PROCEEDINGS OF SCIENCE

$B-\bar{B}$ mixing: decay matrix at high precision

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I review the status of the Standard-Model prediction of the width difference $\Delta\Gamma_s$ among the two B_s meson eigenstates. Ongoing effort addresses three-loop QCD corrections, corresponding to the next-to-next-to-leading order of QCD. With an improved theoretical precision of the ratio $\Delta\Gamma_s/\Delta M_s$, where ΔM_s denotes the mass difference in the $B_s-\bar{B}_s$ system, one can probe new physics in ΔM_s without sensitivity to $|V_{cb}|$, whose value is currently controversial.

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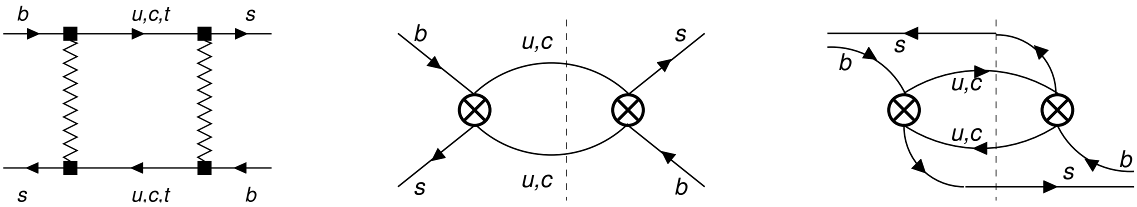

Figure 1: Left: B_s–$\bar{B}_s$ box diagram. A second diagram rotated by 90° is not shown. The right two diagrams are the corresponding diagrams in the effective $|\Delta B| = 1$ theory.

The flavoured neutral mesons $M = K, D, B_d, B_s$ mix with their antiparticles $\bar{M} = \bar{K}, \bar{D}, \bar{B}_d, \bar{B}_s$, with two important consequences: First, the mass eigenstates do not coincide with the flavour eigenstates. Second, a meson produced in the state $|M\rangle$ evolves into a linear superposition of $|M\rangle$ and $|\overline{M}\rangle$. The corresponding time dependence features an oscillatory behaviour in addition to the usual exponential decay law. In this talk I discuss new calculations for $B_s-\bar{B}_s$ mixing, but the results equally apply to $B_d-\bar{B}_d$ mixing with trivial replacements of the corresponding elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

The time evolution of the two-state system $(|B_s\rangle, |\bar{B}_s\rangle)$ is governed by two hermitian 2×2 matrices, the mass matrix M^s and the decay matrix Γ^s. Upon diagonalisation of $M^s - i\Gamma^s/2$ one finds the expression linking the mass eigenstates $|B_L\rangle$ and $|B_H\rangle$ to the flavour eigenstates $|B_s\rangle$,$|\bar{B}_s\rangle$. The mass eigenstates differ in their masses $M_{H,L}$ and decay widths $\Gamma_{H,L}$ with "L" and "H" standing for "light" and "heavy". The mass and width differences $\Delta M_s = M_H - M_L$ and $\Delta\Gamma_s = \Gamma_L - \Gamma_H$ are related to the off-diagonal elements M_{12}^s and Γ_{12}^s as

$$\Delta M_s \simeq 2|M_{12}^s|, \qquad \frac{\Delta\Gamma_s}{\Delta M_s} = -\mathrm{Re}\frac{\Gamma_{12}^s}{M_{12}^s}. \tag{1}$$

