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Uncertainty, Misallocation and the Life-cycle Growth of Firms

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ABSTRACT

We develop a measure of static misallocation that separates uncertainty from misallocation generated by tax-like distortions. In the Finnish firm-level data, uncertainty accounts for the majority of ex post misallocation and explains a strong decreasing age-dependent trend in it. To understand these observations, we set up a life-cycle model of firm growth where new firms have to learn their productivity. We match our model with the salient features of the data and show that our model implies idiosyncratic distortions, in line with our accounting approach. According to our quantitative results, uncertainty suppresses output by 38%, while misallocation has a 26% negative effect on output.

JEL Classification: D24, E23, L11, O47

Keywords: firm dynamics, uncertainty, misallocation

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1 Introduction

In order to understand the determination of aggregate output, it is paramount to have a clear picture of factors determining the aggregate total factor productivity[1]. A seminal paper of Restuccia and Rogerson (2008) illustrates that inefficient allocation of input factors- misallocation- can have severe effects on TFP. To evaluate the empirical relevance of this channel, a popular indirect approach pioneered by Hsieh and Klenow (2009) has been to measure marginal products of labor and capital using firm-level micro data[2]. If the dispersion of marginal products, at least for the part that exceeds the US benchmark, reflects misallocation, the reallocation of input factors could lead to a significant increase in TFP and output. However, there are other factors that can generate dispersion in marginal products that are not necessarily directly related to inefficient allocation of input factors across producers[3].

We show that in a administrative data set covering nearly all Finnish firms, 57% of the variation in the Hsieh and Klenow (2009) style (henceforth HK) revenue distortion is accounted for by uncertainty. The variation in the tax-like distortion, our indirect measure of misallocation[4], explains 13 % of the total variation. In order to disentangle uncertainty from misallocation, we augment the methodology of HK by assuming that firms choose their inputs before they know their productivity (or demand). Moreover, firms also face idiosyncratic revenue distortions. Thus, in our accounting framework, ex post resources might appear inefficiently allocated either because the variation in firms' prediction errors is large (the uncertainty channel), because of the variation in the revenue distortions (the misallocation channel), or because of the covariance between the two.

Using our decomposition conditional on age, we observe a strong age-dependent decreasing trend in uncertainty. The dispersion in the prediction error is more than halved when one moves from new businesses to firms that have been operating for a decade. The strong trend is mirrored in a similar trend in the dispersion of HK revenue wedge[5]. Mis-

[1]See, e.g., Klenow and Rodriguez-Clare 1997 for the importance of TFP in explaining differences in output across countries.

[2]See Bayer et al. (2018) and Cirera et al. (2020) for recent examples of this approach.

[3]For example, measurement errors, different production/demand structures and adjustment costs costs could affect the measured misallocation. See e.g. Bils et al (2020), Rotemberg and White (2020), Gollin and Udry (2019), Haltiwanger et al. (2018), Bartelsman et al. (2013) and Asker et al. (2014).

[4]As in Restuccia and Rogerson (2008), we model idiosyncratic tax-like distortions as the source of misallocation.

[5]This is qualitatively in line with Eslava and Haltiwanger (2020), who observe that idiosyncratic distortions are particularly important for young firms in Colombian establishment-level data.

allocation, measured by variation of the residual wedge, on the other hand, is practically constant for all age brackets. Decomposing HK dispersion conditional on age requires a large set of firms with varying ages, thus observing almost the whole population of firms is important.

We explore the sensitivity of our results by recalculating our decomposition for different industries and years separately. Even though the level of uncertainty varies across industries and years, it is by far the most important component of ex post misallocation. Contrary to uncertainty, ex ante misallocation stays remarkable stable in different subsets of the data. We also extend our approach by allowing heterogeneous markups. This does not affect our measure of ex ante misallocation but alters our uncertainty measure. Even though there is substantial variation in markups in the data, the role of uncertainty is not diminished. In addition, the age-dependent trends in uncertainty and ex post misallocation stay strong, while there are no apparent trends in ex ante misallocation, markup variation nor covariance terms.

Our accounting framework hints that uncertainty, especially in early life-cycle, might play an important role in determining the aggregate TFP. However, to be able to evaluate the relative importance of uncertainty and misallocation for aggregate outcomes, we need to move beyond static calculations. To this end, we set up a life-cycle model of firm growth with entry and exit where the age-size distribution of firms is endogenously determined. Our model augments the learning structure of Jovanovic (1982) to a GE framework similar to Hopenhayn and Rogerson (1993) and Melitz (2003).

The key features of our model are age-dependent uncertainty, convex adjustment costs and tax-like wedges that all reduce the efficiency of resource allocation. In our model, new firms draw a permanent productivity component upon entry. However, in each period a firm's realized productivity is a combination of the permanent component and the transitory one. Firms slowly learn their permanent productivity using Bayesian updating. In line with our static exercise, the inputs are chosen before firms' know their current period TFP. This structure implies that the dispersion of marginal productivity is smaller for older firms. We also allow for adjustment costs which further generate age-dependent dispersion as several authors have suggested this channel of misallocation might be important [6]. In addition, firms' input decisions depend on permanent and

[6]See, e.g., Eslava and Haltiwanger (2018) or Asker et al. (2014).

temporary revenue distortions that we use to model misallocation.

When the distribution that specifies the idiosyncratic productivities is held constant, the aggregate TFP in our model is determined by two channels – selection and allocation of resources across heterogeneous firms. The resource allocation is directly affected by uncertainty, adjustment costs and wedges. However, through equilibrium responses the two channels are tightly knitted together. For example, if adjustment costs and uncertainty reduce the growth of young productive firms, there can be a lower amount of entrants and unproductive firms might be less likely to exit.

Even though our model is relatively parsimonious, it is flexible enough to explain the salient features of the Finnish firm-level data. We utilize this in quantitatively disciplining our model. That is, we match the model to the startup rate, the growth profiles of young and old firms, the size distribution of old firms and the exit rate for new businesses. In addition to being able replicate these targets, our model also generates age-dependent patterns in the first two moments of firm growth that are in line with the Finnish data.

We use our calibrated model to redo our indirect estimates of static misallocation with an artificial data and show that our model is consistent with the observed patterns in the data. First, our model accounts for 49% of the observed variation in the HK revenue distortion. Second, as in the data, the majority of this is due to the uncertainty component. The fraction of ex post misallocation explained by uncertainty is 68%, which is close to the fraction we observe in the Finnish data. In addition, we find that the model generates a decreasing age-dependent trend in uncertainty that is qualitatively similar to the one observed in the data.

By using our quantitative model to do counterfactuals, we can evaluate how important the different components are for aggregate total factor productivity. In line with our static measures, the effects of uncertainty are by far the most profound. When firms have to choose their inputs under uncertainty and learn their permanent productivity component, TFP is 38% lower compared to the benchmark economy without adjustment costs, revenue distortions and information frictions. If firms know their permanent productivity upon entry but do not observe productivity before choosing inputs, the TFP reduction is 21 % relative to the benchmark. Thus, the contribution of early life-cycle learning is about 17 %. Moreover, the idiosyncratic revenue distortions have a substantial effect on TFP, reducing it by around 26%. Finally, adjustment costs also have a role in

TFP determination. Adjustment costs alone reduce TFP by 11%.

2 Related Literature

In a highly influential paper, Restuccia and Rogerson (2008) illustrate that firm-specific tax-like wedges can generate large effects to aggregate total factor productivity in a stylized model of firm-dynamics. They also show that this channel is especially strong if the wedges are correlated with firm-specific TFP. We follow their approach by modeling misallocation with tax-like revenue wedges.

HK develop a method of evaluating misallocation empirically with establishment/firm level data when demand is isoelastic and production technology is homogeneous of degree one. HK quantify the effects of misallocation on aggregate TFP by comparing theoretical static first order conditions with their empirical counterpart and interpreting the difference between the two as an indication of firm-level wedges that distort the optimal allocation. They find that the elimination of distortions would greatly increase TFP in India and China. As the identification of establishment-specific misallocation is based on a residual between first order conditions relevant to a benevolent planer and their empirical counterpart, this measure could also reflect other things besides misallocation. Recognizing this, HK also explore how much China and India could boost their aggregate TFP if they were to reduce the dispersion to the level observed in the US. These gains are still substantial. Several studies have used this approach, finding large potential gains from a more efficient resource allocation.[7]

Our approach of evaluating static misallocation together with uncertainty is related to a branch of new research that relaxes the original assumptions in ways that allow for dispersion in TFPR to also reflect other things besides misallocation. Haltiwanger et al. (2018) consider more general demand and production structures. In their analysis, demand factors explain a large part of the variation in TFPR. Bils et al. (2020) and Rotemberg and White (2020) take into account the possibility of measurement errors. Both of these papers find that the gains of misallocation are reduced substantially after controlling for measurement problems. Gollin and Udry (2019) develop a framework that allows them to separate between measurement error, unobserved heterogeneity and

[7] See, for example, Busso et al. (2013) for Latin America and Cirera et al.(2020) for Africa.

misallocation. They find that heterogeneity and measurement errors account for a large fraction of measured productivity differences in the data covering farms from Tanzania and Uganda[8]. Baqaee and Farhi (2020) allow for flexible input-output linkages and varying substitutability in a non-parametric framework. In relation to this literature, we show that uncertainty, especially in early life-cycle, can explain a large fraction of the dispersion in TFPR.

In trying to quantify the effects of misallocation researchers have also taken an alternative approach, where the role of specific source(s) of "misallocation", variation in marginal (or product(s) of labor and/or capital, is tried to capture with the help of a structural model. Midrigan and Xu (2014), for example, examine the role of financial markets, while Asker et al. (2014) analyze the role of adjustment costs. Bartelsman et al. (2013) study misallocation when capital is quasi-fixed and firms' use overhead labor.

Within the branch of studies that utilize more structural approach, a paper close to our quantitative exercise is David et al. (2016) who investigate the role that information frictions can play for ex post misallocation. As in our setup, firms choose their inputs (or part of their inputs) before they know their fundamentals. However, contrary to our model, at the end of the period uncertainty is revealed. An additional difference is that firms also learn from stock markets in their framework. In our data, a large majority of firms is unlisted, thus we do not allow this type of learning. Moreover, motivated by our accounting exercise, we focus on information frictions over the life-cycle of firms. Thus, in our setup, in addition to choosing their scale, firms also make entry and exit decisions and only slowly learn their profitability. Along these dimensions, the setup of Tian (2020) is close to ours. In her model, firms are always uncertain about the quality of their product and the state of the economy. In contrast to the papers mentioned earlier, her goal is not to evaluate the quantitative importance of uncertainty as a channel of misallocation, but to analyze the connections between the two sources of uncertainty more broadly.

Another paper exploring the role of uncertainty with the help of a structural model is David and Venkateswaran (2019). They develop a tractable framework that allows them to measure several sources of capital misallocation such us adjustment costs and tax-like wedges, jointly with uncertainty. David and Venkateswaran (2019) also explore the role of markup and production heterogeneity. Compared to our model they allow

[8]As they consider a highly homogeneous product the distinction between farm specific TFPR and TFPQ is not relevant.

for a more flexible structure for tax-like wedges in the sense that in their model wedges can be correlated with fundamentals. Regarding to uncertainty the main difference is again the fact we focus on uncertainty over life-cycle. Thus, in our setup information frictions are especially severe for young firm. Moreover, we allow for interaction between selection and different frictions, as firms make endogenous entry and exit decisions in our model. This type of interaction could be important for the aggregate effects of micro-level misallocation as illustrated by, e.g., Yang (forthcoming).

Our paper is also connected to a branch of literature that aims to understand the life-cycle growth of firms. Clementi and Plazzo (2016) incorporate adjustment costs to Hopenhayn's (1992) firm-dynamics model. This generates age-dependent growth and exit rates. Pugsley et al. (2021) use a firm-dynamics model with monopolistic competition similar to ours to understand the determinants of the up-or-out-dynamics that characterize the early life-cycle growth of new businesses. Their results emphasize ex ante heterogeneity over ex post shocks. Within this tradition, a paper closest to ours is Arkolakis et al. (2018). They illustrate that the Jovanovic (1982) style learning combined with a GE framework with monopolistic competition can generate life-cycle profiles that are in line with the US data. The main difference between our paper and these is the fact that our goal is to link life-cycle growth and misallocation. In this respect, our study has similarities with a few recent papers that emphasize the connections between productivity investments and misallocation. Hsieh and Klenow (2014) allow the firms' TFP to evolve endogenously over the life-cycle[9]. In their approach, misallocation can severely discourage investments on the productivity and thus dampen the aggregate TFP. In Bento and Restuccia (2017), firms can invest on productivity not just along the life-cycle but also upon entry. Peters (2019) considers the effects of market power in generating misallocation. In his setup, the market power of firms is endogenous and evolves over the life-cycle.

