Preschool spontaneous focusing on numerosity predicts rational number conceptual knowledge six years later

Jake McMullen [a*], Minna M. Hannula-Sormunen [ab], & Erno Lehtinen [a]

[a] Centre for Learning Research and Department of Teacher Education, University of Turku, Finland

[b] Turku Institute for Advanced Studies, University of Turku, Finland

[*] Corresponding author

Address: Centre for Learning Research, Turun yliopisto, 20014, Turku, Finland

Email: jake.mcmullen@utu.fi

Phone: +358 2 333 8606

Fax: + 358 2 333 8497

Abstract

Recent evidence suggests that early natural number knowledge is a predictor of later rational number conceptual knowledge, even though students' difficulties with rational numbers have also been explained by the overuse of natural number concepts – often referred to as the natural number bias. Hannula and Lehtinen (2005) have shown that children's tendency to spontaneously focus on numerosity (SFON) predicts the development of natural number and arithmetic skills. The present study follows 36 children from the age of six years to the age of twelve years in order to determine how preschool SFON tendency and number sequence skills are related to rational number conceptual knowledge at the age of twelve years. The results show that children's SFON tendency before school age is a strong predictor of later rational numbers conceptual knowledge, even after controlling for preschool number sequence skills. This finding has implications for the understanding of how the transition from reasoning about natural number concepts to reasoning about rational numbers may be influenced by children's self-initiated practice with numbers in everyday situations.

1 Introduction

The difficulties students and many educated adults face with understanding rational numbers has been a subject of deep and extensive study within mathematics education and educational psychology research (Confrey et al. 2009; Jordan et al. 2013; Mazzocco and Devlin 2008; McMullen et al. 2014b; Merenluoto and Lehtinen 2004; Siegler et al. 2013; Vamvakoussi et al. 2012; Vamvakoussi and Vosniadou 2004, 2010; Van Hoof et al. 2015; Vosniadou 2014). One particularly difficult hurdle that must be overcome in understanding rational numbers is the inappropriate use of rules based on the natural number concept when reasoning about rational numbers (e.g. Vamvakoussi and Vosniadou 2004). This overuse of natural number concepts may be explained by a lack of inhibition of intuitive conceptions about numbers (Vamvakoussi et al. 2012; Van Hoof et al. 2013a, 2015; Obersteiner et al. 2013). In this way, it is argued that reasoning about rational numbers is often negatively influenced by this so-called natural number bias, which has its roots in the privileged role natural numbers play in everyday situations.

Understanding rational numbers and the ability to use them in problem solving is not only a key contributor to later mathematical knowledge (Siegler et al. 2012), but also is key for the understanding of a wide range of aspects of everyday life (Reyna and Brainerd 2007). Despite this, few studies have investigated early predictors of rational number conceptual knowledge (Bailey et al. 2014; McMullen et al. 2014a; Vukovic et al. 2014). While rational number conceptual knowledge includes a wide range of dimensions (Hallett et al. 2010; Van Hoof et al. 2015), in the frames of this study we refer to rational number conceptual knowledge as covering particularly those aspects of the size and density of rational numbers which are incongruent with natural numbers. There is sufficient evidence that early spontaneous quantitative focusing tendencies play a role in the development of mathematical skills (e.g. Hannula and Lehtinen 2005). Evidence suggests that the tendency of spontaneous focusing on numerosity (SFON) is a domain-specific predictor of natural number knowledge from the ages of six to eight (Hannula et al. 2010). However, while it is known that SFON tendency promotes the development of natural number knowledge, it remains an important question whether this influence also causes difficulties when reasoning about rational numbers, or if those with a stronger early SFON tendency are more successful with reasoning about rational numbers, suggesting that they are more able to transition from reasoning about natural number concepts to rational number concepts.

1.1 Natural number bias and the development of rational number conceptual knowledge

The natural number bias has important implications for the development of rational number conceptual knowledge, and its impact on the reasoning and use of rational numbers may never fully disappear (Vosniadou 2014). Ni and Zhou (2005) presented an extensive review of the development and implications of the natural number bias. More recently, however, a number of studies have highlighted how the natural number bias affects even adults' reasoning about, and activities with, rational numbers (Vamvakoussi et al. 2012; Van Hoof et al. 2013a; Obersteiner et al. 2013). Specifically, a number of recent studies have highlighted the role inhibition may play in overcoming the natural number bias when reasoning about rational numbers (ibid.). Thus, it has been argued that in order to reason successfully about rational number concepts students, and even many adults, must actively halt intuitive reasoning that uses natural number concepts (e.g. bigger numbers indicates larger magnitude) before solving the task using mathematically correct concepts (e.g. magnitude is determined by relationship between numerator and denominator).

The origins of the natural number bias are still open for debate, and it has been argued to arise from a number of environmental and biological sources (Ni and Zhou 2005). Innate capacities and cultural tools both seem to afford natural numbers a more privileged position in students' reasoning. Some evidence suggests that individual numerical magnitudes may have a discrete representation on the mental number line, suggesting a more innate underpinning for the natural number bias (Gallistel and Gelman 1992; Feigenson et al. 2002). However, some features of the mental number line suggest otherwise (Dehaene et al. 2008). More concretely, the influence of cultural tools, including language, on the natural number bias is more obvious. Children's and even adults' everyday experiences are more often seen through a lens of natural numbers; concepts related to rational numbers are rarely highlighted in cultural tools such as finger counting (Carey 2004; Greer 2004; Andres et al. 2008). Furthermore, early educational experiences almost solely deal with whole numbers and their features.

All of these features of the size of numbers and the use of number in most activities lead to a problematic transition from thinking of numbers as discrete, neatly ordered entities, to the continuous, densely ordered abstractions that constitute the rational numbers (e.g. Vamvakoussi and Vosniadou 2010). In fact, the adjustment from reasoning about natural numbers to rational numbers is so difficult that conceptual change processes may be necessary in order to fully grasp rational number concepts (Vamvakoussi and Vosniadou 2004; Vosniadou and Verschaffel 2004; Vamvakoussi et al 2011; Vosniadou 2014; McMullen et al. 2014b). Indeed, a number of studies have shown that conceptual change theory suits the description of the development of rational number conceptual knowledge, of concepts surrounding the size and density of fractions and decimals (e.g. ibid.). As well, often some prior concepts may never completely disappear and remain either partially dormant or well suppressed when reasoning about related topics (Vamvakoussi et al. 2012; Van Hoof et al. 2013a; Obersteiner et al. 2013; Vosniadou 2014).

