A Hybrid Model of Machine Learning Model and Econometrics' Model to Predict Volatility of KSE-100 Index

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ABSTRACT

Purpose:
The purpose of this paper is to predict the volatility of the KSE-100 index using econometric and machine learning models. It also designs hybrid models for volatility forecasting by combining these two models in three different ways.

Methodology:
Estimations and forecasting are based on an econometric model GARCH (Generalized Auto Regressive Conditional Heteroscedasticity) and a machine learning model NNAR (Neural Network Auto-Regressive model). The hybrid models designed with GARCH and NNAR include GARCH-based NNAR, NNAR-based GARCH, and the linear combination of GARCH and NNAR.

Findings:
In a comparison of the forecasting results of the KSE-100 index over different periods, the least RMSE is found in a linear combination of NNAR and GARCH, followed by NNAR, GARCH, NNAR based GARCH, and GARCH based NNAR models.

Conclusion:
The study concludes that the hybrid model designed with a linear combination of GARCH and NNAR performs better among all the models in forecasting the volatility of the KSE-100 index.

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1. Introduction

The stock market prediction has always been an interesting area for both researchers and investors. Investing in a stock market is one of the most crucial decisions that an investor makes. Before investing in the stock market, decisions are made using multiple mathematical and statistical models that are used to forecast the prices of shares in the upcoming time. Based on this forecasted return, decision-making is done before any trade. But only the prediction of return is not enough in heavy investments, but the risk factor is also required to be forecasted to make better decisions and to do profitable trading. Stock market traders are always eager to know the expected return as well as the expected risk or volatility before taking exposure to the market. Most of the traders in a market are risk avert that aim to have maximum return with minimum possible risk. By predicting volatility, traders can estimate the expected risk. They prefer investing in the asset that bears the least possible risk keeping the return constant and trying to hedge the expected risk. Thus, predicting risk is one of the most significant needs of informed traders when making trading decisions (Huy, D.T.N., & Hang, N.T. 2021).

In the stock market, the future expected value and standard deviation of a financial time series are forecasted to predict the future price and volatility of any particular stock to gain maximum profit and avoid losses. The volatility of a time series is calculated by standard deviation or variance, otherwise, of the expected errors (Hang, N. T., & Nam, V. Q. 2021). It represents the error/white noise of a series and therefore it has very significance in time series analysis. It represents a degree of variation from a mean value at any point of a series. In financial time series, it is quite significant as it represents the risk factor. Volatility modeling provides a simple approach to calculating the value at risk of a financial position. Financial time series also hold the characteristics of time-dependent volatility or heteroscedasticity, which makes it more relevant to forecast. The unpredictable nature of volatility causes heteroscedasticity in a time series (Somarajan, S. et al 2019). It also has a characteristic of volatility clustering; high volatility followed by higher volatility and vice versa. It gradually changes its pace from higher to lower and vice versa, thus showing a tendency of auto-regression, current values depending upon past values. To estimate the heteroscedasticity of a time series, various techniques are used. There were multiple methods used to predict expected return but there was no such statistical model for predicting volatility before the ARCH model (Engle, R. 2001).

As heteroscedasticity is autoregressive, classical time series models can be used to predict future volatility. Among those, econometrics models have been widely used to predict the volatility of financial time series. GARCH-Generalized Auto-Regressive Conditional Heteroscedasticity is one of those econometrics' models that can be used to predict the future volatility of time series. GARCH model is a stochastic model that is widely used for forecasting the volatility of time series. It is an extension of the ARCH (Auto-regressive Conditional Heteroscedasticity) model that deals with the conditional volatility of any heteroscedastic time series. The underlying assumption of this model is stationarity. The mean and variance of a series must be time-invariant to implement the GARCH model, but the conditional variance is time-variant, or we can say that the series may contain volatility clusters. As volatility is autoregressive so it is considered an efficient way to predict it.

On the other side, machine learning modeling is another approach to predicting the volatility of the financial time series. But by the time when machine learning techniques have been used widely in almost every domain, several Machines were used to study time series or sequential data like neural network models, decision tree, or support vector machines. The decision learning algorithms are tree is a classification model that can be used for the classification of time series data (Brunello, A., et al 2019). As this research aims to forecast the volatility of time series for different periods regression on models is selected. Machine Learning algorithms can be used for the prediction of the volatility of financial time series like neural network models can be used to forecast future volatility. NNAR-Neural Network Auto-Regressive model is a neural network model that is particularly used for time series that have autoregressive nature. It uses the linear combination of past values to predict future values.

Another approach is using a hybrid model, which is a combination of two or more models that are combined to obtain better predictive results. The algorithms assembled in the hybrid models can either be the two or more machine learning models, two or more econometric models, or can be a combination of both machine learning and econometric model. These models are designed to achieve higher accuracy and precision as they combine the accuracy of multiple different models. The combined models have enhanced predictivity than the individual models (Hajirahimi, Z., & Khashei, M. 2022). The accuracy of the hybrid model depends on the accuracy of models that are being used in the construction of the hybrid model.

