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A new correlation to determine the Lockhart-Martinelli parameter from vertical differential pressure for horizontal Venturi tube over-reading correction

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Abstract

Venturi tubes are widely used to measure the gas flow rate in a wet-gas stream. To calculate the gas flow rate, wet-gas Venturi tubes require the liquid loading (i.e. Lockhart-Martinelli parameter) to be known as an input to a so-called over-reading correction correlation. It is challenging to derive the Lockhart-Martinelli parameter without installing additional devices in series to the Venturi tube thus adding significant cost and complexity. A relatively low cost and easy to implement method is to derive the Lockhart-Martinelli parameter by measuring the pressure drop across the Venturi tube or along a pipe section. A correlation that predicts the Lockhart-Martinelli parameter by measuring the pressure drop along a vertical pipe section downstream of the Venturi tube is presented. The correlation was obtained by fitting experimental results from TUV-SUD National Engineering Laboratory's wet-gas test facility with nitrogen-water in a 4'' vertical pipe, Lockhart-Martinelli from 0.04 to 0.29, and gas Froude number from 1 to 2.7. Further experiments were conducted at Spirax Sarco Engineering plc to evaluate the performance of the proposed correlation under steam-water conditions. Tests were conducted with steam-water in a vertical 2" pipe, Lockhart-Martinelli from 0.005 to 0.064, and gas Froude number from 0.55 and 1.35. The Lockhart-Martinelli parameter is predicted within ± 0.035 absolute value with the proposed new correlation for both the National Engineering Laboratory and Spirax Sarco datasets. Gas flow rate values within approximately ±5% error are obtained with the proposed correlation together with ISO/TR 11583 over-reading correlation for the NEL dataset. Although further improvements to the proposed method are required, this work demonstrates the efficacy of an agile, simple, and low-cost differential pressure method to aid the Venturi meter to determine the gas mass flow rate in wet-gas flow.

Keywords: Venturi meter; Wet gas; Over-reading correction; Lockhart-Martinelli parameter; Differential Pressure

1. Introduction

Wet-gas flow measurement is relevant for several industrial applications including oil and gas, steam power generation and nuclear. Wet-gas flow can broadly be defined as any gas flowing in a pipe with a small amount of liquid present and with a gas volume fractions greater than 0.95 [1] or alternatively but not equivalently with a Lockhart-Martinelli parameter less than 0.3. The ideal wet-gas flow metering system should accurately measure both gas and liquid flow rates, highly durable and able to adapt to the changing field conditions, affordable to lower capital costs, and able to provide continuous measurements [2].

Venturi tubes are one of the most widely used differential pressure meters and have been extensively used for wet-gas flow measurement as they are less susceptible to wear and erosion, relatively low cost in respect to

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other purpose designed wet-gas meters and produce low pressure loss. However, a Venturi meter over-read the amount of gas flowing in the pipe in the presence of small amount of liquid and its output need to be appropriately corrected to provide accurate gas flow rate measurements. Several wet-gas correction correlations, also called over-reading correlations, were developed during the past few decades [3-8]. Yuan et al. [7] carried out measurements in wet gas flow using a double differential pressure Venturi. The modified wet gas correlation was found to be accurate within ±1% of expected values. Further investigation of wet gas measurements method was carried out by Zheng et al. [3], who designed a new wet-gas measuring device composed of a cyclone and a long-throated venturi. One widely used over-reading correlation is included in ISO/TR 11583 for horizontally installed Venturi meters [5]. Table 1 shows the relative advantages and limitations of general wet-gas correction correlations. Specific details of the correlations developed in the Table 1 can be found in the literatures [4, 5, 8-15].

Although the effect of over-reading on the wet-gas flow measurement of Venturi tubes has been demonstrated by many correlations, however, a common feature of all these is that the liquid flow rate is required as a priori [4] and the liquid flow rate uncertainty has an impact on the correlation accuracy especially at high Lockhart-Martinelli values. Jia et al. [16] developed a method to measure the void fraction in a pipe by measuring the differential pressure along a section of the pipe. As numerous studies [17-19] have attempted to formulate a void fraction expression based on the Lockhart-Martinelli parameter, the differential pressure method developed by Jia et al. also makes it possible in principle to estimate the Lockhart-Martinelli parameter. However, much uncertainty still exists between the vertical differential pressure drop and the Lockhart-Martinelli parameter. What is not yet clear is the impact of factors such as single phase Reynolds number [20] and Froude number [13] on differential pressure method.

The purpose of this paper is to explore the relationship between vertical differential pressure drop and the Lockhart-Martinelli parameter and to develop a new correlation. The new correlation can complement existing wet-gas correction algorithms to provide a cost-effective measurement system. The wet-gas flow test facility at TUV-SUD National Engineering Laboratory (NEL), East-Kilbride, was used to collect a new experimental dataset to assess the performance of Lockhart-Martinelli correlations for wet-gas metering. The results indicate that the modified Lockhart-Martinelli correlation and modified McFarlane correlation are not appropriate for correcting the Venturi meter over-reading and suggest the new proposed method, which accounts for gas Froude numbers to improve the measurement accuracy. To evaluate the performance of the proposed method for flow conditions of actual wet steam, additional experiments were carried out in the wet-steam test facility at Spirax Sarco Engineering plc, Cheltenham. Experiment results demonstrate its potential for use in industrial or commercial applications. The proposed method has the potential to develop a simple, quick and fairly accurate wet-gas measurement method.

Table 1. Wet-gas correction correlations.