Indeed, it appears that even after successfully developing a mathematically correct concept of the nature of number, less mathematically correct reasoning may never fully disappear and will occasionally peek out in intuitive judgments of, for example, the magnitude of fractions. This can be seen in studies of university students, who were no less accurate in comparing the magnitude of fractions that were incongruent with natural number features (e.g. 1/3 vs. 1/4), but took significantly longer to solve these tasks, than on congruent items (e.g. 1/3 vs. 2/3) (Vamvakoussi et al. 2012). As well, these participants were less successful on incongruent items regarding density concepts, and slower at responding, though not significantly so. Surprisingly, some of these results can also be found in expert mathematicians, suggesting that even the most robust mathematical concepts can be overrun by a natural number bias (Obersteiner et al. 2013).

1.2 Spontaneous focusing on numerosity

Spontaneous focusing on numerosity (SFON) is increasingly recognized as an important contributor to mathematical development (Edens and Potter 2013; Hannula and Lehtinen 2005; Hannula et al. 2010; Hannula-Sormunen 2014; Kucian et al. 2012; Potter 2009). SFON tendency is defined as the unguided recognition and use of numerosities in a situation that is not explicitly mathematical. Thus, SFON refers to the spontaneous focusing of attention on numerosity in a situation and not the spontaneous development of any skills or knowledge. It is important to note that individual differences in SFON tendency have been found to not be entirely explained by enumeration ability, suggesting that these differences are the result of attentional processes and not overall skill with enumeration. SFON tendency has been found to be a domain-specific contributor to mathematical development over a six-year period, from six to twelve years old (Hannula-Sormunen et al. submitted). SFON tendency is explained as impacting mathematical development through the promotion of children's self-initiated practice with related mathematical aspects of their everyday environment (Hannula 2005; Lehtinen and Hannula 2006). Children's everyday environment provides opportunities to practise with natural number concepts and this practice presumably has an influence on the natural number bias. The fact that there are individual differences in the tendency to recognize and utilize these mathematical aspects suggests that early SFON tendency might have an influence on the development of rational number conceptual knowledge. The question remains, however, is this a positive or negative relationship?

SFON is positively related to the development of enumeration and number sequence skills before school age (Edens and Potter 2013; Hannula and Lehtinen 2005; Hannula et al. 2007; Hannula 2005; Hannula-Sormunen et al. submitted; Potter 2009), and SFON tendency can be enhanced through social activities in preschool (Hannula et al. 2005; Mattinen 2006). The development of SFON and counting skills from three to six years of age has been shown to be reciprocal (Hannula and Lehtinen 2005). SFON tendency in Kindergarten is a domain-specific predictor of arithmetical, but not reading, skills assessed at the end of Grade 2 (Hannula et al. 2010). It is important to note that individual differences in SFON are not explained by a lack of enumeration skills or other cognitive skills (Hannula 2005; Hannula and Lehtinen 2005). Furthermore, the tendency to focus on other aspects of tasks, such as spatial locations, is a separate process which does not impact the relationship between SFON and counting skills (Hannula et al. 2010). Finally, children's individual differences in SFON before school age have been found to predict mathematical skills six to seven years later in primary school (Hannula-Sormunen et al. submitted). However, the present study is the first to investigate whether early SFON tendency predicts rational number conceptual knowledge.

1.3 Natural number skills

Previously, both SFON and number sequence skills in six-year-olds have been found to be partially related to each other; as well, both have been found to predict general school mathematical ability (Hannula-Sormunen et al. submitted). Number sequence skills, for example counting forward and backward from given numbers, reflect the mental representation of the natural number line (Fuson 1988). Number sequence skills before school age are related to arithmetical skills in kindergarten (Secada et al. 1983; Fuson 1988; Johansson 2005), and they predict later arithmetical skills at school age (Hannula 2005; Lepola et al. 2005; Koponen et al. 2007). In the study by Aunola and colleagues (2004), preschool number sequence and counting skills predicted the overall level and the developmental rate of arithmetical skills. Furthermore, verbal counting skills are key to the development of the natural number concept (Fuson 1988; Gallistel and Gelman 1992). Recently, conceptual knowledge of natural numbers in six-year-olds has been found to be a predictor of conceptual knowledge of fractions in early adolescence (Jordan et al. 2013; Bailey et al. 2014). Thus, the connection between number sequence skills and conceptual knowledge of natural number suggests that number sequence skills at an early age may also be a predictor of rational number conceptual knowledge.

1.4 Research questions and hypotheses

The aim of the present study is to determine how preschool SFON tendency and number sequence skills are related to rational number conceptual knowledge at the age of twelve years. The increased practice with enumeration afforded by a higher SFON tendency is expected to strengthen the noticing and use of natural numbers. Thus, when first considering the relationship between the rational number conceptual knowledge and SFON tendency, an intuitive expectation is that the increase in the fluent use of natural numbers afforded by SFON would lead to greater use of natural number concepts when reasoning about rational numbers. Therefore SFON tendency in early years would be negatively related to rational number conceptual knowledge, especially those aspects which are incongruent with natural number concepts (Hypothesis 1).

However, natural number knowledge in six-year-olds has been found to be a predictor of fraction knowledge in early adolescence, when using rational number items that were both congruent and incongruent with natural number knowledge (Bailey et al. 2014). SFON tendency has previously been found to share a great deal of variance with natural number knowledge (e.g. Hannula et al. 2010). This would lead to the expectation that in fact early SFON tendency, as well as number sequence skills, would predict those aspects of rational number conceptual knowledge that are incongruent with natural numbers (Hypothesis 2).

Finally, SFON tendency has been argued to contribute to the development of mathematical skills through an increase in self-initiated practice with numbers in everyday situations (Hannula and Lehtinen 2005). SFON tendency has been found to correlate with the use of whole numbers in everyday situations in preschool children (Hannula et al. 2005). It is possible that a higher SFON tendency frequently leads children to situations in which natural numbers are recognized as insufficient for describing relevant numerical relationships, such as sharing two cookies among three friends. These everyday experiences may therefore lead to an increase in the awareness of the affordances and, more importantly, the limitations of the natural number system, in turn supporting a smoother and more consistent transition from natural to rational number concepts. In contrast, typical measures of natural number skills, such as number sequence elaboration skills, capture skills learned in more well-defined activities, such as learning the counting sequence. Therefore, it is possible that the unique contribution of early SFON tendency is a stronger predictor of rational number conceptual knowledge than the unique contribution of number sequence skills (Hypothesis 3).

2 Methods

2.1 Participants

Participants were 36 Finnish-speaking children in preschool who had no developmental delays (18 girls). Children were from the ages of 5 years and 9 months to 6 years and 3 months old (*M* = 6 years, 0 months; *SD* = 1.3 months) when the data collection began. In Finland, children begin school in the fall of the year they turn seven. Parents' education and income levels were representative of the urban Finnish population for the same age group. Before the study began parents gave informed consent for their children's participation and, in addition, children gave informed assent at the age of twelve. Ethical guidelines of the University of Turku were followed and the school and daycare administration approved of the study before it began.

