A-Hohfeld: A Language for Robust Structural Representation of Knowledge in the Legal Domain to Build Interpretation-Assistance Expert Systems

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A-Hohfeld: A Language for Robust Structural Representation of Knowledge in the Legal Domain to Build Interpretation-Assistance Expert Systems

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ABSTRACT

The A-Hohfeld language is presented as a set of definitions; it can be used to precisely express legal norms. The usefulness of the A-Hohfeld language is illustrated in articulating 2560 alternative structural interpretations of the four-sentence 1982 Library Regulations of Imperial College and constructing an interpretation-assistance legal expert system for these regulations by means of the general-purpose Interpretation-Assistance legal expert system builder called MINT. The logical basis for A-Hohfeld is included as an appendix.

8.1 INTRODUCTION

The need for more interpretation-assistance (IA) in the efforts to apply Artificial Intelligence (AI) techniques in law has been explored in detail along with the pervasiveness

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of the problem of multiple interpretations of the logical structure of legal rules [AS91]. The efforts to build IA systems are heavily dependent upon having available an adequate language for precisely and unambiguously expressing the various alternative structural interpretations of sets of legal rules. One such language designed for such usage is the A-Hohfeld language specified here. In its present version, A-Hohfeld consists of 34 defined structural terms. Its application is illustrated with respect to a small set of rules expressed by the 1982 Library Regulations of Imperial College—the same rules considered by Andrew Jones and Marek Sergot in their paper for this conference in which they argue that deontic logic in some form must be taken seriously to achieve adequate knowledge representation in the legal domain. This illustration in the library regulations is suggestive that extensions of deontic logic in the form of A-Hohfeld logic can usefully enhance the robustness of the representation of the logical structure of legal rules. The final step of embodying the thousands of different structural interpretations (2560, to be exact) in an IA system is undertaken and described here in the form of an interpretation-assistance system for these library regulations. The IA system generated enables a lawyer-user to dynamically deal with any one or more of these interpretations that the user wishes to consider. The library-regulation interpretation-assistance system was built using a prototype of a general-purpose interpretation-assistance system-generating system called MINT.

8.2 THE DEFINED A-HOHFELD LANGUAGE

The eight fundamental legal conceptions described in Hohfeld's seminal article are the inspiration for the A-Hohfeld[1] language. Hohfeld's conceptions were organized into the following two sets of ideas as pairs of "opposites" (columns) and "correlatives" (rows):

Right Set	Right	Duty	**Power Set**	Power	Liability
	Noright	Privilege		Disability	Immunity

Hohfeld regarded these fundamental legal conceptions as "the lowest common denominators of legal discourse to which all other doctrinal statements of law could be reduced." To the extent that these eight are so comprehensive, they are a good basis to start with in the quest for a robust language for representing knowledge in the legal domain.

In the A-Hohfeld definitions included below and in the logical basis set forth in the appendix, the Hohfeldian concepts of Right, Duty, Noright, and Privilege are extended to include the state of affairs that the Dutyholder is obliged to make so, and Privilege is altered significantly in other ways. In addition, Power, Liability, Disability, and Immunity are altered to become relationships between a person and a legal relation, rather than relationships between legal persons as specified by Hohfeld.

The A-Hohfeld definitions include concepts from other systems of logic, and their relationships to the modified Hohfeldian conceptions are made explicit. The relationships

[1] A-Hohfeld, an abbreviation for "Allen-extended-Hohfeld," is the senior author's extension of the conceptual framework formulated by the legal philosopher, Wesley N. Hohfeld [Hoh13]. See also [All74, AS86].

