Journal of Physics: Conference Series

Black holes in a box

To cite this article: Helvi Witek *et al* 2010 *J. Phys.: Conf. Ser.* **229** 012072

Spanish Relativity Meeting (ERE 2009) IOP Publishing
Journal of Physics: Conference Series **229** (2010) 012072 doi:10.1088/1742-6596/229/1/012072

Black holes in a box

Helvi Witek[1]*, Vitor Cardoso[1,2], Leonardo Gualtieri[3], Carlos Herdeiro[4], Andrea Nerozzi[1], Ulrich Sperhake[5] and Miguel Zilhão[4]

[1] Centro Multidisciplinar de Astrofísica - CENTRA, Dept. de Física, Instituto Superior Técnico, Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal
[2] Department of Physics and Astronomy, The University of Mississippi, University, MS 38677-1848, USA
[3] Dipartimento di Fisica, Università di Roma "Sapienza" and Sezione INFN Roma1, P.A. Moro 5, 00185, Roma, Italy
[4] Centro de Física do Porto e Departamento de Física da Faculdade de Ciências da Universidade do Porto, Rua do Campo Alegre 687, 4169-007 Porto, Portugal
[5] California Institute of Technology, Pasadena, CA 91109, USA

E-mail: `helvi.witek@ist.utl.pt`, `vitor.cardoso@ist.utl.pt`, `Leonardo.Gualtieri@roma1.infn.it`, `crherdei@fc.up.pt`, `andrea.nerozzi@ist.utl.pt`, `sperhake@tapir.caltech.edu`, `mzilhao@fc.up.pt`

Abstract. The evolution of BHs in "confining boxes" is interesting for a number of reasons, particularly because it mimics some aspects of anti-de Sitter spacetimes. These admit no Cauchy surface and are a simple example of a non-globally hyperbolic spacetime. We are here interested in the potential role that boundary conditions play in the evolution of a BH system. For that, we imprison a binary BH in a box, at which boundary we set mirror-like boundary conditions.

1. Introduction

Black holes (BHs) have become of increasing importance in fundamental physics, including not only astrophysics but also high-energy physics and quantum gravity. In high-energy physics, the duality between gauge theory and gravity in anti-de Sitter (AdS) spacetimes has created a powerful framework for the study of strongly coupled gauge theories and found applications in connection with the experimental program on heavy ion collisions at Brookhaven's Relativistic Heavy Ion Collider [1] and at the Large Hadron Collider at CERN [2, 3], among many others. One peculiar feature of asymptotically AdS space is the "active role" played by its boundary. In AdS, null geodesics reach the boundary in finite coordinate time. One thus often refers to an asymptotically AdS space as a box [4, 5, 6]. Following the remarkable breakthroughs of Numerical Relativity (NR) in the course of the past few years, it is now time to explore numerically the dynamics of BHs in different background spacetimes, from higher dimensions [7, 8] to anti-de Sitter backgrounds. Here, we wish to begin exploring BHs in AdS by extending NR methods so as to encompass AdS-like boundary conditions. We will model AdS backgrounds by a confining box with reflecting walls in which a BH binary evolves. This will allow us to study a number of interesting phenomena. The final outcome of a generic BH binary merger is a rotating BH. Since the whole system is inside a box, we expect superradiant effects on the waves generated during the merger to play a role; if the frequency ω of the impinging wave satisfies $\omega < m\Omega$, with the azimuthal number m and the angular velocity of the horizon Ω, this radiation is amplified as it scatters off the BH. Thus, energy and angular momentum of the rotating hole are extracted by superradiant modes. The bouncing back and forth of the waves at the reflecting wall and its subsequent amplification by superradiance close to the ergoregion are expected to turn it into a BH bomb [9, 10, 11]. This mechanism makes small rotating BHs in AdS unstable [10] and may presumably play a role in these simulations.

2. Numerical Setup

The numerical simulations were carried out using Sperhake's LEAN code [12]. We make use of puncture initial data computed by the pseudo-spectral TWOPUNCTURES code [13, 14]. The evolution of the Einstein equations in the "3+1" framework is based on the χ-version of the Baumgarte-Shapiro-Shibata-Nakamura (BSSN) [15, 16] system, with the moving puncture approach [17, 18]. The punctures have initial mass parameter $m_i = 0.483$, initial position parameter $x_i = \pm 3.257$ and linear momentum parameter $p_{y_i} = 0.133$. If not denoted otherwise the simulations have been performed using the resolution $h_c = 1/48M$ near the puncture.

In order to mimic the numerical evolution of BHs in AdS spacetimes, we surround the BH binary by a "mirror-like" box at each spatial hypersurface. We impose a spherical box with reflecting boundary conditions at a certain distance r_B around the BH binary. Because of the non-physical "junk"-radiation the system is evolved using outgoing boundary condition until this pulse has left the numerical domain of interest. Afterwards we switch to reflecting boundary conditions. We consider an inspiral of initially non-spinning BHs, within a "box" with radius $r_B = 48M$.

