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# Morphing Blades for Tidal Turbines: a Theoretical Study

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## Abstract

Tidal energy has the potential to significantly contribute to energy security by providing predictable renewable energy. New technology is needed to decrease the levelised cost of energy and to make this energy sector competitive in the energy market. A key area where technology can contribute to decrease costs is mitigating the hydrodynamic load fluctuations, and thus increasing the fatigue life of the turbine. Here, we formulate a passive morphing blade concept that aims to mitigate the unsteady thrust without affecting the mean torque and thus the harvested power.

We show that a blade with a trailing edge that deflects perfectly elastically can suppress virtually all fluctuations without varying the mean loads. The effect of the hydrodynamic and blade's inertia, the material damping, and the radial shear stress, decrease the performances.

Using a low-order model of the blade, we show that when a gust occurs, the angle of attack experienced by a rigid blade increases, whilst that experienced by a well-designed morphing blade decreases. This counter-intuitive mechanism is what makes morphing blades highly effective. While blades that could passively twist have previously been developed, this theoretical study suggests that chordwise flexibility is a suitable alternative that should be further explored.

*Keywords:* unsteady load mitigation, passive load control, pitch control, morphing, fluid-structure interaction, tidal turbine

28	<b>Nomenclature</b>	52	$C_P^{2D}$ sectional power coefficient [-]
29		53	$C_T$ thrust coefficient [-]
30	$A$ area swept by the blade [m <sup>2</sup> ]	54	$C_T^{2D}$ sectional thrust coefficient [-]
31	$A^{2D}$ perimeter of a blade	55	$D$ drag force [N]
32	annulus [m]	56	$F$ force [N]
33	$C(k)$ Theodorsen's function [-]	57	$F_{Co}$ Coriolis force [N]
34	$C_{am}^c$ circulatory damping	58	$F_{Eu}$ Euler force [N]
35	coefficient [Nm s <sup>-1</sup> deg <sup>-1</sup> ]	59	$F_c$ centrifugal force [N]
36	$C_{am}^{mc}$ non-circulatory damping	60	$I_{am}$ added mass coefficient [Nm
37	coefficient [Nm s <sup>-1</sup> deg <sup>-1</sup> ]	61	s <sup>2</sup> deg <sup>-1</sup> ]
38	$C_\kappa$ stiffness coefficient [-]	62	$J$ blade pitching inertia [kgm <sup>2</sup> ]
39	$C_\kappa^{dy}$ optimal stiffness coefficient	63	$J^{2D}$ blade section inertia [kgm]
40	(dynamic analysis) [-]	64	$J_{tot}$ total inertia [kgm <sup>2</sup> ]
41	$C_\kappa^{qs}$ optimal stiffness coefficient	65	$K_{am}$ added stiffness coefficient
42	(quasi-steady analysis) [-]	66	[Nmdeg <sup>-1</sup> ]
43	$C_{RBM}$ root bending moment	67	$K_{tot}$ total stiffness [Nmdeg <sup>-1</sup> ]
44	coefficient [-]	68	$L$ lift force [N]
45	$C_{tot}$ total damping [Nm s <sup>-1</sup> deg <sup>-1</sup> ]	69	$M$ pitching moment [Nm]
46	$C_\mu$ damping coefficient [-]	70	$M_c^{2D}$ sectional centrifugal
47	$C_D$ drag coefficient [-]	71	moment [N]
48	$C_L$ lift coefficient [-]	72	$M_{dy}^{2D}$ hydrodynamic moment on a
49	$C_M$ foil's pitching moment	73	blade section [N]
50	coefficient [Nm]	74	$M_{qs}^{2D}$ sectional quasi-steady
51	$C_P$ power coefficient [-]	75	hydrostatic moment [N]

76	$M_{RB}$	root bending moment [Nm]	99	$U_\infty$	free stream velocity [m s <sup>-1</sup> ]
77	$M_c$	blade centrifugal moment [Nm]	100	$U_\psi$	tangential component of the
78	$M_{dy}$	hydrodynamic moment on the	101		inflow velocity [m s <sup>-1</sup> ]
79		blade [Nm]	102	$ZZ'$	plane of a generic blade
80	$M_{qs}$	blade quasi-steady	103		section [-]
81		hydrostatic moment [Nm]	104	$\mathbf{A}$	position of the pitching axis
82	$M_s$	spring moment [Nm]	105		of a section [m]
83	$M_s^{2D}$	spring moment	106	$\mathbf{B}$	position of a generic point
84		per unit span [N]	107		on a section [m]
85	$N$	number of blade sections [-]	108	$\overline{\mathbf{AB}}$	distance of a generic point from
86	$N_b$	number of blades [-]	109		the pitching axis of a section [m]
87	$P$	blade power [W]	110	$\delta r$	span of a section [m]
88	$P$	power extracted [W]	111	$\gamma$	angular coordinate on a blade
89	$P^{2D}$	sectional power [Wm <sup>-1</sup> ]	112		section [m]
90	$R$	tip radius [m]	113	$\hat{C}_M$	flat plate's pitching moment
91	$T$	blade thrust force [N]	114		coefficient [Nm]
92	$T$	thrust force [N]	115	$\hat{F}$	Prandtl's tip loss factor [-]
93	$T^{2D}$	sectional thrust force [Nm <sup>-1</sup> ]	116	$\hat{J}$	objective function of the
94	$U$	inflow velocity [m s <sup>-1</sup> ]	117		optimisation problem [-]
95	$U_{hub}$	free stream velocity at	118	$\hat{x}$	chordwise coordinate on a blade
96		hub height [m s <sup>-1</sup> ]	119		section [m]
97	$U_x$	axial component of the inflow	120	$\hat{x}_A$	distance of the pitching axis from
98		velocity [m s <sup>-1</sup> ]	121		the leading edge of a section [m]
			122	Re	Reynolds number [-]
			123	$\mu$	blade mechanical
			124		damping [Nm s <sup>-1</sup> deg <sup>-1</sup> ]

125	$\psi$	blade azimuthal position [deg]	150	$\Omega_N$	system natural frequency
			151		[ $\text{rads}^{-1}$ ]
126	$a$	axial induction factor [-]	152	$\Omega_s$	blade cross-sectional shell
127	$a'$	tangential induction factor [-]	153		area [ $\text{m}^2$ ]
128	$b$	half-chord length [m]	154	$\Omega_w$	blade cross-sectional internal
129	$c$	chord length [m]	155		area [ $\text{m}^2$ ]
130	$d$	distance of the pitching axis	156	$\alpha$	angle of attack [deg]
131		from the half chord [m]	157	$\beta$	pitch angle [deg]
132	$f$	Prandtl's tip loss factor	158	$\beta_0$	twist angle [deg]
133		exponent [-]	159	$\kappa$	stiffness of the spring [ $\text{Nmdeg}^{-1}$ ]
134	$m$	mass of a small blade	160	$\kappa^{2D}$	spring stiffness per unit span
135		element [kg]	161		[ $\text{Ndeg}^{-1}$ ]
136	$r$	radial coordinate [m]	162	$\lambda$	tip speed ratio [-]
137	$t$	time [s]	163	$\lambda_0$	optimal tip speed ratio [-]
138	$x$	horizontal streamwise	164	$\omega$	rotational speed [ $\text{rads}^{-1}$ ]
139		coordinate [m]	165	$\phi$	inflow angle [deg]
140	$xy$	foil thickness in percentage	166	$\rho_s$	density of blade shell [ $\text{kgm}^{-3}$ ]
141		of chord [-]	167	$\rho_w$	density of water [ $\text{kgm}^{-3}$ ]
142	$z$	vertical coordinate from the	168	$\theta$	spring strain angle [deg]
143		seabed [m]	169	$\theta^{\text{dy}}$	spring strain angle (dynamic
144	$z_{\text{hub}}$	height of the hub from the	170		analysis) [deg]
145		seabed [m]	171	$\theta_0^{\text{dy}}$	optimal preload (dynamic
146	$\Gamma$	sectional non-dimensional	172		analysis) [deg]
147		stiffness [-]	173	$\theta^{\text{qs}}$	spring strain angle (quasi-steady
148	$\Gamma_0$	optimal sectional	174		analysis) [deg]
149		non-dimensional stiffness [-]			

175	$\theta_0^{\text{qs}}$	optimal preload	181	HATT	Horizontal Axis Tidal Turbine
176		(quasi-steady analysis) [deg]			
177	$\theta_0$	preload of the spring [deg]	182	HAWT	Horizontal Axis Wind Turbine
178	$\zeta$	damping ratio [-]	183	LCOE	Levelised Cost Of Energy
179	BEMT	Blade Element Momentum			
180		Theory	184	TGL	Tidal Generation Limited

185 Nomenclature note: time derivatives are shown with a dot above the vari-  
186 able; overbars show mean values over a blade revolution; bold symbols describe  
187 vector variables;  $\Delta$  represents the difference between the maximum and mini-  
188 mum values of a fluctuating variable;  $\delta$  represents a small variation; quantities  
189 that refer to the rigid fixed-pitch turbine are indicated by subscripts  $|\kappa \rightarrow \infty$ .

## 190 **1. Introduction**

191 Tidal energy is a promising renewable energy source that could critically  
192 contribute to energy security [1]. It's been estimated that the global theoretical  
193 potential of tidal power is 22000 TWh/year, and that under favourable condi-  
194 tions, by 2050, wave and tidal energy systems could contribute to nearly 10% of  
195 the European electrical demand (350 TWh/year) [2]. The world's first arrays  
196 of tidal turbines (Meygen and Nova's Bluemull Sound, 2016) have only recently  
197 been deployed in Scotland. For this energy sector to develop further, new tech-  
198 nology must be developed to reduce the levelised cost of energy (LCOE), which  
199 is the minimum constant price at which electricity has to be sold in order to  
200 break-even over the lifetime of the project. Technology that enables more reli-  
201 able and cheaper tidal turbines to be built would contribute to a reduction in  
202 the LCOE and it would provide more competitive renewable energy.

203 The large load fluctuations induced by the shear and turbulence of the onset  
204 flow, wave-induced current fluctuations, yaw misalignment, interaction with the  
205 support structures, and wakes of the upstream devices present a major challenge  
206 to the design of tidal turbines [3, 4]. Load fluctuations are transmitted from  
207 the blades to the rest of the turbine making fatigue failures a key limit to  
208 reliability [5]. To increase local blockage and thus yields, tidal turbines might  
209 be placed in close proximity. This will have consequences on, for example,  
210 the foundations and scours [6, 7, 8], but will also result in additional unsteady  
211 load fluctuations [9]. Furthermore, unsteady loads are reflected in power output  
212 fluctuations, which result in over-dimensioned power-take-off systems [10] and in  
213 a lower maximum mean operating power. Therefore, unsteady load mitigation  
214 is critical to reducing LCOE and enhancing the competitiveness of tidal energy.

215 Load fluctuations are currently mitigated by actively varying the turbine  
216 speed, by actively pitching the blades, or by enabling the blade to twist pas-  
217 sively to feather when the fluid dynamic load increases (also known as hydroe-  
218 lastic tailoring) [11]. Unfortunately, the effectiveness of these control systems  
219 decreases with the size of the blade. In fact, there is a trade off between the

220 power that can be spent to actively control the rotor and the highest frequency  
221 that can be mitigated. From the results of Barlas *et al.* [12, 13], for example, it  
222 can be noted that pitch control is up to about 50 times slower than required to  
223 cancel up to the highest load fluctuations experienced by wind turbines. Passive  
224 twist is currently adopted by Schottel Hydro, whose 70 kW turbines are 6.3 m  
225 in diameter. On the other hand, the higher loads on SIMEC Atlantis Energy’s  
226 MW-scale tidal turbines are incompatible with the flexibility required for pas-  
227 sive twist. Additionally, all these mechanisms primarily mitigate the unsteady  
228 load at the tip, which is the major contribution to the torque and thrust, but  
229 they do not prevent large flow separation from occurring near the blade root  
230 [3]. This results in less energy remaining available to the downstream turbines,  
231 and thus to lower efficiency for compacted arrays.

232 Fast-actuated flaps, like those used on aircraft wings, could mitigate higher  
233 frequency fluctuations [12, 13, 14]. These flaps are smaller and therefore could  
234 have a faster response than a whole blade pitching system, but they still re-  
235 quire power electronics, hinges with bearings and actuator mechanisms that are  
236 exposed to debris and biofouling. This additional complexity of the system is  
237 seen by industry as posing a risk to turbine reliability. The cost of maintenance  
238 of both offshore wind and tidal turbines is a major driver compared to onshore  
239 wind. Hence, reliability is paramount [15]. For example, tidal companies such  
240 as Nova Innovation, Nautricity and Schottel Hydro adopt fixed blade turbines  
241 to maximise reliability. Orbital Marine Power and SIMEC Atlantis Energy,  
242 which operate the largest rotors, only use collective pitch control, which can-  
243 not mitigate fast load fluctuations due to, for example, shear, interference, yaw  
244 misalignment, etc.

245 Last year, the European Commission identified blades with built-in chord-  
246 wise flexibility as one of the most promising concepts for reducing the costs of the  
247 blades and the downstream components of turbines [16]. Hence, in this paper  
248 we investigate the underlying principles of load alleviation through morphing  
249 blades and their potential effectiveness.

