

The Mathematics Enthusiast

Manuscript 1605

The Hocus Pocus of Martin Gardner A Review of Martin Gardner's Undiluted Hocus-Pocus: The Autobiography of Martin Gardner

Neil J. Hatfield

Follow this and additional works at: <https://scholarworks.umt.edu/tme>

Let us know how access to this document benefits you.

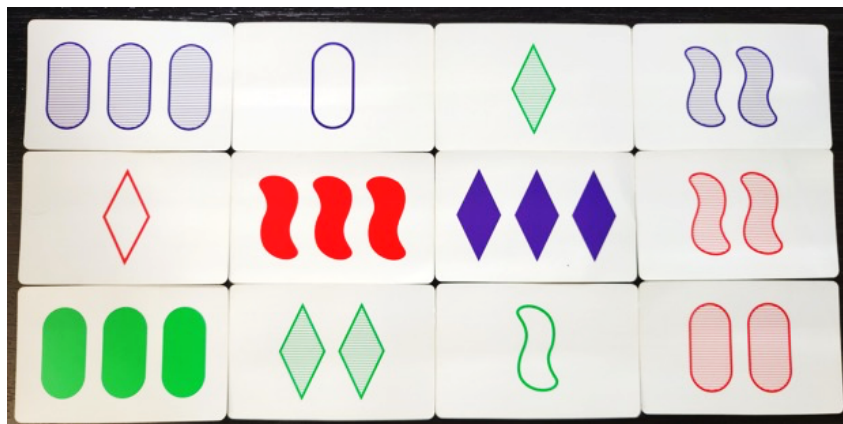
The Hocus Pocus of Martin Gardner
A Review of Martin Gardner's *Undiluted Hocus-Pocus: The Autobiography of Martin Gardner*

Neil J. Hatfield¹
Pennsylvania State University

Begin with a strip of paper 11 inches long and 2.5 inches wide. Orient the strip so that the long sides form the top and bottom. Draw a line along the length of the strip half an inch from the top. Place a mark on this line half an inch from the left side, label A. From A, draw five more marks, each 1.75 inches from the last; label them B-F. Draw one last mark $\frac{7}{8}$ of an inch after F, called G. Draw a perpendicular line from G across the strip widthwise. Draw a diagonal line that is 1.75 inches long from mark F to the line extending from G; call the point of intersection H.

I have always enjoyed games, especially when incorporated into the classroom. I can still remember the game my fifth-grade teachers had us play to explore colonizing a new world. One of the most impactful games I've played is SET®, which has become a go-to for mathematics teachers as a means for exploring geometry, combinatorics, probability, statistics, and vector spaces (Larson Quinn et al., 1999; Waddell Jr., 2017). I first learned the game in the seventh grade and continue to play the game today. How many sets do you see in Figure 1?