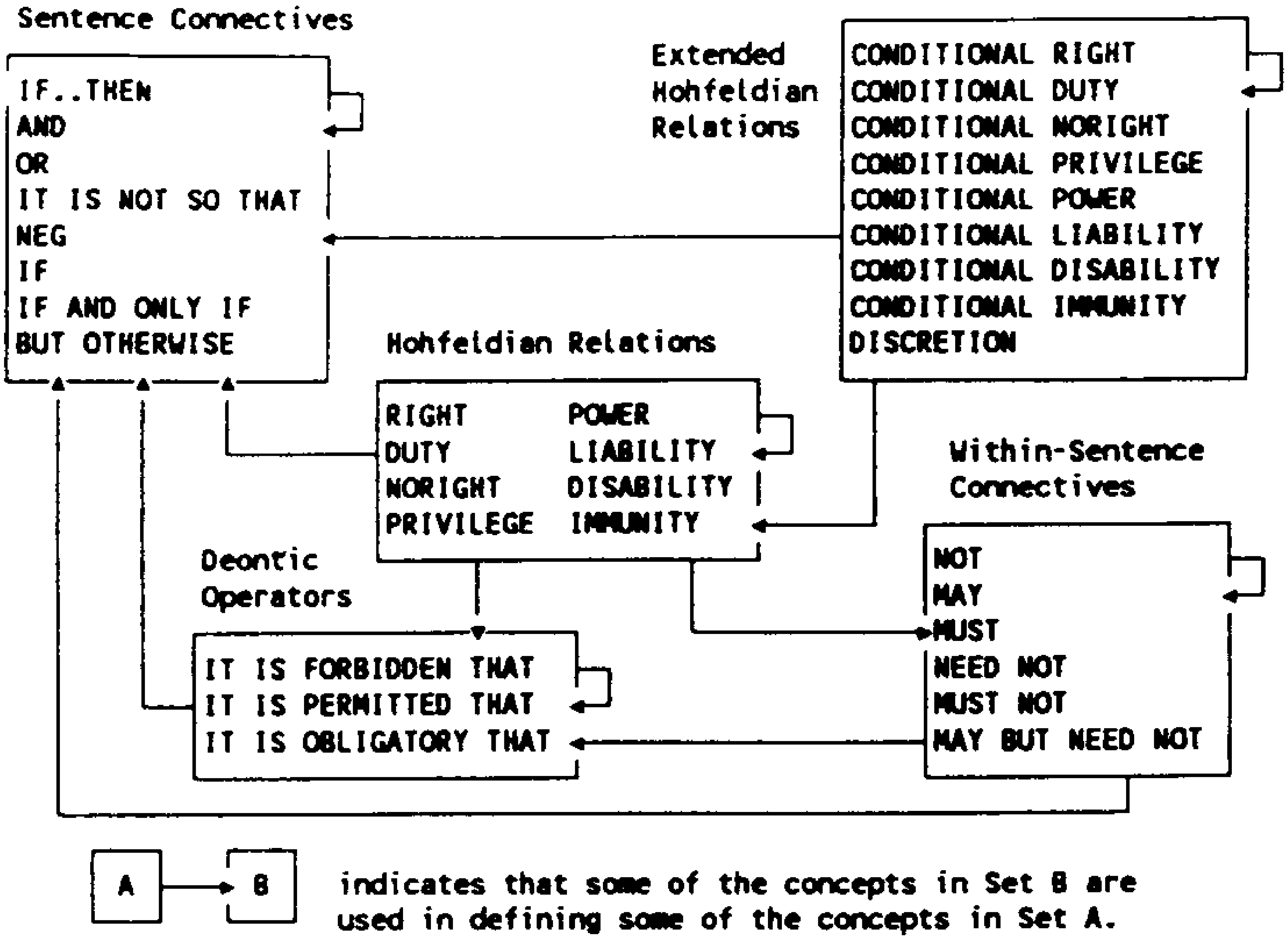

Figure 8.1 Dependence relationships between defined terms of A-Hohfeld language

between the various sets of defined term in the A-Hohfeld language are summarized in figure 8.1.

Each of these terms will be defined in a contextual definition in a format where the defined term appears in capital letters to the left of the symbol '$=\mathcal{DF}$' and the defining clauses appear to the right, where the symbol '$=\mathcal{DF}$' indicates "is by stipulated definition equal to". In these definitions, the lower case letters a,b,c, ... serve as abbreviations for complete sentences, and the alphanumeric combinations a1, a2, a3, ... b1, b2, b3, ... c1, ... serve as abbreviations of parts of sentences. The definitions in the A-Hohfeld language are also simpler than the corresponding more-detailed definitions in A-Hohfeld logic; among other things they include no reference to time.

IF a THEN b. $=\mathcal{DF}$ (1) If the state of affairs described by sentence-a is so then the state of affairs described by sentence-b is so, and (2) if the state of affairs described by sentence-a is not so, then nothing is being said about whether or not the state of affairs described by sentence-b is so.

a AND b. $=\mathcal{DF}$ The state of affairs described by sentence-a is so, and the state of affairs described by sentence-b is so.

a OR b. $=\mathcal{DF}$ The state of affairs described by sentence-a is so, or the state of affairs described by sentence-b is so, or both are so.

IT IS NOT SO THAT a. $=\mathcal{DF}$ It is not so that the state of affairs described by sentence-a is so.

NEG a. $=\mathcal{DF}$ IT IS NOT SO THAT a.

b IF a. $=\mathcal{DF}$ IF a THEN b.

b IF AND ONLY IF a. $=\mathcal{DF}$ IF a THEN b, AND IF NEG a THEN NEG b.

IF a THEN b BUT OTHERWISE c. $=\mathcal{DF}$ IF a THEN b, AND IF NEG a THEN c.

IT IS FORBIDDEN THAT a. $=\mathcal{DF}$ IF (1) the state of affairs described by sentence-a is so, THEN (2) there is a violation, AND (3) the legal system will provide a remedy with respect to the violator.

IT IS PERMITTED THAT a. $=\mathcal{DF}$ IT IS NOT SO THAT IT IS FORBIDDEN THAT a.

IT IS OBLIGATORY THAT a. $=\mathcal{DF}$ IT IS FORBIDDEN THAT NEG a.

a1 NOT [do] a2. $=\mathcal{DF}$ IT IS NOT SO THAT the state of affairs described by sentence-(a1 [do] a2) is so. (where "a1" concatenated with "[do] a2" is a sentence, and the [ ]s indicate that the "do" is optional)

a1 MAY [do] a2. $=\mathcal{DF}$ IT IS PERMITTED THAT a1 [do] a2.

a1 MUST [do] a2. $=\mathcal{DF}$ IT IS OBLIGATORY THAT a1 [do] a2.

a1 NEED NOT [do] a2. $=\mathcal{DF}$ IT IS PERMITTED THAT a1 NOT [do] a2.

a1 MUST NOT [do] a2. $=\mathcal{DF}$ IT IS OBLIGATORY THAT a1 NOT [do] a2.

a1 MAY BUT NEED NOT [do] a2. $=\mathcal{DF}$ a1 MAY [do] a2, AND a1 NEED NOT [do] a2.

p1 has a RIGHT that p2 do b. $=\mathcal{DF}$ IT IS OBLIGATORY THAT p2 do b for p1.

p1 has a DUTY to p2 to do b. $=\mathcal{DF}$ p2 has a RIGHT that p1 do b.

p1 has a NORIGHT that p2 do b. $=\mathcal{DF}$ IT IS NOT SO THAT p1 has a RIGHT that p2 do b.

p1 has a PRIVILEGE with respect to p2 to do b. $=\mathcal{DF}$ IT IS NOT SO THAT p2 has a RIGHT that p1 do NEG b.

p1 has POWER to create legal relation-lr. $=\mathcal{DF}$ Legal relation-lr is NOT so, AND it is naturally possible for p1 to do a1, AND IF p1 does a1 THEN legal relation-lr is created.

Legal relation-lr has LIABILITY of being created by p1. $=\mathcal{DF}$ p1 has POWER to create legal relation-lr.

p1 has DISABILITY to create legal relation-lr. $=\mathcal{DF}$ p1 lacks POWER to create legal relation-lr.

Legal relation-lr has IMMUNITY of being created by p1. $=\mathcal{DF}$ p1 lacks POWER to create legal relation-lr.

p1 has a CONDITIONAL RIGHT that p2 do b. $=\mathcal{DF}$ p1 has a NORIGHT that p2 do b, AND there is an event-e1 such that (1) it is naturally possible for event-e1 to occur, AND (2) IF event-e1 occurs, THEN condition-v is fulfilled, AND (3) IF condition-v is fulfilled, THEN p1's RIGHT that p2 do b is created.

p1 has a CONDITIONAL DUTY to p2 do b. $=\mathcal{DF}$ p2 has a CONDITIONAL RIGHT that p1 do b.

p1 has a CONDITIONAL NORIGHT that p2 do b. $=\mathcal{DF}$ p1 has a RIGHT that p2 do b, AND there is an event-e1 such that (1) it is naturally possible for event-e1 to occur, AND (2) IF event-e1 occurs, THEN condition-v is fulfilled, AND (3) IF condition-v is fulfilled, THEN p1's NORIGHT that p2 do b is created.

p1 has a CONDITIONAL PRIVILEGE that p2 do b. $=\mathcal{DF}$ p2 has a CONDITIONAL NORIGHT that p1 do NEG b.

p1 has CONDITIONAL POWER to create legal relation-lr. $=\mathcal{DF}$ p1 lacks POWER to create legal relation-lr, AND there is an event-e1 such that (1) it is naturally possible for event-e1 to occur, AND (2) IF event-e1 occurs, THEN condition-v is fulfilled, AND (3) IF condition-v is fulfilled, THEN p1's POWER to create legal relation-lr is created.

