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The Stability Diagram of a Single-Electron Transistor

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Abstract-A single-electron transistor (SET) is a promising technology, superseding a traditional transistor. This device needs to be accurately controlled by external voltage sources because this sophisticated structure is operated in the submicron scale. Quantum phenomena control a single-electron flow through the SET framework and keep the electron in this framework. The number of excess electrons is important since it indicates the state of the device. The external potential change can disturb the state constant. To forecast and specify the state, a mathematical model of the state transition was built. The model was then plotted as a stability diagram. The electrostatic energy was considered and analytically solved for the model. In this way, the complicated mathematics involved in this quantum phenomena could be simplified. In the result, the stability diagram is plotted by using the capacitance parameters that is reported by Hofheinz *et al.* (2006) and it strongly correlates with stability diagrams that were previously reported, however, the approach was different. With simplicity method, it may be applied to the type of complex single-electron devices such as single electron pump.

Keywords: Stability diagram, single-electron transistor, and quantum dot

1. Introduction

A single-electron transistor (SET) (Likharev, 1999 ; Devoret *et al.,* 1992 ; Bruijn, 2015 ; Droulers, 2017) is a type of electronic device that has been widely studied and it is preferentially applied in electronic circuits. This device operates like a conventional transistor, but quantum phenomena (Beenakker, 1991 ; Grabert, 2013) are employed to control the flow of an electron. This device has electronic properties that are different from a traditional transistor, such as controlling a single electron. Consequently, the SET has low power consumption and high sensitivity to electromagnetic fields. Therefore, the SET is not limited like a traditional transistor and might be the transistor technology of the future.

The SET device consists of a quantum dot (QD) and three electrodes, namely, the source, the drain, and the gate electrode. This design resembles a field-effect transistor, e.g., a metal-oxide-semiconductor field-effect transistor. The SET operation is, however, influenced by the external voltage at the electrodes. The external voltage induces charges at the electronic junctions that connect the QD with the electrodes. The energy stored as electrostatic energy (Hayt & Buck, 1985) increases when the induced charges are raised. In this way, the stored energy was controlled by adjusting the external voltage. Tunneling phenomena occur when the stored energy equals threshold energy called the charging energy. An individual electron passing through the SET structure can then be controlled. The flow of an individual electron leads to an inconsistent number of electrons in the QD because the excess electrons are instantly trapped within the QD. The number of electrons changing in the QD determines the state of the SET.

The state of the SET is defined by the number of confined excess electrons within the QD. The operating of SET changes the charges in the QD because electron tunneling occurs at the tunneling junctions. This process increases or decreases the number of electrons in the QD. In essence, the state is controlled by the external voltage. Since the SET is extremely sensitive to an electromagnetic field, accurately identifying the state is important. The study aimed to build a mathematical model for the state transition to forecast precisely and specify the state. Although the phenomena of this device could be explained by the quantum physics that lead to complicated mathematics, the classical physics was interested in the macroscopic effects. The electrostatic energy was compared with the tunneling process conditions. An electrostatic energy equation was developed to fit this mathematical model. This simplified the equation. The electrostatic energy was analytically solved to establish the mathematical model and was plotted as a stability diagram of the SET (Scholze *et al*, 2000 ; Wallisser *et al*, 2002 ; Hofheinz *et al*, 2006 ; Amakawa *et al*, 2004).

2. Architecture and Theoretical Background

The SET structure was fabricated with a synthesized quantum dot isolated from the external environment, as illustrated in (Figure 1a). The submicron scale of the QD led to coulomb blockade phenomena (Droulers *et al*, 2017 ; Beenakker, 1991 ; Grabert, 2013) within the structure. These phenomena allowed the SET to control the flow of single electrons from the source to the drain. The QD and the source electrode and the QD and the drain electrode were connected by a tunneling junction. An electron could tunnel through this junction. The QD and the gate electrode were linked with a capacitive junction. The capacitive junction was larger than the tunneling junction, which disabled tunneling electrons. The model of the SET circuit is illustrated in (Figure 1b). Although an electron could not flow through the capacitive junction, the QD could be coupled with the gate electrode using an electric field.

In this setup, the QD was physically separated from the electrodes, but the SET was controlled by the external voltage at the electrodes. The QD was then connected to the electrodes via an electric field. Charges were induced at the junctions by applying a voltage to the parallel plates. Although they were tunneling junctions, they had electronic properties like an ideal capacitor. When the induced energy was equal to the charging energy, the tunneling junctions had resistor-like electronic properties. Thus, an electron could tunnel into or out of the QD.