250 Specifically, we aim to answer the following research questions. (1) How



251 do morphing blades work? What are the physical phenomena that underlie  
252 unsteady load alleviation by morphing blades? (2) Which are the key factors  
253 of a practical implementation that can decrease their effectiveness? To address  
254 these questions, we develop a low-order model of a morphing blade and we  
255 identify the parameters that govern the system performance.

256 This work focuses on the underlying physical mechanism and aims to pave  
257 the way to future designs. The proposed model is kept as simple as possible  
258 to allow insights on the physics, whilst it does not mean to be a design tool.  
259 While the model cannot be currently validated due to the lack of experimental  
260 data on morphing blades, we test our model of rigid and flexible blades against  
261 the numerical simulations of other authors. We consider both the unsteady  
262 fluctuations due to the rotation of the blades in a steady shear current, and  
263 those due to a turbulent shear current in the presence of large waves. The latter  
264 input flow velocity condition is taken from full-scale measurements undertaken  
265 at the European Marine Energy Centre (EMEC) tidal test site [3].

266 The rest of the paper is organised as follows. In Section 2, we present our  
267 methodology and how we modelled the morphing blade. In Sections 4, 5 we  
268 present the model inputs and the turbine geometry, and the model validation  
269 respectively. Section 6 shows the analysis of the morphing blade model, how  
270 we optimised its load alleviation capabilities, we present a parametric study of  
271 the effect of the morphing blade properties on the system performance, and we  
272 show the potential load mitigation of morphing blades on a turbine subjected  
273 to real flow conditions. In Section 7, we summarise our findings.

## 274 **2. Methodology**

275 We present our methodology for a turbine subjected to flow fluctuations  
276 caused by a modelled shear flow. Nonetheless, the method can be readily gener-  
277 alised for flow fluctuations induced by other sources, like turbulence, waves and  
278 wakes, or for real flow conditions, as described in Section 3. The specific flow  
279 conditions, the turbine properties, and the spatial and temporal discretizations

280 are introduced in Section 4.

### 281 2.1. Rigid Blade Model

282 We consider a 3 blades tidal turbine with a rotor diameter of 18 m (blade  
 283 length  $R = 9$  m) operating in a sheared velocity profile, at the optimal tip  
 284 speed ratio of 4.5. The turbine hub is at 20 m from the seabed, the mean flow  
 285 velocity at the hub is  $U_{\text{hub}} = 2 \text{ m s}^{-1}$ , and the velocity profile varies with a  
 286 1/7 power law. Each blade experiences periodic onset flow, such that the inflow  
 287 speed and the angle of attack are maximum for each section when the blade is  
 288 pointing upwards, and minimum when pointing downwards. The 1/7 power law  
 289 is commonly used to describe the velocity profile across a turbulent boundary  
 290 layer [17]. It is often applied to tidal flow in coastal regions [18], and its use  
 291 is recommended by the UK Health and Safety Executive [19] to model velocity  
 292 profiles in coastal regions around the UK.

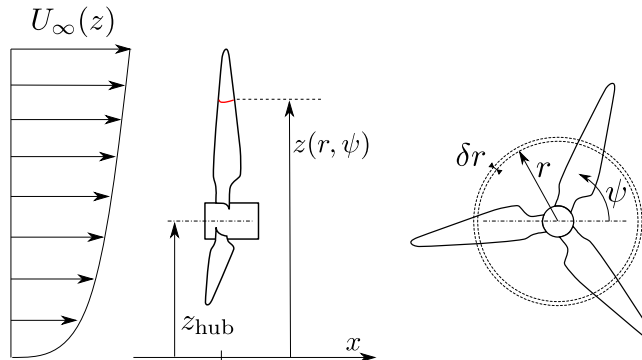


Figure 1: Schematic diagram of a tidal turbine coordinate systems. On the left, the side view shows the interaction of the turbine with a shear flow. On the right, front view of the turbine.

293 We employ Blade Element Momentum Theory (BEMT) [20] to compute the  
 294 axial induction factor  $a$  and the tangential induction factor  $a'$ . The axial ( $U_x$ )  
 295 and tangential ( $U_\psi$ ) components of the flow speed relative to the blade sections  
 296 are depicted in Fig. 2 and they are computed following the approach of Burton  
 297 et al. [20] as

$$\begin{aligned}
U_x &= U_\infty - a\bar{U}_\infty, \\
U_\psi &= \omega r(1 + a'),
\end{aligned}
\tag{1}$$

298 where  $\omega$  is the turbine rotational speed,  $r$  is the section position along the blade,  
299 and  $\bar{U}_\infty$  is the average upstream axial velocity seen by a blade rotating in a shear  
300 flow.

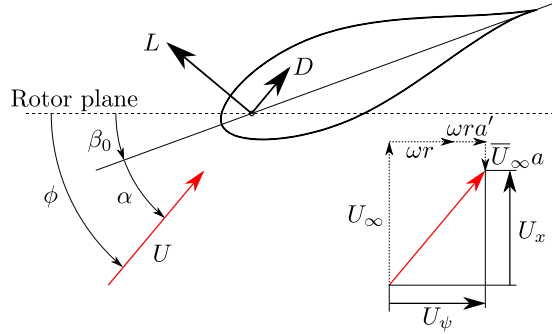


Figure 2: Flow velocities and forces acting on a 2D blade section.

301 From the vectors shown in Fig. 2, the flow relative to the blade is obtained  
302 as

$$\begin{aligned}
U &= \sqrt{U_x^2 + U_\psi^2}, \\
\phi &= \tan^{-1} \left( \frac{U_x}{U_\psi} \right),
\end{aligned}
\tag{2}$$

303 where  $U$  is the magnitude of the relative flow speed experienced by the blade  
304 section and  $\phi$  is the inflow angle (Fig. 2). Let  $\beta_0$  be the twist angle of a blade  
305 section, the angle of attack is

$$\alpha = \phi - \beta_0.
\tag{3}$$

306 We compute the tip loss factor following Burton et al. [20] as

$$\hat{F} = \frac{2}{\pi} \cos^{-1}(e^{-f}),
\tag{4}$$

307 where

$$f = \frac{N_b}{2} \frac{1-r}{r \sin \phi}, \quad (5)$$

308 and  $N_b$  is the number of blades.

309 The BEMT algorithm evaluates the induction factors with an iterative pro-  
 310 cedure that uses the mean loads acting on each section. We define the mean  
 311 inflow conditions as

$$\begin{aligned} \bar{U} &= \sqrt{\bar{U}_x^2 + U_\psi^2}, \\ \bar{\phi} &= \text{atan} \left( \frac{\bar{U}_x}{U_\psi} \right), \end{aligned} \quad (6)$$

312 where  $\bar{U}_x = \bar{U}_\infty(1-a)$ . The average angle of attack experienced by each section  
 313 is

$$\bar{\alpha} = \bar{\phi} - \beta_0. \quad (7)$$

314 The mean lift and drag coefficients,  $C_L(\bar{\alpha})$  and  $C_D(\bar{\alpha})$  respectively, are ini-  
 315 tially evaluated with  $a = 1/3$  and  $a' = 0$  for each section. The resulting lift and  
 316 drag are then used to update the values of the induction factors

$$\begin{aligned} a &= \left( \frac{4\pi r \hat{F} \sin^2 \bar{\phi}}{N_b c (C_L \cos \bar{\phi} + C_D \sin \bar{\phi})} + 1 \right)^{-1}, \\ a' &= \left( \frac{4\pi r \hat{F} \sin \bar{\phi} \cos \bar{\phi}}{N_b c (C_L \sin \bar{\phi} - C_D \cos \bar{\phi})} - 1 \right)^{-1}, \end{aligned} \quad (8)$$

317 where  $c$  is the foil's chord.

318 The procedure is repeated until  $a$ ,  $a'$ ,  $C_L$  and  $C_D$  converge.

319 The performance of the turbine is evaluated in terms of the non-dimensional  
 320 power and thrust coefficients. For a blade section, they are respectively defined  
 321 as

$$\begin{aligned}
C_P^{2D} &= \frac{P^{2D}}{\frac{1}{2}\rho_w U_{\text{hub}}^3 A^{2D}}, \\
C_T^{2D} &= \frac{T^{2D}}{\frac{1}{2}\rho_w U_{\text{hub}}^2 A^{2D}},
\end{aligned}
\tag{9}$$

322 where  $\rho_w$  is the water density,  $A^{2D} = 2\pi r$  is the perimeter swept by the blade  
323 section at spanwise position  $r$ , and the thrust  $T^{2D}$  and the power  $P^{2D}$  are

$$\begin{aligned}
T^{2D} &= L \cos \phi + D \sin \phi, \\
P^{2D} &= \omega r (L \sin \phi - D \cos \phi).
\end{aligned}
\tag{10}$$

324 For a turbine's blade, these coefficients are computed respectively as

$$\begin{aligned}
C_P &= \frac{P}{\frac{1}{2}\rho_w U_{\text{hub}}^3 A}, \\
C_T &= \frac{T}{\frac{1}{2}\rho_w U_{\text{hub}}^2 A},
\end{aligned}
\tag{11}$$

325 where  $A = \pi R^2$  is the area swept by the turbine,  $T = \sum_i T_i^{2D} \delta r_i$  and  $P =$   
326  $\sum_i P_i^{2D} \delta r_i$ , for  $i = 1 \dots N$ , where  $\delta r_i$  is the thickness of the section and is chosen  
327 sufficiently small to ensure the desired accuracy.

## 328 *2.2. Morphing Blade Model*

329 The blade can bend its trailing edge to mitigate the changes in the flow  
330 incidence, thus alleviating the load fluctuations. Let's assume that the loads on  
331 a blade section result in the elastic deformation of the foil as shown in Fig. 3.  
332 We consider the effect of such geometric variation as an equivalent change in  
333 the angle of attack of the original foil. Therefore, the shape deformation is  
334 analogous to the pitch rotation of a rigid foil, and its flexibility can be modelled  
335 by a torsional spring with constant properties that controls the pitching motion  
336 of the blade (Fig. 4). The angular position of the blade is thus determined by  
337 the balance of the moments acting along the pitch axis.

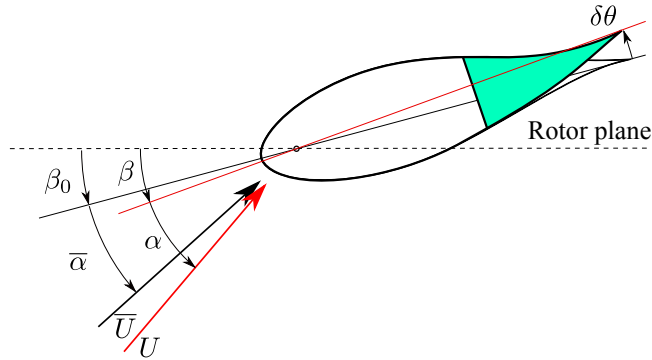


Figure 3: Morphing blade concept. Part of the blade can bend to mitigate inflow fluctuations.

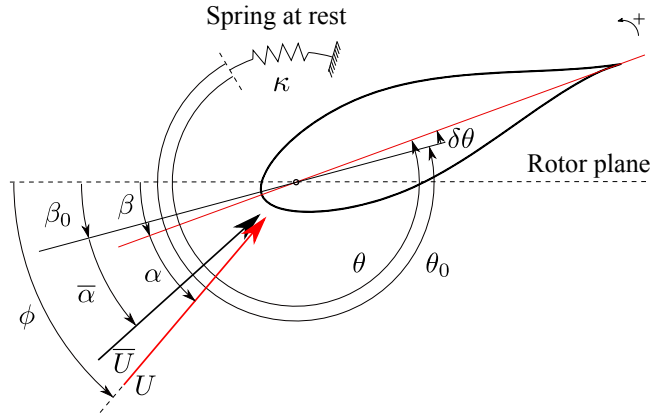


Figure 4: Morphing blade model. The flexibility is modelled by a torsional spring.

338 The analysis is carried out for two different scenarios. Firstly, we consider a  
 339 blade where there is no interaction between the sections such that each section  
 340 moves independently from the others (Fig. 5a). This condition is representa-  
 341 tive, for example, of a blade made of a flexible material with a very low shear  
 342 modulus, such that the shear stresses between the sections are negligible. The  
 343 blade flexibility is represented by a torsional spring for each section, and the  
 344 deflection of each section is determined by the local flow conditions. This model  
 345 is described using a 2-dimensional analysis in Section 2.3, and it is optimised  
 346 for each section of the blade, to allow the same mean load as the rigid foil and

to minimise the fluctuations.

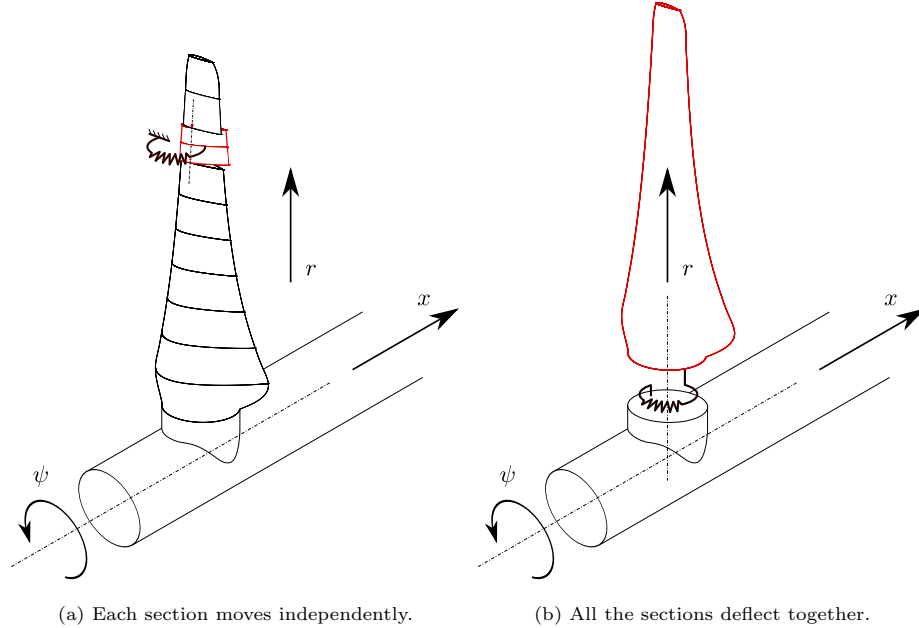


Figure 5: Morphing blade models.

