Mathematics

Research article

Analysis of optical solitons solutions of two nonlinear models using analytical technique

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Abstract: Looking for the exact solutions in the form of optical solitons of nonlinear partial differential equations has become very famous to analyze the core structures of physical phenomena. In this paper, we have constructed some various type of optical solitons solutions for the Kaup-Newell equation (KNE) and Biswas-Arshad equation (BAE) via the generalized Kudryashov method (GKM). The conquered solutions help to understand the dynamic behavior of different physical phenomena. These solutions are specific, novel, correct and may be beneficial for edifying precise nonlinear physical phenomena in nonlinear dynamical schemes. Graphical recreations for some of the acquired solutions are offered.

1. Introduction

Recently, nonlinear evolution equations (NLEEs) has been developed as specific modules of the class of partial differential equations (PDEs). It is distinguished that investigating exact solutions for NLEEs, via many dissimilar methods shows an active part in mathematical physics and has become exciting and rich zones of research analysis for physicist and mathematicians. Lots of significant dynamic processes and phenomena in biology, chemistry, mechanics and physics can be expressed by nonlinear partial differential equations (NLPDEs). In NLEEs, nonlinear wave phenomena of diffusion, dispersion, reaction, convection and dissipation are very important. It is necessary to define

exact traveling solutions for these nonlinear equations to analyze various properties of these equations. Nowadays, NLEEs has become a significant area of research. Mostly, the existence of soliton solutions for NLEEs is of much important because of their widely applications in various areas of mathematical biology, chaos, neural physics, optical fibers and solid state physics etc.

Solitons are considered in the fields as optical communication, plasma, medical imaging, super continuum generation and nonlinear optics etc. They can transmit without changing their amplitude, velocity and wave form for a long distance. Optical soliton forms the excellent transporter minutes in the telecommunication engineering. Nowadays, some methods have been established for discovering exact traveling and solitary wave solutions of NLEEs. Various influential methods for instance, auxiliary equation method [1, 2], homogeneous balance method [3, 4], the Exp-function method [5, 6], the tanh-function method [7], Darboux transformation method [8, 9], the modified extended tanh-function method [10], the first integral method [11, 12], Jacobi elliptic function method [13, 14], the modified simple equation method [15–17], the exp(-F(x))-expansion method [18], the (G'/G)-expansion method [19–26], the variational iteration method [27] the homotopy perturbation method [28–32], the F-expansion method [33–35] and many more [36–41]. Many models are existing to report this dynamic in the structure of optical fibers. The Schrodinger equation, the important model in submicroscopic phenomena and developed a fundamental importance to quantum mechanics. Such model denotes to the form of wave functions that manage the motion of small particles and classifies how these waves are transformed by external impacts. It has been measured and considered in several designs.

In optical fibers, most of these models are frequently stated in the time domain, and when fields at dissimilar frequencies spread through the fiber the common practice is also to transcribe a distance equation for each field component. The nonlinear transformation of dielectric of the fiber termed as the Kerr effect is applied to neutralize the dispersion effect, in this state, the optical pulse might lean to form a steady nonlinear pulse known as an optical soliton. The bit rate of transmission is restricted by the dispersion of the fiber material. Soliton transmission is an area of huge interest since of the wide applications in ultrafast signal routing systems, transcontinental and short-light-pulse telecommunication [42–45]. In this work, we used the generalized Kudryashov method to construct the exact traveling wave solutions for the Kaup-Newell equation (KNE) and Biswas-Arshad equation (BAE). The KNE is a significant model with many applications in optical fibers. The dynamics of solitons in optical tools is observed as an important arena of research in nonlinear optics that has added much attention in the past few decades [46–49]. The transmission of waves in optical tools with Kerr dispersion rests important in construction to the so-called time evolution equations [50–52]. The three models that invent from the Nonlinear Schrodinger equation (NLSE) which are termed as the Derivative Nonlinear Schrodinger Equations (DNLSE) are classified into three classes: I, II, and III. Connected in this study is the DNLSE-I which is in its place known as the KNE. This class will be the focal point of the current study and a lot of inquiries have correctly been approved in the literature. Recently, Biswas and Arshad [53] constructed a model from the NLSE known as Biswas-Arshed equation (BAE). The BAE is one of the important models in the telecommunications industry. The extreme remarkable story of this model is that the self-phase modulation is ignored and likewise GVD is negligibly slight. The plus point of this method is that it offers further novel exact solutions in optical solitons form.

The draft of this paper is organized like this. Section 2, contained the description of the

generalized Kudryashov method. In section 3, application of GKM for KNE is presented. Section 4 presents the application of GKM for BAE. Section 5 contains results and discussion. Conclusion of the paper is discussed in section 6.

2. Description of generalized Kudryashov method

The steps of GKM [54] are as follows
Let NLEE in the form

$$W(q, q_x, q_t, q_{xt}, q_{tt}, q_{xx}, \cdots) = 0, \tag{2.1}$$

where $q = q(x, t)$ is a function.

Step 1. Applying the following wave transformation

$$q(x, t) = g(\eta), \quad \eta = (x - vt), \tag{2.2}$$

into (2.1), so (2.1) converts to nonlinear ODE in the form

$$H(q, q', q'', q''', \cdots) = 0, \tag{2.3}$$

here, q is a function of η and $q' = dq/d\eta$ and v is the wave-speed.

