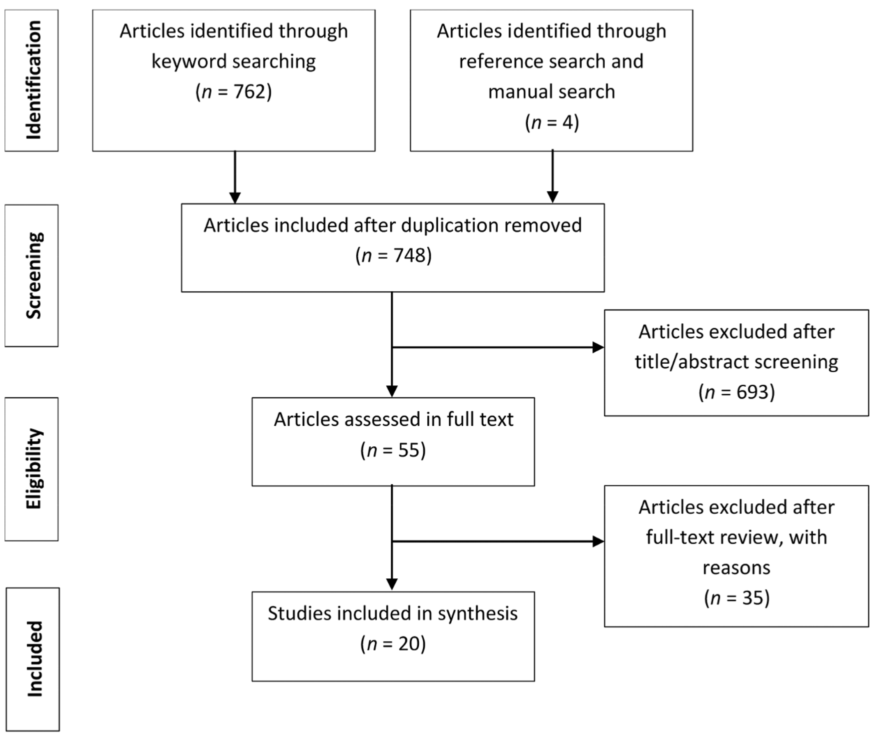

Figure 1. Study identification procedures.

after the end of the intervention. In all studies, the intervention sessions were conducted in one-to-one or small group settings with two or three students. Only one study combined individual instruction with instruction in general education classrooms (Browder et al. 2012).

The procedural fidelity of the studies was high because treatment conditions, procedures and implementation were described and controlled (Tincani and Travers 2018). This is important for understanding which intervention characteristics are pivotal for positive results.

Results

Twenty studies met the inclusion criteria. Table 1 presents the instructional approaches and strategies, targeted mathematical skills, and intensity of the three studies with group comparison design. Table 2 gives an overview of the intervention characteristics of the single-case design studies.

Instructional approaches and strategies

The majority of interventions (k = 15) used systematic or explicit instruction. Explicit instruction involved using strategies such as modelling, where the teacher demonstrated how to solve a task while using think-alouds. Systematic instruction strategies used prompts in the form of simultaneous prompting, system of least prompts, or the provision of prompts when needed. Pictorial (e.g. Zisimopoulos 2010), auditory (e.g. Bouck et al. 2009), or verbal prompts were provide reinforcement through repetition, to control the learning process, and to avoid errors that might frustrate the learner (Browder et al. 2012; Skibo, Mims, and Spooner 2011). A constant time delay was another strategy for eliciting correct responses (e.g. for numerical recognition). This was a fixed component in early numeracy interventions by Browder's research group. Feedback was a component of almost all of the interventions.