347

348 Secondly, we consider a blade where all sections experience the same deflec-  
 349 tion along the span (Fig. 5b). As opposed to the previous scenario, this model  
 350 could represent a blade material with a very high shear modulus. Therefore,  
 351 the stresses caused by the external loads are efficiently redistributed along the  
 352 blade such that each section deforms by the same amount. The blade deflection  
 353 is thus seen as a rigid rotation with the flexibility concentrated at the root of  
 354 the blade. This flexible connection is modelled with a torsional spring placed  
 355 at the root that adds a variable pitch angle  $\delta\theta$  to the rigid blade twist angle  $\beta_0$ .  
 356 This condition is presented in Section 2.4, where the entire blade is considered,  
 357 and the spring's parameters are optimised to minimise the fluctuations in the  
 358 root bending moment without reducing the average power extracted. In Sec-  
 359 tion 2.5, we extend such model to include the dynamics of the blade and the  
 360 unsteadiness of the flow, to check the robustness of the optimal system when

361 the blade dynamics are considered, and to investigate the effect of the blade  
 362 inertia, the unsteady hydrodynamics and the blade mechanical damping on the  
 363 performance of the morphing blade.

### 364 2.3. Quasi-steady Analysis of a Morphing Foil

365 We assume that each section can move independently from the others (Fig. 5a),  
 366 such that every blade section can be optimised separately. The section angular  
 367 position is determined by balancing the moments acting along its pitch axis.  
 368 There are two moments that compete to determine the angular position of a  
 369 blade section: the reaction of the spring

$$M_s^{2D} = -\kappa^{2D}\theta, \quad (12)$$

370 where  $\theta$  is the spring strain and  $\kappa^{2D}$  is the spring stiffness of a 2D section, and  
 371 the hydrostatic pitching moment

$$M_{qs}^{2D} = \frac{1}{2}\rho_w U^2 c^2 C_M(\alpha). \quad (13)$$

372 The blade pitches around the point at the chordwise coordinate  $\hat{x}_A = 0.1c$   
 373 from the leading edge of each section. The moment coefficient  $C_M$  is obtained  
 374 from the quarter-chord moment coefficient  $C_{M_{1/4}}$  by adding the moment con-  
 375 tributions of lift and drag with respect to the pitch axis position  $\hat{x}_A$ , and is  
 376 computed as

$$C_M(\alpha) = -C_{M_{1/4}} - \left(\frac{1}{2} + \frac{d}{b}\right) \frac{1}{2} [C_L \cos(\alpha) + C_D \sin(\alpha)], \quad (14)$$

377 where the location of the pitch axis is indicated with  $d$  (following Theodorsen's  
 378 notation [21]), such that the distance of the pitch axis from the mid-chord is  
 379  $d = -0.8b$ , where  $b = c/2$ .  $C_M$  is positive in the clockwise direction according  
 380 to the sign convention in Fig. 4.

381 Since the passively controlled system works around the average flow condi-  
 382 tion for the rigid turbine (*i.e.*  $\alpha(t) \approx \bar{\alpha}$ ), the hydrostatic pitching moment is  
 383 linearised as



$$C_M(\alpha) = C_{M,0} + C_{M,\alpha}\alpha, \quad (15)$$

384 where  $C_{M,0}$  and  $C_{M,\alpha}$  are the best fitting linear regression coefficients of Eq. 14  
 385 over a 6 deg range of angle of attacks (*i.e.*  $\alpha \in [\bar{\alpha} - 3 \text{ deg}, \bar{\alpha} + 3 \text{ deg}]$ ). Equa-  
 386 tions 13 is thus rewritten as

$$M_{\text{qs}}^{2\text{D}} = \frac{1}{2}\rho_w U^2 c^2 [C_{M,\alpha}\alpha + C_{M,0}]. \quad (16)$$

387 The quasi-steady equations for the foil in Fig. 4 are

$$\begin{cases} M_s^{2\text{D}} + M_{\text{qs}}^{2\text{D}} = 0 & \text{(a)} \\ \phi(t) = \beta(t) + \alpha(t) & \text{(b)} \\ \beta(t) = \beta_0 + \delta\theta(t) & \text{(c)} \end{cases} \quad (17)$$

388 where  $\beta$  is the blade pitch angle and  $\delta\theta(t) = \theta(t) - \theta_0$ .

389 The optimisation problem consists in finding the morphing blade that min-  
 390 imises the flapwise bending moment fluctuations on the blade over its revolution,  
 391 with the requirement that the mean power extracted from the flow is the same  
 392 as for the rigid turbine. For a 2D foil, the condition on the average power is  
 393 restricted to the power generated by a single section of the blade and we at-  
 394 tempt to minimise the fluctuations of the thrust acting on the section  $\Delta C_T^{2\text{D}}$ ,  
 395 as it ultimately contributes to the fluctuations of the flapwise bending moment.  
 396 Hence, in 2D the optimisation problem is

$$\begin{aligned} \min_{\kappa^{2\text{D}}} \hat{J}(\kappa^{2\text{D}}) &= \Delta C_T^{2\text{D}} \\ \text{s.t.} \quad \frac{\partial \bar{C}_P^{2\text{D}}}{\partial \kappa^{2\text{D}}} &= 0, \end{aligned} \quad (18)$$

397 where

$$\bar{C}_P^{2\text{D}} = \frac{1}{2\pi} \int_0^{2\pi} C_P^{2\text{D}} \text{d}\psi. \quad (19)$$

398 The problem is solved using an exhaustive search algorithm.

399 We approximate the average power extracted with the power generated by  
400 average flow conditions, namely  $\overline{C}_P \approx C_P(\overline{\alpha}, \overline{U})$ . The average flow conditions  
401  $\alpha(t) = \overline{\alpha}$  and  $U(t) = \overline{U}$ , thus constant power generation, are achieved for a  
402 constant pitch angle  $\beta(t) = \beta_0$ . The requirement,  $\beta(t) = \beta_0$ , is satisfied with a  
403 spring extended by  $\theta(t) = \theta_0$  where

$$\theta_0 = \frac{1}{2} \rho_w \overline{U}^2 c^2 \left( \frac{C_{M,\alpha} \overline{\alpha} + C_{M,0}}{\kappa^{2D}} \right). \quad (20)$$

404  $\theta_0$  will be referred to as the preload of the spring. The angle of attack is  
405 evaluated from the Eq. 17a where the expression for  $\theta$  is obtained by combining  
406 Eq. 17c with  $\delta\theta = \theta - \theta_0$ . The expression for  $\alpha$  is then rearranged to

$$\alpha(t) = \frac{\kappa^{2D}(\phi(t) - \beta_0 + \theta_0) - \frac{1}{2} \rho_w c^2 U^2 C_{M,0}}{\kappa^{2D} + \frac{1}{2} \rho_w c^2 U^2 C_{M,\alpha}}. \quad (21)$$

407 It's noted that the equation for  $\alpha$  is consistent with the definition of the angle  
408 of attack for a fixed foil, in fact

$$\lim_{\kappa^{2D} \rightarrow \infty} \alpha = \phi - \beta_0. \quad (22)$$

409 The spring deformation  $\theta$  is evaluated by substituting Eq. 17c in Eq. 17b,  
410 such that

$$\theta(t) = \phi(t) - \alpha(t) - \beta_0 + \theta_0. \quad (23)$$

411 For a given value of  $\kappa$ , all the required quantities can be computed and the  
412 loads are assessed.

413 The results for a 2D section will be shown over a range of the non-dimensional  
414 spring stiffness

$$\Gamma = \frac{\kappa^{2D}}{\rho_w c^2 \overline{U}^2}. \quad (24)$$

415  $\Gamma$  is the ratio between the two-dimensional stiffness and a representative  
416 hydrodynamic moment. The latter is taken as the product of the hydrodynamic  
417 force  $\rho_w \overline{U}^2 c$  and the arm  $c$ .

418 *2.4. Quasi-steady Analysis of the Morphing Blade*

419 In this section, we study a morphing blade where the blade deflection at  
 420 every spanwise position is the same. The blade morphing is seen as a rigid  
 421 rotation around the pitch axis and its flexibility is modelled as a torsional spring  
 422 mounted at the root of the blade (Fig. 5b). We study the impact of the proposed  
 423 passive pitch system for a rigid blade pitching around an axis at the chordwise  
 424 coordinate  $\hat{x}_A = 0.1c$ , rotating at the optimal tip speed ratio  $\lambda_0 = 4.5$  for a far-  
 425 field speed  $U_{\text{hub}} = 2 \text{ m s}^{-1}$  at the hub height  $z_{\text{hub}}$ . The pitching equilibrium is  
 426 again dictated by the spring reaction and by the hydrostatic loads. The spring  
 427 acts as a lumped, flexible connection between the blade and the hub and its  
 428 moment reaction is

$$M_s = -\kappa\theta. \quad (25)$$

429 The hydrostatic moment  $M_{\text{qs}}$  takes into account the sum of the moments  
 430 applied on each  $i$ -th section of the blade, and is computed as

$$M_{\text{qs}} = \sum_i \frac{1}{2} \rho_w U_i(t)^2 c_i^2 C_{M_i}(\alpha_i) \delta r_i. \quad (26)$$

431 The quasi-steady equations that describe the motion of the blade are

$$\begin{cases} M_s + M_{\text{qs}} = 0 & \text{(a)} \\ \phi_i(t) = \beta_i(t) + \alpha_i(t) & \text{(b}_i\text{)} \\ \beta_i(t) = \beta_{0_i} + \delta\theta(t) & \text{(c}_i\text{)} \end{cases} \quad (27)$$

432 The optimisation problem is stated similarly to the 2D section case. How-  
 433 ever, since the entire blade is considered, the minimisation is imposed on the  
 434 out-of-plane blade root bending moment coefficient  $C_{\text{RBM}}$ , whereas the con-  
 435 straint considers the power generated by the entire blade. The optimisation  
 436 problem is stated as

$$\begin{aligned}
\min_{\kappa} \hat{J}(\kappa) &= \Delta C_{\text{RBM}} \\
\text{s.t.} \quad \frac{\partial \bar{C}_P}{\partial \kappa} &= 0
\end{aligned} \tag{28}$$

437 where

$$\bar{C}_P = \frac{1}{2\pi} \int_0^{2\pi} C_P(\psi) \, \text{d}\psi, \tag{29}$$

438 and  $\Delta C_{\text{RBM}}$  is the amplitude of  $C_{\text{RBM}} = M_{\text{RB}} / \left( \frac{1}{2} \rho_w U_{\text{hub}}^2 R A \right)$ , with  
439  $M_{\text{RB}} = \sum_i r_i T_i^{2\text{D}} \delta r_i$  the blade root out-of-plane bending moment.

440 The optimisation problem is solved using an exhaustive search approach. To  
441 meet the requirement of constant power extracted, the spring preload is com-  
442 puted similarly to the two-dimensional case as

$$\theta_0 = \frac{1}{\kappa} \sum_i \frac{1}{2} \rho_w \bar{U}_i^2 c_i^2 (C_{M_i, \alpha} \bar{\alpha}_i + C_{M_i, 0}) \delta r_i. \tag{30}$$

443 By combining Eq. 27 with  $\delta\theta = \theta - \theta_0$  we compute the spring deformation  
444  $\theta$  at each time step as

$$\theta(t) = \frac{\sum_i \frac{1}{2} \rho_w c_i^2 U_i^2 (C_{M_i, \alpha} (\phi(t) - \beta_0 + \theta_0) + C_{M_i, 0}) \delta r_i}{\kappa + \sum_i \frac{1}{2} \rho_w c_i^2 U_i^2 C_{M_i, \alpha} \delta r_i}. \tag{31}$$

445 Then, using the complementarity of angles (Eqs. 27b<sub>i</sub>, c<sub>i</sub>), the angle of attack  
446 is determined as

$$\alpha_i(t) = \phi_i(t) - \beta_{0_i} - \theta + \theta_0. \tag{32}$$

447 Using the above equations for  $\theta$  and  $\alpha$  and the preload  $\theta_0$ , all the loads acting  
448 on the blade can be computed.