3 Accounting for Uncertainty and Misallocation

In this section, we develop an accounting framework to jointly measure misallocation and uncertainty. We link our measure to the dispersion of the HK revenue wedge, a

[9]They build on Atkeson and Burstein's (2010) GE model. The approach is close to papers that examine the role investments in organizational capital and customer base. See e.g. Foster et al. (2016)

standard measure of misallocation when there are no distortions that affect capital and labor asymmetrically. We apply our approach to Finnish firm data. Finally, we also extend our approach by allowing variable markups.

3.1 A Theoretical Accounting Framework

This subsection develops a simple way to indirectly measure the wedges that uncertainty and tax-like revenue distortions can generate to static first order conditions. Our framework builds on HK. The main difference between our approach and theirs is that we assume that firms have to choose inputs before they know the current period productivity. Alternatively, we could say that the firms are uncertain about their idiosyncratic demand component[10]. Moreover, in order to use the decomposition also with a simulated data, generated by the GE model developed in Section 3, we use a slightly more stylized demand structure. For the same reason, we focus on revenue distortions[11] and leave the analysis of capital distortions for further research.

There is a large number of firms, indexed by i, each of them producing a differentiated good. Individual goods are aggregated to a single final good with the CES aggregator. Thus, firm i in sector s at time t faces the isoelastic demand curve given by

$$y_{t,s,i} = (\frac{p_{t,s,i}}{P_t})^{-\sigma} Y_t, \tag{1}$$

where $p_{t,s,i}$ is the price of good, P_t is the price index and Y_t is the the amount of final good consumed.

The production technology for each firm is represented by a Cobb-Douglas production function of a firm's TFP, $z_{t,s,i}$, labor, $n_{t,s,i}$, and capital, $k_{t,s,i}$.

$$y_{t,s,i} = z_{t,s,i} n_{t,s,i}^{1-\alpha_s} k_{t,s,i}^{\alpha_s}, \tag{2}$$

where the capital intensity α_s is allowed to vary between sectors.

Given the demand structure and the production function, the objective of firm i is to

[10]Since we do not have data on quantities, these two interpretations are observably equivalent. We illustrated this in subsection 3.5 where we consider a more general demand structure.

[11]Our wedges affect the use of labor and capital in a similar way.

maximize expected profits given by

$$E(\pi_{t,s,i}) = Y_t^{\frac{1}{\sigma}} P_t(1-\tau_{t,s,i})E(z_{t,s,i}^{\frac{\sigma-1}{\sigma}})(n_{t,s,i}^{1-\alpha_s}k_{t,s,i}^{\alpha})^{\frac{\sigma-1}{\sigma}} - w_t n_{t,s,i} - R_t k_{t,s,i}, \tag{3}$$

where $\tau_{t,s,i}$ is a tax-like idiosyncratic distortion that firm i faces in period t.

From the first order condition with respect to labor, we get the following expression for the distortion

$$1-\tau_{t,s,i} = (1-\varphi_{t,s,i})^{-1}\frac{\sigma w_t n_{t,s,i}}{(\sigma-1)(1-\alpha)p_{t,s,i}y_{t,s,i}}, \tag{4}$$

where $1-\varphi_{t,s,i}$ measures the "prediction error" of the firm

$$1-\varphi_{t,s,i} \equiv \frac{E(z_{t,s,i}^{\frac{\sigma_s-1}{\sigma_s}})}{z_{t,s,i}^{\frac{\sigma-1}{\sigma}}}.$$

The derivation of equation (4) is available in Appendix A.

We can also express the "prediction error" with the help of observable variables. This can be done by writing the realized profits, π, when n and k are chosen so that expected profits are maximized. rearranging this equation gives the following expression (see Appendix A for the details):

$$1-\varphi_{t,s,i} = \frac{\sigma}{\sigma-1}\left(\frac{w_t n_{t,s,i}}{w_t n_{t,s,i} + (1-\alpha_s)\pi_{t,s,i}}\right) \tag{5}$$

Taken together equations (4) and (5) state that we can measure firm (and time) specific distortions $(1-\tau_{t,s,i})$ and predictions errors $(1-\varphi_{t,s,t})$ if we observe firm's value added $p_{t,s,i}y_{t,s,i}$, wage stock $w_t n_{t,s,i}$ and profits $\pi_{t,s,i}$.

It is useful to note that, the measure of revenue distortion introduced here does not depend on substitutability between goods. To see this rewrite (4) using (5).

$$1-\tau_{t,s,i} = \frac{w_t n_{t,s,i} + (1-\alpha_s)\pi_{t,s,i}}{(1-\alpha_s)p_{t,s,i}y_{t,s,i}}.$$

Thus, our measure of tax-like distortion is robust to markup variation. We illustrate this further in subsection 3.3 where we consider generalized CES preferences.

The relationship between our measure of misallocation distortion and the one present in HK becomes clear when the expression of our distortion, equation (4), is substituted into the equation that defines the firm's profits:

$$\pi = Y_t^{\frac{1}{\sigma}} P_t (1 - \tau_{t,s,i}^{HK}) z_{t,s,i}^{\frac{\sigma-1}{\sigma}} (n_{t,s,i}^{1-\alpha_s} k_{t,s,i}^{\alpha})^{\frac{\sigma-1}{\sigma}} - w_t n_{t,s,i} - R_t k_{t,s,i}, \tag{6}$$

where

$$(1 - \tau_{t,s,i}^{HK}) = \frac{\sigma w_t n_{t,s,i}}{(\sigma - 1)(1 - \alpha) p_{t,s,i} y_{t,s,i}} \tag{7}$$

is the revenue distortion in HK. Thus, if one ignores uncertainty, ex post it looks like firms were maximizing (6). Equations (7) and (4) also make clear that our approach decomposes the standard measure of revenue distortion

$$\ln(1 - \tau_{t,s,i}^{HK}) = \ln(1 - \varphi_{t,s,i}) + \ln(1 - \tau_{y,t,s,i}) \tag{8}$$

to a component reflecting ex ante distortions $(1 - \tau_{t,s,i})$ and prediction error $(1 - \varphi_{t,s,i})$. For this reason, we also call $\tau_{t,s,i}$ as a residual wedge.

Without capital frictions, variation in the log of HK revenue distortion also gives the variation in the log of TFPR, which is the standard indirect measure of misallocation[12]. We can use the decomposition given by (8) to rewrite the misallocation measure as

$$Var(\ln(1 - \tau^{HK})) = Var(\ln(1 - \varphi)) + Var(\ln(1 - \tau)) + 2Cov(\ln(1 - \varphi), \ln(1 - \tau)). \tag{9}$$

That is, in the presence of uncertainty, the measure of ex post misallocation can be decomposed to components reflecting uncertainty (the variance of prediction error), ex ante misallocation (the variance of tax-like distortion) and the covariance between the two.

3.2 Measuring Misallocation in Finnish Data

We utilize the framework introduced in the previous subsection to analyze the ex post misallocation and its decomposition for Finnish firms. Moreover, we also explore the age-dependent trends in these measures.

We use annual firm-level data from the Financial Statement Statistics and the Busi-

[12]See HK for details.

ness Register data on the establishment level over the period 1989–2012. Both data sets are provided by Statistics Finland. For the period 1995–2012, our data covers the vast majority of Finnish firms in all industries, excluding the financial sector. The coverage varies between 95-99% of all Finnish firms. For the earlier years, the coverage is substantially weaker. This is due to a change in the data collection process. Instead of using their own survey, Statistic Finland started to utilize the Business taxation register data as their primary source of financial statement data in 1995[13]. Thus, we only use periods 1989-1994 in order to determine firms' age.

We focus on industries 15-63 with NACE rev 2 codes. That is, in addition to finance, insurance and real estate, we also omit agriculture and mining industries. As an additional restriction, we only follow firms up to the age of 10 years when we examine age-dependent trends. In theory, the data would allow us to follow some firms up to the age of 24. However, the number of firms for older generations is so low that we omit these cohorts when we do our accounting exercise conditional on firms' age.

The variables we use are value added, employment compensation (wages and salaries plus other personnel expenses), total profits, equity and industry code at three digit level. In our static model, firms rent capital. To get the measure of profits in line with this, we deduce the opportunity cost of a firm's own capital (5% real interest rate times firms total equity) from its total profits. We attached a sector-specific α_s to each firm based on the labor shares at the 3-digit level industries. A firm's age is determined based on the year in which the first establishment appears in the business register data.

We start by calculating HK revenue wedge and its components using equations (4), (5) and (7) for all firms on industries 15-63 for years 1995-2012. To increase the reliability of our misallocation measures, we winsorize the resulting wedges at 0.01-level. Next, we calculate the variance of (log) HK revenue wedge, the variance of its components and their covariance.

The results are given in Table 1. The variance of log HK wedge is 0.656. This is clearly higher than what HK report for the US. However, the data sets used are not directly comparable. Our unit of observation is firm while they use plant-level data. Moreover, our data contains also the majority of small firms for which the distortions are like to be more severe. To illustrate the issue, we redo our accounting exercise for listed companies

[13]Statistic Finland have also added retrospectively the administrative data for 1994. However, the coverage in 1994 is not comparable to the subsequent period.

Table 1: The variance of HK style revenue wedge and its decomposition to uncertainty and residual wedge.

Variable	Value	Share of $\mathrm{Var}(\ln(1-\tau_{HK}))$
$\mathrm{Var}(\ln(1-\tau_{HK}))$	0.656	1.00
$\mathrm{Var}(\ln(1-\tau))$	0.088	0.13
$\mathrm{Var}(\ln(1-\varphi))$	0.371	0.57
$2\mathrm{Cov}(\ln(1-\tau), \ln(1-\varphi))$	0.151	0.23

Notes: Percentages do not necessarily sum up to one because of the winsorization.

only. For this subset of firms, the variance of HK wedge is only around 20% of the variation in the whole sample. The results for listed companies are given in Appendix B. Note also that unlike many others, we did not restrict our sample to manufacturing firms. When exploring only the manufacturing sector, the dispersion of the revenue wedge is smaller (see Table 2).

Moving on to the variance decomposition, we can see that the uncertainty accounts for 57 % of the total variation. The variance of the residual wedge, our indirect measure of misallocation, is making up only 13% of the variation in the HK style misallocation measure. Thus, it seems that for Finnish data, uncertainty is much more relevant in generating ex post misallocation than idiosyncratic revenue distortions. Interestingly, if we restrict our attention to listed companies, uncertainty and misallocation are almost equally important (see Appendix B for details).

Finally, in the pooled data, the covariance of prediction error and revenue wedge is positive and it has a role in accounting for the ex post misallocation. Taken at face value, positive covariance would suggest that firms with high $1-\tau$ are over-optimistic about their productivity. Alternatively, this could also be caused by omitted heterogeneity. We illustrate the latter argument in subsection 3.3, where modify our accounting framework by allowing markup variation, and in subsection 5.5, where we utilize a variant of our structural model and analyze the effects of heterogeneous fixed costs.

Table 2 reproduces the accounting exercise reported in Table 1, but it focuses on one industry at a time. From the table, we see that uncertainty is by far the most important component in explaining the ex post misallocation for all industries, excluding electricity,

Table 2: The variance of HK style revenue wedge and its decomposition to uncertainty and residual wedge for different industries.

Variable	Value	Share of Var($\ln(1-\tau_{HK})$)
D Manufacturing		
$\mathrm{Var}(\ln(1-\tau_{HK}))$	0.504	1.00
$\mathrm{Var}(\ln(1-\tau))$	0.073	0.14
$\mathrm{Var}(\ln(1-\varphi))$	0.284	0.56
$2\mathrm{Cov}(\ln(1-\tau), \ln(1-\varphi))$	0.105	0.21
E Electricity, gas and water supply		
$\mathrm{Var}(\ln(1-\tau_{HK}))$	0.672	1.00
$\mathrm{Var}(\ln(1-\tau))$	0.399	0.60
$\mathrm{Var}(\ln(1-\varphi))$	0.200	0.30
$2\mathrm{Cov}(\ln(1-\tau), \ln(1-\varphi))$	0.034	0.05
F Construction		
$\mathrm{Var}(\ln(1-\tau_{HK}))$	0.710	1.00
$\mathrm{Var}(\ln(1-\tau))$	0.079	0.11
$\mathrm{Var}(\ln(1-\varphi))$	0.418	0.59
$2\mathrm{Cov}(\ln(1-\tau), \ln(1-\varphi))$	0.155	0.22
G Wholesale and retail trade		
$\mathrm{Var}(\ln(1-\tau_{HK}))$	0.565	1.00
$\mathrm{Var}(\ln(1-\tau))$	0.095	0.17
$\mathrm{Var}(\ln(1-\varphi))$	0.306	0.54
$2\mathrm{Cov}(\ln(1-\tau), \ln(1-\varphi))$	0.132	0.23
H Hotels and restaurants		
$\mathrm{Var}(\ln(1-\tau_{HK}))$	0.646	1.00
$\mathrm{Var}(\ln(1-\tau))$	0.086	0.13
$\mathrm{Var}(\ln(1-\varphi))$	0.372	0.58
$2\mathrm{Cov}(\ln(1-\tau), \ln(1-\varphi))$	0.140	0.22
I Transport, storage and communication		
$\mathrm{Var}(\ln(1-\tau_{HK}))$	0.671	1.00
$\mathrm{Var}(\ln(1-\tau))$	0.096	0.14
$\mathrm{Var}(\ln(1-\varphi))$	0.357	0.53
$2\mathrm{Cov}(\ln(1-\tau), \ln(1-\varphi))$	0.187	0.28

Notes: Percentages do not necessarily sum up to one because of the winsorization.

gas and water supply[14]. For these industries, uncertainty's relative importance is always over 50%. From Table 2, it can also be seen that there is some variation in the level of dispersion in the HK revenue wedge between industries. The ex post misallocation is the smallest for firms in manufacturing (0.50) and the highest for firms operating in

[14]Compared to the other industries, the number of firms in electricity, gas and water supply is substantially lower. Moreover, these firms are on average older and larger than in other industries

construction (0.71). Given this variation in the variance of HK revenue wedges, it is interesting that our measure of ex ante misallocation is remarkable stable over different industries, again excluding electricity, gas and water supply. The level of uncertainty, however, is varying a lot between industries. For example, in construction the level of uncertainty is 50% higher than in manufacturing. The relative importance of uncertainty and its substantial variation suggest that differences in the dispersion of the HK revenue wedge across industries are mainly explained by the varying levels of uncertainty. Though, there is also some variation in the covariance term.