Most interventions included instructional strategies such as manipulatives, concrete representations or graphic organisers to make the concepts and operations less abstract. Visual representations were used to help students to acquire and understand mathematical concepts (e.g. Stroizer et al. 2015), solve problems (e.g. Root et al. 2017), and explain procedures and concepts (e.g. Bouck et al. 2018). The learning sequences were highly structured and divided into little steps, and it was possible to repeat each step as often as necessary. Another method for supporting students' understanding of mathematical concepts was the use of a concrete, representational, and abstract method built on explicit instruction (Bouck et al. 2018). It guided students in how to solve mathematics problems, first with concrete manipulatives, then with representations or drawings of objects, and finally showing them how to solve the problem abstractly using symbols.

Teachers adapted the content and instruction to the student's achievement level by, for example, taking into account the pre-test results (Tzanakaki et al. 2014), giving individual feedback, repeating the information, and teaching the next step when the students had acquired the prerequisite skills (e.g. Zisimopoulos 2010). In most interventions, students had to acquire certain early skills before more complex skills were taught

Table 1. Studies with group comparison design.

study	Intervention/ instructional components	Targeted skills	Intervention intensity (CII)	Effect size (Hedges's *g*)
Agaliotis and Teli (2016)	Direct instruction, CRA, FB	Arithmetic: Multiplication, division problems	12 sessions of 30–35 min., 2–3 sessions per week, 4–6 weeks (360)	Multiplication: 0.11
Brankaer, Ghesquière, and de Smedt (2015)	Game	Early numeracy skills: Connecting numbers with magnitudes	8 sessions of 15 min., 2 sessions per week, 4 weeks (120)	0.19
Tzanakaki et al. (2014)	Systematic instruction, SLP, manipulatives, FB	Early numeracy skills: Counting, patterns, addition and subtraction problems with counting strategies	13–33 sessions of 15–20 min., 2–3 sessions per week, 12 weeks (360–540)	TEMA 3: 0.33 Counting: 0.24 Number identification: 0.16

CII = cumulative intervention intensity; SLP = system of least prompts; FB = feedback; CRA = concrete, representational, abstract approach; TEMA 3 = Test of Early Mathematics Ability 3rd edition.

Table 2. Studies with single-case design.

study	Intervention/ instructional components	Targeted skills	Intervention intensity (CII)	Effect size: PND
Sahbaz and Katlav (2018)	CTD	Early numeracy skills: Number identification 1–5	8–9 sessions, 15 trials per session, daily for two weeks (120–135)	88.89%
Skibo, Mims, and Spooner (2011)	SLP	Early numeracy skills: Number identification 1–5	10–22 sessions, 15 trials per session, daily, three weeks (225)	77%, 90%, 100%
Browder et al. (2012)	Explicit instruction with modelling, SLP, GO, FB, CTD, PBMS	Early numeracy skills: Counting, create and add sets, measurement	4 sessions of 30 min. per week, 6 months (2880)	55–100%
Jimenez and Kemmery (2013)	Systematic instruction, GO, SLP, TA, CTD, MLT, PBMS	Early numeracy skills: Counting, create and add sets, measurement	At least 36 sessions (n/a)	100%
Jimenez and Staples (2015)	Systematic instruction, GO, SLP, TA, CTD, PBMS	Early numeracy skills: Counting, create and add sets, measurement	At least 15 sessions (n/a)	72%, 77%, 100%
Jimenez and Besaw (2020)	Systematic instruction, GO, SLP, PBMS	Early numeracy skills: making sets and pattern, measurement	Daily sessions of 30 min., 4–8 weeks (600–1200)	75–90%, 75–100%
Wright et al. (2020)	Systematic instruction, GO, SLP, TA, CTD, PBMS	Early numeracy skills: Counting, create and add sets, measurement	4 sessions of 20 min. per week, 2–6 weeks (160–480)	96%, 81%, 57%, 86%
Hudson, Zambone, and Brickhouse (2016)	Systematic instruction, GO, VOD, SLP, TA, CTD, PBMS	Early numeracy skills: Counting, create and add sets, measurement	3–4 sessions per week, around 15 sessions in 5 weeks (n/a)	100%
Jimenez and Saunders (2019)	Systematic instruction, GO, SP	Early numeracy skills: Subitising, simple addition problems	3–4 trials per week, each trial 5 min, 2–5 weeks (30–120)	91%, 39%, 100%
Calik and Kargin (2010)	Direct instruction: modelling, guided practice, reinforcement	Arithmetic: Single digit addition problems with touch points (counting on strategy)	8–9 sessions, 2 sessions per day (n/a)	100%
Cihak and Foust (2008)	MLT, SLP	Arithmetic: Single digit addition problems with number line or touch points	20–31 sessions with 10 trials, around 6 weeks (240)	92–100%
Stroizer et al. (2015)	Explicit instruction, MLT, CRA	Arithmetic: Addition, subtraction, multiplication facts 0–5	20 sessions of 20–60 min., daily, 4 weeks (400–1200)	90–100%
Zisimopoulos (2010)	Systematic instruction, picture fading technique with 4 steps (SLP), modelling, FB	Arithmetic: Single-digit multiplication facts	3 sessions of 10–15 min. per week, 6 weeks (180–270)	100%
Bouck et al. (2009)	VOD (pentop computer)	Arithmetic: Multiplication problems	3–4 sessions per week, 2–3 weeks (n/a)	100%, 29%, 57%
Bouck et al. (2018)	Explicit instruction, modelling, prompting, CRA, FB	Problem solving: Change-making with coins problems (addition, subtraction)	At least 9–11, 2 sessions of 10–20 min. per week, 11 weeks (180–270)	100%
Browder et al. (2018)	Systematic instruction, visual representation, TA, SI	Problem solving: Addition and subtraction word problems	Up to 100 sessions, daily, 30 min., one school year (3000)	100%
Root et al. (2017)	Explicit instruction, GO, SI, SLP, iPad	Problem solving: Solve compare word problems (How many more?)	6–8 sessions of 15–20 min., daily (113–150)	100%