The Standard Model (SM) predictions for M_{12}^s and Γ_{12}^s are calculated from the dispersive and absorptive parts of the box diagram in Fig. 1, respectively. To find Γ_{12}^s one must therefore only consider diagrams with the light u, c quarks, while box diagrams with one or two internal t quarks will only contribute to M_{12}^s. To properly accomodate strong interaction effects one employs operator product expansions (OPE) to separate the physics from different energy scales. In the first step one matches the SM to an effective theory with $|\Delta B| = 1$ operators [1], where B is the beauty quantum number. The dependence of the SM b decay amplitudes on the masses M_W and m_t is contained in the Wilson coefficients multiplying these operators. The most important operators, *i.e.* those with the largest coefficients, are the current-current operators Q_1 and Q_2 pictorially found by contracting the W boson line connecting the $\bar{b}_L\gamma^\mu c$ and $\bar{c}_L\gamma^\mu s$ currents to a point. Q_1 and Q_2 differ in their colour indices; both operators are needed to properly accomodate QCD corrections. The $B_s-\bar{B}_s$ mixing diagrams (to leading order (LO) in QCD) in the effective $|\Delta B| = 1$ theory are also shown in Fig. 1. The second OPE employed in the calculation is the Heavy Quark Expansion (HQE) [2], which expresses the $B_s \to \bar{B}_s$ transition amplitude as an expansion in $\Lambda_{\mathrm{QCD}}/m_b$, where $\Lambda_{\mathrm{QCD}} \sim 400$ MeV is the fundamental scale of QCD and m_b is the b quark mass. The latter enters the problem as a hard momentum flowing through diagrams in Fig. 1. The HQE involves local $\Delta B = 2$ operators, now found by contracting the hard loop to a point. In the leading order of $\Lambda_{\mathrm{QCD}}/m_b$

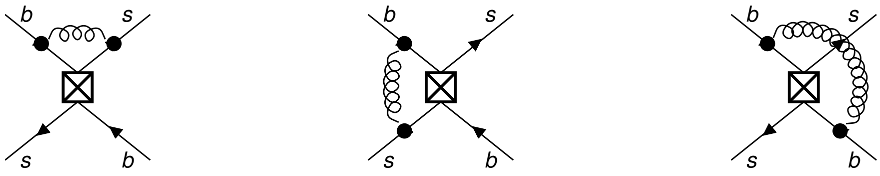

Figure 2: NLO diagrams in the $|\Delta B| = 2$ theory. Infrared singularities cancel with those of the corresponding two-loop diagrams of the $|\Delta B| = 1$ theory, so that the desired coefficients $H^{cc}(z)$ and $\widetilde{H}_S^{cc}(z)$ are infrared-finite.

("leading power") one encounters

$$Q = \bar{s}_i \gamma_\mu (1-\gamma_5) b_i \; \bar{s}_j \gamma^\mu (1-\gamma_5) b_j, \qquad \widetilde{Q}_S = \bar{s}_i (1+\gamma_5) b_j \; \bar{s}_j (1+\gamma_5) b_i \tag{2}$$

where i, j are colour indices. One finally finds

$$\Gamma_{12}^s \;=\; -(V_{cb}V_{cs}^*)^2 \frac{G_F^2 m_b^2}{24\pi M_{B_s}} \left[H^{cc}(z) \langle B_s|Q|\bar{B}_s\rangle + \widetilde{H}_S^{cc}(z) \langle B_s|\widetilde{Q}_S|\bar{B}_s\rangle \right] + \ldots \tag{3}$$

with $z = (m_c/m_b)^2$ and the ellipses denoting higher-order terms in Λ_{QCD}/m_b and CKM-suppressed contributions. To obtain next-to-leading order (NLO) QCD corrections to the Wilson coefficients $H^{cc}(z)$ and $\widetilde{H}_S^{cc}(z)$ one must add a gluon to the diagrams in Fig. 1 and further calculate the one-loop diagrams on the effective theory side with one gluon dressing Q or $\widetilde{Q}_S$, see Fig. 2. The prediction of Γ_{12}^s in Eq. (1) to leading power finally requires the calculation of two non-perturbative quantities, $\langle B_s|Q|\bar{B}_s\rangle$ and $\langle B_s|\widetilde{Q}_S|\bar{B}_s\rangle$ with the help of lattice QCD [3] or QCD sum rules [4, 5].

