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An Analysis of the Implementation Path of Moral Education in Mathematics under Goal Orientation ——Taking "The ellipse and its standard equation" as an example

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Abstract
The practice of moral education in mathematics is an inevitable requirement for the implementation of the fundamental task of establishing moral education. The practice of moral education in mathematics is not conducted blindly. By adopting moral education objectives as the guide, the effectiveness of moral education in the subject can be enhanced. The implementation path of moral education in mathematics was explored in the context of the moral education objectives for the senior secondary level in the "Guidelines for Moral Education in Primary and Secondary Schools", based on the example of the "ellipse and its standard equation". A typical case was used as a carrier; reasonable scenarios were introduced; mathematical thinking and methods were penetrated to focus on the cultural values of mathematics.
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Since the 18th National Congress of the Party, the Ministry of Education has proposed to make moral education the fundamental task of education, and moral education takes a leading position in school teaching. In order to realize the fundamental task of establishing moral education, it is not only necessary to rely on the character education classes offered in schools, but also to permeate moral education in the teaching of all subjects under the leadership of moral education objectives.

1.The feasibility of moral education in mathematics

Moral education in mathematics is a form of education that integrates the teaching of various subjects with moral education, which needs to reflect the essence of moral education and highlight the essential characteristics of the subject. Mathematics is often seen as a natural science with a high degree of abstraction, which may seem far from moral education compared to the social sciences where moral education has obvious connotations. However, in fact, mathematics education is closely linked to the development of moral education. The General High School Mathematics Curriculum Standard states that mathematics education plays the role of implementing the fundamental task of educating people for morality and developing quality education. At the same time, Professor Zhang Dianzhou pointed out that mathematics is also an ideology created by human beings and has a strong humanistic dimension. The aesthetic value of mathematical content, the historical background of mathematics, and the spirit of mathematicians in creating and developing mathematics all have a great impact on students. In addition, as one of the fundamental subjects in schools, mathematics is expected to bring its nurturing value into full play. Therefore, it is both feasible and necessary to infuse moral education through mathematics teaching. At present, many mathematics teachers focus on the development of students' rational thinking, neglecting other aspects of mathematics' nurturing function. In mathematics classroom teaching, it is important not only to emphasize the rational role of mathematics, but also to explore its perceptual cultural inculcation, to infiltrate moral education factors in all aspects, and to implement the moral education function of mathematics subjects.

2.Moral objectives

Active and effective moral education infiltration in the mathematics classroom is the key to implementing moral education in mathematics. Active and effective moral education must be directed and not conducted in a scattered and disorganized manner. There is no doubt that moral education objectives can give teachers direction. The moral education objectives are the key to the whole moral education theory and practice, and teachers' understanding and interpretation of the moral education objectives determine how they implement moral education in the classroom. Therefore, teachers shall understand the objectives of moral education and, under the

guidance of moral education objectives, conduct active and effective moral education infiltration, so as to realize the fundamental task of moral education in the subject and implement the establishment of moral education.

China's Ministry of Education, in the Guidelines for Moral Education in Primary and Secondary Schools (hereinafter referred to as "the Guidelines") issued in 2017, provides us with action guidelines for realizing moral education in the subject. In the high school mathematics classroom, the infiltration of subject moral education can be guided and led by the moral education objectives of the high school level. The author divides the moral education objectives of the senior secondary level proposed in the Guide into four levels: "patriotism", "civic awareness", "scientific method" and "life The four dimensions of the moral education objectives for the senior secondary level have been constructed, as shown in Table 1. Teachers of mathematics in senior secondary schools can implement moral education based on these four dimensions in an orderly and efficient manner.

Table 1 Table of objective dimensions of moral education in the senior high school level

Objective Categories	Specific descriptions in the Guide
Patriotism	Educate and guide students to love the Communist Party of China, adore the motherland and love the people, support the path of socialism with Chinese characteristics, promote the national spirit, and enhance national self-esteem, self-confidence and pride
Civic Consciousness	Enhance civic awareness, a sense of social responsibility and the concept of democracy and the rule of law
Scientific Methodology	Learn to apply basic Marxist views and methods to observe, analyze and solve problems, and develop an attitude and ability to be independent, self-reliant and self-improving
Life Pursuit	Acquire the knowledge to choose the correct path of development in life and form a correct world view, outlook on life and values

3.The implementation of teaching under the orientation of moral education goals

The author takes "ellipse and its standard equation" as an example and uses the moral education objectives of the senior secondary school as a guide to infiltrate moral education into the teaching of mathematics in the senior secondary classroom.

