



Mechanical stress and deformation analyses of pressurized cylindrical shells based on a higher-order modeling

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ABSTRACT

In this research, mechanical stress, static strain and deformation analyses of a cylindrical pressure vessel subjected to mechanical loads are presented. The kinematic relations are developed based on higher-order sinusoidal shear deformation theory. Thickness stretching formulation is accounted for more accurate analysis. The total transverse deflection is divided into bending, shear and thickness stretching parts in which the third term is responsible for change of deflection along the thickness direction. The axisymmetric formulations are derived through principle of virtual work. A parametric study is presented to investigate variation of stress and strain components along the thickness and longitudinal directions. To explore effect of thickness stretching model on the static results, a comparison between the present results with the available results of literature is presented. As an important output, effect of micro-scale parameter is studied on the static stress and strain distribution.

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1. Introduction

Analysis of pressure vessels have great importance in the context of mechanical engineering because of their vast applications in various situations. The cylindrical shells are used in various industrial applications such as chemical reactors, weapon instruments and aerospace technologies. Analysis of pressure vessels are performed using the Newtonian and Lagrangian methods and also using the technical codes proposed by important associations [1]. The Newtonian methods are applied through plane elasticity theory and are mostly valid for long cylindrical shells. The Lagrangian methods based on principle of virtual work and Hamilton's principle are used for more generalized formulation of the cylindrical shells with variable thickness and short length cylinder with axial boundary conditions. There are some applications of cylindrical shells in the various situations subjected to combination of mechanical, thermal and electrical loads [1–6]. A review on the recent works is presented to justify significance of this paper.

Lori Dehsaraji et al. [7] formulated a higher-order thickness

stretched piezoelectric nanoshell through Eringen nonlocal elasticity theory and shear and normal deformation theory. Decomposition of radial displacement into bending, shear and thickness stretching portions was used for kinematic relations. Justification of the results was performed through comparison with some lower order theories and also 3D results. Rezaiee-Pajand et al. [8] employed a higher order element including seven parameters to study nonlinear analysis of various thickness shell of revolution. Thickness stretching was accounted based on the employed element through finite element formulation. Zhang et al. [9] developed an experimental study on the fracture analysis of bituminous coal. Qiu et al. [10] studied effect of thermal load on the mechanical behavior of cylindrical shell as an energy storage.

Accounting concurrent geometric nonlinearity and thickness stretched model was employed by Amabili [11] for large amplitude vibration analysis of a doubly curved shell where all in-plane and transverse nonlinear terms were included in the formulation. The third-order thickness stretching formulation was used for analysis of laminated shell. The results were compared with those higher-order and refined theories that ignored thickness stretching. Lori Dehsaraji et al. [12] developed a modified couple stress model for electroelastic vibration analysis of cylindrical micro and nano shell

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subjected to electromechanical loading through Hamilton's principle and a thickness stretching modeling. Patel et al. [13] investigated dynamic responses of elliptical shell made from functionally graded materials through higher-order modeling considering thickness stretching. To satisfy field consistency and avoid shear as well as membrane locking, the finite element approach was used for formulation. Ganapathi et al. [14] extended higher-order shear deformation theory to dynamic responses of laminated thick cylindrical shell with application in modeling the variation of the in/out of plane displacement and sudden discontinuity in the slope. Finite element method and time integration technique were employed for numerical investigation of the problem. Simo et al. [15] extended shell theory including finite thickness stretch as well as initial variable thickness for analysis of the problems including finite membrane strains, contact, delamination and concentrated surface loads. The numerical results were presented to investigate influence of thickness stretch on the responses of thin shell. Static, dynamic and stability analyses of a composite laminated shell was studied by Zenkour and Fares [16] based on Hamilton–Reissner's mixed variational principle. Polit et al. [17] studied effect of graphene nanoplatelets reinforcement and porosity on the static and stability analyses of curved beam based on higher-order model considering thickness stretching terms. Hamilton's principle and Navier's technique were employed for derivation of motion equations and solution procedure. Effect of thermal and fatigue loading was studied on the structural behavior of new materials and structures [18,19].

Arefi et al. [20] employed two-variable sinusoidal shear deformation theory for static bending analysis of a double curved shell with accounting shear and normal deformation theory. Generalized Hooke's law was used to derive governing equations. After derivation of the governing equations using principle of virtual work and solution using the analytical method, it was shown that accounting thickness stretching leads to a 4% improvement in the results. Amabili et al. [21] developed a new nonlinear model for nonlinear vibration characteristics of a shell made from hyperelastic material with incompressibility including nine parameters. The model was applicable for analysis of a Neo-Hookean material and biomechanics of soft tissues. Capability of the proposed model was justified through comparison of the results with those results without thickness stretching. Amabili [22] studied nonlinear forced vibration analysis of laminated cylindrical shell using the new higher-order shear deformation theory and Lagrange equations in which a harmonic point excitation along the radial direction was applied. Pseudo-arclength continuation method and bifurcation analysis were used to derive governing equations of motion. The accuracy of the proposed formulation was justified using reduction of the formulation to von Kármán formulation and Novozhilov theory. Chu [23] studied large amplitude vibration analysis of thin shell using a higher order shear deformable model and considering nonlinear strain components. Merodio and Haughton [24] studied bifurcation analysis of thick-walled cylindrical shell subjected to Marfan's syndrome. Bert and Birman [25] studied dynamic instability analysis of a cylindrical shell subjected to axial excitation made from orthotropic material reinforced with different fibers. Alsubari et al. [26] studied effect of humidity and thermal loads on the static analysis of anisotropic shell using generalized plane strain assumption and the Murakami zigzag function. There are some important works for referring the basic relations of this work [27–33,47–66].

