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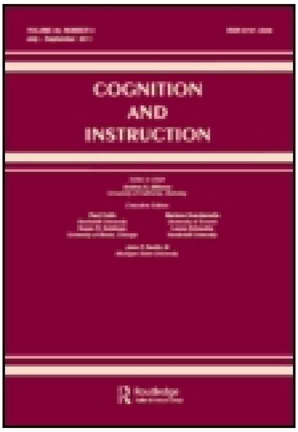

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Spontaneous Focusing on Quantitative Relations in the Development of Children's Fraction Knowledge

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Spontaneous Focusing on Quantitative Relations in the Development of Children's Fraction Knowledge

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While preschool-aged children display some skills with quantitative relations, later learning of related fraction concepts is difficult for many students. We present two studies that investigate young children's tendency of Spontaneous Focusing On quantitative Relations (SFOR), which may help explain individual differences in the development of fraction knowledge. In the first study, a cross-sectional sample of 84 kindergarteners to third graders completed tasks measuring their spontaneous recognition and use of quantitative relations and then completed the tasks again with explicit guidance to focus on quantitative relations. Findings suggest that SFOR is a measure of the spontaneous focusing of attention on quantitative relations and the use of these relations in reasoning. In the second (longitudinal) study, 25 first graders completed measures of SFOR tendency and a measure of fraction knowledge three years later. SFOR tendency was found to predict fraction knowledge, suggesting that it plays a role in the development of fraction knowledge.

Fraction learning has been identified as a key threshold for success with later mathematics (National Mathematics Advisory Panel [NMAP], 2008). A wealth of evidence points to the difficulties students face when learning about fractions, and even adults struggle with many concepts related to fractions (McMullen, Laakkonen, Hannula-Sormunen, & Lehtinen, 2014; Merenluoto & Lehtinen, 2004; Obersteiner, Van Dooren, Van Hoof, & Verschaffel, 2013; Siegler, Fazio, Bailey, & Zhou, 2013; Vamvakoussi, Van Dooren, & Verschaffel, 2012; Vamvakoussi & Vosniadou, 2004). One explanation for these difficulties is that reasoning about fractions requires significant conceptual change in one's understanding of the number concept because many of the characteristics of natural numbers cannot be applied to fractions (Merenluoto & Lehtinen, 2004; Vamvakoussi & Vosniadou, 2004). One fraction concept that requires substantial conceptual change is the representation of the magnitude of fractions (McMullen et al., 2014). A number of studies show that often students and adults are biased toward using natural number concepts when reasoning about fraction magnitudes (Obersteiner et al., 2013; Vamvakoussi et al., 2012).

In order to develop better instructional strategies for dealing with general difficulties and individual differences in fraction learning, it is important to gain a deeper understanding of the

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early predictors of successful fraction learning. Many studies have highlighted early processes with quantitative relations, which are related to later fraction concepts (Boyer, Levine, & Huttenlocher, 2008; Frydman & Bryant, 1988; Jordan et al., 2013; Mix, Levine, & Huttenlocher, 1999; Pitkethly & Hunting, 1996; Pothier & Sawada, 1983; Sophian, 2000, 2007). However, it seems that the early existence of these skills as such is not sufficient for preparing students for the future learning of fractions. Relatedly, studies on the development of natural numbers show that even though young children exhibit skills with discriminating and recognizing quantities (for reviews see Feigenson, 2007; Wynn, 1998), this does not automatically lead to natural number knowledge. Instead, many intervening developmental steps, along with active practice with enumeration, are needed for the development of natural number knowledge (Fuson, 1988; Hannula & Lehtinen, 2005). In particular, earlier studies show that children's tendency to spontaneously focus on numerosity in various situations substantially contributes to the development of natural number skills (Hannula & Lehtinen, 2005). Thus, the aim of the present studies is to investigate if there are differences in the frequency with which children spontaneously use their early skills to deal with quantitative relations, and whether these differences subsequently predict later fraction conceptual knowledge.

EARLY QUANTITATIVE RELATIONS SKILLS AND LATER FRACTION DIFFICULTIES

In the current set of studies, reasoning about quantitative relations is defined as reasoning about the relationship, based on some quantifiable aspect(s), between two or more objects, sets, or symbols. Quantitative relations can refer to a number of mathematical or premathematical relations that may be found in a child's everyday environment. These include exact or approximate proportional relations or ratios, both discrete and continuous; additive and multiplicative relations, in arithmetic and algebraic circumstances; and exact numerical ratios, such as fractions (Frydman & Bryant, 1988; Gallistel & Gelman, 1992; Resnick, 1992; Sophian, 2007; Wynn, 1992). Reasoning about quantitative relations can be seen as extending enumeration processes to a second dimension, expanding these processes to involve determining the relation between two distinct quantities. This expansion of reasoning from single quantities to the relation between quantities is the major focus of the current set of studies. This expansion parallels the (often difficult) progression from treating numbers as discrete entities when reasoning about natural number to treating fractions as relational and continuous quantities (McMullen et al., in press).

A number of studies have found that young children, as early as infancy, display a number of skills for dealing with quantitative relations (Boyer et al., 2008; Feigenson, 2007; Frydman & Bryant, 1988; McCrink & Wynn, 2007; Mix et al., 1999; Singer-Freeman & Goswami, 2001; Sophian, 2000; Spinillo & Bryant, 1999; Wing & Beal, 2004). The early presence of these skills suggests the existence of preliminary understanding of quantitative relations well before a child is formally taught about mathematically relational concepts. These early quantitative relational processes are expected to be important for the development of fraction knowledge (Pitkethly & Hunting, 1996; Pothier & Sawada, 1983; Sophian, 2007).

That young children are successful with reasoning about quantitative relations at an early age suggests that the later learning of fractions should be a fairly straightforward process (Sophian, 2007). However, this is not the case. Instead, students, and many adults, display a wide range of difficulties with reasoning about fractions, even after years of experience working with them in formal mathematical situations (e.g., Vamvakoussi et al., 2012). Notably, fractions are not equally difficult for all students, but there exist substantial individual differences in primary school children's conceptual knowledge of fractions (Jordan et al., 2013; McMullen et al., in press). These differences indicate that some students are successful in developing a mathematically correct conceptualization of rational numbers. Uncovering early predictors of successful fraction learning can provide valuable information for understanding these different developmental outcomes and instructional practices which may support more successful conceptual change with fractions.

As Hannula and Lehtinen (2005) have previously shown, it is important for the study of developmental differences to not only investigate what children can do with their existing mathematical skills, but also investigate what they spontaneously will do in a situation when the opportunity to reason about the mathematical aspects is available, but they are not guided to notice them. Considering the relevance of quantitative relational processes in fraction knowledge, it is crucial to expand the investigations of these early competences to include children's recognition of quantitative relations to mathematically unspecified settings. However, what little evidence exists that details early competences with quantitative relations has relied on tasks that explicitly guide a child's attention to the mathematical nature of the tasks (e.g., Boyer et al., 2008).[1] The investigation of children's spontaneous (i.e., undirected) focusing on quantitative relations has the potential to reveal how much children spontaneously, without adults' guidance, focus their attention on quantitative relations and use them in action.