Wet-gas correction correlations	**Advantages**	**Limitations**	**Ideal environment**
Homogeneous correction	Simple; Assumed the multiphase flow as a single-phase flow to apply the single-phase differential pressure meter	The assumption of homogeneous flow can lead to large error	The slip velocity difference between the fluids should be as small as possible

	equation		
Murdock (1962) [11]	The first extensive wet gas meter model; Simple; Straight-line correlation	Developed based on orifice plates and therefore would need modification to be used for Venturi meters	Ideal for lower liquid rates
Chisholm (1967) [12, 21, 22]	Improved the accuracy of the correlation for the case of higher quality two-phase flows (where $X \leq 1$)	Developed based on orifice plates and therefore would need modification to be used for Venturi meters	Higher quality two-phase flows (where $X \leq 1$)
De Leeuw (1997) [13]	The first Venturi wet gas correlation with the gas densiometric Froude number (Frgas); Considered the effect of flow pattern on error	Assumed equal superficial flow coefficients and can be cancelled in calculation	Flow pattern with $0.5 \leq Fr_{gas} \leq 1.5$ and $Fr_{gas} \geq 1.5$
Steven (2002) [9, 10]	Correlation was able to predict the gas mass flow less than ±3%	Stressed that all the above correlations require the liquid flowrate as an initial input in the correlation	limited the applicability of this correlation to $20 \leq P(bar) \leq 60$ $400 \leq Q_{vg}(am^3/h) \leq 1000$
Reader-Harris and Graham, NEL ISO/TR 11583 model (2008) [5, 23]	High accuracy correlation with an uncertainty of 3% for $X \leq 0.15$ and 2.5% for $0.15 < X \leq 0.30$; Data over both the full standard range of beta ratio for Venturi meters ($0.4 \leq \beta \leq 0.75$), and different liquid types (including hydrocarbons and both cool and very hot water)	Less accuracy for $X > 0.3$. Horizontal Venturi	limited the applicability of this correlation to $0.4 \leq \beta \leq 0.75$ $0 < X \leq 0.3$ $3 < Fr_{g,th}$ $0.02 < \frac{\rho_g}{\rho_l}$ $D > 50\,mm$
Graham et al., NEL ISO/TR 11583 model for vertically installed Venturi tubes (2020) [4]	Modified correlation for vertically installed Venturi tub; Able to predict gas mass flow rate within ±3% error	A larger and extended experimental dataset is needed to develop a suitable over-reading correlation for vertically installed Venturi tubes	Wet gas flows (where $X \leq 0.3$)

2. Wet-gas flow measurement: ISO/TR 11583 over-reading correlation

This section describes how the Venturi meter output is corrected to account for the over-reading effect in the presence of wet-gas flow. Although several correlations were developed in the past decades, in this paper only the ISO/TR 11583 correlation will be covered as it is a widely used and publicly available correlation [5, 15]. Moreover, ISO/TR 11583 was found to give overall better performance than other publicly available over-reading correlations [15]. In two-phase flow conditions, the Venturi meter will measure a value of gas mass flow rate (apparent flow rate) which is greater than the “real” gas flowing in the pipe (real flow rate). This is due the fact that the presence of liquid increases the pressure drop between the inlet and throat tappings of the Venturi tube. The over-reading ($\emptyset$) is the ratio between the uncorrected gas-phase mass flow rate ($m_{apparent}$) and the real gas mass flow rate (m), and is calculated using the equations (1) and (2):

$$\emptyset = \frac{m_{apparent}}{m} \tag{1}$$

$$m_{apparent} = \frac{C\varepsilon\pi d^2\sqrt{2\rho_1\Delta p_1}}{4\sqrt{1-\beta^4}} \tag{2}$$

where ε is the gas expansibility determined from the ISO/TR 5167 [8], d is the Venturi throat diameter, ρ_1 is the upstream gas density, Δp_1 is upstream to throat differential pressure and β is the ratio of the Venturi throat diameter to upstream pipe diameter (i.e. d/D). The dry-gas discharge coefficient, C is equal to 1 for ISO/TR 11583.

The Lockhart-Martinelli parameter (X) is required to determine the over-reading. In this study, it is expressed as the ratio of the gas to liquid densitometric Froude numbers, Fr_{gas} and Fr_{liq}, in line with ISO technical reports [5, 24].

$$X = \frac{Fr_{liq}}{Fr_{gas}} = \frac{m_{liq}}{m_{gas}}\sqrt{\frac{\rho_{gas}}{\rho_{liq}}} \tag{3}$$

$$Fr_{gas} = \frac{m_{gas}}{\rho_{gas}A_D\sqrt{gD}}\sqrt{\frac{\rho_{gas}}{\rho_{liq}-\rho_{gas}}} \tag{4}$$

$$Fr_{liq} = \frac{m_{liq}}{\rho_{liq}A_D\sqrt{gD}}\sqrt{\frac{\rho_{liq}}{\rho_{liq}-\rho_{gas}}} \tag{5}$$

where A_D is the upstream Venturi pipe area, ρ is the phase density at the Venturi tube location and g is the acceleration due to gravity. Reference measurements of gas mass flow rate (m_{gas}) and liquid mass flow rate (m_{liq}) are available at both test facilities, so all physical quantities in the above equations are known. The relation between Lockhart-Martinelli parameter and the over-reading $\emptyset_{ISO}$ is denoted in equation (6).

$$\emptyset_{ISO} = \sqrt{1 + C_{Ch}X + X^2} \tag{6}$$

where C_{ch} is an semi-empirical coefficient based on the work from Chisholm for two-phase flow [12].

$$C_{Ch} = \left(\frac{\rho_{1,gas}}{\rho_{liq}}\right)^n + \left(\frac{\rho_{liq}}{\rho_{1,gas}}\right)^n \tag{7}$$

The exponent n is calculated as follows:

$$n = max\left(0.583 - 0.18\beta^2 - 0.578^{-0.8\frac{Fr_{gas}}{H}}, 0.392 - 0.18\beta^2\right) \tag{8}$$

In equation (8) the term H is a parameter that accounts for the liquid type, for water in these tests it is equal to 1.35. Next, the wet-gas discharge coefficient C_{wet} is determined using equation (9).

$$C_{wet} = 1 - 0.0463e^{-0.05Fr_{gas,th}}\, min\left(1, \sqrt{\frac{X}{0.016}}\right) \tag{9}$$

where $Fr_{gas,th}$ is the gas densiometric Froude number at the throat of the Venturi meter.

$$Fr_{gas,th} = \frac{Fr_{gas}}{\beta^{2.5}} \tag{10}$$

The real mass flow rate of gas as estimated by the ISO/TR 11583 correlation can be determined by equation (11).

$$m_{est} = \frac{C_{wet}}{\sqrt{1-\beta^4}}\varepsilon\frac{\pi}{4}d^2\frac{\sqrt{2\,\Delta p_1\rho_{1,gas}}}{\emptyset_{ISO}} \tag{11}$$

The estimated gas mass flow rate is compared with the reference gas mass flow rate, and the relative error is defined using equation (12).

$$E = 100\left(\frac{m_{est}-m}{m}\right) \tag{12}$$

where E is the percentage relative error, m_{est} is the gas mass flow rate estimated from the ISO/TR 11583 correlation and m is the superficial gas mass flow rate.

