CREAT. MATH. INFORM. Volume **31** (2022), No. 2, Pages 183 - 188

**Online version at https://semnul.com/creative-mathematics/ Print Edition: ISSN 1584 - 286X; Online Edition: ISSN 1843 - 441X** DOI: https://doi.org/10.37193/CMI.2022.02.04

# **Fuzzy Minimal and Maximal** δ**-Open Sets**

S. JOSEPH and D. R. KIRUBAHARAN

ABSTRACT. The aim of this article is to introduce fuzzy minimal  $\delta$ -open and fuzzy maximal  $\delta$ -open sets in fuzzy topological space. Further, we investigate some properties with these new sets.

### 1. INTRODUCTION

Zadeh [6] established fuzzy set in 1965 and Chang [2] introduced fuzzy topology in 1968. Ittanagi and Wali [3] instigated the notions of fuzzy maximal and minimal open sets. The notion of fuzzy  $\delta$ -open set introduced by Supriti Saha [5]. In this paper, we introduce fuzzy minimal  $\delta$ -open and fuzzy maximal  $\delta$ -open sets. Further some of their related results investigated.

A proper nonempty fuzzy open set  $U$  of  $X$  is said to be a fuzzy minimal open [3] set if U and  $0<sub>X</sub>$  are only fuzzy open sets contained in U.

A proper nonempty fuzzy open set  $U$  of  $X$  is said to be a fuzzy maximal open [3] set if  $1<sub>X</sub>$  and U are only fuzzy open sets containing U.

A fuzzy subset  $K$  of a space  $X$  is called fuzzy regular open [1] (resp. fuzzy regular closed) if  $K = \text{Int}(\text{Cl}(K))$  (resp.  $K = \text{Cl}(\text{Int}(K))$ ).

The fuzzy  $\delta$ -interior of a fuzzy subset K of X is the union of all fuzzy regular open sets contained in K. A fuzzy subset K is called fuzzy  $\delta$ -open [5] if  $K = \text{Int}_{\delta}(K)$ . The complement of fuzzy  $\delta$ -open set is called fuzzy  $\delta$ -closed (i.e,  $K = \text{Cl}_{\delta}(K)$ ).

## 2. FUZZY MINIMAL δ-OPEN SETS

**Definition 2.1.** A proper nonzero fuzzy  $\delta$ -open set U in fts  $(X, \tau)$  is said to be fuzzy minimal  $\delta$ -open if fuzzy  $\delta$ -open set contained in U is  $0_X$  or U.

**Lemma 2.1.** *Let*  $(X, \tau)$  *be a fts.* 

*(i) If*  $U_1$  *is fuzzy minimal*  $\delta$ -open and  $U_2$  *is fuzzy*  $\delta$ -open in  $X$ *, then*  $U_1 \cap U_2 = 0_X$  or  $U_1 \subset U_2$ *. (ii)* If  $U_1$  *and*  $U_2$  *are fuzzy minimal*  $\delta$ -open, then  $U_1 \cap U_2 = 0_X$  or  $U_1 = U_2$ .

*Proof.* (i) Let us assume that  $U_2$  is fuzzy  $\delta$ -open in X such that  $U_1 \cap U_2 \neq 0_X$ . Since  $U_1$  is fuzzy minimal  $\delta$ -open, and  $U_1 \cap U_2 \subset U_1$ , then  $U_1 \cap U_2 = U_1$  implies that  $U_1 \subset U_2$ .

(ii) Suppose that  $U_1 \cap U_2 \neq 0_X$ , then clearly from(ii),  $U_1 \subset U_2$  and  $U_2 \subset U_1$  as  $U_1$  and  $U_2$  are fuzzy minimal  $\delta$ -open . Hence  $U_1 = U_2$ .

**Theorem 2.1.** Let U and  $U_i$  are fuzzy minimal  $\delta$ -open sets for any  $i \in M$ . If  $U \subseteq \bigcup U_i$ , then i∈M

 $U = U_j$  for any  $j \in M$ .

Received: 11.10.2021. In revised form: 24.01.2022. Accepted: 31.01.2022

<sup>2010</sup> *Mathematics Subject Classification.* 54A40, 03E72.

