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New classifications of nonlinear Schrödinger model with group velocity dispersion via new extended method

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ABSTRACT

This work investigates the nonlinear Schrödinger equation (NLSE) with group velocity dispersion and second order spatiotemporal dispersion coefficients. The governing model is reduced into the classical nonlinear ordinary differential equation. Extended direct algebraic method (EDAM) is implemented to construct many novel mixed dark, and complex optical solutions. As a result, some important analytical solutions such as travelling mixed dark, and complex travelling wave solutions for the model are extracted.

Introduction

To describe the optical propagation arising in optical fibers is used the generalized nonlinear Schrödinger equation. In the modern century, many analytical models to find more behaviors of optical waves to the literature have been presented [1–52]. Du et al have constructed novel bifurcations of optical wave popagations in Du et al. [53]. Modulation instability analysis of the model has been investigated by Mao et al. [54]. Guan et al have observed the breather properties of generalized nonlinear Schrödinger system with two higher-order [55]. Sun et al. have obtained the rogue waves by virtue of the Kadomtsev-Petviashvili hierarchy reduction to investigate the amplification or absorption of pulses propagating in a monomode optical fiber in Sun et al. [56]. They have also introduced the effects of group-velocity dispersion, nonlinearity, and amplification/absorption coefficients. This model is given by

$$i\left(\frac{\partial q}{\partial x}+\rho\frac{\partial q}{\partial t}\right)+\lambda\frac{\partial^2 q}{\partial t^2}+\varepsilon\frac{\partial^2 q}{\partial x^2}+|q|^2 q=0, \tag{1}$$

where $q=q(x,t),\rho,\lambda$ and ε are defined in [57–60]. $q(x,t)$ is used to symbolize the macroscopic complex valued wave profile. t and x are the temporal and spatialvariables, respectively. Also, $\lambda$and ε are the coefficients of group velocity dispersion and spatial dispersion, ρ, is proportional to the ratio of group speed [61]. The rest of this paper is followed. In section two, we present EDAM. In section three, we obtain a some solutions of Eq. (1). In section four, we conclude the novel properties of NLSE by considering results obtained in this paper.

Fundamentals of the EDAM

Let's give the description of the EDAM [62]. Consider the following NLPDE of the form

$$F(u,u_t,u_x,u_{tt},u_{xx},\ldots)=0 \tag{2}$$

which can be converted to an ODE as the following form

$$G(U,U',U'',\cdots)=0, \tag{3}$$

by using the wave transformation

$$u(x,t)=U(\xi),\qquad \xi=x-\mu t. \tag{4}$$

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where μ is arbitrary constant to be determined later. Suppose that the solution of Eq. (3) can be presented as

$$U(\xi)=\sum_{\iota=0}^{\Omega}\varpi_{\iota}Q^{\iota}(\xi),\quad \varpi_{\Omega}\neq 0, \tag{5}$$

in which $\varpi_{\iota}(0\leqslant\iota\leqslant\Omega)$ are constant coefficients to be determined later and $Q(\xi)$ satisfies the following ODE

$$Q^{'}(\xi)=ln(A)\left(\alpha+\beta Q(\xi)+\sigma Q^{2}(\xi)\right),\quad A\neq 0,1. \tag{6}$$

The solution set of Above ODE are given as follows.

Family-1. When $\beta^2-4\alpha\sigma<0$ and $\sigma\neq 0$,

$$Q_1(\xi)=-\frac{\beta}{2\sigma}+\frac{\sqrt{-(\beta^2-4\alpha\sigma)}}{2\sigma}tan_A\left(\frac{\sqrt{-(\beta^2-4\alpha\sigma)}}{2}\xi\right),$$

$$Q_2(\xi)=-\frac{\beta}{2\sigma}-\frac{\sqrt{-(\beta^2-4\alpha\sigma)}}{2\sigma}cot_A\left(\frac{\sqrt{-(\beta^2-4\alpha\sigma)}}{2}\xi\right),$$

$$Q_3(\xi)=-\frac{\beta}{2\sigma}+\frac{\sqrt{-(\beta^2-4\alpha\sigma)}}{2\sigma}\left(tan_A\left(\sqrt{-(\beta^2-4\alpha\sigma)}\,\xi\right)\pm\sqrt{pq}\,sec_A\left(\sqrt{-(\beta^2-4\alpha\sigma)}\,\xi\right)\right),$$

$$Q_4(\xi)=-\frac{\beta}{2\sigma}+\frac{\sqrt{-(\beta^2-4\alpha\sigma)}}{2\sigma}\left(-cot_A\left(\sqrt{-(\beta^2-4\alpha\sigma)}\,\xi\right)\pm\sqrt{pq}\,csc_A\left(\sqrt{-(\beta^2-4\alpha\sigma)}\,\xi\right)\right),$$

$$Q_5(\xi)=-\frac{\beta}{2\sigma}+\frac{\sqrt{-(\beta^2-4\alpha\sigma)}}{4\sigma}\left(tan_A\left(\frac{\sqrt{-(\beta^2-4\alpha\sigma)}}{4}\xi\right)-cot_A\left(\frac{\sqrt{-(\beta^2-4\alpha\sigma)}}{4}\xi\right)\right)$$