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SPONTANEOUS QUANTITATIVE FOCUSING TENDENCIES

Only a part of the development of basic cognitive skills takes place during formal and guided learning situations (Bransford et al., 2006), while many opportunities to practice and further develop recently learnt skills occur during situations that are informal and unguided (Ericsson, 2006; Lehtinen & Hannula, 2006; Lobato, 2012). Despite this, studies of children's mathematical abilities have almost solely relied on tasks in which children's attention is deliberately and overtly guided toward the mathematical features that are of interest to the researchers, minus a few studies on children's use of mathematical operations in everyday situations (e.g., Ginsburg, Inoue, & Seo, 1999). However, it has been suggested that this is a limited view of the causes of differences in mathematical development (Hannula, 2005; Hannula-Sormunen, in press). Children do not practice their early numerical skills only in explicitly mathematical tasks and situations but also in situations that are not explicitly mathematical and in which they are not guided to do mathematics (Hannula & Lehtinen, 2005). Tasks that can capture individual differences in this amount of self-initiated deliberate practice are needed to understand why some children develop better than others with early mathematical processes (cf. *deliberate practice*, Ericsson, 2006). The investigation of *spontaneous quantitative focusing tendencies* allows for the measurement of these differences, expanding the purview of research on mathematical cognition to include the

[1]Duffy, Huttenlocher, and Levine (2005) presented children with a recognition of relations task in which children must match wooden dowels that are the "exact same size." Crucially, these tasks were made explicitly mathematical by guiding the children toward absolute amount between dowel and tube by these instructions. They were shown the correct answer on two practice trials prior to the experimental phase as well.

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measures of everyday, implicit causes of interindividual differences (Hannula, 2005; Hannula & Lehtinen, 2005; McMullen, Hannula-Sormunen, & Lehtinen, 2013).

Hannula and Lehtinen (2005) have found that it cannot be taken for granted that all children—when not guided to do so—pay attention to exact number as a relevant feature of a situation or task with the same frequency. For example, when faced with a mimicking task involving feeding berries to a stuffed bird, Hannula and Lehtinen found substantial individual differences in the spontaneous recognition and use of exact numerosity among young children. These individual differences in children's unguided attention to numerosity as a feature of a set have been described as differences in Spontaneous Focusing On Numerosity (SFON). SFON tendency not only describes children's activities in laboratory tasks, but observation studies show that SFON tendency is also an indicator of children's spontaneous self-initiated practice of their early numerical skills in their everyday surroundings (Hannula, Mattinen, & Lehtinen, 2005). Studies show that SFON tendency (a) is a domain-specific predictor of mathematical achievement (Hannula & Lehtinen, 2005; Hannula, Lepola, Lehtinen, 2010), (b) can be enhanced through social interaction (Hannula, Mattinen, & Lehtinen, 2005), and (c) is a cross-culturally relevant concept for describing the development of early mathematical skills (Edens & Potter, 2013; Kucian et al., 2012). These results suggest that research into children's spontaneous quantitative focusing tendencies fills a crucial gap in research on children's mathematical competencies.

SPONTANEOUS FOCUSING ON QUANTITATIVE RELATIONS (SFOR)

The present study aims to determine whether a more advanced spontaneous quantitative focusing tendency has a similar role as SFON in the development of mathematical skills by examining SFOR. Differences in SFOR may capture fundamental differences in experiences with reasoning about quantitative relations. Similar to enumeration and SFON tendency, it is expected that interindividual differences in the quality and quantity of practice with quantitative relations may have an impact on the later learning of fraction knowledge.

Capturing spontaneous quantitative focusing tendencies, such as SFON, is methodologically difficult (Hannula, 2005; Hannula-Sormunen, in press). One key to the distinction of these tendencies is that the mathematical and procedural skills required in the tasks should be within all participants' skill range; every participant should be capable of mastering the task once his or her attention is focused on the mathematically relevant aspects. This way, if the participant does not produce or exhibit responses based on the mathematical aspects in the task, it can be concluded that he or she did not focus on these aspects. In addition, there should be a number of different, nonmathematical aspects of the task upon which participants could focus their attention. Studies of SFON rely on a number of tasks that are not explicitly mathematical (Hannula, 2005; Hannula & Lehtinen, 2001, 2005; Hannula, Lepola, & Lehtinen, 2010; Hannula, Räsänen, & Lehtinen, 2007). The instructions of the tasks do not make any mention of numerical aspects, nor are any other mathematical or quantitative concepts brought up prior to completion of the tasks. The numerical or arithmetic aspects of the tasks are also hidden behind the playfulness of the tasks by, for example, feeding berries to an exotic bird. In the verbal instructions of SFON tasks, the child is asked to imitate the tester's action in an open manner (the child is told, "Please, do exactly like

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I did"). This way, when a child is found to have paid attention to numerical or quantitative aspects of the task, it can be said to have occurred without guidance from the researcher and therefore to have arisen spontaneously. Thus, the term *spontaneous*, in studies on spontaneous quantitative focusing tendencies, does not refer to the origins of the skills used or how they were acquired but their unguided use in nonexplicitly mathematical situations. Critically, only very few trials can be used in capturing how a child spontaneously pays attention to quantitative aspects in novel situations.

The tasks used in the present study to measure children's SFOR were previously applied in a study of Finnish-speaking children (McMullen, Hannula-Sormunen, & Lehtinen, 2011, 2013). The sample of this Finnish study was found to be heterogeneous in their spontaneous recognition of quantitative relations, having used quantitative relations, numerosity, and nonmathematical aspects to solve the tasks (McMullen et al., 2013). Using quantitative relations to solve these tasks required calculating the relationship between two sets of objects with different units and determining an unknown quantity that would equate the two sets. This relational reasoning is distinct from solving the tasks using only numerosity, in which case the participants merely matched the number of objects irrespective of the size of the unit. Thus, the difference between focusing on relations and focusing on numerosity is that in SFON tasks, the focus needs to be on the total numerosity of a single set, while in SFOR tasks, one needs to focus attention on the relation of two sets of quantities.

While these previous findings indicate the tasks' potential for capturing spontaneous quantitative focusing tendencies, it remains unclear whether the heterogeneity in participants' responses is explained by the children's ability to use the quantitative relations, or whether the ability to use quantitative relations and the spontaneous use were partly independent (McMullen et al., 2013). Previous studies of SFON (e.g., Hannula & Lehtinen, 2005) have been able to dissociate children's SFON from their actual ability to solve the task when explicitly given instructions to use number. The present study aims to create a similar distinction between the ability to use quantitative relations when guided to do so and the spontaneous recognition and use of quantitative relations.

This distinction will allow for the delineation of SFOR in early primary school children. We expect there to be substantial individual differences in SFOR that are not entirely explained by the ability to use quantitative relations. As well, we expect SFOR tendency to be related to basic arithmetic skills. Finally, we expect that SFOR tendency in early primary school will be related to conceptual knowledge of fractions in late primary school.

STUDY 1

The aim of Study 1 is to determine if SFOR is partially distinct from children's ability to use quantitative relations. Thus, it is expected that there are substantial individual differences in children's spontaneous recognition and use of quantitative relations in tasks that can be attended to in a number of ways—including quantitative relations, numerosity, and nonmathematical aspects. It is expected that these individual differences cannot be entirely explained by the ability to use quantitative relations in these tasks when explicitly guided to do so. Finally, it will be determined if there is a relationship between SFOR and arithmetical ability.

Methods

Participants. Included in the study were 84 English-speaking children (42 female) from a socially and economically diverse school in a medium size city (Population ca. = 125,000) in the southern United States. Participants had no diagnosed learning, attentional, or neurological impairments. The children were between the ages of 5 years and 8 months and 9 years and 8 months old ($M = 7$ years, 9 months; $SD = 13.7$ months), in grades kindergarten through third grade. For analysis, children were grouped according to their grade levels: kindergarten ($n = 23$; $M_{AGE} = 6$ years; 4 months), first grade ($n = 20$; $M_{AGE} = 7$ years; 3 months), second grade ($n = 21$; $M_{AGE} = 8$ years; 4 months), and third grade ($n = 20$; $M_{AGE} = 9$ years; 2 months).