In order to get some insight into the physical properties of the system we consider gravitational radiation as well as properties of the apparent horizon (AH) computed by Thornburg's AHFINDERDIRECT [19, 20]. Gravitational wave information is obtained by computing the Weyl scalar Ψ_4 in the Newman-Penrose formalism and its multipolar components [21].

3. Results

3.1. Wave extraction

In Fig. 1 we present the real part of the dominant $l = m = 2$ mode of Ψ_4, extracted at $r_{ex} = 40M$. We should stress that this is not an ordinary spacetime. As such, quantities such as Ψ_4 do *not* have their usual physical meaning, though for large enough distances they should. The gravitational wave emitted during the inspiral and plunge of the BH binary is reflected back at the boundary, interacts with the final BH and travels outwards again which can be seen in the second and third cycle in Fig. 1.

One of the most important issues regards the well-posedness of this problem. It is not known whether the BSSN evolution system in combination with a reflecting boundary conditions imposes a well-posed initial value boundary problem. We thus perform a convergence analysis using the 22-mode of Ψ_4. The inspiral is simulated at three different resolutions $h_c = 1/48M$, $h_m = 1/52$ and $h_f = 1/56$. The difference between the mid and fine resolution waveform has been scaled by the factor $Q = 1.47$ and $Q = 1.26$ which indicate fourth and second order convergence, respectively (see Fig. 1). We observe fourth order convergence only in the signal emitted throughout the merger. In the first and second after-merger pulse we still obtain second order convergence whereas we start to loose it from the third reflection on. Thus, one important implication of these results is that one *can* extract reliable information from these simulations, as they do converge at least during the first reflections.

We now consider information obtained from the first three cycles. In is apparent from the waveform itself (left Panel in Fig. 1) that there is a broadening of the merger pulse upon each interaction with the hole. A possible explanation relies on superradiant amplification of the low-frequency part of the waveform, though more work is necessary to pinpoint the reason for this. That the pulse contains frequencies both in the superradiant regime and outside of it is clearly

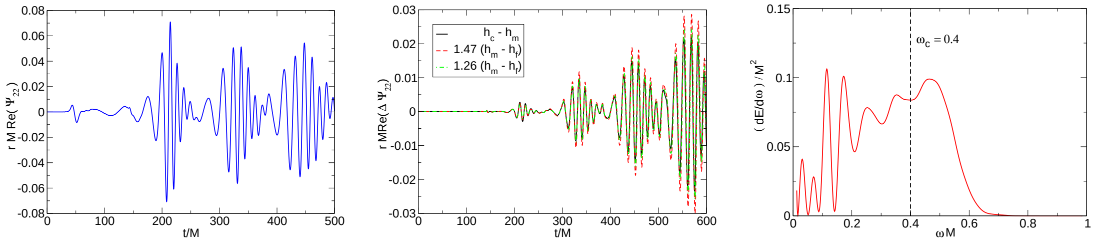

Figure 1. Left: the dominant $l = m = 2$ mode of Ψ_4. Center: convergence analysis. Right: energy spectrum of first emitted pulse.

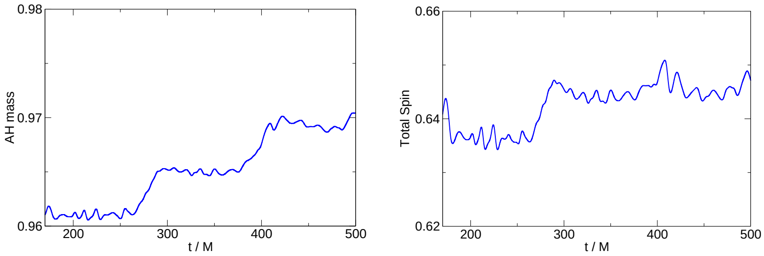

Figure 2. Time evolution of the mass (left) and total spin (right) of the final BH after the merger of the inspiralling BH binary.

seen in the right panel of Fig. 1, where we show the spectrum of the signal emitted throughout the merger. We estimate the critical frequency for superradiance to be around $M\omega_c \sim 0.4$.

3.2. AH data

Although we can get rough estimates of radiated quantities by considering the Newman Penrose scalar Ψ_4 these results should be taken with a grain of salt. Due to our setup we extract the gravitational wave signal only at $r_{ex} = 40M$ which is not yet in the "wave-zone" where the requirement of asymptotically flatness and extraction at spatial infinity is approximately satisfied. Therefore we now turn to look at the AH properties of the final BH, which is a locally defined quantity. The time evolution of AH mass (assuming a Kerr black hole as final state) is depicted in Fig. 2.