Ganapathi et al. [34] used a new higher-order displacement field for dynamic responses of a thick multi-layered shell subjected to thermal and mechanical loads. Kumar et al. [35] employed a higher-order zigzag theory for finite element formulation of composite and sandwich shell. The failure criteria have been examined for the shell subjected to mechanical loadings. Ye et al. [36] combined a higher-order shear deformation theory and a semi-analytical method for formulation and numerical solution of a composite laminated shell using three-dimensional theory of elasticity. Sharifat and Eslami [37] studied dynamic buckling and postbuckling analyses of imperfect cylindrical shell in thermal environment using 3D thermoelasticity relations. Loy and Lam [38] presented higher-order dynamic formulation of a thick cylindrical shell based on elastodynamic relations. Third-order shear deformation theory was used in the framework of motion equations for free vibration responses of a cylindrical shell by Saad et al. [39]. The accuracy of the results was confirmed through comparison with finite element results. Desai and Kant [40] studied effect of sinusoidal higher order shear deformation theory on the bending behaviours of the cylindrical shell. Wavelet transform was used as a mathematical method for analysis of the engineering processes and applications [41–43]. For better and more accurate modeling the shear strains along the thickness direction, Bhimaraddi [44] used lower and higher order shear deformation theories for dynamic analysis of the cylindrical shells. McDaniel and Ginsberg [45] concurrently accounted higher-order shear deformation theory and Ritz expansions for vibration responses of the cylindrical shells. Hirano and Hirashima [46] used infinite power series along the radial direction for three dimensional dynamic analysis of a cylindrical shell based on a higher-order model. There are some applications [47–53] of new materials and technologies in the mechanical engineering such as reinforced materials.

Investigating related works on the recent published papers of cylindrical shells in various environments and using various theoretical formulation and kinetic relations has been performed in the literature survey. It is deduced that stress and deformation analysis of higher-order shear deformable cylindrical pressure vessels are important for more accurate analysis of the shells and should be accounted in new works. The importance of shear stress is confirmed in this paper specially at both ends. Two-variable sinusoidal shear deformation theory is used for description of kinematic relations and the principle of virtual work is used for derivation of the governing equations of motion.

2. Thickness stretching included formulation

An axisymmetric pressurized cylindrical pressure vessel is formulated in this section based on higher-order sinusoidal shear deformation theory and accounting thickness stretching. Based on the axisymmetric model, two axial u_x and radial w_z displacements are assumed in the framework of higher-order shear deformation theory accounting thickness stretching term are assumed as [65,66].

$$\begin{aligned} u_x &= u_0 - z \frac{\partial w_b}{\partial x} - f(z) \frac{\partial w_s}{\partial x} \\ w_z &= w_b + w_s + g(z)\chi \end{aligned} \quad (1)$$

in which, $f(z) = z - h/\pi \sin(\pi z/h)$ and $g(z) = 1 - f'(z)$. Furthermore, w_b, w_s, χ are bending, shear and stretching parts of

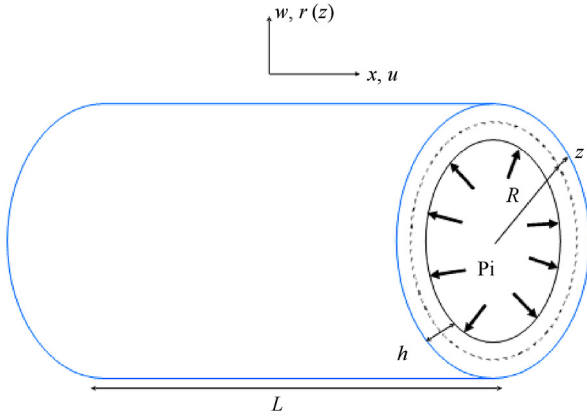
transverse deflection, respectively. Using the kinematic relations in the cylindrical coordinate system, the strain components are derived as [65,66].

$$\begin{aligned}\varepsilon_{xx} &= \frac{\partial u_x}{\partial x} = \frac{\partial u_0}{\partial x} - z \frac{\partial^2 w_b}{\partial x^2} - f(z) \frac{\partial^2 w_s}{\partial x^2} \\ \varepsilon_{\theta\theta} &= \frac{w_z}{r} = \frac{w_b}{r} + \frac{w_s}{r} + \frac{g(z)}{r} \chi \\ \varepsilon_{zz} &= \frac{\partial w_z}{\partial z} = g'(z) \chi \\ \gamma_{xz} &= \frac{\partial u_x}{\partial z} + \frac{\partial w_z}{\partial x} = \{1 - f'(z)\} \frac{\partial w_s}{\partial x} + g(z) \frac{\partial \chi}{\partial x}\end{aligned}\quad (2)$$

The behavioural relations are derived as [7,9,12]

$$\begin{aligned}\begin{Bmatrix} \sigma_{xx} \\ \sigma_{\theta\theta} \\ \sigma_{zz} \\ \tau_{xz} \end{Bmatrix} &= \frac{E}{(1-2\nu)(1+\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 \\ \nu & 1-\nu & \nu & 0 \\ \nu & \nu & 1-\nu & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \\ &\times \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{\theta\theta} \\ \varepsilon_{zz} \\ \gamma_{xz} \end{Bmatrix}\end{aligned}\quad (3)$$

where, normal (shear) stress and strain components are denoted with $\sigma_{ij}(\tau_{xz})$ and $\varepsilon_{ij}(\gamma_{ij})$. Furthermore, E, ν are Young's modulus and Poisson's ratio, respectively.