The prediction of $\Delta M_s \simeq 2|M_{12}^s|$ is conceptually simpler, the box diagram (at NLO dressed with one gluon) is directly matched to Q and the only non-perturbative input needed is $\langle B_s|Q|\bar{B}_s\rangle$. The predictions of both M_{12}^s and Γ_{12}^s are sums of terms of the form

$$(V_{tb}V_{ts}^*)^2 \times \text{perturbative coefficient} \times \text{hadronic matrix element} \tag{4}$$

where $V_{cb}V_{cs}^* \simeq -V_{tb}V_{ts}^*$ has been used. ΔM_s is highly sensitive to new physics stemming from virtual effects of heavy particles, probing particle masses beyond 100 TeV. But the uncertainty of the SM theory prediction exceeds the 0.03% error of the measurment

$$\Delta M_s^{\text{exp}} \;=\; (17.7656 \pm 0.0057)\ \text{ps}^{-1} \qquad [6] \tag{5}$$

by far: The hadronic matrix element [3] contributes 4% to the theory uncertainty and, more importantly, the prediction of ΔM_s is affected by the $|V_{cb}|$ tragedy, *i.e.* the unresolved discrepancy between the values for $|V_{cb}|$ inferred from inclusive [7] and exclusive [8] decays. CKM unitarity determines $|V_{ts}| \simeq |V_{cb}|$ and the $|V_{cb}|$ dispute means 15% error in ΔM_s from the prefactor $|V_{ts}|^2$ in Eq. (4).

The experimental value for the width difference is

$$\Delta\Gamma_s^{\text{exp}} \;=\; (0.082 \pm 0.005)\ \text{ps}^{-1} \qquad [9]\,, \tag{6}$$

where the quoted number for $\Delta\Gamma_s^{\text{exp}}$ is derived from data of LHCb [10], CMS [11], ATLAS [12], CDF [13], and DØ [14]. The CKM factor $|V_{tb}V_{ts}|^2$ drops out from the ratio $\Delta\Gamma_s/\Delta M_s$ and also the hadronic matrix elements largely cancel from this quantity. Thus by confronting $\Delta\Gamma_s^{\rm exp}/\Delta M_s^{\rm exp}$ with a precise theory prediction for $\Delta\Gamma_s/\Delta M_s$ we can both bypass the controversy on $|V_{cb}|$ and eliminate a source of hadronic uncertainty.

Using the NLO results of Refs. [15–18] and state-of-the-art lattice-QCD computations of $\langle B_s|Q|\bar{B}_s\rangle$ and $\langle B_s|\widetilde{Q}_S|\bar{B}_s\rangle$ one finds [19]

$$\begin{aligned} \Delta\Gamma_s &= \left(0.077 \pm 0.015_{\rm pert} \pm 0.002_{\rm had} \pm 0.017_{\Lambda_{\rm QCD}/m_b}\right)\ \text{GeV} \qquad \text{(pole)} \\ \Delta\Gamma_s &= \left(0.088 \pm 0.011_{\rm pert} \pm 0.002_{\rm had} \pm 0.014_{\Lambda_{\rm QCD}/m_b}\right)\ \text{GeV} \qquad (\overline{\rm MS}). \end{aligned} \tag{7}$$

Here "pole" and "$\overline{\rm MS}$" refers to different renormalisation schemes. The three errors denote the perturbative uncertainty, the errors from $\langle B_s|Q|\bar{B}_s\rangle$ and $\langle B_s|\widetilde{Q}_S|\bar{B}_s$, as well as the sub-leading power corrections. The predictions use the calculated $\Delta\Gamma_s/\Delta M_s$ multiplied by $\Delta M_s^{\rm exp}$ in Eq. (5). Both the perturbative error and the scheme dependence indicate that an NNLO calculation is mandatory to match the accuracy of the measurement. Furthermore, better lattice calculations [20] and a NLO calculation of the sub-leading-power corrections are needed to decrease the uncertainty in Eq. (7).

The progress since Refs. [15–18] comprises NNLO corrections enhanced by the number N_f of active quark flavours [19, 21] and two-loop results with one current-current and one penguin operator [22]. The four-quark penguin operators Q_{3-6} have Wilson coefficients which are much smaller than those of $Q_{1,2}$ and the chromomagnetic penguin operator contributes with a suppression factor of α_s. After the conference the remaining missing two-loop contributions, with two insertions of penguin operators [23], and the full NNLO corrections with two current-current operators have been completed [24]. The NNLO corrections of Refs. [19, 21, 24] involve three-loop diagrams which have been calculated in an expansion in $z = (m_c/m_b)^2$ to orders z^0 and z^1. This expansion is also used in Refs. [22, 23].