Procedure. Children participated in a series of video-recorded tasks in one 25–40 minute session conducted in a secluded room at the child's school during morning lessons prior to their lunch break. The present study will report on a portion of these measures; children's basic numerical knowledge was measured but not included in this study as the measures were far too easy for participants and results displayed strong ceiling effects. The first author conducted all data collection. The ethical guidelines of the University of Turku were followed, and the Ethical board of the University of Turku approved the research plan. In addition, all research permissions from the school board, school administration, teachers, and legal guardians of the children were gathered. Participating children gave their informed assent to participate before beginning.

Tasks.

SFOR. Two tasks were created to investigate children's SFOR (McMullen et al., 2011, 2013). In these tasks, children were asked to model how the experimenter fed stuffed animals different pieces of "bread" (made of foam) or spoonfuls of rice. Prior to testing no mention was made to the children about any numerical, quantitative, or mathematical concepts. Teachers, school personnel, and parents were told that the tasks would measure the children's general quantitative skills and were told not to mention the mathematical nature of the tasks to the children. Before and during the tasks, the experimenter made no mention of amount, relations in size, or number aspects. As well, the tasks were introduced without any mention of their quantitative or mathematical nature. The experimenter always made sure that the child's full attention was on the task and that he or she was motivated to complete all trials. He carefully avoided giving any feedback about the child's performance during the task. In order to examine individual differences, the four trials in each task were presented in the same order for all children.

Alternative strategies were controlled for that could lead to a quantitative relational response, but which did not use the intended mathematical relationships. Thus, the total size of the stimuli

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TABLE 1
Materials and Responses That Indicated the Use of Quantitative Relations for Bread and Rice Tasks

	Material		
Trial	*Experimenter*	*Participant*	*Relational Response*
Bread task			
1	2 halves (Gave 1)	4 fourths	2 pieces
2	6 sixths (Gave 2)	3 thirds	1 piece
3	6 sixths (Gave 3)	4 fourths	2 pieces
4	3 thirds (Gave 2)	6 sixths	4 pieces
Rice task			
1	Set A big (Gave 1)	Small A	2 spoons
2	Set B big (Gave 1)	Small B	3 spoons
3	Set B small (Gave 3)	Big B	1 spoon
4	Set A big (Gave 2)	Small A	4 spoons

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was not easily perceived due to the disarrangement of the bread slices and by forbidding direct side-by-side comparison of the spoonfuls (see below, Bread and Rice tasks). Furthermore, the experimenter flipped his plate over to prevent the participant from using the number of pieces or total amount of bread that remained to make a direct comparison of material used. Thus, in order to produce a response that would indicate the use of quantitative relations (see Table 1), the participants must have recognized the relationship between two distinct quantities, calculated an appropriate answer to an unknown in that relationship, and utilized this answer in their response in the task.

Bread task. In this task, two identical stuffed-animal dogs, "Terry and Jerry," 20 cm in height, were fed "bread" made from foam. For each trial, two similar sets of breads were cut from circular pieces, each piece 6.5 cm in diameter, into different proportions with values less than 1 (see Table 1). One set was placed in front of the child and the other placed in front of the researcher. For example, in the first trial, a plate with the same size bread cut into quarters was placed in front of the child, and the researcher took a plate with the bread cut into halves (see Figure 1). The breads were disarranged on the plate in order to prevent the direct mapping of the area of the two sets of bread. The two sets of bread on each trial were the same color.

To begin the task, the experimenter introduced Terry and Jerry as being two friends who always "do everything exactly the same" and now want the same snack. The child was told that the dogs like bread and was shown an example of an uncut, whole piece of the bread. Two plates of the bread were then produced and placed on the table. The child was told, "Watch carefully what I give Terry, and you give Jerry exactly the same." On the first trial, the experimenter then gave one of the two halves of bread to the first stuffed animal, paused a moment, and turned over his plate, covering up the remaining piece. The child was then told, "Now you give Jerry exactly the same as I gave." After the child completed the trial, the experimenter removed the plates without showing either set of remaining pieces and without giving any feedback to the child. There were altogether four trials in the task.

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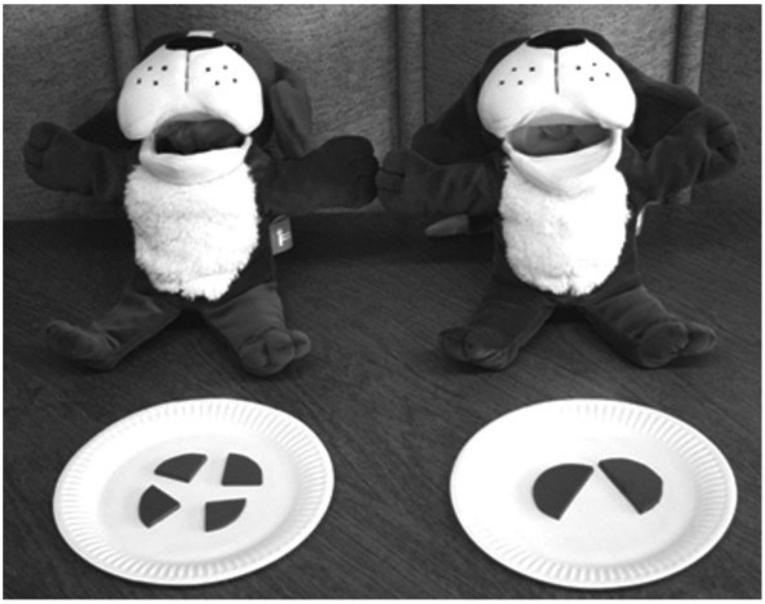

FIGURE 1 Set up and material for Trial 1 of the bread task; the child's plate is on the left.

Rice task. In this task, two identical stuffed monkeys "Glen and Gwen," 20 cm in height, were placed on the table in front of the experimenter and the child. The monkeys were fed rice using different pairs of spoons. The first pair (Set A) was made of plastic, and the spoons were cylinders (Figure 2), with the smaller spoon being half the size of the larger. The small plastic

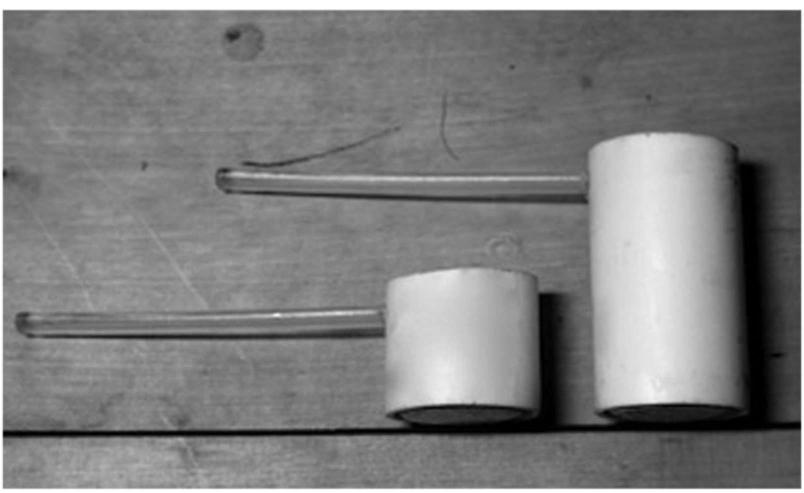

FIGURE 2 Set A spoons for rice task. Larger spoon is double the size of the smaller.

spoon was 3 cm in diameter and 3 cm high, and the large plastic spoon was 3 cm in diameter and 6 cm high. The second pair (Set B) was made of metal and the spoons were rectangular prisms, with the smaller spoon being one third the size of the bigger; the small metal spoon measured 2.5 cm × 2.5 cm × 2.66 cm, and the big metal spoon (three times the size) measured 2.5 cm × 2.5 cm × 8 cm.