Strain energy variation of a cylindrical shell is expressed as [26–30,61,62,67].

$$\begin{aligned}\delta U &= \iiint_{x_1, x_2, z} \left[\sigma_{xx} \left(\frac{\partial \delta u_0}{\partial x} - z \frac{\partial^2 \delta w_b}{\partial x^2} - f(z) \frac{\partial^2 \delta w_s}{\partial x^2} \right) \right. \\ &+ \sigma_{\theta\theta} \left(\frac{\delta w_b}{r} + \frac{\delta w_s}{r} + \frac{g(z)}{r} \delta \chi \right) + \sigma_{zz} g'(z) \delta \chi \\ &\left. + \tau_{xz} \left(\{1 - f'(z)\} \frac{\partial \delta w_s}{\partial x} + g(z) \frac{\partial \delta \chi}{\partial x} \right) \right] dV\end{aligned}\quad (4)$$

Through definition of resultant components, we will have the strain energy in the updated form as

$$\begin{aligned}\delta U &= 2\pi \int \left[N_{xx} \frac{\partial \delta u_0}{\partial x} - M_{xx} \frac{\partial^2 \delta w_b}{\partial x^2} - S_{xx} \frac{\partial^2 \delta w_s}{\partial x^2} + N_{\theta\theta} \delta w_b + N_{\theta\theta} \delta w_s \right. \\ &\left. + S_{\theta\theta} \delta \chi + S_{zz} \delta \chi + N_{xz} \frac{\partial \delta w_s}{\partial x} + S_{xz} \frac{\partial \delta \chi}{\partial x} \right] dx\end{aligned}\quad (5)$$

where the resultant components are defined as [7,9,12,67]:

$$\begin{aligned}\{N_{xz}, M_{xx}, S_{xx}\} &= \int r \sigma_{xx} \{1, z, f(z)\} dz, \{N_{\theta\theta}, S_{\theta\theta}\} \\ &= \int \sigma_{\theta\theta} \{1, g(z)\} dz, S_{zz} = \int \sigma_{zz} r g(z) dz \\ \{N_{xz}, S_{xz}\} &= \int r \tau_{xy} \{1 - f'(z), g(z)\} dz\end{aligned}\quad (6)$$

Integration by part leads to

$$\begin{aligned}\delta U &= 2\pi \int \left[-\frac{\partial N_{xx}}{\partial x} \delta u_0 + \left(-\frac{\partial^2 M_{xx}}{\partial x^2} + N_{\theta\theta} \right) \delta w_b + \right. \\ &\left. \left(-\frac{\partial^2 S_{xx}}{\partial x^2} - \frac{\partial N_{xz}}{\partial x} + N_{\theta\theta} \right) \delta w_s + \left(S_{\theta\theta} + S_{zz} - \frac{\partial S_{xz}}{\partial x} \right) \delta \chi \right] dx\end{aligned}\quad (7)$$

The resultant components are defined in terms of displacement functions as

$$\begin{aligned}N_{xx} &= \wp_1 \frac{\partial u_0}{\partial x} - \wp_2 \frac{\partial^2 w_b}{\partial x^2} - \wp_3 \frac{\partial^2 w_s}{\partial x^2} + \wp_4 w_b + \wp_4 w_s \\ &+ \wp_5 \chi + \wp_6 \chi \\ M_{xx} &= \wp_7 \frac{\partial u_0}{\partial x} - \wp_8 \frac{\partial^2 w_b}{\partial x^2} - \wp_9 \frac{\partial^2 w_s}{\partial x^2} + \wp_{10} w_b + \wp_{10} w_s \\ &+ \wp_{11} \chi + \wp_{12} \chi \\ S_{xx} &= \wp_{13} \frac{\partial u_0}{\partial x} - \wp_{14} \frac{\partial^2 w_b}{\partial x^2} - \wp_{15} \frac{\partial^2 w_s}{\partial x^2} + \wp_{16} w_b + \wp_{16} w_s \\ &+ \wp_{17} \chi + \wp_{18} \chi \\ N_{\theta\theta} &= \wp_{19} w_b + \wp_{19} w_s + \wp_{20} \chi + \wp_{21} \frac{\partial u_0}{\partial x} - \wp_{22} \frac{\partial^2 w_b}{\partial x^2} \\ &- \wp_{23} \frac{\partial^2 w_s}{\partial x^2} + \wp_{24} \chi \\ S_{\theta\theta} &= \wp_{25} w_b + \wp_{25} w_s + \wp_{26} \chi + \wp_{27} \frac{\partial u_0}{\partial x} - \wp_{28} \frac{\partial^2 w_b}{\partial x^2} \\ &- \wp_{29} \frac{\partial^2 w_s}{\partial x^2} + \wp_{30} \chi \\ S_{zz} &= \wp_{31} \chi + \wp_{32} w_b + \wp_{32} w_s + \wp_{33} \chi + \wp_{34} \frac{\partial u_0}{\partial x} - \wp_{35} \frac{\partial^2 w_b}{\partial x^2} \\ &- \wp_{36} \frac{\partial^2 w_s}{\partial x^2} \\ N_{xz} &= \wp_{37} \frac{\partial w_s}{\partial x} + \wp_{38} \frac{\partial \chi}{\partial x} \\ S_{xz} &= \wp_{39} \frac{\partial w_s}{\partial x} + \wp_{40} \frac{\partial \chi}{\partial x}\end{aligned}\quad (8)$$

where the integration constants are computed in Appendix A.

The external work due to internal and external pressures and an outer Pasternak's foundation is computed as follows [34–36,40–42].

$$\delta W = 2\pi \int \int \left[P_i \delta w_{z=-\frac{h}{2}} - P_o \delta w_{z=+\frac{h}{2}} - F_f \delta w_{z=+\frac{h}{2}} \right] dz dx \quad (9)$$

Substitution of transverse displacement into above relations leads to

$$\delta W = 2\pi \int \int \left[(P_i - P_o - F_f) \delta w_b + (P_i - P_o - F_f) \delta w_s \right] dz dx \quad (10)$$

The governing equations are derived as

$$\begin{aligned} \delta u_0 : \frac{\partial N_{xx}}{\partial x} &= 0 \\ \delta w_b : -\frac{\partial^2 M_{xx}}{\partial x^2} + N_{\theta\theta} &= P_i - P_o - F_f \\ \delta w_s : -\frac{\partial^2 S_{xx}}{\partial x^2} - \frac{\partial N_{xz}}{\partial x} + N_{\theta\theta} &= P_i - P_o - F_f \\ \delta \chi : S_{\theta\theta} + S_{zz} - \frac{\partial S_{xz}}{\partial x} &= 0 \end{aligned} \quad (11)$$