Family-2. When $\beta^2-4\alpha\sigma>0$ and $\sigma\neq 0$,

$$Q_6(\xi)=-\frac{\beta}{2\sigma}-\frac{\sqrt{\beta^2-4\alpha\sigma}}{2\sigma}tanh_A\left(\frac{\sqrt{\beta^2-4\alpha\sigma}}{2}\xi\right),$$

$$Q_7(\xi)=-\frac{\beta}{2\sigma}-\frac{\sqrt{\beta^2-4\alpha\sigma}}{2\sigma}coth_A\left(\frac{\sqrt{\beta^2-4\alpha\sigma}}{2}\xi\right),$$

$$Q_8(\xi)=-\frac{\beta}{2\sigma}+\frac{\sqrt{\beta^2-4\alpha\sigma}}{2\sigma}\left(-tanh_A\left(\sqrt{\beta^2-4\alpha\sigma}\,\xi\right)\pm i\sqrt{pq}\,\mathrm{sech}_A\left(\sqrt{\beta^2-4\alpha\sigma}\,\xi\right)\right),$$

$$Q_9(\xi)=-\frac{\beta}{2\sigma}+\frac{\sqrt{\beta^2-4\alpha\sigma}}{2\sigma}\left(-coth_A\left(\sqrt{\beta^2-4\alpha\sigma}\,\xi\right)\pm\sqrt{pq}\,\mathrm{csch}_A\left(\sqrt{\beta^2-4\alpha\sigma}\,\xi\right)\right),$$

$$Q_{10}(\xi)=-\frac{\beta}{2\sigma}-\frac{\sqrt{\beta^2-4\alpha\sigma}}{4\sigma}\left(tanh_A\left(\frac{\sqrt{\beta^2-4\alpha\sigma}}{4}\xi\right)+coth_A\left(\frac{\sqrt{\beta^2-4\alpha\sigma}}{4}\xi\right)\right)$$

Family-3. When $\alpha\sigma>0$ and $\beta=0$,

$$Q_{11}(\xi)=\sqrt{\frac{\alpha}{\sigma}}tan_A\left(\sqrt{\alpha\sigma}\,\xi\right),$$

$$Q_{12}(\xi)=-\sqrt{\frac{\alpha}{\sigma}}cot_A\left(\sqrt{\alpha\sigma}\,\xi\right),$$

$$Q_{13}(\xi)=\sqrt{\frac{\alpha}{\sigma}}\left(tan_A\left(2\sqrt{\alpha\sigma}\,\xi\right)\pm\sqrt{pq}\,sec_A\left(2\sqrt{\alpha\sigma}\,\xi\right)\right),$$

$$Q_{14}(\xi)=\sqrt{\frac{\alpha}{\sigma}}\left(-cot_A\left(2\sqrt{\alpha\sigma}\,\xi\right)\pm\sqrt{pq}\,csc_A\left(2\sqrt{\alpha\sigma}\,\xi\right)\right),$$

$$Q_{15}(\xi)=\frac{1}{2}\sqrt{\frac{\alpha}{\sigma}}\left(tan_A\left(\frac{\sqrt{\alpha\sigma}}{2}\xi\right)-cot_A\left(\frac{\sqrt{\alpha\sigma}}{2}\xi\right)\right)$$

Family-4. When $\alpha\sigma<0$ and $\beta=0$,

$$Q_{16}(\xi)=-\sqrt{-\frac{\alpha}{\sigma}}tanh_A\left(\sqrt{-\alpha\sigma}\,\xi\right),$$

$$Q_{17}(\xi)=-\sqrt{-\frac{\alpha}{\sigma}}coth_A\left(\sqrt{-\alpha\sigma}\,\xi\right),$$

$$Q_{18}(\xi)=\sqrt{-\frac{\alpha}{\sigma}}\left(-tanh_A\left(2\sqrt{-\alpha\sigma}\,\xi\right)\pm i\sqrt{pq}\,\mathrm{sech}_A\left(2\sqrt{-\alpha\sigma}\,\xi\right)\right),$$

$$Q_{19}(\xi)=\sqrt{-\frac{\alpha}{\sigma}}\left(-coth_A\left(2\sqrt{-\alpha\sigma}\,\xi\right)\pm\sqrt{pq}\,\mathrm{csch}_A\left(2\sqrt{-\alpha\sigma}\,\xi\right)\right),$$

$$Q_{20}(\xi)=-\frac{1}{2}\sqrt{-\frac{\alpha}{\sigma}}\left(tanh_A\left(\frac{\sqrt{-\alpha\sigma}}{2}\xi\right)+coth_A\left(\frac{\sqrt{-\alpha\sigma}}{2}\xi\right)\right)$$