Participants were told that the monkeys were brother and sister and like to have the same for lunch. Two large bowls, each roughly two-thirds full of rice, were placed on the table in front of the monkeys. Two smaller opaque bowls were placed in between the bowls of rice and the monkeys. The first pair of spoons, the plastic ones, was held up next to each other for possible comparison, and the child was told that "we will use these spoons" and that "Glen and Gwen always want full spoonfuls." On the first trial, the smaller spoon was placed in the rice bowl in front of the child. The experimenter held the larger spoon and then said that the child should "watch carefully how I give Glen rice, and you give Gwen exactly the same." The experimenter took one spoon of rice from the rice bowl, put it in the empty bowl and then asked the child to "give exactly the same as I gave." After the first trial was complete, Set A spoons were put away, and Set B spoons, the metal pair, were introduced. No feedback was given to the child after they completed a trial, and children were not able to see the total amount of rice given by either themselves or the researcher. There were altogether four trials in the task (see Table 1).

Guided focusing on quantitative relations. Those children who did not spontaneously use quantitative relations on any of the trials on the Bread or Rice tasks were given the guided versions of both tasks at the end of the session. The inclusion of this guided condition allows for the distinction between the children's ability to solve the tasks using quantitative relations and SFOR. Thus, the guided tasks measured children's ability to use quantitative relations.

Altogether, 38 children did not focus spontaneously on quantitative relations on either the Bread or the Rice task. At the end of the testing session, these children were asked to repeat the tasks. In the guided condition, children were told explicitly to solve the tasks by matching the total amounts not the total number of pieces of bread or spoonfuls of rice given by the researcher.

For the guided versions of the tasks, children were told that this time they would be doing something "completely different." This time they should give "exactly the same amount of bread" and that even with the different sizes of the breads or spoons they need to give "just as much." Practice trials were included for both tasks in order to make sure children understood the instructions. On the practice trial for the Bread task, the experimenter gave a whole piece of bread, while the child had halves. On the practice trial for the Rice task, the experimenter gave two spoons with the smaller plastic spoon (half the size), and the child had the bigger spoon. If the child did not give the same amount, the experimenter said that they did not get the same amount of bread or rice and asked to child to try again. Thus, it was indicated that the child should not match the number of pieces given by the researcher but the total amount of bread or rice. The same trials as in the spontaneous versions of the tasks were then repeated.

Arithmetic fluency. The Math Fluency Test (WJ-Math Fluency) of the Woodcock-Johnson Tests of Achievement (WJ III®) was administered (Woodcock, McGrew, & Mather, 2001).

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Children complete as many arithmetic problems (addition, subtraction, multiplication, division) as possible in 3 minutes. The sample's maximum was 91 correct responses. Normalized scores were calculated and used in further analysis.

Analysis.

SFOR tasks. All relevant actions and utterances were coded from video-recordings of the tasks to determine whether a child responded using quantitative relations in that trial or not.

Ninety-nine percent of all bread trials and 98.6% of all rice trials were scored based on the amount of bread or rice the participant gave. Participants made utterances on 10.1% of trials for the bread task and on 6.8% of trials for the rice task. However, utterances were used as a basis for the evaluation of children's spontaneous focusing on only 0.9% of the trials for the bread task and only 1.4% of the trials for the rice task.

The child was scored as responding based on quantitative relations if they gave the same amount of bread or rice as the experimenter gave and/or made any *relational utterances*. Relational utterances were any utterances including relational words or phrases (e.g., "Cause a half is equal to two fourths") or other comments referring either to amounts or relations (e.g., "I gave too much!").

Typical *nonrelational responses* included giving the same number of pieces of bread or spoons of rice as the experimenter, giving all pieces of bread, giving an inordinate amount of spoonfuls of rice, or giving one piece of bread and/or spoonful of rice for every trial. Children's *numerical responses* were coded as well. The child was scored as responding based on numerosity in a trial if she or he gave the same number of pieces of bread or spoons of rice as the experimenter and/or if she or he was observed making any numerical utterances.

The maximum sum score for spontaneously focusing on quantitative relations for each task was 4. Two independent raters analyzed 20% of the trials, representative of the sample, to determine whether a participant spontaneously focused on quantitative relations or not. The two raters agreed on 97% of the trials for both tasks, so the primary rater continued analyzing with the same criteria for the remainder of the trials.

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Results

In order to look at the consistency of participants' relational responses on the SFOR tasks, the average intraclass correlation for responses for the eight trials of both the Bread and Rice tasks were run. These indicated that relational responses were highly consistent ($r = .86$) across the two tasks. Thus, participants' responses on the two tasks were combined for further analysis.

General results reveal that participants completed the tasks using a variety of mathematical aspects, including quantitative relations and numerosity, and nonmathematical aspects in their responses. Table 2 details the descriptive statistics for the tasks reported in this study. These results suggest that participants had different interpretations of the nature of the SFOR tasks. A comparison of responses in the spontaneous and guided conditions and a more detailed analysis of the individual and age-related differences in relational responses follows.

TABLE 2
Descriptive Statistics of the Variables by Grade Level

		Relational Responses					
		Spontaneous		*Guided*[a]		*WJ-Math Fluency*	
	n	M	SD	M	SD	M	SD
All participants	84	4.06	3.14	4.79	2.66	105.9	13.9
Kindergarten	23	1.30	1.99	4.32	3.05	102.6	13.3
First grade	20	3.75	3.52	5.67	1.80	109.3	12.6
Second grade	21	4.90	2.55	4.00	2.16	107.9	15.7
Third grade	20	6.65	1.49	7.00	1.41	103.7	14.0

Note. [a]$n = 38$. There were 21 kindergarteners, nine first graders, six second graders, and two third graders.

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Spontaneous and Guided Focusing on Quantitative Relations. In order to determine whether SFOR is separate from the requisite skills needed to solve the tasks, children's relational responses on the guided version of the bread and rice tasks (Table 2) were compared with their spontaneous relational responses. Results of the guided conditions reveal that those children who did not use quantitative relations in the spontaneous condition nevertheless were able to successfully complete the tasks. The mean frequency of relational responses across both guided tasks was 4.79 ($SD = 2.66$). Three kindergarten children were the only children in the sample who were unable to utilize quantitative relations at all in the guided condition. Overall, those children who completed the guided condition gave relational responses significantly more in the guided condition than in the spontaneous condition, $t(34) = 9.28$, $p < .001$. This finding was replicated when looking at kindergarteners, $t(17) = 5.83$, $p < .001$; first graders, $t(8) = 9.43$, $p < .001$; and second graders, $t(5) = 3.64$, $p = .036$; only two third graders completed the guided condition, and therefore no significant differences were found. While a small number of the youngest children displayed an inability to use quantitative relations on these tasks, these results suggest that, overall, children's differences in spontaneous relational responses were not caused by differences in the ability to solve the tasks using quantitative relations.

Individual and Age-Related Differences in SFOR. Within age groups, there were substantial interindividual differences in spontaneous relational responses (see Table 2 for descriptive statistics). These individual differences were most apparent in kindergarteners' ($M = 1.30$, $SD = 1.99$) and first graders' ($M = 3.75$, $SD = 3.52$) spontaneous relational responses. There is also moderate variation between individuals in second graders' relational responses ($M = 4.90$, $SD = 2.55$). This interindividual variation in spontaneous relational responses indicates that there were substantial individual differences in children's SFOR.