Finally, the governing equations in terms of primary functions are developed as

$$\begin{aligned} \delta u_0 : -\wp_1 \frac{\partial^2 u_0}{\partial x^2} + \wp_2 \frac{\partial^3 w_b}{\partial x^3} - \wp_4 \frac{\partial w_b}{\partial x} + \wp_3 \frac{\partial^3 w_s}{\partial x^3} - \wp_4 \frac{\partial w_s}{\partial x} - (\wp_5 + \wp_6) \frac{\partial \chi}{\partial x} &= 0 \\ \delta w_b : -\wp_7 \frac{\partial^3 u_0}{\partial x^3} + \wp_{21} \frac{\partial u_0}{\partial x} + \wp_8 \frac{\partial^4 w_b}{\partial x^4} - (\wp_{10} + \wp_{22} + K_p) \frac{\partial^2 w_b}{\partial x^2} + (\wp_{19} + K_w) w_b + \wp_9 \frac{\partial^4 w_s}{\partial x^4} - (\wp_{10} + \wp_{23} + K_p) \frac{\partial^2 w_s}{\partial x^2} \\ &+ (\wp_{19} + K_w) w_s - (\wp_{11} + \wp_{12}) \frac{\partial^2 \chi}{\partial x^2} + (\wp_{20} + \wp_{24}) \chi = P_i - P_o \\ \delta w_s : -\wp_{13} \frac{\partial^3 u_0}{\partial x^3} + \wp_{21} \frac{\partial u_0}{\partial x} + \wp_{14} \frac{\partial^4 w_b}{\partial x^4} - (\wp_{16} + \wp_{22} + K_p) \frac{\partial^2 w_b}{\partial x^2} + (K_w + \wp_{19}) w_b + \wp_{15} \frac{\partial^4 w_s}{\partial x^4} - (\wp_{16} + \wp_{23} + \wp_{37} + K_p) \frac{\partial^2 w_s}{\partial x^2} \\ &+ (K_w + \wp_{19}) w_s - (\wp_{17} + \wp_{18} + \wp_{38}) \frac{\partial^2 \chi}{\partial x^2} + (\wp_{20} + \wp_{24}) \chi = P_i - P_o \\ \delta \chi : (\wp_{34} + \wp_{27}) \frac{\partial u_0}{\partial x} - (\wp_{28} + \wp_{35}) \frac{\partial^2 w_b}{\partial x^2} + (\wp_{25} + \wp_{32}) w_b - (\wp_{29} + \wp_{36} + \wp_{39}) \frac{\partial^2 w_s}{\partial x^2} + (\wp_{25} + \wp_{32}) w_s - \wp_{40} \frac{\partial^2 \chi}{\partial x^2} \\ &+ (\wp_{26} + \wp_{30} + \wp_{31} + \wp_{33}) \chi = 0 \end{aligned} \quad (12)$$

3. Numerical results

Numerical solution procedure is explained in this section for simply-supported boundary conditions. The solution of governing equations is expressed based on trigonometric functions as

Table 1

Comparison between the dimensionless transverse deflection \bar{w} between present results and those results of Refs. [31–33].

$S = R/h$	Soedel [32]	Khdeir et al. [33]	Nwojia and Ani [31]	Present
100	0.0019	0.0019	0.0019	0.00195
20	0.0206	0.0212	0.0214	0.0215
10	0.0301	0.0334	0.034	0.0342
4	0.0629	0.0616	0.0629	0.0632

$$\begin{pmatrix} u_0 \\ w_b \\ w_s \\ \chi \end{pmatrix} = \sum_n \sum_m \begin{pmatrix} U \cos(\lambda_m x) \\ W_b \sin(\lambda_m x) \\ W \sin(\lambda_m x) \\ \chi \sin(\lambda_m x) \end{pmatrix} \quad (13)$$

in which $\lambda_m = \frac{m\pi}{L}$. Two dimensional results including stresses, strains and displacements are explored along the radial and axial directions.

A validation through comparison of the present results with those results available in literature is presented. Listed in Table 1 is comparison between the dimensionless transverse deflection of the present results and those results of Refs. [31–33]. A good agreement between present results with literature results is observed.

The results including displacements, strains and stresses components are plotted along the axial and axial and radial directions. The results are presented for two different geometries of cylindrical

shell. The first case is a long cylindrical shell: $R = 1$ m, $L = 5$ m, $t = 0.05$ m and the second case is a short cylindrical shell: $R = 1$ m, $L = 2$ m, $t = 0.1$ m.

Axial displacement is shown in Fig. 1 along the radial and axial directions for a pressurized cylindrical pressure vessel. The results of Fig. 1(a) and Fig. 1(b) are presented for $R = 1$ m, $L = 5$ m, $t = 0.02$ m and $R = 1$ m, $L = 2$ m, $t = 0.1$ m respectively. It is observed

