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Abstract. We explore Einstein-Podolsky-Rosen steering, measured by steering robustness, in the ground states of serval typical models that exhibit a quantum phase transition. For the anisotropic XY model, steering robustness approaches to zero around the critical point and vanishes in the ferromagnetic phase despite the fact that there exist other quantum nonlocality, e.g. quantum entanglement. For the Heisenberg XXZ model, steering robustness exhibits some similar behavior as entanglement around the infinite order quantum phase transition point $\Delta = 1$, e.g. reaching its maximum. As a further example, we also consider steering robustness in the Lipkin-Meshkov-Glick collective spin model. It is then shown that steering robustness disappears at the transition point and remains to zero in the fully polarized symmetric phase, just like the behavior of entanglement and Bell nonlocality.

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1. Introduction

At absolute zero temperature, the ground state properties of a many-body system may change dramatically and qualitatively owing to the pure quantum fluctuation. This phenomenon, known as quantum phase transition (QPT), which comes from the interplay between the different orders associated with competing interactions in the system[1]. This topic attracts much attention in many branches of physics. Traditionally, a QPT is described in the framework of Landau-Ginzburg paradigm that the transition from one phase to another is usually accompanied by symmetry breaking. In recent years, many works paid attention to this problem from the quantum-information perspective[2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21](e.g. fidelity[16, 17],quantum echo[18], quantum quench dynamics[20]). Specially, quantum systems possess genius nonlocal correlations, which are fundamental to varying applications of quantum information theory and technology. Since correlations among the subsystems of many-body systems are closely related to the emergence of the QPT, it is natural to investigate the link between this phenomena and nonlocal correlations[2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. Particularly, nonlocal correlations measures such as entanglement and Bell nonlocality have been employed to investigate QPT in numerous quantum spin systems. For instance, pairwise entanglement close to the QPT of one-dimensional Ising model[2, 3] and entanglement entropy approach to the quantum critical phenomena in *XY* and *XXZ* spin chains were addressed[4, 5, 7]. And Bell nonlocality measured by Clauser-Horne-Shimony-Holt (CHSH) function has also been introduced to examine the QPT in several spin systems[6, 8, 9]. All these results seemly state that the quantum nonlocality as a nontrivial tool to characterize the QPT of a quantum system does function.

Recently, Einstein-Podolsky-Rosen (EPR) steering has been attracted much interest both theoretically and experimentally[22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41]. Apart from the fundamentality of steering in quantum theory itself[22, 23, 24, 25, 26, 27, 28], there are also quite a few of application motivations in the quantum information technology (e.g. quantum key distribution[38] and secure teleportation[39]). Now we know that EPR steering is an intermediate type of quantum nonlocality between entanglement and Bell nonlocality[22]. Contrary to entanglement and Bell nonlocality, quantum steering possesses a fundamental asymmetry due to the facts that the two observers hold different positions in the steering test[29, 30, 32]. Since both entanglement and Bell nonlocality play an important role in the QPT, so it's an interesting issue to investigate the behavior of EPR steering in critical systems. In this work, we will study the behavior of steering robustness around the critical points in three kinds of spin systems. By calculating the steerable robustness, we find that the bipartite reduced state for the Ising chain is unsteerable in the ferromagnetic phase, and steering robustness approaches to zero around the critical point. For the *XXZ* model, steering robustness reaches its maximum at the critical point, and for the Lipkin-Meshkov-Glick (LMG) collective spin ones, steering robustness disappears at the critical point and remains to be zero in the symmetric polarized phase.

The remainder of this work is organized as follows. In Section 2, we introduce EPR steering and steering robustness. In Sections 3, we study the steering robustness in several typical critical systems. Finally, we give a summary of our main results.

2. Einstein-Podolsky-Rosen steering and Robustness

For a bipartite state ρ_{AB} shared by Alice and Bob, quantum steering commonly refers to the ability of one observer (e.g. Alice) can nonlocally change (i.e. steer) the remote other observer's (e.g. Bob) state through local measurements. In 2007, Wiseman et al. gave quantum steering a rigorous definition in an operational way from a quantum information perspective[22, 23]. And their scenario is proposed as follows: Alice performs a series of measurements described by positive operator values measures (POVMs) $\{M_{a|x}\}_{a,x}$ (i.e. $M_{a|x} \geq 0$ and $\sum_a M_{a|x} = I$ for all a, x. Here, x and a denote the measurement setting and its corresponding outcome, respectively) on her subsystem, then Bob is left with a non-normalized state assemblage

$$\sigma_{a|x} := tr_A[(M_{a|x} \otimes I)\rho_{AB}], \quad (1)$$

where I is the identity operator on Bob's subsystem. The quantum assemblages must satisfy the no-signaling requirement

$$\sum_a \sigma_{a|x} = \sum_a \sigma_{a|x'} \quad \forall x, x^{'}, \quad (2)$$

and the normalization condition

$$tr \sum_a \sigma_{a|x} = 1 \quad \forall x. \quad (3)$$

If all the above unnormalized states in the assemblage $\{\sigma_{a|x}\}_{a,x}$ can be formulated in the form