Legal relation-lr has CONDITIONAL LIABILITY of being created by p1. $=\mathcal{DF}$ p1 has CONDITIONAL POWER to create legal relation-lr.

p1 has CONDITIONAL DISABILITY to create legal relation-lr. $=\mathcal{DF}$ p1 has POWER to create legal legal relation-lr, AND there is an event-e1 such that (1) it is naturally possible for event-e1 to occur, AND (2) IF event-e1 occurs, THEN condition-v is fulfilled, AND (3) IF condition-v is fulfilled, THEN p1's DISABILITY to create legal relation-lr is created.

Legal relation-lr has CONDITIONAL IMMUNITY of being created by p1. $=\mathcal{DF}$ p1 has CONDITIONAL DISABILITY to create legal relation-lr.

p1 has DISCRETION with respect to p2 as to whether or not to do b. $=\mathcal{DF}$ p1 has a PRIVILEGE with respect to p2 to do b, AND p1 has a PRIVILEGE with respect to p2 to do NEG b.

The motivation for picking this set of 34 definitions is primarily to assure the Hohfeldian goal of having a language that is sufficiently expressive to describe all legal states of affairs as well as all changes in legal states of affairs. The sentence connectives are used in defining the concepts in all four other sets, and the deontic concepts are used in defining both the Hohfeldian relations and the within-sentence connectives. The Hohfeldian relations, in turn, are used in defining the extended Hohfeldian relations, which are needed to achieve the goal of comprehensive expressivity.

One example of the multitude of counterexamples of changes in legal states of affairs that cannot be expressed by Hohfeldian relations alone is the creation of a right-to-payment of an insured with respect to an insurance company if a fire occurs when a fire insurance policy is in force. Such right-to-payment would not be created by the exercise of a power of some legal person, which is the only means of affecting legal change in a system that includes only the Hohfeldian relations. On the other hand, in the extended A-Hohfeld system, such change is readily expressible as the fulfilment of the condition of a conditional right-to-payment, which produces the resulting right-to-payment.

The current 34 definitions are obviously not in any sense a minimal set for achieving comprehensiveness; the redundancy provides alternative ways of describing legal states of affairs that are handy in the sense that the alternatives frequently coincide with the way those states of affairs are often described in natural English prose.

With this set of 34 definitions that comprise the current version of the A-Hohfeld language available to express the logical structure of legal norms unambiguously, each of 2560 different structural interpretations of the four sentences that constitute the Library Regulations for Imperial College in 1982 can be stated with precision and clarity. The MINT program that specifies the 2560 alternative interpretations requires as input a specification of the individual structural ambiguities that occur in the rules and the alternative interpretations of those ambiguities. The heart and most difficult parts of the system-builder's task are (1) to formulate the questions about those ambiguities that will, in turn, be used (2) to construct the input file to MINT. In the next section we turn to the task of formulating the questions about the individual ambiguities.

8.3 INTERPRETATION OF THE 1982 LIBRARY REGULATIONS OF IMPERIAL COLLEGE

In order to specify the various alternative structural interpretations, it is necessary to formulate questions about the structural ambiguities in the current wording of the library regulations. In the alternative interpretations presented in the questions below, the definitions, NOT, MAY, MUST, NEED NOT, MUST NOT, DUTY, POWER, and DISCRETION are used. Note that in this particular example, only 8 of the 34 definitions are involved in expressing the 2560 different structural interpretations of these library regulations. In other sets of rules other combinations of the various defined structural terms will be used.

The differences among some of these defined terms is subtle and requires close attention. The difference between a MUST-statement and a DUTY-statement is that the DUTY-statement indicates to whom the obligation is owed. The difference between a MUST-statement and a POWER-statement in which the POWER MUST be exercised is that the POWER-statement indicates the legal relation that is created when the obligation is fulfilled. There may be significant differences in the legal results that occur when a person has POWER compared to when that person lacks POWER. For example, consider the persons in the following pair of circumstances: (A) one who has POWER to borrow more items but MUST NOT exercise that POWER, and (B) one who lacks POWER to borrow more items and MUST NOT engage in action that would exercise such POWER. The B-type borrower who takes items from the library may be stealing and subject to the penalties for theft—whereas the A-type borrower, although violating the library rule not to "borrow", will nevertheless have the book in a "borrowed" state rather than a "stolen" state—and be subject only to the penalties for violating the library rule and not to the penalties for theft.

The complete text of the 1982 version of the Imperial College Library Regulations that the questions address is the following:

1. A separate form must be completed by the borrower for each volume borrowed.
2. Books should be returned by the date due.
3. Borrowers must not exceed their allowance of books on loan at any one time.
4. NO BOOK WILL BE ISSUED TO BORROWERS WHO HAVE BOOKS OVERDUE FOR RETURN TO THE LIBRARY.

Book allowances	Undergraduates	6
	Postgraduates	10
	Academic staff	20

The structural ambiguities in these regulations that we detected on this first attempt appear below.

Q1 INTERPRETATION OF THE TERM "MUST"

Whether or not it is appropriate to ask this first question may well depend upon whether the library system is a manual one or an appropriately computerized one. There may not be any practical difference between these three alternative structural interpretations for purposes of an effectively operating computerized library system. But for a manual system, the remedies available to universities for violations may well differ depending upon which interpretation is picked as the appropriate one. The A-

Hohfeldian alternative interpretations may also have different referents for a manual system than they have for a computerized system. In a manual system they clearly refer to LEGAL RELATIONS, while for a computerized system they may be referring to some form of "control" rules for what the computer does. Consider the interpretation of the term "must" in the following specified statement:

that borrower must complete a separate form for each such item

(where an item is a book, a volume, a periodical, a chart, a map, a record, a diskette, an audio cassette, a video cassette, or anything else that has been authorized to be borrowed from the library).

In the most appropriate interpretation of the term "must", which of the following lettered alternatives should the specified statement be interpreted as asserting:

A) that borrower MUST complete a separate form for each such item

B) that borrower has a DUTY to the university to complete a separate form for each such item

C) that borrower has POWER to complete a separate form for each such item and MUST exercise that POWER

D) that borrower has POWER to complete a separate form for each such item and a DUTY to the university to exercise that POWER

Q2 INTERPRETATION OF THE TERM "SHOULD"

Consider the interpretation of the term "should" in the following specified statement:

Books should be returned by the date due.