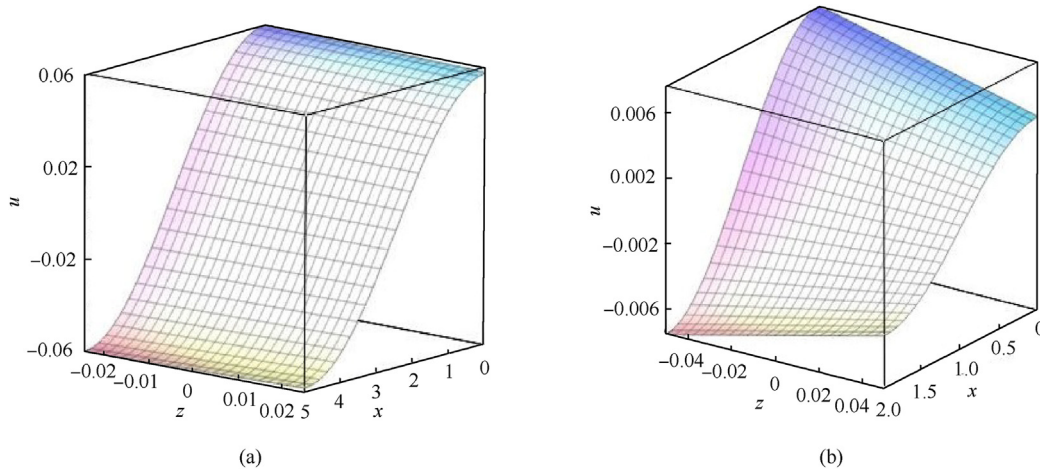


Fig. 1. Changes of axial displacement u along the radial and axial directions: (a) $R = 1$ m, $L = 5$ m, $t = 0.05$ m; (b) $R = 1$ m, $L = 2$ m, $t = 0.1$ m.

that for a long cylindrical pressure vessel, the axial displacement is approximately unchanged along the thickness direction. Unlike this case, the results of figure b are computed for a short and thick shell in which the radial displacement is changed along the thickness direction. It is confirmed that the higher-order shear deformation

theory and thickness stretching leads to an efficient result for short and thick shells.

Shown in Fig. 2(a), Fig. 2(b) are two dimensional changes of radial displacements along the thickness and axial directions for cylindrical pressure vessel. The results are presented for two

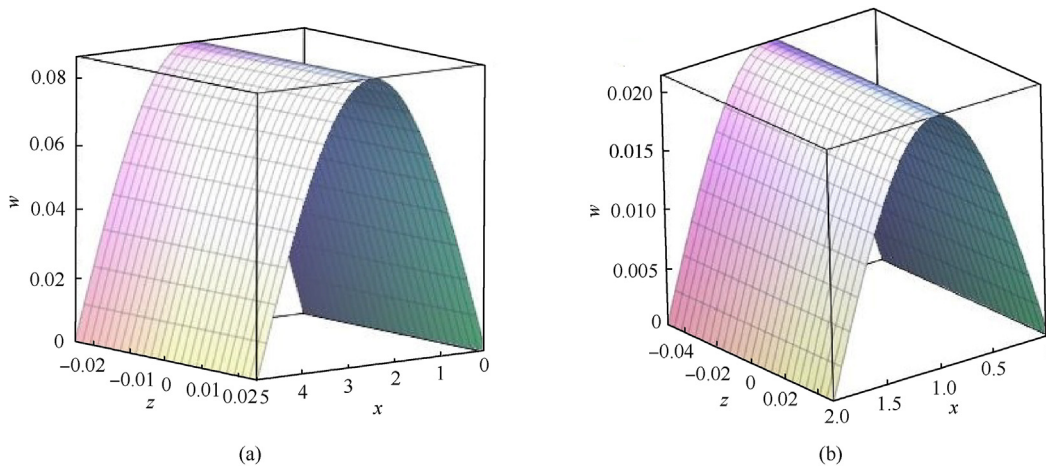


Fig. 2. Changes of transverse deflection w along the radial and axial directions: (a) $R = 1$ m, $L = 5$ m, $t = 0.05$ m; (b) $R = 1$ m, $L = 2$ m, $t = 0.1$ m.

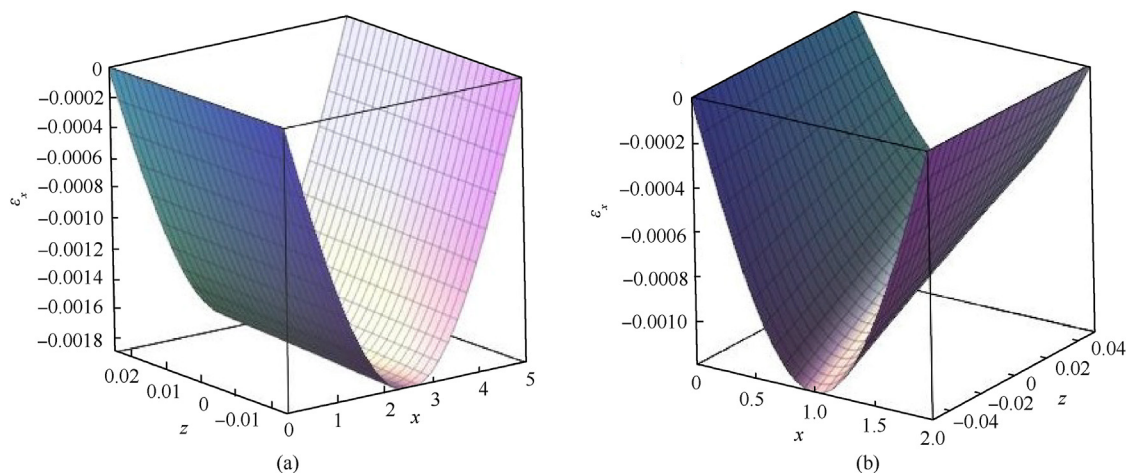


Fig. 3. Changes of axial strain ϵ_x along the radial and axial directions: (a) $R = 1$ m, $L = 5$ m, $t = 0.05$ m; (b) $R = 1$ m, $L = 2$ m, $t = 0.1$ m.

geometric cases. A significant decrease in the radial displacement is observed for a short length cylindrical shell $R = 1$ m, $L = 2$ m, $t = 0.1$ m respect to a long shell $R = 1$ m, $L = 5$ m, $t = 0.05$ m.