An ANOVA of spontaneous relational responses indicated that there are substantial differences between age groups in SFOR, $F(3, 80) = 17.55$, $p < .001$, $\eta_p^2 = .40$. Tukey-HSD post hoc tests reveal that kindergarteners had significantly less spontaneous relational responses than all other grade levels (Mean differences < –2.45, $p < .01$). As well, first graders significantly differed from third graders in their spontaneous relational responses (Mean difference = –2.90, $p = .002$) but did not differ from second graders (Mean difference = -1.15, $p = .451$). Finally, second graders

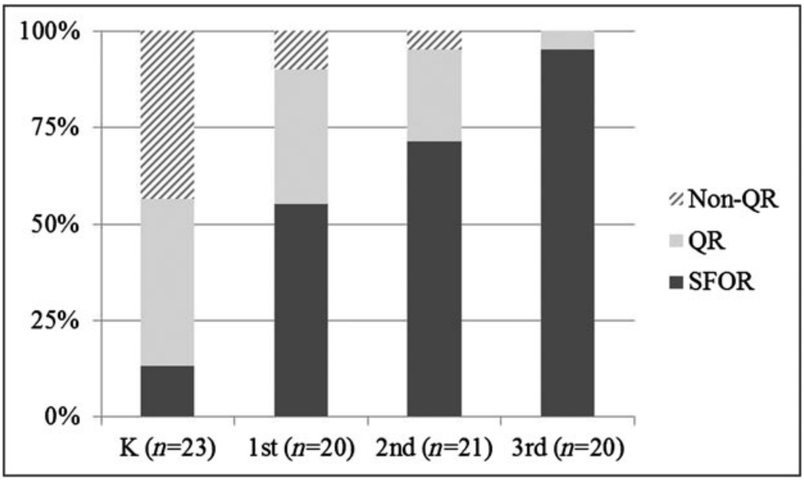

FIGURE 3 Proportion of participants classified into SFOR, QR, and Non-QR groups in each grade.

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and third graders did not significantly differ in spontaneous relational responses (Mean difference = –1.75, $p = .120$).

In order to further investigate SFOR, participants were classified into groups based on their response patterns (Figure 3). Participants who used relational responses on more than half the trials in the spontaneous condition were considered to have reliably displayed SFOR tendency and were therefore placed in the *SFOR* group. Children who were able to utilize quantitative relations on more than half of the trials in the guided condition but did not consistently do so in the spontaneous condition were considered to have the ability to reason about quantitative relations, but lacked SFOR tendency; these children were therefore placed in the *QR* group (i.e., quantitative relations group). Those children who failed to use quantitative relations on at least half of the tasks in both the spontaneous and guided condition were placed in the *Non-QR* group (i.e., no quantitative relations group). Overall, 48 children (57.1%) were classified into the SFOR group, 23 children (27.4%) were classified into the QR group, and 13 children (15.5%) were classified into the Non-QR group.

The means and standard deviations of spontaneous and guided relational responses for each group are available in Table 3. While there were significant differences in the use of relational responses in the guided condition across the three groups, $F(2, 81) = 100.91$, $p < .001$, planned post hoc analysis revealed that this was caused by differences between the Non-QR group and the other groups (QR group: Mean difference = 4.21, $p < .001$; SFOR group: Mean difference = 4.48, $p < .001$). There were no differences in the guided focusing on relational responses between the QR group and the SFOR group (Mean difference = 0.27, $p = .60$).

SFOR in Relation to Arithmetic Skills. An ANCOVA was run in order to measure the between-subject differences in the groups' arithmetic skills after controlling for age in months. The WJ-Math Fluency task was analyzed as the dependent variable (see Table 2). Response

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TABLE 3
Mean and Standard Deviation of Spontaneous and Guided Relational Responses by Response Type Grouping

		Relational Responses			
		Spontaneous		*Guided*	
Response Group	n	M	SD	M	SD
SFOR	48	6.65	1.01	6.68	1.00
QR	23	0.86	1.28	6.41	1.01
Non-QR	15	0.60	1.12	2.20	1.42

type was found to have a significant impact on WJ-Math Fluency, $F(2, 81) = 6.05$, $p = .004$, $\eta_p{}^2 = .131$. Planned pairwise comparisons were used to investigate group-level differences. SFOR participants displayed higher WJ scores than both QR (Mean Difference = 9.3; $p = .029$) and Non-QR (Mean Difference = 16.3; $p < .001$) participants, even after adjusting for age differences. However, there were no significant differences in WJ-Math Fluency scores between the QR and Non-QR groups (Mean Difference = 0.6; $p > .05$).

Conclusions

There were substantial individual and age-related differences in participants' spontaneous relational responses. These differences were not entirely explained by participants' ability to solve the tasks using quantitative relations in the guided condition. SFOR is therefore defined as the spontaneous (i.e., undirected) focusing of attention on quantitative relations and the use of these relations in reasoning in situations that are not explicitly mathematical. Furthermore, SFOR tendency is found to be related to children's arithmetical skills. However, further investigation into SFOR tendency's role in the development of mathematical skills, fraction knowledge in particular, is necessary.

STUDY 2

SFOR tasks were previously implemented in a Finnish cross-sectional study (McMullen et al., 2011, 2013). Study 2 consists of a follow-up of the first graders from this study. Participants completed a measure of fraction knowledge three years later while in fourth grade. This allows for the exploration of the long-term effects of early SFOR tendency on the understanding of fractions. Participants' grades in math class were also collected.

Methods

Participants. Participants were 25 Finnish-speaking children (12 female) from two schools from middle-class areas in a medium sized city in southwest Finland (Population = ca. 175,000).

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Participants had no diagnosed learning impairments and had home languages of Finnish. (Four children were unable to participate in the original study because of diagnoses of learning impairments, a different home language, or no parental permission. One child was not included in the follow-up measures because he or she had previously repeated a grade.) At the start of the study, participants were from the ages of 7 years and 2 months to 8 years and 4 months ($M = 7$ years; 8 months; $SD = 3.5$ months). In Finland, children start school during the fall of the year they turn 7 years-old. Parents' educational attainment revealed a sample representative of urban parents in Finland.

Procedures. Similar procedures were followed as in Study 1 for the collection of SFOR measures and other mathematical skills in first grade (see also McMullen et al., 2013). Follow-up data was collected in the spring of 2013, three years after the initial testing. First, participants' math grades from the Fall 2012 semester were obtained from their teachers. Finally, all participants completed a test of fraction conceptual knowledge in their normal classrooms under the direction of a trained female researcher.

Tasks.

SFOR measures. Participants completed Finnish-language versions of the Bread and Rice tasks. The tasks were identical to the tasks in Study 1.

Mathematical skills in first grade. Participants also completed a number of tasks aimed at measuring their general mathematical abilities, including arithmetic and number sequence skills.

Nonsymbolic arithmetic. Basic arithmetic skills were assessed through a nonsymbolic subtraction task eliciting a verbal response. Materials included glass beads (1.5 cm in diameter) and an opaque box. In the first trial, the children was shown three beads and told, "Here are [3] candies, I'll put them under this box. And then, I'll take some [1] away. How many candies are under the box now?" The task repeated with subtraction problems: 6 – 3, 9 – 4, and 13 – 5. The maximum score for this task was 4.

