Young children's recognition of quantitative relations in mathematically unspecified settings

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Abstract

Children have been found to be able to reason about quantitative relations, such as non-symbolic proportions, already by the age of 5 years. However, these studies utilize settings in which children were explicitly guided to notice the mathematical nature of the tasks. This study investigates children's spontaneous recognition of quantitative relations on mathematically unspecified settings. Participants were 86 Finnish-speaking children, ages 5 to 8. Two video-recorded tasks, in which participants were not guided to notice the mathematical aspects, were used. The tasks could be completed in a number of ways, including by matching quantitative relations, numerosity, or other aspects. Participants' matching strategies were analyzed with regard to the most mathematically advanced level utilized. There were substantial differences in participants' use of quantitative relations, numerosity and other aspects in their matching strategies. The results of this novel experimental setting show that investigating children's spontaneous recognition of quantitative relations provides novel insight into children's mathematical thinking and furthers the understanding of how children recognize and utilize mathematical aspects when not explicitly guided to do so.

Keywords: Mathematical development; Quantitative relations; Focusing attention; Young children

1. Introduction

Children do not learn basic skills, such as counting, solely through explicit teaching; children often spontaneously engage with their environment, which provides them with rich and robust learning opportunities (Bransford et al., 2006; Ginsburg, Inoue, & Seo, 1999). Typically,adults are limited in opportunities to give children explicit tasks or guide children's attention towards mathematical aspects in everyday situations (Carey, 2004; Gunderson & Levine, 2011; Tudge & Doucet, 2004). In fact, previous studies have found that some children pay more attention the number of objects or events in their everyday environment already during the early development before school age, while other children may involve themselves much less with these spontaneous numerical activities (Hannula & Lehtinen, 2005).

1.1. Spontaneous Focusing On Numerosity (SFON)

Previous studies indicate the tendency of spontaneous, or unprompted, focusing on an exact number of objects and/or events (i.e. numerosity) is a predictor of later learning of natural number and arithmetical skills (Hannula & Lehtinen, 2005; Hannula, Lepola, & Lehtinen, 2010; Potter, 2009; Edens & Potter, 2012). The term "spontaneous" refers to a non-prompted action in a certain situation, not to the origins of SFON tendency (Hannula & Lehtinen, 2005). Hannula and Lehtinen (2001; 2005) found that it is not self-evident that all children would spontaneously utilize exact numerosity in tasks in which they were not explicitly guided to do so. These individual differences in children as young as 3 years old have been attributed to the existence of a distinct attentional process, Spontaneous Focusing On Numerosity (SFON), which has been found to be a domain-specific contributor to mathematical development (Hannula & Lehtinen, 2005; Hannula et al, 2010). A higher SFON tendency suggests a person is more likely to focus attention on numerosity, recognize and use exact number without being prompted to do so (Hannula, 2005). Children with a higher SFON tendency acquire a greater amount of self-initiated practice with enumeration to the benefit of their mathematical development (cf. deliberate practice; Ericsson & Lehmann, 1996). This influence may in fact be reciprocal, as early numerical skills in turn predict later strength of SFON tendency (Hannula & Lehtinen, 2005).

SFON tendency was measured based on the frequency with which the participants recognized and utilized numerosity in completing the tasks without any guidance to focus on numerical aspects of the tasks (Hannula, 2005; Hannula & Lehtinen, 2005; Hannula, Lepola & Lehtinen, 2010; Hannula, Räsänen & Lehtinen, 2007). For example, in one of the SFON tests (Hannula & Lehtinen, 2005), the experimenter introduced a toy parrot and his favorite berries to the child and said: "Watch what I do carefully, and then you do it just like I did." Then the experimenter put two berries, one at a time into the parrot's mouth and asked the child to do exactly like she had done. Since participants are not directed to focus on the numerical aspects of the tasks, those participants who did recognize and utilize numerical aspects for the completion of the tasks can be said to have spontaneously – self-initiated, without outside direction – done so. Importantly, children were able to complete the tasks while being explicitly guided to focus on number, thus it was determined that all children were capable of handling the procedural requirements, such as enumeration skills, necessary for success on the task. It can be said that this task was able to differentiate children's SFON tendency from their ability to enumerate (Hannula, 2005).

In the present study, we expand these previous studies of SFON tendency to explore if there are similar spontaneous tendencies which may be related to more advanced mathematical aspects. Children's development of more demanding and complex arithmetic and numerical skills, such as reasoning about proportions and other quantitative relations, may similarly be related to spontaneous quantitative focusing tendencies. Therefore, the current study aims to investigate if and how children spontaneously recognize quantitative relations in situations which are not explicitly mathematical.

1.2. Reasoning about quantitative relations in young children

The study of reasoning about quantitative relations is essential for the understanding of children's development of mathematical skills (Resnick, 1992; Sophian, 2007; Squire & Bryant, 2002). Reasoning about quantitative relations is defined within this study as reasoning about the relationship between two or more objects, sets, or symbols based on some quantifiable aspect(s). The term "quantitative relations" refers to a number of different mathematical or pre-mathematical relations that children could perceive in their everyday environment. Quantitative relations can include a) exact or approximate proportional relations or ratios, including discrete and continuous quantities, b) additive and multiplicative relations, and c) exact numerical ratios, such as fractions (Frydman & Bryant, 1988; Gallistel & Gelman, 1992; Resnick, 1992; Sophian, 2007; Wynn, 1992).