Two-dimensional variation of axial strain ϵ_x along the radial and axial directions is presented in Fig. 3(a), Fig. 3(b) for a pressurized

cylindrical shell for two geometries. A decrease in axial strain ϵ_x is observed for a thick and short cylindrical shell. The maximum values of axial strain are observed at middle length of cylindrical shell due maximum bending moment at same section. Fig. 4, Fig. 5(a), Fig. 5(b) show two dimensional variation of radial ϵ_r and

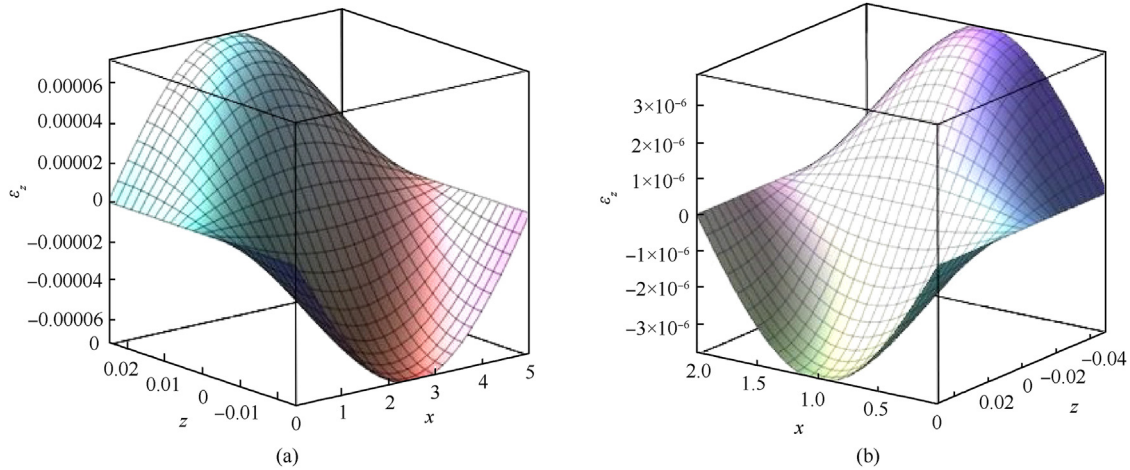


Fig. 4. Changes of radial strain ϵ_r along the radial and axial directions: (a) $R = 1$ m, $L = 5$ m, $t = 0.05$ m; (b) $R = 1$ m, $L = 2$ m, $t = 0.1$ m.

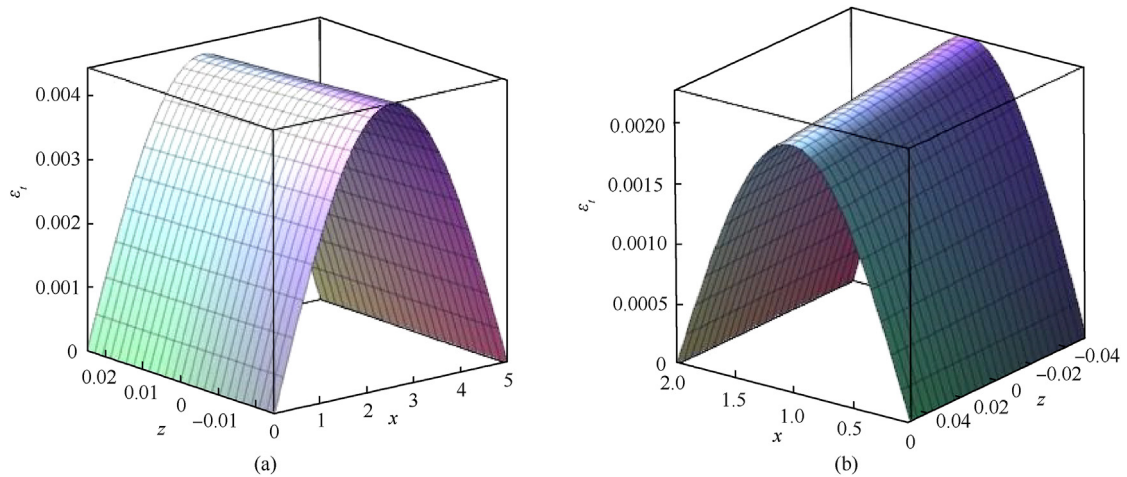


Fig. 5. Changes of hoop strain ϵ_θ along the radial and axial directions: (a) $R = 1$ m, $L = 5$ m, $t = 0.05$ m; (b) $R = 1$ m, $L = 2$ m, $t = 0.1$ m.

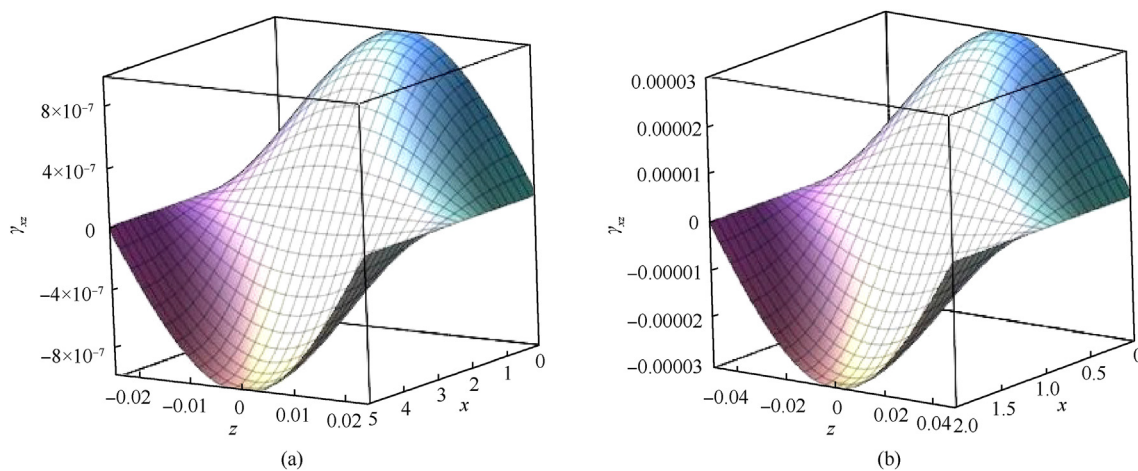


Fig. 6. Changes of shear strain γ_{xz} along the radial and axial directions: (a) $R = 1$ m, $L = 5$ m, $t = 0.05$ m; (b) $R = 1$ m, $L = 2$ m, $t = 0.1$ m.

circumferential ε_t strain components along the radial and axial directions. The results show that the radial strain is zero at both ends of cylindrical shell as well as middle surface.