Family-5. When $\beta=0$ and $\sigma=\alpha$,

$$Q_{21}(\xi)=tan_A(\alpha\xi),$$

$$Q_{22}(\xi)=-cot_A(\alpha\xi),$$

$$Q_{23}(\xi)=tan_A(2\alpha\xi)\pm\sqrt{pq}sec_A(2\alpha\xi),$$

$$Q_{24}(\xi)=-cot_A(2\alpha\xi)\pm\sqrt{pq}csc_A(2\alpha\xi),$$

$$Q_{25}(\xi)=\frac{1}{2}\left(tan_A\left(\frac{\alpha}{2}\xi\right)-cot_A\left(\frac{\alpha}{2}\xi\right)\right)$$

Family-6. When $\beta=0$ and $\sigma=-\alpha$,

$$Q_{26}(\xi)=-tanh_A(\alpha\xi),$$

$$Q_{27}(\xi)=-coth_A(\alpha\xi),$$

$$Q_{28}(\xi)=-tanh_A(2\alpha\xi)\pm i\sqrt{pq}\mathrm{sech}_A(2\alpha\xi),$$

$$Q_{29}(\xi)=-coth_A(2\alpha\xi)\pm\sqrt{pq}\mathrm{csch}_A(2\alpha\xi),$$

$$Q_{30}(\xi)=-\frac{1}{2}\left(tanh_A\left(\frac{\alpha}{2}\xi\right)+coth_A\left(\frac{\alpha}{2}\xi\right)\right)$$

Family-7. When $\beta^2=4\alpha\sigma$,

$$Q_{31}(\xi)=\frac{-2\alpha((lnA)\beta\xi+2)}{(lnA)\beta^2\xi}.$$

Family-8. When $\beta=\eta$, $\alpha=\Theta\eta$ $(\Theta\neq 0)$and

$$Q_{32}(\xi)=A^{\eta\,\xi}-\Theta.$$

Family-9. When $\beta=\sigma=0$,

$$Q_{33}(\xi)=(lnA)\alpha\xi.$$

Family-10. When $\beta=\alpha=0$,

$$Q_{34}(\xi)=\frac{-1}{(lnA)\sigma\xi}.$$

Family-11. When $\alpha=0$, and $\beta\neq0$, then

$$Q_{14}(\xi)=-\frac{p\beta}{\sigma(cosh_A(\beta(\xi))-sinh_A(\beta(\xi))+p)},$$

$$Q_{15}(\xi)=-\frac{\beta(sinh_A(\beta(\xi))+cosh_A(\beta(\xi)))}{\sigma(sinh_A(\beta(\xi))+cosh_A(\beta(\xi))+q)}.$$

Family-12. When $\beta=\eta$, $\sigma=\Theta\eta$ $(\Theta\neq0)$ and $\alpha=0$,

$$Q_{16}(\xi)=\frac{pA^{\eta\,\xi}}{p-\Theta qA^{\eta\,\xi}},$$

Subrogating Eqs. (5) and (6) into Eq. (4) and equating the coefficients of $Q^{\iota}(\xi)$ to zero, one gets nonlinear algebraic system in ϖ_ι $(\iota=0,1,\ldots,N)$ and μ. Then putting the obtained values of constants and solution set of Eq. (6) into Eq. (5) by using the wave transform (3), we get the exact wave solutions for Eq. (2).

Application of the method

To begin, we take the travelling wave transformation as:

$$q(x,t)=U(\xi)e^{i\phi},\quad \xi=x-\nu t,\quad \phi=-\kappa x+\omega t+\theta_0. \tag{7}$$

where

$$\begin{aligned}q_x&=(U'-i\kappa U)e^{i\phi},\\ q_t&=(-\nu U'+i\omega U)e^{i\phi},\\ q_{xx}&=(U''-2i\kappa U'-\kappa^2U)e^{i\phi},\\ q_{tt}&=(\nu^2U''-2i\omega\nu U'-\omega^2U)e^{i\phi}.\end{aligned} \tag{8}$$

Putting Eq. (7) into Eq. (1), we have

$$i(1-\rho\nu)U'-(\rho\omega-\kappa)U+(\lambda\nu^2+\varepsilon)U''-2i(\omega\nu\lambda+\varepsilon\kappa)U'-(\lambda\omega^2+\varepsilon\kappa^2)U+U^3=0. \tag{9}$$

$$q_1^{\pm}(x,t)=\pm\frac{(lnA)\sqrt{2\Delta(\beta^2-4\alpha\sigma)}}{2}\times tan_A\left(\frac{\sqrt{-(\beta^2-4\alpha\sigma)}}{2}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2-2(ln^2A)\lambda\Delta(\beta^2-4\alpha\sigma)+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2-2(ln^2A)\lambda\Delta(\beta^2-4\alpha\sigma)+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right]$$

Imaginary part:

$$1-\rho\nu-2(\omega\nu\lambda+\varepsilon\kappa)=0\quad\Rightarrow\quad\nu=\frac{1-2\varepsilon\kappa}{\rho+2\omega\lambda}, \tag{10}$$