Proportional reasoning, the foundation for more advanced mathematical skills, is seen as the basic understanding of ratios and fractions (Boyer, Levine & Huttenlocher, 2008; Frydman & Bryant, 1988; Pothier & Sawada, 1983). Proportional reasoning involves "co-variation between two variables", which can manifest itself in multiple forms (Spinillo & Bryant, 1999, p. 182). In order to reason proportionally in sharing situations, children must reason according to the similar "pattern of relations" of the analogous proportions (English & Halford, 1995, p. 304). Children as young as the age of 4 years have been shown to have a number of skills related to reasoning about proportional relations (Frydman & Bryant, 1988; Mix et al., 1999; Singer-Freeman & Goswami, 2001; Sophian 2000; Spinillo & Bryant, 1999; Wing & Beal, 2004). Already at the age of 5 years, children are able to distinguish and match relations between quantities, such as the relation between concentrate and water in a juice mixture (Boyer et al., 2008), and account for the different unit sizes in their allocation of equal shares, such as accounting for different sizes in sharing with chocolate bars (Frydman & Bryant, 1988; Singer-Freeman and Goswami, 2001; Wing and Beal, 2004). Furthermore, Mix and colleagues (1999) concluded that 4- and 5-year-old children are able to add and subtract non-symbolic proportional quantities smaller than 1 (i.e. ¼, ½, ¾). Spinillo and Bryant (1999) found that 6-year-old children are able to match quantitative relations represented by continuous quantities.

In the present study, participants complete tasks that are based on these previous studies of children's explicit use of proportional reasoning. However, unlike these previous studies, participants must spontaneously recognize the mathematical aspects of our tasks without explicit guidance from the experimenter. The nature of the quantitative relations that can be used to solve the tasks may be interpreted in a number of ways, including proportional relations, part-whole relations, and many-to-one correspondence (see Section 2.4 for more detail). Because the tasks are open to interpretation by the participants, the term *quantitative relations* is used to describe the mathematical aspects involving relations between two or more quantitative, which may be most recognizable as proportional reasoning. The use of the term quantitative relations also allows for the consideration that the measurement of spontaneous recognition of quantitative relations may not necessarily be dependent on the type of quantitative relation being used in the tasks. Therefore, we use the generic term quantitative relations also in consideration that we can not yet differentiate between the wide range quantitative relations detailed above.

1.3. Confounding effects of natural numbers on reasoning about quantitative relations

Despite the ability to reason about quantitative relations in many different contexts at an early age, evidence suggests that early-on children display an overreliance on using exact number to complete a task in which quantitative relations and numerical ratios could be seen as most appropriate (Ni & Zhou, 2005; Sophian, 2000; Sophian et al., 1995), and even adults display similar tendencies (Vamvakoussi, Van Dooren, & Verschaffel, 2012). For example, children matched exact number when asked to match sets of characters and objects, when proportional relations would have been appropriate to take into account (Sophian et al., 1995). However, debate surrounds the origins of this natural number bias. Natural number's privilege may originate in an innate representation of natural numbers that is discrete and therefore incommensurable with the infinitely dense nature of rational numbers (Gallistel & Gelman, 1992, Wynn, 1992). In contrast, children display a number of skills with proportional reasoning suggesting early knowledge of pre-fractional mathematical concepts. This suggests the dominance of natural number comes from issues surrounding the cultural privilege, representations, and teaching of natural number (Andres, DiLuca, & Pesenti, 2008; Carey, 2004 Feigenson, Carey, & Spelke, 2002; Lehrer & Lesh, 2003; Merenluoto & Lehtinen, 2005; Mix et al., 1999; Nunes & Bryant, 1996; Sophian et al., 1997). These questions are crucial for the understanding of the origins of difficulties with fraction learning (Boyer et al., 2008; Gallistel & Gelman, 1992; Merenluoto & Lehtinen, 2004; Ni & Zhou, 2005; Sophian, 2000; Vamvakoussi & Vosniadou, 2004; Vosniadou & Verschaffel, 2004).

The divergence between aspects of quantitative relations and exact numbers may also present multiple options for action in a child's everyday experiences. In some situations both the exact numerosity and relations between numerosities could be seen as mathematically relevant. Differences in children's recognition and use of numerosity and quantitative relations in mathematically unspecified settings can shed light on the question of whether natural numbers also gain, or retain, privilege through children's spontaneous recognition of different mathematical aspects. Thus, an examination of young children's recognition of quantitative relations in mathematically unspecified settings may further the understanding of the interaction between knowledge about natural numbers and quantitative relations such as proportions and (eventually) fractions.

In previous studies of children's reasoning about natural number and quantitative relations the participant's attention has been deliberately guided towards the mathematical aspects of the task. For example, in the study by Frydman and Bryant (1988) children were explicitly guided to notice the quantitative relation between the single and double or triple size pieces. In the current study, by applying similar principles as SFON tasks (Hannula, 2005; Hannula & Lehtinen, 2005), two tasks were developed in order to investigate children's spontaneous recognition of quantitative relations. In these tasks, participants are not directed to notice the mathematical aspects of the task and can complete the tasks using any combination of quantitative relations, numerosity, or other (non-mathematical) aspects. We therefore ask: (1) how do children utilize quantitative relations, exact numerosity, and/or other aspects in mathematically unspecified settings? (2) What are the age-related differences in these matching strategies?