Step 2. Let (2.3) has the solution in form

$$g(\eta) = \frac{\sum_{i=0}^{N} a_i S^i(\eta)}{\sum_{j=0}^{M} b_j S^j(\eta)}, \tag{2.4}$$

where a_i, b_j are constants and $(i \le 0 \le N)$, $(j \le 0 \le M)$ such that $a_N \ne 0$, $b_M \ne 0$.

$$S(\eta) = \frac{1}{1 + Ae^\eta}, \tag{2.5}$$

is the solution in the form

$$\frac{dS(\eta)}{d\eta} = S^2(\eta) - S(\eta), \tag{2.6}$$

where A is constant.

Step 3. Using balancing rule in (2.3) to obtain the values of N and M.

Step 4. Utilizing (2.4) and (2.6) into (2.3), we get an expression in S^i, where $(i = 0, 1, 2, 3, 4, \cdots)$. Then collecting all the coefficients of S^i with same power(i) and equating to zero, we get a system of alebraic equations in all constant terms. This system of algebraic equations can be solved by Maple to unknown parameters.

3. Kaup-Newell equation

The governing equation [55] is given as:

$$iq_t + aq_{xx} + ib(|q|^2 q)_x = 0. \tag{3.1}$$

Here, $q(x, t)$ is a complex valued function, indicates the wave pfofile and rests on variables, space x and time t. It includes the non-Kerr dispersion, evolution and and GVD terms. Also, a is the coefficient of GVD and b is the coefficient of self-steepening term.

Suppose (3.1) has the following solution

$$q(x, t) = g(\eta)e^{i\phi(x, t)}, \tag{3.2}$$

where

$$\eta = \alpha x - ct \ , \ \ \phi(x, t) = -\kappa x + \omega t. \tag{3.3}$$

Here $g(\eta)$, κ, c and ω are the amplitude, frequency, speed and wave number of the pulse, respectively. Putting (3.2) into (3.1), and splitting into imaginary and real parts.

The imaginary part has the form

$$-cg' - 2a\kappa\alpha g' + 3b\alpha g^2 g' = 0. \tag{3.4}$$

We can get easily the value of c as under

$$c = -2\alpha\kappa a,$$

and the constraint condition as under

$$3\alpha b g^2 = 0.$$

The real part has the form

$$a\alpha^2 g'' + \kappa b g^3 - (a\kappa^2 + \omega)g = 0. \tag{3.5}$$

Now, balancing the g'' and non-linear term g^3 in (3.5), we get $N = M + 1$. So for $M = 1$, we get $N = 2$.

3.1. Application of GKM

The solution of (3.5) by generalized Kudryashov method as given in (2.4), reduces to the form

$$g(\eta) = \frac{a_0 + a_1 S(\eta) + a_2 S^2(\eta)}{b_0 + b_1 S(\eta)}, \tag{3.6}$$

a_0, a_1, a_2, b_0 and b_1 are constants. Subtitling the (3.6) into (3.5) and also applying (2.6), we get an expression in $S(\eta)$. Collecting the coefficients of same power of S^i and equating to zero, the system of equations is obtained, as follows.

$$\begin{cases}
b\kappa a_0^3 + (-a\kappa^2 - \omega)a_0 b_0^2 = 0, \\
3b\kappa a_0^2 a_1 + (-a\kappa^2 - \omega)a_1 b_0^2 + a\alpha^2 a_1 b_0^2 + 2(-a\kappa^2 - \omega)a_0 b_0 b_1 - a\alpha^2 a_0 b_0 b_1 = 0, \\
3b\kappa a_0 a_1^2 + 3b\kappa a_0^2 a_2 - 3a\alpha^2 a_1 b_0^2 + (-a\kappa^2 - \omega)a_2 b_0^2 + 4a\alpha^2 a_2 b_0^2 + 3a\alpha^2 a_0 b_0 b_1 \\
\quad + 2(-a\kappa^2 - \omega)a_1 b_0 b_1 - a\alpha^2 a_1 b_0 b_1 + (-a\kappa^2 - \omega)a_0 b_1^2 + a\alpha^2 a_0 b_1^2 = 0, \\
b\kappa a_1^3 + 6b\kappa a_0 a_1 a_2 + 2a\alpha^2 a_1 b_0^2 - 10a\alpha^2 a_2 b_0^2 - 2a\alpha^2 a_1 b_0 b_1 + a\alpha^2 a_1 b_0 b_1 \\
\quad + 2(-a\kappa^2 - \omega)a_2 b_0 b_1 + 3a\alpha^2 a_2 b_0 b_1 - a\alpha^2 a_0 b_1^2 + (-a\kappa^2 - \omega)a_1 b_1^2 = 0, \\
3b\kappa a_1^2 a_2 + 3b\kappa a_0 a_2^2 + 6a\alpha^2 a_2 b_0^2 - 9a\alpha^2 a_2 b_0 b_1 + (-a\kappa^2 - \omega)a_2 b_1^2 + a\alpha^2 a_2 b_1^2 = 0, \\
3b\kappa a_1 a_2^2 + 6a\alpha^2 a_2 b_0 b_1 - 3a\alpha^2 a_2 b_1^2 = 0, \\
b\kappa a_2^3 + 2a\alpha^2 a_2 b_1^2 = 0.
\end{cases} \tag{3.7}$$

By solving the above system, we get various types of solutions. These solutions are deliberated below.