$$\sigma_{a|x} = \sum_{\xi} D_{\xi}(a|x)\sigma_{\xi} \quad \forall a, x \tag{4}$$

with

$$tr \sum_{\xi} \sigma_{\xi} = 1, \quad \sigma_{\xi} \geq 0 \ \ \forall \xi, \tag{5}$$

where ξ is a classical random variable held by Alice, $D_{\xi}(a|x)$ are deterministic single-party conditional probability distributions for Alice, then we call such an assemblage unsteerable (local hidden state (LHS) model) and the set of unsteerable assemblage is denoted by $\{\sigma^{US}_{a|x}\}_{a,x}$. Otherwise, any assemblage that can not be written in the form of above expression is called steerable, and the set of steerable assemblage is denoted by $\{\sigma^{S}_{a|x}\}_{a,x}$. With semidefinite program (SDP)[25], it is possible to test whether a given assemblage belongs to the set of unsteerable assemblage or belongs to the set of steerable assemblage.

Another interesting question considered in the previous literatures is how to quantify EPR steering, and there are many scenarios were proposed recently. For instance, Skrzypczyk et al. have proposed a quantity called steering weight to measure the steerability of quantum assemblage[24] by separating an assemblage into a steerable part $\{\sigma^{S}_{a|x}\}_{a,x}$ and an unsteerable one $\{\sigma^{US}_{a|x}\}_{a,x}$ ($\sigma_{a|x} = \mu\sigma^{US}_{a|x} + (1-\mu)\sigma^{S}_{a|x}, \quad \forall\ a, x, \quad 0 \leq \mu \leq 1$). And Chen et al, extended this scenario to its temporal analogue[28]. Analogously to the robustness of entanglement, Piani et al. proposed another alternative approach for the quantification of steering, called steering robustness $\mathcal{R}(\mathcal{A})$ by asking how much noise one has to add to a given assemblage $\mathcal{A} = \{\sigma_{a|x}\}_{a,x}$ in order for it to have an LHS model[26]. Generally, a $\mathcal{N}$-robustness of an assemblage $\{\sigma_{a|x}\}_{a,x}$ can be defined as[25, 26],

$$\begin{aligned} \mathcal{R}(\sigma_{a|x}) = \min_{\sigma_{a|x},\sigma_{\lambda},t} & \ t \quad s.t. \\ & \frac{\sigma_{a|x} + t\tau_{a|x}}{1+t} = \sigma^{LHS}_{a|x} \quad \forall a, x \\ & \sigma^{LHS}_{a|x} = \sum_{\lambda} D(a|x,\lambda)\sigma_{\lambda} \ \ \forall a, x \\ & \tau_{a|x} \in \mathcal{N}, \sigma_{\lambda} \geq 0 \quad \forall \lambda. \end{aligned} \tag{6}$$

Here, $\mathcal{N}$ is any subset of assemblages characterised by positive semi-definite constraints and linear matrix inequalities, which will determine the specific type of noise and the corresponding robustness quantifier. For instance, the choices of the noise $\mathcal{N}$ can be consider the set of LHS assemblages ($\mathcal{N} = \{\tau_{a|x} | \tau_{a|x} = \sum_{\lambda} D(a|x,\lambda)\sigma_{\lambda} \ \ \forall a, x, \sigma_{\lambda} \geq 0 \quad \forall \lambda, tr \sum_{\lambda} \sigma_{\lambda} = 1\}$), corresponding to the LHS-robustness[27]; or $\mathcal{N}$ can be composed by a single assemblage (e.g, the maximally mixed assemblage, $\mathcal{N} = \{I/(d_B o_A) \quad \forall a, x\}$. Here, o_A and d_B denote the number of outputs of Alice's measurements and the dimension of Bob's Hilbert space, respectively), corresponding to the random steering robustness. Also, we can take a particularly case that the set $\mathcal{N}$ corresponds to the set of all valid assemblages (i.e., $\mathcal{N} = \{\tau_{a|x} | \sum_a \tau_{a|x} = \sum_a \tau_{a|x'} \ \ \forall x, x', tr \sum_a \tau_{a|0} = 1\}$), in which case the quantifier was named the steering robustness $\mathcal{R}$ simply[25, 26]. The steering robustness of $\mathcal{A}$, which is nonzero if and only if $\mathcal{A}$ is steerable, is a measure of the minimal noise needed to destroy the steerability of the assemblage $\mathcal{A}$.