### 449 2.5. Dynamic Analysis of the Morphing Blade

450 We consider a hollow blade made of composite layers then filled with water  
451 [22]. The inertia  $J^{2\text{D}}$  of each blade section is obtained by

$$J^{2D} = \rho_s \int_{\Omega_s} \overline{AB}^2 d\Omega + \rho_w \int_{\Omega_w} \overline{AB}^2 d\Omega, \quad (33)$$

452 where  $\rho_s$  and  $\rho_w$  are the density of the blade shell and the density of water  
 453 respectively,  $\Omega_s$  is the area of the blade shell on the section,  $\Omega_w$  is the area  
 454 occupied by the water within and  $\overline{AB}$  represents the relative position of a  
 455 generic section element from the pitching axis (Fig. 6). A blade usually presents  
 456 a box spar placed near the pitch axis. Such spar has little influence on the blade  
 457 inertia, it becomes relevant only for structural consideration, and it is therefore  
 458 neglected. The inertia  $J$  of the entire blade is then computed as

$$J = \sum_i J_i^{2D} \delta r_i, \quad (34)$$

459 for  $i = 1 \dots N$ , with  $N$  the number of equally spaced blade sections.

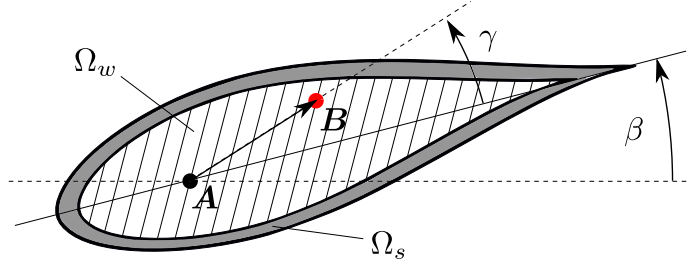


Figure 6: The blade is composed by a composite shell filled with water.

460 The blade rotates around the rotor axis with angular velocity  $\omega$ . This leads to  
 461 the Euler force  $\mathbf{F}_{Eu} = -m \frac{d\omega}{dt} \wedge \mathbf{r}$ , the Coriolis force  $\mathbf{F}_{Co} = -m\omega \wedge \frac{d\mathbf{r}}{dt}$  and the  
 462 centrifugal force  $\mathbf{F}_c = -m\omega \wedge (\omega \wedge \mathbf{r})$ . In this case, the Euler force disappears  
 463 because we consider a constant rotation rate ( $\frac{d\omega}{dt} = 0$ ). The Coriolis force  
 464 is also equal to zero because solid points on the blade do not move radially  
 465 ( $\frac{d\mathbf{r}}{dt} = 0$ ). Conversely, there is a component of  $\mathbf{F}_c$  that lies in the section plane  
 466 which generates a pitching moment (Fig. 7). For a generic point  $B$  on a blade  
 467 section, this force,  $\mathbf{F}_c^B$ , generates a moment  $\mathbf{M}_c^B = \overline{AB} \wedge \mathbf{F}_{c,\psi}^B$  that tends to  
 468 align the blade with the rotor plane.

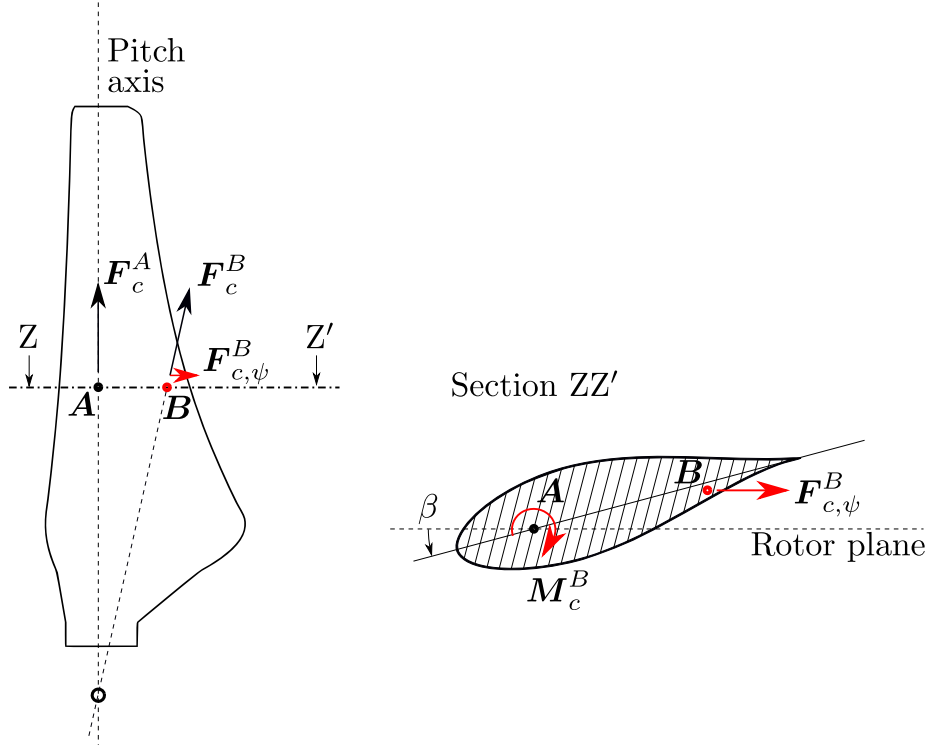


Figure 7: A generic small element of a section that does not lie on the pitch axis is subjected to a centrifugal moment  $M_c^B$ .

469 The centrifugal moment is evaluated for each section as

$$\begin{aligned}
 M_c^{2D}(\beta) = & -\frac{1}{2}\omega^2\rho_s \int_{\Omega_s} \overline{AB}^2 \sin(2(\beta + \gamma)) \, d\Omega \\
 & -\frac{1}{2}\omega^2\rho_w \int_{\Omega_s} \overline{AB}^2 \sin(2(\beta + \gamma)) \, d\Omega,
 \end{aligned} \tag{35}$$

470 where  $\beta + \gamma$  is the angular distance of a blade element from the rotor axis  
 471 (Fig. 6).

472 A Taylor-Young first-order expansion is used to approximate the centrifugal  
 473 moment as

$$M_c^{2D} \approx M_c^{2D}(\beta_0) + \frac{\partial M_c^{2D}}{\partial \beta} \delta\beta. \tag{36}$$

474 This approximation is valid as long as  $\beta \approx \beta_0$ , a condition that is met by im-  
 475 posing the requirement of constant power generated (Eq. 28). By construction,  
 476  $\delta\beta \equiv \delta\theta$  (Fig. 4). Hence, Eq. 36 is rearranged to make explicit the dependency  
 477 of the centrifugal moment to the spring deformation. The centrifugal moment  
 478 acting on the entire blade is

$$M_c \approx \sum_i (M_{c_i,0} + M_{c_i,\theta} \delta\theta) \delta r_i, \quad (37)$$

479 where  $M_{c,0} = M_c^{2D}(\beta_0)$  and  $M_{c,\theta} = \frac{\partial M_c^{2D}}{\partial \theta}$ .

480 The preload angle is computed for average flow conditions, and pitch angle  
 481  $\beta(t) = \beta_0$ . Under these conditions, the centrifugal moment is the only dynamic  
 482 contributions to the preload angle. Equation 30 is thus modified as

$$\theta_0 = \frac{1}{\kappa} \sum_i \left[ \frac{1}{2} \rho_w \bar{U}_i^2 c_i^2 (C_{M_i,\alpha} \bar{\alpha}_i + C_{M_i,0}) + M_{c_i,0} \right] \delta r_i. \quad (38)$$

483 A detailed calculations of the centrifugal and inertial terms on a rotating  
 484 blade can be found in Lanczos [23].

485 The contribution of the gravity force is neglected. The composite blade shell  
 486 is filled with water [22], the density of the whole blade is assumed to match that  
 487 of the surrounding water, and the gravitational force is in equilibrium with the  
 488 buoyancy force.

### 489 2.5.1. Unsteady Hydrodynamics

490 The oscillations due to the shear flow are periodic but not harmonic, and  
 491 to estimate the dynamic loads acting on the blade using Theodorsen's theory,  
 492 we have to consider sinusoidal variations of the angle of attack. We determined  
 493 the fluctuations of the axial velocity ( $U_x$ ) such that the angle of attack would  
 494 fluctuate harmonically with the same amplitude and the same mean value as  
 495 the inflow oscillations due to the shear profile (Fig. 8). This procedure can  
 496 be readily extended to generic inflow conditions, where the signal is decom-  
 497 posed in its Fourier harmonic components, and their effects are superimposed  
 498 via Theodorsen's linear theory. The variations in the angle of attack and in

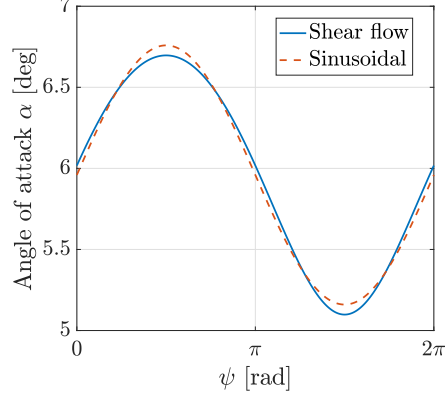


Figure 8: The fluctuations of the angle of attack are approximated by a sinusoidal shape (shown here for  $r = 0.75R$ ).

499 the axial velocity lead to additional pitching moment contributions. Following  
500 Theodorsen [21], the moment on a 2D foil pitching sinusoidally is

$$\begin{aligned}
M_{\text{dy}}^{2\text{D}} = & \rho_w b^3 \pi \left[ \left( \frac{1}{2} - \frac{d}{b} \right) U \delta \dot{\alpha} + b \left( \frac{1}{8} + \left( \frac{d}{b} \right)^2 \right) \delta \ddot{\theta} \right] \\
& - 2 \rho_w U b^3 \pi \left( \frac{1}{2} + \frac{d}{b} \right) C(k) \left( \frac{1}{2} - \frac{d}{b} \right) \delta \dot{\theta} \\
& + \frac{1}{2} \rho_w U^2 c^2 C_{M,\alpha} C(k) \delta \alpha \\
& + \frac{1}{2} \rho_w U^2 c^2 [C_{M,\alpha} \bar{\alpha} + C_{M,0}],
\end{aligned} \tag{39}$$

501 where  $\delta \alpha = \alpha - \bar{\alpha}$ ,  $C(k)$  is Theodorsen's circulation function that is expressed  
502 in terms of Hankel functions of the reduced frequency  $k = \frac{\omega b}{U}$ . The unsteady  
503 pitching moment for the whole blade accounts for the contributions of each  
504 section and it is computed as

$$M_{\text{dy}} = \sum_i M_{\text{dy}_i}^{2\text{D}} \delta r_i, \tag{40}$$

505 The aerodynamic lift  $L$  for the quasi-steady analysis accounts only for the  
506 hydrostatic loads (Eq. ??) whereas, in the dynamic analysis, it accounts for



507 both the dynamic and the hydrostatic contributions and is computed as

$$L = \pi\rho_w b^2 \left[ U\delta\dot{\alpha} - b\left(\frac{d}{b}\right)\delta\ddot{\theta} \right] + 2\pi\rho_w U b^2 C(k) \left( \frac{1}{2} - \frac{d}{b} \right) \delta\dot{\theta} \quad (41)$$

$$+ \frac{1}{2}\rho_w U^2 c [C(k)C_L(\alpha) + (1 - C(k))C_L(\bar{\alpha})].$$

508 The dynamic equations for the blade are

$$\begin{cases} M_s + M_c + M_{dy} - \mu\delta\dot{\theta} = J\delta\ddot{\theta} & \text{(a)} \\ \phi_i(t) = \beta_i + \alpha_i(t) & \text{(b}_i\text{)} \\ \beta_i(t) = \beta_{0_i} + \delta\beta_i(t), & \text{(c}_i\text{)} \end{cases} \quad (42)$$

509 where  $\mu$  is a parameter that accounts for the mechanical damping of the blade.