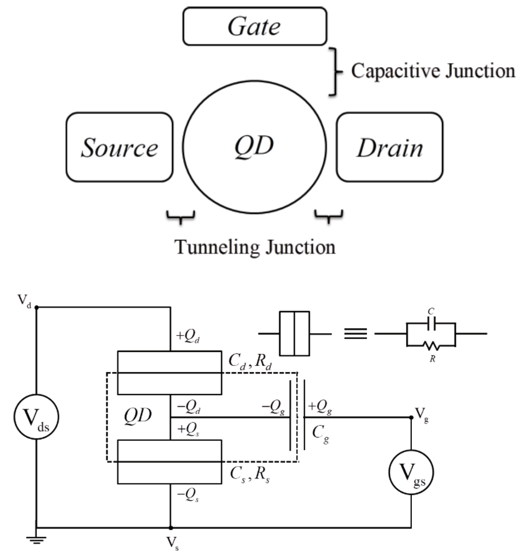

Figure 1. The diagram of the single-electron transistor: (a) the SET setup, (b) the equivalent circuit.

When a single-electron transistor was biased by the external voltage supplies, $V_{ds}>0$ and $V_{gs}>0$, the induced charges have occurred at the electric junctions, as shown in (Figural 1b). The cumulative charges affected to store some energy in the QD as the electrostatic energy (Hayt & Buck, 1985). The energy was dependent on the total net charges and potential in the QD and was calculated following equation (1)

$$E_i=Q_iV_i \quad (1)$$

where Q_i was the net charge and V_i was the potential within the QD. The occurred energy has affected the energy levels within the QD. These levels thus could be controlled by applying the external voltage. Single-electron tunneling could be controlled by biasing. According to the calculus method, the change of the electrostatic energy could be exactly calculated by

$$dE_\Delta=Q_idV_i + V_idQ_i \quad (2)$$

Equation (2) referred to the change of the electrostatic energy. This energy changing was divided into two cases, i.e., the Q_i was constant while the V_i changed, and the V_i was constant while the Q_i was altered. With the Coulomb blockade phenomena (Likharev, 1999 ; Devoret *et al*, 1992) in the SET structure, this device could have momentarily stayed on the state when the SET was biased by the external voltage supplies. Thus, the electron could not tunnel into or out of the QD. The charge in the QD become constant (dQ=0). In this case, varying the external potential would be affected by the stored energy in the QD. According to equation (2), the change of the electrostatic energy of this case could be calculated by

$$dE_{\Delta}=Q_i dV_i \tag{3}$$

On the other way, when the stored energy was equal to the charging energy, the energy level would be equal to the electron energy level at the electrodes ; source or drain electrode. The electron could freely tunnel into or out of the QD. The state of SET was changed. In this case, the potential of the QD was constant. According to equation (2), the change of the electrostatic energy of this case could be calculated by

$$dE_{\Delta}=V_i dQ_i \tag{4}$$

3. Mathematical Modeling

Equation (4) referred to the change of electrostatic energy when the single-electron transistor changed its state. In this case, the potential of the QD was constant and calculated by the difference between the potential of the electrodes and the potential of the QD as expressed by

$$dE_{\Delta}=(V_i-V_i)dQ_i \tag{5}$$

where V_t was the voltage at the source or the drain electrodes, and V_i was the voltage at the QD. Thus, the total change of energy was calculated using the following equation (6).

$$E_{\Delta}=\int_{Q_1}^{Q_2}(V_i-V_t)dQ_i \tag{6}$$

where Q_1 is the initial charge, and Q_2 is the final charge in the QD, as shown in equation (7a) and (7b), respectively

$$Q_1=-ne \tag{7a}$$

$$Q_2=-(n \pm 1)e \tag{7b}$$

The final charge, Q_2, was divided into two cases, an electron tunneling into the QD and an electron tunneling out of the QD, which is indicated by a plus and minus sign, respectively. The tunneling of an electron into the QD made the state of SET to be increased. The initial state "n" became the final state "n+1". The tunneling of an electron out of the QD made the state of SET decrease. The initial state "n" became the final state "n-1".