2.2 Procedure

Children's SFON and number sequence elaboration skills were tested at the age of six years. Furthermore, six years later, at the age of twelve, their rational number conceptual knowledge was assessed. The tasks were given individually in a familiar room at the child's kindergarten or school. Tasks were presented in the same order for all children. In order to make sure the mathematical nature of the study was not revealed to participants, SFON tasks were completed before any other numerical tasks.

2.3 Tasks

2.3.1 SFON

Detailed discussion of the nature of SFON measurements can also be found in a number of texts by the second and third authors (Hannula 2005; Hannula and Lehtinen 2005). In the present study, when presenting the SFON tasks, the experimenter sat next to the child, on the child's right side at the table. She first made sure the participant's attention was on the task. The experimenter did not use any phrases or other contextual hints that would suggest that the task was somehow mathematical or quantitative. Furthermore, no feedback about the child's performance was given during the task situation. The number of items used in the task was always well within participants' enumeration level, thus allowing differentiation of the variable of spontaneous focusing on numerosity from enumeration skills. All of a participant's (a) utterances involving number words, (b) use of fingers to express numbers, (c) counting acts, like a whispered number word sequence and indicating acts by fingers and/or head, d) other comments referring either to quantities or counting (e.g. "Oh, I miscounted them"), or e) interpretation of the goal of the task as quantitative (e.g. "I stamped an accurate number of them") were noted. The participant was scored as focusing on numerosity if she or he produced the correct number of items on that trial, and/or if she or he was observed doing any of the aforementioned (a–e) quantifying acts. All SFON tasks included three separate trials, which were coded as SFON or not independently of each other. The maximum score for each task was three. Inter-rater reliability of two independent raters, who scored 30 percent of the video recordings of the SFON tasks, varied from .96 to 1.00.

Imitation task. Materials included a post-box, which was on the table in front of the child, a set of ten, red, closed, blank envelopes placed to the left of the post-box, and a pile of ten, blue, closed, blank envelopes on the right, 10 cm from the red envelopes.

The experimenter first pointed to the post-box and the piles of envelopes, and said: "This is a post-box, and there are red envelopes here and blue envelopes here. Watch carefully what I do, and then you do just like I did." The experimenter then put, one at a time, two red envelopes and one blue envelope into the post-box. Then the child was told: “Now you do exactly like I did.” On the second test item two red and three blue envelopes were used and on the third item, 3 blue and 2 red envelopes.

Model task. Materials were A4-sized line-drawings of dinosaurs, and three stamps: a circular stamp called a “node” and two types of triangular stamps called “spikes”. Similar pictures of dinosaurs were in front of the experimenter and the child. The experimenter then explained that she would make her dinosaur into a model and then turn the model upside down so that the child could not see the model. Then the child was told to make his or her dinosaur look exactly the same as the model one. The experimenter then said: “Now, watch carefully, I am making this dinosaur into a model.” After stamping six nodes beginning at the head of the dinosaur, the experimenter turned her model upside down and gave the stamp to the child, and said: “Now, you make your dinosaur look exactly the same as the model dinosaur.” The procedure was repeated with a seven-spiked dinosaur as the second item and a five-spiked dinosaur as the third item.

Finding task. Materials were 27 cone-shaped wooden hats (diameter 16mm at the bottom, and height 16mm) placed side-by-side in a semicircle (28cm in diameter) on a mat, a small troll, and his gold ingot (4mm x 4mm x 1mm). The experimenter introduced the materials and said: “Now you may watch while I hide this gold ingot under a hat. Then you can tell the troll where his gold ingot is hidden. Now watch under which hat I hide the gold ingot.” The experimenter lifted up the target hat and placed the gold ingot in its place. Then the experimenter counted silently to four, allowing the participant time to note the location of the gold ingot, and covered the gold ingot with the hat. To prevent the participant from visual marking the target hat, he/she was asked to look up at the right and left corners of the ceiling. The participant was also not allowed to use his/her finger to mark the target hat. Then the participant was asked to indicate which hat the gold ingot was under. If the participant did not find the gold ingot, the experimenter lifted the target hat, and removed the gold ingot in order to start the next trial. The ingot was hidden under the 6th hat from the right, under the 7th hat from the left, and finally under the 5th hat from the right.

The participant was interviewed after the task about the strategies she or he used to remember where the gold ingot was hidden, in order to determine whether the child had used numerical information of some kind to find the target hat. The SFON scoring of the task was based on analyses of video-recordings, the interview, and the hat pointed to by the child. The participant was not given a SFON score if the analyses of the child's behaviour and the interview revealed that the participant had found the target hat by accident, irrespective of whether the target hat was found or not.

SFON sum score. A SFON tendency sum score was created by applying item analysis with the aim of identifying a reliable uni-dimensional SFON score with as little random variance as possible (Metsämuuronen 2006). Observing the inter-total correlations for all items across the three tasks, three of the nine original items had low correlation coefficients (< .2) with the other items. Subsequently, the reliability was calculated as these three items were removed one-by-one, with each step leading to a higher reliability. In all, 6 items from the three SFON tasks were included in the final sum variable (Items 1 and 3 from the Imitation task, Item 1 from the Model task, and items 1, 2, and 3 from the Finding task). The maximum of the sum score was 6. The resulting reliability was acceptable (Cronbach's alpha = .72).

2.3.2 Number sequence elaboration skills

Number sequence elaboration skills forwards and backwards. A modified version of the test of Salonen et al. (1994) was used to assess children's number sequence elaboration skills (e.g. Fuson 1988). First the child was asked to count forwards from 3, 8, 12, 19, and 24 (at least 4 correct number words), and second from 2 to 7, from 6 to 11, from 14 to 19, and from 18 to 25, to assess his or her number sequence elaboration skills forwards. Correspondingly, the items for counting backwards were as follows: counting backwards from 4, 8, 12, 19, and 24 (at least 4 correct number words), and counting backwards from 6 to 3, from 13 to 8, from 19 to 15, and from 23 to 18. The child got a score for each trial in which he or she recited all required numbers correctly. Maximum of the task was 18. Cronbach's alpha for the measure in a robust sample reported by Salonen et al. (1994) was .88.