Real part:

$$(\lambda\nu^2+\varepsilon)U''+(\kappa-\rho\omega-\lambda\omega^2+\varepsilon\kappa^2)U+U^3=0. \tag{11}$$

By applying Eq. (10) in Eq. (11), we get

$$\Delta U''+(\kappa-\rho\omega-\lambda\omega^2+\varepsilon\kappa^2)U+U^3=0, \tag{12}$$

where

$$\Delta=\lambda\left(\frac{1-2\varepsilon\kappa}{\rho+2\omega\lambda}\right)^2+\varepsilon.$$

Balancing the terms U'' and U^3 by using homogeneous principle yields $\Omega=1$.So, Eq. (13) has a formal solution of the form

$$U(\xi)=\varpi_0+\varpi_1 Q(\xi). \tag{13}$$

By substituting (13) into Eq. (12) and collecting all terms with the same order of $Q(\xi)$together, the left-hand side of (12) are converted into polynomial in $Q(\xi)$. Setting each coefficient of each polynomial to zero, wederive a set of algebraic equations for ϖ_0, ϖ_1 and ωas follows:

Coefficients of $Q^{\iota}(\xi)$

$$Q^0(\xi):\quad \Delta b_1(ln^2A)\alpha\beta+\kappa b_0-\rho\omega b_0-\lambda\omega^2b_0+\varepsilon\kappa^2b_0+b_0^3=0,$$

$$Q^1(\xi):\quad b_1\left(\Delta(ln^2A)\beta^2-\rho\omega+\kappa+\varepsilon\kappa^2-\lambda\omega^2+2\Delta(ln^2A)\alpha\sigma+3b_0^2\right)=0,$$

$$Q^2(\xi):\quad 3b_1\left(\Delta(ln^2A)\beta\sigma+b_0b_1\right)=0,$$

$$Q^3(\xi):\quad b_1\left(2\Delta(ln^2A)\sigma^2+b_1^2\right)=0.$$

Solving the above system of equations for ϖ_0, ϖ_1 and ω we obtain the following values:

$$\begin{aligned}&\varpi_0=\pm\frac{1}{2}(lnA)\beta\sqrt{-2\Delta},\qquad \varpi_1=\pm(lnA)\sqrt{-2\Delta}\sigma,\\ &\omega=\frac{-\rho\mp\sqrt{\rho^2-2(ln^2A)\lambda\Delta(\beta^2-4\alpha\sigma)+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}.\end{aligned} \tag{14}$$

The solutions of (1) corresponding to (7), (13) and (14) are singular solutions given by

being as constrain condition $\beta^2-4\alpha\sigma<0$ and $\sigma\neq0$. The graphs of the solution q_1 is shown in Fig. 1 of absolute valued together with the real valued solution in Fig. 2. To this purpose, we elect some special values of the constants obtained for $q_1$when $\beta=3,\alpha=2.5,\sigma=1,A=e,$ $\theta=-1,k=1,p=1,q=1,\rho=1.5,\lambda=-1,\varepsilon=1.$

$$q_2^{\pm}(x,t) = \mp\frac{(lnA)\sqrt{2\Delta\left(\beta^2-4\alpha\sigma\right)}}{2}$$
$$\times cot_A\left(\frac{\sqrt{-\left(\beta^2-4\alpha\sigma\right)}}{2}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2-2\left(ln^2A\right)\lambda\Delta\left(\beta^2-4\alpha\sigma\right)+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)$$
$$\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2-2\left(ln^2A\right)\lambda\Delta\left(\beta^2-4\alpha\sigma\right)+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right],$$

$$q_3^{\pm}(x,t) = \pm\frac{(lnA)\sqrt{2\Delta\left(\beta^2-4\alpha\sigma\right)}}{2}$$
$$\times\left(tan_A\left(\sqrt{-\left(\beta^2-4\alpha\sigma\right)}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2-2\left(ln^2A\right)\lambda\Delta\left(\beta^2-4\alpha\sigma\right)+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\right.$$
$$\left.\pm\sqrt{pq}\,sec_A\left(\sqrt{-\left(\beta^2-4\alpha\sigma\right)}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2-2\left(ln^2A\right)\lambda\Delta\left(\beta^2-4\alpha\sigma\right)+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\right)$$
$$\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2-2\left(ln^2A\right)\lambda\Delta\left(\beta^2-4\alpha\sigma\right)+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right],$$

$$q_4^{\pm}(x,t) = \pm\frac{(lnA)\sqrt{2\Delta\left(\beta^2-4\alpha\sigma\right)}}{2}$$
$$\times\left(-cot_A\left(\sqrt{-\left(\beta^2-4\alpha\sigma\right)}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2-2\left(ln^2A\right)\lambda\Delta\left(\beta^2-4\alpha\sigma\right)+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\right.$$
$$\left.\pm\sqrt{pq}\,csc_A\left(\sqrt{-\left(\beta^2-4\alpha\sigma\right)}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2-2\left(ln^2A\right)\lambda\Delta\left(\beta^2-4\alpha\sigma\right)+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\right)$$
$$\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2-2\left(ln^2A\right)\lambda\Delta\left(\beta^2-4\alpha\sigma\right)+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right],$$