Case 1.

$$a_0 = 0, \ a_1 = a_1, \ a_2 = -a_1, \ b_0 = -\frac{1}{2\,b_1}, \ b_1 = b_1,$$

$$a = a, \ \kappa = -\frac{2\,a\,\alpha^2\,b_1^2}{a_1^2\,b}, \ \omega = -\frac{a\,\alpha^2(4\,a^2\,\alpha^2\,b_1^4 - a_1^4\,b^2)}{a_1^4\,b^2}.$$

Case 1 corresponds the following solution for Kaup-Newell equation

$$q(x,t) = \left(\frac{-\frac{a_1}{(1+Ae^{\alpha x - ct})} - \frac{a_1}{(1+Ae^{\alpha x - ct})^2}}{-\frac{1}{2\,b_1} + \frac{b_1}{(1+Ae^{\alpha x - ct})}} \right) \times e^{\iota \left(\frac{2\,a\,\alpha^2\,b_1^2}{a_1^2\,b} x - \frac{a\,\alpha^2(4\,a^2\,\alpha^2\,b_1^4 - a_1^4\,b^2)}{a_1^4\,b^2} t \right)}. \tag{3.8}$$

Case 2.

$$a_0 = a_0, \ a_1 = -\frac{(b_1 + 2b_0)a_0}{b_0}, \ a_2 = 0, \ b_0 = b_0, \ b_1 = b_1.$$

$$a = a, \ \kappa = -\frac{a\,\alpha^2\,b_0^2}{2\,b\,a_0^2}, \ \omega = -\frac{a\,\alpha^2(2b^2\,a_0^4 + b_0^4 a_2 \alpha^2)}{4\,b^2\,a_0^4}.$$

Case 2 corresponds the following solution for Kaup-Newell equation

$$q(x,t) = \left(\frac{a_0 - \frac{(b_1 + 2b_0)a_0}{b_0(1+Ae^{\alpha x - ct})}}{b_0 + \frac{b_1}{(1+Ae^{\alpha x - ct})}} \right) \times e^{\iota \left(\frac{a\,\alpha^2\,b_0^2}{2\,b\,a_0^2} x - \frac{a\,\alpha^2(2b^2\,a_0^4 + b_0^4 a_2 \alpha^2)}{4\,b^2\,a_0^4} t \right)}. \tag{3.9}$$

Case 3.

$$a_0 = -\frac{b_0 a_2}{2b_1}, \ a_1 = \frac{a_2(2b_0 - b_1)}{2b_1}, \ a_2 = a_2, \ b_0 = b_0, \ b_1 = b_1.$$

$$a = a, \ \kappa = -\frac{2\,a\,\alpha^2\,b_1^2}{b\,a_2^2}, \ \omega = -\frac{a\alpha^2(b^2 a_2^4 + 8a^2\alpha^2 b_1^4)}{2b^2 a_2^4}.$$

Case 3 corresponds the following solution for Kaup-Newell equation

$$q(x,t) = \left(\frac{-\frac{b_0 a_2}{2b_1} + \frac{a_2(2b_0 - b_1)}{2b_1(1+Ae^{\alpha x - ct})} + \frac{a_2}{(1+Ae^{\alpha x - ct})^2}}{b_0 + \frac{b_1}{(1+Ae^{\alpha x - ct})}} \right) \times e^{\iota \left(\frac{2\,a\,\alpha^2\,b_1^2}{b\,a_2^2} x - \frac{a\alpha^2(b^2 a_2^4 + 8a^2\alpha^2 b_1^4)}{2b^2 a_2^4} t \right)}. \tag{3.10}$$

Case 4.

$$a_0 = \frac{1}{2}a_2, \ a_1 = -a_2, \ a_2 = a_2, \ b_0 = -\frac{1}{b}b_1, \ b_1 = b_1.$$

$$a = a, \ \kappa = -\frac{2a\alpha^2 b_1^2}{ba_2^2}, \ \omega = -\frac{2a\alpha^2(b^2 a_2^4 + 2a^2\alpha^2 b_1^4)}{b^2 a_2^4}.$$

Case 4 corresponds the following solution for Kaup-Newell equation

$$q(x,t) = \left(\frac{\frac{1}{2}a_2 - \frac{a_2}{(1+Ae^{\alpha x - ct})} + \frac{a_2}{(1+Ae^{\alpha x - ct})^2}}{-\frac{1}{b}b_1 + \frac{b_1}{(1+Ae^{\alpha x - ct})}} \right) \times e^{\iota \left(\frac{2a\alpha^2 b_1^2}{ba_2^2} x - \frac{2a\alpha^2(b^2 a_2^4 + 2a^2\alpha^2 b_1^4)}{b^2 a_2^4} t \right)}. \tag{3.11}$$

4. Biswas-Arshed equation

The BAE with Kerr Law nonlinearity [56] is

$$\alpha_1 q_{xx} + \alpha_2 q_{xt} + i q_t + i(\beta_1 q_{xxx} + \beta_2 q_{xxt}) = i(\lambda(|q|^2 q)_x + \mu(|q|^2)_x q + \theta|q|^2 q_x). \tag{4.1}$$

Here $q(x, t)$ representing the wave form. On the left of (4.1) α_1 and α_2 are the coefficients of GVD and STD, respectively. β_1 and β_2 are the coefficients of 3OD and STD, respectively. On the right of (4.1) μ and θ represents the outcome of nonlinear disperssion and λ represents the outcome of self-steepening in the nonappearance of SPM.