For this paper, the KSE-100 index is considered, the representative index of the stock market of Pakistan. This research aims to estimate the volatility of the KSE-100 index by using GARCH, NNAR, and a hybrid model designed by using the two models. The research includes five different models altogether; GARCH, NNAR, and three hybrid models that are designed by combing NNAR and GARCH using three different techniques. The research further extends by comparing the predictivity of volatility from all the models.

From the literature review, it was found that the hybrid models using NNAR and GARCH have never been designed to predict the volatility of the KSE 100 index.

2. Literature Review

GARCH model is a stochastic model that is widely used for forecasting the volatility of time series. It is an autoregressive model as it allows the conditional variance to change with time as a function of previous errors and volatility. It is an extension of the ARCH (Auto-regressive Conditional Heteroscedasticity) model that deals with the conditional volatility of a heteroscedastic time series, and it considers conditional variance only as a function of past errors (Bollerslev, T. 1986). GARCH is an econometric model which is especially useful in analyzing the volatility of financial time series, such as stock prices, inflation rates, and exchange rates. A distinguishing feature of this model is that the error variance is correlated over time because of the phenomenon of volatility clustering which means that the large and small errors tend to cluster together (McNees, S. 1973). This allows, in addition to the average value of the studied parameters, to model the dynamics of its variance simultaneously. Therefore, such a model can correctly describe phenomena such as clustering of volatility and asymmetric information (Tsay, R. S. 2005). The underlying assumption of this model is stationarity. The mean and variance of a series must be time-invariant to implement the GARCH model, but the conditional variance is time-variant, or we can say that the series may contain volatility clusters (Tsay, R. S. 2005; Satchell, S., & Knight, J. 2011). As this model deals with the volatility of time series, it makes certain assumptions about volatility that sometimes are beyond the nature of time series. Therefore, the accuracy of the model may compromise. Say, for instance, the GARCH model assumes the same effects of positive and negative shocks on the volatility of time series. But in real-world data negative and positive shocks respond to the volatility in a different manner. Furthermore, as the model uses previous values for future predictions, the error increases for the prediction of a longer period. In a simple word, we can say that the number of days to be predicted and the error has a direct relation. In the

GARCH model, the conditional variance is converged towards the unconditional variance which is why the predictions of GARCH are more relevant in short term (Hamilton 1994).

Kanasro et al. found a high presence of volatility in the KSE-100 index in a clustering manner and with stochastic nature. It is analyzed using ARCH and GARCH models (Kanasro, H. A., et al 2009). M Fakhfekh, A Jeribi compared five different GARCH models to predict the volatility of most popular cryptocurrencies and found that Threshold GARCH (TGARCH) is the best model among them to predict volatility (Fakhfekh, M., & Jeribi, A. 2020). An artificial Neural network (ANN) is a machine learning model that can be used in various fields. ANN works on the working behavior of the human brain; it stores events and learns from data. It designs the same neural network just like a neural network of the human brain. Therefore, it is named as artificial neural network (Basheer, I. A., & Hajmeer, M. 2000). Neural Network Auto-Regressive (NNAR) model is a neural network model that uses lagged values of a time series as an input and gives the predicted values for the future as an output. It feeds a forward neural network with three layers; an input layer, a hidden layer, and an output layer (Maleki, A.et al 2018).

Khan et al. used GRNN-Generalized Regression Neural Network and simple and exponential moving average models to study the forecasting ability of these models to predict the KSE-100 index and found that artificial neural network is better in dealing with the behavior of stock prices (Khan, M. A., et al. 2017). Islam and Nguyen presented a comparative analysis of three models for predicting stock price; ARIMA, ANN, and Geometric Brownian motion, and found that for next-day prediction conventional statistical and stochastic models are better than ANN (Islam, M. R., & Nguyen, N. 2020). Hybrid modeling is another mathematical technique to design models for desirable outcomes. Hybrid models or ensemble models are formed by combining two or more models. The models can be combined in multiple ways to form a new model. These models can be stochastic, statistical, or machine learning models (Khashei, M., & Bijari, M. 2012).

Mergani and Khairalla have designed a hybrid model from a statistical model and ANN and concluded that the hybrid model is better than the two themselves in predicting returns of financial time series (Khairalla, M., & AL-Jallad, N. T. 2017). Josip Arnerić, Tea Poklepović, and Zdravka Aljinović have been worked on a hybrid model of GARCH and ANN for predicting exchange rates using daily closing prices of the Zagreb Stock Exchange and found the hybrid model much more accurate than individual models (Arnerić, J., 2014). Monfared, Almasi and Enkehave also worked on the hybrid model in 2014 in which they built a GARCH-based ANN model and ANN-based GARCH model, for predicting the volatility of the market for the next two months (Monfared, S. A., & Enke, D. 2014). Fatima et al., designed hybrid financial systems using ARIMA, ANN, and ARCH/ GARCH for the forecasting risk and return of the KSE-100 index and found that ANN ARCH/ GARCH is a better predictive model for the KSE-100 index than simple ANN and ANN ARIMA (Fatima S, Hussain G. 2006).