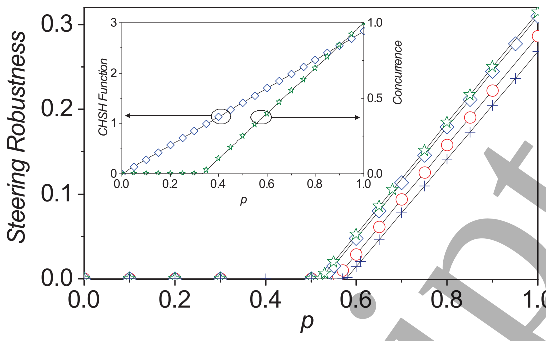

Figure 1. (Color online) Main panel: The biggest steering robustness among 500 randomly generated assemblages with respect to the parameters p for the two-qubit Werner state $\rho_{AB} = p|\phi\rangle\langle\phi| + (1-p)I \otimes I/4$. Results for different number of measurements in each assemblage are displayed. k = 3(cross), 4(circle), 6(diamond) and 8(pentagram). Insets: Entanglement and Bell nonlocality vanish at the points 1/3 and 0.707, respectively. Hereafter, we take the number of measurements $k = 8$ and sample 500 random generated assemblages without additional remarks.

First, we take the two-qubit Werner state $\rho_{AB} = p|\phi\rangle\langle\phi| + (1-p)I \otimes I/4$ ($|\phi\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$ is the singlet state) as an example to investigate the steering robustness. By taking a measurement of the three Pauli operators X, Y, and Z, we find that the steering robustness decreases monotonically with decreasing p and further the state becomes unsteerable exactly when $p = 1/\sqrt{3}$. The results are showed in figure 1. Now, we choose a given number k of random measurements on the Bloch sphere. For $k = 4, 6$ and 8, we sampled over 500 random generated assemblages for various values of p. We can find that steering robustness increases as k increasing. And we can also see that the points where steering robustness vanishes depend on the choice of number of measurement k and would approach to 1/2 with an increasing k. So the choice of number of measurement k and sample will be very important in the following research. As a comparison, we also plot some other quantum nonlocality in figure 1, which show that entanglement vanishes at the point $p = 1/3$ and Bell nonlocality (CHSH functions less then 2) disappears around the point $p \simeq 0.707$.

3. Results

In the following, we investigate the behavior of steering robustness in the three typical spin systems. Since we take steering robustness to describe the quantum nonlocality, it is necessary to derive the pairwise reduced density matrix. For both the XY model and XXZ one, the pairwise reduced

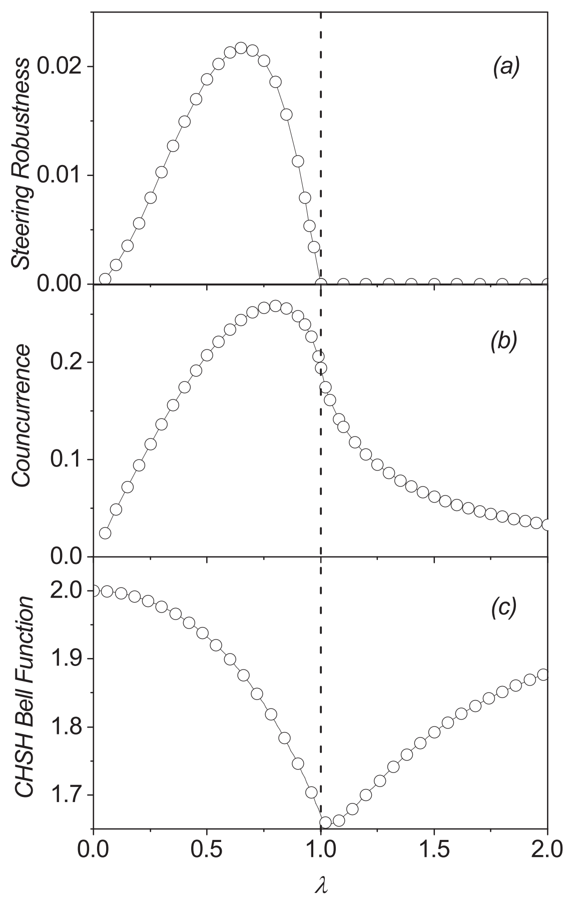

Figure 2. (a)Steering Robustness with respect to the parameters λ for the nearest-neighbor reduced density state in the Ising model. Steering Robustness approaches to zero around the critical points $\lambda = 1$ and remains to be zero in the ferromagnetic phase. (b) Entanglement and (c) CHSH function for the nearest-neighbor reduced density state in the Ising model.

density matrix in the basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ can be given by[2, 5, 15]

$$\rho_{ij} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix} \tag{7}$$

disregarding spontaneous symmetry breaking effects. Here, all the elements of the matrix ρ_{ij} can be expressed by the spin-spin correlation functions, seeing the Appendix.