SLP = system of least prompts; SP = simultaneous prompting; TA = task analysis; CTD = constant time delay; GO = graphic organiser; PBMS = problem-based maths story; MLT = model-lead test, VOD = voice-output device; SI = self-instruction; FB = feedback; CRA = concrete, representational, abstract approach; n/a = information not available.

(e.g. Bouck et al. 2018). If prerequisites were necessary for certain targeted skills, those skills were repeated at the beginning of the intervention (Browder et al. 2012, 2018; Jimenez and Kemmery 2013; Bouck et al. 2018; Tzanakaki et al. 2014). Almost all of the interventions included scripted lesson plans to help instructors adapt to students' achievement levels, implement the use of systematic or explicit instruction, give feedback and structure the lesson.

A few studies supported the mathematical learning of students with ID with the help of a game (Brankaer, Ghesquière, and de Smedt 2015) or specialised software (Bouck et al. 2009; Jimenez and Besaw 2020).

Targeted mathematical skills

Eleven interventions focused on early numeracy skills, in six interventions the students were taught arithmetical skills, and three interventions targeted arithmetical problem solving. Early numeracy intervention often included basic arithmetic tasks and arithmetical problem solving and one problem solving intervention included early numeracy skills (Root et al. 2017). The arithmetic interventions covered all four basic operations. Two of them focused on fact retrieval (Agaliotis and Teli 2016; Zisimopoulos 2010) and four interventions aimed to foster the use of strategies (e.g. Stroizer et al. 2015). In two problem solving interventions, students practiced the four problem solving steps (Browder et al. 2018; Root et al. 2017). Bouck et al. (2018) focused on solving arithmetic problems with and without manipulatives.