2.3.3 Rational number test

The rational number test (RNT) consisted of 28 multiple choice and short answer items. Participants completed the test in a one-on-one situation. In order to capture the range of participants' knowledge, the testing situation was modelled on the concept of engaging the participant's zone of proximal development in their rational number conceptual knowledge (Brown 1992). Therefore, the experimenter asked probing questions based on the participant's responses after they completed certain items. At three points during the test participants were asked to explain their thinking behind their responses on the previously completed items. Questions were as follows: 1) "Why is this one [points to one fraction] smaller than that one [points to a neighbouring fraction]?"; 2) "How do you know that this is the correct order for these numbers [points to decimals on paper]?"; and 3) "How do you know which number is bigger [points to fraction and decimal]?" Out of 1008 trials, participants changed their answers on 6.5% of the tasks: in 23 trials the participant changed a correct answer to an incorrect answer, in 13 trials they changed an incorrect answer to another incorrect answer, and in 30 trials they changed an incorrect answer to a correct one. Participants' final scores were based on both their final written responses and their verbal reasoning.

The test consisted of three types of problems aimed at capturing participants' knowledge of those rational number concepts which are most often conflated by natural number concepts, namely (a) the size of fractions and decimals and (b) the density of the set of fractions and decimals. The size concepts were covered by comparison items and ordering items. Density items were all open-ended questions. In all items the features of the rational numbers were incongruent with natural number features.

Comparison items were multiple-choice including: 5 items comparing two fractions (e.g. "Circle the larger fraction. If the numbers are equal circle both": 5/8; 4/3); 7 items comparing two decimals (e.g. "Circle the larger decimal…": 0.36; 0.5); and 7 items comparing fractions and decimals (e.g. "Circle the larger number…": 1/8; 0.8). Each item was scored as correct (1 point) or incorrect (0 points); the maximum for comparison items was 19. Cronbach's alpha for comparison items was .88.

Ordering items were short-answer responses including: 1 item ordering fractions ("Put the numbers in order from smallest to largest": 5/6, 1, 1/7, and 4/3); and 3 items ordering decimals (e.g. "Put the numbers in order from smallest to largest": 6.79; 6.796; 6.4). For the fraction item, 2 points were given for a correct answer, 1 point given if the fractions were in the correct order but the "1" was at the beginning or end, and 0 points for an otherwise incorrect answer. For the decimal items, each item was scored as correct (1 point) or incorrect (0 points); the maximum score of the ordering items was 5. Cronbach's alpha for ordering items was .72.

Density items were short-answer responses including: 2 items on fraction density (e.g. "Are there other fractions between 3/5 and 4/5? If so, how many?"); and 3 items on decimal density (e.g. "Are there other decimal numbers between 0.3 and 0.4? If so, how many?"). Each item was scored as incorrect (0 points), partially correct (1 point), or fully correct (2 points). *Fully Correct* responses were a mathematically correct concept of the density of rational numbers, stating that there are an infinite or indescribable number of numbers between any two rational numbers, or that it is not possible say how many there are (e.g. "There are an infinite number [of fractions between 3/5 and 4/5.]"). *Partially correct* responses had some basic understanding that there are some fractions or decimals in between any two fractions or decimals, but did not include notions of infinity (e.g. "10", "many", "a lot", etc.). *Incorrect* responses stated that there are no numbers in between the two rational numbers (e.g. "There are no numbers [in between 0.3 and 0.4.]"). The maximum score for the density portion of the test was 10. Cronbach's alpha for density items was .89.

3 Results

Table 1 presents the descriptive statistics for the variables measured at the ages of six and twelve years. As can be seen, there is substantial variation in SFON scores among the participants at the age of six years.

Correlations between the sum variables indicate that SFON (r = .55, p = .001) and number sequence elaboration skills (r = .58, p < .001) at the age of six years are related to rational number conceptual knowledge at the age of twelve. As well, number sequence elaboration skills are related to SFON (r = .50, p = .002).

Table 1 Descriptive statistics for variables at age six and twelve years (N = 36)

	Mean	SD	Skewness	Kurtosis	Range
Number sequence elaboration at 6	11.53	5.13	−0.68	−0.99	2–18
SFON at 6	2.58	1.76	0.35	−0.88	0–6
RNT at 12	19.61	7.29	0.00	−0.83	4–32
Comparison	14.44	4.47	−0.70	−0.41	3–19
Ordering	3.53	1.73	−0.71	−0.96	0–5
Density	1.64	2.50	1.59	1.56	0–8

With a relationship apparent between all three variables, we then aimed to determine exactly how SFON and number sequence elaboration skills at age six uniquely predict rational number conceptual knowledge at age twelve. Due the large amount of covariance between SFON and number sequence elaboration skills, standardized residual scores of SFON and number sequence elaboration were calculated. This provides a measure of SFON tendency without the variance that can be attributed to number sequence skills, and vice-versa (see Hallett et al. 2010 for a detailed explanation of the use of residual scores). The residualized SFON score represents the difference between an individual's observed SFON score and the SFON score that is predicted by his/her number sequence elaboration score. Therefore a positive residual SFON score indicates a higher SFON score than would be expected based on the number sequence elaboration skill, and a negative indicates a SFON score lower than would be expected (Lehtinen et al. submitted).

These so-called "pure" number sequence elaboration and early SFON scores were then entered step-wise as independent variables into a regression with rational number conceptual knowledge at age twelve as the dependent variable. Rational number conceptual knowledge score was calculated by taking the sum of the standardized scores for the comparison, ordering, and density portions of the rational number test. "Pure" number sequence elaboration was entered in the first step and "pure" SFON was added in the second step. As can be seen in Table 2, overall 43% of the variance on the rational number test is explained by number sequence elaboration and SFON at six years. For the whole test, while number sequence elaboration skills explain some of the variance in rational number conceptual knowledge (change in R^2 = .13; β = .68), SFON explains even more precisely this variance (change in R^2 = .30; β = .64). In order to see in more detail the nature of how SFON and number sequence elaboration skills predict rational number conceptual knowledge, a standardized sum score of the comparison and ordering parts (referred to as size) and a sum score of the density part of the rational number test were also entered as dependent variables in similar regressions (Table 2). In total, number sequence skills and early SFON explained 40% of the variance on the size portion of the rational number test and 28% of the variance on the density portion. While for the items dealing with the size of rational numbers a similar pattern emerged, interestingly only SFON improved the explanation of density scores.

Table 2 Stepwise regression analyses: specific effects of residualized number sequence elaboration skills and SFON at age six years on rational number conceptual knowledge at age twelve years

	RNT		Size		Density	
Variable entered by step	β	R^2 change	β	R^2 change	β	R^2 change
1. Num Seq Elab	.68***	.13*	.68***	.15*	.44**	.03
2. SFON	.64***	.30***	.59**	.26**	.51**	.19**
Total R^2		.43		.40		.23

Note: *p < .05, **p < .01, ***p < .001

In order to more directly investigate whether SFON predicts rational number conceptual knowledge, we first investigated whether those with higher-than-expected early SFON scores at the age of six are more likely to have higher rational number conceptual knowledge. Participants were grouped based on their rational number conceptual knowledge at twelve years and separately grouped based on their residualized SFON scores. Cross-tabulations were then used to determine if SFON predicts rational number conceptual knowledge groupings.