3.1. One-dimensional anisotropic XY model

Now we consider the anisotropy XY model with an external field. The Hamiltonian is written as follows,

$$H = -\sum_{i}^{L}\left[\lambda\left(\frac{1+\gamma}{2}\sigma_i^x\sigma_{i+1}^x + \frac{1-\gamma}{2}\sigma_i^y\sigma_{i+1}^y\right) + \sigma_i^z\right]. \tag{8}$$

Here, λ denotes the relative strength of the nearest-neighbor coupling with respect to the external field, and taking $\lambda \geq 0$ without losing generality. γ is an anisotropy parameter which can be varied in the region [0, 1], and can separate the Ising model($\gamma = 1$) from the isotropic XX model($\gamma = 0$). In the limit $\lambda \to \infty$, the ground state is a product of spins pointing in the positive x direction, $|0^+\rangle_{\lambda\to\infty} \approx \cdots|\rightarrow\rangle_i|\rightarrow\rangle_{i+1}\cdots$ or negative ones $|0^-\rangle_{\lambda\to\infty} \approx \cdots|\leftarrow\rangle_i|\leftarrow\rangle_{i+1}\cdots$ with the global phase flip, so the ground state is two-fold degenerate and is an equal mixture of these two states(i.e. a thermal ground state $\rho = \frac{1}{2}|0^+\rangle_{\lambda\to\infty}\langle 0^+| + \frac{1}{2}|0^-\rangle_{\lambda\to\infty}\langle 0^-|$, which is a mixed state that does not contain any type of quantum correlations between any parties). While in the limit $\lambda \to 0$, the transverse Ising model ground state becomes a product of spins pointing in the positive z direction, $|0\rangle_{\lambda\to 0} \approx \cdots|\uparrow\rangle_i|\uparrow\rangle_{i+1}\cdots$, which is also a product state that there does not exit any type of correlations between any parties[3]. Around the quantum phase transition point, the system's ground state changes from degenerate to non-degenerate.

In figure 2(a), we display steering robustness for nearest-neighbor spins in the Ising chain as a function of λ at zero temperature. The results demonstrate that there is a remarkable difference for the steering robustness between the regions $0 < \lambda < 1$ and $\lambda > 1$. In the paramagnetic phase ($\lambda < 1$), steering robustness increases first and reaches a maximum then decreases with increasing λ. Around the point $\lambda = 1$, it disappears and remains to be zero in the whole region of ferromagnetic phase ($\lambda > 1$), which is very different to the entanglement. Here, it should be mentioned that Zhang et al. shown that the bipartite state would be unsteerable around the point $\lambda = 0.8$[37]. The difference mainly come from the special three measurements in their calculation. As a comparison, in figure 2(b) and (c), we display the behavior of entanglement and Bell nonlocality. We know that the first-order derivative of entanglement is divergent[2] and CHSH function takes its local minimum[8] at the critical point $\lambda = 1$, which mean that these three kinds of quantum nonlocality exhibit different behavior around the critical point. Also, we can find that there does not exist Bell nonlocality both in ferromagnetic phase and paramagnetic one (CHSH function is less then 2) even there are some quantum states which are highly entangled. All these results state that there exits a strict hierarchy among EPR steering, entanglement and Bell nonlocality in different phases for this model. In facts, Bell nonlocality implies nonclassical correlations that can not be described by local hidden variable theory (i.e. Alice's and Bob's joint probability satisfies $P(a,b|A,B,\rho_{AB}) = \sum_\xi P(a|A,\xi)P(b|B,\xi)P_\xi$ for any Alice's measurements A with output a and Bob's measurements B with output b) for a given quantum state[23, 35]; EPR steering describes correlations that can not be formulated by local hidden state theory (i.e. Alice's and Bob's joint probability and the marginal probability satisfy $P(a,b|A,B,\rho_{AB}) = \sum_\xi P(a|A,\xi)P_Q(b|B,\xi)P_\xi$ with $P_Q(b|B,\xi) = Tr[\prod_b^B \rho_\xi^B]$ for any A and B) and entanglement is one whose joint probability cannot be expressed by any