Table 3: The variance of HK style revenue wedge and its decomposition to uncertainty and residual wedge for different years.

Variable	Value	Share of $\text{Var}(\ln(1-\tau_{HK}))$
1996		
$\text{Var}(\ln(1-\tau_{HK}))$	0.618	1.00
$\text{Var}(\ln(1-\tau))$	0.086	0.14
$\text{Var}(\ln(1-\varphi))$	0.368	0.60
$2\text{Cov}(\ln(1-\tau), \ln(1-\varphi))$	0.155	0.25
2001		
$\text{Var}(\ln(1-\tau_{HK}))$	0.494	1.00
$\text{Var}(\ln(1-\tau))$	0.078	0.16
$\text{Var}(\ln(1-\varphi))$	0.277	0.56
$2\text{Cov}(\ln(1-\tau), \ln(1-\varphi))$	0.119	0.24
2006		
$\text{Var}(\ln(1-\tau_{HK}))$	0.724	1.00
$\text{Var}(\ln(1-\tau))$	0.090	0.12
$\text{Var}(\ln(1-\varphi))$	0.398	0.55
$2\text{Cov}(\ln(1-\tau), \ln(1-\varphi))$	0.166	0.23
2011		
$\text{Var}(\ln(1-\tau_{HK}))$	0.862	1.00
$\text{Var}(\ln(1-\tau))$	0.098	0.11
$\text{Var}(\ln(1-\varphi))$	0.496	0.58
$2\text{Cov}(\ln(1-\tau), \ln(1-\varphi))$	0.190	0.22

Notes: Percentages do not necessarily sum up to one because of the winsorization.

In Table 3 we measure misallocation in different years. The dispersion of the residual wedge, our measure of ex ante misallocation, is again stable over the subsets of the data. Uncertainty, however, is varying over the years. In 1996 and in 2006 uncertainty is close to the value reported in Table 1. However, in 2011, at the height of the European debt crisis, uncertainty is substantially higher. In 2001, a year in the middle of a strong growth period in Finland, uncertainty is lower compared to the other years. Consistently with the variation in uncertainty, the dispersion of the HK revenue wedge is high in 2011 and low in 2001.

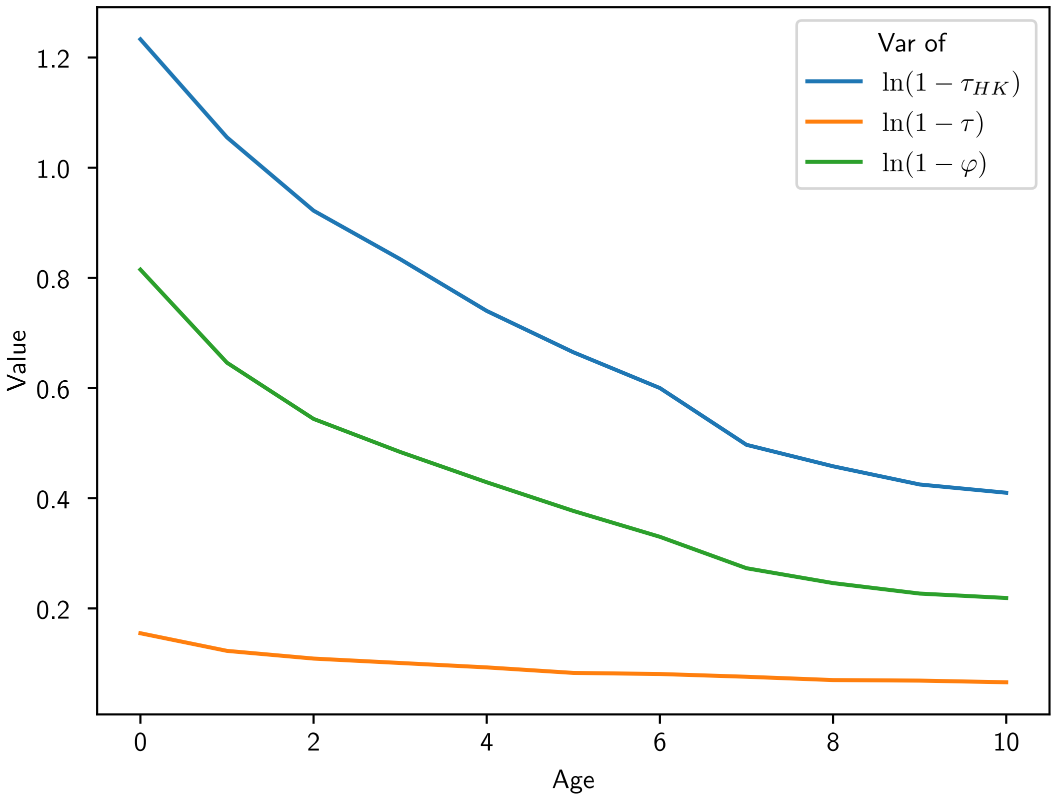

Figure 1: Measuring uncertainty and misallocation for Finnish firms conditional on firms' age.

Finally, we explore the life-cycle aspects of misallocation. We calculate the variance of the HK revenue wedge conditional on firms' age. We also redo our decomposition separately for all age groups and report the variances of prediction errors and the residual wedges.The results of this exercise are reported in Figure 1. As stated earlier, there is a strong negative trend in the ex post misallocation (the blue line). The variance is more than twice as high for entrants than for firms that are ten years old. This trend is almost entirely accounted for by the decreasing uncertainty (the green line). Note that, the orange line that gives the dispersion in the residual wedge is practically constant.

The age decomposition hints that slowly reducing uncertainty about fundamentals could play a significant role in understanding the life-cycle patterns of resource allocation. In addition, the strongly decreasing trend in uncertainty is in line with Jovanovic (1982) style mechanism of firm growth where learning plays an important role in explaining the observed up or out style life-cycle patterns.

3.3 Markup Heterogeneity

Up to this point, we have assumed that markups do not vary between firms. The new methods of measuring markups at the micro-level pioneered by De Locker and Warzynski (2012) have increased the interest in connections between aggregate economy and markups [15]. De Loecker et al. (2020) highlight the substantial and, for the past decades, increasing spread in markups in the US. This variation in markups could potentially have a substantial effect on misallocation. In line with this, David and Venkateswaran (2019) show that markups account for a non-negligible fraction of the variation of TFPR in the US, while Baqaee and Farhi (2020) find that eliminating misallocation caused my heterogeneous markups could increase aggregate TFP by 15%. The effects are more moderate in Edmond et al. (2018). For us, markup heterogeneity could potentially be important as it could distort our measures of uncertainty and misallocation.

Unlike many others (see, e.g., De Locker et al., 2020 or David and Venkateswaran, 2019), we do not follow the cost minimization approach of De Locker and Warzynski (2012). Instead, we relax the assumption of homogeneous markups and allow the substitutability of goods to vary between different producers. We do this by applying preferences similar to, e.g., Caliendo et al. (2020) and assume that (inverse) demand for a good is determined by generalized CES (see Spence, 1976)

$$p_{t,s,i} = \Lambda z^d_{t,s,i} y_{t,s,i}^{-(\frac{1}{\sigma_{t,s,i}})} \tag{10}$$

where Λ is an aggregate demand sifter and $z^d_{t,s,i}$ is a firm-specific demand shock. We need to specify the demand structure since we want to evaluate how heterogeneous markups affect our measure of uncertainty. However, like most of the studies using the cost min-

[15] See, Syverson (2019) for a survey.

imization approach, we rely on the firms' use of raw materials as a way to identify the markups. This means that we need to introduce a more general production function where firms also use intermediate inputs, m, that is

$$y_{t,s,i} = z^{y}_{t,s,i} k^{\alpha_s}_{t,s,i} n^{\xi_s-\alpha_s}_{t,s,i} m^{1-\xi_s}_{t,s,i} \tag{11}$$

Given the demand structure and the production function, the objective of firm i is to maximize expected profits

$$\max_{n,k} \Bigg\{ E\left[\max_{m} (1-\tau_{t,s,i}) \Lambda z_{t,s,i} (k^{\alpha_s}_{t,s,i} n^{\xi_s-\alpha_s}_{t,s,i} m^{1-\xi_s}_{t,s,i})^{\frac{\sigma_{t,s,i}-1}{\sigma_{t,s,i}}} - (1-\tau_{t,s,i}) p^m_t m_{t,s,i} \right] \\ - w_t n_{t,s,i} - R_t K_{t,s,i} \Bigg\} \tag{12}$$

where $z_{t,s,i}$ is a convolution of demand and productivity shocks, and p^m_t is the price of raw materials. To identify markups, we assume that the firm chooses its raw materials after uncertainty has been revealed. Moreover, we also assume that the revenue tax does not directly affect its use of raw materials.

The first order condition with respect to m pins down the markup for firm i at time t (see Appendix C for details)

$$\frac{\sigma_{t,s,i}}{\sigma_{t,s,i}-1} = (1-\xi_s) \frac{p_{t,s,i} y_{t,s,i}}{p^m_t m_{t,s,i}} \tag{13}$$

Thus, like De Locker and Warzynski (2012), we only need to observe the share of revenues that goes to raw materials in order to observe the firm-specific markup once we have fixed ξ_s.

As before, using the first order condition with respect to labor and the firm's profit equation when inputs are chosen ex ante optimally, allow us to determine the prediction error and the revenue distortion for firm i. The details are available in Appendix C.

The tax-like wedge for firm i is now given by

$$(1-\tau_{t,s,i}) = \frac{w_t n_{t,s,i} + \frac{\xi_s-\alpha_s}{\xi_s} \pi_{t,s,i}}{\frac{\xi_s-\alpha_s}{\xi_s} p_{t,s,\bar{i}} y_{t,s,i}} \tag{14}$$

where $p_{t,s,\bar{i}} y_{t,s,i}$ is the value added. This is equivalent to our original measure of tax-like distortion. Thus, as suggested earlier our measure of ex ante distortion is robust to markup heterogeneity. The measure of prediction error, however, is affected by varying markups. Now firm i's prediction error in period t, $1-\varphi_{t,s,i}$ is given by

$$(1-\varphi_{t,s,i}) = \frac{w_t n_{t,s,i}(\frac{\sigma_{t,s,i}}{\sigma_{t,s,i}-1} - (1-\xi_s))}{(\xi_s-\alpha_s)\pi_{t,s,i} + \xi_s w_t n_{t,s,i}} \tag{15}$$

Finally, we can rewrite HK wedge as

$$(1-\tau^{HK}_{t,s,i}) = (1-\tau_{t,s,i})(1-\varphi_{t,s,i})(\frac{\sigma_{t,s,i}}{\sigma_{t,s,i}-1} - (1-\xi_s))^{-1}\frac{\sigma}{\sigma-1}\xi_s \tag{16}$$

This allows us to decompose the ex post measure of misallocation, $Var(\ln(1-\tau^{HK}_{t,s,i})$, to uncertainty, ex ante misallocation, a component that reflects variation in markups and their covariances:

$$\begin{aligned}
Var(\ln(1-\tau^{HK}_{t,s,i})) =& Var(\ln(1-\tau_{t,s,i})) + Var(\ln(1-\varphi_{t,s,i})) \\
&+ Var(\ln(\frac{\frac{\sigma_{t,s,i}}{\sigma_{t,s,i}-1} - (1-\xi_s)}{\xi_s})) + 2Cov(\ln(1-\tau_{t,s,i}), \ln(1-\varphi_{t,s,i})) \\
&- 2Cov(\ln(1-\tau_{t,s,i})), \ln(\frac{\frac{\sigma_{t,s,i}}{\sigma_{t,s,i}-1} - (1-\xi_s)}{\xi_s})) \\
&- 2Cov(\ln(1-\varphi_{t,s,i})), \ln(\frac{\frac{\sigma_{t,s,i}}{\sigma_{t,s,i}-1} - (1-\xi_s)}{\xi_s}))
\end{aligned}$$

Table 4 assembles the results of this decomposition for the pooled data. To begin with, the introduction of variable markups does not diminish the importance of uncertainty. In fact, the variation of prediction error is higher than with constant markups. As for the markups, the variation in the component mainly reflecting markups is substantial making up almost 50% of the variation in the ex post misallocation. Finally, the co-variation between prediction error and ex ante misallocation is now substantially smaller than what it was was earlier. Next note that the covariance between tax-like distortion and markup component is negative and close to zero. Contrary to the other covariance terms, there is a substantial amount of co-variation between markup term and uncertainty.