Škorna, B. Š. & Tomanová, P. ensemble GARCH, and ANN to predict the volatility of S&P 500 to have better predictions in a sudden downturn and relative stability (Škorna, B. Š., & Tomanová, P. 2020). Koo, Eunho, and Kim designed a hybrid model of GARCH and LSTM along with volume up distribution from original volatility to predict the volatility of the stock market and obtained 21.03% performance gain in comparison with the already designed hybrid model of GARCH and LSTM (Koo, E., & Kim, G. 2022). In contrast with the available related work, the novelty of this research is that it aims to design a better predictive model to forecast the volatility of the KSE-100 index by combining GARCH and NNAR as NNAR-based GARCH, GARCH-based NNAR, and a linear combination of NNAR and GARCH.

3. Research Methodology

3.1. Data Set

The data used in this paper is a time-series data of daily closing values of the KSE-100 index from January 2000 to July 2019 collected from yahoo finance. It is split into two parts, test and train. From 2000 to 2018, data is used for training purposes. The remaining data from January 2019 to July 2019 is taken as test data. For this research, daily volatility is required, which is calculated from daily log returns calculated from the closing price.

3.2. Pre-Testing

The closing values of KSE-100 form a non-stationary time series, observed from its graph Figure.1. The graph of closing price shows an increasing trend. Since there is a trend in the graph, it shows that the series is not stationary.

To convert it into stationary series, the log difference was calculated using the formula mentioned in equation (2).

$$r_t = \ln\left({}^{y_{t-1}}/_{y_t}\right) r_t = \ln\left({}^{y_{t-1}}/_{y_t}\right) \tag{2}$$

Where, $r_t r_t$ is return in period 'tt' and subscript $p_{t-1} p_{t-1}$ and $p_t p_t$ are the prices at time '$t-1t-1$' and 'tt' respectively.

Dickey-Fuller test is used for checking the stationarity of a time series (Tsay, R.S. 2005). On performing this test, it is found that the log return is stationary. Log return in Figure 2: Log closing price of the KSE-100 index shows stationarity with no trend and seasonality.

Figure.1. Daily Closing Price of KSE-100 Index from January 2000 to July 2019
Source: Daily closing price collected from yahoo finance

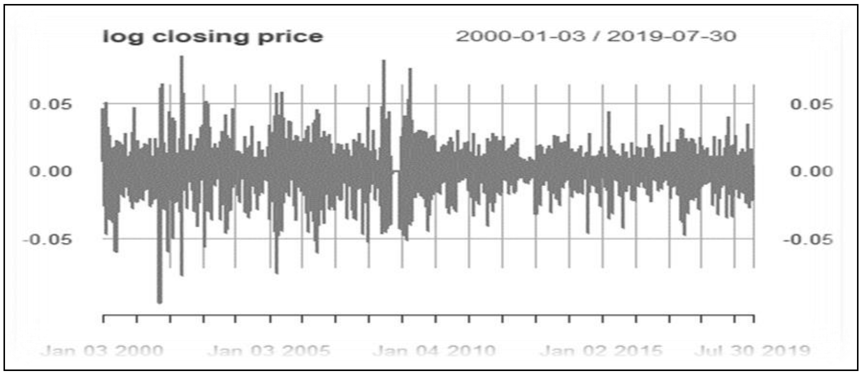

Figure.2. Log Closing Price of KSE-100 Index
Source: Author's own elaboration

From Figure 2, it can be observed that the series is more volatile from 2000 to 2010 and is comparatively less volatile from 2010 to 2019. Besides, there are clusters visible in the figure. It indicates that the variance changes over time.

3.3. Volatility of KSE-100 Index

Volatility is estimated by using the estimated error in the return series. The model used to estimate returns and consequently the errors, is Autoregressive Moving Average (ARMA). ARMA is applied on stationary time series. Equation 3 shows the ARMA (p, q). In order to apply the ARMA model, the optimal order of ARMA is identified using Auto Correlation Function (ACF) and Partial Auto Correlation (PACF) plots. Volatility is then calculated by calculating the absolute estimated error (Lin, F.n.d.). Figure 3 shows the absolute values of estimated error (volatility) of return on the KSE-100 index from 2000 to 2019.

$$r_t = \propto_0 + \sum_{j=1}^{p} \propto_j r_{t-j} - \sum_{k=1}^{q} \beta_k a_{t-k} + a_t$$

$$r_t = \propto_0 + \sum_{j=1}^{p} \propto_j r_{t-j} - \sum_{k=1}^{q} \beta_k a_{t-k} + a_t \tag{3}$$

Where $r_t r_t$ is return at time '*t*', $\propto_0, \propto_j$ and $\beta_k \propto_0, \propto_j$ and $\beta_k$are parameters and $a_t a_t$ is white noise at time '*t*'.