Figure 1. A SET® game board ready for play



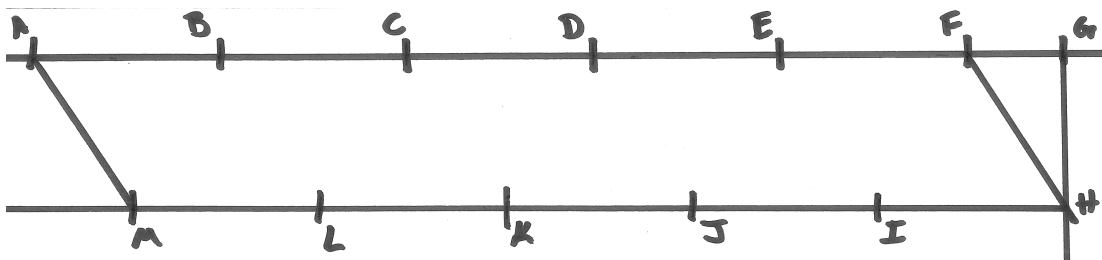
¹ njh5464@psu.edu

The double opening might seem a bit strange, but they have their purposes. The first I'll leave as a bit of mystery except to say these are instructions I've come up with based upon Gardner's (1988). Games, or more broadly play, can be a powerful tool to transform the classroom from an instructor-centered session to a student-centered and engaging environment. Games of a mathematical nature fall into the realm of recreational mathematics. Sumpter notes that recreational mathematics is a powerful "tool to educate people, *all people*, and not just a selected elite" (2015, p. 135 italics in original). I can think of no individual who had a greater impact on recreational mathematics than Martin Gardner. Gardner wrote nearly 300 *Mathematical Games* columns for *Scientific American*. *Undiluted Hocus-Pocus: The Autobiography of Martin Gardner* (Gardner, 2015) provides a look at the man behind the columns.

The Man Behind the Curtain

From H, draw a line the length of the strip, perpendicular to the line of GH. Along this new line, create new marks, every 1.75 inches starting from H; label them I-M. Draw a diagonal line from A to M; the line should be 1.75 inches long and parallel to the line FH. (See Figure 2)

Figure 2. Drawn parallelogram



Undiluted Hocus-Pocus is written in the style of a beloved grandparent telling a story with lots of tangents. While at first glance, the diversions feel out of place, they play a role in bringing the entire tapestry together. Gardner shares with the reader his earliest memories from growing up in Tulsa, Oklahoma, living in Chicago and New York, before finally living out his twilight years in Norman, Oklahoma.

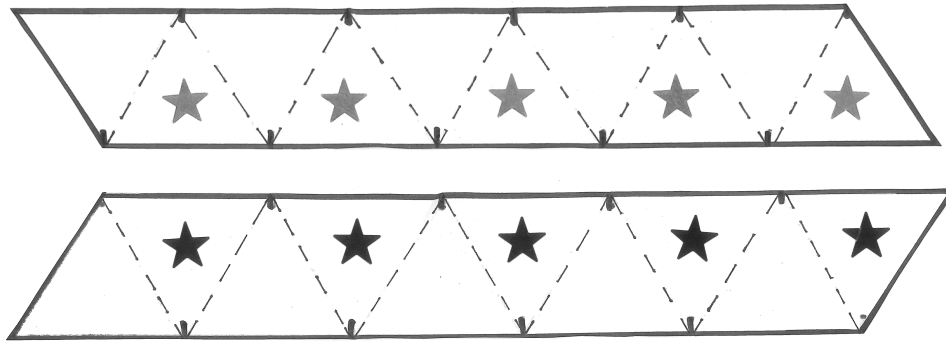
There are several aspects of Gardner that come through: his love of writing and persistent curiosity. Poetry and literature flow throughout Gardner's stories, particularly his love for the worlds of Oz and Wonderland (he even wrote his own tales involving these worlds and their characters). Writing was something much more than a means to make a living for Gardner; it was an expression of joy and learning.

Many people think of Gardner as a scientist and mathematician. After all, he is best known for his column in *Scientific American*. He was also friends with a number of scientists and mathematicians including John Tukey, Fred Mosteller, Roger Penrose, Benoit Mandelbrot, Donald Knuth, and John Conway. What might be surprising is that Gardner majored in philosophy and took no math courses while attending university. He also admits to only somewhat understanding calculus, being the most advanced math class, he ever took. His curiosity shows itself through his drive to understand things around him. Whether it is a breakthrough in the sciences, mathematics, a puzzle, a game, or a magic trick, he dives in headfirst. Many of the famous mathematicians and scientists he got to know, he did so through connections he made by studying magic tricks (another life-long passion that inspired the title of the autobiography). A lasting message from *Hocus-Pocus* is that exploring and building your own understanding of the world is possible for anyone; anyone can become a mathematician and scientist.