510 The sectional hydrodynamic torque  $M_{dy_i}^{2D}$  is a function of the average angle  
511 of attack  $\bar{\alpha}_i$  and of its fluctuations  $\delta\alpha_i$  (Eq. 39). Using the complementarity of  
512 the angles in Eqs. 42b<sub>i</sub> and 42c<sub>i</sub>, it is possible to express  $\delta\alpha_i$  as a function of  $\delta\theta$ ,  
513 and to obtain an expression for the blade hydrodynamic torque in Eq. 40 which  
514 depend only on  $\delta\theta$  and its first and second time derivative. By substituting Eqs.  
515 25, 37, 40 into Eq. 42a, the morphing blade dynamic equilibrium is formulated  
516 as:

$$\begin{aligned} & \left( J - \sum_i I_{am_i} \delta r_i \right) \delta\ddot{\theta} + \left( \mu + \sum_i (C_{am_i}^{mc} - C_{am_i}^c) \delta r_i \right) \delta\dot{\theta} \\ & \quad + \left( \kappa + \sum_i (K_{am_i} - M_{c_i,\theta}) \delta r_i \right) \delta\theta \quad (43) \\ & = \sum_i \left( C_{am_i}^{mc} \delta\dot{\phi}_i + K_{am_i} \delta\phi_i + M_{qs_i}^{2D} (\bar{\phi}_i - \beta_{0_i}) + M_{c_i,0} \right) \delta r_i - \kappa\theta_0, \end{aligned}$$

517 where

$$\begin{aligned}
I_{\text{am}} &= \rho_w b^4 \pi \left( \frac{1}{8} + \left( \frac{d}{b} \right)^2 \right), \\
C_{\text{am}}^c &= -\rho_w U b^3 \pi \left( \frac{1}{2} - \frac{d}{b} \right) \left( 1 + 2 \frac{d}{b} \right) C(k), \\
C_{\text{am}}^{nc} &= -\rho_w U b^3 \pi \left( \frac{1}{2} - \frac{d}{b} \right), \\
K_{\text{am}} &= \frac{1}{2} \rho_w U^2 c^2 C_{M,\alpha} C(k), \\
M_{\text{qs}_i}^{2\text{D}}(\bar{\alpha}) &= \frac{1}{2} \rho_w U^2 c^2 [C_{M,\alpha} \bar{\alpha} + C_{M,0}],
\end{aligned} \tag{44}$$

518 The fluctuations are defined by  $\delta\phi = \frac{\Delta\phi}{2} \exp(j\omega t)$  and  $\delta\alpha = \frac{\Delta\alpha}{2} \exp(j\omega t)$ ,  
519 where  $\Delta\phi$  and  $\Delta\alpha$  represent the amplitude of the fluctuations of the respective  
520 angles and  $j$  is the imaginary unit. The blade is expected to oscillate with  
521 the same period, namely  $\delta\theta = \frac{\Delta\theta}{2} \exp(j\omega t + \chi)$ . By substituting  $\delta\dot{\theta} = j\omega\delta\theta$ ,  
522  $\delta\ddot{\theta} = -\omega^2\delta\theta$ ,  $\delta\dot{\phi} = j\omega\delta\phi$  and  $\delta\ddot{\phi} = -\omega^2\delta\phi$ , Eq. 43 is rewritten as

$$\begin{aligned}
& \left[ -\omega^2 \left( J - \sum_i I_{\text{am}_i} \delta r_i \right) + j\omega \left( \mu + \sum_i (C_{\text{am}_i}^{nc} - C_{\text{am}_i}^c) \delta r_i \right) \right. \\
& \quad \left. + \left( \kappa + \sum_i (K_{\text{am}_i} - M_{c_i,\theta}) \delta r_i \right) \right] \delta\theta \tag{45} \\
& = \sum_i \left( (j\omega C_{\text{am}_i}^{nc} + K_{\text{am}_i}) \delta\phi_i + M_{\text{qs}_i}^{2\text{D}}(\bar{\phi}_i - \beta_{0_i}) + M_{c_i,0} \right) \delta r_i - \kappa\theta_0.
\end{aligned}$$

523 The dynamics of  $\delta\theta$  are determined by the algebraic solution of Eq. 45 for  
524 each time step and the equilibrium position of the blade is computed as  $\theta =$   
525  $\theta_0 + \delta\theta$ . The solution method is inspired by the work of Medina and Hemati  
526 [24].

527 The optimisation problem is defined as for the quasi-steady case (Eq. 28),  
528 where the loads account for the unsteady contributions described in Eq. 41.  
529 The angle of attack is determined as for the quasi-steady analysis using Eq. 32,  
530 hence all the loads acting on the blade can be computed.

531 In the Results (Sections 6.2, 6.3), we present the spring stiffness  $\kappa$  and the

532 material damping  $\mu$  in terms of the nondimensional coefficients

$$\begin{aligned} C_\kappa &= \frac{\kappa}{\rho_w U_{\text{hub}}^2 A c}, \\ C_\mu &= \frac{\mu}{\rho_w U_{\text{hub}} A c^2}. \end{aligned} \tag{46}$$

533

534 Akin to  $\Gamma$ , also  $C_\kappa$  and  $C_\mu$  are made nondimensional using a hydrodynamic  
535 force and the length scale  $c$ . Here, the force is that acting on the rotor disk  
536 and it is proportional to  $\rho_w U_{\text{hub}}^2 A$ . It is noted that both the coefficients are  
537 based on the force on the rotor disk and do not account for the hydrodynamic  
538 force associated with the tangential velocity. Hence, as the tip speed ratio  
539 increases, the optimum  $\kappa$  and  $C_\kappa$  are expected to increase to match the higher  
540 hydrodynamic force associated with the rotation.

### 541 3. Real inflow conditions

542 For this numerical study, we use a flow sample measured at EMEC during a  
543 flood tide on November 22nd 2014. This sample was originally chosen by Scarlett  
544 *et al.* [3] to investigate numerically the unsteady hydrodynamic response of the  
545 Tidal Generation Limited (TGL) turbine subjected to large waves and opposing  
546 current. The significant wave height in the sample is 4.2 m, the maximum  
547 observed height is about 5 m, and the wave period is 10 s. The turbine is  
548 operating at the optimal tip speed ratio 4.5, the magnitude of the inflow speed  
549 averaged over the area and the sample time period is  $2.77 \text{ m s}^{-1}$ , and the  
550 rotational speed of the turbine is  $\omega = 1.38 \text{ rads}^{-1}$ . Figure 9 shows the time  
551 history of the angle of attack at the blade spanwise position  $r/R = 0.75$ , over  
552 10 rotational periods ( $T_r = 2\pi/\omega$ ). Further details about the measured flow  
553 conditions and the measurement system are found in Scarlett *et al.* [3].

554 The blade deflection at every spanwise position is computed as described  
555 in Section 2.4, the blade flexibility is modelled as a torsional spring mounted  
556 at the root, and the blade pitches around an axis at the chordwise coordinate  
557  $\hat{x}_A = 0.1c$ . The pitching equilibrium is described using the quasi-steady analysis

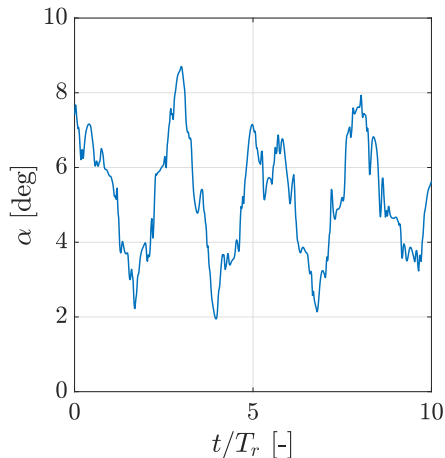


Figure 9: Time history of the angle of attack at  $r/R = 0.75$ . Data from ReDAPT project re.

558 in Eq. 27. The optimisation problem is defined in Eq. 28 and solved using an  
 559 exhaustive search algorithm varying the input spring stiffness. For any stiffness,  
 560 the preload is computed using Eq. 30. The optimum is represented by the stiff-  
 561 ness and preload that minimise the fluctuations of the out-of-plane blade root  
 562 bending moment  $C_{\text{RBM}}$ , whilst keeping constant the average power generated  
 563 by the turbine.

#### 564 4. Input parameters

565 The aerodynamics ( $C_L$ ,  $C_D$ ) for the blade sections profiles is taken from  
 566 Grettton and Ingram [25] for a Reynolds number  $\text{Re} = 3 \times 10^6$  which matches  
 567 the flow on a full-scale tidal turbine. The shape of the blade sections is the  
 568 NACA  $63_{(318)} - 4xy$ , based on the NACA 6-series designation given by, for  
 569 instance, Abbott and Von Doenhoff [26]. The thickness  $xy$  in percentage of  
 570 the foil chord varies from the NACA  $63_{(318)} - 455$  near the root to the NACA  
 571  $63_{(318)} - 418$  at the blade tip. An overview of the blade geometry is showed  
 572 in Fig. 10 and data is provided in Table 1. The blades are thicker at the root,  
 573 which results in a smooth force variation near the stall angle. For the blade

574 sections with thickness up to 40%, the foil characteristics are interpolated from  
 575 the data provided by Gretton and Ingram [25]. Instead, for thicker sections,  
 576 the  $C_L$  and  $C_D$  of the 40%-thick section are used. This approximation will not  
 577 affect the results significantly since the root of the blade does not contribute  
 578 much to the power extraction compared to the mid span and the tip.

579 The quarter-chord pitching moment  $C_{M_{1/4}}$ , which is not provided by Gretton  
 580 and Ingram [25], is taken from Abbott and Von Doenhoff [26] for a NACA  
 581  $63_3 - 418$  (Fig. 11). The parameters used in the simulations are resumed in  
 582 Table 2.

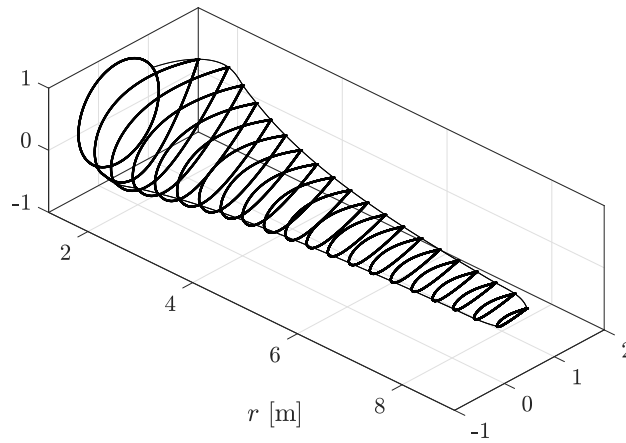


Figure 10: TGL blade geometry adapted from Gretton and Ingram [25].

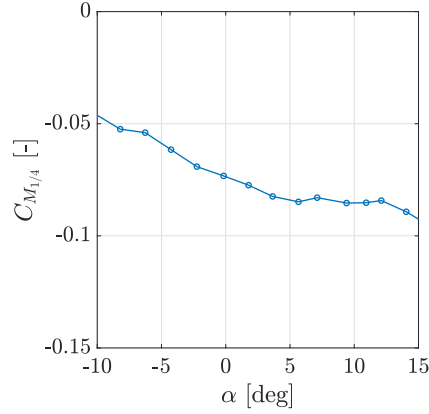


Figure 11: Quarter chord pitching moment coefficient for a NACA 633 – 418 as a function of  $\alpha$ , at  $\text{Re} = 3 \times 10^6$ . Data taken from Abbott and Von Doenhoff [26].

583 **5. Code Validation**

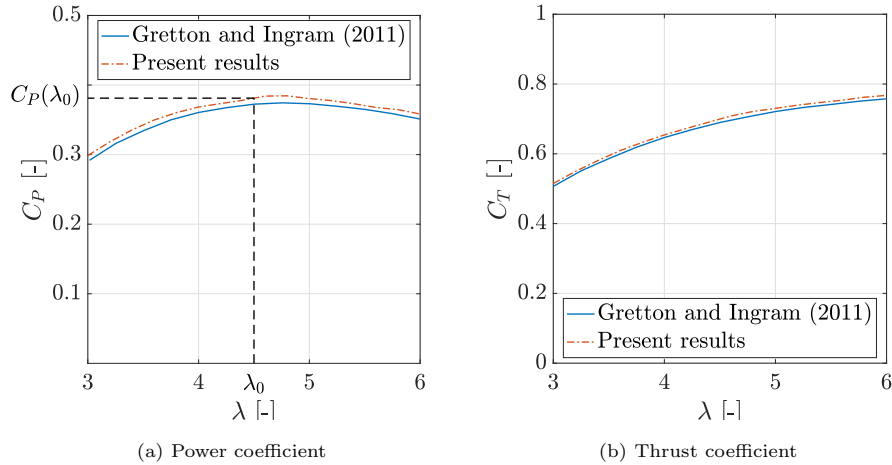


Figure 12: Comparison of the BEMT results of this study and the CFD results from Gretton and Ingram [25] using  $U_{\text{hub}} = 2 \text{ m s}^{-1}$ .  $C_P$  and  $C_T$  refer to the entire turbine.

584

585 First, we assess the accuracy of the numerical code for a turbine subjected  
 586 to steady flow conditions, against the data from Gretton and Ingram [25],

Radius [m]	Twist [deg]	Chord [m]	Thickness ratio	Profile
1.25	32.5	1.612	1.000	Circle
2.05	23.2	2.271	0.550	63 <sub>(318)</sub> – 455
2.45	19.9	2.119	0.533	63 <sub>(318)</sub> – 453
2.85	17.2	1.962	0.511	63 <sub>(318)</sub> – 451
3.25	14.9	1.813	0.485	63 <sub>(318)</sub> – 449
3.65	13.1	1.677	0.454	63 <sub>(318)</sub> – 445
4.05	11.5	1.556	0.422	63 <sub>(318)</sub> – 442
4.45	10.2	1.447	0.390	63 <sub>(318)</sub> – 439
4.85	9.1	1.351	0.359	63 <sub>(318)</sub> – 436
5.25	8.1	1.265	0.330	63 <sub>(318)</sub> – 433
5.65	7.2	1.189	0.306	63 <sub>(318)</sub> – 431
6.05	6.4	1.120	0.286	63 <sub>(318)</sub> – 429
6.45	5.8	1.058	0.275	63 <sub>(318)</sub> – 427
6.85	5.2	1.003	0.267	63 <sub>(318)</sub> – 427
7.25	4.6	0.953	0.255	63 <sub>(318)</sub> – 426
7.65	4.2	0.907	0.243	63 <sub>(318)</sub> – 424
8.05	3.7	0.865	0.227	63 <sub>(318)</sub> – 423
8.45	3.3	0.827	0.208	63 <sub>(318)</sub> – 421
8.85	3.0	0.792	0.188	63 <sub>(318)</sub> – 419
9	2.8	0.600	0.180	63 <sub>3</sub> – 418

Table 1: Blade geometry.