In the most appropriate interpretation of the term "should", which of the following lettered alternatives should the specified statement be interpreted as asserting:

A) items MAY, but NEED NOT, be returned by the date due

B) items MUST be returned by the date due

C) that borrower MAY, but NEED NOT, return the items by the date due

D) that borrower MUST return the items by the date due

E) that borrower has DISCRETION with respect to the university of whether or not to return items by the date due

F) that borrower has a DUTY to the university to return items by the date due

G) that borrower has POWER to return items by the date due and MAY, but NEED NOT, exercise that POWER

H) that borrower has POWER to return items by the date due and MUST exercise that POWER

I) that borrower has POWER to return items by the date due and DISCRETION with respect to the university of whether or not to exercise that POWER

J) that borrower has POWER to return items by the date due and a DUTY to the university to exercise that POWER

Q3 INTERPRETATION OF THE TERM "MUST NOT"

Whether or not it is appropriate to ask this third question, also, may well depend upon whether the library system is a manual one or an appropriately computerized one. There may not be any practical difference between the alternative structural interpretations for purposes of an effectively operating computerized library system. But for a manual system, the remedies available to libraries for violations may well differ depending upon which interpretation is picked as the appropriate one. The A-Hohfeldian alternative interpretations may also have different referents for a manual system than they have for a computerized system. In a manual system they clearly refer to LEGAL RELATIONS, while for a computerized system they may be referring to some form of "control" rules for what the computer does. Consider the interpretation of the term "must not" in the following specified statement:

that borrower must not borrow any more items.

In the most appropriate interpretation of the term "must not", which of the following lettered alternatives should the specified statement be interpreted as asserting:

A) that borrower MUST NOT borrow any more items

B) that borrower has POWER to borrow more items but MUST NOT exercise that POWER

C) that borrower lacks POWER to borrow any more items and MUST NOT engage in action that would exercise such POWER

D) that borrower lacks POWER to borrow any more items and MAY, but NEED NOT, engage in action that would exercise such POWER

E) that borrower has a DUTY to the university NOT to borrow any more items

F) that borrower has POWER to borrow more items but has a DUTY to the university NOT to exercise that POWER

G) that borrower lacks POWER to borrow any more items and has a DUTY to the university NOT to engage in action that would exercise such POWER

H) that borrower lacks POWER to borrow any more items and has DISCRETION with respect to the university on whether or not to engage in action that would exercise such POWER

Q4 INTERPRETATION OF THE TERM "NO ... WILL BE"

Consider the interpretation of the term "no ... will be" in the following specified statement:

no other item will be issued to that borrower.

In the most appropriate interpretation of the term "no ... will be", which of the following lettered alternatives should the specified statement be interpreted as asserting:

A) no other item MAY be issued to that borrower

B) some library employees have POWER to issue items to that borrower but MUST NOT exercise that POWER

C) every library employee lacks POWER to issue any item to that borrower and MUST NOT engage in action that would exercise such POWER

D) every library employee lacks POWER to issue any item to that borrower and MAY, BUT NEED NOT, engage in action that would exercise such POWER

E) every library employee has a DUTY to the university NOT to issue any item to that borrower

F) some library employees have POWER to issue items to that borrower but have a DUTY to the university NOT to exercise that POWER

G) every library employee lacks POWER to issue any item to that borrower and has a DUTY NOT to engage in action that would exercise such POWER

H) every library employee lacks POWER to issue any item to that borrower but has DISCRETION with respect to the university on whether or NOT to engage in action that would exercise such POWER

Since there are 4 specified different structural interpretations to Q1, and 10, 8, and 8 to Q2, Q3, Q4, respectively, there are a total of $4 \times 10 \times 8 \times 8 = 2560$ different structural interpretations of the complete set of rules—specifically those named by AAAA ... AAAH ... AAHH ... AJHH ... DJHH. In the library regulation interpretation-assistance system generated by the MINT (an acronym for Multiple INTerpretation) general-purpose interpretation-assistance system-building system to be considered next, a user can obtain a consultation with as many of these different structural interpretations as the user wishes to explore. MINT will construct dynamically "on the fly" whichever interpretation the user wants to look at in the situation being analyzed.

8.4 MINT—A SYSTEM FOR GENERATING MULTIPLE-INTERPRETATION LEGAL EXPERT SYSTEMS

One form that a multiple-interpretation expert system could take is to provide a lawyer-user with the capacity to explore the effects of various alternative interpretations of the set of rules that are embodied in the system and define it. A prototype of an IA system that enables such users to explore all of the structural interpretations of such a set of rules is the one described here called MINT. As such, MINT is itself an expert system with Interpretation Assistance (IA) capabilities that provide a lawyer-user with assistance in structurally interpreting the involved set of rules and enable the user to easily modify (and explore) such interpretations. With MINT the lawyer-user can easily and quickly investigate, not only an expert system for each particular structural interpretation, but also normalized arrow diagrams and outlines of the structure of the interpretation [AS85].

In the building and using of MINT IA systems there are three identifiable roles with assigned tasks being performed by each. The system-builder formulates the structural questions about the set of rules that are the basis for the IA system and constructs, with the assistance of the legal expert, the multiple-interpretation input file to the MINT program. The legal expert (in the legal domain of the IA system rule-set) chooses the default interpretation incorporated in the IA system, answers and helps to refine the structural questions, and helps the system-builder to construct the multiple-interpretation input file. The lawyer-user characterizes the legal situation being dealt with in the consultation by providing answers to the situation questions presented by the IA system. These roles and tasks will be illustrated below in the description of the process of building a MINT IA system for the set of rules expressed in the four sentences that constitute the 1982 Library Regulations of Imperial College.

From this text of the library regulations the system-builder specifies the questions about its structural interpretation and all of the constituent sentences of all such interpretations for use in the multiple-interpretation input file [APS90, AS88a, AS88b]. For this provision there are the four structural questions specified in the previous section.

Using as a guide the alternative structural interpretations specified by these structural questions, along with the original provision and assistance from the legal expert, the system-builder constructs the input file to MINT for these library regulations, which includes the following three parts: (1) the specification of the logical structure of the multiple interpretations (2) the structural questions, and (3) each possible structural interpretation of each constituent sentence of the provision.

The most difficult task for the system-builder is specifying the logical structure for all of the possible structural interpretations. This is done by using abbreviations for the constituent sentences of such interpretations and logical operators to indicate the logical relationships among them. The system-builder must carefully construct an expression that precisely represents the structure of each of the various alternative structural interpretations.