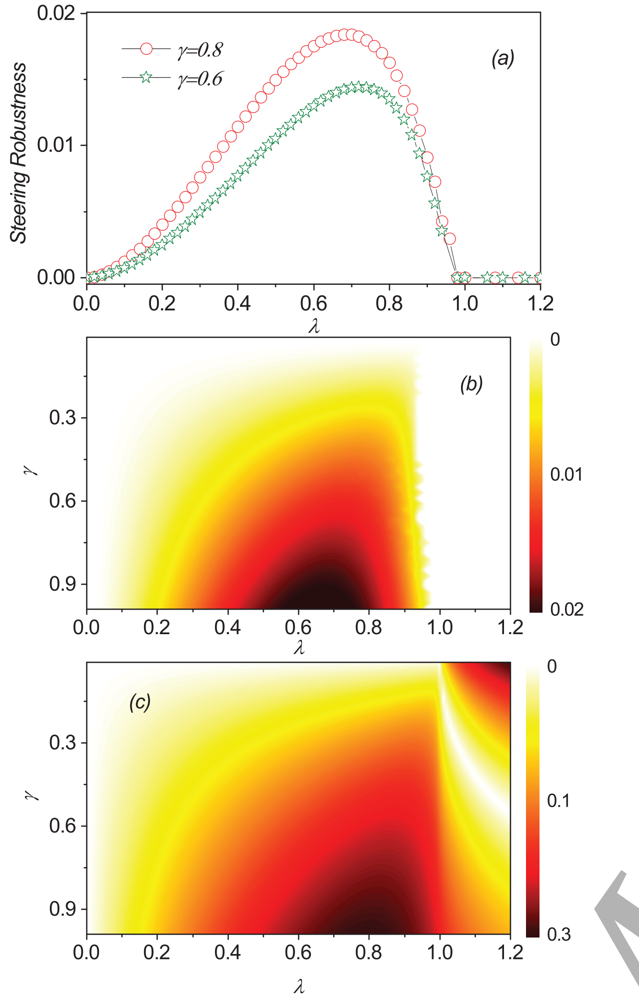

Figure 3. (Color online) Steering Robustness with respect to the parameters λ for the nearest-neighbor reduced density state in the anisotropic XY model for different γ. (b) Steering Robustness with respect to the parameters λ and γ in the whole region. (c) Entanglement with respect to the parameters λ and γ in the whole region. There exist some regions where the quantum states are highly entangled and steering robustness is absent in the ferromagnetic phase.

separable model(i.e. Alice's and Bob's joint probability and the marginal probability satisfy $P(a,b|A,B,\rho_{AB}) = \sum_\xi P_Q(a|A,\xi)P_Q(b|B,\xi)P_\xi$ with $P_Q(a|A,\xi) = Tr[\prod_a^A \rho_\xi^A]$ and $P_Q(b|B,\xi) = Tr[\prod_b^B \rho_\xi^B]$ for any A and B). Obviously, the condition for no steering can imply the condition for Bell locality (By setting $P(b|B,\xi) = P_Q(b|B,\xi)$) and the condition for separability can also imply the condition for no steering (By setting $P(a|A,\xi) = P_Q(a|A,\xi)$). Thus steerability is strictly stronger than nonseparability, and strictly weaker than Bell nonlocality[23]. For the nearest-neighbor reduced state $\rho_{i,i+1}$ in the Ising model, entanglement can exist both in the ferromagnetic phase and paramagnetic ones, while EPR steering only can exist in the paramagnetic phase. As far as the Bell nonlocality is concerned, there does not exist such kind of quantum correlation in the whole region.

As is well known that the quantum XY chain belongs to the same quantum Ising universality class for non-zero γ. To confirm the universality, we need to check the behaviors of steering robustness for different values of γ. Comparing with the results in figure 2(a), we choose to plot steering robustness as function of λ with $\gamma = 0.6$ and $\gamma = 0.8$ in figure 3(a). Two curves of steering robustness with respect to λ show some similar trend as $\gamma = 1.0$ qualitatively, which mean that one can distinguish the ferromagnetic phase from the paramagnetic ones by calculation of steering robustness for the whole XY family. Figure 3(b) display the results for whole region $0 < \gamma \leq 1$. It clearly show that the steering robustness approaches to zero when λ cross the critical point 1. Therefore, we can reasonably state that the quantum critical behavior can be characterized by the steering robustness in this special model. As a comparison, we also plot the entanglement in the region $\lambda \in [0, 1.2]$ and $\gamma \in (0, 1]$ in figure 3(c). Obviously, there exist some regions where the quantum states are highly entangled but steering robustness are absent in the ferromagnetic phase, which imply that whether a quantum state is steerable depend on its structure, which is related to the phases of system.

3.2. One-dimensional Heisenberg spin-1/2 XXZ model

The spin-1/2 XXZ chain is one of important and fundamental models in the study of QPT, the Hamiltonian is given by

$$H = \sum_i (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \Delta \sigma_i^z \sigma_{i+1}^z), \qquad (9)$$

where $\sigma_i^\alpha, \alpha = x, y, z$ are the Pauli matrices acting on the ith site and Δ is the anisotropy parameter. The model has two critical points that separate three different phases[4, 5, 6, 7, 8]: the first critical point is $\Delta_c = -1$, a first-order phase transition point that separates the ferromagnetic phase for $\Delta < -1$ from the gapless one for $-1 < \Delta < 1$; the second one is $\Delta_c = 1$, an infinite-order phase transition one that separates the gapless phase from the anti-ferromagnetic one for $\Delta > 1$.

Here, we first review some previous results of entanglement and Bell nonlocality in this model. Around the first-order phase transition point $\Delta = -1$, entanglement of the nearest neighbors suddenly appears when the system enters the gapless phase[6, 8], and then achieves its maximum at the infinite-order QPT point $\Delta = 1$[5, 6, 7, 8], and slowly decreases in antiferromagnetic phase ($\Delta > 1$). The first derivative of entanglement diverges at the first-order phase transition point $\Delta = -1$. Justino et al have shown that the first-derivative of CHSH function diverges both at the first-order phase transition point and the infinite-order one[6].