2. METHOD

2.1. Participants

Participants were 86 Finnish-speaking children (50% Female) with no diagnosed learning impairments. Participants came from two day-care centers and three schools from middle-class areas in a medium sized city in southwest Finland (Pop= ca. 175,000). Nine children did not participate in the study because of previous diagnoses of learning impairments, different home language, or no parental permission. The participants were between the ages of 4 years and 5 months to 8 years and 4 months (M= 6y; 8m; SD=1.0 years) at the time of testing. Parents' educational attainment revealed a sample representative of urban parents in Finland, chi-square tests revealed that parents' highest level of educational attainment did not significantly differ from Finland averages (Mother: $X^2 = 5.45$, $p = 0.24$; Father; $X^2 = 4.58$, $p = 0.33$).

2.2. Design

Participants took part in a series of video-recorded tasks which were conducted over two 30-minute sessions. Sessions were held in a secluded room at the participant's day-care or school before noon. A male experimenter conducted all data collection. The ethical guidelines of the University of Turku were followed, and thus all research permissions from the school and day care administration, schools, day care centers, and legal guardians of the participants were gathered. Participants gave their verbal agreement to participate before beginning. Before the testing no mention of any counting, number, or mathematical concepts related to the study were made to the participants. Day-care and school personnel and parents were told that the tasks would measure the participants' general quantitative skills. For analysis, participants were split into three age-groups based on their placement in either Kindergarten (n=31; M_{AGE}=5y; 6m), Pre-school (n=27; M_{AGE}=6y; 9m), or Grade One (n=28; M_{AGE} =7y; 9m).[1]

2.3. Tasks

The participants completed two sets of tasks, each lasting approximately thirty minutes, aimed to measure a variety of mathematical skills. For the purpose of the current study only two of these tasks will be reported, those meant to measure participants' spontaneous recognition of quantitative relations. These tasks were completed before any tasks that explicitly measured mathematical skills.

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[1] In Finland, children begin formal schooling ("Grade One") in the fall of the year they turn 7-years-old. Before this is an optional pre-schooling year ("Pre-school") with 700 hours of preschool education covering all main areas of children's academic skill development. Before this is children can go to kindergarten ("Kindergarten"), which focuses more on supporting children's overall development than their specific academic skills.

2.3.1. Spontaneous recognition of quantitative relations

Two tasks were created to investigate the spontaneous recognition of quantitative relations, through the examination of participants' matching strategies in mathematically unspecified settings. In both tasks the participant fed stuffed animals with either different sized slices of bread or different sized spoonfuls of rice. The tasks were introduced to the participant without any mention of the quantitative or mathematical nature of the tasks; no mention of amount, relations in size, or number aspects were made before or during these tasks. Alternative strategies were controlled for that could lead to a correct relational or numerical response. It was not possible for the children to use the total area, the number, or amount of remaining pieces, or the direct comparison of size in determining their answer. As well, the experimenter always made sure that the participant's attention was on the task and he or she was motivated to complete all trials. He carefully avoided giving any feedback about the participant's performance during the task situation.

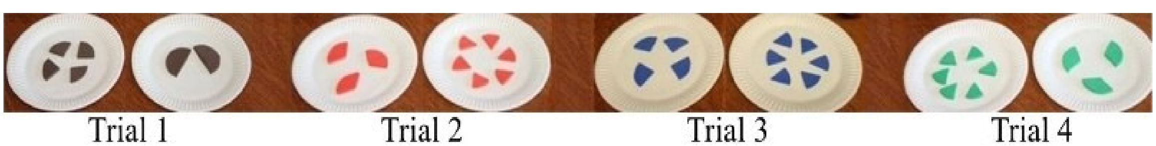

Figure 1. Bread Trials 1 – 4, child's plate on the left, researchers plate on the right.

Bread task. In this task, two identical stuffed-animal dogs, "Nassu and Tassu", 20 cm in height, were fed 'bread' by the researcher and the participant. The bread was made of circular foam pieces 6.5cm in diameter, which were a different color for each trial. The whole breads were originally the same size but has been previously cut into different proportions (halves, thirds, quarters or sixths), and presented to the participant disarranged on the plate. To begin the task, the experimenter introduced Nassu and Tassu as being two friends who always “do everything in exactly the same way” (e.g. jump the same way, run the same way, eat the same way). The participant was told that the dogs like bread and was shown an example of a whole-piece of the bread. Two plates of the breads (see Figure 1) were then placed on the table, one in front the participant, and one in front of the researcher. The participant was told, “Watch what I do carefully, and you do it in exactly the same way.” On the first trial, the experimenter gave one of the two pieces of bread to the first stuffed animal, paused a moment, and turned over his plate to hide the remaining pieces. The participant was then told, “Now you give to Tassu in exactly the same way.” After the participant completed the trial, the experimenter removed the plates and introduced the new set of breads. No feedback was given to the participant after they completed a trial. There were altogether 4 trials in the task (see Table 1).

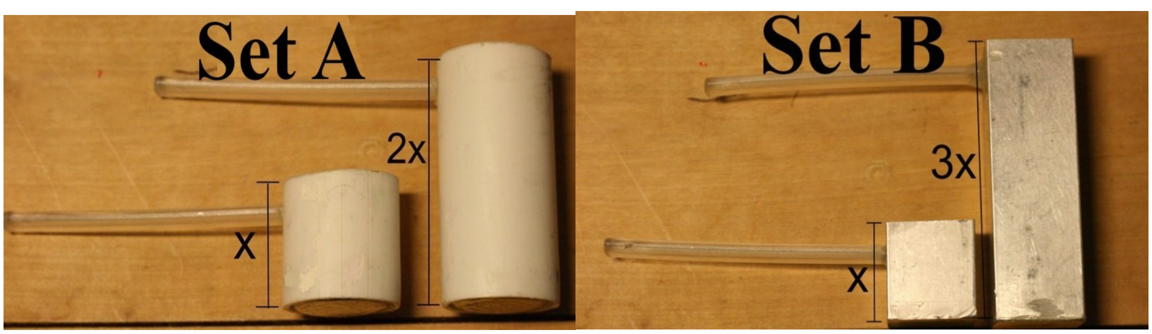

Figure 2. Rice task – Sets of spoons. See Table 1 for more information.