Key words and phrases. *Fuzzy regular open set, Fuzzy* δ*-open set, Fuzzy minimal* δ*-open set, Fuzzy maximal* δ*-open set.*

Corresponding author: S. Joseph; jjalenjose@gmail.com

*Proof.* Suppose  $U \subseteq \bigcup$  $\bigcup_{i\in M} U_i$ , then  $U = U \cap \left(\bigcup_{i\in M}$  $\bigcup_{i\in M}U_i\bigg)=\bigcup_{i\in M}$  $\bigcup_{i\in M} (U\cap U_i)$ . By deploying lemma 2.1(ii),  $U \cap U_i = 0_X$  or  $U = U_i$  as U and  $U_i$  are fuzzy minimal  $\delta$ -open sets. If  $U \cap U_i = 0_X$ , then  $U = 0_X$  which contradicts that U is a fuzzy minimal δ-open set. Hence if  $U \cap U_i \neq 0_X$ then  $U = U_j$  for any  $j \in M$ .

**Theorem 2.2.** If U and  $U_i$  are fuzzy minimal  $\delta$ -open sets for any  $i \in M$  and  $U \neq U_i$ , then  $U \cap \Big( \bigcup$  $\bigcup_{i\in M} U_i$   $\bigg) = 0$  *x* for any  $i \in M$ .

*Proof.* Let  $U \cap \Big( \ \bigcup$  $\bigcup_{i\in M} U_i$   $\Big) \neq 0_X$ , then  $U\cap U_i \neq 0_X$  for any  $i\in M$ . By deploying lemma  $\sqrt{2}$  $\setminus$ 

2.1(ii), 
$$
U = U_i
$$
 contradictory to  $U \neq U_i$ . Hence  $U \cap \left(\bigcup_{i \in M} U_i\right) = 0_X$ .

**Theorem 2.3.** If  $U_i$  is a fuzzy minimal  $\delta$ -open for any  $i \in M$  ( $|M| \geq 2$ ) and  $U_i \neq U_j$  for any *distinct* i, j <sup>∈</sup> <sup>M</sup>*, then* S  $\bigcup_{i\in M\setminus\{j\}} U_i$  $\setminus$  $\cap U_j = 0_X$  for any  $j \in M$ .

*Proof.* Let 
$$
\left(\bigcup_{i \in M \setminus \{j\}} U_i\right) \cap U_j \neq 0_X
$$
. Then  $\bigcup_{i \in M \setminus \{j\}} (U_i \cap U_j) \neq 0_X \Rightarrow (U_i \cap U_j) \neq 0_X$ . By

lemma 2.1(ii),  $U_i = U_j$ , a contradiction. Hence  $\begin{pmatrix} & & \ & \end{pmatrix}$  $\bigcup_{i\in M\setminus\{j\}} U_i$  $\bigcap U_j = 0_X$  for any  $j \in M$ .  $\square$ 

**Theorem 2.4.** If  $U_i$  is a fuzzy minimal  $\delta$ -open for any  $i \in M$ , ( $|M| \geq 2$ ) and  $U_i \neq U_j$  for any *distinct* i, j <sup>∈</sup> <sup>M</sup>*. If* <sup>K</sup> *is a proper fuzzy set of* <sup>M</sup>*, then* S  $\bigcup_{i\in M\setminus K} U_i$  $\setminus$ n ( u  $\bigcup_{s\in K} U_s\bigg)=0_X.$ 

*Proof.* Let  $\left(\begin{array}{c}$  $\bigcup_{i\in M\setminus K} U_i$  $\setminus$ ก( บ  $\Bigcup_{s\in K} U_s \Big) \neq 0_X.$  It implies that  $\bigcup \left( U_i \cap U_s \right) \neq 0_X$  for  $i \in M \backslash K$ and  $s \in K$  implies that  $U_i \cap U_s \neq 0$ <sub>X</sub> for some  $i \in M$  and  $s \in K$ . By lemma2.1(ii),  $U_i = U_s$ , which is a contradiction. Hence S  $\bigcup_{i\in M\setminus K} U_i$  $\setminus$  $\cap$ (U  $\bigcup_{s\in K} U_s$  = 0x.