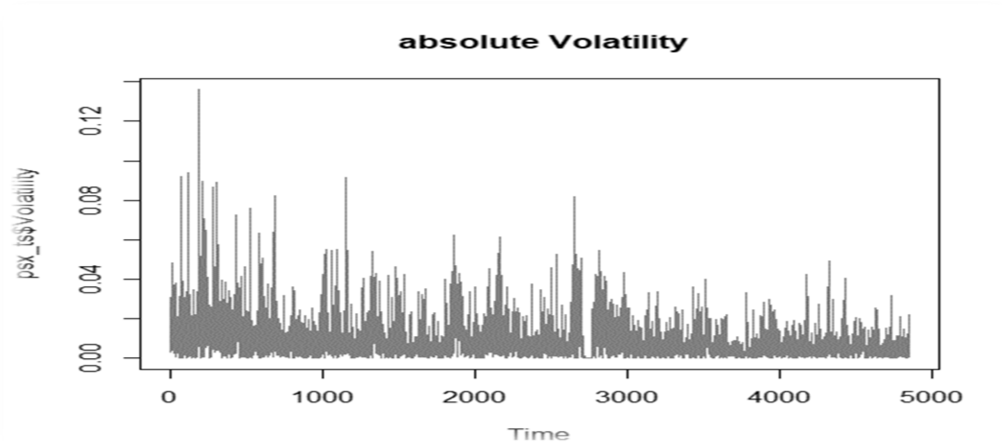

Figure 3: Daily Volatility
Source: Author's own elaboration

3.4. GARCH on KSE-100 Index

GARCH model is applied to the volatility, the series shown in Figure 3. The optimal model of GARCH for the series is determined by AIC (Akaike Information Criteria). Equation 4 shows how GARCH (m, s) can be used to estimate the volatility of the series.

$$a_t = \sigma_t \in_t , \qquad \sigma_t^2 = \propto_0 + \sum_{i=1}^{m} \propto_i a^2{}_{t-i} + \sum_{j=1}^{s} \beta_j \sigma^2{}_{t-j} \tag{4}$$

Where $a_t a_t$ is white noise at time 't', $\propto_0, \propto_i$ and $\beta_j \propto_0, \propto_i$ and $\beta_j$are parameters, $\sigma^2{}_t \sigma^2{}_t$ is conditional variance and $\in_t \in_t$ is variable having standard normal distribution (McNees, S.S. 1973).

3.5. NNAR on KSE-100 Index

The Neural Network Auto-Regressive model is a feed-forward neural network. Since the volatility of a time series is autoregressive in nature this Neural Network is used to forecast volatility. NNAR is a three-layered model with one input layer, one hidden layer, and one output layer. This model involves a linear combination and an activation function (Oh, K. J., et al 2011). The mathematical representation of a linear function is given in equation 5. The activation function is a sigmoid function, mentioned in equation 6.

$$y_t = w_0 + \sum_{j=1}^{h} W_j \cdot g\left(w_{0,j} + \sum_{i=1}^{n} w_{i,j} \cdot y_{t-i}\right) + \in_t$$

$$y_t = w_0 + \sum_{j=1}^{h} W_j \cdot g\left(w_{0,j} + \sum_{i=1}^{n} w_{i,j} \cdot y_{t-i}\right) + \in_t \tag{5}$$

$f(x) = \frac{1 \quad 1}{1+e^{-x}1+e^{-x}}$

(6)

As in the data preparation phase, the time series has already been made stationary; hence it seems not to contain any seasonal effect. However, an optimal number of seasonal and non-seasonal lags is used to design an efficient model to predict the volatility of time series.

3.6. Hybrid Models of GARCH and NNAR

As previous literature proves that hybrid models enhance the forecasting ability of the individual models so three different hybrid models are designed (Taskaya-Temizel, T. et al 2005). To design a model that contains characteristics of a machine learning model and an econometric model, hybrid models are designed to predict the volatility of the KSE-100 index. In this regard, three methods are used to ensemble GARCH and NNAR and then tested to determine the efficient one.