Figure 2 shows the variance terms conditional on firms' age. Again the main conclusion from Figure 1 does not change; there is a strong decreasing trend in the variation of HK wedge, which is accounted for by a similar trend in uncertainty. There is no substantial variation in markups conditional on age.

Table 4: The variance of HK style revenue wedge and its decomposition to uncertainty, residual wedge and a component reflecting variation in markups.

Variable	Value	Share of Var($\ln(1-\tau_{HK})$)
Var($\ln(1-\tau_{HK})$)	0.656	1.00
Var($\ln(1-\tau)$)	0.088	0.135
Var($\ln(1-\varphi)$)	0.918	1.401
Var($\ln((\frac{\sigma}{\sigma-1}-(1-\xi))/\xi)$)	0.312	0.476
$2\text{Cov}(\ln(1-\varphi),\ln(1-\tau))$	0.08	0.124
$-2\text{Cov}(\ln(1-\varphi),\ln((\frac{\sigma}{\sigma-1}-(1-\xi))/\xi))$	-0.717	-1.094
$-2\text{Cov}(\ln(1-\tau),\ln((\frac{\sigma}{\sigma-1}-(1-\xi))/\xi))$	0.036	0.055

Notes: Percentages do not necessarily sum up to one because of the winsorization.

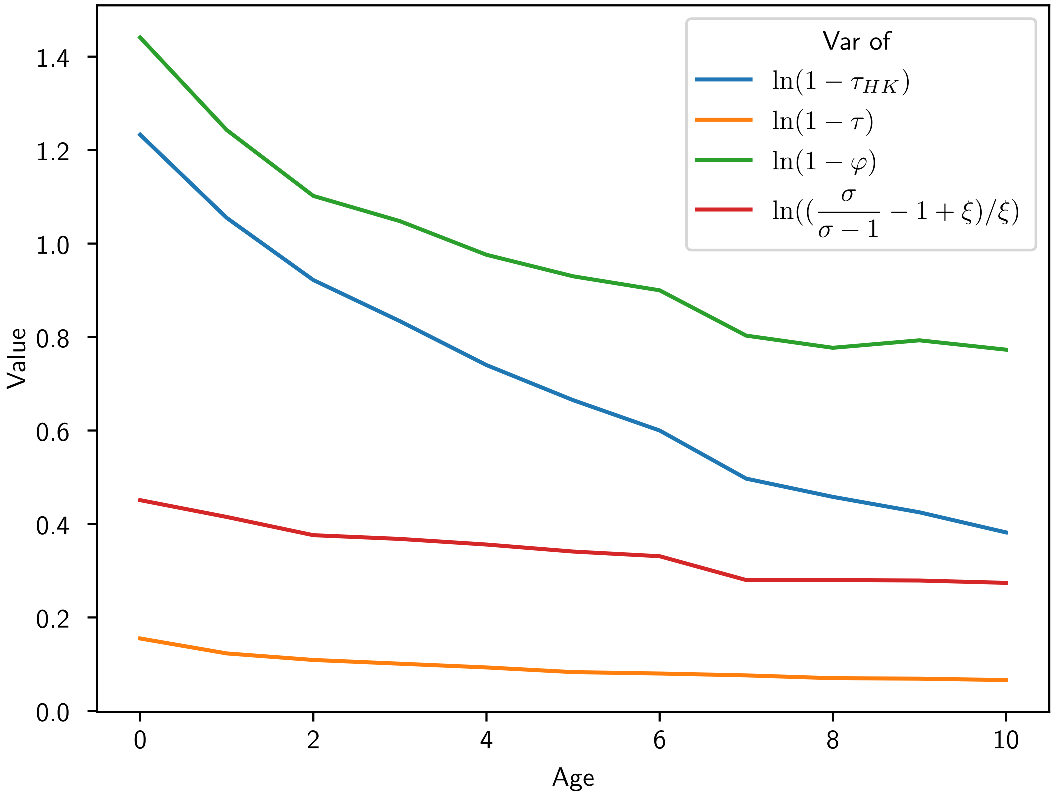

Figure 2: Measuring uncertainty and misallocation for Finnish firms conditional on firms' age.

4 Model

Motivated by the results from our accounting exercise, we set up a general equilibrium model where firms have to choose their input without full information about the current period productivity. To allow for the observed age-dependent trend in the prediction errors, we add Jovanovic's (1982) learning mechanism to a general equilibrium framework with monopolistic competition similar to Melitz (2003).

4.1 Households

There is a unit mass of risk neutral infinitely lived households who derive utility from consumption and supply labor inelastically. The behavior of households can be summarized with a representative household whose preferences are given by $\sum_{t=0}^{\infty} \beta^t C_t$, where C_t is a consumption basket compiled from individual goods with the CES aggregator such that

$$C_t = \left(\int_{\Omega_t} c_{i,t}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{17}$$

where Ω_t is the amount of goods available. The household owns the firms and thus the budget constraint is given by

$$\int_{\Omega_t} p_{i,t} c_{i,t} = w_t \bar{N} + \Pi_t \tag{18}$$

where Π_t are aggregate profits and w_t is the wage rate. We focus on stationary equilibrium thus from now on we drop time indexes. Moreover, we use labor as the numéraire.

4.2 Incumbent

There is an endogenous measure of incumbent firms denoted by Ω. Each firm produces a unique good and faces a demand in line with (17) and (18),

$$y_i = \left(\frac{p_i}{P} \right)^{-\sigma} C \tag{19}$$

The production function of a firm is given by linear technology

$$y_i = e^{z_i} n_i \tag{20}$$

where e^{z_i} is firm's TFP and n_i employment hired by firm i. The firm-specific TFP in the current period is given in logs as

$$z_i = z_{i,p} + z_{i,t} \tag{21}$$

where $z_{i,p}$ is a permanent productivity component drawn in the first period of the firms' life-cycle. It is distributed according to

$$z_{i,p} \sim N(\mu_{z_p}, \sigma^2_{z_p}) \tag{22}$$

The other component, $z_{i,t}$, is temporary productivity that is drawn from a normal distribution in each period such that

$$z_{i,t} \sim N(0, \sigma^2_{z_t}),$$

for all i.

Firm i observes z_i but is unable to decompose it. This implies that the firm only learns its permanent productivity slowly as it needs to be extracted from the noisy signal, namely the observed TFP. The firm uses Bayesian learning to update its expectation. Given the log-normality, we get the standard Kalman filter with the following recursive representation for the mean, m, and variance of the firm's expectation of its permanent productivity,Σ,

$$m_i = (1-K)m_{i,-} + Kz_i \tag{23}$$

$$K = \frac{\Sigma}{\Sigma + \sigma^2_\epsilon}, \quad \Sigma' = \frac{\Sigma\sigma^2_\epsilon}{\Sigma + \sigma^2_\epsilon}, \quad m_{i,0} = \mu_{z_p}, \quad \Sigma_0 = \sigma^2_{z_p} \tag{24}$$

(See e.g. Ljungqvist and Sargent, 2018, for details). We assume that all firms start with common priors, i.e., with unconditional permanent productivity distribution. Given this, the firm's next period belief is distributed as

$$m_i' \sim N(m_i, K'\Sigma')$$

In addition,

$$z_i = m_i + (z_{i,p} - m_i) + z_{i,t} \sim N(m, \Sigma + \sigma^2_{z_t}). \tag{25}$$

Following, e.g., Restuccia and Rogerson (2008), we do not explicitly model the sources of misallocation but assume that these frictions can be summarized by an idiosyncratic distortion, $1-\tau_{i,t}$, that appears in firms profit maximization problem as revenue tax would. We also assume that the firm-specific distortion contains a permanent and a transitory component. That is,

$$ln(1-\tau) = \tau_{i,p} + \tau_{i,t} \tag{26}$$

where $\tau_{i,p}$ and $\tau_{i,t}$ are both normally distributed. The permanent part, $\tau_{i,p}$, is drawn upon entry and memoryless transitory part, $\tau_{i,t}$, in each period. In addition, operating firms also have to pay periodic fixed costs c_f.

Given the demand structure, the productivity process and the wedges, a firm's objective is to maximize its lifetime profits by making an optimal exit/stay decision and conditional on stay to choose the current period employment. In line with the analysis in the previous section, we assume that this decision is made before the current period productivity is known but after the firm has observed the current period revenue wedge τ_a. Thus, firms' marginal productivities will differ due to wedges and expectation error. In addition to these channels, we also allow for convex adjustment costs that some authors (see, for example, Eslava and Haltiwanger, 2018) consider as a potential explanation for the age-dependent trend in misallocation.

The intra-period timing is summarized in Figure 3. At the beginning of the period, an incumbent firm chooses whether it wants to exit or not. A firm, that decides to continue, observes the current period revenue wedge and pays periodic fixed cost c_f. Next, it chooses its employment. If a continuing firm decides to adjust its scale, the firm has to pay adjustment cost $\lambda(\frac{n-n_-}{\bar{n}})^2\bar{n}$ where $\bar{n} = \frac{n+n_-}{2}$. After choosing its employment level, the firm produces and observes the current period (combined) productivity. Finally, at the end of the period, the firm may be forced to exit due to an exogenous shock that happens with probability γ.

Taken all together, a firm's problem at the beginning of the period can be summarized by the following recursive expression

$$V(m, a, n_-, \tau_p) = \max\{E_{\tau_t}(W(m, a, n_-, \tau_p, \tau_t)), -2\lambda n_-\} \tag{27}$$

where the relevant state variables are the firm's belief about its permanent productivity m, firm's age a, its employment in the previous period and the permanent distortion τ_p.

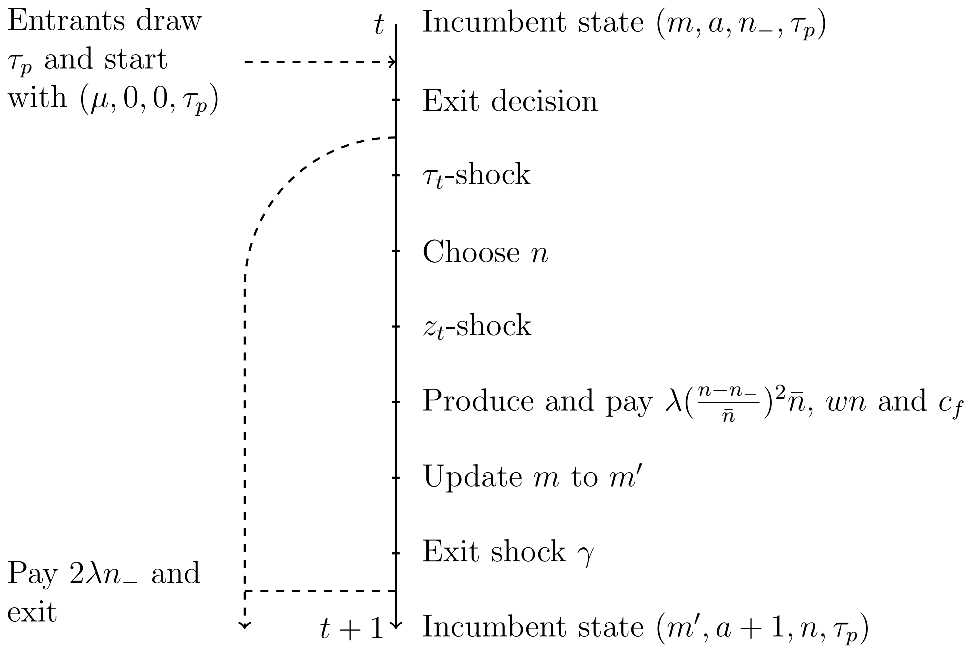

Figure 3: Intra-period timing

The value at the time when employment is decided is given by

$$W(m, a, n_-, \tau_p, \tau_t) = \Bigg[\max_n C^{\frac{1}{\sigma}} P e^{\tau_p + \tau_t} E_\epsilon[(e^z)^{\frac{\sigma-1}{\sigma}}] n^{\frac{\sigma-1}{\sigma}} - n - c_f - \lambda(\frac{n - n_-}{\bar{n}})^2 \bar{n} + \beta(1-\gamma) \int V(m', a+1, n, \tau_p) dF(m'|m, K'\Sigma') \Bigg] \tag{28}$$

The solution to this Bellman equation gives exit policy $x(m, a, n_-, \tau_p, \tau_t)$ taking value 1 if firm chooses to exit and taking value 0 if firm chooses to continue, and an employment policy $n(m, a, n_-, \tau_p, \tau_t)$.