3.1Contextual introduction, understanding ellipses

Scenario 1: Show students the elliptical orbit of Chang'e 5 around the moon to experience the application of ellipses in spaceflight, and to appreciate the rapid development of China's space industry.

Scenario 2: Show students the cross-section of the oil tanker to experience the application of ellipse in life practice.

Design Intention: Using Chang'e 5 as an example, students can appreciate the increasing strength of China's science and technology, enhance their national pride and inspire their patriotism, so as to achieve the moral education objective of "patriotism".

3.2Analogous conjectures, exploring definitions

Use the geometry drawing board to show students an animation of a circle compressed into an ellipse, so that they can appreciate the connection between circles and ellipses and initially develop the idea of guessing an ellipse by analogy with a circle.

Question 1: What is the definition of a circle that we have learnt before?

Students: The trajectory of a point in the plane whose distance from a fixed point is equal to a fixed length (set).

Question 2: What do you find when you analyze the definition of a circle in mathematical terms?

Students: There is a certain point O inside the circle and the distance from the moving point P to O is a constant.

Question 3: We find that an ellipse can be obtained by compressing a circle and that there is a certain connection between the two, by analogy with the definition of a circle. By changing which of the conditions in the definition of a circle, the trajectory is changed to an ellipse?

Students: Change in-plane to in-space and one fixed point to two fixed points.

Teacher: As the ellipse is a plane figure, it is not advisable to change the plane into space, try one fixed point to two fixed points.

Question 4: By changing one fixed point to two fixed points, what relationship can we study between the moving point and the two fixed points?

Students: Study the trajectory of the point where the sum, difference, product and quotient of the

distance from the moving point to the two fixed points is equal to the fixed length.

Teacher: Sum is the most basic form of arithmetic. We conjecture that the trajectory of the point whose sum of the distances from the moving point to the two fixed points is a constant is an ellipse, and next we have to verify its correctness.

Question 5: Given two fixed points F_1 and F_2, the moving point is P and $|PF_1|+|PF_2|=l$, l is a constant. If you want to explore the trajectory of the moving point P, you can first find a point P_0 that satisfies the conditions. How to find the point P_0? Discuss among the group.

Students: Since $|PF_1|+|PF_2|=l$, l is a constant and the distance $|PF_1|$ from the point P to one of the fixed points F_1 is fixed, then the distance $|PF_2|$ from the point P to the other fixed point F_2 is also fixed. Based on the knowledge of circles, draw two circles $\odot F_1$ and $\odot F_2$ that satisfy the condition that one of the intersections of these two circles satisfies the condition, respectively.

The geometric drawing board is used to demonstrate that a moving point P is selected on a fixed length line segment *l*. Two circles $\odot F_1$ and $\odot F_2$ are obtained with F_1 and F_2 as centers, PF_1 and PF_2 as radii respectively, and the intersection of the two circles is found to be P_0, as shown in Figure 1.

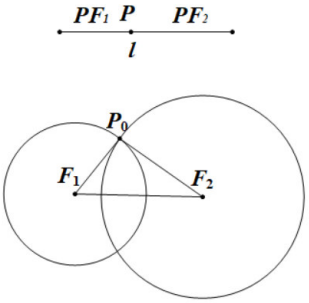

Figure 1 Diagram of the intersection of two circles

Using the geometry drawing board, the point P is dragged on the line *l* to change the size of PF_1 and PF_2, and to trace the trajectory of the intersection point P_0. The trajectory where the point P_0 is found to be an ellipse. Plot the trajectory of the ellipse as shown in Figure 2.

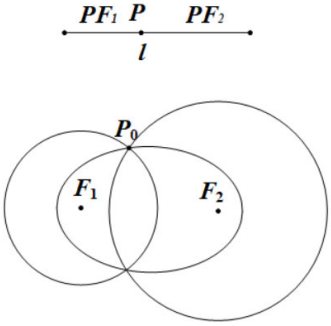

Fig. 2 Elliptical trajectory diagram

Question 6: We first find a certain moving point and the geometry drawing board is utilized to verify that the trajectory of that moving point is an ellipse. Is it possible to obtain a definition of an ellipse from this? Discuss among the group.

Students: We find that the trajectory exists only if the moving point is always the intersection of two circles.

Question 7: How can this finding be expressed in mathematical terms?

Students: The relationship between the two circles can be expressed by the position of the two circles. When the line segment is $l<|F_1F_2|$, the two circles are separated and the intersection point does not exist and cannot form a trajectory; when the line segment is $l=|F_1F_2|$, the two circles are tangent and the trajectory at the intersection point is a straight line; when the line segment is $l>|F_1F_2|$, the two circles intersect and the trajectory at the intersection point is an ellipse. Therefore, the line segment is $l>|F_1F_2|$.