3.7. Linear Combination of NNAR and GARCH

The hybrid model is designed by using simple regression that is applied to the outputs of the GARCH and NNAR models. The idea of linear combination came from (Khairalla, M., 2017) in which three different models were combined linearly. After the implementation of simple regression, the significance of each parameter is checked, and only significant parameters are used in the prediction model. Mathematically the designed model is expressed in equation (7).

$$V_t = w_1 + w_2 V_{NNAR} + w_3 V_{GARCH} V_t = w_1 + w_2 V_{NNAR} + w_3 V_{GARCH} \tag{7}$$

Where $V_t V_t$ is predicted volatility from GARCH and NNAR. $V_{NNAR} V_{NNAR}$ and V_{GARCH} V_{GARCH} are predicted volatilities from NNAR and GARCH respectively. $w_1, w_2 \text{ and} w_3$ $w_1, w_2 \text{ and} w_3$are the parameters.

3.8. NNAR Based GARCH

In this form of a hybrid model, firstly NNAR is applied to actual volatility. The model is then used to predict volatility. The actual volatility is then compared with the predicted volatility obtained from NNAR. The absolute difference between actual and predicted volatility is the error of the NNAR model and this error is a new time series that is taken as an input of the GARCH model. GARCH model is then applied to that to estimate the error of NNAR. The estimated error of GARCH (output of GARCH) and predicted volatility of NNAR (output of NNAR) is added to get the final volatility which is considered the output of the hybrid (NNAR-based GARCH) model. The actual volatility is then compared with the predicted volatility of the hybrid model to compute the error of the hybrid model and to check its efficiency.

3.9. GARCH Based NNAR

In this form of a hybrid model, firstly GARCH is applied to the volatility series of the KSE-100 index. The forecasted volatility from this model (output of GARCH) is compared with actual volatility to compute the error of the GARCH. This error of GARCH is then used as an input of NNAR. NNAR model is applied to that to estimate the error of GARCH. The estimated error (output of NNAR) and predicted volatility (output of GARCH) are added, and a new measure of volatility is computed that is considered as the result of this hybrid model. In simple words, by adding the predicted volatility of GARCH and predicted error of NNAR we get the overall output volatility of the hybrid model. This output is compared with the actual volatility of the KSE-100 index to calculate the error of the hybrid model to analyze its efficiency.

4. Results and Findings

4.1. GARCH Model

ARMA (1, 9) is found as the best order to calculate the mean of the series. Table 1 shows the result of the ARMA (1, 9) model on the log daily closing price of the KSE-100 index. The optimal order of the ARMA model is found using ACF and PACF plots.

Table.1. Result of ARMA (1,9)

Coefficient	Value of Coefficient	Standard Error
ar1	0.7879**	0.0977
ma1	-0.6960**	0.0984
ma2	-0.0355	0.0201
ma3	0.0081	0.0182
ma4	-0.0092	0.0182
ma5	0.0046	0.0176
ma6	-0.0148	0.0179
ma7	0.0319	0.0176
ma8	-0.0290	0.0177
ma9	0.0409*	0.0156
Intercept	0.0007*	0.0003

Source: Author's own elaboration

GARCH (5, 4) is found as the optimal model to predict the volatility of the KSE-100 index from AIC. 2 represents the result of GARCH (5,4) to predict the volatility of the KSE-100 index.

Table.2. Result of GARCH (5,4)

Coefficient	Value of Coefficient	Standard Error
α_1	1.3418**	0.0221
α_2	0.9250**	0.0747
α_3	0.5087**	0.0350
α_4	1.4662**	0.0334
α_5	0.7998**	0.0125
β_1	1.3766**	0.3544
β_2	1.3290**	0.1931
β_3	1.4903**	0.2074
β_4	1.3848**	0.1664
Intercept	1.0895**	0.0004(10^{-2})

Source: Author's own elaboration

The trained model of GARCH (5,4) is then tested on test data of volatility from January 2019 to July 2019. The designed model is tested in two ways i.e., the model's accuracy for short-term prediction of volatility and long-term prediction. RMSE (Root Mean Squared Error) is used to compare one model with another. Furthermore, the predicted volatility (represented by the red line in the graph [Figure 4]) from GARCH seems acceptable for a shorter period but it becomes a straight linear line as the number of days increases [Figure 5] and the reason behind this behavior of GARCH is that the conditional variance is convergent towards unconditional variance. We can see this convergence in the long-term prediction of volatility from the GARCH model.

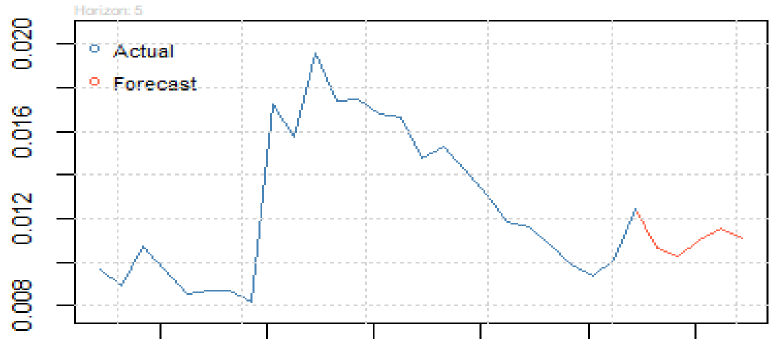

Figure.4. Short-term (5 days) Forecasting using GARCH
Source: Author's own elaboration