587 who studied a full-scale turbine prototype from TGL at different tip speed  
588 ratio  $\lambda$ , where the free stream velocity is uniform with  $U_\infty(z) = U_{\text{hub}}$  and  
589  $U_x = U_\infty(1 - a)$ . We compare the thrust and power coefficients in Fig. 12. The  
590 difference of the computed power coefficient with the CFD data from Gretton  
591 and Ingram [25] is always smaller than 3%, whereas the thrust differs by less

	Blade section	
	$r = 0.5R$	$r = 0.75R$
Mean angle of attack $\bar{\alpha}$ [deg]	9.58	5.96
Twist angle $\beta_0$ [deg]	10.17	5.39
Chord $c$ [m]	1.445	1.019
Foil type	NACA 63 <sub>(318)</sub> 439	NACA 63 <sub>(318)</sub> 427
Axial induction factor $a$ [-]	0.1835	0.3112
Tangential induction factor $a'$ [-]	0.0182	0.0164
Mean inflow speed $\bar{U}$ [m s <sup>-1</sup> ]	4.82	6.98
Blade length $R$ [m]	9	
Far field flow speed $U_{\text{hub}}$ [m s <sup>-1</sup> ]	2	
Hub height $z_{\text{hub}}$ [m]	20	
Pitch axis position $\hat{x}_A/c$ [-]	0.1	
Rotational speed $\omega$ [rad s <sup>-1</sup> ]	1	
Blade shell thickness [mm]	56.5 (root) - 14.5 (tip)	
Blade shell density $\rho_s$ [kg m <sup>-3</sup> ]	570 (root) - 660 (tip)	
Sea water density $\rho_w$ [kg m <sup>-3</sup> ]	1025	

Table 2: Simulation parameters.

592 than 2% over the considered range  $\lambda \in [3, 6]$ , including the optimal operating  
593 point  $\lambda_0 = 4.5$  where we study the performance of the morphing blade.

594 Next, we consider the accuracy of the model at predicting loads fluctuations  
595 when the turbine is subjected to unsteady flow. The predictions of our model  
596 are compared to the results of Scarlett *et al.* [3]. They developed a model  
597 based on blade element momentum theory coupled with a dynamic stall model,  
598 and they validated it against data from AeroDyn, an opensource aerodynamic  
599 software developed by NREL, and wind tunnel data. To aid the modelling  
600 of dynamic stall phenomena, Scarlett *et al.* [3] considered the blade made of



601 NREL S814 airfoils, since a large database of empirical dynamic stall parameters  
 602 is available for a series of NREL airfoils [27]. In particular, the NREL S814  
 603 generates a similar power coefficient to the NACA 63-418, which represents the  
 604 tip airfoil of the TGL turbine blades. Therefore, the comparison is made for  
 605 the lift experienced by the tip section ( $r/R = 1$ ), over 56 rotational periods.  
 606 In Fig. 13, we show the time history of the lift coefficient over 5 periods, and  
 607 also the predictions from Scarlett *et al.* [3] when the turbine with rigid blades  
 608 is subjected to the real flow conditions, measured at the EMEC tidal test site  
 609 on November 22nd 2014. Our results match qualitatively both the unsteady  
 610 and quasi-steady lift from Scarlett *et al.* [3]. The average discrepancies over  
 611 56 periods are 6% compared to Scarlett’s quasi-steady model, and less than  
 612 5% compared to the unsteady model, with maximum deviations of 18% and  
 613 40% respectively. Discrepancies with Scarlett’s quasi-steady model might be  
 614 partially due to geometric differences between the two airfoils, as the NREL  
 615 S814 has double curvature and 3% maximum camber, whereas the NACA 63-  
 616 418 has simple curvature and 2.2% maximum camber. Bigger differences are  
 617 noticed against Scarlett’s unsteady model, however, these are due to unsteady  
 618 phenomena and are noticed also between both of Scarlett’s models.

619 There is no experimental data for a turbine with a morphing concept such as  
 620 that discussed in this paper. Hence, the load predictions are validated for a rigid  
 621 blade and cannot be validated for the morphing blade. However, Dai *et. al* [28]  
 622 performed a 2-dimensional CFD investigation of the performance of a passively  
 623 pitching NACA 63-427, which corresponds to the airfoil at  $r/R = 0.75$  of the  
 624 turbine blade used in the current study. They considered a rigid and an airfoil  
 625 pitching passively around the axis at  $\hat{x}_A = 0.1c$  and subjected to a periodically  
 626 varying inflow speed representing a blade rotating in shear flow. The passive  
 627 pitch was modelled with an akin mass-damper-spring system to that in this  
 628 paper. The CFD predicted a reduction of the amplitude of the load fluctuations  
 629 within 7% of the predictions of the present analytical model (Fig. 14).

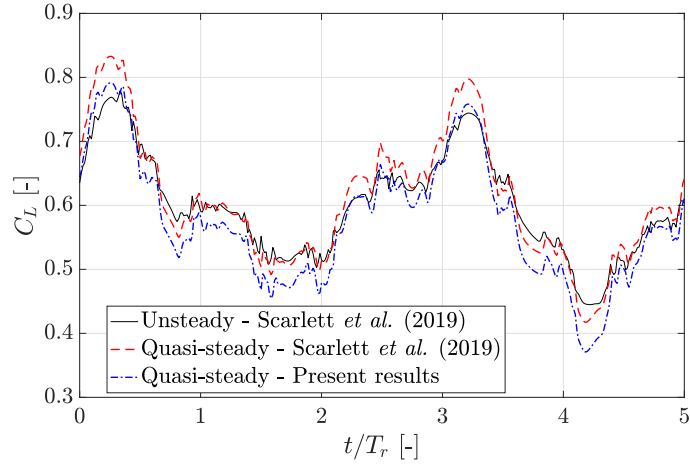


Figure 13: Comparison of lift coefficient between Scarlett *et al.* [3] unsteady results (black line), quasi-steady results (red dashed line), and present quasi-steady results (blue dashed-dotted line).

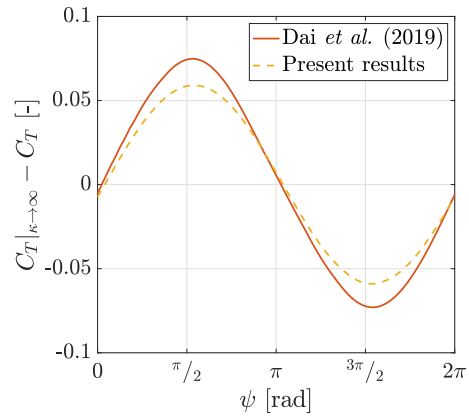


Figure 14: Comparison of CFD and analytical results showing the difference of  $C_T$  between rigid and morphing blades.

630 **6. Results**

631 *6.1. Quasi-steady Analysis of a Morphing foil*

632 The results for the passive pitch control applied to a 2D foil are presented  
 633 here. Figure 15 shows the evolution of  $\alpha(t)$  and  $C_T^{2D}(t)$  for the foil at  $r = 0.5R$   
 634 over the azimuthal coordinate  $\psi$  (Fig. 1), whereas Fig. 16 refers to the blade  
 635 section at  $r = 0.75R$ . Both cases show the fluctuations over one revolution  
 636 of the foils equipped with springs that are optimised to mitigate the thrust  
 637 fluctuations for the respective flow conditions (Table 3). The non-dimensional  
 638 stiffness  $\Gamma$  is defined as the ratio between spring stiffness  $\kappa$  and the hydrostatic  
 639 stiffness  $\rho_w c^2 \bar{U}^2$  (Eq. 24), and  $\Gamma_0$  is the optimal value. On both blade sections,  
 640 the thrust fluctuations are almost perfectly cancelled.

641 Similar performance can be achieved for any blade section as long as the  
 642 stiffness and preload are optimised for the flow condition that the section ex-  
 643periences. Therefore, if each section can be equipped with the optimal stiffness  
 644 and can deflect independently of the neighbouring sections, perfect unsteady  
 645 load cancellation is possible also for the full blade.

646 Such optimal behaviour is possible thanks to the angle of attacks that varies  
 647 in opposition to the fluctuations of the inflow angle. In fact when the inflow  
 648 angle  $\phi$  reaches a peak value  $\alpha$  is at a minimum, as observed by Shen *et al.* [29].

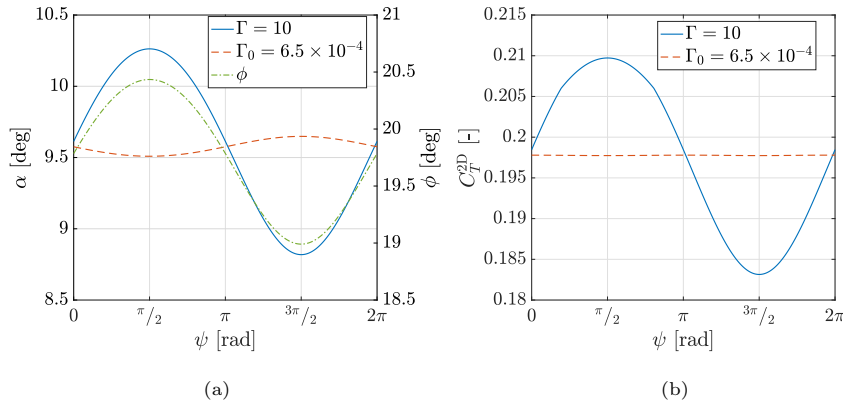


Figure 15: Section at  $r = 0.5R$ : evolution of angle of attack (a) and thrust coefficient (b).

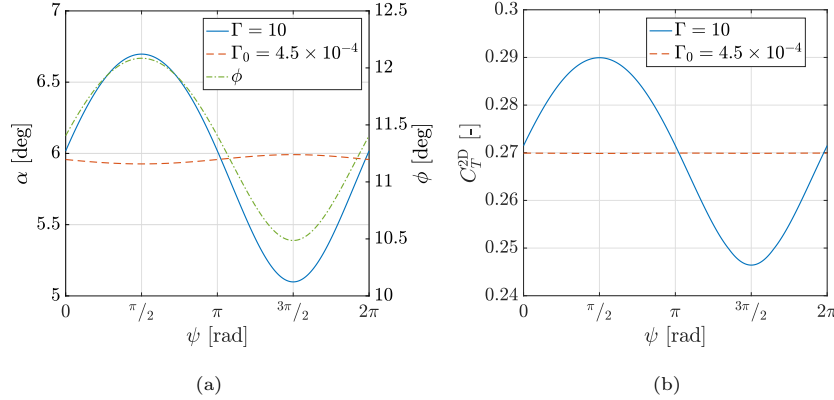


Figure 16: Section at  $r = 0.75R$ : evolution of angle of attack (a) and thrust coefficient (b).

Blade section	Optimal spring properties		Thrust fluctuations $\Delta C_T^{2D}$	
	$\Gamma_0$ [-]	$\theta_0$ [deg]	% mean $C_T^{2D}$	% reduction
$r = 0.5R$	$6.5 \times 10^{-4}$	132	0.03 (13.46)	99.74
$r = 0.75R$	$4.5 \times 10^{-4}$	233	0.02 (16.12)	99.85

Table 3: Minimum load fluctuations using optimal springs for two blade sections. The number in parenthesis refer to the respective rigid blade sections.

649

650 This behaviour is explained in Fig. 17. The loads on the foil change because  
651 of the changes in the angle of attack but also in the flow velocity. When the blade  
652 is in the upper position, the foil pitches in order to compensate the incidence  
653 increase, such that both  $\alpha$  and  $C_T^{2D}$  are restored to their initial values (Fig. 17b).  
654 However, the inflow speed increases as well causing higher hydrostatic moments  
655 which further pitch the foil (Fig. 17c). Essentially, the angle of attack is reduced  
656 to a value which is lower than its initial value to compensate the increase in  
657 the inflow speed. Similarly, when the blade is in the lower position, the foil  
658 pitches to a higher incidence in order to cancel the effect of the reduction in the

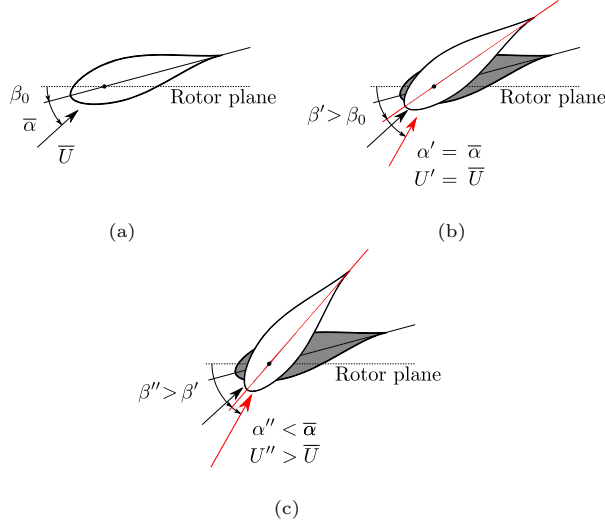


Figure 17: Reaction of the morphing foil to an increase of angle of attack and flow speed; (a) reference condition; (b) effect of the angle of attack only and (c) combined effect of the increase of both the angle of attack and flow speed.

659 incoming flow speed. Therefore, the spring that minimises the fluctuations of  
 660  $C_T^{2D}$  is different from the one that minimises the oscillations of  $\alpha$ . In particular,  
 661 Fig. 18 shows the amplitude of the fluctuations of  $C_T^{2D}$  (top) and  $\alpha$  (bottom)  
 662 for the blade section at  $r = 0.75R$ , for a range of values of  $\Gamma$ . It's shown that  
 663 a spring that minimises the fluctuations of  $\alpha$  is stiffer than the spring that  
 664 minimises the fluctuations of  $C_T^{2D}$ .