2.3.1.1. *Rice task.* In this task, two identical stuffed monkeys “Pate and Miina”, 20 cm in height, were fed rice. The monkeys were fed using two pairs of spoons (see Figure 2), which were proportional in height. The first pair was made of plastic cylinders, the small spoon was 3cm in diameter and 3cm high, and the larger spoon (twice the size) was

3cm in diameter and 6cm high. The second pair was made of metal rectangular prisms, the small spoon measured 2.5cm x 2.5cm x 2.66cm, and the big spoon (three times the size) measured 2.5cm x 2.5cm x 8cm.

Table 1.

Bread and Rice task trials. Experimenter's and participant's materials and possible responses.

Trial	Researcher	Participant	Quantitative Relations	Numerosity	Other
Bread Task					
1	2 Halves (Gave 1)	4 Fourths	2 pieces	1 piece	3-4 pieces
2	6 Sixths (Gave 2)	3 Thirds	1 piece	2 pieces	3-6 pieces
3	6 Sixths (Gave 3)	4 Fourths	2 pieces	3 pieces	1, 4-6 piece(s)
4	3 Thirds (Gave 2)	6 Sixths	4 pieces	2 pieces	1, 3, 5-6 piece(s)
Rice Task					
1	Set A Big (Gave 1)	Small A	2 spoons	1 spoon	3+ spoons
2	Set B Big (Gave 1)	Small B	3 spoons	1 spoon	2, 4+ spoons
3	Set B Small (Gave 3)	Big B	1 spoon	3 spoons	2, 4+ spoons
4	Set A Big (Gave 2)	Small A	4 spoon	2 spoons	1, 3, 5+ spoons

Participants were told that the monkeys were brother and sister who were having lunch together. Two bowls, each filled with a kilogram of rice, were placed on the table in front of the monkeys. Two opaque empty bowls with the animals' names on them were placed in front of the bowls of rice. The first pair of spoons, the plastic ones, was held up next to each other for possible comparison and the participant was told that "we will use these spoons" and that "Pate and Miina always want full spoonfuls". For the first trial, the smaller spoon was placed in the bowl in front of the participant. The experimenter held the larger spoon and then said that the participant should "watch carefully what I do, and you do in exactly the same way." The experimenter gave one spoon of rice to Pate and then asked the participant to "give in exactly the same way" to Miina. After the first trial was completed, the experimenter put the first pair of spoons away and introduced the second pair, the metal spoons, by holding them next to each other, in front of the participant, so that he or she could see the both properly. No feedback was given to the participant after they completed a trial. In total there were four trials using the two sets of spoons in different combinations (see Table 1).

2.4. Analysis

Video-recordings of the tasks and interviews were analyzed in order to determine the participant's matching strategies. On each trial participants' responses were coded based on the most mathematically advanced aspect of their matching strategy; quantitative relations[2] being the most advanced, then numerosity, and finally other aspects. This hierarchy is not considered mutually exclusive, with each step up in complexity possibly containing lower steps. For example, the recognition of quantitative relations may also include the recognition of numerosity and other aspects.

Participants made utterances on 8.7% of trials for the Bread task and on 10.8% of trials for the Rice task. 99.5% of Bread trials and 98.3% were scored based on the amount of bread or rice the participant gave. Additionally, verbal utterances were used as a basis for

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[2] The term quantitative relations is used here to account for there being multiple ways to for children to view the relational aspects of these tasks, including proportional relations, part-whole relations, and many-to-one correspondence.

the evaluation of participants' matching strategies on 0.5% of the trials for the Bread task and 1.7% of the trials for the Rice task.

All of a participant's (a) utterances including relational words or phrases (e.g., *"You put 2 of those big ones, so I should put 4 small ones*"), or (b) other comments referring either to amounts or relations (e.g., "*You gave 1 whole one, then I give 1 more, then it's even*.") were identified. The participant was scored as matching using *quantitative relations* if they gave the same total amount as the experimenter and/or made any utterances about relations of the quantities in the trials.

As well, all of a participant's (a) utterances including number words (e.g., "*Now just one!*"), or b) other comments referring either to quantities or counting (e.g., "*How many pieces were there?*") were identified. The participant was scored as matching based on *numerosity* in a trial if she or he produced the same number of items in the trial as the experimenter did, and/or if she or he was observed making any numerical utterances (for details, see Hannula & Lehtinen, 2005).

If the participant gave no evidence of responding based on quantitative relations or numerosity he or she was scored as matching based on other aspects, for example matching the way the animals were fed.

The maximum sum score for responding based on quantitative relations, numerosity and other aspects for each task was 4.Two independent raters analyzed 21% of the trials (N=144) and agreement on the most advanced strategy used was found on 98% of the trials for the Bread task, and 97% of the trials on the Rice task. Therefore, the primary rater continued with the same criteria for the remainder of the trials.