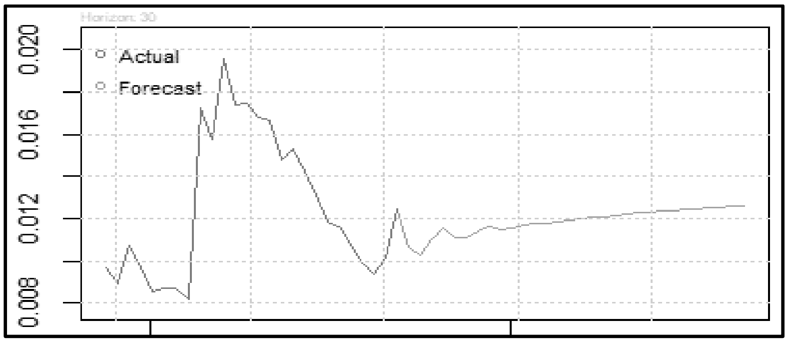

Figure.5. Long-term (60 days) Prediction using GARCH
Source: Author's own elaboration

4.2. NNAR Model

NNAR model is trained on stationary series of the volatility of the KSE-100 index. Since the series is stationary therefore there is no seasonality in the series. Seasonal lag is taken at 0 and non-seasonal lag is taken at 28, identified using AIC. The trained model of NNAR is then tested on a test series of volatility from January 2019 to July 2019. The accuracy of the model is calculated for both periods, short-term forecasting and long-term forecasting. RMSE (Root Mean Squared Error) is used to compute the model's error and to determine the efficient model among all designed models.

NNAR gives lesser RMSE as compared to the GARCH model [Table 4]. Figure 6 and Figure 7 show short and long-term predictions of volatility from the NNAR model respectively. The blue line is representing actual volatility while the orange line is showing predicted volatility. The model also seems fair in predicting long-term volatility unlike the GARCH model since it remains fluctuated throughout the period due to its intrinsic characteristics [Figure 7]. The variation is observed in the graphs of volatility in the case of different models is due to the difference in the period of data as depicted in graphs and the multiple measures of volatility used in the case of hybrid models. However, for comparison, RMSE is used on the difference between actual and predicted values of the volatility of the KSE-100 index.

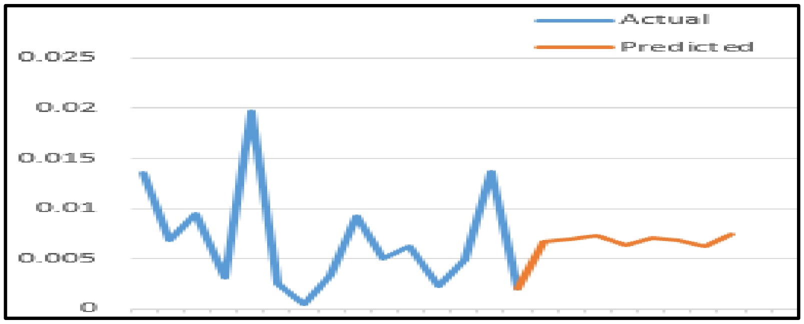

Figure.6. Short-term (5 days) Forecasting using NNAR
Source: Author's own elaboration

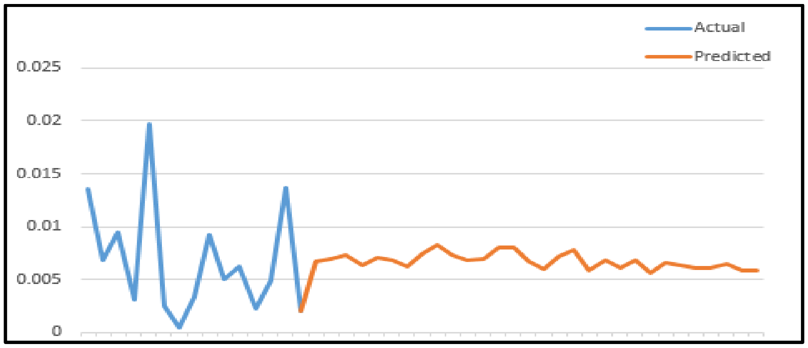

Figure.7. Long-term (60 days) prediction from NNAR
Source: Author's own elaboration

4.3. Hybrid Models

As mentioned earlier those three different hybrid models are designed from NNAR and GARCH to predict the volatility of the KSE-100 index. After designing the models, each model is used to forecast short-term and long-term volatility. The predicted volatility is then compared to the actual volatility of test data. RMSE is calculated to compute error and to analyze the most efficient model. The behavior of GARCH that it converges for long-run prediction continues in the hybrid model as well.