665 The results for 2D sections are summarised in Fig. 19, which shows the  
 666 evolution of  $C_T^{2D}$  and  $C_P^{2D}$  during the revolution of the blade section at  $r =$   
 667  $0.75R$  for different values of the non-dimensional stiffness  $\Gamma$ . The dotted lines  
 668 on Fig. 19a show the blade azimuthal position. The maximum value  $\Gamma = 10$   
 669 represents the condition of a rigid foil, where the spring becomes too stiff to  
 670 bend and the loads on the section are not effectively alleviated.

671 In Fig. 19b, the range of  $\Gamma$  is narrowed to focus the reader attention around  
 672 the optimal value  $\Gamma_0 = 4.5 \times 10^{-4}$  which is highlighted by the dotted line. The

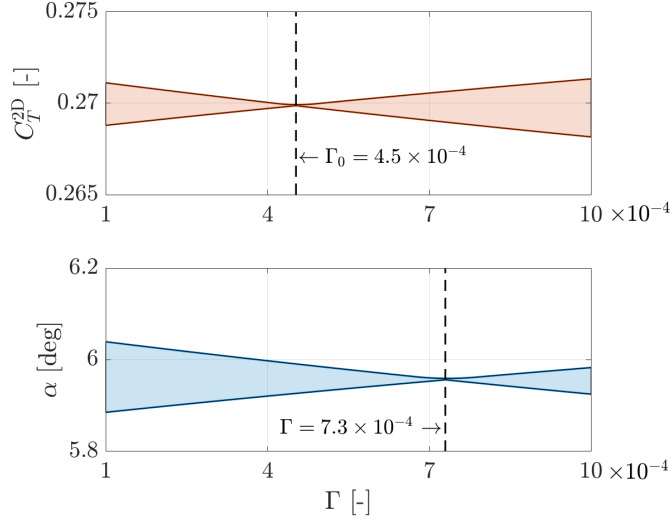


Figure 18: Fluctuations of  $C_T^{2D}$  (top) and  $\alpha$  (bottom) for the section at  $r = 0.75R$ , for a range of values of  $\Gamma$ .

673 optimal stiffness is represented by a slightly curved line. Therefore, the  $C_T^{2D}$  still  
674 exhibits small fluctuations throughout each revolution, although these fluctua-  
675 tions are negligible compared to the mean value and compared to fluctuations  
676 experienced by the rigid foil. The quasi-steady analysis shows that, for very low  
677 stiffness values, the variations of the thrust coefficient is reversed. The preload  
678  $\theta_0$  is inversely proportional to  $\Gamma$  (Eq. 20), which means that the lower the value  
679 of  $\Gamma$  the higher needs to be the preload in order to balance the hydrostatic mo-  
680 ment. Therefore, reducing the stiffness of the spring leads quickly to very high  
681 values of  $\theta_0$ .  $C_P^{2D}$  always oscillates around the same value whilst the amplitude  
682 of the oscillations is slightly reduced.

### 683 6.2. Quasi-steady Analysis of the 3D Morphing Blade

684 The analysis presented in Section 6.1 is extended to the entire blade. A  
685 spring with quasi-steady non-dimensional stiffness (Eq. 46)  $C_{\kappa}^{qs} = 6.0 \times 10^{-3}$   
686 and quasi-steady preload angle  $\theta_0^{qs} = 313$  deg leads to variations of the thrust

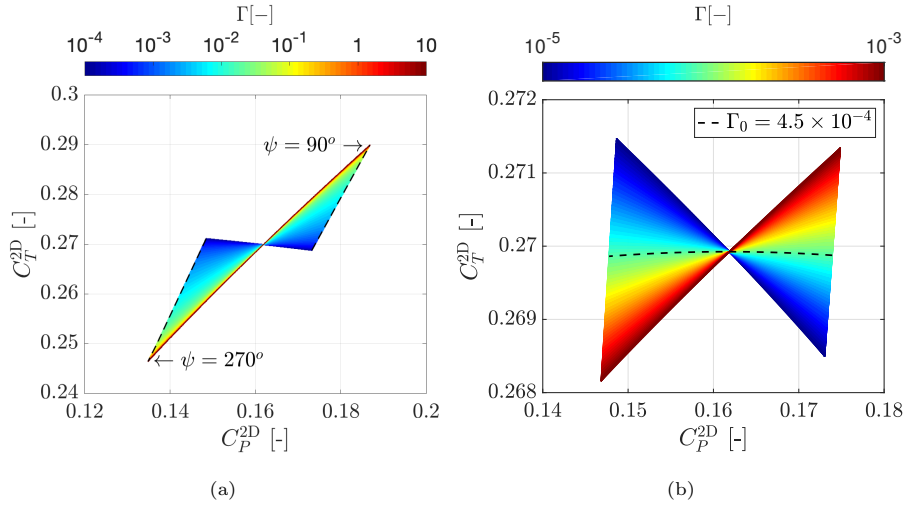


Figure 19: Variations of the power and thrust coefficients as the inflow angle varies through the rotation of the blade, (a) for a large range of values  $\Gamma \in [10^{-5}, 10]$  and (b) around  $\Gamma_0 = 4.5 \times 10^{-4}$ .

687 coefficient at each section that are negligible, whilst the mean power generated  
688 in one revolution is unaffected. Figure 20 shows the thrust fluctuations over the  
689 area swept by the blade for a rigid (Fig. 20a) and a morphing blade (Fig. 20b).  
690 In particular, we plot the thrust coefficient of the rigid blade  $C_T^{2D}|_{\kappa \rightarrow \infty}$  and of  
691 the optimal morphing blade  $C_T^{2D}$  divided by the maximum sectional thrust of  
692 the rigid blade. The load is concentrated in the outer half of the blade and  
693 it drops at the tips and towards the hub. The high thrust experienced by the  
694 blade when  $\psi = 90$  deg (Fig. 20a) is reduced thanks to the blade flexibility,  
695 whereas the lower thrust at  $\psi = 270$  deg is increased.

696 The effectiveness of the blade flexibility is finally checked for the flapwise  
697 blade root bending moment shown in Fig. 21. Since the changes in the amplitude  
698 of the thrust fluctuations for the blade in the upward/downward positions are of  
699 opposite sign, the bending moment is kept nearly constant over each revolution.  
700 The morphing blade experiences RBM fluctuations with amplitude smaller than  
701 0.5% of the average bending moment. As the amplitude of the fluctuations

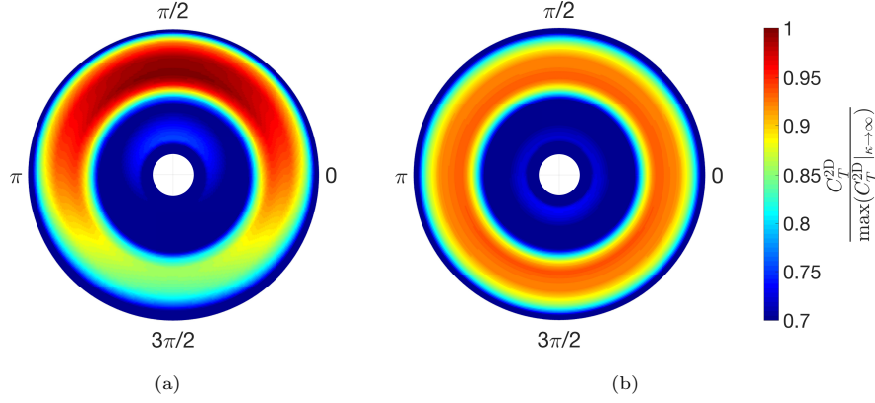


Figure 20: Map of the thrust coefficient over the rotor disc for (a) a rigid blade and (b) a morphing blade.

702 on the rigid blade is 18%, the morphing blade mitigated 98% of the RBM  
 703 fluctuations. Compared to the 2D analysis, the performance is slightly lower.  
 704 Each section experiences a different load fluctuation amplitude that requires a  
 705 different blade deflection. As the morphing blade is optimised in a global fashion,  
 706 it affects mainly the sections that experience stronger load fluctuations, and it  
 707 is thus sub-optimal for the sections that contribute the least to the blade loads  
 708 causing a performance reduction.

### 709 6.3. Dynamic Analysis of the 3D Morphing Blade

710 We study to what extent the dynamics of the blade and the unsteadiness  
 711 of the fluid loads affect the performances of the passive control system. From  
 712 Eq. 42, we consider the centrifugal moment  $M_c$ , the hydrodynamic moment  
 713  $M_{dy}$  and that due to the mechanical damping of the blade  $\mu\dot{\theta}$ . The blade non-  
 714 dimensional damping is initially set to  $C_\mu = 10^{-3}$  (Eq. 46), which represents  
 715 a negligible value. Its effect will be discussed in Section 6.4. Figure 22 shows  
 716 that the effectiveness of the morphing blade in reducing the RBM oscillations is  
 717 only marginally affected by the blade inertia and the unsteadiness of the flow.  
 718 Figure 23 shows that the blade inertia and the fluid-induced damping have a  
 719 small effect on the system effectiveness.



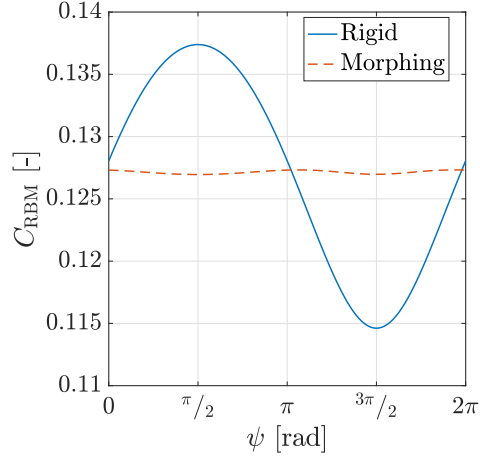


Figure 21: Quasi-steady analysis of the flapwise one-blade root bending moment  $C_{\text{RBM}}$ . The blade is equipped with a spring with  $C_{\kappa}^{\text{qs}} = 6.0 \times 10^{-3}$  and  $\theta_0^{\text{qs}} = 313$  deg.

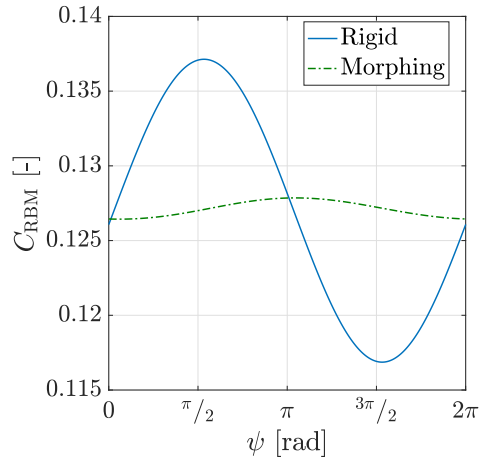


Figure 22: Dynamic analysis of the evolution of  $C_{\text{RBM}}$ . The blade is equipped with a spring with  $C_{\kappa}^{\text{dy}} = 1.1 \times 10^{-2}$  and  $\theta_0^{\text{dy}} = 171$  deg.

720        Accounting for the additional moments, we find that the optimal non-dimensional  
721        stiffness and preload obtained for the dynamic model are  $C_{\kappa}^{\text{dy}} = 1.1 \times 10^{-2}$  and  
722         $\theta_0^{\text{dy}} = 171$  deg. The optimal spring is stiffer than the one in the previous sec-

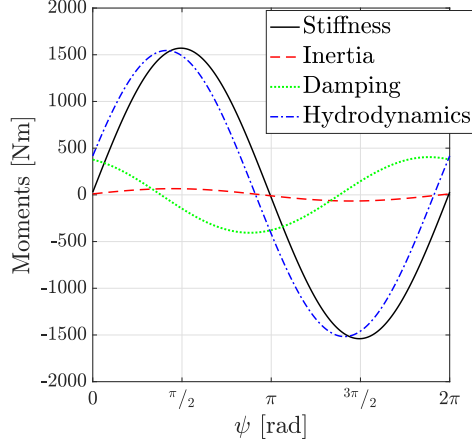


Figure 23: Dynamic contributions to the moment equilibrium of the blade, for  $C_{\kappa}^{dy} = 1.1 \times 10^{-2}$  and  $\theta_0^{dy} = 171$  deg.

723 tion because of the presence of unsteady loads. Table 4 shows the reduction in  
724 the bending moment fluctuations. As the optimal spring for the dynamic case  
725 is different than the optimal spring obtained for the quasi-steady model, it is  
726 difficult to understand whether the difference in the performance are due to a  
727 different spring or to dynamic effects. For ease of comparison, in Table 4 we  
728 display two sets of results for the dynamic model and for the quasi-steady one  
729 (in parenthesis), both using the same spring optimised for the dynamic model.  
730 The performance estimated with the quasi-steady model is slightly lower than  
731 before as the spring is suboptimal. The system dynamics affect the blade perfor-  
732 mance only marginally, as the reduction of RBM fluctuations is only 5% lower  
733 compared to the best quasi-steady performance previously shown. It must be  
734 noted that the mean power generated is the same as for the rigid blade.