Let us assumed that the solution of (4.1) is as under

$$q(x, t) = g(\eta) \, e^{i\phi(x, t)}. \tag{4.2}$$

where

$$\phi(x, t) = -\kappa x + \omega t + \theta_0, \quad \eta = x - vt. \tag{4.3}$$

Here $g(\eta)$ shows amplitude, $\phi(x, t)$ is phase component. Also κ, v, θ_0, ω denote the soliton frequency, speed, phase constant and wave number, respectively.

Substituting (4.2) into (4.3) and splitting it into imaginary and real parts.

The imaginary part has the form

$$(\beta_2 v\kappa^2 - 3\beta_1\kappa^2 + 2\beta_2\omega\kappa - v + 2\alpha_2 v\kappa - 2\alpha_1\kappa + \alpha_2\omega)g' + (-\beta_2 v + \beta_1)g''' - (2\mu + \theta + 3\lambda)g^2 g' = 0.$$

We can get easily the value of v as under

$$v = \frac{\beta_1}{\beta_2},$$

and the constraints conditions as under

$$\begin{cases} \beta_2 v\kappa^2 + 2\beta_2\omega\kappa - 3\beta_1\kappa^2 - v - 2\alpha_1\kappa + 2\alpha_2 v\kappa + \alpha_2\omega = 0, \\ 3\lambda + 2\mu + \theta = 0. \end{cases}$$

The real part has the form

$$(\alpha_1 - \alpha_2 v + 3\beta_1\kappa - 2\beta_2 v\kappa - \omega\beta_2)g'' - (\omega + \alpha_1\kappa^2 + \beta_1\kappa^3 - \alpha_2\omega\kappa - \beta_2\omega\kappa^2)g - (\lambda + \theta)\kappa g^3 = 0. \tag{4.4}$$

Using balancing principal on (4.4), we attain $M + 1 = N$. So for $M = 1$, we obtain $N = 2$.

4.1. Application of GKM

Hence, solution of (4.4) by GKM as given in (2.4) will be reduced into the following form

$$g(\eta) = \frac{a_0 + a_1 S(\eta) + a_2 S^2(\eta)}{b_0 + b_1 S(\eta)}. \tag{4.5}$$

a_0, a_1, a_2, b_0 and b_1 are constants.

Substituting (4.5) into (4.4) and also applying (2.6), we acquire an expression in $S(\eta)$. Collecting the coefficients of with same power S^i and equating to zero, the following system equations is got.