Participants were classified into one of four categories based on the patterns of their knowledge of rational number concepts. McMullen and colleagues (2014b) identified a four-class model as most appropriate for this age group for classifying students' rational number conceptual knowledge of the size and density of fractions and decimals using a similar test. These groupings were used as a guide in classifying the participants of the present study into distinct rational number conceptual knowledge groupings. In the "Low" group (n = 15) were students who had below-average scores for all three problem types. "Average" students (n = 11) were those with at least one problem type above average but no problem types above one standard deviation above the mean. "Representation" students (n = 5) had a comparison and/or ordering item score above +1 SD, but a density item score below +1 SD. "High" students (n = 5) had a comparison and/or ordering item score above +1 SD and a density item score above +1 SD. Figure 1 shows the group means for the comparison, ordering, and density items on the rational number test, as well as residualized early SFON scores.

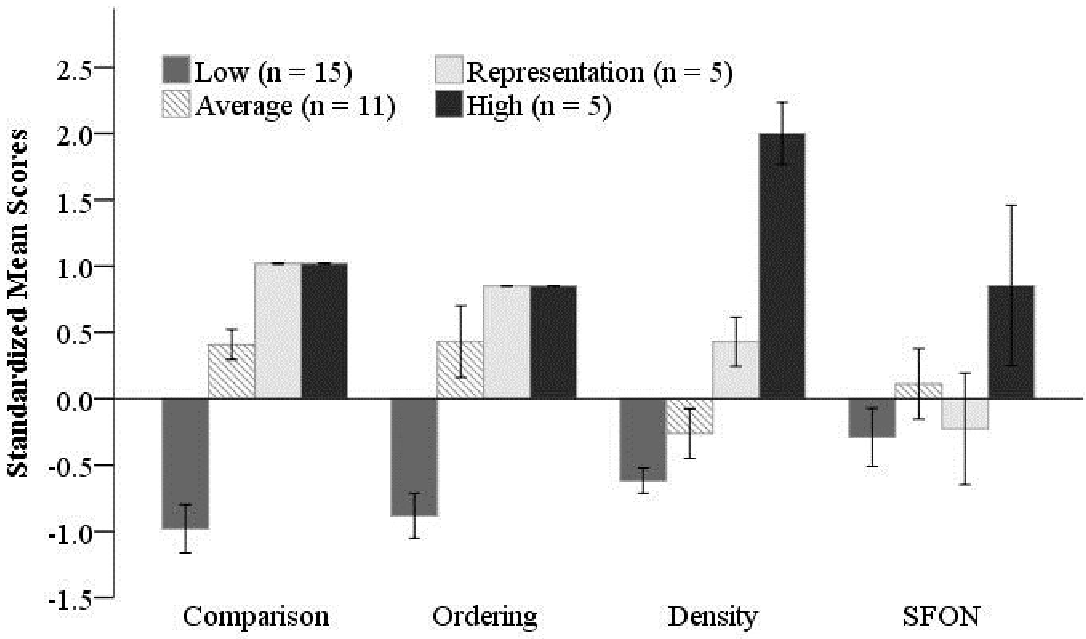

Fig. 1 Standardized rational number group means for rational number test items and residualized SFON (+/– 2 S.E.)

In order to determine if early SFON tendency had a direct impact on rational number group membership, participants were split into two SFON groups. Those with a positive SFON residual could be said to have a higher-than-expected SFON score based on number sequence elaboration skills and were placed in the "High SFON" group. Those participants with a negative SFON residual score could be said to have a lower-than-expected SFON score and were placed in the "Low SFON" category. Table 3 presents the results of the cross-tabulation. An Ordinal by Ordinal directional measure revealed that those students in the "High SFON" group were more likely to end up in higher rational number test groups than those in the "Low SFON" group (Somers' d = .40, Approx. T = 2.39, p = .017). As can be seen, most of the High rational number conceptual knowledge group also had higher-than-expected SFON scores. Likewise, very few participants with lower-than-expected SFON scores were found outside of the Low or Average rational number conceptual knowledge groups.

Table 3 Cross-tabulation of SFON grouping at age six years and rational number conceptual knowledge group membership at age twelve years

	Rational Number Conceptual Knowledge Group			
SFON at age 6 Group	Low	Average	Rep	High
Low (n = 20)	11	6	2	1
High (n = 16)	4	5	3	4
Total (N = 36)	15	11	5	5

4 Discussion

The present study provides new evidence of the role of early SFON in the development of mathematical skills. For the first time, a connection between early SFON tendency and rational number conceptual knowledge in late primary school has been identified. Children's SFON tendency is a strong predictor of later rational number conceptual knowledge, even after taking into account early number sequence skills. This finding has implications for the understanding of how the transition from reasoning about natural number concepts to reasoning about rational numbers may be influenced by children's self-initiated practice with numbers in everyday situations.

An intuitive hypothesis was the first proposed in the present study: that a higher early SFON tendency, and the increased amount of practice with whole numbers that comes with it, would lead to a more substantial natural number bias causing more difficulties with learning rational number concepts (Hypothesis 1). However, the results of the present study indicate that there is a positive relationship between SFON tendency at the age of six and rational number conceptual knowledge at the age of twelve. This indicates that this intuitive hypothesis is not supported and that a more analytic hypothesis was supported, which stated that, due to its relationship to number sequence skills, which predict rational number conceptual knowledge, SFON would also predict this knowledge (Hypothesis 2).

Despite evidence that early SFON tendency did predict rational number conceptual knowledge, it was still entirely possible that most of the variance explained by SFON was actually due to shared variance with number sequence skills (Bailey et al. 2014; Hannula et al. 2010). It was, however, expected that the unique contribution of SFON tendency could be even greater than that of number sequence skills (Hypothesis 3), due to the nature of the self-initiated practice in varying situations afforded by SFON. This hypothesis was not confirmed by the multiple regression analyses. However, these results indicated that SFON tendency explained more precisely and was similarly important in explaining the variance in rational number conceptual knowledge than number sequence skills. Furthermore, this result was replicated for both the size and density portions of the rational number test. Finally, those with a higher-than-expected SFON tendency were found to be more likely to end up with more advanced rational number conceptual knowledge groups. This indicates that those with a higher SFON tendency at six years old may be more likely to overcome the natural number bias when reasoning about rational numbers at twelve years old.