Intensity of intervention

The number of sessions per intervention ranged from six to 100 (M = 25.33, SD = 26.61) and the total intervention duration ranged from one week to one school year (M = 7.39, SD = 7.88). Session length ranged from 5–60 minutes with an average of 20 minutes (SD = 9.85). There were daily intervention sessions in seven studies, three to four sessions per week in six studies, and two sessions per week in four studies. Not all authors reported session lengths, the number of sessions per week, or the total intervention duration. In order to compare intervention intensities, the CII (see Methods) was calculated where possible. CII values range from 113 to 3000, demonstrating the large variability in the intensity of the interventions (M = 798.53, SD = 1253.85). The mean and SD are skewed because four of the interventions had particularly frequent (daily), long (30–40 min.), and long-term (up to a year) intervention sessions. These studies had very high CII scores, between 800 and 4500. Ten interventions had high CII values between 120 and 510. They had sessions lengths of around 20 minutes and took place daily or at least twice per week for 3–12 weeks (e.g. Zisimopoulos 2010; Wright et al. 2020).

Effect size

The students made significant progress in all of the group intervention studies, but only two studies revealed significant differences in the targeted skills between the intervention and a control group (Brankaer, Ghesquière, and de Smedt 2015; Tzanakaki et al. 2014). Most of the interventions in single-case studies had a PND over 70% however the PND of two interventions was under 50%, and three interventions had a PND between 50 and 70%. That means that the intervention was ineffective in two cases and showed unclear results in three cases (Scruggs and Mastropieri 2013). No correlation of effect size and intervention intensity was found.

Discussion

This review examined the characteristics of 20 mathematics interventions for students with ID aged 5 to 12 reported in studies published between 2008 and 2020. It revealed that interventions with systematic and explicit instruction were effective instructional approaches for teaching early numeracy, arithmetic, and arithmetical problem solving skills. A common feature of effective interventions was a high intensity of intervention.

Instructional approaches

In line with the findings of previous reviews and meta-analyses, our results confirm that explicit and systematic instruction are effective methods for teaching mathematics to students with ID (e.g. Browder et al. 2008; Lemons et al. 2015). Successful interventions are generally conducted in one-to-one or small group settings with an instructor who adapts the lessons to the student's achievement level by providing prompts, feedback, and repetitions. These strategies, which are significant components of systematic instruction, are also important for activating students, ensuring that time on task and students' attention to the learning topic are given (Hughes et al. 2017).

Prompting procedures, such as constant time delay, system of least prompts and simultaneous prompting, are also effective instructional strategies for teaching mathematics to children with ID. However, according to a review by Shepley, Lane, and Ault (2019), a strategy of least prompts is not effective for students under 13. They suggest that the strategy is too complex for young students with ID because the learner has to select the level of support by him- or herself. Therefore, single-prompt strategies, such as a constant time delay and simultaneous prompting are more suitable and effective when teaching academic skills to younger students. In the interventions reported in this review, systems of least prompts were always implemented together with other instructional strategies. Thus, the impact of this prompting strategy remains unclear.

Surprisingly, the use of technology in mathematics instruction for students with ID was rarely investigated and therefore it is not possible to make a statement on its effectiveness. This result is in contrast to reviews by Hudson, Rivera, and Grady (2018) and Spooner et al. (2019) which highlight technology and multimedia as effective instructional aids. Further research is necessary to investigate the effects of technology-aided instruction for students with ID.

Manipulatives and visual representations

Effective mathematics interventions for students with ID employ manipulatives, visual representations and graphic organisers. This strategy, which is a component of explicit instruction, was used in most of the studies analysed. Our findings are in line with those of previous reviews (Bowman et al. 2019) and this strategy is recommended for the mathematics instruction of students with special educational needs (Browder et al. 2012; Bouck et al. 2018). Van Garderen et al. (2018) found that the use of manipulatives is challenging for teachers, especially when they have to use them to identify patterns or explain an answer. These difficulties were not reported in the studies included in our review, possibly because the teachers usually had scripted lesson plans detailing how to use the manipulatives (e.g. Browder et al. 2012).