In this specification, alternative choices are indicated by a set of expressions in which a name is followed by an equals sign, which in turn, is followed by the alternatives separated by commas and terminated by a semicolon. Names that begin with lowercase letters are names of constituent sentences and names that begin with question marks are names that refer to the structural questions of the indicated number. The characters that indicate the logical relationships between the named entities in the portion above are ">", and "&", and "|", and "BO" indicating "IF..THEN..", "AND", and "OR", and "BUT OTHERWISE" respectively. The specification, for example, of the structure for the first part of the library regulations is the following:

```
{Logical Structure of the 1982 Imperial College Library Regulations}
GOAL = uc > ?1 BO not_apply;        {Comments are enclosed in braces}
DEFAULT = AAAA;
?1 = aoa1: (a2 > (aoa1 & ?2 & (((c1&c2)|(c4&c5)|(c6&c7))>(?3&?4)))),
     bdua1: (a2 > (bdua1 & ?2 & (((c1&c2)|(c4&c5)|(c6&c7))>(?3&?4)))),
     cpwoa1: (a2 > (cpwoa1 & ?2 & (((c1&c2)|(c4&c5)|(c6&c7))>(?3&?4)))),
     dpwdua1: (a2 > (dpwdua1 & ?2 & (((c1&c2)|(c4&c5)|(c6&c7))>(?3&?4))));
```

The system-builder can put the structural questions into the MINT input file in a form similar to way that they were presented in the section above. For example, the first question and its constituent sentences in the input file are:

```
[?1:INTERPRETATION OF THE TERM "MUST"
Whether or not it is appropriate to ask this first question may well depend upon
whether the library system is a manual one or an appropriately computerized one.
There may not be any practical difference between these three alternative
structural interpretations for purposes of an effectively operating computerized
library system.  But for a manual system, the remedies available to universities
for violations may well differ depending upon which interpretation is picked as
the appropriate one.  The A-Hohfeldian alternative interpretations may also have
different referents for a manual system than they have for a computerized system.
In a manual system they clearly refer to LEGAL RELATIONS, while for a computerized
system they may be referring to some form of "control" rules for what the computer
does.

Consider the interpretation of the term "must" in the following specified statement:

    that borrower must complete a separate form for each such item

In the most appropriate interpretation of the term "must", which of the following
lettered alternatives should the specified statement be interpreted as asserting:]

[aoa1:that borrower MUST complete a separate form for each such item]
```

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[bdual:that borrower has a DUTY to the university to complete a separate form
   for each such item]

[cpwoal:that borrower (1) has POWER to change what otherwise might be
   appropriately characterized as theft of that item into characterizing it as
   a legitimate borrowing by completing a separate form for each such item and
   (2) MUST exercise that POWER]