For those students with a lower early SFON tendency, the vast majority of experiences dealing with natural numbers may solely come from typical school mathematics tasks. In these tasks there is rarely an instance where natural numbers do not suffice, until rational numbers are explicitly introduced in the curriculum. This may encourage a more disjointed view of the number system, with natural and rational numbers cleanly separated and only used with like numbers, or when the problem explicitly calls for their use. This may lead to a proficiency in dealing with typical school math tasks, a sort of routine expertise with the number system. However, a student with a higher SFON tendency may instead see natural and rational numbers as a part of a more well-connected number system, allowing for a more adaptive expertise, in which previous skills and knowledge can be flexibly applied to new situations (Baroody 2003; Heinze et al. 2009; Threlfall 2009; Hatano and Oura 2012).

One possibility is that a higher early SFON tendency increases opportunities to recognize numerical possibilities in everyday situations which are not clearly defined nor clearly mathematical. These situations provide novel experiences with reasoning about mathematical aspects, in contrast to typical school math tasks, which almost always have a clear question, given numbers to calculate with, and come to a well-defined solution. More experiences with the messy mathematics that can be found in everyday situations, as afforded by SFON, may particularly prepare students for recognizing that, in some situations, natural number features are not sufficient for finding a solution. Indeed, in many everyday situations whole number solutions are inadequate, such as trying to split two cookies between three people, and those with a higher SFON tendency may be more likely to encounter these situations than those with a lower SFON tendency.

The present study includes no direct measure of the inhibition of the natural number bias (e.g. reaction time paradigm), and thus, the link between inhibition and SFON cannot be explicitly made. However, it is possible to speculate on the nature of this relationship based on the substantial relationship between SFON and rational number conceptual knowledge revealed in the present study. The increased awareness or sensitivity to novel, convoluted, and "messy" aspects of the number system facilitated by a higher SFON tendency may be particularly relevant for the inhibition of intuitive conceptions related to the natural number bias when reasoning about rational numbers. Inhibition can be seen from a broader perspective beyond typical reaction-time measures. Merenluoto and Lehtinen (2004) identified a model for the inhibition of mathematically incorrect concepts of number leading to conceptual change. In this instance, inhibiting less-mathematically correct concepts, such as using incongruent natural number features to reason about rational numbers, occurs when a student is able to overcome this natural number bias and use mathematically correct concepts in their reasoning.

A starting point for the path towards this inhibition, they argued, lies with an experience of conflict between prior-held concepts and an experienced phenomenon. The personal attributes which lead to experiencing this conflict include the sensitivity to new aspects of a phenomenon, as well as cognitive and motivational contributors to this sensitivity. In this way, a higher tendency to notice specific features of a situation, such as features of natural or rational numbers which are incongruent with a held concept or experienced phenomenon, may more often lead to the inhibition of intuitive conceptualizations, such as a longer decimal number means a larger magnitude. In other words, a tendency to recognize that a conception of numbers is conflicting with an aspect of a given task may support the inhibition of this prior conception, possibly in favour of a mathematically correct conception. This process may be one way that SFON tendency leads to greater success on the rational number test.

It should be noted that the results of the present study represent a first step in determining the role of early SFON tendency in the development of rational number conceptual knowledge. Due to the small sample size the statistical analyses reported here should be taken as exploratory, and the results as tentative. These results are especially threatened by a lack of power, leading to the possibility that some effects were substantial but not statistically significant. That said, the strength of the relationships found here, even over such a long period of time, suggest that it is possible to be cautiously optimistic of the veracity of the conclusions drawn within this study. Further investigations with a larger sample would be necessary to confirm the results. As well, the investigation of the role of inhibition in SFON tendency would clarify whether inhibitory processes are involved in both SFON tendency and rational number conceptual knowledge, leading to the relation found in the present study.

There are other issues that should be taken into account in future studies on this topic. First, there is the possibility of confounding variables which may have inflated the relationships found between early SFON and rational number conceptual knowledge. A number of predictors of rational number conceptual knowledge have recently been identified (Bailey et al. 2014; Jordan et al. 2013). Therefore, including more measures of, for example, whole number arithmetic and magnitude estimation, would be valuable to determine SFON tendency's exact role in rational number development. Furthermore, confounding variables at the age of twelve may also explain some of the variance that SFON tendency seems to explain, and more measures at this age, such as non-verbal reasoning, arithmetic skills, and procedural knowledge of rational numbers (Lehtinen et al. submitted), would also be appropriate. As well, the rational number test instrument only includes items in which there is incongruence between natural number features and rational number features; it would be important to have a more comprehensive measure that includes both types of items (see Van Hoof et al. 2013b).

One more problematic feature of the present study (and any study measuring density knowledge in students of this age) is the relative instability of the "High" rational number conceptual knowledge grouping in previous research (McMullen et al. 2014b), indicating that sustaining mathematically correct knowledge of the density of the set of rational numbers is particularly difficult, even among those who at one time or another display a more mathematically correct conception of rational number density. This finding is in line with conceptual change theory and reaction time research (Vamvakoussi et al. 2012; Van Hoof et al. 2013a; Obersteiner et al. 2013; Vosniadou 2014). This suggests that those who are in the "High" group in the present study may not have been so a few months before or after this, and not all those who have at least some mathematically correct understanding of the density of the rational number set were in the "High" group in this study. This suggests that multiple measurements could prove valuable for the more valid capturing of rational number conceptual understanding. It is worth noting, however, that even after taking with a grain of salt the finding that higher-than-expected early SFON leads to higher rational number conceptual knowledge, the present study still identified that lower-than-expected SFON tendency more often leads to the “Low” rational number conceptual knowledge category, which itself was shown to be worryingly stable (McMullen et al. 2014b).

Despite all of these limitations, there remain important implications of the finding that early SFON tendency predicts rational number conceptual knowledge over a six-year period. The tentative result that SFON tendency predicts rational number conceptual knowledge even after six years suggests that individual differences in SFON may have a long-reaching impact on differences in mathematical development. Previously, the tendency of spontaneous focusing on quantitative relations (SFOR), defined as the spontaneous (i.e. undirected) focusing of attention on quantitative relations and the use of these relations in situations that are not explicitly mathematical, has been identified as a predictor of rational number conceptual development (Lehtinen et al. submitted; McMullen et al. 2014a; McMullen 2014). SFOR in seven-year-olds was found to predict fraction knowledge at the age of ten (McMullen et al. 2014a). As well, SFOR predicted the development of rational number conceptual knowledge over two school years in late primary school (Lehtinen et al. submitted). That both SFON and SFOR predict rational number conceptual knowledge suggests that further study of the relationship between SFON and SFOR tendency in early years is important for the understanding both of the nature of spontaneous quantitative focusing tendencies in general, and of how SFON and SFOR are related to each other.