Number sequence elaboration skills. Children's number word sequence production was tested with a modified version of Salonen and colleagues' (1994) test. This included three subtests:

1. Highest count. Children were asked to count from 1 as high as they can and were stopped at 50. Children were stopped after one failure, and this was then repeated. The highest number counted was recorded, with 50 being the top score.
2. Count backward. Children were asked to count backwards from 4, 8, 12, 19, and 23. Children were stopped after four correct numbers. A child's total number of complete and correct sequences was recorded, with 5 being the highest score possible.

3. Nonsymbolic subtraction. Non-symbolic subtraction skills were assessed through eliciting a verbal response. Materials included glass beads (1.5 cm in diameter) and an opaque box. In the first trial, the children was shown 3 beads and told, "Here are three candies, I'll put them under this box. And then, I'll take one away. How many candies are under the box now?" The task repeated with problems of 6-3, 9-4, and 13-5. The maximum score for this task was 4, when the child gave all correct responses.

Math measures at fourth grade. Follow-up data was collected from the participants when they were in fourth grade. Mathematical achievement was measured by collecting the participants' math grades from the Fall 2012 semester. Furthermore, participants completed a test of their fraction conceptual knowledge.

Fraction test. Students were given 15 minutes to complete the test. The test consisted of six items measuring conceptual knowledge of comparison and ordering of fractions. Items were multiple choice and short answer with three items comparing two fractions (e.g., "Circle the larger fraction. If the numbers are equal circle both: 5/8; 4/3"), and three items ordering three fractions (e.g., "Put the numbers in order from smallest to largest: 6/8; 2/2; 1/3"). Each item was scored as correct or incorrect with a maximum score of 6.

Analysis. The same procedures were followed in analyzing children's responses to the Bread and Rice tasks as were followed in Study 1. A second rater also independently coded 21% of the trials in the Finnish-language analysis in order to determine the reliability of the coding scheme. Agreement on the most advanced strategy used was found on 98% of the trials for the Bread task and 97% of the trials on the Rice task. The primary rater then completed the rest of the analysis using the original scheme.

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Results

Table 4 presents the descriptive statistics for the variables measured at both time points in Study 2. Bread SFOR and rice SFOR scores were only moderately related to each other in the present sample ($\alpha = 0.57$), therefore the two tasks were kept separate. Participants were said to have displayed SFOR based on whether they were able to complete each task using relational responses above what would be expected by chance (two or more relational responses per task). For the Bread task, eight children were classified as displaying SFOR, with 17 who did not. For the Rice task, eight children were classified as displaying SFOR, with 17 who did not.

Next, in order to measure SFOR tendency's contribution to fraction knowledge a ANCOVA was run with Bread SFOR and Rice SFOR categories as independent variables and the Fraction Test as the dependent variable, again controlling for Highest Count, Count Backwards, and Nonsymbolic Subtraction scores. Rice SFOR responses did significantly predict conceptual knowledge of fractions, $F(1, 18) = 7.74$, $p = .01$, $\eta_p{}^2 = .30$, while Bread SFOR responses did not significantly predict any subtest scores. Analysis of the estimated marginal means reveals that those participants

TABLE 4
Descriptive Statistics for Study 2 Measures ($N = 25$)

Variables	M	SD	*Skew*	*Kurt*	*Range*
First Grade					
Relational Responses					
Bread Task	1.24	1.56	0.78	−1.14	0–4
Rice Task	1.08	1.53	1.00	−0.64	0–4
Math Measures					
Highest Count	45.72	9.32	−2.42	5.36	16–50
Count Backward	4.60	0.71	−2.31	6.66	2–5
Nonsymbolic Subtraction	3.68	0.63	−1.86	2.46	2–4
Fourth Grade					
Fraction Test	3.44	2.14	−0.38	−1.34	0–6
Math Grade	8.76	0.66	−1.56	2.84	7–10

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who exhibited higher than chance SFOR responses on the Rice task had significantly higher fraction conceptual knowledge than their peers (mean difference = 3.01).

In order to test whether SFOR tendency predicts math achievement in general, an ANCOVA was run with bread SFOR and rice SFOR category as independent variables and Math Grade as dependent variable, controlling for Highest Count, Count Backwards, and Nonsymbolic Subtraction. Results reveal that neither bread SFOR nor rice SFOR categories significantly predict math grades at fourth grade.

Conclusions

SFOR, as measured in the rice task in first grade, was revealed to predict conceptual knowledge of fractions in fourth grade, while the bread task did not significantly predict fraction knowledge. This finding parallels previous evidence of the contribution of SFON tendency to mathematical skills development. Interestingly, SFOR on either task did not predict math achievement, as tested by given math grades in fourth grade. This finding suggests that SFOR is potentially related to a more specific aspect of mathematical development, namely the development of conceptual knowledge of fractions, not merely a measure of mathematical achievement.

The present study had limited statistical power due to the small number of participants. Even still, those students who spontaneously focused on quantitative relations in the rice task in first grade were found to have done better on a test of fraction conceptual knowledge in fourth grade, even after taking into account mathematical skills in first grade. However, SFOR responses on the bread task did not predict fraction knowledge, suggesting further investigation with a larger sample is necessary.

GENERAL DISCUSSION

We found that there were both substantial age-related and interindividual differences in the extent of 5- to 9-year-old children's SFOR on nonexplicitly mathematical tasks, even when taking into account children's ability to use quantitative relations when they were guided to do so. In

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other words, individual differences in children's performance on SFOR tasks could not be fully explained by the children's ability to use the relations to solve the task when verbally instructed to do so. Furthermore, we found that SFOR tendency predicted conceptual knowledge of fractions in fourth grade. Even after taking into account prior mathematical skills, those children who spontaneously focused on quantitative relations in the rice task had higher fraction knowledge three years later.

SFOR is therefore defined as the spontaneous (i.e., undirected) focusing of attention on quantitative relations and the use of these relations in situations that are not explicitly mathematical. Quantitative relations are defined as the relationship, based on some quantifiable aspect(s), between two or more objects, sets, or symbols. In this way, SFOR can be seen as the spontaneous focusing of attention on a quantifiable relationship between two or more objects, while SFON pertains to focusing on the numerosity of a single set (Hannula and Lehtinen, 2005). The expression of SFOR tendency is used to describe a child's general tendency to spontaneously focus on quantitative relations in a wide variety of task contexts. It is hypothesized that SFOR tendency indicates the amount of spontaneous practice with the reasoning and use of quantitative relations in natural surroundings.

Within age groups there appeared a significant disparity in SFOR when compared with the ability to use quantitative relations. Kindergarten, first, and second grade participants used quantitative relations significantly less in the spontaneous condition than in the guided condition. The delay in SFOR tendency when compared with the ability to use quantitative relations suggests that there are many children who possess the skills needed to use quantitative relations on these tasks but do not do so unless explicitly guided. While previous studies have found that children have a number of well developed skills involving quantitative relations (e.g., Boyer et al., 2008; Frydman & Bryant, 1988; Sophian, 2000), these findings show that it is not entirely clear that all children use these skills spontaneously with the same frequency in situations where they are not explicitly guided to do so.