3. Results

Results indicate that on the majority of trials for both tasks, participants were found to have matching strategies based on exact number (see Table 2 and 3). A smaller number of trials indicated matching strategies based on quantitative relations. However, a large number of participants did have at least one matching strategy based on quantitative relations on either task, with over half of the First graders utilizing quantitative relations in their matching strategy on at least one trial (see Table 3).

Table 2.

Percentages of matching strategies based on quantitative relations, numerosity, and other aspects in Bread and Rice Tasks ($N_{bread\ \&\ rice}$=344; N_{total}=688)

Task	Quantitative Relations	Numerosity	Other
Bread	17.2 %	61.9 %	20.9 %
Rice	14.8 %	66.3 %	18.9 %
Total	16.0 %	64.1 %	19.9 %

On the Bread task 31% of the participants were found to have matching strategies based on quantitative relations on at least one trial, with 27% of the participants matching based on quantitative relations on at least one trial on the Rice task. For both tasks combined, 42% of the participants used matching strategies based on quantitative relations on at least one trial. Participant's matching strategies were found to be consistent across the two tasks (Pearson's r correlation: quantitative relations, $r=0.65$, $p<0.001$; numerosity, $r=0.65$, $p<0.001$; other, $r=0.52$, $p<0.001$).

A 2 x 2 x 3 [Task (Bread, Rice) x Matching Strategy (Quantitative relations, Numerosity) x Age-group (Kindergarten, Pre-school, Grade 1)] ANOVA was run on participants' responses, including Tukey's post-hoc tests. Significant main effects were found for Matching Strategy, $F(1, 83) = 66.56$, $p<0.001$ and Age-Group, $F(2, 83) = 5.36$, $p<0.01$. Participants had a significantly higher number of matching strategies based on numerosity than quantitative relations. As well, participants increased in the use of matching strategies based on either mathematical aspect (quantitative relations or numerosity) with age, in other words, matching strategies based only on other aspects decreased with age. Significant interaction effects were found for Task x Age-Group, $F(2, 83) = 3.65$, $p<0.05$ and Matching Strategy x Age-Group, $F(2, 83) = 3.35$, $p<0.05$. Tukey post-hoc test of Age-Group revealed that Kindergarteners used other aspects in responding significantly more than First Graders (mean difference: 0.42, $p<0.01$).

Table 3.

Frequencies of matching strategies based on quantitative relations, Numerosity, and other aspects per participant in Bread and Rice Tasks (N=86)

Age Group	Frequency of response	Bread Task			Rice Task		
		Quant. Rel	Numerosity	Other	Quant. Rel	Numerosity	Other
All Grades	0	69%	20%	55%	73%	17%	56%
	1	15%	10%	27%	10%	5%	27%
	2	2%	6%	3%	6%	14%	8%
	3	7%	30%	6%	5%	23%	0%
	4	7%	34%	8%	6%	41%	8%
Kind. (n=31)	0	74 %	13 %	45 %	77 %	23 %	26 %
	1	19 %	10 %	32 %	13 %	3 %	45 %
	2	6 %	6 %	6 %	10 %	19 %	10 %
	3	0 %	45 %	3 %	0 %	35 %	0 %
	4	0 %	26 %	13 %	0 %	19 %	19 %
Pre-School (n=27)	0	81 %	11 %	59 %	81 %	7 %	70 %
	1	11 %	15 %	19 %	11 %	4 %	19 %
	2	0 %	7 %	4 %	0 %	11 %	7 %
	3	4 %	22 %	11 %	4 %	22 %	0 %
	4	4 %	44 %	7 %	4 %	56 %	4 %
Grade 1 (n=28)	0	50 %	36 %	64 %	61 %	21 %	79 %
	1	14 %	7 %	29 %	7 %	7 %	14 %
	2	0 %	4 %	0 %	7 %	11 %	7 %
	3	18 %	21 %	4 %	11 %	11 %	0 %
	4	18 %	32 %	4 %	14 %	50 %	0 %

Pair-wise t-test comparisons were run in order to further investigate the within-subject effects. Task by Age-Group comparisons revealed that, within age-groups, there were no significant differences in the use of relational responses between the two tasks. However, differences did appear when comparing matching strategies based on quantitative relations and those based on numerosity within the age-groups. Both Kindergarteners and Pre-schoolers displayed significantly fewer quantitative relational matching strategies than numerosity matching strategies (Kindergarten: $t(30)=-7.94$, $p<0.001$; Pre-school: $t(26)=-7.30$, $p<0.001$). However, First graders' matching strategies based on quantitative relations and numerosity did not significantly differ.