**Theorem 2.5.** If  $U_i$  is a fuzzy minimal  $\delta$ -open for any  $i \in M$  such that  $U_i \neq U_j$  for any distinct  $i,j\in M.$  If  $S$  is a proper nonzero fuzzy set of  $M$ , then  $\begin{bmatrix} \quad \mid \end{bmatrix}$  $\bigcup_{i\in M\setminus k} U_i$ 1 ∩∫u  $\left[\bigcup_{k\in S} U_k\right] = 0_X.$ 

*Proof.* Assume that  $\cup [U_i \cap U_k] \neq 0$ <sub>X</sub> for  $i \in M \setminus k$ ,  $k \in S$ . Clearly, for some  $i \in M$ ,  $k \in S$  we have  $[U_i \cap U_k] \neq 0_X$ . By deploying lemma 2.1(ii)  $U_i = U_k$ , a contradiction.  $□$ 

**Theorem 2.6.** *If*  $U_i$  *and*  $U_k$  *are fuzzy minimal*  $\delta$ -open sets for *any*  $i \in M$  *and*  $k \in S$  *and* if  $\exists$  *an*  $n \in S$  such that  $U_i \neq U_n$  for any  $i \in M$  , then  $\Big[ \begin{array}{c} \bigcup \ \end{array} \Big]$  $\left[\bigcup_{n\in K}U_n\right]\not\subset \left[\bigcup_{i\in K}$  $\bigcup_{i\in M} U_i\bigg].$ 

*Proof.* Assume that  $\exists$  an  $n \in S$  such that  $U_i \neq U_n$  for any  $i \in M$ , then  $\Big[ \bigcup_{i=1}^n V_i \Big]$  $\bigcup_{n\in K} U_n\bigg] \subset$  $\lceil \ \cup$  $\bigcup_{i\in M} U_i\bigg].$ 

 $\Rightarrow U_n \subset \left[ \begin{array}{c} \bigcup \ \ \end{array} \right]$  $\left[\bigcup_{i\in M} U_i\right]$  for some  $n \in K$ .

 $\Rightarrow U_i \neq U_n$  for any  $i \in M$ , by theorem 2.1, which is a contradiction. Hence  $\Big[ \,\, \bigcup_{i=1}^n A_i \Big]$  $\bigcup_{n\in K}U_n\bigg]\not\subset$  $\lceil \ \cup$  $\bigcup_{i\in M}U_i\Bigg]$ . □

**Theorem 2.7.** If  $U_i$  is a fuzzy minimal  $\delta$ -open for any  $i \in M$  such that  $U_i \neq U_j$  for any distinct  $i,j\in M$  , then  $\Big[\bigcup$  $\left[\bigcup_{k\in K} U_k\right] \subsetneqq \left[\bigcup_{i\in K}$  $\left[\bigcup_{i\in M}U_i\right]$  for any proper nonzero subset  $K$  of  $M.$ 

*Proof.* Let  $m \in M\backslash K$ , then  $U_m$  is a fuzzy minimal  $\delta$ -open set of the family  $\{U_m|m \in K\}$  $\mathbb{M}\backslash K\}$  of fuzzy minimal  $\delta$ -open sets. Clearly  $U_m\cap \left[\begin{array}{c} 0 \end{array} \right]$  $\left[\bigcup_{k\in K}U_k\right]=\bigcup_{k\in K}$  $\bigcup_{k\in K} [U_m \cap U_k] = 0_X$ . Also

$$
U_m \cap \left[\bigcup_{i \in M} U_i\right] = \bigcup_{i \in M} [U_m \cap U_i] = U_m.
$$
  
If 
$$
\left[\bigcup_{k \in K} U_k\right] = \left[\bigcup_{i \in M} U_i\right], \text{ then } U_m = 0_X \text{ which is a contradiction that } U_m \text{ is a fuzzy minimal } \delta \text{-open set. Hence } \left[\bigcup_{k \in K} U_k\right] \subsetneq \left[\bigcup_{i \in M} U_i\right].
$$

**Theorem 2.8.** If  $U_i$  is a fuzzy minimal  $\delta$ -open set for any  $i \in M$  such that  $U_i \neq U_j$  for any *distinct*  $i, j \in M$ *, then* 

(i) 
$$
U_j \subset \left[\bigcup_{i \in M \setminus \{j\}} U_i\right]^c
$$
 for some  $j \in M$ .  
\n(ii)  $\bigcup_{i \in M \setminus \{j\}} U_i \neq 1_X$  for any  $j \in M$ .