3. Correlations for Lockhart-Martinelli parameter

3.1 Void fraction from differential pressure

Tait et al. [2] derived a new expression for gas void fraction based on differential pressure measurement in a vertical pipe (ΔP). Under the assumption that frictional contribution to the pressure drop is negligible compared to the hydrostatic contribution, an accurate value of gas void fraction can be achieved by:

$$\alpha_{gas,v} \cong \frac{\Delta P}{(\rho_{gas}-\rho_{liq})gh} - \frac{\rho_{liq}}{(\rho_{gas}-\rho_{liq})} \tag{13}$$

where ρ is the density of the fluid, subscripts *gas* and *liq* refer receptively to the gas and liquid phase, g is the gravity acceleration and h is the distance between two pressure tapping points along the vertical pipe.

3.2 Correlation between Lockhart-Martinelli parameter and gas void fraction in homogeneous flow

The gas volume to the overall pipe volume is known as the gas volume fraction. The gas void fraction (α_{gas}) and volume fraction ($\varphi_{g,hom}$) are equal under the homogeneous no-slip flow assumption, i.e. the liquid and gas are travelling at the same velocity.

$$\alpha_{gas} \cong \varphi_{g,hom} = \frac{Q_{gas}}{Q_{gas}+Q_{liq}} \tag{14}$$

The volumetric flow rates of gas (Q_{gas}) and liquid (Q_{liq}) can be found by dividing the mass flow rate by the fluid density:

$$Q_{liq} = \frac{m_{liq}}{\rho_{liq}} \tag{15}$$

$$Q_{gas} = \frac{m_{gas}}{\rho_{gas}} \tag{16}$$

The ratio of liquid and gas volumetric flow rate can be expressed as:

$$\frac{Q_{liq}}{Q_{gas}} = \frac{m_{liq} \cdot \rho_{gas}}{m_{gas} \cdot \rho_{liq}} \tag{17}$$

Substituting equation (3) into equation (17) gives the new expression for the ratio of liquid and gas volumetric flow rate:

$$\frac{Q_{liq}}{Q_{gas}} = X\sqrt{\frac{\rho_{gas}}{\rho_{liq}}} \tag{18}$$

The relationship between the α_{gas} and X can be expressed as:

$$\alpha_{gas} = \frac{1}{1+\frac{Q_{liq}}{Q_{gas}}} = \frac{1}{1+X\sqrt{\frac{\rho_{gas}}{\rho_{liq}}}} \tag{19}$$

Equation (19) can be rearranged to give:

$$X = (\frac{1}{\alpha_{gas}} - 1)\sqrt{\frac{\rho_{liq}}{\rho_{gas}}} \tag{20}$$

3.3 McFarlane correlation between Lockhart-Martinelli parameter and gas void fraction

McFarlane [18] proposed an empirical correlation for the gas void fraction (α_{gas}) as a function of the Lockhart-Martinelli parameter. The McFarlane correlation is expected to predict the gas mass flow with uncertainties less than ±3% [18].

$$\alpha_{gas} = 1 - \left[1 + \frac{21}{X} + \frac{1}{X^2}\right]^{-0.5} \tag{21}$$

A void fraction expression of Lockhart-Martinelli parameter is formulated based on data fit. Equation (21) can be rearranged as follow:

$$X = 35.9 * {\alpha_{gas}}^2 - 68 * \alpha_{gas} + 32.3 \tag{22}$$

3.4 Correlation between Lockhart-Martinelli and differential pressure

Differential pressure can be devised for directly calculating the Lockhart-Martinelli parameter, thus the calculation step of the void fraction is bypassed. A correlation between pressure drop and Lockhart-Martinelli parameter can be obtained by combining equations (13) and (20).

$$X = \frac{\rho_{gas} gh - \Delta P}{\Delta P - \rho_{liq} gh} \cdot \sqrt{\frac{\rho_{liq}}{\rho_{gas}}} \tag{23}$$

This correlation is valid under the assumption of homogeneous no-slip flow and negligible frictional pressure drop which unfortunately is hardly valid under field conditions.

Alternatively, substituting equation (13) into equation (22) allows for the McFarlane correlation to be further rearranged into a more complex relation between differential pressure and X.

$$X = 35.9 * \left(\frac{\Delta P}{(\rho_{gas}-\rho_{liq})gh} - \frac{\rho_{liq}}{(\rho_{gas}-\rho_{liq})}\right)^2 - 68 * (\frac{\Delta P}{(\rho_{gas}-\rho_{liq})gh} - \frac{\rho_{liq}}{(\rho_{gas}-\rho_{liq})}) + 32.3 \tag{24}$$

However, the assumption of negligible frictional pressure drop in respect to gravitational pressure drop is still required for this second correlation. Frictional pressure drop will have a greater impact on the void fraction for relatively high vapour to liquid density ratios leading to greater errors.

4. Experiment Setup and Operating Procedure

4.1 Wet-gas test at TUV-SUD NEL

The TUV-SUD NEL high-pressure wet-gas facility operated with water and oxygen-free nitrogen at a temperature of 20° C over a pressure range of 10 bar_g to 63 bar_g. A schematic diagram of the facility is depicted in Figure 1. During this experimental campaign, a vertical custom pipe section was manufactured to contain the fittings and tapping points for all sensing equipment required to measure the vertical pressure drop. A schematic representation of the Venturi installation and instrument setup are presented in Figure 2. The vertical pipe was installed downstream of the Venturi tube and after a blind tee which aimed to improve mixing between the phases. The pressure drop in the vertical pipe section was measured over a 500 mm length with a Yokogawa transmitter and logged by NEL. Note that the Venturi pipe diameter was 102.36 mm while the diameter of the vertical pipe was 97.2 mm. Table 2 shows an overview of wet-gas test conditions.