$$
\begin{cases}
-2\omega\beta_2 b_1^2 a_2 - 4\beta_2 v\kappa b_1^2 a_2 + \kappa\theta a_2^3 + 2\alpha_1 b_1^2 a_2 + 6\beta_1\kappa b_1^2 a_2 - 2\alpha_2 v b_1^2 a_2 + \kappa\lambda a_2^3 = 0, \\
3\omega\beta_2 b_1^2 a_2 + 3\alpha_2 v b_1^2 a_2 + 6\beta_2 v\kappa b_1^2 a_2 - 9\beta_1\kappa b_1^2 a_2 + 6\alpha_1 b_0 a_2 b_1 - 12\beta_2 v\kappa b_0 a_2 b_1 - 3\alpha_1 b_1^2 a_2 \\
+3\kappa\theta a_2 a_2^2 + 3\kappa\lambda a_1 a_2^2 + 18\beta_1\kappa b_0 a_2 b_1 - 6\alpha_2 v b_0 a_2 b_1 - 6\omega\beta_2 b_0 a_2 b_1 = 0, \\
\alpha_1 b_1^2 a_2 + \beta_2\omega\kappa^2 b_1^2 a_2 + 3\kappa\lambda a_0 a_2^2 - \alpha_2 v b_1^2 a_2 + \alpha_2\omega\kappa b_1^2 a_2 + 18\beta_1\kappa a_2 b_0^2 \\
+3\kappa\lambda a_1^2 a_2 + 3\kappa\theta a_1^2 a_2 + 6\alpha_1 a_2 b_0^2 + 3\kappa\theta a_0 a_2^2 - \beta_1\kappa^3 b_1^2 a_2 - 6\omega\beta_2 a_2 b_0^2 - 27\beta_1\kappa b_0 a_2 b_1 \\
-2\beta_2 v\kappa b_1^2 a_2 - 9\alpha_1 b_0 a_2 b_1 + 3\beta_1\kappa b_1^2 a_2 - 6\alpha_2 v a_2 b_0^2 - 12\beta_2 v\kappa a_2 b_0^2 + 9\alpha_2 v b_0 a_2 b_1 - \\
\alpha_1\kappa^2 b_1^2 a_2 - \omega\beta_2 b_1^2 a_2 + 18\beta_2 v\kappa b_0 a_2 b_1 - \omega b_1^2 a_2 + 9\omega\beta_2 b_0 a_2 b_1 = 0, \\
-2\beta_2 v\kappa b_1 a_1 b_0 - 6\beta_2 v\kappa b_0 a_2 b_1 + 4\beta_2 v\kappa b_0 b_1 a_0 + 2\alpha_2\omega\kappa b_0 b_1 a_2 + 2\beta_2\omega\kappa^2 b_0 b_1 a_2 \\
+\omega\beta_2 b_1^2 a_0 - 2\omega\beta_2 b_0^2 a_1 - 2\omega b_0 b_1 a_2 - \alpha_1\kappa^2 b_1^2 a_1 - \beta_1\kappa^3 b_1^2 a_1 + \alpha_1 b_1 a_1 b_0 + 3\alpha_1 b_0 a_2 b_1 \\
-2\alpha_1 b_0 b_1 a_0 + 10\alpha_2 v a_2 b_0^2 + \alpha_2 v b_1^2 a_0 - 2\alpha_2 v b_0^2 a_1 - 30\beta_1\kappa a_2 b_0^2 - 3\beta_1\kappa b_1^2 a_0 + 6\beta_1\kappa b_0^2 a_1 \\
+10\omega\beta_2 a_2 b_0^2 - 10\alpha_1 a_2 b_0^2 - \omega b_1^2 a_1 + \kappa\theta a_1^3 + \kappa\lambda a_1^3 + 2\alpha_1 b_0^2 a_1 - \alpha_1 b_1^2 a_0 - 2\alpha_1\kappa^2 b_0 b_1 a_2 \\
-2\beta_1\kappa^3 b_0 b_1 a_2 + \alpha_2\omega\kappa b_1^2 a_1 + \beta_2\omega\kappa^2 b_1^2 a_1 - \alpha_2 v b_1 a_1 b_0 - 3\alpha_2 v b_0 a_2 b_1 + 2\alpha_2 v b_0 b_1 a_0 \\
+3\beta_1\kappa b_1 a_1 b_0 + 9\beta_1\kappa b_0 a_2 b_1 - 6\beta_1\kappa b_0 b_1 a_0 + 20\beta_2 v\kappa a_2 b_0^2 + 2\beta_2 v\kappa b_1^2 a_0 - 4\beta_2 v\kappa b_0^2 a_1 \\
-\omega\beta_2 b_1 a_1 b_0 - 3\omega\beta_2 b_0 a_2 b_1 + 2\omega\beta_2 b_0 b_1 a_0 + 6\kappa\lambda a_0 a_1 a_2 + 6\kappa\theta a_0 a_1 a_2 = 0, \\
2\beta_2 v\kappa b_1 a_1 b_0 - 6\beta_2 v\kappa b_0 b_1 a_0 + 2\alpha_2\omega\kappa b_0 b_1 a_1 + 2\beta_2\omega\kappa^2 b_0 b_1 a_1 - \omega\beta_2 b_1^2 a_0 + 3\omega\beta_2 b_0^2 a_1 \\
+3\kappa\lambda a_0^2 a_2 + 3\kappa\lambda a_0 a_1^2 + 3\kappa\theta a_0^2 a_2 + 3\kappa\theta a_0 a_1^2 - 2\omega b_0 b_1 a_1 - \alpha_1\kappa^2 b_0^2 a_2 - \alpha_1\kappa^2 b_1^2 a_0 \\
-\beta_1\kappa^3 b_0^2 a_2 - \beta_1\kappa^3 b_1^2 a_0 - \alpha_1 b_1 a_1 b_0 + 3\alpha_1 b_0 b_1 a_0 - 4\alpha_2 v a_2 b_0^2 - \alpha_2 v b_1^2 a_0 + 3\alpha_2 v b_0^2 a_1 \\
+12\beta_1\kappa a_2 b_0^2 + 3\beta_1\kappa b_1^2 a_0 - 9\beta_1\kappa b_0^2 a_1 - 4\omega\beta_2 a_2 b_0^2 + 4\alpha_1 a_2 b_0^2 - \omega b_1^2 a_0 - \omega b_0^2 a_2 - 3\alpha_1 b_0^2 a_1 \\
+\alpha_1 b_1^2 a_0 - 2\beta_1\kappa^3 b_0 b_1 a_1 + \alpha_2\omega\kappa b_0^2 a_2 + \alpha_2\omega\kappa b_1^2 a_0 + \beta_2\omega\kappa^2 b_0^2 a_2 + \beta_2\omega\kappa^2 b_1^2 a_0 + \alpha_2 v b_1 a_1 b_0 \\
-3\alpha_2 v b_0 b_1 a_0 - 3\beta_1\kappa b_1 a_1 b_0 + 9\beta_1\kappa b_0 b_1 a_0 - 8\beta_2 v\kappa a_2 b_0^2 - 2\beta_2 v\kappa b_1^2 a_0 + 6\beta_2 v\kappa b_0^2 a_1 \\
+\omega\beta_2 b_1 a_1 b_0 - 3\omega\beta_2 b_0 b_1 a_0 - 2\alpha_1\kappa^2 b_0 b_1 a_1 = 0, \\
-\omega b_0^2 a_1 - \alpha_1 b_0 b_1 a_0 - \omega\beta_2 b_0^2 a_1 - \alpha_2 v b_0^2 a_1 + \omega\beta_2 b_0 b_1 a_0 + \alpha_2\omega\kappa b_0^2 a_1 - \beta_1\kappa^3 b_0^2 a_1 \\
-2\beta_2 v\kappa b_0^2 a_1 - 2\beta_2 v\kappa b_0^2 a_1 - 2\alpha_1\kappa^2 b_0 b_1 a_0 + \alpha_2 v b_0 b_1 a_0 + 3\beta_1\kappa b_0^2 a_1 - 2\beta_1\kappa^3 b_0 b_1 a_0 \\
-2\omega b_0 b_1 a_0 + \alpha_1 b_0^2 a_1 - \alpha_1\kappa^2 b_0^2 a_1 + 3\kappa\theta a_0^2 a_1 - 3\beta_1\kappa b_0 b_1 a_0 + 2\beta_2 v\kappa b_0 b_1 a_0 \\
+2\alpha_2\omega\kappa b_0 b_1 a_0 + \beta_2\omega\kappa^2 b_0^2 a_1 + 2\beta_2\omega\kappa^2 b_0 b_1 a_0 + 3\kappa\lambda a_0^2 a_1 = 0, \\
\alpha_2\omega\kappa b_0^2 a_0 + \beta_2\omega\kappa^2 b_0^2 a_0 + \kappa\theta a_0^3 - \alpha_1\kappa^2 b_0^2 a_0 - \beta_1\kappa^3 b_0^2 a_0 + \kappa\lambda a_0^3 - \omega b_0^2 a_0 = 0.
\end{cases}
\tag{4.6}
$$

On solving above system, get various types of solutions. These solutions are deliberated below.