In figure 4(a),(b) and (c), we depict the dependence of the entanglement, steering robustness and CHSH functions on the control parameter Δ. Obviously, both entanglement

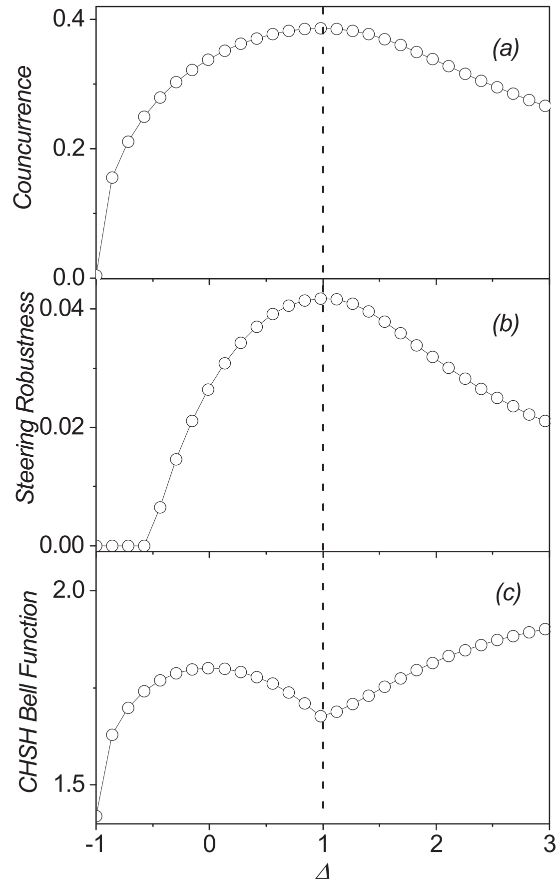

Figure 4. (a) Entanglement and (c) CHSH function for the nearest-neighbor reduced density state in the Heisenberg spin-1/2 *XXZ* model. Entanglement reached to its maximum and CHSH function obtain its minimal locally around the critical point. (b)Steering Robustness with respect to the parameters Δ for the nearest-neighbor reduced density state. Steering Robustness takes its maximal value at the critical point Δ = 1.

and CHSH function exhibit the same results as the previous work[5, 6, 7, 8]. Here it should be mentioned that although CHSH function can signal the critical points $\Delta = -1$ and $\Delta = 1$, there does not exit Bell nonlocality in this model as well as *XY* one. Oliveira et al. stated that nonviolation of Bell's inequality in translation invariant systems should trace back to the monogamy trade-off obeyed by bipartite Bell correlations[11]. Surprisedly, steering robustness exhibits some similar behavior as entanglement around the infinite order phase transition point $\Delta = 1$, i.e. increasing monotonically with respect to Δ in the region $\Delta < 1$ and decreasing monotonically in the region $\Delta > 1$, and reaching its maximum at the critical point. Around the first-order transition point, we observe that steering robustness does not emerge at the point $\Delta = -1.0$ exactly where a first-order phase transition happen. This phenomenon may come from the facts that the calculating process for the steering robustness depend on the number of measure k heavily. The point that the steering robustness emerge would approach to the critical point as we increasing k. However, a large k will cost too much CPU time.

3.3. LMG model

Finally, we analyze steering robustness in the ground state of the LMG model. In its spin 1/2 representation, LMG model can be regarded as a chain of N spin particles with infinite-ranged interactions, and each spin is subject to an external transverse magnetic field λ. The Hamiltonian can be written as

$$H_{LMG} = -\frac{1}{N}(S_x^2 - S_y^2) + \lambda S_z. \tag{10}$$

The model has a second-order quantum phase transition at λ_c between a symmetric phase ($\lambda > 1$) and a broken phase ($\lambda < 1$). Alternatively, this model can also be seen as a two-level Fermi system $\{|+\rangle, |-\rangle\}$ with each level having degeneracy Ω. And the ground state of the LMG model for $N \to \infty$ can be obtained by using the Hartree-Fock (HF) approach. In this work, we study the pairwise ground state density matrix for general modes ($+m$) and ($-n$) in the HF ground state of the LMG model by taking the scenario in the work[15].

In the previous study, some typical quantum correlations (e.g. entanglement, quantum discord) have been investigated carefully[8, 14, 15]. And the results state that all these quantum correlations measures are equal for the density matrix of the ground state for the ($+m$,$-m$) modes due to the fact that those modes of the ground state is pure[15]. In figure 5 (a), (b) and (c), we display the calculated quantum nonlocality (entanglement, steering robustness and Bell nonlocality) with respect to λ between the modes ($+m$) and ($-m$) in the HF ground state of the such model. All these quantum nonlocality do exist in the broken phase which process a twofold degenerate ground state for $\lambda < 1$. And we also can find that all these quantum nonlocality approach to zero around the critical point and disappear in the symmetric phase which process fully polarized in the direction of the field for $\lambda > 1$. And all the results state that three kinds of quantum correlations exhibit a similar behavior qualitatively even under a different phase for the pure state in this model. Here it should be mentioned that the results would be different if the pairwise density matrix in the LMG model is evaluated directly for qubits (i.e., spins)[14].