After analogy, conjecture and verification, the definition of an ellipse was obtained, i.e. the trajectory of a point in the plane whose sum of distances to two fixed points F_1 and F_2 is a constant $2a$ ($2a > |F_1F_2|$) is an ellipse. These two fixed points F_1 and F_2 are called the foci of the ellipse, and the distance between the two foci F_1 and F_2 is called the focal length of the ellipse.

Design Intent: Students are led to conjecture ellipse definitions by analogous to the definition of a circle. Geometric artboards are used to validate conjectures and refine and supplement defined conditions. The inquiry process needs to be closely related to the "analogy-conjecture-verification" method of argumentative reasoning. This helps to cultivate students' logical thinking ability and scientific inquiry ability, so that students can form a thinking of "finding problems through analogies, predicting conclusions according to conjectures, verifying conclusions through experience, and finally solving problems", so as to cultivate students' independent, self-reliant and self-reliant attitudes and abilities to achieve the moral education goal at the level of "scientific concept of development".

3.3Cooperative inquiry to derive equations

By analogy with the derivation of the standard equation of a circle, students can naturally think of finding the standard equation of an ellipse by establishing a right angle co-ordinate system.

Question 8: How do you set up a right-angle co-ordinate system?

Students: The x-axis is the line at the focus of the ellipse, and the y-axis is the perpendicular bisector of the line joining the two foci of the ellipse.

In accordance with the student's statement, a plane right-angle co-ordinate system is established as shown in Figure 3.

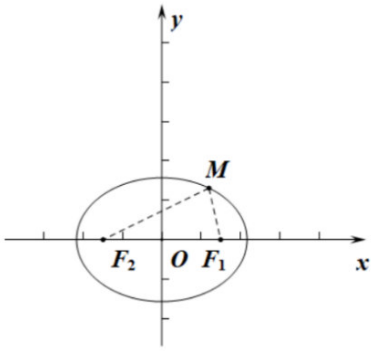

Fig. 3 Planar Cartesian coordinate system

In the plane rectangular coordinate system xOy , give the coordinates of the focal point F_1 、 F_2 as $F_1(-c,0)$ 、 $F_2(c,0)$. $M(x,y)$ is any point on the ellipse. According to $|MF_1|+|MF_2|=2a$, $\sqrt{(x+c)^2+y^2}+\sqrt{(x-c)^2+y^2}=2a$ can be obtained from the formula for the distance between two points.

Question 9: How to simplify an equation containing two radicals?

Students: Shift the terms and square both sides at the same time to collate to get $a^2-cx=a\sqrt{(x-c)^2+y^2}$, and square both sides again at the same time to collate to get $(a^2-c^2)x^2+a^2y^2=(a^2-c^2)a^2$, i.e. $\frac{x^2}{a^2}+\frac{y^2}{a^2-c^2}=1$.

In order to make the equation symmetrical and concise, students are guided to introduce the paramete b , which is $b^2=a^2-c^2$, to obtain the standard equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b>0)$.

The method for deriving the standard equation for an ellipse above is the 'quadratic method', which is the method used in most textbooks. Although it gives a general method for simplifying equations containing two radicals, the process of simplification is tedious. Next, the methods used by mathematicians in the history of mathematics to derive the standard equation of an ellipse are given so that students can learn to see the problem from multiple

perspectives.

• The Lopita Method of Harmonization

Because of $|MF_1|+|MF_2|=2a$, assume that $|MF_1|=a+z, |MF_2|=a-z$, and z is parameter. Using the formula for the distance between two points, it gives:

$$|MF_1|^2=(a+z)^2=(x+c)^2+y^2 \quad (1)$$

$$|MF_2|^2=(a-z)^2=(x-c)^2+y^2 \quad (2)$$

(1) Subtract (2) to get $4az=4cx$, which can be simplified:

$$z=\frac{cx}{a} \quad (3)$$

Substituting equation (3) into equation (1), it gives:

$$a^2+2cx+\frac{c^2x^2}{a^2}=x^2+2cx+c^2+y^2$$

Thus, the ellipse equation is obtained（Assuming $b^2=a^2-c^2$ ）

$$y^2=\frac{b^2}{a^2}(a^2-x^2) \quad (4)$$

Simplify to obtain the standard equation of the ellipse:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b>0)$$

• Wright's "Square Difference Method"