$$q_5^{\pm}(x,t) = \pm\frac{(lnA)\sqrt{2\Delta\left(\beta^2-4\alpha\sigma\right)}}{4}$$
$$\times\left(tan_A\left(\frac{\sqrt{-\left(\beta^2-4\alpha\sigma\right)}}{4}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2-2\left(ln^2A\right)\lambda\Delta\left(\beta^2-4\alpha\sigma\right)+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\right.$$
$$\left.-cot_A\left(\frac{\sqrt{-\left(\beta^2-4\alpha\sigma\right)}}{4}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2-2\left(ln^2A\right)\lambda\Delta\left(\beta^2-4\alpha\sigma\right)+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\right)$$
$$\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2-2\left(ln^2A\right)\lambda\Delta\left(\beta^2-4\alpha\sigma\right)+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right]$$

For the next solutions,

$$q_6^{\pm}(x,t) = \mp\frac{\sqrt{-2\Delta\left(\beta^2-4\alpha\sigma\right)}}{2}$$
$$\times tanh_A\left(\frac{\sqrt{\beta^2-4\alpha\sigma}}{2}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2-2\left(ln^2A\right)\lambda\Delta\left(\beta^2-4\alpha\sigma\right)+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)$$
$$\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2-2\left(ln^2A\right)\lambda\Delta\left(\beta^2-4\alpha\sigma\right)+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right],$$

being constrain conditions are $\beta^2 - 4\alpha\sigma > 0$ and $\sigma \neq 0$,

$$q_7^{\pm}(x,t) = \mp\frac{\sqrt{-2\Delta\left(\beta^2-4\alpha\sigma\right)}}{2}$$
$$\times coth_A\left(\frac{\sqrt{\beta^2-4\alpha\sigma}}{2}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2-2\left(ln^2A\right)\lambda\Delta\left(\beta^2-4\alpha\sigma\right)+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)$$
$$\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2-2\left(ln^2A\right)\lambda\Delta\left(\beta^2-4\alpha\sigma\right)+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right]$$

$$q_8^{\pm}(x,t) = \pm\frac{(lnA)\sqrt{-2\Delta\left(\beta^2-4\alpha\sigma\right)}}{2}$$
$$\left(-tanh_A\left(\sqrt{\beta^2-4\alpha\sigma}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2-2\left(ln^2A\right)\lambda\Delta\left(\beta^2-4\alpha\sigma\right)+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\right.$$
$$\left.\pm i\sqrt{pq}\,\mathrm{sech}_A\left(\sqrt{\beta^2-4\alpha\sigma}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2-2\left(ln^2A\right)\lambda\Delta\left(\beta^2-4\alpha\sigma\right)+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\right)$$
$$\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2-2\left(ln^2A\right)\lambda\Delta\left(\beta^2-4\alpha\sigma\right)+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right],$$

$$q_9^{\pm}(x,t) = \pm\frac{(lnA)\sqrt{-2\Delta\left(\beta^2-4\alpha\sigma\right)}}{2}$$
$$\left(-coth_A\left(\sqrt{\beta^2-4\alpha\sigma}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2-2\left(ln^2A\right)\lambda\Delta\left(\beta^2-4\alpha\sigma\right)+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\right.$$
$$\left.\pm\sqrt{pq}\,\mathrm{csch}_A\left(\sqrt{\beta^2-4\alpha\sigma}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2-2\left(ln^2A\right)\lambda\Delta\left(\beta^2-4\alpha\sigma\right)+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\right)$$
$$\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2-2\left(ln^2A\right)\lambda\Delta\left(\beta^2-4\alpha\sigma\right)+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right],$$

$$q_{10}^{\pm}(x,t) = \mp\frac{(lnA)\sqrt{-2\Delta\left(\beta^2-4\alpha\sigma\right)}}{4}$$
$$\left(tanh_A\left(\frac{\sqrt{\beta^2-4\alpha\sigma}}{4}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2-2\left(ln^2A\right)\lambda\Delta\left(\beta^2-4\alpha\sigma\right)+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\right.$$
$$\left.+coth_A\left(\frac{\sqrt{\beta^2-4\alpha\sigma}}{4}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2-2\left(ln^2A\right)\lambda\Delta\left(\beta^2-4\alpha\sigma\right)+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\right)$$
$$\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2-2\left(ln^2A\right)\lambda\Delta\left(\beta^2-4\alpha\sigma\right)+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right]$$