4. summary

In summary, we have investigated the quantum steering in several typical spin systems including one-dimensional *XY* model, *XXZ* model and LMG collective spin one by calculating steering robustness. The results show that the behavior of steering robustness are model depend in different phases. As a comparison, we also calculated two other typical quantum nonlocality (e.g. entanglement and Bell nonlocality). The results imply that there exit a strict hierarchy among EPR steering, entanglement and Bell nonlocality in different quantum phases for these models.

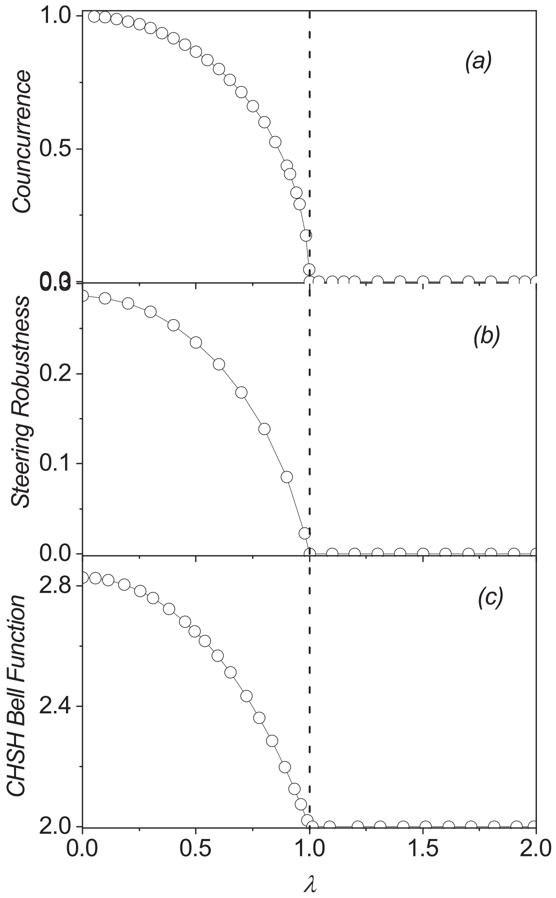

Figure 5. (a) Entanglement, (b) Steering Robustness and (c) CHSH function with respect to the parameters λ for the nearest-neighbor reduced density state in the LMG model. Steering Robustness approach to zero around the critical points $\lambda = 1$ and remains to be zero in the symmetric phase. Three kinds of quantum nonlocality exhibit a similar behavior in this model.

For the XY model, the results show that steering robustness increase firstly and then decrease in the paramagnetic phase, which is very different from the results by using steering weight as a measures[37]. When the parameter λ approaches to the critical point, steering robustness disappears exactly and then remains to be zero in the whole ferromagnetic phase. These characters are quite different from the information provided by other quantum correlations(e.g. entanglement). For the XXZ model, the results show that steering robustness reaches its maximum at the critical point, indicating the occurrence of QPT, which agrees with the prediction provided by the entanglement. In the both above two models, although CHSH function exhibits a behavior of local minimum around the critical points, there does not exit Bell nonlocality due to the monogamy trade-off obeyed by bipartite Bell correlations. For the LMG model, EPR steering does exist in the broken phase for $\lambda < 1$, and approaches to zero around the critical point then disappear in the symmetric phase for $\lambda > 1$, just liking the behavior of entanglement and Bell nonlocality.

5. Acknowledgments

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Appendix A. Elements of reduced density matrix

Appendix A.1. Elements of reduced density matrix for XY model

For the one-dimensional anisotropic XY model, the elements of the reduced density matrix can be calculated as[3]

$$\rho_{11} = \frac{1}{4} + \frac{\langle\sigma^z\rangle}{2} + \frac{\langle\sigma_i^z\sigma_j^z\rangle}{4}$$
$$\rho_{44} = \frac{1}{4} - \frac{\langle\sigma^z\rangle}{2} + \frac{\langle\sigma_i^z\sigma_j^z\rangle}{4}$$
$$\rho_{22} = \rho_{33} = \frac{1}{4} - \frac{\langle\sigma_i^z\sigma_j^z\rangle}{4}$$
$$\rho_{23} = \rho_{32} = \frac{\langle\sigma_i^x\sigma_j^x\rangle + \langle\sigma_i^y\sigma_j^y\rangle}{4}$$
$$\rho_{14} = \rho_{41}\frac{\langle\sigma_i^x\sigma_j^x\rangle - \langle\sigma_i^y\sigma_j^y\rangle}{4} \tag{A.1}$$