These results have important implications for the teaching of rational numbers and early mathematical skills. First, they suggest that SFON tendency may be important for a broader range of mathematical development than previously thought. The early encouragement of SFON may have a long-lasting impact on some of the most difficult topics in mathematics. Previous research suggests that already at an early age SFON tendency can be enhanced through social interaction (Hannula et al. 2005). This increase in SFON tendency led to gains in enumeration skills as well. More research into the effects of SFON training could prove promising for the later development of rational number conceptual knowledge. In any case, teachers and maths educators should take into account the limitations of typical classroom maths from a broader perspective. The results of the present study suggest that it is possible that working with "messy" maths may provide long-term benefits for overcoming inappropriate intuitions about the nature of numbers.

References

Andres, M., Di Luca, S., & Pesenti, M. (2008). Finger counting: the missing tool? *Behavioral and Brain Sciences*, *31*, 642. doi:10.1017/S0140525X08005578.

Aunola, K., Leskinen, E., Lerkkanen, M.-K., & Nurmi, J.-E. (2004). Developmental dynamics of math performance from preschool to grade 2. *Journal of Educational Psychology*, *96*, 699–713. doi:10.1037/0022-0663.96.4.699.

Bailey, D., Siegler, R., & Geary, D. C. (2014). Early predictors of middle school fraction knowledge. *Developmental Science*. doi: 10.1111/desc.12155.

Baroody, A. J. (2003). The development of adaptive expertise and flexibility: the integration of conceptual and procedural knowledge. In A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Constructing adaptive expertise* (pp. 1–33). London: Lawrence Erlbaum.

Brown, A. L. (1992). Design experiments: theoretical and methodological challenges in creating complex interventions in classroom settings. *The Journal of the Learning Sciences*, *2*, 141–178.

Carey, S. (2004). Bootstrapping & the origin of concepts. *Daedalus*, *133*, 59–68.

Confrey, J., Maloney, A., Nguyen, K., Mojica, G., & Myers, M. (2009). Equipartitioning/splitting as a foundation of rational number reasoning using learning trajectories. In *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education*, *1*, 1–8.

Dehaene, S., Izard, V., Spelke, E., & Pica, P. (2008). Log or linear? Distinct intuitions of the number scale in Western and Amazonian indigene cultures. *Science*, *320*, 1217–1220. doi:10.1126/science.1156540.

Edens, K. M., & Potter, E. F. (2013). An exploratory look at the relationships among math skills, motivational factors and activity choice. *Early Childhood Education Journal*, *41*, 235–243.

Feigenson, L., Carey, S., & Spelke, E. (2002). Infants' discrimination of number vs. continuous extent. *Cognitive psychology*, *44*, 33–66. doi:10.1006/cogp.2001.0760.

Fuson, K. C. (1988). *Children's counting and concepts of number*. New York: Springer. doi: 10.1017/s0140525x08005724.

Gallistel, C. R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition*, *44*, 43–74. doi:10.1016/0010-0277(92)90050-R.

Greer, B. (2004). The growth of mathematics through conceptual restructuring. *Learning and Instruction*, *14*, 541–548. doi:10.1016/j.learninstruc.2004.06.018.

Hallett, D., Nunes, T., & Bryant, P. (2010). Individual differences in conceptual and procedural knowledge when learning fractions. *Journal of Educational Psychology*, *102*, 395–406. doi:10.1037/a0017486.

Hannula, M. M. (2005). *Spontaneous focusing on numerosity in the development of early mathematical skills.* Painosalama, Turku, Finland.

Hannula, M. M., & Lehtinen, E. (2005). Spontaneous focusing on numerosity and mathematical skills of young children. *Learning and Instruction*, *15*, 237–256. doi:10.1016/j.learninstruc.2005.04.005.

Hannula, M. M., Mattinen, A., & Lehtinen, E. (2005). Does social interaction influence 3-year-old children's tendency to focus on numerosity? A quasi-experimental study in day care. In L. Verschaffel, E. De Corte, G. Janselaar, & M. Valcke (Eds.), *Powerful environments for promoting deep conceptual and strategic learning* (pp. 63–80). Amsterdam: Elsevier.

Hannula, M. M., Räsänen, P., & Lehtinen, E. (2007). Development of counting skills: role of spontaneous focusing on numerosity and subitizing-based enumeration. *Mathematical Thinking and Learning*, *9*, 51–57. doi:10.1207/s15327833mtl0901_4.

Hannula, M. M., Lepola, J., & Lehtinen, E. (2010). Spontaneous focusing on numerosity as a domain-specific predictor of arithmetical skills. *Journal of Experimental Child Psychology*, *107*, 394–406. doi:10.1016/j.jecp.2010.06.004.

Hannula-Sormunen, M. M. (2014). Spontaneous focusing on numerosity and its relation to counting and arithmetic. In R. Cohen-Kadosh & A. Dowker (Eds.), *Oxford handbook of mathematical cognition*. doi:10.1093/oxfordhb/9780199642342.013.018.

Hannula-Sormunen, M. M., Lehtinen, E., & Räsänen, P. (submitted). Children's preschool subitizing, spontaneous focusing on numerosity and counting skills as predictors of mathematical performance 6–7 years later at school.

Hatano, G., & Oura, Y. (2012). Commentary: Reconceptualizing school learning using insight from expertise research. *Educational Researcher*, *32*, 26–29.

Heinze, A., Star, J. R., & Verschaffel, L. (2009). Flexible and adaptive use of strategies and representations in mathematics education. *ZDM Mathematics Education*, *41*, 535–540. doi:10.1007/s11858-009-0214-4.

Johansson, B. S. (2005). Number-word sequence skill and arithmetic performance. *Scandinavian Journal of Psychology*, *46*, 157–167. doi:10.1111/j.1467-9450.2005.00445.x.

Jordan, N., Hansen, N., & Fuchs, L. (2013). Developmental predictors of fraction concepts and procedures. *Journal of Experimental Child Psychology*, *116*, 45–58. doi:10.1016/j.jecp.2013.02.001.

Koponen, T., Aunola, K., Ahonen, T., & Nurmi, J. E. (2007). Cognitive predictors of single-digit and procedural calculation skills and their covariation with reading skill. *Journal of Experimental Child Psychology*, *97*, 220–241. doi:10.1016/j.jecp.2007.03.001.

Kucian, K., et al. (2012). Kinder mit Dyskalkulie fokussieren spontan weniger auf Anzahligkeit [Children with developmental dyscalculia focus spontaneously less on numerosities]. *Lernen und Lernstörungen*, *1*, 241–253.

Lehtinen, E., & Hannula, M. M. (2006). Attentional processes, abstraction and transfer. In L. Verschaffel, F. Dochy, M. Boekaerts, & S. Vosniadou (Eds), *Instructional psychology: Past, present and future trends*, (pp. 39–55). Amsterdam: Elsevier.

Lehtinen, E., McMullen, J., & Hannula-Sormunen, M. M. (submitted). The influence of spontaneous focusing on quantitative relations on the development of rational number knowledge.