Table 2. TUV-SUD NEL test conditions.

Test Parameter	Selected test condition
Pressure (bar_g)	10, 25
Temperature (°C)	20
Gas volumetric flow rates (m^3/h)	112, 250, 450
Water volumetric flow rates (m^3/h)	1 – 25
Nominal pipe diameter (inch)	4
Gas phase	Nitrogen
Liquid phase	Water
Flow patterns	Annular flow

Fig. 1. TUV-SUD NEL wet-gas flow facility.

The wet-gas flow facility is a closed loop, where the gas is circulated by a 200 kW fully encapsulated gas blower. The gas reference volumetric flow rate is measured using traceable calibrated 6” and 4” SICK Maihak FLOWSIC600 Quattro gas ultrasonic flow meters. The water mass flow rate is measured using a set of Coriolis meters. Positions of the pressure and temperature measurements are given in Figure 2.

The test was carried out at two static pressures of 10 bar_g and 25 bar_g. Different gas and water volumetric flow rates used in the test programme covered the range from 110 m^3/h to 688 m^3/h. These rates corresponded to a gas densiometric Froude number (*Fr_{gas}*) range of 0.6 to 4. The water volumetric flow rates required for the wet gas tests covered the range from 1 m^3/h to 25 m^3/h, and the Lockhart-Martinelli parameter varied from 0.045 to 0.58. The differential pressure (DP) cell is located at least 10 times pipe diameter (d) downstream of the blind tee (l) to allow the flow to fully develop. However, the developing region of $10<l/d<100$ is considered insufficient in vertical multiphase flow measurement. This may affect the universality of the proposed correlation as is used in different flow loop conditions.

During the experiment, NEL collected all gas reference, liquid reference and Venturi meter data as well as the differential pressure transmitter installed in the vertical pipe. In both dry-gas and wet-gas conditions, all data points were logged at each test condition for 180 seconds at a sampling rate of 1 Hz. Before logging each test condition, test flow, pressure, and temperature states were allowed to stabilise for several minutes.

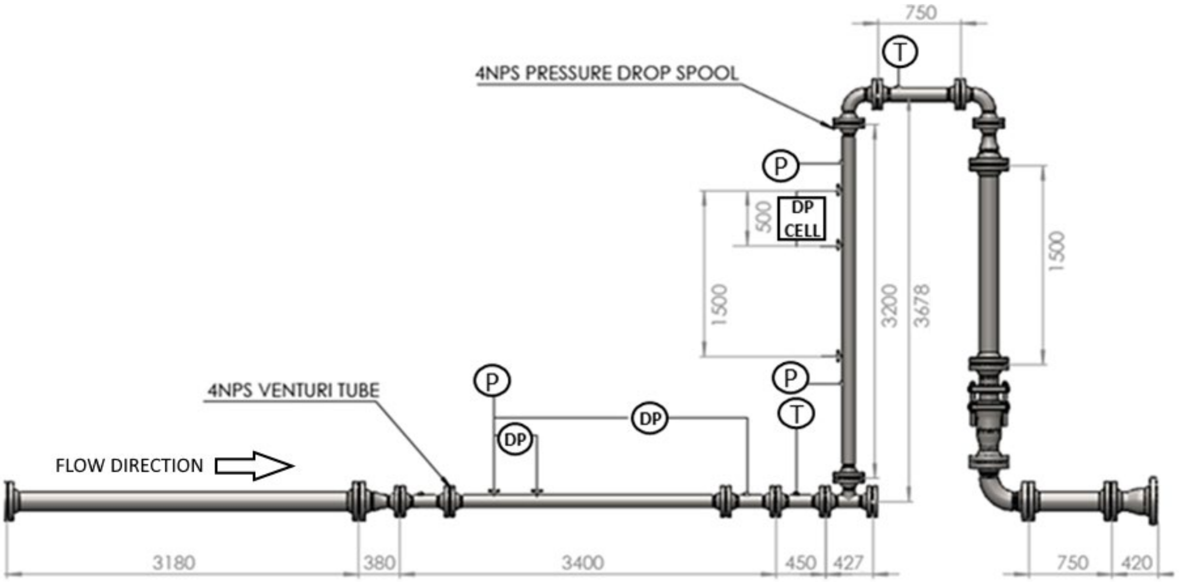


Fig. 2. Schematic representation of the Venturi tube and vertical pipe installation.

4.2 Wet-steam test in Spirax Sarco Engineering plc

Test were carried out at the Spirax Sarco wet-steam flow loop to evaluate the performance of the differential pressure method. The test section is a carbon steel pipe with an inner diameter of 52.48 mm. The flow pressure was maintained at 15 bar_g. The test matrix was designed to as close as possible to real geothermal well flow while also covering as many conditions as were available using the Spirax Sarco test rig. A dryness fraction or steam mass fraction of 0.6 is larger than what would be expected in a real geothermal well, but this is the smallest gas fraction available at Spirax Sarco. Additional tests with steam superficial velocities up to 30 m/s and dryness fractions up to 0.97 were also carried out to make full use of the range of test conditions available at Spirax Sarco. The vertical pipe was installed downstream of the Venturi tube and after a 90° elbow. Because the wet gas flow was fully before differential pressure measurement in both experiment campaigns, it is anticipated that the effect of the shape of the elbow/Tee is insignificant. Differential pressure sensor tapping points are located 500 mm apart. A detailed overview of the wet steam flow test conditions is shown in Table 3. The test section of the flow rig is laid out in Figure 3. The steam/water mix first flows through the Venturi meter before proceeding into the vertical part of the test section, which measures differential pressure drop to get gas void fraction.

Table 3. Spirax Sarco test conditions.