4.3 Entry

There is a continuum of potential entrants. Each of them has to pay entry cost c_e if they want to start operating. After paying the entry cost, a new firm starts as an incumbent firm in the next period with age zero and $m = \mu_{z_p}$. The entrant learns its permanent revenue wedge τ_p only after entry. The amount of entrants is such that the expected gains are equal to the entry cost:

$$c_e = \beta \int V(\mu_{z_p}, 0, 0, \tau_p) H(d\tau_p) \tag{29}$$

where $H(\cdot)$ is the density function for permanent revenue wedge.

4.4 Stationary Equilibrium

Using firms' exit and hiring policies, $x(\cdot)$ and $n(\cdot)$, we can define the evolution of the firm distribution measured at the beginning of each period, $\Psi(dm, a, dn_-, d\tau_p)$. At the stationary equilibrium $\Psi(dm, a, d_n-, d\tau_p)$ is given by

$$\Psi(\mathcal{M}', A', \mathcal{N}, T_p) = \sum_{a|a+1\in A'} \iint_{(m,n_-,\tau_p,\tau_t)|n(\cdot)\in\mathcal{N}, m'\in\mathcal{M}', \tau_p\in T_p)} \\ [Q(m,\mathcal{M}')(1 - x(m,a,n_-,\tau_p))\Psi(dm,a,dn_-,d\tau_p)dF(\tau_t) \\ + M\mathbb{I}(a+1=0)\mathbb{I}(m'=\mu)\mathbb{I}(n=0)\int_{T_p} H(d\tau_p)] \tag{30}$$

where $Q(m, \mathcal{M}')$ is the transition function for beliefs, each $(m, a, n_-, \tau_p, \tau_t)$ is such that $n(m, n_-, \tau_p, \tau_t) \in \mathcal{N}$, $\tau_p \in T_p$ and $dF(\tau_t)$ is the density function for transitory wedges. Moreover, M denotes the measure of entrants, $\mathbb{I}(a+1 = 0)$ is an indicator function taking the value one if $0 \in A'$ and $\mathbb{I}(n = 0)$ takes value one if $0 \in \mathcal{N}$.

Given the measure of firms, we can express the labor demand as

$$N = \sum_a \iint [n(m,a,n_-,\tau_p,\tau_t) + c_f + \lambda(\frac{n-n_-}{n_-})^2 n_-]\Psi(dm,a,dn_-,d\tau_p)dF(\tau_t) + c_e M. \tag{31}$$

The stationary equilibrium can be defined with policy functions $x(\cdot)$ and $n(\cdot)$, Price index, aggregate output, a stationary distribution of firms and a mass of entrants such that

1. The policy rules $x(\cdot)$ and $n(\cdot)$ solve the firms problem given by (27) and (28)
2. The price level and aggregate output are such that the free entry condition holds
3. The stationary measure of firms is given by (30)
4. the mass of new entrants is such that the labor market clears, i.e., N given by (31) is equal to fixed labor supply $\bar{N}$

5 Quantitative Analysis

In this section, we match our model with the Finnish firm-level data. We then redo our static calculations using simulated data and explore the importance of different frictions for the determination of the aggregate TFP.

5.1 Calibration Strategy

We use the method of simulated moments to fix the parameters (directly) related to the firms' problem by setting our model to match the prime observable features of the Finnish firm-level data, e.g., growth patterns and size distribution of established firms. We calculate our targets from the same data set that we used in Section 3. The parameters governing the preferences of the representative household are calibrated directly.

In line with our data, the model's period is set to one year. We assume 5% real interest rate and thus fix β to 0.95. We follow HK and set the elasticity of substitution, σ, to 3. In addition, we normalize the productivity process by fixing $\mu_{z_p} = 0$.

We are left with eight parameters that we calibrate internally. That is, we still need to determine values for the variances of permanent and transitory productivity components, $\sigma^2_{z_p}$ and $\sigma^2_{z_t}$,; the variances of revenue wedges, $\sigma^2_{\tau_p}$ and $\sigma^2_{\tau_t}$; entry costs, c_e; fixed costs , c_f; the parameter governing the adjustment costs, λ; and the exogenous exit rate, γ. To fix the values of these parameters, we minimize the squared relative distance between moments calculated from the Finnish firm-level data and the same moments generated by our model using the identity matrix as a weighting matrix. We target 11 moments related to the growth rates of young firms (age < 5) and old firms (age ≥ 5), the entry rate of firms, the exit rate of young firms, variation in profits controlling the size of firms and the size distribution of old firms. Due to a complicated equilibrium setup, the parameter values are defined jointly. However, next, we give a heuristic argument about which moment is the most relevant for which parameter.

When a firm that has learned its permanent productivity chooses its employment level, the only uncertainty relates to the temporary productivity shock. Without adjustment costs, the log of profits for these firms would be given by

$$\ln(\pi) = ln(n^*) + \ln\left[\frac{E(Z) - \sigma(E(Z) - Z)}{(\sigma - 1)E(Z)}\right] \tag{32}$$

Thus, without adjustment costs, regressing (log) profits with (log) employment for these firms and calculating the variance of the residual term would directly pin down the variance of temporary TFP shock. With adjustment costs and without knowing which firms have learned their permanent productivity, this relationship is more complicated. Nevertheless, the variance of the residual calculated using all firms still informs us about the variation of the temporary TFP component.

Young firms adjust their scale when they receive new information about their permanent productivity. With fixed values for the variance of temporary shocks and the adjustment cost parameter, more variation in the permanent productivity component across firms increases the variance of the growth rates of young firms. That is, the variation in the growth rates helps us to identify the variance of the permanent productivity component.

Holding the parameters governing the learning constant, altering the adjustment costs changes the speed at which young firms reach their optimal scale. Thus, the mean growth rate of young firms is informative about the adjustment cost parameter. In our setup, old firms do not adjust their scale on average. They are, however, reacting to changes in the temporary distortion. This means that the standard deviation of growth rates for old firms helps us to identify the variance of temporary revenue distortions.

Reducing (increasing) entry costs increases (decreases) competition and prices. This sifts the size distribution to the right. The distribution of old firms is also conveying information about permanent revenue distortions. Moreover, the size distribution gives additional information about the distribution of permanent productivity.

In the model, young firms choose to exit from the market if they believe that their productivity is not high enough to cover the fixed costs. Therefore, the exit rate of young firms is sensitive to the level of fixed costs. Finally, the exogenous exit rate affects the effective discount factor and thus the expected profits of potential entrants. Alternatively, one may think that the exogenous exit rate pins down the aggregate exit rate in the economy, which at the steady state equilibrium is equal to the entry rate.

5.2 Fit of the Model

Table 5 gives the data targets and the model counterparts. The associated parameter values are given in Table 6. Overall, the model fits the data quite well, especially taking into account the over-identification. To be more precise, our model generates a size distribution that is highly similar to the Finnish data. In addition, the fraction of new firms entering into the economy and exit rate for young firms are close to their observed values. The model also captures the targeted growth patterns; the mean growth rates for young firms and the average variation in growth rates for young and old firms are matched exactly. The only target for which the model's fit is somewhat weaker, in absolute terms, is the standard deviation of profit residual. The variation of profits, after controlling the size, is not at the level observed in the data.

Table 5: Targets and model counterparts

Moment	Data	Model
Std of profit residual	1.23	1.15
Firm Growth		
Mean growth rate, young firms	0.10	0.10
Std of growth rates, young firms	0.46	0.46
Std of growth rates, old firms	0.28	0.28
Size distribution, old firms		
Firms with employment < 5	0.67	0.68
Firms with employment ≥ 5 and < 10	0.16	0.15
Firms with employment ≥ 10 and < 20	0.09	0.09
Firms with employment ≥ 20 and < 50	0.05	0.06
Firms with employment > 50	0.02	0.02
Selection		
Exit rate, young firms	0.09	0.08
Entry rate	0.09	0.10

Notes: Firms are called young (old) when they are under (over) 5 years old.

Figure 4 and Figure 5 illustrate our model's ability to replicate life-cycle growth patterns beyond the targets. As we can see from Figure 4, our calibration generates age-dependent mean growth rates closely resembling the ones observed in the data. Note also that, on average only the young firms are growing. Both in the data and the model, the growth rates are close to zero after the first four years.

Figure 5 highlights the dispersion at the age-dependent growth rates. Consistent with the learning channel, there is a strong non-linear decreasing trend in the standard devi-

Table 6: Parameter values

Parameter	Value
c_e	12.59
c_f	0.31
σ_z	0.65
σ_ϵ	0.58
σ_{τ_p}	0.41
σ_{τ_t}	0.13
λ	0.09
γ	0.07

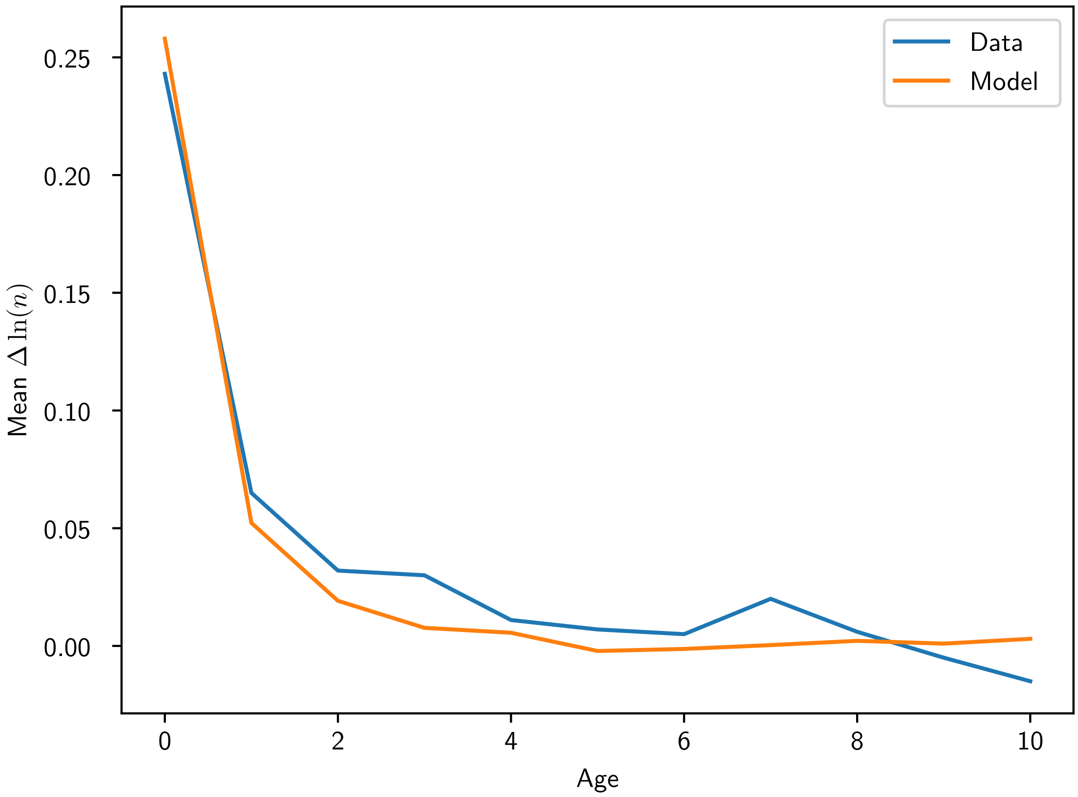

Figure 4: Mean growth rates conditional on age

ations of growth rates. Interestingly, the convergence of the standard deviations is much slower than the convergence of the mean growth rates. As with the first moments, our model is able to replicate the age-related patterns in the data remarkably well, given that we did not explicitly target these moments.