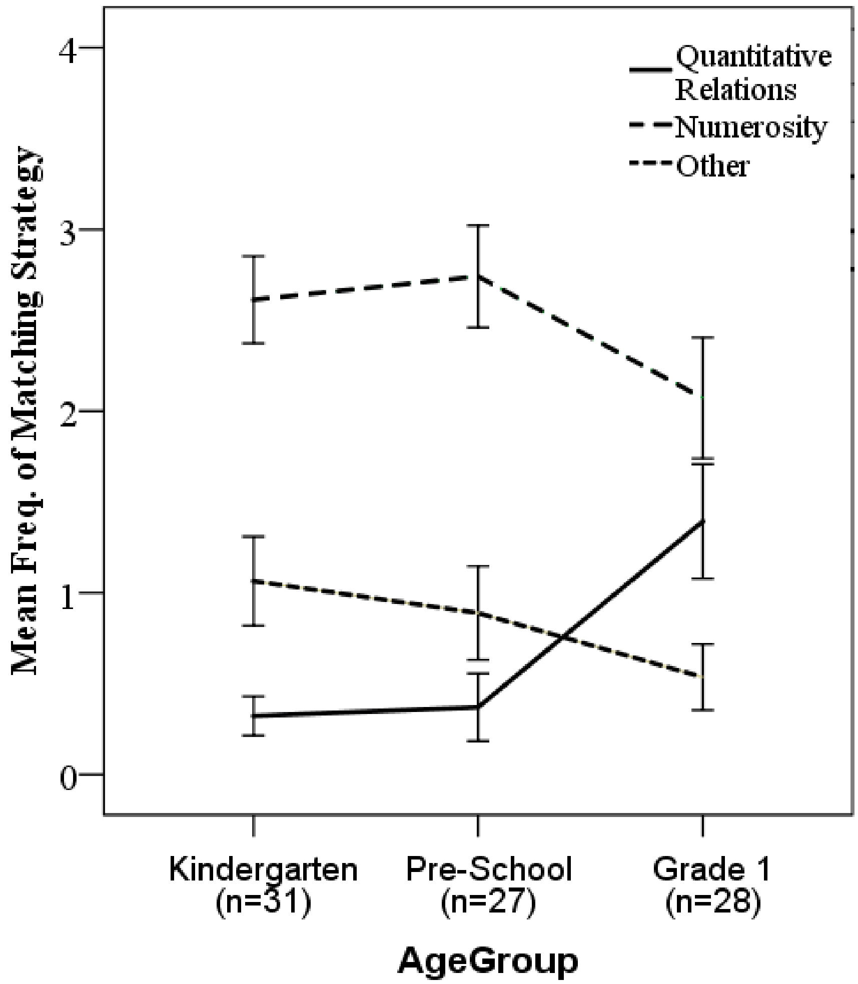

Figure 3. Bread task mean frequencies of most mathematically advanced matching strategies by age group.

One-way ANOVA analyses were used to further analyze the differences between the age-groups for all matching alternatives. For the Bread task (see Figure 3), a significant main effect of age group was found for the frequency of matching strategies based on quantitative relations $F(2, 83) = 7.86$, $p<0.001$. Tukey's post-hoc comparisons between the age groups revealed that the First graders had significantly higher quantitative relational matching strategies on the Bread task than both the Kindergarteners (mean difference = 1.07; $p<0.01$) and Pre-schoolers (mean difference = 1.02; $p<0.01$). On the

Bread task, no effect was found for age-grouping for the frequency of numerosity matching strategies, F (2, 83) = 1.52, p=ns., or other aspects matching strategies F (2, 83) = 1.37, p=ns.

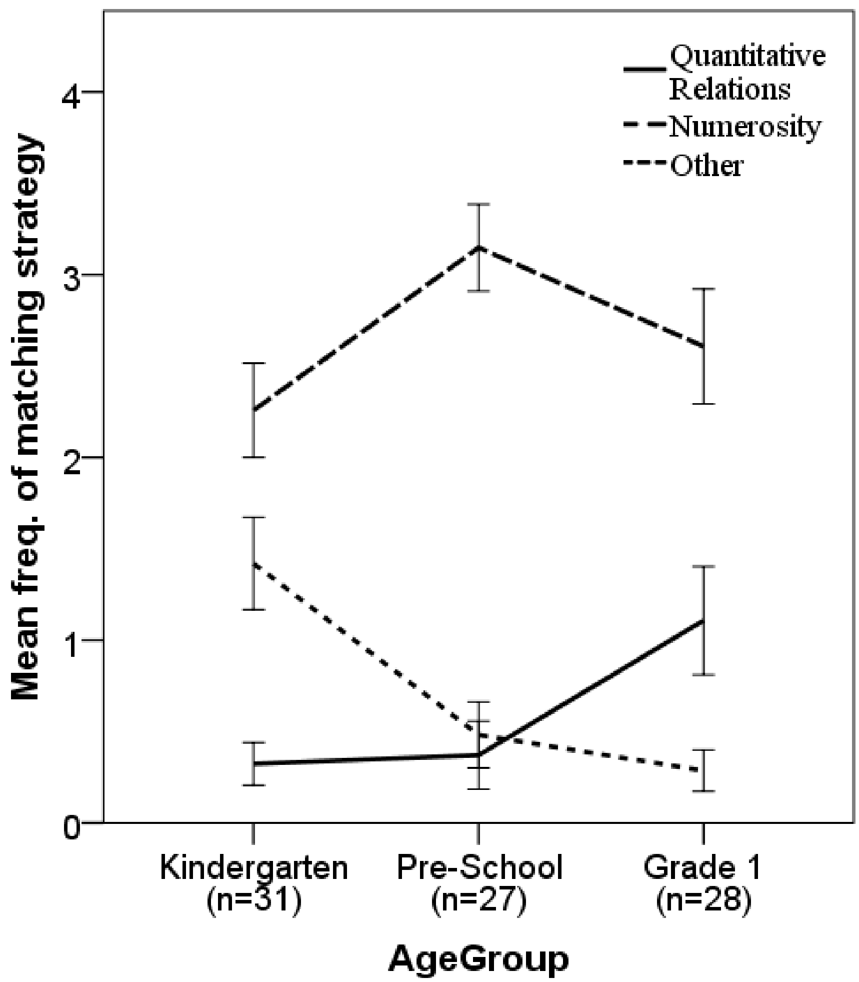

Figure 4. Rice task mean frequencies of most mathematically advanced matching strategy by age group.