Test Parameter	**Selected test condition**
Pressure (bar$_g$)	15
Temperature (°C)	201
Dryness fraction (-)	0.6 – 0.97
Steam flow velocity (m/s)	4 – 10
Nominal pipe diameter (inch)	2
Gas phase	Steam
Liquid phase	Water
Flow patterns	Churn flow

Fig. 3. Test setup side view. Venturi meter is located in the middle and vertical differential pressure section is located on the right.

5. Results and discussion

A vertical pipe differential pressure drop correlation was developed to determine the Lockhart-Martinelli parameter and in turn correct the Venturi over-reading when subjected to wet-gas flows. Flow regime measurement is determined using relative values of gas phase and liquid phase momentum flux. Tait et al. [2] mapped all flow conditions tested on the diagram of Hewitt and Roberts [25], which indicated that the flow patterns for NEL test and Spirax Sarco test were annular flow and churn flow, respectively. In Figure 5 the experimental Lockhart-Martinelli parameter and predicted Lockhart-Martinelli parameter is plotted versus the square of the experimental pressure drop. Calculation of the gas Froude number require gas and liquid densities and gas and liquid superficial velocities to be known. The NEL and Spirax Sarco test centres have logged single-phase stream reference measurements. This allows the Lockhart-Martinelli parameter predicted by the proposed correlation to be compared against the reference one. The relative error in gas mass flow rate calculated by the new correlation together with ISO 11583 over-reading correlation is also compared with the reference gas mass flow rate measured at NEL to assess the overall correlation performance.

5.1. NEL test results

Before developing a new correlation the performance of two available correlations, equation (23) and equation (24), were assessed in Figure 4. Overall, these two correlations cannot accurately estimate the Lockhart-Martinelli parameter for this dataset. Furthermore, as the Lockhart-Martinelli parameter increases, the difference between the reference Lockhart-Martinelli parameter and the value predicted by the correlation increases. A possible explanation for this might be that the energy losses caused by gravity increase as Lockhart-Martinelli parameter increases. It is apparent from Figure 4 that results from equation (23) exhibit different relationships at different pressures. This result may be explained by the fact that the density of the gas varies greatly at different pressures. As shown in Figure 5, the Lockhart-Martinelli parameter is dependent on pressure drop. Therefore, a new correlation is developed as a function of pressure drop and the gas Froude number.

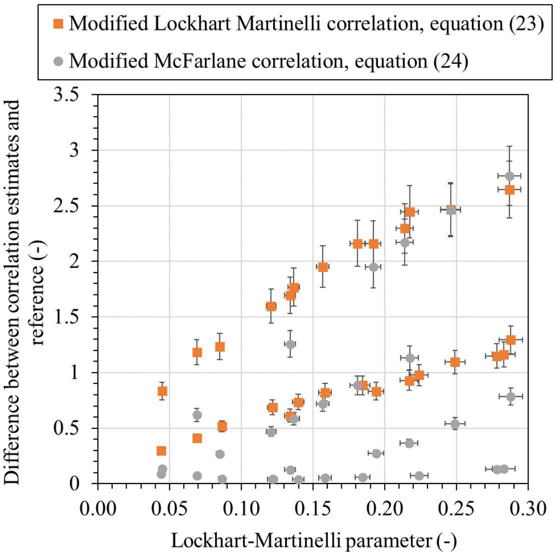


Fig. 4. Sample of NEL wet gas data showing difference between estimates from correlation estimates and reference.

Based on the NEL wet-gas dataset, a new correlation is developed by minimizing the root mean square error between the reference Lockhart-Martinelli parameter and the predicted Lockhart-Martinelli parameter at each gas Froude number. After fitting the Lockhart-Martinelli parameter as a function of the squared pressure drop and the gas Froude number, the Lockhart-Martinelli parameter can be approximated as follows:

$$X = 50\, {Fr_{gas}}^{-1.7} \frac{\Delta P^2}{\left[\left(\rho_{gas} - \rho_{liq}\right) g h\right]^2} \frac{D}{h} \tag{25}$$

where g is the gravity acceleration and h is the distance between the two vertical pressure tapping points across which the differential pressure is measured (in this case 500 mm). The new correlation in equation (25) is valid for gas Froude numbers varying from 1 to 2.7, and Lockhart-Martinelli parameters between 0.05 and 0.3. Figure 5 shows the reference Lockhart-Martinelli parameters and the estimated Lockhart-Martinelli parameters at different gas Froude numbers. There is a strong linear relationship between Lockhart-Martinelli parameter and squared pressure drop. The result suggests that the gas Froude number has a significant impact on the Lockhart-Martinelli parameter. As the gas Froude number increases, the gradient of the Lockhart-Martinelli parameter versus pressure drop square decreases. The uncertainty in reference Lockhart-Martinelli parameter and predicted Lockhart-Martinelli parameter is 2.8% and 4.8%, respectively. The most important experimental uncertainty source comes from the Coriolis meters for of reference liquid mass flow rate measurement, which is 1.2%.