For the next trigonometric solutions

$$q_{11}^{\pm}(x,t)=\pm(lnA)\sqrt{-2\Delta\alpha\sigma}\times tan_A\left(\sqrt{\alpha\sigma}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha\sigma+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha\sigma+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right],$$

$$q_{12}^{\pm}(x,t)=\mp(lnA)\sqrt{-2\Delta\alpha\sigma}\times cot_A\left(\sqrt{\alpha\sigma}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha\sigma+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha\sigma+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right],$$

$$q_{13}^{\pm}(x,t)=\pm(lnA)\sqrt{-2\Delta\alpha\sigma}\times\left(tan_A\left(2\sqrt{\alpha\sigma}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha\sigma+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\pm\sqrt{pq}sec_A\left(2\sqrt{\alpha\sigma}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha\sigma+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\right)\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha\sigma+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right],$$

$$q_{14}^{\pm}(x,t)=\pm(lnA)\sqrt{-2\Delta\alpha\sigma}\times\left(-cot_A\left(2\sqrt{\alpha\sigma}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha\sigma+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\pm\sqrt{pq}csc_A\left(2\sqrt{\alpha\sigma}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha\sigma+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\right)\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha\sigma+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right],$$

$$q_{15}^{\pm}(x,t)=\pm\frac{(lnA)\sqrt{-2\Delta\alpha\sigma}}{2}\times\left(tan_A\left(\frac{\sqrt{\alpha\sigma}}{2}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha\sigma+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)-cot_A\left(\frac{\sqrt{\alpha\sigma}}{2}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha\sigma+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\right)\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha\sigma+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right]$$

being constrain conditions are $\beta^2-4\alpha\sigma>0$ and $\sigma\neq0$. If $\alpha\sigma<0$ and $\beta=0$,

$$q_{16}^{\pm}(x,t)=\mp\sqrt{2\Delta\alpha\sigma}\times tanh_A\left(\sqrt{-\alpha\sigma}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha\sigma+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha\sigma+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right],$$

$$q_{17}^{\pm}(x,t)=\mp\sqrt{-2\Delta\alpha\sigma}\times coth_A\left(\sqrt{-\alpha\sigma}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha\sigma+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha\sigma+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right],$$

$$q_{18}^{\pm}(x,t)=\pm(lnA)\sqrt{2\Delta\alpha\sigma}\left(-tanh_A\left(2\sqrt{-\alpha\sigma}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha\sigma+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\pm i\sqrt{pq}\,\mathrm{sech}_A\left(2\sqrt{-\alpha\sigma}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha\sigma+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\right)\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha\sigma+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right],$$

$$q_{19}^{\pm}(x,t)=\pm(lnA)\sqrt{2\Delta\alpha\sigma}\left(-coth_A\left(2\sqrt{-\alpha\sigma}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha\sigma+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\pm\sqrt{pq}\,\mathrm{csch}_A\left(2\sqrt{-\alpha\sigma}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha\sigma+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\right)\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha\sigma+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right],$$

$$q_{20}^{\pm}(x,t)=\mp\frac{(lnA)\sqrt{2\Delta\alpha\sigma}}{2}\left(tanh_A\left(\frac{\sqrt{-\alpha\sigma}}{2}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha\sigma+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)+coth_A\left(\frac{\sqrt{-\alpha\sigma}}{2}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha\sigma+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\right)\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha\sigma+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right],$$

If $\beta=0$ and $\sigma=\alpha$,

$$q_{21}^{\pm}(x,t)=\pm(lnA)\alpha\sqrt{-2\Delta}\times tan_A\left(\alpha\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha^2+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha^2+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right],$$

$$q_{22}^{\pm}(x,t)=\mp(lnA)\alpha\sqrt{-2\Delta}\times cot_A\left(\alpha\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha^2+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha^2+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right],$$

$$q_{23}^{\pm}(x,t)=\pm(lnA)\alpha\sqrt{-2\Delta}\times\left(tan_A\left(2\alpha\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha^2+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\pm\sqrt{pq}\,sec_A\left(2\alpha\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha^2+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\right)\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha^2+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right],$$

$$q_{24}^{\pm}(x,t)=\pm(lnA)\alpha\sqrt{-2\Delta}\times\left(-cot_A\left(2\alpha\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha^2+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\pm\sqrt{pq}\,csc_A\left(2\alpha\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha^2+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\right)\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha^2+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right],$$

$$q_{25}^{\pm}(x,t)=\pm\frac{(lnA)\alpha\sqrt{-2\Delta}}{2}\times\left(tan_A\left(\frac{\alpha}{2}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha^2+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)-cot_A\left(\frac{\alpha}{2}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha^2+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\right)\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2+8\left(ln^2A\right)\lambda\Delta\alpha^2+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right]$$