Case 1.

$$\kappa = \kappa, \ \lambda = -\theta,$$

$$\omega = -\alpha_2 v - \omega\beta_2 + \beta_2\omega\kappa^2 - 2\beta_2 v\kappa - \beta_1\kappa^3 + \alpha_2\omega\kappa + 3\beta_1\kappa + \alpha_1 - \alpha_1\kappa^2,$$

$$a_0 = 0 \, , a_1 = a_1, \ a_2 = 0, \ b_0 = -b_1, \ b_1 = b_1.$$

Above these values correspond to the following solution for Biswas-Arshed equation.

$$q(x, t) = -\frac{a_1}{b_1}\left[\frac{1}{Ae(x - vt)}\right] \times e^{i\{-\kappa x + \omega t + \theta_0\}}. \tag{4.7}$$

Case 2.

$$\kappa = \kappa, \lambda = \frac{1}{2} \times \frac{-2\kappa\theta a_0^2 - \alpha_1 b_0^2 - 3\beta_1\kappa b_0^2 + \omega\beta_2 b_0^2 + \alpha_2 v b_0^2 + 2\beta_2 v\kappa b_0^2}{\kappa a_0^2},$$

$$\omega = -\frac{1}{2}\alpha_1 - \frac{3}{2}\beta_1\kappa + \frac{1}{2}\omega\beta_2 - \alpha_1\kappa^2 + \frac{1}{2}\alpha_2 v - \beta_1\kappa^3 + \beta_2\omega\kappa^2 + \beta_2 v\kappa + \alpha_2\omega\kappa,$$

$$a_0 = a_0, a_1 = \frac{-a_0(b_1 + 2b_0)}{b_0}, \ a_2 = 0, \ b_0 = b_0, b_1 = b_1.$$

Above these values correspond to the following solution for Biswas-Arshed equation

$$q(x,t) = \frac{a_0}{b_0}\left[\frac{b_0 A e^{(x-vt)} - (b_0 + b_1)}{b_0 A e^{(x-vt)} + (b_0 + b_1)}\right] \times e^{i\{-\kappa x + \omega t + \theta_0\}}. \tag{4.8}$$

Case 3.

$$\kappa = \kappa, \ \lambda = \frac{-2\alpha_1 b_1^2 - \kappa\theta a_2^2 - 6\beta_1\kappa b_1^2 + 2\omega\beta_2 b_1^2 + 2\alpha_2 v b_1^2 + 4\beta_2 v\kappa b_1^2}{\kappa a_2^2},$$

$$\omega = \alpha_2\omega\kappa + \beta_2\omega\kappa^2 - \alpha_1\kappa^2 - \beta_1\kappa^3 - 2\alpha_1 - 6\beta_1\kappa + 2\omega\beta_2 + 2\alpha_2 v + 4\beta_2 v\kappa,$$

$$a_0 = \frac{1}{2}a_2, \ a_1 = -a_2, a_2 = a_2, \ b_0 = -\frac{1}{2}b_1, \ b_1 = b_1.$$

Above these values correspond to the following solution for Biswas-Arshed equation

$$q(x,t) = \frac{a_2}{b_1}\left[\frac{1 + A^2 e^{2(x-vt)}}{1 - A^2 e^{2(x-vt)}}\right] \times e^{i\{-\kappa x + \omega t + \theta_0\}}. \tag{4.9}$$

Case 4.

$$\kappa = 0, \lambda = \lambda, \ \omega = \alpha_1 - \omega\beta_2 - \alpha_2 v,$$

$$a_0 = -a_1, \ a_1 = a_1, \ a_2 = 0, \ b_0 = 0, \ b_1 = b_1.$$

Above these values correspond to the following solution for Biswas-Arshed equation

$$q(x,t) = -\frac{a_1}{b_1}\left[A e^{(x-vt)}\right] \times e^{i\{\omega t + \theta_0\}}. \tag{4.10}$$

Case 5.

$$\kappa = -\frac{-\alpha_1 + \omega\beta_2 + \alpha_2 v}{-3\beta_1 + 2\beta_2 v}, \lambda = -\theta,$$