One possible explanation for the positive association between SFOR tendency and later fraction knowledge is that those children who have a higher SFOR tendency may get more self-initiated deliberate practice with the use of quantitative relations in their everyday environment (cf. deliberate practice; Chi, Glaser, & Farr, 1988; Ericsson, 2006). Previous evidence indicates that there are substantial individual differences in the tendency with which children spontaneously paid attention to exact number when not guided to do so (e.g., Hannula & Lehtinen, 2005). Importantly, it was found that SFON in laboratory tasks was significantly related to children's spontaneous use of exact number in their everyday environment, suggesting that what is captured is a general spontaneous numerical focusing tendency, which is not limited to the SFON laboratory tasks (Hannula et al., 2005). An increase in the amount of self-initiated practice with the use of exact number or quantitative relations may help children to learn and/or utilize formal mathematical skills. Previously, SFON tendency has been found to progress iteratively together with counting skills (Hannula & Lehtinen, 2005). A longitudinal study of SFOR and fraction knowledge with a pre- or posttest design would provide valuable information into the contribution of SFOR tendency to the development of fraction concepts. Furthermore, previous studies show that children can be trained by daycare professionals to increase their tendency of spontaneous focusing on numerical aspects of their environment (Hannula et al., 2005). The success of these previous intervention studies indicates the development of a SFOR training program could be beneficial for children's learning of fractions.

Previously, SFON tendency has been found to be a distinct contributor to numerical skills development (Hannula & Lehtinen, 2005; Hannula et al., 2010), and exact number has been found to be a strong distracter from relational aspects (Jeong, Levine, & Huttenlocher, 2007; Ni & Zhou, 2005; Vamvakoussi et al. 2012). However, in the present study, solely focusing on numerosity was not the most mathematically appropriate response, as there was a more advanced mathematical aspect that could be used to complete the task—quantitative relations. Ericsson and Lehman (1996) argue that those who become experts in a field must constantly be pushing their level of practice to the upper boundary of their knowledge and skills. Acquiring advanced skills requires that the learner is able to identify situations in which they are able to practice these emerging skills (Lobato, 2012). In the present study, children who spontaneously focused on quantitative relations can be seen as engaging in more advantageous self-initiated practice with mathematical skills than those who solely focused on other aspects, such as exact number. Thus, it is not entirely surprising that students who more readily utilize the most mathematically advanced aspects of the Rice task, in this case quantitative relations, display more advanced knowledge of fraction concepts later. However, it is worth noting that those children who displayed SFOR in these tasks must have also spontaneously focused on numerosity.

While most previous studies on quantitative relational processes investigated children's reasoning with proportional relations between two different total amounts, for example two blue marbles and four red, versus one blue and two red (e.g., Boyer et al., 2008), the present study utilizes tasks in which the proportional relation is between the same amount (e.g., 2/6 of a circle versus one third of the same size circle). Still, in order to solve the tasks using the same amount of bread or rice, participants were obligated to establish a relationship between the participant's and experimenter's breads or spoons, which were different sizes. In addition, the direct mapping of the target amount of bread or rice was hindered in a number of ways. For the bread task, the breads were arranged on the plates in such a way that no direct mapping could be made between the two sets. The researcher's bread was hidden from view after his or her animal had been fed as well. For the rice task, the children were not allowed to make a direct comparison of the spoonfuls of rice, and the total amount of rice placed in the bowls was not visible. The inability of the participants to make a direct magnitude comparison between the breads or spoons supports the use of the term quantitative relations. The unguided nature of the SFOR tasks requires consideration of the threshold for recognition and use of the quantitative relations. In attempting to capture individual differences in children's tendency to use quantitative relations in these tasks, it was determined that having different amounts would overly direct children's attention to the proportional relations.

In the tasks measuring SFOR, we aim to provide a similar level of ambiguity of the quantitative relations in these tasks as the ambiguity of numerosity was in previous studies measuring SFON. Previous studies using the same tasks in 5- to 7-year-old children suggest that there was a sufficient level of ambiguity (McMullen et al., 2011, 2013). Children were found to use a variety of aspects to solve the bread and rice tasks, including quantitative relations, numerosity, and/or nonmathematical aspects. While spontaneous responses based on the recognition of exact number (e.g., giving the same number of pieces of bread as the researcher despite the sizes) were the most common nonrelational responses, there were a sizable number of participants who also used nonmathematical aspects to solve the tasks. These nonmathematical aspects included the shape of the breads, the manner of feeding, or giving one piece or spoon every time (McMullen et al.,

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2011). These results suggest that it cannot be assumed that all children equally recognize the relevance of the quantitative relations in these tasks when not guided to do so.

The finding that those children who spontaneously focused on quantitative relations in first grade had higher conceptual knowledge of fractions in fourth grade has potential implications for the investigation of conceptual change with fractions (McMullen et al., 2014; Vamvakoussi et al., 2012; Vamvakoussi & Vosniadou, 2004). Natural number conceptualizations have been found to be a hindrance in learning about fractions. The finding that those children who more readily pay attention to quantitative relations have a better-developed conception of fraction representations in fourth grade encourages further studies on how SFOR tendency relates to the development of fractions. In particular, it is of value to determine if these findings could be replicated in a larger sample.

Methodologically, the present study demonstrates an experimental means to probe into SFOR in a situation or task. It is encouraging that it is possible to capture this more complex spontaneous quantitative focusing tendency, extending the SFON study methodology outlined first time in Hannula and Lehtinen (2001, 2005). Study 1 also shows a partial dissociation of SFOR from the requisite skills. The present SFOR tasks displayed a ceiling effect with older children, suggesting that the relational aspects embedded in the tasks were too apparent for this age group. Further studies should be conducted to more optimally adjust the perceptibility of these aspects for a given age. Despite this, both studies reported here were able to capture substantial differences in how participants attended to these tasks, and these results suggest that studies of individual differences in SFOR tendency can help explain developmental differences in mathematical skills, particularly in fraction knowledge.

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REFERENCES

Boyer, T. W., Levine, S. C., & Huttenlocher, J. (2008). Development of proportional reasoning: Where young children go wrong. *Developmental Psychology*, *44*(5), 1478–90.

Bransford, J. D., Barron, B., Pea, R., Meltzoff, A., Kuhl, P., Bell, P., . . . Sabelli, N. (2006). Foundations and opportunities for an interdisciplinary science of learning. In K. Sawyer (Ed.), *The Cambridge handbook of the learning sciences* (pp. 19–34). New York, NY: Cambridge University Press.

Chi, M. T. H., Glaser, R., & Farr, M. J. (Eds.). (1988). *The nature of expertise*. Hillsdale, NJ: Erlbaum.

Duffy, S., Huttenlocher, J., & Levine, S. (2005). It is all relative: How young children encode extent. *Journal of Cognition and Development*, *6*(1), 51.

Edens, K. M., & Potter, E. F. (2013). An exploratory look at the relationships among math skills, motivational factors and activity choice. *Early Childhood Education Journal*, *41*(3), 235–243.

Ericsson, K. A. (2006). The influence of experience and deliberate practice on the development of superior expert performance. In K. A. Ericsson, N. Charness, P. Feltovich, & R. R. Hoffman (Eds.), *Cambridge handbook of expertise and expert performance* (pp. 685–706). Cambridge, UK: Cambridge University Press.

Ericsson, K. A., & Lehman, A. C. (1996). Expert and exceptional performance: Evidence of maximal adaptation to task constraints. *Annual Review of Psychology*, *47*, 273–305.

Feigenson, L. (2007). The equality of quantity. *Trends in Cognitive Sciences*, *11*(5), 185–187.

Frydman, O., & Bryant, P. (1988). Sharing and the understanding of number equivalence by young children. *Cognitive Development*, *3*, 323–339.

Fuson, K. (1988). *Children's counting and concepts of number*. New York, NY: Springer Verlag.

Gallistel, C. R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition*, *44*, 43–74.

Ginsburg, H. P., Inoue, N., & Seo, K.-H. (1999). Preschoolers doing mathematics: Observations of everyday activities. In J. Copley (Ed.), *Mathematics in the early years* (pp. 88–99). Reston, VA: National Council of Teachers of Mathematics.