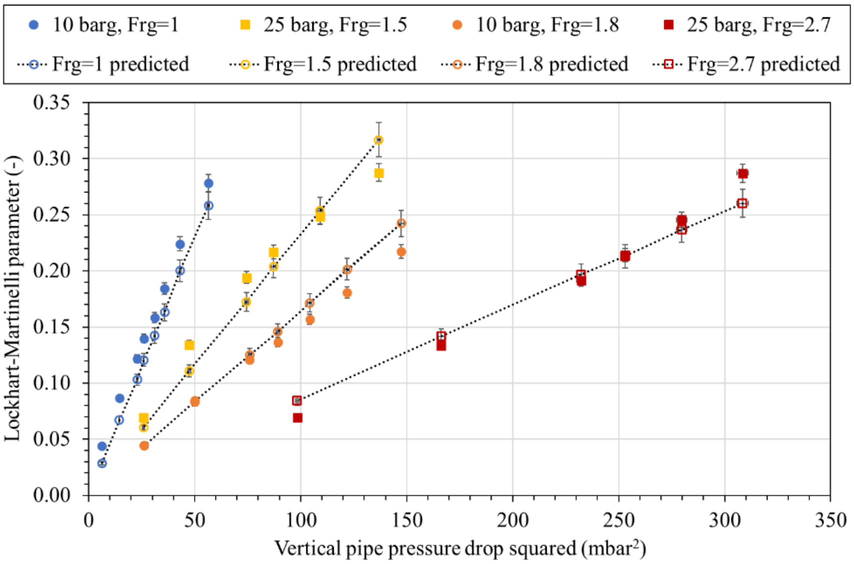
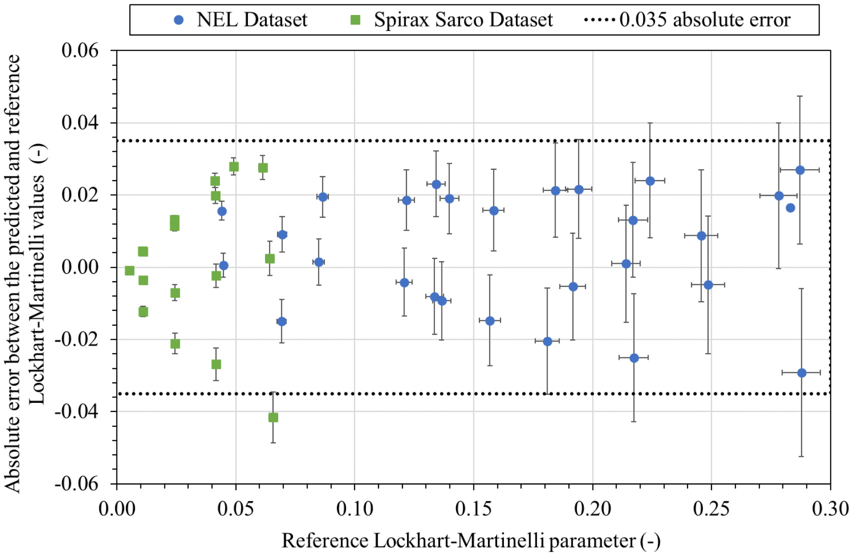


Fig. 5. Experimental and predicted with new correlation Lockhart-Martinelli parameter values as a function of the squared pressure drop across a vertical pipe section and the gas Froude number. NEL dataset

A plot of the absolute and relative error between the predictions from the new proposed correlation and the Lockhart-Martinelli reference experimental values is shown in Figure 6. In Figure 6 and Figure 7, the reference gas mass flow rate measured is used to calculate the gas Froude number for wet gas flow with Lockhart-Martinelli between 0 and 0.3. The derived Lockhart-Martinelli parameter was given in input to solve ISO 11583 equation [1] via calculation procedure introduced at section 2. If the reference gas mass flow rate cannot be obtained in practical application, the gas mass flow rate can be estimated using an iterative process. An example iterative procedure is demonstrated in the Appendix. In Figure 6, it can be observed that the deviation of the new proposed correlation is within ±0.035 absolute error, which is much smaller than the other two correlations previously discussed in Figure 4. Figure 7 compares the relative errors in gas mass flow rate when the predicted Lockhart-Martinelli parameter is used together with ISO 11583 over-reading correlation to determine the gas mass flow rate of the Venturi meter. The results show that most of the estimated gas mass flow rates have relative error within ±5%, while by using the reference experimental Lockhart-Martinelli values (i.e. Lockhart-Martinelli known a priori) an error within ±3% is obtained.



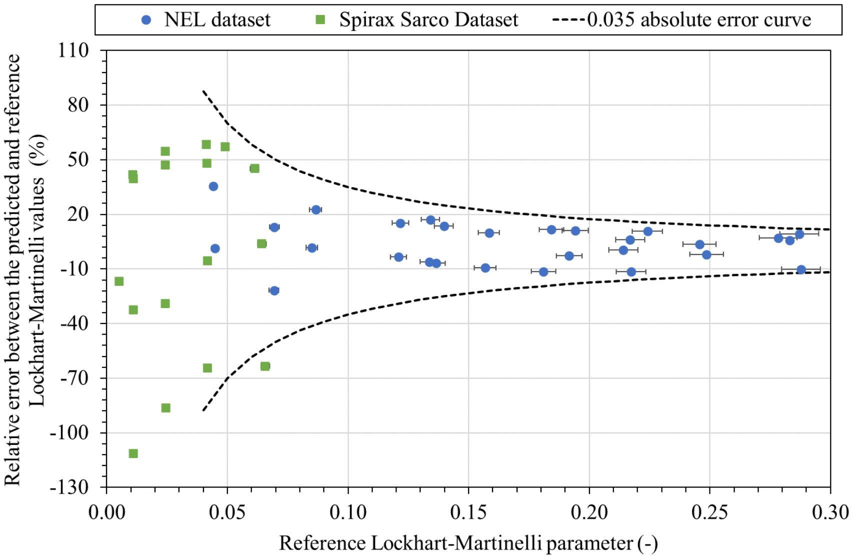


Fig. 6. Absolute error (top) and percentage relative error (bottom) between Lockhart-Martinelli predicted with the new proposed correlation and the NEL (blue dots) and Spirax Sarco (green squares) reference values

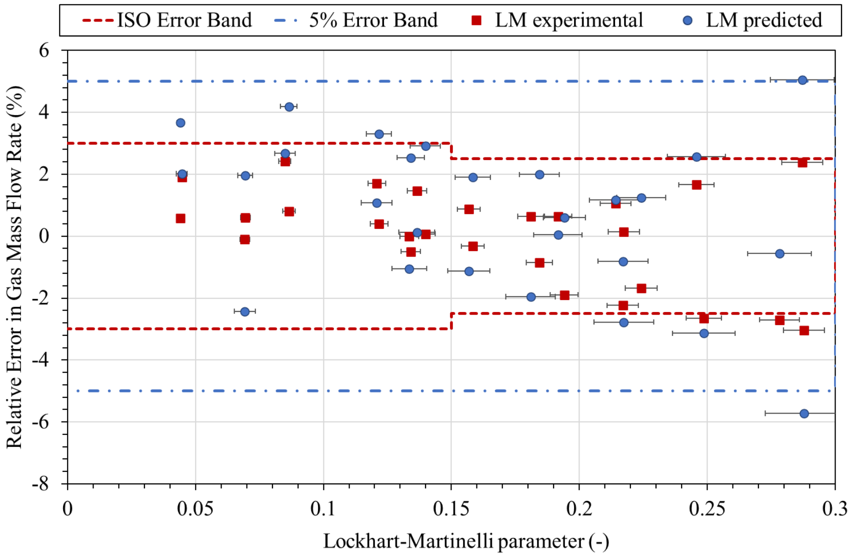


Fig. 7. Relative gas mass flow rate error when using ISO/TR 11583 over-reading correlation with reference experimental Lockhart-Martinelli values (red squares) and predicted values from the new proposed correlation (blue dots). NEL dataset