Recreational Mathematics and Play

Cut out the parallelogram. Fold the parallelogram to make 10 equilateral triangles; you can use your marks from earlier (B-F, H-M) to make the diagonal folds. Make the folds both forwards and back to make them easier to work with. Decorate the triangles as you see fit. (See Figure 3)

Figure 3. Cut, folded (dashed lines), and decorated parallelogram (upper: the front; lower, the back)



Martin Gardner was a pioneer of recreational mathematics. While *Hocus-Pocus* is rather silent on the impact Gardner had on mathematics education, there is a body of work on the role of recreational mathematics in the education. Sumpter (2015) explored historical texts and educational policies around the issue of recreational mathematics and found a common theme of making mathematics fun. Bradfield (1970) suggested that teachers use amusing, mystifying, and/or dramatic problems to spark student interest in the classroom. While Bradfield's article is 50 years old, its recommendations are still insightful for determining whether some activity might be enjoyable to our students. However, using fun, amusing, mystifying, and dramatic as the indicators for recreational mathematics results in an overly broad realm. I find the following working definition useful:

Recreational mathematics is a type of play which is enjoyable and requires mathematical thinking or skills to engage with; typically, it is accessible to a wide range of people and can be effectively used to motivate engagement with and develop understanding of mathematical ideas or concepts. (Rowlett et al., 2019, p. 978)

This working definition is broad enough to encompass tasks such as games, puzzles, as well as problem solving. Further, it places an emphasis on learning aspects (student understandings, reasoning, and thinking). While there are still subjective elements to this definition (enjoyable, motivating), as educators we can embrace the Piagetian notion of decentering: the act of stepping out of ourselves and into the role of another to gain insight to their actions (Montangero & Maurice-Naville, 1997). This would allow us to view the

subjective elements through our students' thinking. For both pre-service and in-service teachers, working with them to decenter on what students might view as enjoyable/motivating is lower stakes and potentially easier than how students might be thinking about multiplication, solving equations, logical implications, or randomness. Thus, this could serve as a starting point to help teachers begin to use decentering to think about student thinking. This could ultimately allow them to engage in conceptual analysis (Thompson, 2008) both of potential recreational mathematics tasks and student work.

One powerful tool for recreational mathematics is the idea of Math Circles. These social gatherings provide the opportunity for participants of all ages and backgrounds to tackle mathematical problems in a congenial environment (Kennedy & Smolinsky, 2016). Wieggers and White (2016) detail the history of Math Circles in the United States and provide some sample problems. Wieggers, Lai, and White (2016) followed students participating in six different math circles in San Francisco over four years. They found that participants expressed higher interest in mathematics. Further, their results suggest that long-term participants appear to have higher levels for doing well in math for intrinsic and extrinsic reasons, pointing towards a cumulative effect. Perhaps more importantly was the impact of the math circles on students' perceptions of mathematicians: "these participants diversified their descriptions of mathematicians to include tattoos, women, and more people with an outward appearance similar to the students" (Wieggers et al., 2016, p. 106). The benefits of math circles are not limited to just students. Math teacher circles have also had some positive impacts on teachers' mathematical knowledge for teaching (White et al., 2013).

I wish to return to what is perhaps the most important part of recreational mathematics: play. Piaget classified play into four types: exercise (play for the joy of it), symbolic, rule games (e.g., tic-tac-toe, SET®), and construction games (Montangero & Maurice-Naville, 1997; Piaget, 1995). Piaget (1995) wrote:

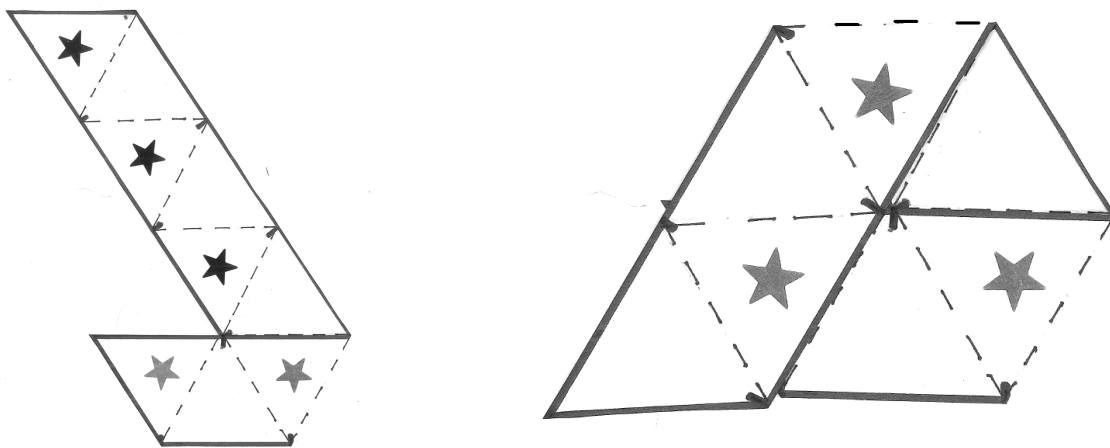
It is indispensable to his [a child's] affective and intellectual equilibrium, therefore, that he have available to him an area of activity whose motivation is not adaptation to reality but, on the contrary, assimilation of reality to the self, without coercions or sanctions. Such an area is play, which transforms reality by assimilation to the needs of the self. (p. 492)

Symbolic play enables a child to relive events so that they might resolve conflicts and deal with unmet needs. For example, through symbolic play a child might have a doll speak up and answer a question (show bravery) when the child did not in class. Play lets students explore their world. Out of symbolic play rise construction games “initially imbued with play symbolism but tend later to constitute genuine adaptations (mechanical constructions, etc.) or solutions to problems and intelligent creations” (Piaget, 1995, p. 493). Recreational mathematics provides an opportunity for individuals to engage in these various forms of play, culminating in games of construction that lead to new and/or revised understandings. To learn more about recreational mathematics, I recommend checking out Gathering 4 Gardner (www.gathering4gardner.com); they hold monthly (virtual) events and have a wealth of resources.

Closing the Loop

Fold the strip along KL, so that the rest of the strip is behind the first three triangles (Figure 4 left). Fold again at DJ (on reverse) down so that you cover first three triangles Figure 4 right). Fold the tail triangle (FI) backwards to wrap around the first triangle. Glue the two sides facing each other together. You’ve now created a hexagonal Möbius strip.

Figure 4. Folding the hexaflexagon (left: first fold; right: second fold)

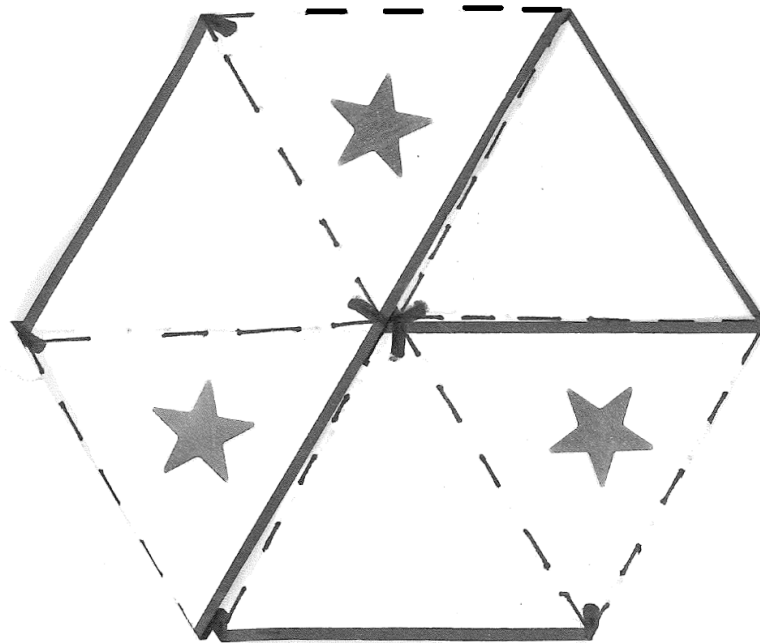


Undiluted Hocus-Pocus provides a look behind the curtain of Gardner’s magic. The instructions that I began in the first opening and the subsequent italicized paragraphs

between the sections of this article layout the construction of a hexaflexagon (Gardner, 1988). This particular hexaflexagon is a trihexaflexagon which will display three different faces. The final paragraph will help you to expose/change these faces. Gardner's columns provide a wealth of material for bringing recreational mathematics into classrooms at any level. Flexagons were Gardner's second article for *Scientific American* and the impetus for his *Mathematical Games* column. What better magic trick than to end at the beginning?