From the general properties of the method as a constrain conditions, if we consider $\beta=0$ and $\sigma=-\alpha$, as we reach

$$q_{26}^{\pm}(x,t)=\mp\alpha\sqrt{-2\Delta}\times tanh_A\left(\alpha\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2-8\left(ln^2A\right)\lambda\Delta\alpha^2+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2-8\left(ln^2A\right)\lambda\Delta\alpha^2+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right],$$

$$q_{27}^{\pm}(x,t)=\mp\alpha\sqrt{-2\Delta}\times coth_A\left(\alpha\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2-8\left(ln^2A\right)\lambda\Delta\alpha^2+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2-8\left(ln^2A\right)\lambda\Delta\alpha^2+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right],$$

$$q_{28}^{\pm}(x,t)=\pm(lnA)\alpha\sqrt{-2\Delta}\left(-tanh_A\left(2\alpha\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2-8\left(ln^2A\right)\lambda\Delta\alpha^2+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\pm i\sqrt{pq}\,\mathrm{sech}_A\left(2\alpha\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2-8\left(ln^2A\right)\lambda\Delta\alpha^2+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\right)\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2-8\left(ln^2A\right)\lambda\Delta\alpha^2+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right],$$

$$q_{29}^{\pm}(x,t)=\pm(lnA)\alpha\sqrt{-2\Delta}\left(-coth_A\left(2\alpha\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2-8\left(ln^2A\right)\lambda\Delta\alpha^2+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\pm\sqrt{pq}\,\mathrm{csch}_A\left(2\alpha\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2-8\left(ln^2A\right)\lambda\Delta\alpha^2+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\right)\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2-8\left(ln^2A\right)\lambda\Delta\alpha^2+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right],$$

$$q_{30}^{\pm}(x,t)=\mp\frac{(lnA)\alpha\sqrt{-2\Delta}}{2}\left(tanh_A\left(\frac{\alpha}{2}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2-8\left(ln^2A\right)\lambda\Delta\alpha^2+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)+coth_A\left(\frac{\alpha}{2}\left(x\mp\frac{1-2\varepsilon\kappa}{\sqrt{\rho^2-8\left(ln^2A\right)\lambda\Delta\alpha^2+4\lambda\kappa(1+\varepsilon\kappa)}}t\right)\right)\right)\times exp\left[i\left(-\kappa x+\left(\frac{-\rho\mp\sqrt{\rho^2-8\left(ln^2A\right)\lambda\Delta\alpha^2+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t+\theta_0\right)\right],$$

When $\beta^2=4\alpha\sigma$,

$$q_{31}^{\pm}(x,t) = \pm\frac{\sqrt{-2\Delta}}{\left(x \mp \frac{1-2\varepsilon\kappa}{\sqrt{\rho^2+4\lambda\kappa(1+\varepsilon\kappa)}}\,t\right)}$$
$$\times exp\left[i\left(-\kappa x + \left(\frac{-\rho \mp \sqrt{\rho^2+4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t + \theta_0\right)\right]$$

Once it is considered as $\alpha = 0$, and $\beta \neq 0$, then, it is obtained that

$$q_{32}^{\pm}(x,t) = \pm(LnA)\beta\sqrt{-2\Delta}$$
$$\times\left(\frac{1}{2} - \frac{p}{cosh_A(\beta\xi) - sinh_A(\beta\xi) + p}\right)$$
$$\times exp\left[i\left(-\kappa x + \left(\frac{-\rho \mp \sqrt{\rho^2 - 2\left(Ln^2A\right)\lambda\Delta\beta^2 + 4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t + \theta_0\right)\right],$$

$$q_{33}^{\pm}(x,t) = \pm(LnA)\beta\sqrt{-2\Delta}\left(\frac{1}{2} - \frac{sinh_A(\beta(\xi)) + cosh_A(\beta(\xi))}{sinh_A(\beta(\xi)) + cosh_A(\beta(\xi)) + q}\right)$$
$$\times exp\left[i\left(-\kappa x + \left(\frac{-\rho \mp \sqrt{\rho^2 - 2\left(Ln^2A\right)\lambda\Delta\beta^2 + 4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t + \theta_0\right)\right],$$

where $\xi = x \mp \frac{1-2\varepsilon\kappa}{\sqrt{\rho^2 - 2(Ln^2A)\lambda\Delta\beta^2 + 4\lambda\kappa(1+\varepsilon\kappa)}}t$.

The geometric behaviour of the solutions of q_{33} are studied next drawing the 3-dimensional Figs. 3 and 4 of absolute valued together with the real valued solution. To this purpose, we elect some special values of the constants obtained for q_{33} when $\beta = 1, \alpha = 0, \sigma = 2, A = e$, $\theta = -1, k = 1.5, p = 0.95, q = 0.88, \rho = 2, \lambda = 1, \varepsilon = 2$.

When $\beta = \eta$, $\sigma = \Theta\eta$ $(\Theta \neq 0)$ and $\alpha = 0$,

$$q_{34}^{\pm}(x,t) = \pm(LnA)\eta\sqrt{-2\Delta}\left(\frac{1}{2} + \frac{\Theta p A^{\eta\left(x \mp \frac{1-2\varepsilon\kappa}{\sqrt{\rho^2 - 2\left(Ln^2A\right)\lambda\Delta k^2 + 4\lambda\kappa(1+\varepsilon\kappa)}}\right)}}{p - \Theta q A^{\eta\left(x \mp \frac{1-2\varepsilon\kappa}{\sqrt{\rho^2 - 2\left(Ln^2A\right)\lambda\Delta k^2 + 4\lambda\kappa(1+\varepsilon\kappa)}}\right)}}\right)$$
$$\times exp\left[i\left(-\kappa x + \left(\frac{-\rho \mp \sqrt{\rho^2 - 2\left(Ln^2A\right)\lambda\Delta k^2 + 4\lambda\kappa(1+\varepsilon\kappa)}}{2\lambda}\right)t + \theta_0\right)\right]$$