*Proof.* (i) By hypothesis,  $U_i \neq U_j$  for any distinct  $i, j \in M$ . By theorem 2.2,  $\begin{bmatrix} \cup \end{bmatrix}$  $\left[\bigcup_{i\in M}U_i\right]\cap U_j=0_X$  which is true for any  $j\in M.$ ⇒ U  $\bigcup_{i\in M} [U_i \cap U_j] = 0_X$  $\Rightarrow$   $U_i \cap U_j = 0_X$  (By Lemma 2.1(ii))  $\Rightarrow U_i \subset U_j^c$ ⇒ S  $\bigcup_{i\in M\setminus\{j\}} U_i \subset U_j^c$ . Hence proved. (ii) Let  $j \in M$  such that  $\bigcup$  $\bigcup_{i\in M\setminus\{j\}} U_i = 1_X$  $\Rightarrow U_i = 0_X$ 

 $\Rightarrow U_i$  is not a fuzzy minimal  $\delta$ -open set, a contradiction. Hence  $\bigcup$  $\bigcup_{i\in M\setminus\{j\}} U_i\neq 1_X$  for any  $j \in M$ .

**Corollary 2.1.** If  $U_i$  is a fuzzy minimal  $\delta$ -open set for any  $i \in M$  such that  $U_i \neq U_j$  for any *distinct*  $i, j \in M$ , then  $U_i \cup U_j \neq 1_X$  for any distinct  $i, j \in M$ .

*Proof.* Similar to that of "Theorem 2.8 (ii)." □

**Theorem 2.9.** If  $U_i$  is a fuzzy minimal  $\delta$ -open sets for any  $i \in M$  such that  $U_i \neq U_j$  for any distinct  $i,j\in M$ , then  $U_j=\left\lceil\,\,\bigcup\,\right\rceil$  $\bigcup_{i\in M} U_i\bigg] \cap$  $\sqrt{ }$ U  $\bigcup_{i\in M\setminus\{j\}} U_i$  $\overline{1}^c$ *for any*  $j \in M$ .

186  
\n*Proof.* For any 
$$
j \in M \Rightarrow \left[\bigcup_{i \in M} U_i\right] \cap \left[\bigcup_{i \in M \setminus \{j\}} U_i\right]^c = \left[\bigcup_{i \in M \setminus \{j\}} U_i \cup U_j\right] \cap \left[\bigcup_{i \in M \setminus \{j\}} U_i\right]^c\right]
$$
  
\n
$$
= \left[\left(\bigcup_{i \in M \setminus \{j\}} U_i\right) \cap \left(\bigcup_{i \in M \setminus \{j\}} U_i\right)^c\right] \cup \left[U_j \cap \left(\bigcup_{i \in M \setminus \{j\}} U_i\right)^c\right]
$$
\n
$$
= 0_X \cup U_j
$$
\n
$$
= U_j \text{ for any } j \in M.
$$

**Proposition 2.1.** *Let* G *be a fuzzy minimal*  $\delta$ -open set. If  $x_\alpha \in G$ , then  $G \subset G_1$  for any fuzzy *open neighbourhood*  $G_1$  *of*  $x_\alpha$ *.* 

*Proof.* Let  $G_1$  be an fuzzy  $\delta$ -open neighborhood of  $x_\alpha$  such that  $G \not\subset G_1$ . Clearly  $G \cap G_1$  is an fuzzy  $\delta$ -open such that  $G \cap G_1 \subsetneq G$  and  $G \cap G_1 \neq 0_X$ . This implies that G is a fuzzy minimal  $\delta$ -open set which a contradiction.  $\Box$ 

**Proposition 2.2.** *Let* G *be a fuzzy minimal* δ*-open set. Then*  $G = \bigcap \{G_1: G_1$  *fuzzy e-open neighbourhood of*  $x_\alpha$  *for any*  $x_\alpha \in G\}$  .