For the Rice task (see Figure 4), a significant main effect of age group was found for the frequency of matching strategies based on quantitative relations $F(2, 83) = 4.40$, $p<0.05$. Tukey post-hoc analysis revealed a similar pattern as in the Bread task, with First graders having a significantly higher frequency of matching strategies based on quantitative relational than both Kindergarteners (mean difference = 0.78; $p<0.05$) and Pre-schoolers (mean difference = 0.74; $p<0.05$). No main effect of age group was found for the frequency of numerosity matching strategies on the Rice task, $F(2, 83) = 2.71$, p=ns. However, there was a significant main effect of age group for matching strategies based on other aspects, $F(2, 83) = 9.86$, $p<0.001$. Tukey post-hoc testing revealed that Kindergarteners responded using other aspects significantly more than Pre-schoolers (mean difference = 0.94; $p<0.01$) or First graders (mean difference = 1.13; $p<0.01$). Due to slightly high Skewness and Kurtosis for the Pre-schoolers relational responses analysis were repeated using Non-parametric Mann-Whitney tests, which produced similar results of the statistical testing as the parametric tests.

3.1. Stimulated recall responses

After the initial testing, both tasks were repeated using the same materials and trials in a later session. This time the tasks included a stimulated recall interview question aimed at capturing participants' explicit reasoning in their matching strategy. Participants' were asked: "*How do you know, from this bread/rice (points to child's bread/rice) what to give…*?", after the experimenter had given his bread or rice and before the participant gave his/hers. Previous results suggest that very few children are able to verbalize their reasoning regarding their use of exact number, but a similar percentage of children utilize quantitative relations and make responses based on quantitative relations (McMullen, Hannula-Sormunen & Lehtinen, 2011).

This association was further investigated in the present study, by analyzing the relationship between children's matching strategies on the original task and the stimulated recall verbal responses. It was discovered that matching strategies based on quantitative relations were highly correlated with quantitative relations verbalizations, $r = 0.70$, $p < 0.001$. However, matching strategies based on numerosity were not correlated with verbalizations of exact number, $r = 0.15$, $p = 0.18$. Other matching strategies were only slightly correlated with non-mathematical verbalizations, $r = 0.22$, $p = 0.04$.

4. **Discussion**

The aim of this study was to examine children's spontaneous matching strategies in tasks that could be responded to in a number of different ways, including based on quantitative relations, exact number, or other aspects. Results show there was a significant increase in the use of quantitative relations with age, as First graders used quantitative relations significantly more than younger participants. In fact, First graders were just as likely to respond based on quantitative relations as they were to respond based on numerosity, while Kindergarteners and Pre-schoolers were more likely to respond based on numerosity than quantitative relation or other aspects.

Previous research has found that children of the same age have been successful at similar tasks when guided to use quantitative relations. (Frydman & Bryant, 1988; Mix et al., 1999; Singer-Freeman & Goswami, 2001; Sophian 2000; Spinillo & Bryant, 1999; Wing & Beal, 2004). In the present study, participants must recognize the quantitative aspects of the task without being guided to do so. This lack of prompting is a key difference in the experimental settings between the current study and previous studies of children's reasoning about quantitative relations, which may have caused the disparity in the use of quantitative relations.

The spontaneous recognition of quantitative relations may appear later than the related quantitative relational skills due to the privileged position natural numbers hold in young children's reasoning (Gallistel & Gelman, 1992; Ni & Zhou, 2005; Sophian, 2000; Sophian et al., 1995). In the present study, younger participants show an over-reliance on the natural number stimuli and fail to make the appropriate leap to utilizing the quantitative relations in completing the tasks. These results suggest that the natural number bias that appears in children's mathematical reasoning may be impacted by children's spontaneous recognition of different mathematical aspects. The overwhelming use of exactly the same number of pieces of bread or spoons of rice in the present study, especially by younger children, emphasizes the stronger position natural numbers hold in children's everyday experiences (Andres, DiLuca, & Pesenti, 2008; Carey, 2004). The greater amount of practice children receive with the use of exact number of objects and events in everyday situations, as a result of a stronger tendency to spontaneously recognize these aspects, may be in a feedback loop with the natural number bias. Numerosity may be more readily apparent to young children than relational aspects, leading to more spontaneous recognition of natural numbers, which leads to more practice with these aspects, thereby making them more likely to be recognized in the future (see Hannula & Lehtinen for a detailed account of the feedback loop of SFON tendency and enumeration skills).

The present study reports on a novel experimental setting, which is partially based on previous studies of Hannula and Lehtinen (2001; 2005). By not guiding participants towards the mathematical nature of these tasks, those participants who utilized quantitative relations or numerosity can be said to have done so spontaneously. Therefore, the differences in participants' use of quantitative relations on these tasks may be a result of differences in the spontaneous recognition of quantitative relations. Children's SFON tendency has been shown to be an attentional process that is not entirely explained by enumeration ability, as well as being a domain specific contributor to the development of numerical skills (Hannula & Lehtinen, 2005, Hannula, Lepola & Lehtinen, 2010). The current findings suggest that the spontaneous recognition of quantitative relations may include a spontaneous attentional process analogous to the role of SFON in the recognition of exact number. However, more specific methods are needed in order to fully extrapolate this attentional process from the spontaneous recognition of quantitative relations.