5.2. Spirax Sarco test results

Figure 8 shows the calculated and predicted with the new proposed correlation Lockhart-Martinelli parameters for the Spirax Sarco dataset at different gas Froude numbers. The single most striking observation is that for the $Fr_g \approx 0.7$, there is a significant positive correlation between the Lockhart-Martinelli parameter, and the vertical squared pressure drop. However, the estimated Lockhart-Martinelli parameter is not consistent with these reference results for $Fr_g \approx 0.55$, $Fr_g \approx 1$ and $Fr_g \approx 1.35$. The possible reasons for large variation are the significant difference in test conditions between NEL and Spirax Sarco, as well as the very low Lockahrt-Martinelli values at Spirax Sarco (below 0.06) and the very low measured pressure drop. Owing that the proposed new correlation is derived from the NEL dataset, even with similar operating pressure, tests with a Froude gas number beyond 1 to 2.7 range could lead to poorer performance. However, the absolute error is still found within ± 0.035 for the Spirax Sarco dataset too in Figure 6, providing confidence that the proposed correlation gives a reasonably stable fit although requiring further improvement and assessment.

The gas mass flow rate error when using the new proposed correlation together with ISO 11583 over-reading correlation is not presented for the steam-water tests, since the measured Venturi tube differential pressure were not found reliable for the steam-water tests due to a non-optimal design of the impulse lines leading to condensation.

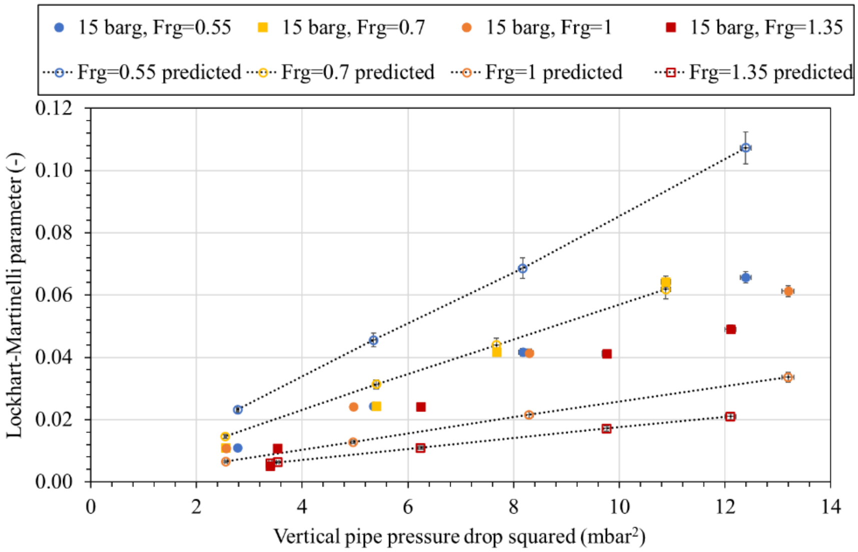


Fig. 8. Experimental and predicted with new correlation Lockhart-Martinelli parameter values as a function of the squared pressure drop across a vertical pipe section and the gas Froude number. Spirax Sarco dataset

6. Conclusion

A new correlation to determine the Lockhart-Martinelli parameter by measuring the differential pressure drop across a vertical pipe section and correct the Venturi tube over-reading in wet-gas flow is developed. The correlation is obtained by fitting experimental dataset, gathered at the TUV-SUD NEL wet-gas test facility. The proposed correlation describes the Lockhart-Martinelli parameter as a function of the measured squared differential pressure and the gas Froude number. Even without gas reference mass flow rate in practical application, it is possible to use proposed correlation with iterative process to estimate gas mass flow rate. Test results at NEL have demonstrated that the proposed correlation together with ISO/TR 11583 over-reading correlation could correct the Venturi tube gas mass flow rate to within $\pm 5\%$ of the reference gas mass flow rate measurement. The findings in this study provide a new method for Lockhart-Martinelli parameter prediction using vertical differential pressure drop measurement and gas Froude number. Although the proposed method is derived from a small database under specific flow conditions, wet-gas data at Spirax Sarco with steam-water suggests that the method is overall robust. However, it is strongly suggested to carry out further development and assessment to validate and improve the proposed method using a broader range of data and by performing a rigorous dimensional analysis and theoretical derivation. Further research is required to investigate the effect of flow patterns on the correlation.

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Appendix A. Example calculation

A.1 Input data and assumptions

The input data for iterations calculation are provided in Table 4. The reference gas mass flow and reference liquid mass flow rate are assumed unknown (real value can be found in Table 2). Therefore, an iterative process is required to improve the accuracy of gas mass flow rate estimate.