*Proof.* By deploying proposition 2.1 and as G is an fuzzy  $\delta$ -open neighborhood of  $x_{\alpha}$ , we have  $G \subset \bigcap \{G_1 : G_1 \text{ fuzzy }$  e-open neighbourhood of  $x_\alpha \subset G\}$  . This completes the proof.  $\Box$ 

**Theorem 2.10.** *Let* G *be a fuzzy minimal* δ*-open set. Then the following conditions are equivalent. (i)* G *is fuzzy minimal* δ*-open set.*

 $(ii)G \subset eCl(K)$  *for any nonzero fuzzy subset* K *of G*.  $(iii)eCl(G) = eCl(K)$  *for any nonzero fuzzy subset* K *of G*.

*Proof.* (i)  $\Rightarrow$  (ii): By deploying "proposition 2.1"for any  $x_{\alpha} \in G$  and fuzzy  $\delta$ -open neighborhood M of  $x_{\alpha}$ , we have  $K = (G \cap K) \subset (M \cap K)$  for any proper nonzero fuzzy subset  $K \subset G$ . Therefore, we have  $(M \cap K) \neq 0_X$  and  $x_\alpha \in eCl(K)$ . It follows that  $G \subset eCl(K)$ .

 $(ii) \Rightarrow (iii)$ : For any proper fuzzy subset K of G,  $eCl(G) \subset eCl(K)$ . Also by $(ii)$   $eCl(G) \subset$  $eCl(eCl(K)) = eCl(K)$ . Hence proved.

 $(iii) \Rightarrow (i)$ : Let us assume that G is not fuzzy minimal  $\delta$ -open. Then there exists a proper fuzzy  $\delta$ -open D such that  $D \subset G$ . Then  $\exists y_{\alpha} \in G$  such that  $y_{\alpha} \notin D$ . Then  $eCl({y_{\alpha}}) \in D^c$  implies that  $eCl({y_{\alpha}}) \neq eCl(G)$ , a contradiction. This completes our proof.  $\Box$ 

#### 3. FUZZY MINIMAL  $\delta$ -OPEN SETS AND ITS PROPERTIES

**Definition 3.2.** A proper nonzero fuzzy  $\delta$ -open set V of a fts  $(X, \tau)$  is said to fuzzy maximal δ-open if any fuzzy δ-open set which contains V is either V or  $1_X$ .

**Example 3.1.** Let  $X = \{a, b, c, d\}$ . Then fuzzy sets  $\mathcal{F}_1 = \frac{0.3}{a} + \frac{0.4}{b} + \frac{0.3}{c} + \frac{0.4}{d}$ ;  $\mathcal{F}_2 =$  $\frac{0.5}{a} + \frac{0.5}{b} + \frac{0.5}{c} + \frac{0.5}{d}$ ;  $\mathcal{F}_3 = \frac{0.7}{a} + \frac{0.6}{b} + \frac{0.7}{c} + \frac{0.6}{d}$ ; are defined as follows: Consider the fuzzy topology  $\tau = \{0_X, \mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, 1_X\}$ . Here  $\mathcal{F}_1$  is fuzzy minimal  $\delta$ -open and  $\mathcal{F}_3$  fuzzy maximal  $δ$ -open set.

**Lemma 3.2.** Let  $(X, \tau)$  be a fts. Then

(i) If  $V_1$  is a fuzzy maximal  $\delta$ -open and  $V_2$  is fuzzy  $\delta$ -open in X, then  $V_1 \cup V_2 = 1_X$  or  $V_2 \subset V_1$ .

(ii) If  $V_1$  and  $V_3$  are fuzzy maximal  $\delta$ -open sets, then either  $V_1 \cup V_3 = 1_X$  or  $V_1 = V_3$ .

*Proof.* (i) Assume that  $V_2 \not\subset V_1$ . Clearly,  $V_1 \subset (V_1 \cup V_2)$  a contrary to  $V_1$  is a fuzzy maximal δ-open set if  $V_1$  ∪  $V_2$  ≠ 1<sub>X</sub>. Hence,  $V_1$  ∪  $V_2$  = 1<sub>X</sub>.

(ii) Let  $V_1$  and  $V_3$  are fuzzy maximal  $\delta$ -open sets. Then from(i)  $V_3 \subset V_1$  and  $V_1 \subset V_3$ implies that  $V_1 = V_3$ .