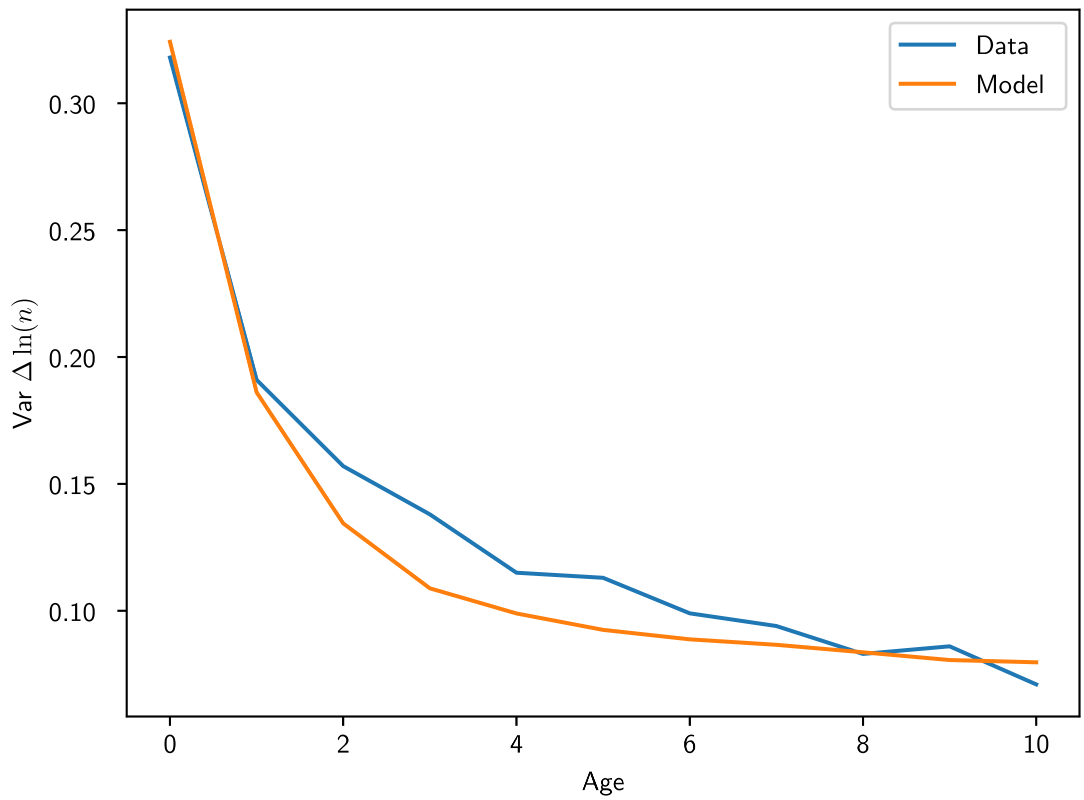

Figure 5: The dispersion of growth rates conditional on age.

5.3 Static Misallocation in the Simulated Data

We now explore the misallocation patterns in simulated data generated by our model and compare these with the ones we observed in Finnish data. In this regard, we redo our calculations of static misallocation that allowed us to decompose ex post misallocation to ex ante misallocation and uncertainty.

Table 7 reports the indirect measure of ex post misallocation and its decomposition in data generated with our calibrated model. The variance of the HK revenue wedge is 0.318. In the Finnish data, it was 0.653 when looking at all industries and years jointly. Thus, our model is able to explain around 49% of the observed variation in the HK revenue wedge.

Looking at the components of ex post misallocation, one observes that the model implied variance of $1-\varphi$ is 0.215. In the pooled data, this was 0.371. That is, our model is able to account for 58% of the variation in prediction errors. Note also that the level of uncertainty in the simulated data is not far from the level of uncertainty observed in

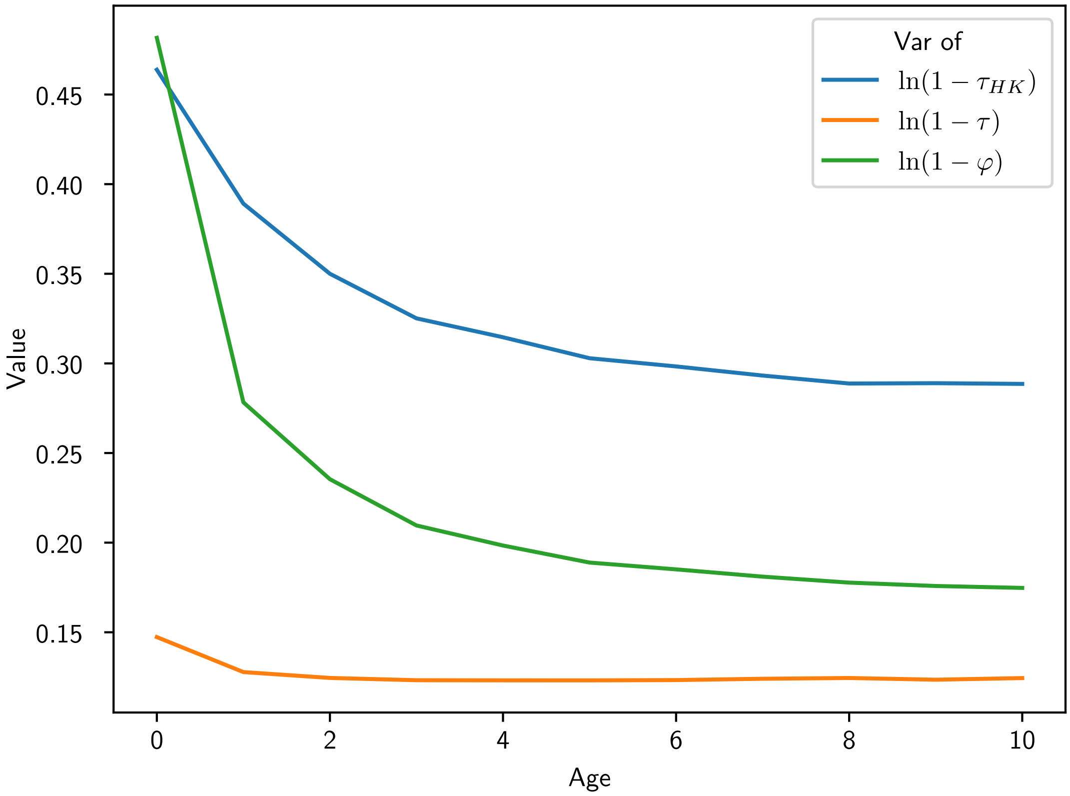

Figure 6: Measuring uncertainty and misallocation in the simulated data

manufacturing or wholesale and retail trade (0.284 and 0.306, respectively). As with the actual data, uncertainty is by far the most important component here, making up 68% of the variance of HK revenue wedge. This is relatively close to the one observed in the pooled data (57%).

Our model also implies ex ante misallocation that is close to the one we observed in section 3.2; the variance of the tax-like wedge is 0.132 while it was 0.088 in the pooled data. However, due to the other components, the relative importance of ex ante misallocation is higher than in the Finnish data (42% vs. 13%).

Qualitatively the main difference between the model and the pooled data is in the covariance term, which was 0.151 with homogeneous markups and 0.07 with heterogeneous markups, while here it is -0.03. A small negative covariance in the model is due to the adjustment costs. In Subsection 5.5, we introduce heterogeneity in fixed costs that, when correlated with ex ante misallocation, allows us to have a covariance term that is better in line with the data.

Table 7: The variance of HK style revenue wedge and its decomposition to uncertainty and residual wedge calculated from the simulated data

Variable	Value	Share of $\text{Var}(\ln(1-\tau_{HK}))$
$\text{Var}(\ln(1-\tau_{HK}))$	0.318	1.00
$\text{Var}(\ln(1-\tau))$	0.132	0.42
$\text{Var}(\ln(1-\varphi))$	0.215	0.68
$2\text{Cov}(\ln(1-\tau), \ln(1-\varphi))$	-0.030	-0.09

In Figure 6, we replicate the exercise shown in Figure 1. We calculate the variance of the HK revenue wedge conditional on the age of the firms and decompose it to a component reflecting uncertainty and the dispersion in the residual wedge.

A comparison of Figure 1 and Figure 6 reveals that our model is generating age-dependent patterns that are qualitatively similar to what we observed for the Finnish data. The dispersion of the HK revenue wedge is decreasing condition on age. Though, the level of ex post misallocation for new firms is clearly lower here. As in the data, this decreasing trend is explained by decreasing uncertainty.

For new firms, our model is able to generate about 60% of the variance of $\ln(1-\varphi)$. For the old firms that have been operating for ten years, the level of uncertainty is 66% of the observed uncertainty. However, uncertainty is decreasing faster here than in Section 3.

As in the data, there is a slight decrease in ex ante misallocation after the first period. Otherwise, and in line with our accounting exercise, the misallocation term is practically independent of the firms' age. Overall, a lower level at the uncertainty, especially for the entrants together with slightly negative covariance, due to adjustment costs, explain why ex post misallocation is lower here than in Section 3.

5.4 The Quantitative Significance of Uncertainty, Misallocation and Adjustment Costs

In order to evaluate how different factors affect productivity, we set up a benchmark economy without distortions or uncertainty. In this economy, firms pay the entry cost and immediately learn their permanent productivity. In each period, firms, first observe their productivity and then choose their labor input. Thus, there is no learning nor intraperiod uncertainty. Moreover, there are no adjustment costs or tax-like revenue distortions. Otherwise, we use our calibrated parameter values. We normalize the aggregate TFP of this economy to 100. We then illustrate how adding frictions affects the aggregate TFP in relative terms. Since that the labor supply is fixed, this also gives the output responses. Table 8 assembles the results of these counterfactuals. In all cases, we use the parameter values given in Table 6 to fix the level of different frictions.

Consistent with the static calculations reported in the previous subsection, uncertainty is by far the most important individual element in reducing aggregate TFP. When firms do not know their permanent productivity and have to choose their labor before knowing their current productivity, aggregate TFP is reduced over 38% relative to the benchmark.

We can further decompose the effects of uncertainty by exploring the periodic uncertainty when there is no early life-cycle learning. We do this by calculating aggregate TFP for an economy where firms learn their permanent productivity immediately after paying the entry cost but still have to choose their employment level before knowing their current productivity. Compared with the benchmark, aggregate TFP is reduced by approximately 21% in this case. Thus, the early life-cycle uncertainty is important. However, the uncertainty channel relevant to all firms has a somewhat larger effect on the aggregate TFP. This part of the uncertainty is similar to the one considered by David et al. (2016) when they explore a setup where both capital and labor are chosen under information frictions. Their quantitative results for the listed firms in the US suggest that uncertainty reduces TFP by 40%. If only capital is affected by uncertainty, and labor can be adjusted afterward, they find that the TFP reduction is substantially smaller. The results in David and Venkateswaran (2019) are similar. If only capital is affected, uncertainty only has a minor effect on aggregate TFP. However, if all input factors are chosen under imperfect information, TFP reduction is substantial.

Also misallocation has a substantial effect on aggregate TFP. Compared with the benchmark, an introduction of the tax-like distortions leads to a 26% reduction in productivity. Comparing the impacts of permanent and transitory wedges, it can be seen that permanent wedges are by far more important. The permanent component of misallocation reduces aggregate TFP by over 24%, while temporary wedges alone reduce TFP by less than 3%. These results are a bit stronger than what David and Venkateswaran (2019) find for the US. Different modeling decisions (e.g., whether distortions are correlated with fundamentals and whether entry and exit are allowed to interact with misallocation) make it hard to directly compare the results. Nevertheless, part of the differences is likely reflecting the fact that our sample of firms is quite different to theirs. While they focus on the listed firms for the US, our data covers nearly all Finnish firms from which a vast majority are unlisted. Thus, for example, financial frictions that increase misallocation are likely to be more of an issue in our sample partly just because firms are smaller and younger.

Finally, adjustment costs alone drop aggregate productivity by 11%. The lice-cycle aspects of our model, i.e., entry, exit and uncertain growth of young firms, emphasize the role of adjustment costs for aggregate outcomes. The non-negligible role of adjustment costs also illustrates the potential problems that could be associated with static evaluations of misallocation.

Comparing the calibrated model and our benchmark economy, one observes that uncertainty, misallocation and adjustment costs jointly reduce the economy's TFP by 55%. It is also noteworthy that the interaction of different frictions is important. For example, evaluating misallocation and uncertainty based on their added individual effects would lead one to overestimate their combined effects by around 9%. Moreover, even though adjustment costs alone have a substantial impact on TFP, adding adjustment costs to a setup where we already have misallocation distortions and uncertainty do not lead notable reduction in TFP.

Table 8: Results: the frictions and aggregate TFP

	No uncertainty	Partial uncertainty	Uncertainty
No distortions or adjustment costs	100.00	78.74	61.58
Adjustment cost	89.30	76.74	59.96
Misallocation	74.12	58.58	45.11
Misallocation and adjustment costs	67.06	57.86	44.57

Notes: Uncertainty refers to a case where firms have learn their permanent productivity and choose their labor before they know the current period productivity. In column partial uncertainty refers to a case where only temporary productivity is uncertain, firms learn their permanent productivity upon entry but choose their labor before knowing their current productivity. In order to shut down tax-like distortions, we set $\sigma_{\tau_p} = 0$ and $\sigma_{\tau_t} = 0$.