The mass of the AH increases upon each interaction with the gravitational radiation pulse. We estimate that in the course of the first and second interaction about $\approx 15\%$ of the incident pulse is absorbed by the BH. In Fig. 2 we also show the time evolution of the total spin J of the rotating BH, obtained assuming a Kerr BH. Not shown here is the dimensionless spin parameter, which we estimate to be around $J/M^2 \sim 0.69$ at the time of formation of the final black hole (therefore in good agreement with simulations of the analogous setup using outgoing boundary conditions [21]). In the course of the first interaction between the spinning BH and the ingoing gravitational wave angular momentum is absorbed. However, during the second interaction the spin of the central BH remains approximately constant. This process can be explained if we consider two competing phenomena, namely absorption of energy and angular momentum of high frequency modes of the radiation by the BH on the one hand and amplification of low frequency modes due to the superradiance effect on the other hand. This behaviour is in qualitative agreement with linearized studies [22], but clearly deserves a more thorough investigation.

4. Conclusions and Outlook

The dynamics of BHs in generic spacetimes is a fascinating, yet terribly complex problem. By considering a BH binary in a "boxed" spacetime we have performed the first successful steps towards a full numerical simulation of BHs in AdS spacetimes. We have studied an initial configuration corresponding to the inspiral of an equal-mass, non-spinning binary yielding a spinning final BH, plus gravitational radiation.

Our results are consistent with expectations, namely a pulse of radiation travelling back and forth between the wall and the black hole, part being absorbed and part being amplified by the BH. Our results show some evidence for the absorption of energy and angular momentum during each interaction with the BH, but they are also consistent with parts (presumably the low-frequency part) being amplified. In future work we plan to investigate this stability studies further by considering a highly spinning, final BH produced by the inspiral of spinning BHs.

Perhaps the most important conclusion of the present work is that these simulations *can* be done and represent the first step to a full numerical evolution of BHs in AdS spacetimes.

Acknowledgments

The authors wish to thank David Hilditch for useful suggestions and discussions. We also thank the participants of the V Iberian Cosmology Meeting, the XII Marcel Grossman Meetings and the Spanish Relativity Meeting 2009 for useful feedback. V.C acknowledges financial support from Fundação Calouste Gulbenkian through a short-term scholarship. V.C and C.H. are supported by a "Ciência 2007" research contract. H.W.'s work was supported by FCT-Portugal through grant SFRH/BD/46061/2008. M.Z. is funded by FCT through grant SFRH/BD/43558/2008. A.N. is funded by FCT through grant SFRH/BPD/47955/2008. This work was partially supported by FCT - Portugal through projects PTDC/FIS/64175/2006, PTDC/FIS/098025/2008, PTDC/FIS/098032/2008, CERN/FP/109306/2009, CERN/FP/109290/2009, NSF grants PHY-090003, PHY-0900735 and the Fairchild foundation to Caltech. Computations were performed at Magerit in Madrid, part of the Barcelona Supercomputing Center.

References

[1] Mateos D 2007 *Class. Quantum Grav.* **24** S713
[2] Arkani-Hamed N, Dimopoulos S and Dvali G R 1998 *Phys. Lett.* B **429** 263
[3] Randall L and Sundrum R 1999 *Phys. Rev. Lett.* **83** 4690
[4] Wald R M 1980 *J. Math. Phys.* **21** 2802
[5] Ishibashi A and Wald R M 2003 *Class. Quantum Grav.* **20** 3815
[6] Ishibashi A and Wald R M 2004 *Class. Quantum Grav.* **21** 2981
[7] Yoshino H and Shibata M 2009 *Phys. Rev.* D **80** 084025
[8] Zilhão M, Witek H, Sperhake U, Cardoso V, Gualtieri L, Herdeiro C and Nerozzi A *Preprint* arXiv:1001.2302
[9] Press W H and Teukolsky S A 1972 *Nature* **238** 211
[10] Cardoso V and Dias O J C 2004 *Phys. Rev.* D **70** 084011
[11] Cardoso V, Dias O J C, Lemos J P S and Yoshida S 2004 *Phys. Rev.* D **70** 044039
[12] Sperhake U 2007 *Phys. Rev.* D **76** 104015
[13] Brandt S and Bruegmann B 1997 *Phys. Rev. Lett.* **78** 3606
[14] Ansorg M, Bruegmann B and Tichy W 2004 *Phys. Rev.* D **70** 064011
[15] Baumgarte Th W and Shapiro S L 1999 *Phys. Rev.* D **59** 024007
[16] Shibata M and Nakamura T 1995 *Phys. Rev.* D **52** 5428
[17] Campanelli M, Lousto C O, Marronetti P and Zlochower Y 2006 *Phys. Rev. Lett.* **96** 111101
[18] Baker J G, Centrella J, Choi D I, Koppitz M and van Meter J 2006 *Phys. Rev. Lett.* **96** 111102
[19] Thornburg J 2004 *Class. Quantum Grav.* **21** 743
[20] Thornburg J 1996 *Phys. Rev.* D **54** 4899
[21] Berti E, Cardoso V, Gonzalez J A, Sperhake U, Hannam M, Husa S and Bruegmann B 2007 *Phys. Rev.* D **76** 064034
[22] Teukolsky S A and Press W H 1974 *Astrophys. J.* **193** 443