**Theorem 3.11.** If  $V_1, V_2$  and  $V_3$  are fuzzy maximal  $\delta$ -open sets such that  $V_1 \neq V_2$  and  $(V_1 \cap V_2) \subset V_3$ , then either  $V_1 = V_3$  or  $V_2 = V_3$ .

*Proof.* Suppose that  $V_1$ ,  $V_2$  and  $V_3$  are fuzzy maximal  $\delta$ -open sets with  $V_1 \neq V_2$ ,  $(V_1 \cap V_2) \subset$  $V_3$  and if  $V_1 \neq V_3$ , then

 $(V_2 \cap V_3) = V_2 \cap (V_3 \cap 1_X)$  $= V_2 \cap [V_3 \cap (V_1 \cup V_2)]$ , by lemma 3.2(ii)  $= V_2 \cap [(V_3 \cap V_1) \cup (V_3 \cap V_2)]$  $=[V_2 \cap V_3 \cap V_1] \cup [V_2 \cap V_3 \cap V_2]$  $=[V_2 \cap V_1] \cup [V_2 \cap V_3]$  $= V_2 \cap [V_1 \cup V_3]$  $= V_2 \cap 1_X$  $= V_2$  $(V_2 \cap V_3) = V_2 \Rightarrow V_2 \subset V_3$ . As  $V_2$  and  $V_3$  are fuzzy maximal  $\delta$ -open sets,  $V_3 \subset V_2$ .

Hence  $V_2 = V_3$ .

**Theorem 3.12.** *For any distinct fuzzy maximal*  $\delta$ *-open sets*  $V_1, V_2, V_3$  $[V_1 \cap V_2] \not\subset [V_1 \cap V_3]$ .

*Proof.* Consider  $[V_1 \cap V_2] \subset [V_1 \cap V_3]$  for any distinct fuzzy maximal  $\delta$ -open sets  $V_1, V_2$  and  $V_3$ . Then  $[V_1 \cap V_2] \cup [V_2 \cap V_3] \subset [V_1 \cap V_3] \cup [V_2 \cap V_3]$  $=[V_1 \cup V_3] \cap V_2 \subset [V_1 \cup V_2] \cap V_3$  $= 1_X \cap V_2 \subset 1_X \cap V_3$  $= V_2$  is contained in  $V_3$ a contradiction to  $V_1, V_2$  and  $V_3$  are distinct . Hence  $[V_1 \cap V_2] \not\subset [V_1 \cap V_3]$ .

**Remark 3.1.** Proofs of "Theorem 3.13, Corollary 3.2, Theorem 3.14 and Theorem 3.15" are similar to proofs of "Theorem 2.8, Corollary 2.1, Theorem 2.9 and Theorem 2.7" respectively. Hence the proofs are omitted.

**Theorem 3.13.** If  $V_i$  is a fuzzy maximal  $\delta$ -open sets for any  $i \in M, M$  is a finite set and  $V_i \neq V_j$ *for any distinct*  $i, j \in M$ *, then* 

*(i)*  $\bigcap$  $\bigcap_{i\in M\setminus\{j\}} V_i$  $\mathbb{1}^{\dot{c}}$  $\subset V_j$  *for any*  $j ∈ M$  $(ii) \cap V_i \neq 0$ <sub>X</sub> for any  $j \in M$ .  $i\in M\backslash\{j\}$ 

**Corollary 3.2.** If  $V_i$  is a fuzzy maximal  $\delta$ -open sets for any  $i \in M, M$  is a finite set and  $V_i \neq V_j$ *for any distinct*  $i, j \in M$  *then*, $V_i \cap V_j \neq 0$ <sub>X</sub> for any distinct  $i, j \in M$ .