[dpwdual:that borrower (1) has POWER to change what otherwise might be
   appropriately characterized as theft of that item into characterizing it as
   a legitimate borrowing by completing a separate form for each such item and
   (2) a DUTY to the university to exercise that POWER]
```

As exemplified with respect to Question 1 above, the language of the constituent sentences for all of the structural interpretations of the library regulations will be determined by the structural questions asked. Each constituent sentence is enclosed in square brackets and has an alphanumeric name associated with it.

In most customary legal expert systems, the help that a lawyer-user could obtain would end there with the results that the one structural interpretation led to and the reasons why. What is added in IA systems of the MINT sort is that a lawyer-user who is unhappy with the results obtained in the situation from the default interpretation will be able to call for alternative interpretations and explore the results that those alternative interpretations lead to in that situation. That user will be stimulated to think about which of the interpretations that lead to the results most favored by the client are the most likely to be accepted by the decision-maker in the forum where the case is being argued (or will be likely to be the most influential in negotiations with opposing counsel).

The building of IA systems with MINT for dealing with the pervasive problem of multiple structural interpretations of legal rules is heavily dependent upon having available an adequate language like A-Hohfeld to precisely and unambiguously express the various alternative interpretations.

8.5 CONCLUSION

The defined terms provided in the A-Hohfeld language give both legal experts and expert system builders the ability to specify precisely which one of the many possible structural interpretations is desired when working with legal norms. This precision is useful for communication between human experts and it is indispensable when attempting to construct expert systems embodying interpretations of legal norms. An example of the use of some of the A-Hohfeld defined terms was presented using the 1982 Library Regulations of Imperial College as the norms being interpreted. An expert system was built from the resulting rules using the MINT interpretation-assistance legal expert system builder. We believe that using the A-Hohfeld language can substantially increase the precision with which interpretations of legal norms may be specified and that the expert systems from which they are built can more accurately model the desired interpretations.

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APPENDIX

A-Hohfeld Logic

The formulation of A-Hohfeld logic that follows is presented in fuller detail with its constituent propositional-, alethic-, deontic-, and predicate-logic subsystems and action theory in an earlier article [AS86]. There are some modifications and notational changes in the version here. A-Hohfeld logic is a variation and extension of the relevance logics of Anderson and Belnap [AB74][2].

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ALPHABET
Object Language                                  Metalanguage Variables
   Variables                                        Individual          x y z x4 ...
      Sentence                 p q r s s5 s6 ...    Predicate           f g h f4 ...
      Numerical Subscripts                          Formula             d e e3 e4 ...
         Individual            [i]                  WFF                 u v w w4 ...
         Set                   a b
         Logical Sum           aUb
         Logical Difference    a-b
      Individual               x y z x4 ...
      Predicate                f g h f4 ...
   Constants
      Sentence                 Z V V1 V2 V3 ... S1 S2 S3 ...
      Individual               a b c a4 ...
      Predicate                Q1 Q2 Q3 ... D D2 D4 ...
   Connectives                 K G A N L M R B U
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FORMATION RULES
   FR1  If a formula is a sentence variable or a constant, then it is a WFF.
   FR2  If formulas d and e are WFFs, then so are Kde, Gde, and Ade, and Ne.
   FR3  If formula e is a WFF, then so are Le and Me.
   FR4  If formula e is a WFF, then so are Re and Be.
   FR5  If e is an n-adic predicate symbol (where n = 1, 2, 3, . . .) then e
        x1 x2 x3 . . . xn is a WFF.
   FR6  If e is a WFF, and x is an individual variable, then Uxe is a WFF.
   FR7  If x is an individual variable ranging over persons and formula e
        is a WFF, then De, D2ex, and D4xe are WFFs.
   FR8  If x and y are individual variables ranging over persons, t1 t2 t3 ... etc.
        are individual variables ranging over time intervals, LR LR2 LR3 ... etc. are
        sentence variables ranging over propositional functions about legal relations,
        CLR CLR2 CLR3 ... etc. are sentence variables ranging over propositional
        functions about conditional legal relations, and formula e is a WFF,
        then D24(e(t2),x,y,t1), D42(e(t2),x,y,t1), LR(t1), CLR(t1), and
        PO(e(t2),LR(t2),x,t1) are WFFs.
   FR9  If a formula is not a WFF by one of the above rules, then it is not a WFF.
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[2] The theses of standard deontic logics, D1-D4 and RO->, which McArthur assumes essential for any reasonable deontic system, are expressed in A-Hohfeld as: D1. I-OIpq-IOpOq, D2. I-OKpq-KOpOq, D3. I-KOpOq-OKpq, D4. I-Op-NONp, and RO->. (...Ivw) —** (...IOvOw). These are all provable in A-Hohfeld except D4, but a weaker variation, D4'. G-Op-NONp, is provable. However, (...w) —o (...Ow), i.e., RO. (...w) —** (...Ow) is not—which we believe is appropriate for deontic systems for law. For true entailments McArthur regards the provability of T-Tpq-OTpq and T-OTpq-Tpq as desirable, where T represents entailment. However, the corresponding I-Ipq-OIpq and I-OIpq-Ipq of A-Hohfeld are not provable. For law, we think that appropriate; and also the implication that the natural implication, I, is not an entailment [McA81].

TRANSFORMATION RULES

Name of Rule	Statement of Rule
Ko':	Kvw(a) --* v(a), w(a). where 'a' indicates the set of numerical subscripts in 'Kvw' that is carried along to 'v' and 'w'. (From 'Kvw(a)' (i.e. 'v and w') it is assumed to be valid to infer 'v(a)' and it is assumed to be valid to infer 'w(a)'.)
Ki':	v(a), w(aUb) --* Kvw(aUb). where 'a' indicates that the set of numerical subscripts on 'v' and 'aUb' indicates the logical sum of 'a' and 'b', the set of subscripts on 'w' (which indicates that 'v' and 'w' share at least one subscript), and on 'Kvw' (which indicates that the 'aUb' sum of subscripts is carried along to 'Kvw'). (From 'v(a)' and 'w(aUb)', it is assumed to be valid to infer 'Kvw(aUb), (i.e., 'v and w').)
Go':	Gvw(a), v(b) --* w(aUb). where 'a' and 'b' indicate that the sets of numerical subscripts on 'Gvw' and 'v' may be different and 'aUb' indicates that the set of subscripts carried along to 'w' is the logical sum of 'a' and 'b'. (From 'Gvw(a)' (i.e., 'v genuinely implies w') and 'v(b)', it is assumed to be valid to infer 'w(aUb)'.)