The voltage of the QD, V_i occurred from the induced charges that was stored in the QD. The total charge thus could be calculated from the individual induced charged of each junction. It was expressed as

$$Q_i=Q_s-Q_{g-}Q_d \tag{8}$$

where Q_s, Q_d, and Q_g were the induced charge at the source, drain, and gate junctions. These charges depended on the potential difference across the capacitor. The total charges of the QD were calculated by

$$Q_i=C_s(V_i-V_s)-C_d(V_d-V_i)-C_g(V_g-V_i) \tag{9}$$

Generally, the potential at the source electrode was set to be ground, and the potential V_s could be defined to zero. Thus, the total V_i was showed as equation (10).

$$V_i = \frac{\left(Q_i + C_g V_g + C_d V_d\right)}{C_t} \quad (10)$$

where $C_t = C_g + C_s + C_d$ was a total capacitance of the SET circuit. The V_i in equation (10) had both parameters of the voltage and the charge. For simplicity, the parameter V was changed in the team of Q with the relation of Q=CV. It could be expressed by

$$V_i = \frac{\left(- n_i e + n_g e + n_d e\right)}{C_t}, \quad (11)$$

where $Q_i = -n_i e$, $Q_g \cong C_g V_g = n_g e$ and $Q_d \cong C_d V_d = n_d e$. Thus, the energy for a state transition was calculated as

$$E_{\Delta}^{i\to j} = \frac{e^2}{C_t}(n - n_g - n_d \pm \frac{1}{2}) + eV_t \quad (12)$$

The state transition of a single-electron transistor from the initial state n_i to the final state n_j could be indicated into four cases

1) The electron tunneled into the QD from the source electrode. The requirement for this tunneling was

$$\Delta E_{\Delta}^{i\to j} = \frac{e^2}{C_t}(n - n_g - n_d + \frac{1}{2}) + eV_s \quad (13a)$$

2) The electron tunneled out of the QD to the source electrode. The requirement for this tunneling was

$$\Delta E_{\Delta}^{i\to j} = \frac{e^2}{C_t}(n - n_g - n_d - \frac{1}{2}) + eV_s \quad (13b)$$

3) The electron tunneled into the QD from the drain electrode. The requirement for this tunneling was

$$\Delta E_{\Delta}^{i\to j} = \frac{e^2}{C_t}(n - n_g - n_d + \frac{1}{2}) + eV_d \quad (13c)$$

4) The electron tunneled out of the QD to the drain electrode. The requirement for this tunneling was

$$\Delta E_{\Delta}^{i\to j} = \frac{e^2}{C_t}(n - n_g - n_d - \frac{1}{2}) + eV_d \quad (13d)$$

4. Results and Discussions

Equation (13a-13d) referred to the change of the electrostatic energy as a single-electron transistor changed its state. The tunneling of an electron causes the changed state of SET into or out of the QD. When an electron tunneled into the QD, the state of SET at the initial state n becomes the final state n+1. Or another case, when an electron tunneled out of the QD, the state of SET at the initial state n becomes the final state n-1. With the tunneling of an electron in the SET architecture, the energy difference between the initial and the final state was equal. Thus, the energy $E_{\Delta}^{i\to j}$ became zero. It meant that an electron could freely tunnel into or out of the QD. The SET with the initial state n_i could change to the final state n_j. With this condition, there were four possibilities of the changed state, as illustrated in (Table 1).

Table 1. The conditions for the changed state of a single-electron transistor.

Tunneling Process	Initial state (n_i)	Final state (n_j)	Conditions
From source to QD	n	n + 1	$0 = \frac{e^2}{C_t}(n - n_g - n_d + \frac{1}{2}) + eV_s$
From QD to source	n	n-1	$0 = \frac{e^2}{C_t}(n - n_g - n_d - \frac{1}{2}) + eV_s$
From drain to QD	n	n + 1	$0 = \frac{e^2}{C_t}(n - n_g - n_d + \frac{1}{2}) + eV_d$
From QD to drain	n	n-1	$0 = \frac{e^2}{C_t}(n - n_g - n_d - \frac{1}{2}) + eV_d$

The relation between the charges and the voltage that was occurred by supplying a voltage to a capacitor, Q=CV was used changing the drain voltage to the induced charge, $Q_d \cong C_d V_d = n_d e$ but the source voltage could be omitted since it was set to the ground of the circuit. The models could be rewritten as

$$n_d = -n_g + \left(n + \frac{1}{2}\right) \quad (14a)$$

$$n_d = -n_g + \left(n - \frac{1}{2}\right) \quad (14b)$$

$$n_d = -\frac{C_d}{C_s + C_g} n_g + \frac{C_d}{C_s + C_g}\left(n + \frac{1}{2}\right) \quad (14c)$$

$$n_d = -\frac{C_d}{C_s + C_g} n_g + \frac{C_d}{C_s + C_g}\left(n - \frac{1}{2}\right) \quad (14d)$$

The equation (14a-14d) referred to the relation of the potential that was applied at the drain and the gate electrodes in the form of the induced charges to the exceed charges that occurred in the QD. With the simplicity to indicate the state of SET, these models were plotted as the stability diagram. For example, the capacitance parameters reported by Hofheinz *et al.* (2006) were used to plot as depicted in (Figure 2). Since the capacitance parameters were C_s=42 aF, C_d=32 aF, and C_g=13.6 aF, the capacitance C_t was 87.6 aF (the prefix "a" means 10^{-18}).