In conclusion, $B_s-\bar{B}_s$ mixing is highly sensitive to virtual effects of new physics, with a reach to particle masses of 100 TeV and more. The theory prediction for ΔM_s is currently limited by the uncertainties of $|V_{cb}|$ and, to less extent, of the hadronic matrix element $\langle B_s|Q|\bar{B}_s\rangle$. The precise measurement of $\Delta\Gamma_s$ calls for a better theory prediction; with the results of Refs. [19, 21, 24] one of the two major uncertainties is pushed below the experimental error. The other one stems from the power-suppressed contributions and will be reduced once the lattice calculations [20] become more accurate and the Wilson coefficientes of power-suppressed operators are calculated to NLO. With precise experimental and theoretical values for $\Delta\Gamma_s$ the ratio $\Delta\Gamma_s/\Delta M_s$ will be an excellent quantity to probe new physics in ΔM_s, because this ratio is not affected by the exclusive-vs-inclusive controversy on the value of $|V_{cb}|$. For the latest numerical theory predictions for $\Delta\Gamma_s/\Delta M_s$ I refer the reader to Refs. [23, 24].

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References

[1] F. J. Gilman and M. B. Wise, *Effective Hamiltonian for* $\Delta S = 1$ *Weak Nonleptonic Decays in the Six Quark Model,* Phys. Rev. D **20** (1979), 2392 doi:10.1103/PhysRevD.20.2392.

[2] V. A. Khoze, M. A. Shifman, N. G. Uraltsev and M. B. Voloshin, *On Inclusive Hadronic Widths of Beautiful Particles,* Sov. J. Nucl. Phys. **46** (1987), 112.

[3] R. J. Dowdall, C. T. H. Davies, R. R. Horgan, G. P. Lepage, C. J. Monahan, J. Shigemitsu and M. Wingate, *Neutral B-meson mixing from full lattice QCD at the physical point,* Phys. Rev. D **100** (2019) no.9, 094508 doi:10.1103/PhysRevD.100.094508 [arXiv:1907.01025 [hep-lat]].

[4] M. Kirk, A. Lenz and T. Rauh, *Dimension-six matrix elements for meson mixing and lifetimes from sum rules,* JHEP **12** (2017), 068 [erratum: JHEP **06** (2020), 162] doi:10.1007/JHEP12(2017)068 [arXiv:1711.02100 [hep-ph]].

[5] D. King, A. Lenz and T. Rauh, *B_s mixing observables and $|V_{td}/V_{ts}|$ from sum rules,* JHEP **05** (2019), 034 doi:10.1007/JHEP05(2019)034 [arXiv:1904.00940 [hep-ph]].

[6] R. Aaij *et al.* [LHCb], *Precise determination of the B^0_s–$\overline{B}{}^0_s$ oscillation frequency,* Nature Phys. **18** (2022) no.1, 1-5 doi:10.1038/s41567-021-01394-x [arXiv:2104.04421 [hep-ex]].

[7] P. Gambino, K. J. Healey and S. Turczyk, *Taming the higher power corrections in semileptonic B decays,* Phys. Lett. B **763** (2016), 60-65 doi:10.1016/j.physletb.2016.10.023 [arXiv:1606.06174 [hep-ph]].

[8] Y. Aoki, T. Blum, G. Colangelo, S. Collins, M. Della Morte, P. Dimopoulos, S. Dürr, X. Feng, H. Fukaya and M. Golterman, *et al. FLAG Review 2021,* [arXiv:2111.09849 [hep-lat]].

[9] *Heavy Flavor Averaging Group (HFLAV),*
`https://hflav-eos.web.cern.ch/hflav-eos/osc/PDG_2020/\# DMS`

[10] R. Aaij *et al.* [LHCb], *Updated measurement of time-dependent CP-violating observables in $B^0_s \to J/\psi K^+K^-$ decays,* Eur. Phys. J. C **79** (2019) no.8, 706 [erratum: Eur. Phys. J. C **80** (2020) no.7, 601] [arXiv:1906.08356 [hep-ex]].

[11] A. M. Sirunyan *et al.* [CMS], *Measurement of the CP-violating phase ϕ_s in the $B^0_s \to J/\psi\, \phi(1020) \to \mu^+\mu^-K^+K^-$ channel in proton-proton collisions at $\sqrt{s} = 13$ TeV,* Phys. Lett. B **816** (2021), 136188, doi:10.1016/j.physletb.2021.136188 [arXiv:2007.02434 [hep-ex]].