Lepola, J., Niemi, P., Kuikka, M., & Hannula, M. M. (2005). Cognitive-linguistic skills and motivation as longitudinal predictors of reading and arithmetic achievement: a follow-up study from kindergarten to grade 2. *International Journal of Educational Research*, *43*, 250–271. doi:10.1016/j.ijer.2006.06.005.

Mattinen, A. (2006). Huomio lukumääriin: Tutkimus 3-vuotiaiden lasten matemaattisten taitojen tukemisesta päiväkodissa [Focus on numerosities: a study on supporting 3-year-old children's mathematical development in day care]. Painosalama, Turku, Finland.

Mazzocco, M. M. M., & Devlin, K. T. (2008). Parts and "holes": gaps in rational number sense among children with vs. without mathematical learning disabilities. *Developmental Science*, *11*, 681–691. doi:10.1111/j.1467-7687.2008.00717.x.

McMullen, J. (2014). Spontaneous focusing on quantitative relations and the development of rational number conceptual knowledge. Painosalama, Turku.

McMullen, J., Hannula-Sormunen, M. M., & Lehtinen, E. (2014a). Spontaneous focusing on quantitative relations in the development of children's fraction knowledge. *Cognition and Instruction*, *32*, 198–218.

McMullen, J., Laakkonen, E., Hannula-Sormunen, M. M., & Lehtinen, E. (2014b). Modeling the developmental trajectories of rational number concept(s). *Learning and Instruction*. doi:10.1016/j.learninstruc.2013.12.004.

Merenluoto, K., & Lehtinen, E. (2004). Number concept and conceptual change: towards a systemic model of the processes of change. *Learning and Instruction*, *14*, 519–534. doi:10.1016/j.learninstruc.2004.06.016.

Metsämuuronen, J. (2006). *Tutkimuksen tekemisen perusteet ihmistieteissä. 3. laitos. 2. korjattu painos.* Jyväskylä, Finland: Gummeruksen kirjapaino Oy.

Ni, Y., & Zhou, Y.-D. (2005). Teaching and learning fraction and rational numbers: the origins and implications of whole number bias. *Educational Psychologist*, *40*, 27–52. doi:10.1207/s15326985ep4001_3.

Obersteiner, A., Van Dooren, W., Van Hoof, J., & Verschaffel, L. (2013). The natural number bias and magnitude representation in fraction comparison by expert mathematicians. *Learning and Instruction*, *28*, 64–72. doi:10.1016/j.learninstruc.2013.05.003.

Potter, E. F. (2009). Spontaneous focusing on numerosity: motivational and skill correlates in young children in a public preschool and kindergarten program. In S. L. Swars, D. W. Stinson, & S. Lemons-Smith (Eds.), *Proceedings of the 31st annual meeting of the North American chapter of the International Group for the Psychology of Mathematics Education* (pp. 152–155). Atlanta: Georgia State University Press.

Reyna, V. F., & Brainerd, C. J. (2007). The importance of mathematics in health and human judgment: numeracy, risk communication, and medical decision making. *Learning and Individual Differences*, *17*, 147–159. doi:10.1016/j.lindif.2007.03.010.

Salonen, P., et al. (1994). Diagnostiset testit 3. Motivaatio, metakognitio ja matematiikka. [Diagnostic tests 3. Motivation, metacognition and mathematics]. Centre for Learning Research, University of Turku.

Secada, W. G., Fuson, K. C., & Hall, J. W. (1983). The transition from counting-all to counting-on in addition. *Journal for Research in Mathematics Education*, *14*, 47–57.

Siegler, R., Thompson, C., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology*, *62*, 273–96. doi:10.1016/j.cogpsych.2011.03.001.

Siegler, R., et al. (2012). Early predictors of high school mathematics achievement. *Psychological Science*, *23*, 691–697. doi:10.1177/0956797612440101.

Siegler, R., Fazio, L., Bailey, D., & Zhou, X. (2013). Fractions: the new frontier for theories of numerical development. *Trends in Cognitive Sciences*, *17*, 13–19. doi:10.1016/j.tics.2012.11.004.

Threlfall, J. (2009). Strategies and flexibility in mental calculation. *ZDM Mathematics Education*, *41*, 541–555. doi:10.1007/s11858-009-0195-3.

Vamvakoussi, X., & Vosniadou, S. (2004). Understanding the structure of the set of rational numbers: a conceptual change approach. *Learning and Instruction*, *14*, 453–467. doi:10.1016/j.learninstruc.2004.06.013.

Vamvakoussi, X., & Vosniadou, S. (2010). How many decimals are there between two fractions ? Aspects of secondary school students' understanding of rational numbers and their notation. *Cognition and Instruction*, *28*, 181–209. doi:10.1080/07370001003676603.

Vamvakoussi, X., Christou, K. P., Mertens, L., & Van Dooren, W. (2011). What fills the gap between discrete and dense? Greek and Flemish students' understanding of density. *Learning and Instruction*, *21*, 676–685. doi:10.1016/j.learninstruc.2011.03.005.

Vamvakoussi, X., Van Dooren, W., & Verschaffel, L. (2012). Naturally biased? In search for reaction time evidence for a natural number bias in adults. *The Journal of Mathematical Behavior*, *31*, 344–355. doi:10.1016/j.jmathb.2012.02.001.

Van Hoof, J., Lijnen, T., Verschaffel, L., & Van Dooren, W. (2013a). Are secondary school students still hampered by the natural number bias? A reaction time study on fraction comparison tasks. *Research In Mathematics Education*, *15*, 154–164. doi:10.1080/14794802.2013.797747.

Van Hoof, J., Verschaffel, L., & Van Dooren, W. (2013b). *Inhibiting natural number knowledge in rational number tasks: Towards a comprehensive test.* Paper presented at the 15th Biennual Meeting of the European Association for Research on Learning and Instruction (EARLI).

Van Hoof, J., Janssen, R., Verschaffel, L., & Van Dooren, W. (2015). Inhibiting natural knowledge in fourth graders: towards a comprehensive test. *ZDM Mathematics Education, 47*(5) (this issue). doi:10.1007/s11858-014-0650-7

Vosniadou, S. (2014). Examining cognitive development from a conceptual change point of view: the framework theory approach. *European Journal of Developmental Psychology*. doi:10.1080/17405629.2014.921153.

Vosniadou, S., & Verschaffel, L. (2004). Extending the conceptual change approach to mathematics learning and teaching. *Learning and Instruction*, *14*, 445–451. doi:10.1016/j.learninstruc.2004.06.014.

Vukovic, R. K., et al. (2014). Sources of individual differences in children's understanding of fractions. *Child Development*. doi: 10.1111/cdev.12218.