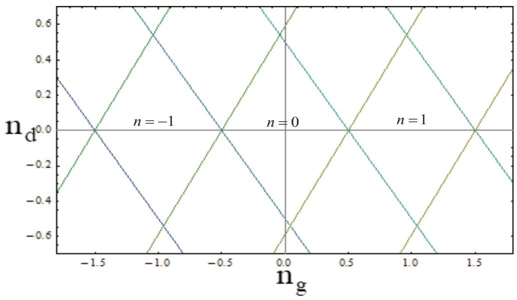

Figure 2. The stability diagram of a single-electron transistor. This diagram was plotted based on the work of Hofheinz *et al*. (2006). The parameters were C_s=42 aF, C_d=32 aF, and $C_{g=}$13.6 aF.

The stability diagram in (Figure 2) showed the stable regions that demonstrated the diamond shape. There regions were indicated by the state of SET ; n=-1, n=0 and n=1. These regions indicated the constant of the charge. Thus, the voltage biasing of this area, the different charges become zero, dQ=0. The transition lines indicated the changed voltage of the drain and the gate voltages of each state. The cut points between the state n=-1 and n=0 or the state n=0 and n=1 were showed as the boundary of the state transition of SET.

5. Conclusion

The state transition of a single-electron transistor was investigated by calculating the electrostatic energy. With this method, the mathematical model for expressing the state transition was easy to build. The model was analytically solved and ignored the complicated mathematics that occurred from the quantum phenomena. It was clear to show the boundary of the state transition and the stable regions. The mathematical model was plotted as the stability diagram by using the capacitance parameters that were reported by Hofheinz *et al.* (2006) and Amakawa *et al.* (2004). This result was agreeable with the work that was reported too.

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7. References

Amakawa, S., Tsukagoshi, K., Nakazato, K., Mizuta, H., & Alphenaar, B. W. (2004). *Single-electron logic based on multiple-tunnel junctions.* Mesoscopic Tunneling Devices.

Beenakker, C.W. (1991). Theory of Coulomb-blockade oscillations in the conductance of a quantum dot, *Physical Review B*, *44*, 1646.

Bruijn, D.S. (2015). *Mapping electron tunnelling in a nanoparticle network to a cellular neural network* [Bachelor's thesis]. University of Twente.

Devoret, M.H., Esteve, D., & Urbina, C. (1992). Single-electron transfer in metallic nanostructures, *Nature*, *360*, 547-553.

Droulers, G., Ecoffey, S., Pioro-Ladrière, M., & Drouin, D. (2017). Metallic single electron transistors: Impact of parasitic capacitances on small circuits, *IEEE Transactions on Electron Devices*, *64*, 5202-5208.

Grabert, H., & Devoret, M.H. (2013). *Single charge tunneling: Coulomb blockade phenomena in nanostructures*. Springer Science & Business Media.

Hayt W.H., & Buck, J.A. (1985). *Energy and Potential in Engineering Electromagnetics*. Mc Graw-Hill.

Hofheinz, M., Jehl, X., Sanquer, M., Molas, G., Vinet, M., & Deleonibus, S. (2006). Simple and controlled single electron transistor based on doping modulation in silicon nanowires. *Applied Physics Letter*, *89*, 143504.

Likharev, K.K. (1999). Single-electron devices and their applications, *Proceedings of the Institute of Electrical and Electronics Engineers*, *87* (4), 606-632.

Scholze, A., Schenk, A., & Fichtner, W. (2000). Single-electron device simulation. *IEEE Transactions on Electron Devices*, *47,* 1811-1818.

Wallisser, C., Limbach, B., Vom Stein, P., Schäfer, R., Theis, C., Göppert, G., & Grabert, H. (2002). Conductance of the single-electron transistor: A comparison of experimental data with Monte Carlo calculations, *Physical Review B*, *66*, 125314.