In NNAR-based GARCH, GARCH is applied to the error of NNAR. Figure 8 and Figure 9 show the short and long-term prediction of NNAR based GARCH hybrid model in which the red line is showing actual volatility and the blue line is representing predicted volatility.

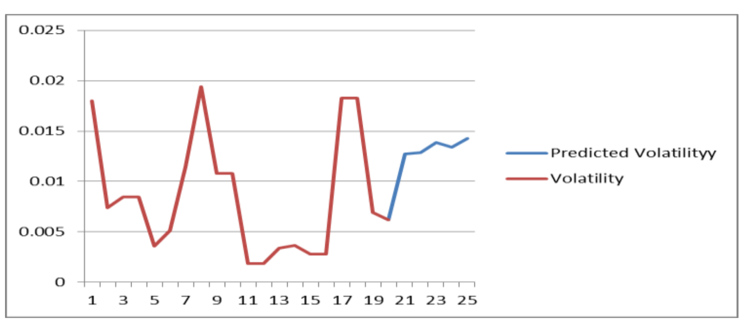

Figure.8. Short-term Prediction of NNAR-base GARCH
Source: Author's own elaboration

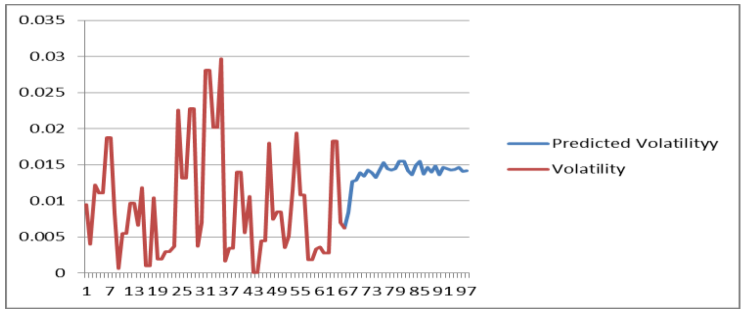

Figure.9. Long-term Prediction of NNAR based GARCH
Source: Author's own elaboration

In GARCH based NNAR, NNAR model is trained on a series of errors of GARCH model. Figure 10 and 11 show the short and long-run prediction of GARCH based NNAR hybrid model.

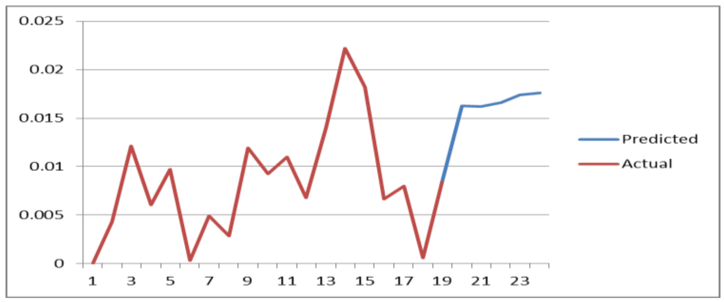

Figure.10. Short-term Prediction of GARCH based NNAR
Source: Author's own elaboration

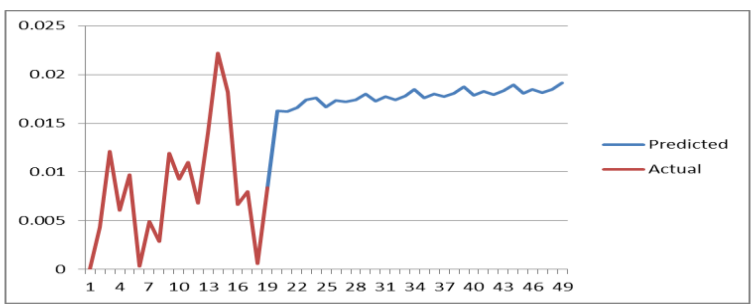

Figure.11. Long-term Prediction of GARCH based NNAR
Source: Author's own elaboration

In a linear combination of GARCH and NNAR, the predicted volatility obtained from both of the models is combined linearly. Table 3 shows the result obtained from the linear combination of the NNAR and GARCH model. From the predicted volatility and RMSE of the model it is found that the behavior of prediction of this model is the same as that of NNAR. The reason behind this is that the significant variable in the regression is the output of NNAR only, but not the output of GARCH.

Table.3. Result of Hybrid Model (Linear Combination of GARCH and NNAR)

Coefficient	Value of Coefficient	Standard Error
W_1	-0.0011*	0.0004
W_2	1.0539***	0.0282
W_3	0.0435	0.0264

Source: Author's own elaboration

Figure 12and Figure 13 show the short and long-term prediction of volatility respectively, of the KSE-100 index from the hybrid model of linearly combined GARCH and NNAR.