[12] G. Aad *et al.* [ATLAS], *Measurement of the CP-violating phase ϕ_s in $B^0_s \to J/\psi\phi$ decays in ATLAS at 13 TeV,* Eur. Phys. J. C **81** (2021) no.4, 342 doi:10.1140/epjc/s10052-021-09011-0 [arXiv:2001.07115 [hep-ex]].

[13] T. Aaltonen *et al.* [CDF], *Measurement of the Bottom-Strange Meson Mixing Phase in the Full CDF Data Set,* Phys. Rev. Lett. **109** (2012), 171802, doi:10.1103/PhysRevLett.109.171802 [arXiv:1208.2967 [hep-ex]].

[14] V. M. Abazov *et al.* [D0], *Measurement of the CP-violating phase $\phi_s^{J/\psi\phi}$ using the flavor-tagged decay $B_s^0 \to J/\psi\phi$ in 8 fb^{-1} of $p\bar{p}$ collisions,* Phys. Rev. D **85** (2012), 032006, doi:10.1103/PhysRevD.85.032006 [arXiv:1109.3166 [hep-ex]].

[15] M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste, *Next-to-Leading Order QCD Corrections to the Lifetime Difference of B_s Mesons*, Phys. Lett. B **459** (1999), 631-640 [arXiv:hep-ph/9808385 [hep-ph]].

[16] M. Ciuchini, E. Franco, V. Lubicz, F. Mescia and C. Tarantino, *Lifetime Differences and CP Violation Parameters of Neutral B Mesons at the Next-to-Leading Order in QCD*, JHEP **08** (2003), 031 [arXiv:hep-ph/0308029 [hep-ph]].

[17] M. Beneke, G. Buchalla, A. Lenz and U. Nierste, *CP asymmetry in flavour-specific B decays beyond leading logarithms*, Phys. Lett. B **576** (2003), 173-183 [arXiv:hep-ph/0307344 [hep-ph]].

[18] A. Lenz and U. Nierste, *Theoretical update of B_s–$\bar{B}_s$ mixing*, JHEP **06** (2007), 072 [arXiv:hep-ph/0612167 [hep-ph]].

[19] H. M. Asatrian, H. H. Asatryan, A. Hovhannisyan, U. Nierste, S. Tumasyan and A. Yeghiazaryan, *Penguin contribution to the width difference and CP asymmetry in B_q-$\bar{B}_q$ mixing at order $\alpha_s^2 N_f$,* Phys. Rev. D **102** (2020) no.3, 033007 doi:10.1103/PhysRevD.102.033007 [arXiv:2006.13227 [hep-ph]].

[20] C. T. H. Davies *et al.* [HPQCD], *Lattice QCD matrix elements for the B_s–$\bar{B}_s$ width difference beyond leading order*, Phys. Rev. Lett. **124** (2020) no.8, 082001 [arXiv:1910.00970 [hep-lat]].

[21] H. M. Asatrian, A. Hovhannisyan, U. Nierste and A. Yeghiazaryan, *Towards next-to-next-to-leading-log accuracy for the width difference in the B_s–$\bar{B}_s$: fermionic contributions to order $(m_c/m_b)^0$ and $(m_c/m_b)^1$*, JHEP **10** (2017), 191 [arXiv:1709.02160 [hep-ph]].

[22] M. Gerlach, U. Nierste, V. Shtabovenko and M. Steinhauser, *Two-loop QCD penguin contribution to the width difference in $B_s-\overline{B}_s$ mixing,* JHEP **07** (2021), 043 doi:10.1007/JHEP07(2021)043 [arXiv:2106.05979 [hep-ph]].

[23] M. Gerlach, U. Nierste, V. Shtabovenko and M. Steinhauser, *The width difference in $B-\overline{B}$ beyond mixing at order α_s and,* JHEP **04** (2022), 006 doi:10.1007/JHEP04(2022)006 [arXiv:2202.12305 [hep-ph]].

[24] M. Gerlach, U. Nierste, V. Shtabovenko and M. Steinhauser, in preparation.