G()oGi':	G(v([i]): . . . w(a)) --* Gvw(a-[i]). where '[i]' indicates a numerical subscript assigned to supposition 'v' which is distinct from the numerical subscript assigned to any other supposition, 'a' is a set of subscripts which contains '[i]' and 'a-[i]' is a set of subscripts comprised of those in 'a' with '[i]' deleted. (From the derivability in a G-restricted subproof of 'w(a)', given that 'v([i])' is assumed to be true, it is assumed to be valid to infer 'Gvw(a-[i])' (i.e., 'v' genuinely implies 'w').)
G()i':	w(a) --* G(v([i]): . . . w(a)), G(. . . w(a)). where 'a' indicates the set of numerical subscripts on 'w' that is carried along on reiteration into a G-restricted subproof and '[i]' indicates a numerical subscript assigned to supposition 'v' which is distinct from the numerical subscript assigned to any other supposition. (From 'w(a)', it is assumed to be valid in a G-restricted subproof to infer 'w(a)', given that 'v([i])' is assumed to be true, and it is assumed to be valid in a G-restricted subproof to infer 'w(a)'.)
GoNo':	Gvw(a), Nw(b) --* Nv(aUb). where 'a' and 'b' indicate that the sets of numerical subscripts on 'Gvw' and 'Nw' may be different and 'aUb' indicates that the set of subscripts carried along to 'Nv' is the logical sum of 'a' and 'b'. (From 'Gvw(a)' (i.e., 'v genuinely implies w') and 'Nw(b)' (i.e., 'not w'), it is assumed to be valid to infer 'Nv(aUb)' (i.e., 'not v').)
G()oNi':	G(v([i]): . . . w(a), Nw(b)) --* Nv(aUb-[i]). where 'a' and 'b' indicate that the sets of numerical subscripts on 'w' and 'Nw' may be different, '[i]' indicates a numerical subscript assigned to supposition 'v' which is distinct from the numerical subscript assigned to any other supposition and is contained in both 'a' and 'b', and 'aUb-[i]' indicates that the set of subscripts carried along to 'Nv' is comprised of those in 'aUb' with '[i]' deleted. (From the derivability in a G-restricted subproof of 'w(a)' and 'Nw(b)', given that 'Nv([i])' is assumed to be true, it is assumed to be valid to infer 'Nv(aUb-[i])'.)
Rp':	w(a) --* w(a). (From 'w(a)', it is assumed to be valid to infer 'w(a)'.)
AoNKi':	Avw(a) --* NKNvNw(a). (From 'Avw(a)' (i.e., 'v or w'), it is assumed to be valid to infer 'NKNvNw(a)' (i.e., 'not (not v and not w)').)
NKoAi':	NKNvNw(a) --* Avw(a). (From 'NKNvNw(a)' (i.e., 'not (not v and not w)'), it is assumed to be valid to infer 'Avw(a)' (i.e., 'v or w').)

KoAi2': KuAvw(a) --* AKuvw(a).
(From 'KuAvw(a)' (i.e., 'u and (v or w)'), it is assumed to be valid to infer 'AKuvw(a)' (i.e., '(u and v) or w').)

NNo': NNw(a) --* w(a).
(From 'NNw(a)' (i.e., 'not not w'), it is assumed to be valid to infer 'w(a)'.)

Ai': w(a) --* Avw(a), Awv(a).
(From 'w(a)', it is assumed to be valid to infer 'Avw(a)' (i.e., 'v or w') and to be valid to infer 'Awv(a)' (i.e., 'w or v').)

KoNNKi': Kvw(a) --* NNKvw(a).
(From 'Kvw(a)' (i.e., 'v and w'), it is assumed to be valid to infer 'NNKvw(a)' (i.e., 'not not (v and w)').)

Lo': Lw(a) --* w(a).
(From 'Lw(a)' (i.e., 'it is logically necessary that w'), it is assumed to be valid to infer 'w(a).')

L()oLi': L(. . . w(a)) --* Lw(a).
(From an L-restricted subproof that has no suppositions and that has 'w(a)' as an item, it is assumed to be valid to infer 'Lw(a)' (i.e., 'it is logically necessary that w').)

LoL(L)i': Lw(a) --* L(v([i]): ... Lw(a)), L(. . . Lw(a)).
where 'a' indicates the set of numerical subscripts on 'Lw' that is carried along upon reiteration into an L-restricted subproof and '[i]' indicates a numerical subscript assigned to supposition 'v' which is distinct from the numerical subscript assigned to any other supposition.
(From 'Lw(a)' (i.e., 'it is logically necessary that w'), it is assumed to be valid in an L-restricted subproof to infer 'Lw(a)', given that 'v([i])' is assumed to be true, and it is assumed to be valid in an L-restricted subproof that has no suppositions to infer 'Lw(a).')

LoLNNi': Lw(a) --* LNNw(a).
(From 'Lw(a)' (i.e. 'it is logically necessary that w'), it is assumed to be valid to infer 'LNNw(a)' (i.e., 'it is logically necessary that not not w').)

MoNLNi': Mw(a) --* NLNw(a).
(From 'Mw(a)' (i.e., 'it is logically possible that w'), it is assumed to be valid to infer 'NLNw(a) (i.e., 'it is not logically necessary that not w').)

NLNoMi': NLNw(a) --* Mw(a).
(From 'NLNw(a)' (i.e., 'it is not logically necessary that not w'), it is assumed to be valid to infer 'Mw(a)' (i.e., 'it is logically possible that w').)

LoR(L)i': Lw(a) --* R(v([i]): . . . Lw(a)), R(. . . Lw(a)).
where 'a' indicates the set of numerical subscripts on 'Lw' that is carried along upon reiteration into an R-restricted subproof and '[i]' indicates a numerical subscript assigned to supposition 'v' which is distinct from the numerical subscript assigned to any other supposition.
(From 'Lw(a)' (i.e., 'it is logically necessary that w(a)', it is assumed to be valid in an R-restricted subproof to infer 'Lw(a)', given that 'v([i])' is assumed to be true, and it is assumed to be valid in an R-restricted subproof that has no suppositions to infer 'Lw(a)'.)

GoNBNi': GZw(a) --* NBNw(a).
(From 'GZw(a)' (i.e., 'the laws of nature genuinely imply that w'), it is assumed to be valid to infer 'NBNw(a) (i.e., 'it is not naturally possible that not w').) [See Fitch for a discussion of the sense in which the idea of Z (laws of nature) is used here and its relationship to the concept of logical necessity. [Fit52]]

MKi': w(a) --* MKZw(a).
(From 'w(a)', it is assumed to be valid to infer 'MKZw(a)' (i.e., 'it is logically possible for both the laws of nature and w to be true').)

MKoNGi': MKZw(a) --* NGZNw(a).
(From 'MKZw(a)' (i.e., 'it is logically possible for both the laws of nature and w to be true'), it is assumed to be valid to infer 'NGZNw(a)' (i.e.,'it is not so that the laws of nature genuinely imply that not w').)

RoGi': Rw(a) --* GZw(a).
(From 'Rw(a)' (i.e., 'it is naturally necessary that w'), it is assumed to be valid to infer 'GZw(a)'
(i.e., 'the laws of nature (Z) genuinely imply that w').)

GoRi': GZw(a) --* Rw(a).
(From 'GZw(a)' (i.e., 'the laws of nature genuinely imply that w'), it is assumed to be valid to infer 'Rw(a)'
(i.e., 'it is naturally necessary that w').)

R()oRi': R(. . . w(a)) --* Rw(a).
(From an R-restricted subproof that has no suppositions and that has 'w(a)' as an item, it is assumed to be valid to infer 'Rw(a)'
(i.e., 'it is naturally necessary that w').)

RoNNRi': Rw(a) --* NNRw(a).
(From 'Rw(a)' (i.e., 'it is naturally necessary that w'), it is assumed to be valid to infer 'NNRw(a)'
(i.e., 'it is not not naturally necessary that w').)

RoR(R)i': Rw(a) --* R(v([i]): . . . Rw(a)), R(. . . Rw(a)).
where 'a' indicates the set of numerical subscripts on 'Rw' that is carried along upon reiteration into an R-restricted subproof and '[i]' indicates a numerical subscript assigned to supposition 'v' which is distinct from the numerical subscript assigned to any other supposition.
(From 'Rw(a)' (i.e., 'it is naturally necessary that w'), it is assumed to be valid in an R-restricted subproof to infer 'Rw(a)', given that 'v([i])' is assumed to be true, and it is assumed to be valid in an R-restricted subproof that has no suppositions to infer 'Rw(a).')

BoMKi': Bw(a) --* MKZw(a).