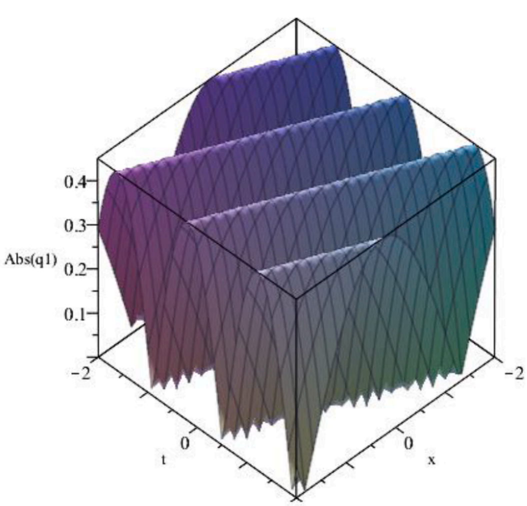

Fig. 1. The 3D graph, $|q_1|$.

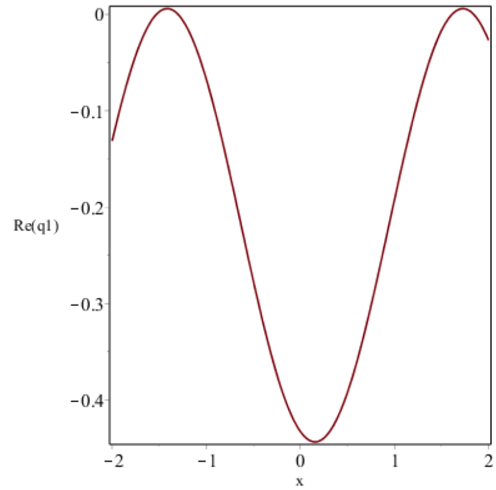

Fig. 2. The 2D graph, q_1 with $t = 2$.

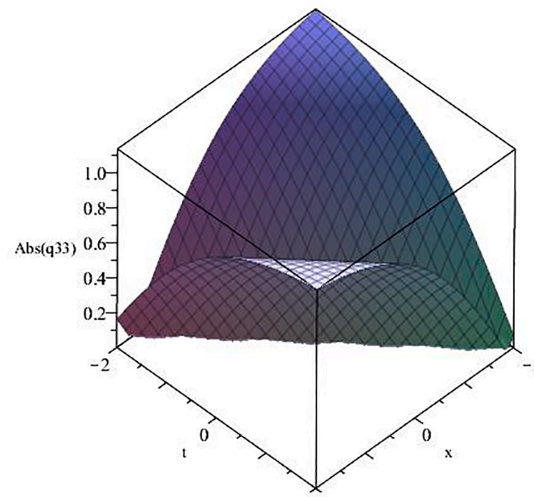

Fig. 3. The 3D graph, $|q_{33}|$.

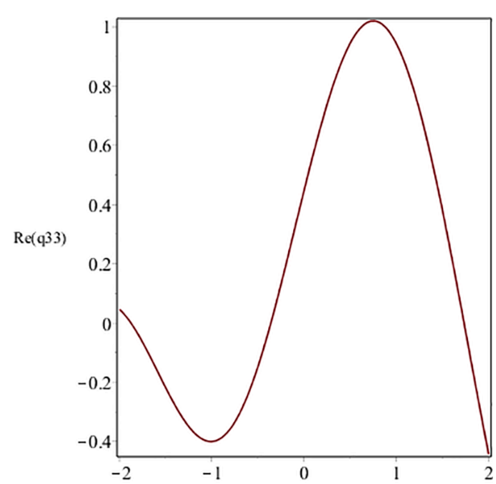

Fig. 4. The 2D graph, q_{33} with $t = 2$.

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Conclusions

In this paper, we have successfully applied the EDAM to the NLSE with group velocity dispersion with second order spatiotemporal dispersion coefficients. We have obtained many new mixed dark, bright and complex function solutions to the governing model, under the satisfying conditions coming from the general properties of the method. It can be also observed that all results satisfied the governing model. We presented also several simulations of the results obtained in this paper. It may be observed from the figures that these results have shown estimated simulation physically of the NLSE with group velocity dispersion.

CRediT authorship contribution statement

Haci Mehmet Baskonus: Conceptualization. **Wei Gao:** Formal analysis. **Hadi Rezazadeh:** Data curation, Writing – original draft. **S.M. Mirhosseini-Alizamini:** Validation. **Jamel Baili:** Formal analysis, Funding acquisition, Resources. **Hijaz Ahmad:** Writing – review & editing. **Tuan Nguyen Gia:** Writing – original draft.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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