Two dimensional variation of shear strain is presented in

Fig. 6(a), Fig. 6(b) along the radial and axial directions. It is confirmed that the shear strain is zero at top and bottom surfaces. This condition has been justified using the proposed shear deformation theory.

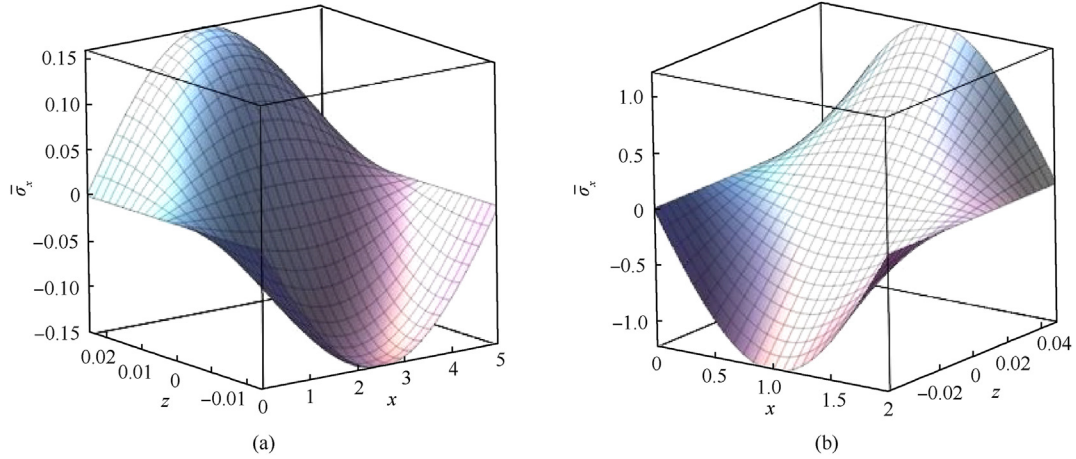


Fig. 7. Changes of non-dimensional axial stress $\bar{\sigma}_x$ along the radial and axial directions: (a) $R = 1$ m, $L = 5$ m, $t = 0.05$ m; (b) $R = 1$ m, $L = 2$ m, $t = 0.1$ m.

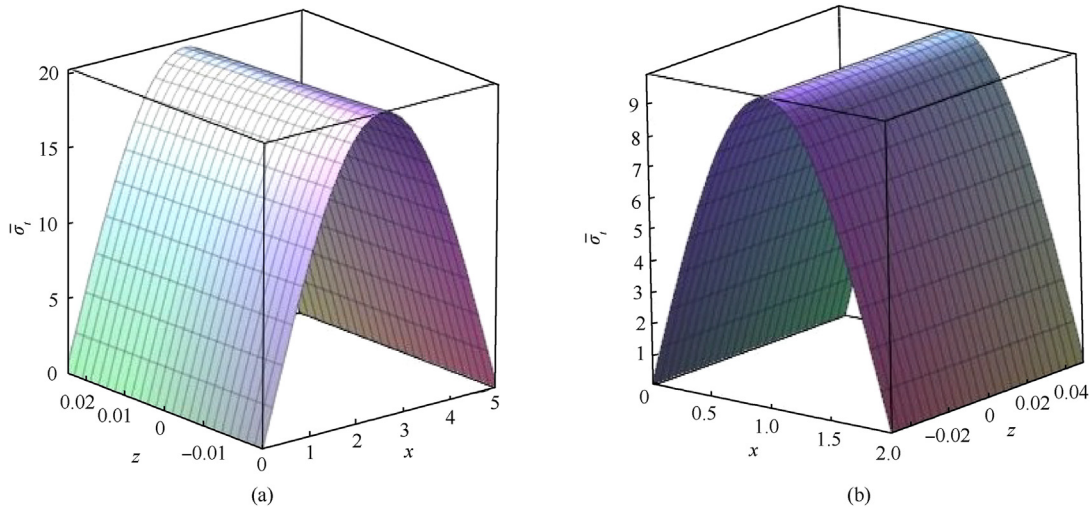


Fig. 8. Changes of non-dimensional hoop stress $\bar{\sigma}_r$ along the radial and axial directions: (a) $R = 1$ m, $L = 5$ m, $t = 0.05$ m; (b) $R = 1$ m, $L = 2$ m, $t = 0.1$ m.