The magnetization of the spin$-1/2$ XY chain is given by,

$$\langle\sigma_z\rangle = -\int_0^\pi \frac{(1+\lambda\cos\phi)}{2\pi\omega_\phi}d\phi \tag{A.2}$$

with $\omega_\phi = ((\gamma\lambda\sin\phi)^2 + (1+\lambda\cos\phi)^2)^{\frac{1}{2}}$. The two-point spin correlation functions read,

$$\langle\sigma_0^x\sigma_r^x\rangle = \begin{pmatrix} G_{-1} & G_{-2} & \cdots & G_{-n} \\ G_0 & G_{-1} & \cdots & G_{-n+1} \\ \vdots & \vdots & \ddots & \vdots \\ G_{n-2} & G_{n-3} & \cdots & G_{-1} \end{pmatrix}$$
$$\langle\sigma_0^y\sigma_r^y\rangle = \begin{pmatrix} G_1 & G_0 & \cdots & G_{-n+2} \\ G_2 & G_1 & \cdots & G_{-n+3} \\ \vdots & \vdots & \ddots & \vdots \\ G_n & G_{n-1} & \cdots & G_1 \end{pmatrix} \tag{A.3}$$

and

$$\langle\sigma_0^z\sigma_r^z\rangle = \langle\sigma^z\rangle^2 - G_rG_{-r}. \tag{A.4}$$

Here,

$$G_r = \int_0^\pi \frac{1}{2\pi\omega_\phi}\{\cos(r\phi)(1+\lambda\cos\phi) - \gamma\lambda\sin(r\phi)\sin\phi\}d\phi \tag{A.5}$$

r denotes the distance between the site i and j.

Appendix A.2. Elements of reduced density matrix for XXZ model

For the one-dimensional Heisenberg spin-1/2 *XXZ* model, the elements of the reduced density matrix can be written as[5, 6]

$$\begin{aligned}\rho_{11} &= \rho_{44} = \frac{1}{4} + \frac{\langle\sigma_i^z\sigma_j^z\rangle}{4}\\ \rho_{22} &= \rho_{33} = \frac{1}{4} - \frac{\langle\sigma_i^z\sigma_j^z\rangle}{4}\\ \rho_{23} &= \rho_{32} = \frac{\langle\sigma_i^x\sigma_j^x\rangle}{2}\\ \rho_{14} &= \rho_{41} = 0 \end{aligned} \quad \text{(A.6)}$$

Here,

$$\langle\sigma_i^z\sigma_{i+1}^z\rangle = 4\frac{\partial e_0(\Delta)}{\partial\Delta}, \quad \text{(A.7)}$$

and

$$\langle\sigma_i^x\sigma_{i+1}^x\rangle = \frac{1}{2}[4e_0(\Delta) - \Delta\langle\sigma_i^z\sigma_{i+1}^z\rangle] \quad \text{(A.8)}$$

The energy e_0 determined by the infinite size Bethe ansatz is given by[7]

$$e_0 = \frac{\Delta}{4} \quad \text{(A.9)}$$

for $\Delta \leq -1$, and

$$e_0 = \frac{\Delta}{4} - \frac{1}{2}(1-\Delta^2)\int_{-\infty}^{\infty}\frac{dx}{\cosh\pi x[\cosh(2x\arccos\Delta) - \Delta]} \quad \text{(A.10)}$$

for $\Delta > -1$. At $\Delta = 1$ the integrand is not well defined, and one needs to take an appropriate limit.

Appendix A.3. Reduced density matrix for LMG model

The ground state of the LMG model for $N \to \infty$ can be obtained by using the HF approach. Then the pairwise ground state density matrix for general modes ($+m$) and ($-n$) in the HF ground state of the LMG model can be written as[15],

$$\rho_{m,-n} = \begin{pmatrix} \langle M_{+m}M_{-n}\rangle & 0 & 0 & 0\\ 0 & \langle M_{+m}N_{-n}\rangle & \langle c_{+m}^\dagger c_{-n}\rangle & 0\\ 0 & \langle c_{-n}^\dagger c_{+m}\rangle & \langle N_{+m}M_{-n}\rangle & 0\\ 0 & 0 & 0 & \langle N_{+m}N_{-n}\rangle \end{pmatrix} \quad \text{(A.11)}$$

Here, the matrix elements can be given as

$$\begin{aligned}\langle M_{+m}M_{-n}\rangle &= (1-\delta_{mn})\sin^2\alpha\cos^2\alpha)\\ \langle M_{+m}N_{-n}\rangle &= \delta_{mn}\cos^2\alpha + (1-\delta_{mn})\cos^4\alpha\\ \langle N_{+m}M_{-n}\rangle &= \delta_{mn}\sin^2\alpha + (1-\delta_{mn})\sin^4\alpha\\ \langle N_{+m}N_{-n}\rangle &= (1-\delta_{mn})\sin^2\alpha\cos^2\alpha)\\ \langle c_{+m}^\dagger c_{-n}\rangle &= \delta_{mn}\cos\alpha\sin\alpha \end{aligned} \quad \text{(A.12)}$$

with $\alpha = \arccos\lambda/2$ for $\lambda < 1$ and $\alpha = 0$ for $\lambda \geq 1$.

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