$$\omega = -\frac{1}{(-3\beta_1 + 2\beta_2 v)^3} \times \Bigg[-4\alpha_1^2\alpha_2 v^2\beta_2 + 2\alpha_1\alpha_2^2 v^3\beta_2 - 9\alpha_2\omega\alpha_1\beta_1^2 +$$
$$9\alpha_2\omega^2\beta_2\beta_1^2 + 9\alpha_2^2\omega v\beta_1^2 + 2\alpha_2^2\omega v^3\beta_2^2 + 6\omega\beta_2\alpha_1^2\beta_1 - 6\omega\beta_2^2\alpha_1^2 v - 6\omega^2\beta_2^2\alpha_1\beta_1$$
$$+ 6\omega^2\beta_2^3\alpha_1 v + 3\beta_1\alpha_1^2\alpha_2 v + 2\omega^3\beta_2^3\beta_1 - 2\omega^3\beta_2^4 v - \beta_1\alpha_2^3 v^3 + 2\alpha_1^3\beta_2 v + 4\alpha_2\omega\alpha_1\beta_2^2 v^2$$
$$- 9\alpha_2\omega^2\beta_2^2\beta_1 v - 12\alpha_2^2\omega v^2\beta_1\beta_2 - 2\beta_1\alpha_1^3 + 6\alpha_2\alpha_1\beta_1\beta_2 v \Bigg],$$

$$a_0 = 0, \ a_1 = a_1, \ a_2 = a_2, \ b_0 = b_0, \ b_1 = b_1.$$

Above these values correspond to the following solution for Biswas-Arshed equation

$$q(x,t) = \left[\frac{a_1 A e^{(x-vt)} + (a_1 + a_2)}{A^2 e^{2(x-vt)} + (2b_0 + b_1)A e^{(x-vt)} + (b_0 + b_1)}\right] \times e^{i\{-\kappa x + \omega t + \theta_0\}}. \tag{4.11}$$

5. Results and discussions

In this study, we effectively construct novel exact solutions in form of optical solitons for Kaup-Newell equation and Biswas-Arshed equation using the generalized Kudryashov method. This method is considered as most recent scheme in this arena and that is not utilized to this equation earlier. For physical analysis, 3-dim, 2-dim and contour plots of some of these solutions are included with appropriate parameters. These acquired solutions discover their application in communication to convey information because solitons have the capability to spread over long distances without reduction and without changing their forms. Acquired results are novel and distinct from that reported results. In this paper, we only added particular figures to avoid overfilling the document. For graphical representation for KNE and BAE, the physical behavior of (3.8) using the proper values of parameters $\alpha = 0.3$, $a_1 = 0.65$, $b_1 = 0.85$, $p = 0.98$, $q = 0.95$, $k = 2$, $A = 3$, $b = 2$, $c = 4$. and $t = 1$ are shown in Figure 1, the physical behavior of (3.9) using the appropriate values of parameters $\alpha = 0.75$, $a_0 = 1.5$, $b_0 = 1.7$, $b_1 = 0.98$, $A = 3$, $b = 1.6$, $a_0 = 2$, $c = 2.5$. and $t = 1$ are shown in Figure 2, the physical behavior of (3.11) using the proper values of parameters $\alpha = 0.75$, $a_0 = 1.5$, $b_0 = 1.7$, $b_1 = 0.98$, $A = 3$, $b = 1.6$, $a_0 = 2$, $c = 2.5$. and $t = 1$ are shown in Figure 3, the absolute behavior of (4.9) using the proper values of parameters $\alpha = 0.75$, $a_0 = 1.5$, $b_0 = 1.7$, $A = 2.3$, $b = 1.6$, $c = 2.5$, $v = 2.5$, $\theta_0 = 4$. and $t = 1$ are shown in Figure 4.
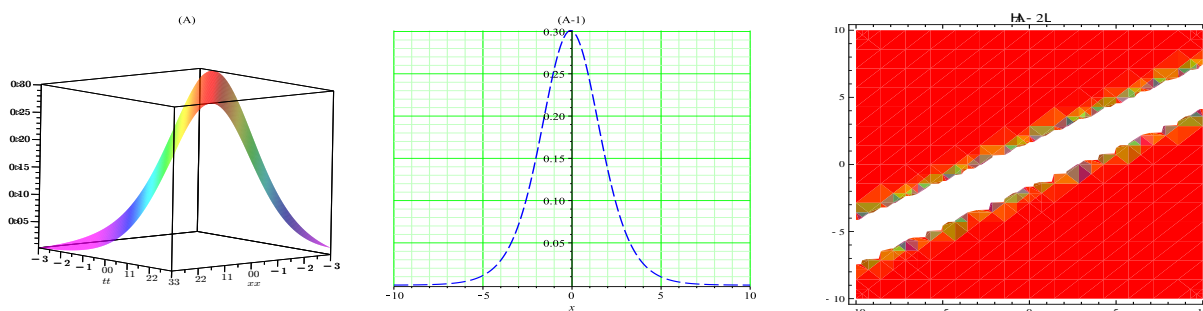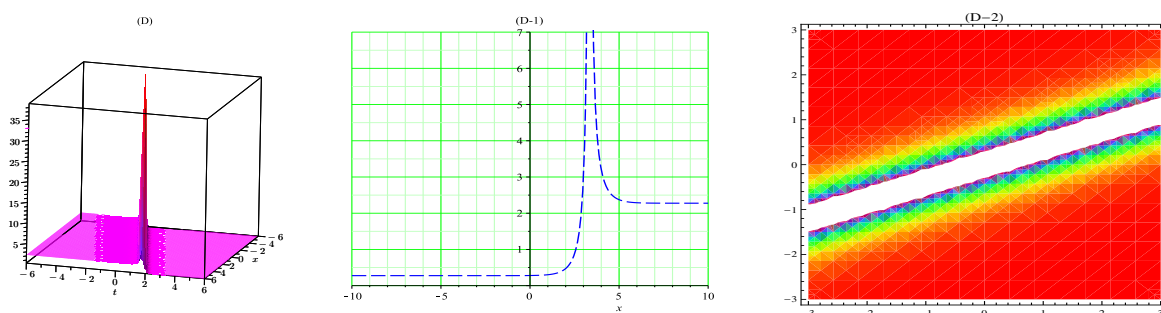