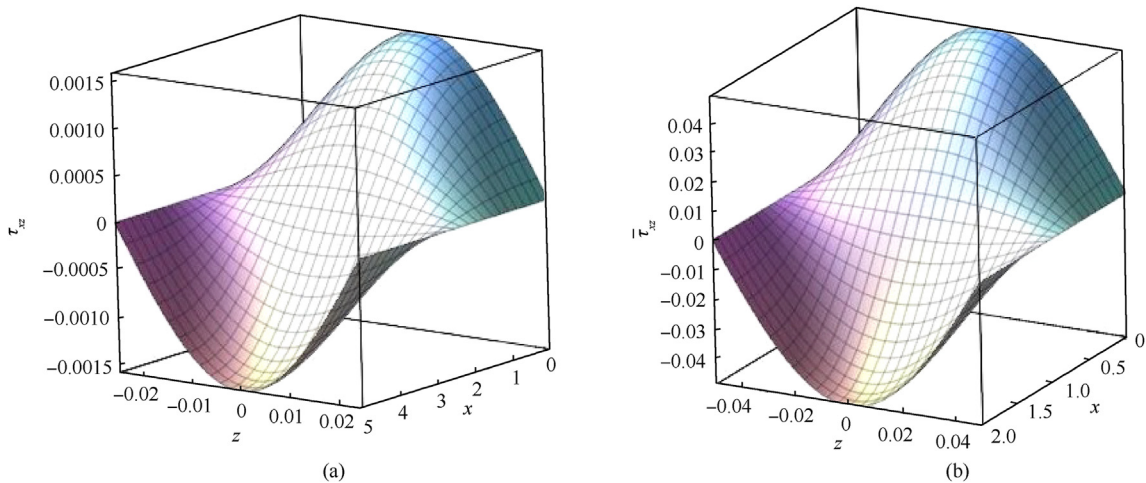


Fig. 9. Changes of non-dimensional shear stress $\bar{\tau}_{xz}$ along the radial and axial directions: (a) $R = 1$ m, $L = 5$ m, $t = 0.05$ m; (b) $R = 1$ m, $L = 2$ m, $t = 0.1$ m.

Shown in Fig. 7(a), Fig. 7(b) are two dimensional variation of non-dimensional axial stress along the radial and axial directions. It is observed that the maximum stresses are occurred at the middle of the shell and the top and bottom surfaces. Existence of maximum normal stress at middle length of the shell is due to maximum bending moment at same section.

Shown in Fig. 8(a), Fig. 8(b) are changes of non-dimensional circumferential stress $\bar{\sigma}_t$ along the radial and axial directions for both long and short cylindrical shells. The maximum circumferential stress is occurred at the middle length of the shell where the maximum deflection is occurred. Small change of circumferential stress is observed along the thickness direction.

Shown in Fig. 9(a), Fig. 9(b) are changes of non-dimensional shear stress $\bar{\tau}_{xz}$ along the radial and axial directions for both long and short cylindrical shells. It is observed that the shear stress is zero at top and bottom surfaces. Furthermore, the maximum shear stress is occurred at the middle surface.

4. Conclusions

Higher-order shear deformable model and thickness stretching formulation are employed in this research for mechanical stress, strain and deformation analyses of a cylindrical pressure vessel subjected to mechanical loads resting on Pasternak's foundation. The transverse deflection of the shell is assumed as the combination of bending, shear and stretching functions, in which the last term is responsible for variation of deflection along the thickness direction. After derivation of the governing equations through the principle of virtual work, the analytical solution is developed for stress and strain analyses. The mechanical stress, strain and

deformation are presented along the length and thickness directions for two long and short cylindrical shells. A sinusoidal shape function is used for exact and accurate modeling of the shear stress along the thickness direction. The main results of this paper are classified as follows:

- The changes of results along the thickness direction indicates that there is no significant changes for all stress and strain components except shear stress and strain the are changed significantly.
- The results show that the transverse shear stress and strains are satisfied zero stress conditions at top and bottom surfaces.
- The results indicate that the main stress is hoop stress that should be accounted in design procedure.
- Based on the outputs, it is concluded that the shear strain and stress become very important for short cylindrical shells.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A

$$\{\varphi_1, \varphi_2, \varphi_3\} = \int_z r \Xi (1 - \vartheta) \{1, z, f(z)\} dz, \{\varphi_4, \varphi_5\} = \int_z \Xi \vartheta \{1, g(z)\} dz, \{\varphi_6\} = \int_z r \Xi \vartheta g'(z) dz$$

$$\{\varphi_7, \varphi_8, \varphi_9\} = \int_z zr \Xi (1 - \vartheta) \{1, z, f(z)\} dz, \{\varphi_{10}, \varphi_{11}\} = \int_z z \Xi \vartheta \{1, g(z)\} dz, \{\varphi_{12}\} = \int_z zr \Xi \vartheta g'(z) dz$$

$$\{\varphi_{13}, \varphi_{14}, \varphi_{15}\} = \int_z f(z) r \Xi (1 - \vartheta) \{1, z, f(z)\} dz, \{\varphi_{16}, \varphi_{17}\} = \int_z f(z) \Xi \vartheta \{1, g(z)\} dz, \{\varphi_{18}\} = \int_z f(z) r \Xi \vartheta g'(z) dz$$

$$\{\varphi_{19}, \varphi_{20}\} = \int_z \frac{\Xi (1 - \vartheta)}{r} \{1, g(z)\} dz, \{\varphi_{21}, \varphi_{22}, \varphi_{23}\} = \int_z \Xi \vartheta \{1, z, f(z)\} dz, \{\varphi_{24}\} = \int_z \Xi \vartheta g'(z) dz$$

$$\{\varphi_{25}, \varphi_{26}\} = \int_z \frac{\Xi (1 - \vartheta)}{r} g(z) \{1, g(z)\} dz, \{\varphi_{27}, \varphi_{28}, \varphi_{29}\} = \int_z \Xi \vartheta g(z) \{1, g(z)\} \{1, z, f(z)\} dz, \{\varphi_{30}\} = \int_z \Xi \vartheta g(z) g'(z) dz$$

$$\varphi_{31} = \int_z r g'(z) \Xi (1 - \vartheta) g'(z) dz, \{\varphi_{32}, \varphi_{33}\} = \int_z g'(z) \Xi \vartheta \{1, g(z)\} dz, \{\varphi_{34}, \varphi_{35}, \varphi_{36}\} = \int_z r g'(z) \Xi \vartheta \{1, z, f(z)\} dz$$

$$\{\varphi_{37}, \varphi_{38}\} = \int_z \Xi \frac{1 - 2\vartheta}{2} r \{1 - f'(z)\} \{1 - f'(z), g(z)\} dz$$

$$\{\varphi_{39}, \varphi_{40}\} = \int_z \Xi \frac{1 - 2\vartheta}{2} r g(z) \{1 - f'(z), g(z)\} dz$$

Appendix B. Supplementary data

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.dt.2022.07.003>.

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