The spontaneous recognition of quantitative relations requires both spontaneous focusing on quantitative relations (i.e. an attentional process that triggers the recognition of quantitative relations) and the skills needed to complete the task using quantitative relations. However, participants' skills in utilizing quantitative relations when explicitly guided to do so were not controlled for in the present study (cf. Hannula & Lehtinen, 2005). This leaves open the possibility that the results of the current study may have been a result of differences in the attentional component or skill component of the spontaneous recognition of quantitative relations, or a combination of both.

Similar to Baroody et al. (2008), the current study did not differentiate between the spontaneous attentional processes and the quantitative skills required by the task. In their studies of children's SFON, Hannula and Lehtinen (2005) showed that even though children did not produce accurate numbers in the SFON tests, they were able to do so once their attention was guided towards the numerical aspect of the tasks. Confronting the participants of the current study who did not spontaneous recognize quantitative relations in the task with guided versions of the same tasks would allow for more substantial claims regarding the existence of an attentional component in the task performance of the participants.

If the individual differences in matching strategies were a mainly a result of differences in skills with using quantitative relations, it can be expected that relational responses would be consistent with trial difficulty. However, when comparing the Rice task trials that involved multiples of 3 compared with multiples of 2, there was no significant difference in the use of quantitative relations. Similarly, the use of quantitative relations in participants' matching strategies was not consistent with trial difficulty on the Bread task. Trial C (see Table 1), involving the most complex quantitative relations (two-fourths with three-sixths), did not prove to be most difficult.

Evidence from previous studies suggests that the manner of representation, either as discrete or continuous quantities may have an impact on children's recognition and utilization of quantitative relations and numerosity (e.g. Jeong et al., 2007; Spinillo & Bryant, 1999). On the Rice task, relations were represented by direct comparison of continuous quantities, though the number of spoons could be regarded as discrete representations. However, in the Bread task participants could use either discrete or continuous representations for the matching of relational aspects. The similarity in the frequencies of response types and the consistency in participants' responses across the bread and rice tasks suggest that the recognition of quantitative relations may not be fundamentally dependent on specific task features or representational type (cf. SFON; Hannula, 2005). A detailed analysis of participants' verbalizations on the stimulated recall version of these tasks may be needed to clarify whether participants are recognizing more often or more easily the discrete or continuous relational aspects of these tasks.

Additionally, schooling may have an important role in the recognition of quantitative relations. The increase in the use of quantitative relations in First grade may be related to the fact that this is when children begin formal schooling in Finland and their mathematical knowledge begins to be formalized. However, the spontaneous recognition of quantitative relations was not found for all First graders, indicating that there are other factors beyond schooling effects contributing to these differences.

Even when taking into consideration differences in the ability to reason about quantitative relations, it is unsurprising that participants' tendency to recognize quantitative relations seems to be over-shadowed by their SFON tendency in this sample. First, children of this age have basic enumeration skills that are more developed than their reasoning about quantitative relations. As well, reasoning about quantitative relations is more complex and demanding than reasoning about numerosity. In fact, it is necessary to first spontaneously focus on number in order to spontaneously recognize quantitative relations. Thus, the spontaneous recognition of quantitative relations requires a number of mathematical skills beyond those needed for SFON – including recognizing the related quantities, discerning the relation and solving for the missing value – along with the recognition that these skills should be utilized, and finally the utilization of these skills in completing the task. These extra steps required in the spontaneous recognition of quantitative relations require a strong awareness of the relevance of quantitative relations in the situation or task. Those children who have a higher tendency to spontaneously recognize quantitative relations may acquire an increased amount of self-initiated practice in reasoning about quantitative relations and this increased practice may be related to the enhanced development of formal mathematical concepts, including fractions.

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Appendix

Table 4. Mean frequencies of responses based on quantitative relations, numerosity and other aspects by age group (Max = 4)

	Quantitative. Relations		*Numerosity*		*Other*	
Age Group	*Bread*	*Rice*	*Bread*	*Rice*	*Bread*	*Rice*
All Children						
Mean	**0.69**	**0.59**	**2.48**	**2.65**	**0.84**	**0.76**
St. Dev	1.24	1.16	1.53	1.48	1.24	1.16
Skewness	1.76	1.96	-0.61	-0.79	1.55	1.80
Kurtosis	1.79	2.68	-1.18	-0.80	1.30	2.58
Range	0-4	0-4	0-4	0-4	0-4	0-4
Kindergarten						
Mean	**0.32**	**0.32**	**2.61**	**2.26**	**1.06**	**1.42**
St. Dev	0.60	0.65	1.33	1.44	1.36	1.41
Skewness	1.74	1.87	-0.94	-0.56	1.31	1.02
Kurtosis	2.15	2.26	-0.27	-1.00	0.56	-0.23
Range	0-2	0-2	0-4	0-4	0-4	0-4
Pre-school						
Mean	**0.37**	**0.37**	**2.74**	**3.15**	**0.89**	**0.48**
St. Dev	0.97	0.97	1.46	1.23	1.34	0.94
Skewness	3.02	3.02	-0.80	-1.50	1.36	2.50
Kurtosis	8.96	8.96	-0.83	1.49	0.51	7.09
Range	0-4	0-4	0-4	0-4	0-4	0-4
Grade 1						
Mean	**1.39**	**1.11**	**2.07**	**2.61**	**0.54**	**0.29**
St. Dev	1.66	1.57	1.75	1.66	0.96	0.60
Skewness	0.62	0.98	-0.16	-0.67	2.45	2.04
Kurtosis	-1.45	-0.74	-1.85	-1.29	6.53	3.23
Range	0-4	0-4	0-4	0-4	0-4	0-4