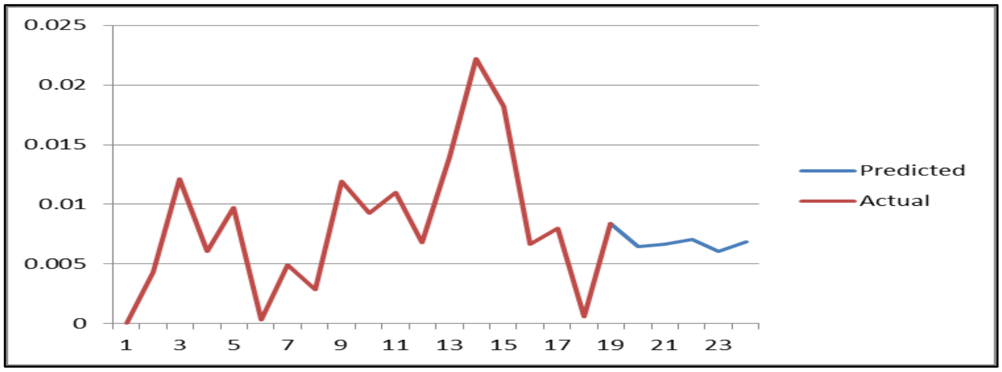

Figure.12. Short-term Prediction of Linear Combination of GARCH and NNAR
Source: Author's own elaboration

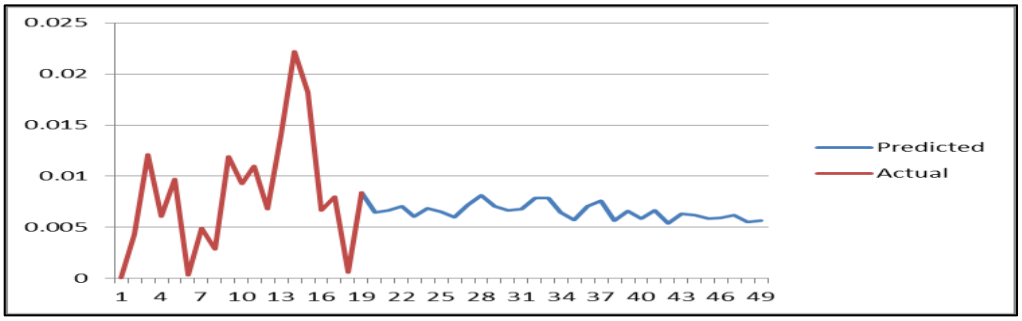

Figure.13. Long-term Prediction of Linear Combination of GARCH and NNAR
Source: Author's own elaboration

It is found that the linear combination of NNAR and GARCH has the least RMSE values and therefore is a better model than NNAR based GARCH and GARCH based NNAR in predicting volatility of the KSE-100 index for both long-term and short-term periods [Table 4]. On the other hand, GARCH based NNAR and NNAR based GARCH models have almost the same RMSE values and are not efficient predictive models as compared to linear combination of GARCH and NNAR.

Table.4. RMSE of all Models (Values are Presented in 10^{-3}).

Model	15-days ahead	30-days ahead	60-days ahead	6-months ahead	Remarks
Linear Combination of GARCH and NNAR	6.05	5.06	5.07	7.36	Robust
NNAR	6.07	5.56	5.44	7.57	Good
GARCH	8.28	8.09	8.18	8.18	Average
NNAR Based GARCH	9.11	9.35	9.50	9.63	Average
GARCH based NNAR	11.6	11.9	12.2	11.7	Poor

Source: Author's own elaboration

5. Conclusion and Recommendations

Volatility of time series can be predicted using various different techniques. Econometric models, machine learning models and hybrid models all can be used to predict the volatility of financial time series or a risk. As the working algorithms of these models are different, therefore the results of the models also vary. It can be observed that among five models; out of which one is econometric model (GARCH), one is machine learning model (NNAR) and three are hybrid models (linear combination of NNAR and GARCH, GARCH based

NNAR and NNAR based GARCH), all models give different RMSE over different forecasting periods. Out of all five models, our designed hybrid model which is a linear combination of GARCH and NNAR is the best predictive model for predicting volatility of KSE-100 index for both short-term and long-term forecasting of risk with minimum RMSE [Table 4]. After linear combination of NNAR and GARCH, NNAR is proved to be the best for the prediction of volatility of KSE-100 index followed by GARCH, NNAR based GARCH, and GARCH based NNAR models for the prediction of volatility of KSE-100 index. In order to check the robustness of the designed models' the data size of the training data set is halved and it is observed that again the linear combination of GARCH and NNAR is the best model for predicting the volatility of the KSE-10 index.

The research can be further extended by combining these models in other ways. As NNAR and GARCH are combined linearly in this research, the non-linear combination can also be explored to design new hybrid combinations. Other methods can also be used to design hybrid models to predict the volatility of financial time series.

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