5.5 Heterogeneous Fixed Costs

An evident qualitative difference between static misallocation in the simulated data and the Finnish data is in the covariance between prediction errors and ex ante misallocation. In our model, the covariance is slightly negative, while in section 3, it was positive. Obviously, it is possible that firms with high (low) $1 - \tau$ are overoptimistic (overpessimistic) about their future. However, in our framework, which relies on Bayesian learning, such a cognitive bias would be hard to explain. An alternative explanation is that some unobserved heterogeneity that correlates with both our measure of uncertainty and tax-like wedges is responsible for this covariance. Our static calculations with heterogeneous markups support the latter argument. In subsection 3.3, we illustrated that when markups are allowed to vary, the covariance between the revenue distortion and the prediction error is almost halved compared to the case where markups are assumed to be constant (see Table 3 vs. Table 1).

Table 9: The variance of HK style revenue wedge and its decomposition to uncertainty and residual wedge calculated from the simulated data

$c_f(\tau_p + \tau_t) = \bar{c_f}(\exp\{(\tau_t + \tau_p)\varepsilon\}$	$\varepsilon = 2$ (Calibrated)		$\varepsilon = 2$ & $\lambda = 0$	
Variable	Value	Share of $\mathrm{Var}(\ln(1-\tau_{HK}))$	Value	Share of $\mathrm{Var}(\ln(1-\tau_{HK}))$
$\mathrm{Var}(\ln(1-\tau_{HK}))$	0.370	1.00	0.377	1.00
$\mathrm{Var}(\ln(1-\tau))$	0.092	0.25	0.089	0.24
$\mathrm{Var}(\ln(1-\varphi))$	0.253	0.68	0.236	0.63
$2\mathrm{Cov}(\ln(1-\tau), \ln(1-\varphi))$	0.026	0.07	0.053	0.14

Adjustment costs further illustrate how omitted factors can alter the covariance term between prediction error and ex ante wedge. In the simulated data generated by our calibrated model, the covariance term was slightly negative. If we set the adjustment costs to zero and redo our static calculations, this covariance term disappears.

In this subsection, we further examine the issue by introducing production heterogeneity into our model. To be more precise, we allow for heterogeneous fixed costs that depend on $1 - \tau$:

$$c_{f,i}((1-\tau_i) = c_f e^{ln(1-\tau_i)\varepsilon},$$

where c_f is now an average fixed cost and ε is a scale factor that links a (relative) change in revenue distortions to a (relative) change in fixed costs. When $\varepsilon > 0$, firms with $1 - \tau$ above the average also pay above the average fixed costs. For example, it could be that managers can appropriate a larger share of profits when firms get "revenue tax subsidies".

To illustrate how the proposed mechanism can alter the covariance in static calculations, we set ε to 2. Implying that, for example, 10% "revenue tax subsidy" will increase (or decrease) fixed costs by 20%. With this specification, we redo our calibration with the same targets as before. The model's fit and resulting parameter values are reported in Appendix D. The mean of the fixed costs stays close to the value of homogeneous fix costs in our baseline calibration. As for the other parameters, also these values are pretty close to our original calibration. In addition, the fit of the model stays more or less unaltered. Thus, given the relatively modest importance of fixed costs, setting ε to 2 does not change the behavior of our model.

Table 9 collects the results from the static accounting exercise. In columns one and two, we use the calibrated value for the adjustment cost parameter, λ, and in columns three and four, we set adjustment costs to zero. In both cases, the covariance is now positive though lower than in the Finnish data for pooled industries (0.15). However, without adjustment cost, the covariance is within the industry variation (see Table 2). Looking at the variance decomposition, the covariance term accounts for 7-14% of the total variation of the HK wedge. Again this somewhat lower than the observed value in the actual data.

As for the other components explaining variation in the HK wedge, ex ante misallocation is now a little lower than in our baseline model and close to the value we reported in Table 1 (the pooled data over industries and years). Due to the increased covariance term, the relative importance of misallocation is also more in line with the data. Uncertainty is slightly higher than in our original calibration, though still lower than in the pooled data (0.25-0.23 vs. 0.37). However, the values are again within the industry variation. Finally, due to increased covariance, the ex post misallocation is now a bit higher (0.37-0.38) but still clearly smaller than in Table 1 (0.66).

To sum it up, it seems that allowing fixed costs to correlate with misallocation allows one to generate a covariance term between ex ant misallocation and uncertainty that is better in line with our baseline accounting exercise. Moreover, it seems that this change does not alter the model's behavior along other dimensions. Interestingly, it also seems that our model generates static misallocation that is relatively close to what we observe for manufacturing firms.[16]

6 Conclusions

In this paper, we develop an indirect measure of misallocation that allows us to separate misallocation from uncertainty. We apply our approach to Finnish firm-level data. In the data, ex post misallocation is substantial. However, this is mainly accounted for by uncertainty, while ex ante misallocation seems to be less important, explaining only about 13% of the variation in ex post misallocation. We also show that when one compares different industries or different years, it seems that our measure of ex ante misallocation stays relatively stable. Ex post misallocation and uncertainty, however, vary substantially across years and industries. Moreover, we also observed a strong decreasing age-dependent trend in uncertainty.

To explore the quantitative significance of these issues, we build a life-cycle model of firm dynamics. We set our model to match the salient features of Finnish firm-level data and use our calibrated model to evaluate the importance of uncertainty and misallocation for aggregate TFP. As suggested by our static accounting exercise, uncertainty is by far the most important channel reducing TFP by almost 40 percent. However, also ex ante inefficient resource allocation, modeled with tax-like distortions, has a substantial effect on the aggregate productivity. Misallocation alone has a 26 percent negative effect on total productivity.

The substantial role of uncertainty in explaining ex post misallocation and in the determination of aggregate TFP raises the question of whether there exist some policies or practices that could reduce uncertainty, especially for young firms. This could mean boosting practices that would give firms additional information about the viability of their business plans. One potential avenue would be to increase the amount of venture capitalists, whose role in Finland has traditionally been small. Given that these investors would have high incentives to provide accurate information for firms, this could reduce the firms' reliance on their private information. In addition, training and consultation aimed at new entrepreneurs could also be helpful, especially for young firms. Policies

[16]For manufacturing firms the variance of HK wedge is 0.50 with its components taking the following values: $Var(ln(1-\tau)) = 0.07$, $Var(ln(1-\varphi) = 0.28$ and $Cov(ln(1-\tau), ln(1-\varphi)) = 0.10$.

that would increase the relative amount of listed companies could also, in theory at least, reduce uncertainty, as firms could learn from stock prices. However, the work of David et al. (2016), where they focus on these types of companies, suggests that additional information from stock markets plays a relatively minor role.

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Appendix A. A Static Measure of Uncertainty and Misallocation

The firm's static problem is given by

$$\begin{aligned}
\max_{n,k} E(\pi_{t,s,i}) &= E(p_{t,s,i}y_{t,s,i}) - w_t n_{t,s,i} - R_t k_{t,s,i} \\
&= \max_{l,k} Y^{\frac{1}{\sigma}} P_t (1-\tau_{t,s,i}) E(y_{t,s,i}^{\frac{\sigma-1}{\sigma}}) - w_t n_{t,s,i} - R_t k_{t,s,i} \\
&= \max_{l,k} Y^{\frac{1}{\sigma}} P_t (1-\tau_{t,s,i}) E(z_{t,s,i}^{\frac{\sigma-1}{\sigma}}) (n_{t,s,i}^{1-\alpha_s} k^{\alpha_s})^{\frac{\sigma-1}{\sigma}} - w_t n_{t,s,i} - R_t k_{t,s,i}
\end{aligned}$$

where we have used the fact that $p_{t,s,i} = y_{t,s,i}^{-\frac{1}{\sigma}} Y_t^{\frac{1}{\sigma}} P_t$. The first order conditions of this problem are

$$Y_t^{\frac{1}{\sigma}} P_t (1-\tau_{t,s,i}) E(z_{t,s,i}^{\frac{\sigma-1}{\sigma}}) (n_{t,s,i}^{1-\alpha_s} k_{t,s,i}^{\alpha_s})^{\frac{\sigma-1}{\sigma}-1} \frac{\sigma-1}{\sigma} (1-\alpha_s) n_{t,s,i}^{-\alpha_s} k_{t,s,i}^{\alpha_s} = w_t \tag{33}$$

$$Y_t^{\frac{1}{\sigma}} P_t (1-\tau_{t,s,i}) E(z_{t,s,i}^{\frac{\sigma-1}{\sigma}}) (n_{t,s,i}^{1-\alpha_s} k_{t,s,i}^{\alpha_s})^{\frac{\sigma-1}{\sigma}-1} \frac{\sigma-1}{\sigma} \alpha_s n_{t,s,i}^{\alpha_s} k_{t,s,i}^{\alpha_s-1} = R_t. \tag{34}$$

Multiplying eq (33) with $z_{t,s,i}^{\frac{\sigma-1}{\sigma}} / z_{t,s,i}^{\frac{\sigma-1}{\sigma}}$ and $n_{t,s,i}$ we get

$$Y_t^{\frac{1}{\sigma}} P_t (1-\tau_{t,s,i}) \frac{E(z_{t,s,i}^{\frac{\sigma-1}{\sigma}})}{z_{t,s,i}^{\frac{\sigma-1}{\sigma}}} (z_{t,s,i} n_{t,s,i}^{1-\alpha_s} k_{t,s,i}^{\alpha_s})^{\frac{\sigma-1}{\sigma}} \frac{\sigma-1}{\sigma} (1-\alpha_s) = w_t n_{t,s,i}.$$

Using $p_{t,s,i} = y_{t,s,i}^{-\frac{1}{\sigma}} Y_t^{\frac{1}{\sigma}} P_t$ the previous equation reduces to

$$(1-\tau_{t,s,i}) = (1-\varphi_{t,s,i})^{-1} \frac{\sigma}{\sigma-1} \frac{w_t n_{t,s,i}}{(1-\alpha_s) p_{t,s,i} y_{t,s,i}} \tag{35}$$

where

$$(1-\varphi_{t,s,i}) \equiv \frac{E(z_{t,s,i}^{\frac{\sigma-1}{\sigma}})}{z_{t,s,i}^{\frac{\sigma-1}{\sigma}}}.$$

This gives us formula (4) in the main text.

To measure firms expectation error $(1-\varphi_{t,s,i})$, we write realized profits, $\pi_{t,s,i}$, after uncertainty has been revealed as

$$\pi_{t,s,i} = (1-\tau_{t,s,i})p_{t,s,i}y_{t,s,i}\left(1-\frac{\sigma-1}{\sigma}\frac{E(z_{t,s,i}^{\frac{\sigma-1}{\sigma}})}{z_{t,s,i}^{\frac{\sigma-1}{\sigma}}}\right).$$

Substituting the expression for $(1-\tau_{t,s,i})$ into the previous equation we get

$$\begin{aligned}\pi_{t,s,i} &= \frac{z_{t,s,i}^{\frac{\sigma-1}{\sigma}}}{E(z_{t,s,i}^{\frac{\sigma-1}{\sigma}})}\frac{\sigma}{\sigma-1}\frac{w_t n_{t,s,i}}{(1-\alpha_s)}\left(1-\frac{\sigma-1}{\sigma}\frac{E(z_{t,s,i}^{\frac{\sigma-1}{\sigma}})}{z_{t,s,i}^{\frac{\sigma-1}{\sigma}}}\right)\\ &= \frac{w_t n_{t,s,i}}{(1-\alpha_s)}\left(\frac{z_{t,s,i}^{\frac{\sigma-1}{\sigma}}}{E(z_{t,s,i}^{\frac{\sigma-1}{\sigma}})}\frac{\sigma}{\sigma-1}-1\right).\end{aligned}$$

Solving $(1-\varphi_{t,s,i})$ from the previous equation gives

$$(1-\varphi_{t,s,i}) \equiv \frac{E(z_{t,s,i}^{\frac{\sigma-1}{\sigma}})}{z_{t,s,i}^{\frac{\sigma-1}{\sigma}}} = \frac{\sigma}{\sigma-1}\left(\frac{w_t n_{t,s,i}}{w_t n_{t,s,i}+(1-\alpha_s)\pi_{t,s,i}}\right) \tag{36}$$

which is expression (5) in the main text.

Appendix B. Only Listed Companies

When compared to existing literature, our results differ quite significantly. Based on the earlier literature we would expect, for instance, much lower values for the ex-post misallocation. Another difference to earlier studies is that we find large significance for the uncertainty. One reason for the differences lies potentially in the sample that we consider as the sample contains much larger set of firms in comparison to earlier studies. However, it might be also the case that Finland is just significantly different from other countries. To address the potential caveat we separately analyze the subset of listed firms.

We repeat the calculation of our main table with only firms that were listed even one year between 2000-2012 in Finland[17]. The results in table 10 show that listed firms are much less misallocated and that the ex-ante misallocation is more important than the uncertainty for the listed firms. Splitting the sample to listed and unlisted firms reveals the potential bias that might occur if we would consider only the listed firms. The

[17] We consider the whole time period and 1995-2012 for those firms.