Assuming $|MF_1|=r_1$, $|MF_2|=r_2$, According to the definition of ellipse, it gives:

$$r_1+r_2=2a \quad (5)$$

and

$$r_1^2=(x+c)^2+y^2 \quad (6)$$

$$r_2^2=(x-c)^2+y^2 \quad (7)$$

(6) Formula minus (7) to get $r_1^2-r_2^2=4cx$, which is $(r_1+r_2)(r_1-r_2)=4cx$, thus:

$$r_1-r_2=\frac{2cx}{a} \quad (8)$$

From equations (5) and (8), it gives:

$$r_1=a+\frac{cx}{a},\quad r_2=a-\frac{cx}{a} \quad (9)$$

Substituting equation (9) into equation (6), and assuming $b^2=a^2-c^2$, The standard equation of the ellipse is obtained by rectifying:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b>0)$$

Question 10: What do you find when you compare the above two methods with the 'quadratic method' we use?

Students: The latter two methods construct a system of equations, which we solve using one equation.

Question 11: How is the system of equations constructed?

Students: The unknown quantity is introduced.

Question 12: What do the sums and differences refer to in the method 'sum and difference'?

Students: Sum and difference mean $|MF_1|=a+z, |MF_2|=a-z$ respectively.

The teacher concludes by pointing out that mathematicians have thought differently about the derivation of the standard equation of an ellipse. Using the "sum and difference" method, students are introduced to the story of Lopitha, who studied mathematics night and day despite being a nobleman.

Design intention: (1) Guide students to derive the standard equation of an ellipse and show them the methods used by mathematicians throughout the history of mathematics. Students can compare and analyze these three methods, so that they can learn to analyze and solve problems with a developmental perspective and achieve the moral objective of "scientific method". (2) Learn the derivation methods of mathematicians so that students can appreciate that the perfection and development of mathematics is the result of the joint efforts of mathematicians. Through the struggles of mathematicians, students are able to understand the mission given to them in the new era and to achieve the moral objective of "civic consciousness". (3) Through the example of Lopitha, students are taught to understand that they should have their own pursuit in life, helping them to establish a correct world view, outlook on life and values, and to realize the moral education objective of "pursuit of life".

3.4 Summarize and consolidate

What is the most impressive thing about this lesson? What have you learnt and understood?

3.5 Post-lesson reflections, emotional sublimation

After the lesson, search for the history of the ellipse, the origin of the ellipse and the different methods of deriving the standard equation of the ellipse.

4.Pathways for implementing moral education in mathematics

In the above-mentioned teaching design of "Ellipse and its Standard Equation", the moral education objectives of the senior secondary level are used as a guide, and the theme of moral education in mathematics is closely followed. Admittedly, the implementation of moral education in mathematics cannot depend on a particular lesson alone, but should be integrated throughout the whole of mathematics education. The author, guided by the moral education objectives of the senior secondary school and based on the characteristics of mathematics, summarizes the following paths for the implementation of moral education in mathematics.

4.1Using typical cases as a carrier

The spirit embodied in mathematicians can have a positive impact on students. Our mathematicians, both in ancient and modern periods, have made significant contributions to the development of mathematics. In classroom teaching, examples of ancient mathematicians in China shall be highlighted in order to enhance students' national self-confidence, cultivate their patriotism and achieve the moral education objective of "patriotism".

4.2Reasonable use of scenarios to introduce

In the introduction of the scenario, content related to current affairs of the country can be reasonably chosen according to the content of the class, so that students can appreciate the rapid development of the country. This not only cultivates students' patriotism, but also enables them to understand the responsibility and mission given to them by the times, thus enhancing their civic consciousness and sense of social responsibility and achieving the moral education objective of "civic consciousness".

4.3Penetration of mathematical thinking and methods

The mathematical method of thought is the scientific method of thought, which contains the elements of materialistic dialectics. Many mathematical thinking methods experienced in the process of mathematical learning, such as special and general, induction and deduction, abstraction and generalization, etc., all reflect the materialistic dialectical factors. The infiltration of mathematical thinking methods can help students learn to use scientific points of view and methods to solve problems, and achieve the moral education goals at the level of "scientific method view".

4.4Attention to the cultural value of mathematics

In addition to the rigid nature of mathematics, there are also a large number of mathematical cultural knowledge with humanistic atmosphere.The mathematical culture contains the spirit of hard work, rigorous logical reasoning thinking and unremitting exploration spirit and creativity of mathematicians. In classroom teaching, paying attention to the cultural value of mathematics can help students establish a correct world view, outlook on life and values, so that students can strive towards their own life pursuits, so as to achieve the moral education goals at the level of "life pursuit".

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