Pick any two adjacent triangles in the hexaflexagon and pinch them together by folding the crease between them downwards. Push the vertex opposite of the pinched sides down towards the center to make a three-pointed shape. One of the edges you're not currently holding should be openable; open it and release the sides you're holding. As you open, gently push the paper through to turn the hexaflexagon inside out. Repeat as desired to create new patterns and designs. The more you play with the hexaflexagon, the easier it will be to manipulate it.

Figure 5. A (tri-) hexaflexagon



References

- Bradfield, D. L. (1970). Sparking interest in the mathematics classroom. *The Arithmetic Teacher*, 17(3), 239–242. <https://doi.org/10.5951/AT.17.3.0239>
- Gardner, M. (1988). Hexaflexagons. In *Hexaflexagons and other mathematical diversions: The first Scientific American book of puzzles & games*. University of Chicago Press. https://www.maa.org/sites/default/files/pdf/pubs/focus/Gardner_Hexaflexagons12_1956.pdf
- Gardner, M. (2015). *Undiluted hocus-pocus: The autobiography of Martin Gardner*. <http://www.vlebooks.com/vleweb/product/openreader?id=none&isbn=9781400847983>
- Kennedy, E., & Smolinsky, L. (2016). Math circles: A tool for promoting engagement among middle school minority males. *Eurasia Journal of Mathematics, Science & Technology Education*, 12(4), 717–732.
- Larson Quinn, A., Koca Jr., R. M., & Weening, F. (1999). Developing mathematical reasoning using attribute games. *The Mathematics Teacher*, 92(9), 768–775.
- Montangero, J., & Maurice-Naville, D. (1997). *Piaget, or, The advance of knowledge*. Lawrence Erlbaum Associates, Inc.
- Piaget, J. (1995). *The essential Piaget* (100th Anniversary ed). J. Aronson.
- Rowlett, P., Smith, E., Corner, A. S., O’Sullivan, D., & Waldock, J. (2019). The potential of recreational mathematics to support the development of mathematical learning. *International Journal of Mathematical Education in Science and Technology*, 50(7), 972–986. <https://doi.org/10.1080/0020739X.2019.1657596>
- Sumpter, L. (2015). Recreational mathematics—Only for fun? *Journal of Humanistic Mathematics*, 5(1). <https://doi.org/10.5642/jhummath.201501.07>
- Thompson, P. W. (2008). Conceptual analysis of mathematical ideas: Some spadework at the foundation of mathematics education. *Proceedings of the Annual Meeting of the International Group for the Psychology of Mathematics Education*, 1, 45–64. <http://bit.ly/10YE9al>
- Waddell Jr., G. (2017, April 10). Geometry using the game of SET. *Mathematics Teacher*. <https://www.nctm.org/Publications/Mathematics-Teacher/Blog/Geometry-Using-the-Game-of-Set/>
- White, D., Donaldson, B., Hodge, A., & Ruff, A. (2013). Examining the effects of math teachers’ circles on aspects of teachers’ mathematical knowledge for teaching. *International Journal for Mathematics Teaching & Learning*, 14, 1–28.
- Wieggers, B., Lai, Y., & White, D. (2016). Exploring the effects of four years of a math circle on middle school and high school students’ mathematical task value. In M. J. Mohr-Schroeder & J. N. Thomas (Eds.), *Proceedings of the 115th Annual Convention of the School Science and Mathematics Association*. School Science and Mathematics Association.

Wiegers, B., & White, D. (2016). The establishment and growth of Math Circles in America. In M. Zack & E. Landry (Eds.), *Research in History and Philosophy of Mathematics* (pp. 237–248). Springer International Publishing.
https://doi.org/10.1007/978-3-319-46615-6_17