#### 735 6.4. Effect of Blade Mechanical Damping

736 In this section, we consider the effects of the mechanical damping  $\mu$ . Figure  
737 24 shows the reduction of the fluctuations of the bending moment coefficient for  
738 a range of values of the stiffness coefficient  $C_{\kappa}$  and of the damping coefficient

	Optimal spring		RBM fluctuations $\Delta C_{\text{RBM}}$	
	$C_{\kappa}^{\text{dy}}$ [-]	$\theta_0^{\text{dy}}$ [deg]	% mean $C_{\text{RBM}}$	% reduction
Rigid	–	–	16(18)	–
Morphing	$1.1 \times 10^{-2}$	171 (177)	2.6 (< 1)	93 (96)

Table 4: Optimal dynamic results for the morphing blade model. The numbers within the parenthesis refer to the quasi-steady analysis that employed the spring optimised for the dynamic case.

739  $C_{\mu}$ . In particular, the amplitude of the bending moment fluctuation  $\Delta C_{\text{RBM}}$  is  
740 divided by the amplitude of the fluctuations for the rigid blade  $\Delta C_{\text{RBM}}|_{\kappa \rightarrow \infty}$ .

741 The optimal working point for each curve is their minimum. When  $C_{\mu}$  is  
742 large compared to the hydrodynamic damping, the efficacy of the morphing  
743 blade is poor. Conversely, for  $C_{\mu} = 1$  the system performance improves consid-  
744 erably, and the fluctuations of the blade root bending moment are reduced by  
745 98%. Increasing the damping, the lowest point of each curve moves to higher  
746 values of  $C_{\kappa}$ , hence the optimal performance of the passive control system is  
747 achieved with stiffer springs. For high values of  $C_{\kappa}$ , all the curves in Fig. 24  
748 collapse and converge asymptotically to 1. The system is not very sensitive to  
749 small changes in the spring stiffness, as the reduction in the  $\Delta C_{\text{RBM}}$  does not  
750 vary significantly near the optimum.

751 The blade does not experience any resonance phenomena, and, even for  
752 negligible mechanical damping  $C_{\mu}$ , the fluid acts as a damper, the motion of  
753 the blade is effectively damped. In fact, by considering the blade damping as a  
754 lumped parameter, it is possible to define a damping ratio as

$$\zeta = \frac{C_{\text{tot}}}{2\sqrt{J_{\text{tot}}K_{\text{tot}}}}, \quad (47)$$

755 where  $J_{\text{tot}}$ ,  $C_{\text{tot}}$  and  $K_{\text{tot}}$  represent the inertia, the damping and the stiffness  
756 of the system respectively. Similarly, we define the natural frequency of such  
757 system as

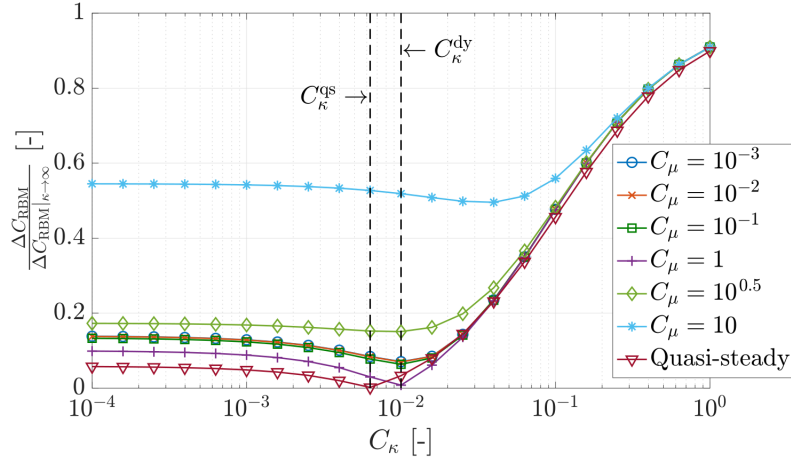


Figure 24: Normalised amplitude of the root bending moment for a range of values of  $C_{\kappa}$  and  $C_{\mu}$ .

$$\Omega_N = \sqrt{\frac{K_{\text{tot}}}{J_{\text{tot}}}}, \quad (48)$$

758 where  $J_{\text{tot}}$  is the sum of all the inertial terms in Eq. 43,  $C_{\text{tot}}$  is the sum of all the  
759 terms that multiply  $\dot{\delta}\theta$  and  $K_{\text{tot}}$  is the sum of the terms that multiply  $\delta\theta$ , the  
760 damping ratio describes the coupled mechanical-hydrodynamic system. For the  
761 range of values of  $C_{\kappa}$  considered, the natural frequency of the coupled system  
762 is always greater than 2.8 rad/s. Since the frequency of the load fluctuations  
763 is  $\omega = 1$  rad/s, the blades are likely to experience no resonance. The damping  
764 ratio  $\zeta$  changes with the spring stiffness, and at the minimum of each curve, it  
765 always takes values greater than 0.4. Therefore, the hydrodynamic contribution  
766 alone dampens the motion of the blade substantially. The effect of the damping  
767 can be observed in Fig. 25, which shows the blade motion amplitude  $\Delta\theta$  over  
768 the same range of  $C_{\kappa}$  and  $C_{\mu}$ . All curves converge for stiff springs, for which  
769  $\Delta\theta$  tends to zero. For very flexible springs,  $\Delta\theta$  is never higher than 2 deg and  
770 it decreases for increasing damping values. On the other hand, the preload  $\theta_0$   
771 is inversely proportional to the spring stiffness and increases rapidly from zero  
772 to more than  $10^4$  deg for  $C_{\kappa} < 10^{-4}$ .

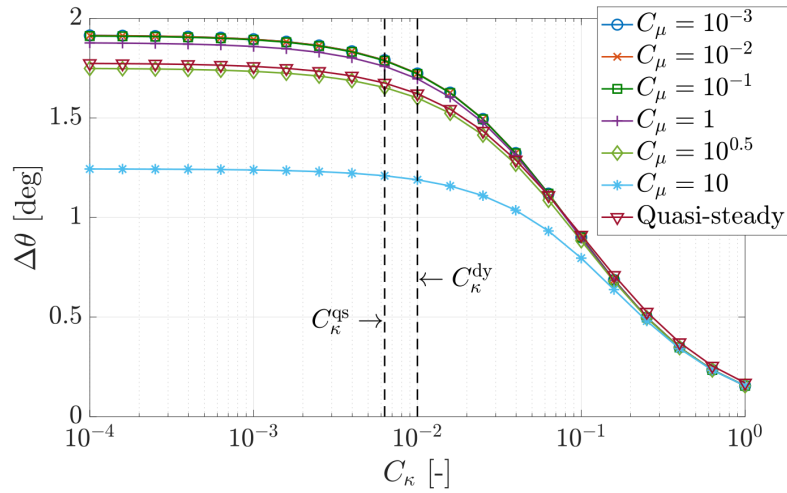


Figure 25: Evolution of  $\Delta\theta$  for a range of values of  $C_\kappa$  and  $C_\mu$ .

773 6.5. Morphing blade performance in real flow conditions

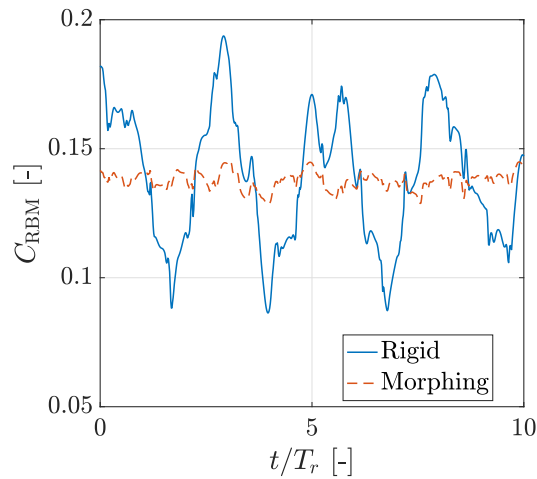


Figure 26: Root bending moment under real flow conditions for a rigid blade (blue line) and a morphing blade (red dashed line).

774 The performance of morphing blades is estimated again, this time using

775 realistic inflow conditions, measured during a flood tide at the EMEC tidal  
776 testing site on the 22nd of November 2014. Figure 26 shows the root bending  
777 moment coefficient  $C_{\text{RBM}}$  simulated over 10 rotational periods, for a rigid blade  
778 and a morphing blade that has been optimised to minimise the fluctuation  
779 of  $C_{\text{RBM}}$ . Using a spring with quasi-steady non-dimensional stiffness  $C_{\kappa}^{\text{qs}} =$   
780  $5.1 \times 10^{-2}$ , and quasi-steady preload angle  $\theta_0^{\text{qs}} = 32$  deg, the fluctuations of the  
781 root bending moment are reduced by 82%, whilst the average power coefficient  
782 has decreased by 2%. Results are resumed in Table 5.

	Optimal spring		RBM fluctuations $\Delta C_{\text{RBM}}$	
	$C_{\kappa}^{\text{qs}}$ [-]	$\theta_0^{\text{qs}}$ [deg]	% mean $C_{\text{RBM}}$	% reduction
Rigid	—	—	101	—
Morphing	$5.1 \times 10^{-2}$	32	18	82

Table 5: Optimal morphing blade model when subjected to real flow conditions.

783 The efficacy of the morphing blade to mitigate the fluctuations induced by  
784 real inflow conditions is slightly lower than that shown in Sections 6.2 and 6.3 for  
785 a modelled shear flow. The shear flow induces periodic fluctuations at the blade  
786 rotational frequency, whereas the real flow conditions induce load fluctuations  
787 over a wide range of frequencies. Since the load fluctuations are dominated by  
788 the high-amplitude, low-frequency waves, the optimal morphing blade is tuned  
789 to operate around that frequency. Therefore, the system is not as effective at  
790 alleviating the high-frequency, low-amplitude fluctuations, as shown in Fig. 26.

791 We have shown that morphing blades are capable of alleviating unsteady  
792 load fluctuations caused by shear flow and large waves. In our model, flow  
793 fluctuations are experienced by the blade as oscillations of the inflow speed and  
794 of the angle of attack, and a morphing blade mitigates the loads by inverting  
795 the sign of the angle of attack variations. The mechanism underlying load  
796 mitigation by morphing blade is independent of the flow condition. Hence, we  
797 believe that similar performance can be achieved for a wide range of unsteady

798 flow conditions, including turbulence, yaw misalignment, wakes of upstream  
799 turbines, and the proximity of other devices. As the frequency of the fluctuation  
800 increases, such as for small-scale turbulence, the inertia of the blade leads to  
801 a reduction of the system efficiency. Moreover, the model is not capable to  
802 consider the effect of high-frequency flow fluctuations with a period of the order  
803 of  $c/(\omega R)$ , such that the flow cannot be considered uniform along the chord. In  
804 such cases, a more sophisticated model should be used.

## 805 **7. Conclusions**

806 In this paper we discuss the underlying mechanism by which morphing blades  
807 can mitigate unsteady load fluctuations on tidal turbines.

808 By neglecting mass and damping effects, and by further assuming indepen-  
809 dent blade sections at any spanwise location, we show that a morphing blade  
810 can completely cancel the thrust fluctuations ( $> 99\%$  reduction) without affect-  
811 ing the mean torque and thus the energy harvested. This is possible because,  
812 when a gust occur, the increased flow speed makes the blade to pitch and this  
813 result in a reduction in the angle of attack. The optimum blade flexibility is  
814 such that the load increase due to the higher flow speed is cancelled by the load  
815 reduction due to the lower angle of attack.

816 The condition that each blade section deflects independently of those at other  
817 spanwise locations is not critical for the effectiveness of the system. The opposite  
818 limiting condition is when all blade sections must pitch by the same amount,  
819 which is saying that the blade is rigid and has a passive pitching mechanism at  
820 the root. In this case, we show that the root bending moment fluctuations are  
821 decreased by more than 98% for a 18 m diameter, 1 MW turbine in a sheared  
822 tidal current operating at a tip speed ratio of 4.5.

823 The effectiveness of the system is partially decreased by both the mass and  
824 damping of the system.

825 Accounting for the unsteady hydrodynamics effects and the blade inertia,  
826 the root bending moment fluctuations are reduced by more than 93%. Their

827 effect, however, depends on the onset flow conditions. Hence, we also model  
828 the morphing blade in the tidal flow conditions measured at the EMEC site  
829 with more than 4 m significant wave height with a period of 10 s. In these  
830 conditions, the morphing blades enable a root bending moment reduction of  
831 more than 82%. Hence, we conclude that unsteady hydrodynamic effects and  
832 blade inertia are important, but do not undermine the general effectiveness of  
833 the morphing blades.

834 On the other hand, a high mechanical damping can undermine the ability to  
835 mitigate unsteady loads. The damping depends on the design of the morphing  
836 blade and can be due to the viscous dissipation of the flexible material, or on  
837 the mechanical damping of a passive pitch mechanism. Our parameter study  
838 can be used by future researchers to estimate the unsteady load mitigation of  
839 different blade design concepts.

840 This conceptual study does not consider the practicalities arising from the  
841 design and manufacturing of a morphing blade. Before this technology can be  
842 adopted by the industry, more research is needed to address the need for a  
843 new structural design, to identify the materials that can be adopted, and the  
844 associated manufacturing and the supply chain. Since tidal turbine blades are  
845 typically made of a composite structure reinforced with a boxspar, the use of a  
846 novel structure and materials will require extensive testing and characterisation  
847 to guarantee reliability and survivability of the blade. Overall this paper con-  
848 tributes by providing insights on the underlying mechanism of morphing blades  
849 and the blade behaviour that future designs need to achieve.

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853 **References**

854 **References**

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