Table 10: The variance of HK style revenue wedge and its decomposition to uncertainty and residual wedge for listed firms. Percentages do not necessarily sum up to one because of the winsorization.

Variable	Value	Share of Var($\ln(1-\tau_{HK})$)
Var($\ln(1-\tau_{HK})$)	0.130	1.00
Var($\ln(1-\tau)$)	0.168	1.29
Var($\ln(1-\varphi)$)	0.142	1.09
2Cov($\ln(1-\tau), \ln(1-\varphi)$)	-0.168	-1.29

obtained result also gives support to our claim that large share of the differences with our results in comparison to earlier studies stem from the differences in the firm types rather than differences in the considered country.

Appendix C. A Static Measure of Uncertainty, Ex Ante Misallocation and Markups

This one extends our basic static model to also include heterogeneity in markups. Firm's problem is

$$\begin{aligned}
&\max_{n,k}\left\{E\left[\max_{m}(1-\tau_{t,s,i})p_{t,s,i}y_{t,s,i}-(1-\tau_{t,s,i})p_t^m m_{t,s,i}\right]-w_t n_{t,s,i}-R_t K_{t,s,i}\right\}\\
&p_{t,s,i}=\Lambda z_{t,s,i}y_{t,s,i}^{-\frac{1}{\sigma_{t,s,i}}}\\
&y_{t,s,i}=k_{t,s,i}^{\alpha_s}n^{\xi_s-\alpha_s}m^{1-\xi_s}
\end{aligned}$$

At the second stage the first order condition with respect to intermediate inputs is given by

$$\Lambda z_i k^{\alpha_1}n^{\alpha_2}m^{\alpha_3}(1-\xi_s)(1-\frac{1}{\sigma_{t,s,i}})=p_t^m m_{t,s,i}, \tag{37}$$

where $\alpha_1=\alpha_s(1-\frac{1}{\sigma_{t,s,i}})$, $\alpha_2=(\xi_s-\alpha_s)(1-\frac{1}{\sigma_{t,s,i}})$ and $\alpha_3=(1-\xi_s)(1-\frac{1}{\sigma_{t,s,i}})$. From the previous equation we can solve the firm-specific markups

$$\frac{\sigma_{t,s,i}}{\sigma_{t,s,i}-1}=(1-\xi_s)\frac{p_{t,s,i}y_{t,s,i}}{p_t^m m_{t,s,i}} \tag{38}$$

Moreover, from the first order condition we get

$$m_{t,s,i} = \left[\frac{\alpha_3}{p_t^m} z_{t,s,i} \Lambda k_{t,s,i}^{\alpha_1} n_{t,s,i}^{\alpha_2}\right]^{\frac{1}{1-\alpha_3}}$$

The first stage problem can now be written as

$$\begin{aligned}
&\max_{n,k}[(1-\tau_{t,s,i})Ep_{t,s,i}y_{t,s,i} - (1-\tau_{t,s,i})\alpha_3 Ep_{t,s,i}y_{t,s,i}] - w_t n_{t,s,i} - R_t k_{t,s,i} \\
&= \max_{n,k}(1-\tau_{t,s,i})(1-\alpha_3)E\left\{\Lambda z_{t,s,i} k_{t,s,i}^{\alpha_1} n_{t,s,i}^{\alpha_2}\left[\frac{\alpha_3}{p_t^m} z_{t,s,i}\Lambda k_{t,s,i}^{\alpha_1} n_{t,s,i}^{\alpha_2}\right]^{\frac{\alpha_3}{1-\alpha_3}}\right\} - w_t n_{t,s,i} - R_t k_{t,s,i} \\
&= \max_{n,k}(1-\tau_{t,s,i})(1-\alpha_3)\Lambda^{\frac{1}{1-\alpha_3}} E z_{t,s,i}^{\frac{1}{1-\alpha_3}} \left(\frac{\alpha_3}{p_t^m}\right)^{\frac{\alpha_3}{1-\alpha_3}} k_{t,s,i}^{\frac{\alpha_1}{1-\alpha_3}} n_{t,s,i}^{\frac{\alpha_2}{1-\alpha_3}} - w_t n_{t,s,i} - R_t k_{t,s,i}
\end{aligned}$$

Let's use following notation: $\tilde{a}_i = \frac{\alpha_i}{1-\alpha_3}$. Now the first order condition with respect to n is

$$\begin{aligned}
\tilde{\alpha_2}(1-\tau_{t,s,i})(1-\alpha_3)\Lambda^{\frac{1}{1-\alpha_3}} E z_{t,s,i}^{\frac{1}{1-\alpha_3}} \left(\frac{\alpha_3}{p_t^m}\right)^{\tilde{\alpha_3}} k_{t,s,i}^{\tilde{\alpha_1}} n_{t,s,i}^{\tilde{\alpha_2}} &= w_t n_{t,s,i} \\
\tilde{\alpha_2}(1-\tau_{t,s,i})Ep_{t,s,\bar{i}}y_{t,s,i} &= w_t n_{t,s,i} \\
\tilde{\alpha_2}(1-\tau_{t,s,i})\frac{E z_{t,s,i}^{\frac{1}{1-\alpha_3}}}{z_{t,s,i}^{\frac{1}{1-\alpha_3}}} p_{t,s,\bar{i}}y_{t,s,i} &= w_t n_{t,s,i} \qquad (39) \\
(1-\tau_{t,s,i}) = \frac{1}{\tilde{\alpha_2}}\frac{w_t n_{t,s,i}}{p_{t,s,\bar{i}}y_{t,s,i}}(1-\varphi)^{-1} & \qquad (40)
\end{aligned}$$

where $p_{t,s,\bar{i}}y_{t,s,i} \equiv (1-\alpha_3)p_{t,s,i}y_{t,s,i}$ and $1-\varphi_{t,s,i} \equiv \frac{E z_{t,s,i}^{\frac{1}{1-\alpha_3}}}{z_{t,s,i}^{\frac{1}{1-\alpha_3}}}$. Note that $p_{t,s,\bar{i}}y_{t,s,i}$ is value added for firm i.

The first order condition with respect to capital is given by

$$\tilde{\alpha_1}(1-\tau_{t,s,i})(1-\alpha_3)\Lambda^{\frac{1}{1-\alpha_3}} E z_{t,s,i}^{\frac{1}{1-\alpha_3}} \left(\frac{\alpha_3}{p_t^m}\right)^{\tilde{\alpha_3}} k_{t,s,i}^{\tilde{\alpha_1}} n_{t,s,i}^{\tilde{\alpha_2}} = R_t k_{t,s,i} \qquad (41)$$

As in Appendix A, we can write realized profits with the help of optimal policies:

$$\pi_{t,s,i} = (1-\tau_{t,s,i})(1-\alpha_3)p_{t,s,i}y_{t,s,i} - (\tilde{\alpha_1}+\tilde{\alpha_2})\frac{Ez_{t,s,i}^{\frac{1}{1-\alpha_3}}}{z_{t,s,i}^{\frac{1}{1-\alpha_3}}}(1-\tau_{t,s,i})(1-\alpha_3)p_{t,s,i}y_{t,s,i}$$
$$\pi_{t,s,i} = \frac{w_t n_{t,s,i}}{\tilde{\alpha_2}}[(1-\varphi_{t,s,i})^{-1} - (\tilde{\alpha_1}+\tilde{\alpha_2})]$$

where we have used expression (40) in the last equation. Rearranging this gives

$$(1-\varphi_{t,s,i}) = \frac{w_t n_{t,s,i}}{[\pi_{t,s,i} + w_t n_{t,s,i}(1+\frac{\tilde{\alpha_1}}{\alpha_2}]\tilde{\alpha_2}}$$
$$(1-\varphi_{t,s,i}) = \frac{w_t n_{t,s,i}(1-\alpha_3)}{\alpha_2\pi_{t,s,i} + (\alpha_1+\alpha_2)w_t n_{t,s,i}} \tag{42}$$
$$(1-\varphi_{t,s,i}) = \frac{w_t n_{t,s,i}(\frac{\sigma_{t,s,i}}{\sigma_{t,s,i}-1} - (1-\xi_s))}{(\xi_s-\alpha_s)\pi_{t,s,i} + \xi_s w_t n_{t,s,i}} \tag{43}$$

This is our measure of prediction error with heterogeneous markups in the main text.

To get a measure for ex ante misallocation, plug equation (42) into (40)

$$(1-\tau_{t,s,i}) = \frac{1}{\alpha_2}\frac{\alpha_2\pi_{t,s,i} + (\alpha_1+\alpha_2)w_t n_{t,s,i}}{p_{t,s,\bar{i}}y_{t,s,i}}$$
$$= \frac{w_t n_{t,s,i} + \frac{\xi_s-\alpha_s}{\xi_S}\pi_{t,s,i}}{\frac{\xi_s-\alpha_s}{\xi_s}p_{t,s,\bar{i}}y_{t,s,i}} \tag{44}$$

This is equal to our measure of $1-\tau_{t,s,i}$ with homogeneous markups (see Appendix A).

Finally to link $(1-\tau_{t,s,i})$, $(1-\varphi_{t,s,i})$ and markups to $1-\tau^{HK}_{t,s,i}$, start with (40)

$$(1-\tau_{t,s,i}) = \frac{1-(1-\xi_s)(1-\frac{1}{\sigma_{t,s,i}})}{\xi_s-\alpha_s}\frac{\sigma_{t,s,i}}{\sigma_{t,s,i}-1}\frac{w_t n_{t,s,i}}{p_{t,s,\bar{i}}y_{t,s,i}}(1-\varphi_{t,s,i})^{-1}$$
$$= \frac{\sigma}{\sigma-1}\frac{1}{\frac{\xi_s-\alpha_s}{\xi_s}}\frac{w_t n_{t,s,i}}{p_{t,s,\bar{i}}y_{t,s,i}}\frac{\sigma-1}{\sigma}\frac{1}{\xi_s}(\frac{\sigma_{t,s,i}}{\sigma_{t,s,i}-1} - (1-\xi_s))(1-\varphi_{t,s,i})^{-1}$$
$$(1-\tau_{t,s,i}) = (1-\tau^{HK}_{t,s,i})(1-\varphi_{t,s,i})^{-1}(\frac{\sigma_{t,s,i}}{\sigma_{t,s,i}-1} - (1-\xi_s))\frac{\sigma-1}{\sigma}\frac{1}{\xi_s}$$
$$(1-\tau^{HK}_{t,s,i}) = (1-\tau_{t,s,i})(1-\varphi_{t,s,i})(\frac{\frac{\sigma_{t,s,i}}{\sigma_{t,s,i}-1} - (1-\xi_s)}{\xi_s})^{-1}\frac{\sigma}{\sigma-1} \tag{45}$$

Taking logs and evaluating variance of the ex post post misallocation a la HK gives

$$\begin{aligned}Var(\ln{(1-\tau^{HK}_{t,s,i})}) =&Var(\ln{(1-\tau_{t,s,i})}) + Var(\ln{(1-\varphi_{t,s,i})})\\ &+ Var(\ln\frac{\frac{\sigma_{t,s,i}}{\sigma_{t,s,i-1}}-(1-\xi_s)}{\xi_s}) + 2Cov(\ln{(1-\tau_{t,s,i})},\ln{(1-\varphi_{t,s,i})})\\ &- 2Cov(\ln{(1-\tau_{t,s,i})}),\ln{(\frac{\frac{\sigma_{t,s,i}}{\sigma_{t,s,i-1}}-(1-\xi_s)}{\xi_s}}))\\ &- 2Cov(\ln{(1-\varphi_{t,s,i})}),\ln{(\frac{\frac{\sigma_{t,s,i}}{\sigma_{t,s,i-1}}-(1-\xi_s)}{\xi_s}})))\end{aligned} \tag{46}$$

Appendix D. Calibration with Heterogeneous Fixed Costs

Table 11: Targets and model counterparts $\varepsilon = 2$

Moment	Data	Model
Std of profit residual	1.23	1.11
Firm Growth		
Mean growth rate, young firms	0.10	0.09
Std of growth rates, young firms	0.46	0.46
Std of growth rates, old firms	0.28	0.27
Size distribution, old firms		
Firms with employment < 5	0.67	0.70
Firms with employment ≥ 5 and < 10	0.16	0.14
Firms with employment ≥ 10 and < 20	0.09	0.08
Firms with employment ≥ 20 and < 50	0.05	0.06
Firms with employment > 50	0.02	0.02
Selection		
Exit rate, young firms	0.09	0.08
Entry rate	0.09	0.10

Notes: Firms are called young (old) when they are under (over) 5 years old.

Table 12: Parameter values $\varepsilon = 2$

Parameter	Value
c_e	11.55
c_f	0.34
σ_z	0.70
σ_ϵ	0.57
σ_{τ_p}	0.41
σ_{τ_t}	0.12
λ	0.09
γ	0.06

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