Hannula, M. M. (2005). *Spontaneous focusing on numerosity in the development of early mathematical skills*. Turku, Finland: Painosalama.

Hannula, M. M., & Lehtinen, E. (2001). Spontaneous tendency to focus on numerosities in the development of cardinality. In M. Panhuizen-Van Heuvel (Ed.), *Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education* (pp. 113–120). Drukkerij Wilco, The Netherlands: Amersfoort.

Hannula, M. M., & Lehtinen, E. (2005). Spontaneous focusing on numerosity and mathematical skills of young children. *Learning and Instruction*, *15*, 237–256.

Hannula, M. M., Lepola, J., & Lehtinen, E. (2010). Spontaneous focusing on numerosity as a domain-specific predictor of arithmetical skills. *Journal of Experimental Child Psychology*, *107*, 394–406.

Hannula, M. M., Mattinen, A., & Lehtinen, E. (2005). Does social interaction influence 3-year-old children's tendency to focus on numerosity? A quasi-experimental study in day-care. In L. Verschaffel, E. De Corte, G. Kanselaar, & M. Valcke (Eds.), *Powerful learning environments for promoting deep conceptual and strategic learning. Studia Paedagogica*, 41 (pp. 63–80). Leuven, Belgium: Leuven University Press.

Hannula, M. M., Räsänen, P., & Lehtinen, E. (2007). Development of counting skills: Role of spontaneous focusing on numerosity and subitizing-based enumeration. *Mathematical Thinking and Learning*, *9*, 51–57.

Hannula-Sormunen, M. M. (in press). Spontaneous focusing on numerosity and its relation to counting and arithmetic. In A. Dowker & R. Cohen Kadosh (Eds.), *Oxford handbook of mathematical cognition*. Oxford, UK: Oxford University Press.

Jeong, Y., Levine, S., & Huttenlocher, J. (2007). The development of proportional reasoning: Effect of continuous vs. discrete quantities. *Journal of Cognition and Development*, *8*, 237–256.

Jordan, N. C., Hansen, N., Fuchs, L. S., Siegler, R. S., Gersten, R., & Micklos, D. (2013). Developmental predictors of fraction concepts and procedures. *Journal of Experimental Child Psychology*, *116*(1), 45–58.

Kucian, K., Kohn, J., Hannula-Sormunen, M. M., Richtmann, V., Grond, U., Käser, T., ... von Aster, M. (2012). Kinder mit dyskalkulie fokussieren spontan weniger auf anzahligkeit [Children with developmental dyscalculia focus spontaneously less on numerosities]. *Lernen und Lernstörungen*, *1*(4), 241–253.

Lehtinen, E., & Hannula, M. M. (2006). Attentional processes, abstraction and transfer in early mathematical processes. In L. Verschaffel, F. Dochy, M. Boekaerts, & S. Vosniadou (Eds.), *Instructional psychology: Past, present and future trends (Advances in learning and instruction series)* (pp. 39–55). Amsterdam, The Netherlands: Elsevier.

Lobato, J. (2012). The actor-oriented transfer perspective and its contributions to educational research and practice. *Educational Psychologist*, *47*(3), 232–247.

McCrink, K., & Wynn, K. (2007). Ratio abstraction by 6-month-old infants. *Psychological Science*, *18*, 740–745.

McMullen, J., Hannula-Sormunen, M. M., & Lehtinen, E. (2011). Young children's spontaneous focusing on quantitative aspects and their verbalizations of their quantitative reasoning. In B. Ubuz (Ed.), *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education* (pp. 217–224). Ankara, Turkey: PME.

McMullen, J., Hannula-Sormunen, M. M., & Lehtinen, E. (2013). Young children's recognition of quantitative relations in mathematically unspecified settings. *The Journal of Mathematical Behavior*, *32*(3), 450–460.

McMullen, J., Laakkonen, E., Hannula-Sormunen, M. M., & Lehtinen, E. (2014). Modeling the developmental trajectories of rational number concept(s): A latent variable approach. *Learning and Instruction*. doi:10.1016/j.learninstruc.2013.12.004

Merenluoto, K., & Lehtinen, E. (2004). Number concept and conceptual change: Towards a systemic model of the processes of change. *Learning and Instruction*, *14*, 519–534.

Mix, K. S., Levine, S. C., & Huttenlocher, J. (1999). Early fraction calculation ability. *Developmental Psychology*, *35*, 164–174.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education.

Ni, Y., & Zhou, K. Y. (2005). Teaching and learning fraction and rational numbers: The origins and implications of whole number bias. *Educational Psychologist*, *40*, 27–52.

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Obersteiner, A., Van Dooren, W., Van Hoof, J., & Verschaffel, L. (2013). The natural number bias and magnitude representation in fraction comparison by expert mathematicians. *Learning and Instruction*, *28*, 64–72.
Pitkethly, A., & Hunting, R. P. (1996). A review of recent research in the area of initial fraction concepts. *Educational Studies in Mathematics*, *30*, 5–38.
Pothier, Y., & Sawada, D. (1983). Partitioning: The emergence of rational number ideas in young children. *Journal for Research in Mathematics Education*, *14*, 307–317.
Resnick, L. B. (1992). From protoquantities to operators: Building mathematical competence on a foundation of everyday knowledge. *Analysis of Arithmetic for Mathematics Teaching*, *19*, 275–323.
Salonen, P., Lepola, J., Vauras, M., Rauhanummi, T., Lehtinen, E., & Kinnunen, R. (1994). *Diagnostiset testit 3. Motivaatio, metakognitio ja matematiikka. [Diagnostic tests 3. Motivation, metacognition and mathematics]*. Turku, Finland: Centre for Learning Research, University of Turku.
Siegler, R. S., Fazio, L. K., Bailey, D. H., & Zhou, X. (2013). Fractions: The new frontier for theories of numerical development. *Trends in Cognitive Sciences*, *17* (1), 13–19.
Singer-Freeman, K. E., & Goswami, U. (2001). Does half a pizza equal half a box of chocolates? Proportional matching in an analogy task. *Cognitive Development*, *16*, 811–829.
Sophian, C. (2000). Perceptions of proportionality in young children: Matching spatial ratios. *Cognition*, *75*, 145–170.
Sophian, C. (2007). *The origins of mathematical knowledge in childhood*. New York, NY: Lawrence Erlbaum.
Spinillo, A. G., & Bryant, P. (1999). Proportional reasoning in young children: Part–part comparisons about continuous and discontinuous quantity. *Mathematical Cognition*, *5*, 181–197.
Vamvakoussi, X., Van Dooren, W., & Verschaffel, L. (2012). Naturally biased? In search for reaction time evidence for a natural number bias in adults. *The Journal of Mathematical Behavior*, *31*(3), 344–355.
Vamvakoussi, X., & Vosniadou, S. (2004). Understanding the structure of the set of rational numbers: A conceptual change approach. *Learning and Instruction*, *14*, 453–467.
Wing, R. E., & Beal, C. R. (2004). Young children's judgments about the relative size of shared portions: The role of material type. *Mathematical Thinking and Learning*, *6*, 1–14.
Woodcock, R. W., McGrew, K. S., & Mather, N. (2001). *Woodcock-Johnson tests of achievement*. Itasca, IL: Riverside Publishing.
Wynn, K. (1992). Addition and subtraction in human infants. *Nature*, *358*, 749–750.
Wynn, K. (1998). Numerical competence in infants. In C. Donlan (Ed.), *The development of mathematical skills* (pp. 3–25). East Sussex, UK: Psychology Press.

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