**Theorem 3.14.** If  $V_i$  is a fuzzy maximal  $\delta$ -open sets for any  $i \in M, M$  is a finite set and  $V_i \neq V_j$ for any distinct  $i,j\in M$  , then  $V_j=\left\lceil\ \bigcap\ j\right\rceil$  $\left[\bigcap_{i\in M} V_i\right] \cup$  $\sqrt{ }$  $\cap$  $\bigcap_{i\in M\setminus\{j\}} V_i$  $\overline{1}^c$ *for any* j ∈ M*.*

**Theorem 3.15.** If  $V_i$  is a fuzzy maximal  $\delta$ -open sets for any  $i \in M, M$  is a finite set and  $V_i \neq V_j$ *for any distinct*  $i, j \in M$  *and if K is a proper nonzero fuzzy subset of M, then*  $\bigcap_{i \in M} V_i \subsetneqq \bigcap_{k \in I}$  $\bigcap_{k\in K} V_k$ .

**Theorem 3.16.** If  $V_i$  is a fuzzy maximal  $\delta$ -open sets for any  $i \in M, M$  is a finite set and  $V_i \neq V_j$ for any distinct  $i, j \in M$  and if  $\bigcap\limits_{i \in M} V_i$  is a fuzzy subset, then  $V_j$  is a fuzzy subset for any  $j \in M.$ 

*Proof.* By "Theorem 3.14", we have  $V_j = \left\lceil \; \bigcap \right\rceil$  $\left[\bigcap_{i\in M}V_i\right]\cup$  $\sqrt{ }$  $\cap$  $\bigcap_{i\in M\setminus\{j\}} V_i$  $\overline{1}^{\overline{c}}$ for any  $j \in M$ .  $V_j=\left\lceil \begin{array}{c} \bigcap \end{array} \right\rceil$  $\left[\bigcap_{i\in M}V_i\right]\cup$  $\lceil$ U  $V_i^c$ 1 .

 $i\in M\backslash\{j\}$ Since M is finite,  $\bigcup \overline{V_i^c}$  is fuzzy  $\delta$ -closed. Hence  $V_j$  is fuzzy  $\delta$ -closed for any  $j \in$  $i\in M\setminus\{j\}$  $M$ .

**Theorem 3.17.** If  $V_i$  is a fuzzy maximal  $\delta$ -open set for any  $i \in M$ , M is a finite set and  $V_i \neq V_j$ *for any distinct*  $i, j \in M$ *. If*  $\bigcap_{i \in M} V_i = 0_X$ ,then  $\{V_i / i \in M\}$  *is the set of all fuzzy maximal*  $\delta$ *-open* 

*sets of fts* X*.*

*Proof.* Suppose that ∃another fuzzy maximal  $\delta$ -open  $V_k$  of a fts  $X$  such that  $V_k \neq V_i, \forall i \in M$ . Clearly,  $0_X = \bigcap$  $\bigcap_{i\in M}V_i=\bigcap_{i\in (M\cup k)}$  $i \in (M \cup k) \setminus \{k\}$  $V_i \neq 0_X$ , by Theorem 3.13(ii), a contradiction.

Hence  $\{V_i/i \in M\}$  is the family of all fuzzy maximal  $\delta$ -open sets of fts X.

#### **REFERENCES**

- [1] Azad, K. K. On fuzzy semi-continuity, fuzzy almost continuity and fuzzy weakly continuity. *J. Math. Anal. Appl.* **82** (1981), 14–32.
- [2] Chang, C. L. Fuzzy topological spaces. *J. Math.Anal.Appl.* **24** (1968), 182–190.
- [3] Ittanagi, B. M.; Wali, R. S. On fuzzy minimal open and fuzzy maximal open sets in fuzzy topological spaces. *Int. j. math. sci. appl.* **1** (2011), no. 2
- [4] Joseph, S.; Kirubaharan, D. R.*Fuzzy minimal and maximal* δ*-open sets*(communicated).
- [5] Supriti, S. Fuzzy δ-continuous mappings. *J. Math. Anal. Appl.* **126** (1987), 130–142.
- [6] Zadeh, L. A. Fuzzy sets. *Inf Comput.*, **8** (1965), 338–353.

DEPARTMENT OF MATHEMATICS RESEARCH SCHOLAR PRIST DEEMED TO BE UNIVERSITY THANJAVUR,TAMIL NADU-613403, INDIA *Email address*: jjalenjose@gmail.com

DEPARTMENT OF MATHEMATICS PRIST DEEMED TO BE UNIVERSITY THANJAVUR,TAMIL NADU-613403, INDIA *Email address*: jagkans@yahoo.com