Figure 1. (A): 3D graph of (3.8) with $\alpha = 0.3$, $a_1 = 0.65$, $b_1 = 0.85$, $p = 0.98$, $q = 0.95$, $k = 2$, $A = 3$, $b = 2$, $c = 4$. (A-1): 2D plot of (3.8) with $t = 1$. (A-2): Contour graph of (3.8).
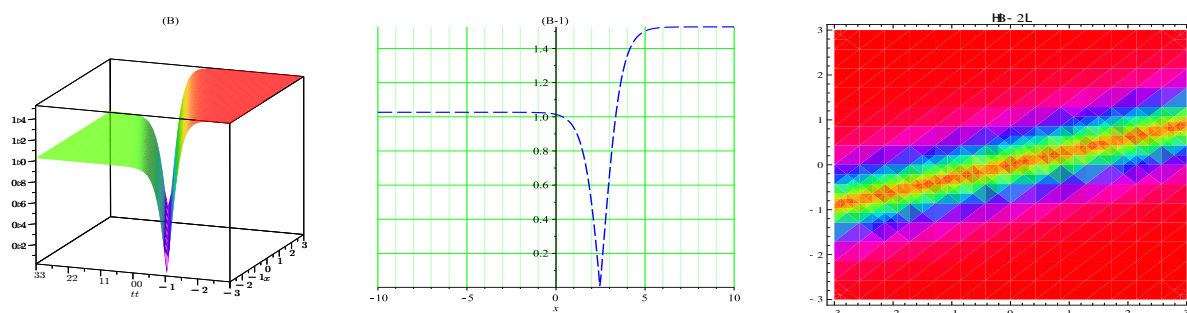
Figure 2. (B): 3D graph of (3.9) with $\alpha = 0.75$, $a_0 = 1.5$, $b_0 = 1.7$, $b_1 = 0.98$, $A = 3$, $b = 1.6$, $a_0 = 2$, $c = 2.5$. (B-1): 2D plot of (3.9) with $t = 1$. (B-2): Contour graph of (3.9).
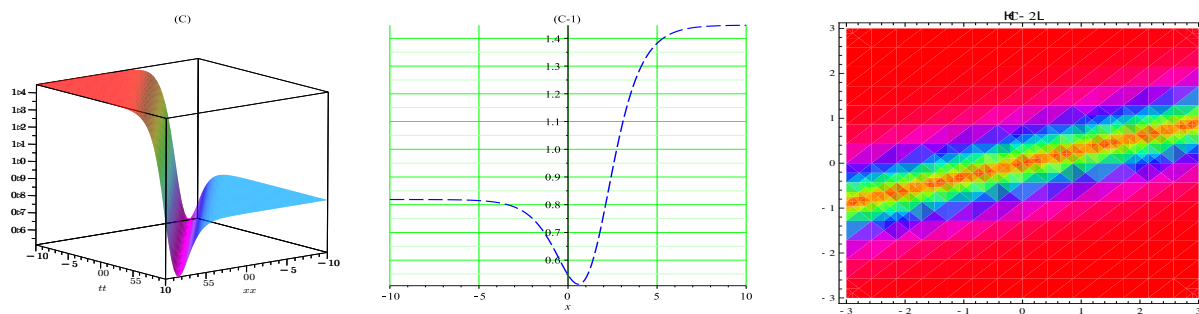
Figure 3. (C): 3D graph of (3.11) with $\alpha = 0.75$, $a_0 = 1.5$, $b_0 = 1.7$, $b_1 = 0.98$, $A = 3$, $b = 1.6$, $a_0 = 2$, $c = 2.5$. (C-1): 2D plot of (3.11) with $t = 1$. (C-2): Contour graph of (3.11).

Figure 4. (D): 3D graph of (4.9) with $\alpha = 0.75$, $a_0 = 1.5$, $b_0 = 1.7$, $A = 2.3$, $b = 1.6$, $c = 2.5$, $v = 2.5$, $\theta_0 = 4$. (D-1): 2D plot of (4.9) with $t = 1$. (D-2): Contour graph of (4.9).

6. Conclusions

The study of the exact solutions of nonlinear models plays an indispensable role in the analysis of nonlinear phenomena. In this work, we have constructed and analyzed the optical solitons solutions of the Kaup-Newell equation and Biswas-Arshad equation by using Kudryashove method. The transmission of ultrashort optical solitons in optical fiber is modeled by these equations. We have achieved more general and novel exact solutions in the form of dark, singular and bright solitons. The obtained solutions of this article are very helpful in governing solitons dynamics. The constructed solitons solutions approve the effectiveness, easiness and influence of the under study techniques. we plotted some selected solutions by giving appropriate values to the involved parameters. The motivation and purpose of this study is to offer analytical techniques to discover solitons solutions which helps mathematicians, physicians and engineers to recognize the physical phenomena of these models. This powerful technique can be employed for several other nonlinear complex PDEs that are arising in mathematical physics. Next, the DNLSE classes II and III will be scrutinized via the similar methods to more evaluate them, this definitely will offer a huge understanding of the methods along with the classes of DNLSE. These solutions may be suitable for understanding the procedure of the nonlinear physical phenomena in wave propagation.

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Conflict of interest

The authors declare that they have no conflict of interest.

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