Table 4: Initial inputs for example calculation (NEL test data)

Parameter	**Initial value**
Venturi tube throat diameter d, (mm)	61.416
Upstream internal pipe diameter (or upstream diameter of a Venturi tube) D, (mm)	102.36
Diameter ratio β, (-)	0.6
Upstream Venturi pipe area A_D, (m^2)	0.008229
Atmospheric pressure, (Pa)	98500
Venturi throat differential pressure Δp, (Pa)	7468.8
Test pressure p, (Pa)	1070000
Test temperature t, (°C)	20.47
Gas test density $\rho_{1,gas}$, (kg/m^3)	13.44
Gas test expansibility ε, (-)	0.9959
Gas reference mass flow rate m_{gas}, (kg/s)	0.926 (assume unknown)
Pipe Reynolds number, (-)	6.49E+05
Liquid test density ρ_{liquid}, (kg/m^3)	998.14

Liquid reference viscosity, (Pa.s)	0.001
Liquid reference mass flow rate m_{liq}, (kg/s)	2.220 (assume unknown)
The Venturi tube is located in a place where g, (m/s^2)	9.81
Parameter that accounts for the liquid type H, (-)	1.35
Distance between the two vertical pressure tapping points h, (mm)	500
Vertical pipe differential pressure ΔP, (Pa)	751.9
Wet-gas discharge coefficient C_{wet}, (-)	1
Over-reading correction factor $\emptyset$, (-)	1

A.2 First iteration

From equation (11), the value of gas mass flow rate can be calculated. Assume: $C_{wet} = 1, \emptyset_{ISO} = 1$

$$m_{gas} = \frac{1}{\sqrt{1-0.6^4}} 0.9959 \frac{3.14}{4} 0.061416^2 \frac{\sqrt{2\times 7468.8\times 13.44}}{1} = 1.41693\ kg/s$$

From equation (4), gas densiometric Froude number can be calculated.

$$Fr_{gas} = \frac{1.41693}{13.44\times 0.008229\times\sqrt{9.81\times 0.10236}} \sqrt{\frac{13.44}{998.14-13.44}} = 1.49365$$

From equation (10), gas densiometric Froude number at the Venturi tube throat can be calculated.

$$Fr_{gas,th} = \frac{1.49365}{0.6^{2.5}} = 5.35636$$

From equation (9), assume $X > 0.016$, wet gas discharge coefficient can be calculated.

$$C_{wet} = 1 - 0.0463 e^{-0.05\times 5.35636} = 0.96458$$

From equation (8), the exponent *n* is calculated as follows:

$$n = max\left(0.583 - 0.18\times 0.6^2 - 0.578\times e^{-0.8\frac{1.49365}{1.35}}, 0.392 - 0.18\times 0.6^2\right) = 0.32720$$

Then from equation (7), the semi-empirical coefficient C_{ch} based on the work from Chisholm for two-phase flow [12] is calculated as follows:

$$C_{Ch} = \left(\frac{13.44}{998.14}\right)^{0.32720} + \left(\frac{998.14}{13.44}\right)^{0.32720} = 4.33804$$

From equation (25), Lockhart-Martinelli parameter can be calculated.

$$X = 50\times 1.49365^{-1.7} \frac{751.9^2}{[(13.44-998.14)\times 9.81\times 0.5]^2} \frac{0.10236}{0.5} = 0.12541$$

Finally, from equation (6), the over-reading $\emptyset_{ISO}$ is calculated as follows:

$$\emptyset_{ISO} = \sqrt{1 + 4.33804\times 0.12541 + 0.12541^2} = 1.24891$$

A.3 Second iteration

The results of over-reading $\emptyset_{ISO}$ on the first iteration will be used as an input for equation (11).

$$m_{gas} = \frac{0.96458}{\sqrt{1-0.6^4}} 0.9959 \frac{3.14}{4} 0.061416^2 \frac{\sqrt{2\times 7468.8\times 13.44}}{1.24891} = 1.09435\ kg/s$$

From equation (4)

$$Fr_{gas}\frac{1.09435}{13.44\times 0.00822\times\sqrt{9.81\times 0.10236}}\sqrt{\frac{13.44}{998.14-13.44}}=1.15360$$

From equation (10)

$$Fr_{gas,th}=\frac{1.15360}{0.6^{2.5}}=4.13691$$

From equation (9)

$$C_{wet}=1-0.0463e^{-0.05\times 4.13691}=0.96235$$

From equation (8)

$$n=max\left(0.583-0.18\times 0.6^2-0.578\times e^{-0.8\frac{1.15}{1.35}},0.392-0.18\times 0.6^2\right)=0.32720$$

From equation (7)

$$C_{Ch}=\left(\frac{13.44}{998.14}\right)^{0.3272}+\left(\frac{998.14}{13.44}\right)^{0.3272}=4.33804$$

From equation (25)

$$X=50\times 1.15360^{-1.7}\frac{751.9^2}{[(13.44-998.14)\times 9.81\times 0.5]^2}\frac{0.10236}{0.5}=0.19457$$

Finally, from equation (6)

$$\emptyset_{ISO}=\sqrt{1+4.34\times 0.1956+0.1956^2}=1.37183$$

A.4 Final results

The following steps are repeated, and the final results are obtained until the iteration is converged. On the 13th iteration the values in Table 5 are obtained.

Table 5: ISO/TR 11583 predicted results

Parameter	**Final results**
m_{gas}, (kg/s)	0.91551
Fr_{gas}, (-)	0.96508
$Fr_{gas,th}$, (-)	3.46086
C_{wet}, (-)	0.96106
n, (-)	0.32720
C_{Ch}, (-)	4.33804
$\emptyset_{ISO}$, (-)	1.48748
E without iteration, (%)	18.18035
E with iteration, (%)	-1.13283

The correlation-derived actual gas mass flow rate estimates have been compared with the reference gas mass flow rate, and the relative errors calculated using equation (12).

$$E = 100\left(\frac{0.91551-0.926}{0.926}\right) = -1.13283\%$$

Table 5 compares the errors when using the iterative process and without using iterative process. This shows that the iterative process provides a robust gas mass flow rate estimate and significantly reduces the errors compared without using the iterative process.

A.5 Effectiveness and applicable range

The correlation can be used to determine the gas mass flowrate under the following conditions [5].

$0.4 \leq \beta \leq 0.75$

$0 < X \leq 0.3$

$Fr_{gas,th} > 3$

$\frac{\rho_{gas}}{\rho_{liquid}} > 0.02$

D ≥ 50 mm

The overall uncertainty in the gas mass flowrate is ±4% if the Lockhart-Martinelli parameter and pressure is known without error [5].

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