Targeted mathematical skills

Arithmetic problem solving was the focus of three of the interventions and a part of at least five early numeracy and arithmetic interventions. The number of studies focusing on arithmetical problem solving has increased in the period since 2008 as compared to the years before (Browder et al. 2008). An increased focus on problem solving could indicate a shift towards a stronger link between mathematics instruction and daily life, solving real-world problems, as well as an emphasis on conceptual understanding (Hudson, Rivera, and Grady 2018). According to Browder et al. (2018), arithmetical problem solving skills are now accorded greater importance, and therefore have more prominence, in curricula, instruction, and research.

Intensity of intervention

High cumulative intensity is an important characteristic of effective mathematics interventions for students with ID. The most effective mathematical interventions in our review involved at least two sessions per week. We used CII values to compare the intensity of intervention between the different studies (Warren, Fey, and Yoder 2007), however only 14 of the 20 studies were used in our analysis as the other six had not reported the information needed for the calculation. So far, not many studies have investigated the effectiveness of interventions for students with special educational needs in terms of their intensity. Neil and Jones (2019) discuss intensity in terms of the spacing between interventions and conclude that one intervention per week is insufficient for effecting improvements in speech and language skills in children with ID.

Limitations

Although the literature search was conducted in multiple steps, it is possible that not all relevant intervention studies were identified. Another possible limitation is publication bias; only studies with large and significant effects are published (Gage, Cook, and Reichow 2017). In addition, the interventions in this review covered only the selected skills of early numeracy, arithmetic and arithmetical problem solving. The results are based on an analysis of 20 studies, which is a relatively small number from which to draw generalised conclusions.

Most of these studies had a single-case design with small sample size making it difficult to draw conclusions about treatment effects (Tincani and Travers 2018). However, small sample studies enable the reporting of more detail about the development, prior knowledge, and characteristics of each participant than large sample group-comparisons (Horner et al. 2005).

A further limitation is the evaluation of the effectiveness of these studies based on the PND. Most interventions were very effective, especially when the student's prior knowledge was very low. In some studies, the students had even never been taught the targeted skills before the intervention (Sahbaz and Katlav 2018; Tzanakaki et al. 2014). In such cases, answering one item correctly resulted in a PND of 100%.

Most of the interventions in this review were conducted in small groups or one-to-one settings and only four studies considered how interventions might be implemented during general classroom instruction (Browder et al. 2012; Bouck et al. 2009; Brankaer, Ghesquière, and de Smedt 2015; Jimenez and Besaw 2020). This is a significant limitation because an increasing number of students with ID are attending general classrooms (Wehmeyer and Shogren 2017), and future interventions will need to be adapted for these settings.

A final, and very important, consideration is the maintenance and longevity of learning so that the skills acquired can be used in daily life (McDonnell et al. 2020). Only eight of the studies included in this review included maintenance assessments in their design.

Implications for practice and further research

Our systematic review shows that students with ID aged 5 to 12 can successfully acquire skills in the areas of early numeracy, arithmetic and arithmetical problem solving. Systematic and explicit instruction with feedback and the use of manipulatives are effective instructional approaches and strategies for teaching these students. Mathematics instruction should have well-structured, high intensity learning sequences which are individualised and adapted to the students' achievement levels and scripted lesson plans for teaching specific skills can also be helpful in providing instruction with these characteristics. However, further research is necessary in order to evaluate which instructional strategy is effective for imparting which skills. Further intervention studies should be conducted with larger samples of students with ID in order to better data on effective mathematics instruction for this important group of learners. Group comparison studies do use larger samples, but the heterogeneity of the students in the groups complicates any evaluation. One solution could be to conduct more studies with matched pairs and wait-list control groups. Cluster analyses can also help to group students with similar characteristics (Sermier Dessemontet, Moser Opitz, and Schnepel 2020). In order to evaluate long-term effects, studies with follow-up or maintenance assessments are necessary. Finally, future research must investigate how to implement mathematics interventions for students with ID, especially high intensity interventions, in general education classrooms.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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