(From 'Bw(a)' (i.e., 'it is naturally possible that w'), it is assumed to be valid to infer 'MKZw(a)' (i.e., 'it is logically possible that both the laws of nature and w are true').)

MKoBi': MKZw(a) --* Bw(a).
(From 'MKZw(a) (i.e., 'it is logically possible that both the laws of nature and w are true'), it is assumed to be valid to infer 'Bw(a)'
(i.e., 'it is naturally possible that w(a)).)

xG()oUi': xG(. . . fx(a)) --* Uxfx(a).
where 'a' indicates the usual subscript convention used here.
(From an xG-restricted subproof that has 'fx(a)' as an item that is not a supposition, it is assumed to be valid to infer 'Uxfx(a)'
(i.e., 'everything has the property f').)

Uo.x/y': Uxfx(a) --* fy(a).
where 'a' indicates the usual subscript convention used here.
(From 'Uxfx(a)' (i.e., 'everything has the property f'), it is assumed to be valid to infer 'fy(a)' (i.e., 'y has the property f').)

xG()': wx'(a) --* xG(. . . wx'(a)), xG(v([i]): . . . wx'(a)).
where 'a' indicates the usual subscript convention used here and "wx'" is a WFF that does not mention 'x'.
(From 'wx'(a)', it is assumed to be valid in an xG-restricted subproof that has no suppositions to infer 'wx'(a)', and it is assumed to be valid in an xG-restricted subproof to infer 'wx'(a)', given that 'v([i])' is assumed to be true.)

OD2oDNoD2Ni': OD2wx(a), DNw(b), --* D2Nwx(aUb).
where 'a', 'b', and 'aUb' indicate the usual subscript conventions used here.
(From 'OD2wx(a)' (i.e., 'it is obligatory that w be done by x') and 'DNw(b)' (i.e.,'not w has been done'), it is assumed to be valid to infer 'D2Nwx(aUb)' (i.e., 'not w has been done by x').)

OD4oDNoD4Ni': OD4wx(a), DNw(b) --* D4Nwx(aUb).
where 'a', 'b', and 'aUb' indicate the usual subscript conventions used here.
(From 'OD4wx(a)' (i.e., 'it is obligatory that w be done for x') and 'DNw(b)' (i.e., 'not w has been done') it is assumed to be valid to infer 'D4Nwx(aUb)' (i.e., 'not w has been done for x').)

D2NoND2i': D2Nwx(a) --* ND2wx(a).
where 'a' indicates the usual subscript conventions used here.
(From 'D2Nwx(a)' (i.e., 'not w has been done by x'), it is assumed to be valid to infer 'ND2wx(a)' (i.e.,'it is not so that w has been done by x').)

D4NoND4i': D4Nwx(a) --* ND4wx(a).
where 'a' indicates the usual subscript conventions used here.
(From 'D4wx(a)' (i.e., 'not w has been done for x'), it is assumed to be valid to infer 'ND4wx(a)' (i.e.,'it is not so what w has been done for x').)

D2NNoD2i': D2NNwx(a) --* D2wx(a).
where 'a' indicates the usual subscript conventions used here.
(From 'D2NNwx(a)' (i.e., 'not not w has been done by x') it is assumed to be valid to infer 'D2wx(a)' (i.e., 'w has been done by x').)

D4NNoD4i': D4NNwx(a) --* D4wx(a).
where 'a' indicates the usual subscript conventions used here.
(From 'D4NNwx(a)' (i.e., 'not not w has been done for x') it is assumed to be valid to infer 'D4wx(a)' (i.e., 'w has been done for x').)

D2oD2NNi': D2wx(a) --* D2NNwx(a).
where 'a' indicates the usual subscript conventions used here.
(From 'D2wx(a)' (i.e., 'w has been done by x') it is assumed to be valid to infer 'D2NNwx(a)' (i.e., 'not not w has been done by x').)

D4oD4NNi': D4wx(a) --* D4NNwx(a).
where 'a' indicates the usual subscript conventions used here.
(From 'D4wx(a)' (i.e., 'w has been done for x') it is assumed to be valid to infer 'D4NNwx(a)' (i.e., 'not not w has been done for x').)

AXIOMS

BNVa: BNV. It is naturally possible that there is no violation.

Za: Z. The laws of nature are true.

DEFINITIONS

LG()d: LG(v: . . .) =df L(G(v: . . .)).
An LG-restricted subproof with a supposition 'v' is equal to by definition (that is, by stipulated definition means) an L-restricted subproof of a G-restricted subproof with a supposition 'v'.

RG()d: RG(v: . . .) =df R(G(v: . . .)).
An RG-restricted subproof with a supposition 'v' is equal to by definition an R-restricted subproof of a G-restricted subproof with a supposition 'v'.

Id: Ivw =df RGvw.
'v naturally implies w' is equal to by definition 'it is naturally necessary that v genuinely implies w'.

Vd: V =df AA . . . AV(1)V(2) . . . V(n+1) where the number of A's is n
('There is a violation' is equal to by definition 'there is a violation of particular norm #1 or there is a violation of particular norm #2, . . . , or there is a violation of particular norm #(n+1), where there are just n+1 norms in the legal system.)

Od: Ow =df INwV.
'It is obligatory that w' is equal to by definition 'not w naturally implies that there is a violation'.

Pd: Pw =df NONw.
'It is permitted that w' is equal to by definition 'it is not so that it is obligatory that not w'.

Fd: Fw =df ONw.
'It is forbidden that w' is equal to by definition 'it is obligatory that not w'.

Sd: Sxw =df NUxNw.
'There is an x such that w' is equal to by definition 'It is not so that for all x not w'.

xG()d: xG(v: . . .) =df x(G(v: . . .)).
An xG-restricted subproof with a supposition 'v' is equal to by definition an x-restricted subproof of a G-restricted subproof with a supposition 'v'.

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D24d:     D24wxy  =df  K-D2wx-D4wy.
          'w has been done by x for y' is equal to by definition 'w has been done
          by x and w has been done for y'.
D42d:     D42wxy  =df  K-D4wx-D2wy.
          'w has been done for x by y' is equal to by definition 'w has been done
          for x and w has been done by y'.
     (In the definitions below that involve time, t1 precedes t2, t2 precedes t3,
      etc.)
RId:      DEFINITION OF RIGHT   RI(w(t2),x,y,t1)  =df  OD24(w(t2),x,y,t1)
          'Person-y has a RIGHT at time-t1 that person-x do w at time-t2' is equal
          to by definition 'it is obligatory at time-t1 that w be done by person-x
          at time-2 for person-y'.
DUd:      DEFINITION OF DUTY   DU(w(t2),x,y,t1)  =df  RI(w(t2),y,x,t1)
          'Person-y has a DUTY at time-t1 to person-x to do w at time-t2' is equal
          to by definition 'Person-x has a RIGHT at time-t1 that person-y do w at
          time-t2'.
NOd:      DEFINITION OF NORIGHT   NO(w(t2),x,y,t1)  =df  NRI(w(t2),x,y,t1)
          'Person-y has a NORIGHT at time-t1 that person-x do w at time-t2' is
          equal to by definition 'IT IS NOT SO THAT person-y has a RIGHT at
          time-t1 that person-x do w at time-t2'.
PRd:      DEFINITION OF PRIVILEGE   PR(w(t2),x,y,t1)  =df  NRI(Nw(t2),y,x,t1)
          'Person-y has a PRIVILEGE at time-t1 with respect to person-x to do w at
          time-t2' is equal to by definition 'IT IS NOT SO THAT person-x has a
          RIGHT at time-t1 that person-y do not w at time-t2'.
LRd:      DEFINITION OF LEGAL RELATION (Recursive Definition)
              (1) RI(w(t2),x,y,t1) is a LR(t1).
              (2) IF u(t1) is a LR(t1) AND w(t2) is a LR(t2), THEN Nu(t1) is a
                  LR(t1) AND CLR(v(t2),w(t2),t1) is a LR(t1), AND so is
                  PO(v(t2),w(t2),x,t1).
              (3) IF w(t1) is NOT a LR(t1) by (1) or (2), THEN w(t1) is NOT a
                  LR(t1).
          (1) 'Person-y has a RIGHT at time-t1 that person-x do w at time-t2' is
              a LEGAL-RELATION-LR at time-t1.
          (2) IF 'u' at time-t1 is a LEGAL-RELATION-LR at time-t1, AND 'w' at
              time-t2 is a LEGAL-RELATION-LR at time-t2, THEN
                 (A) 'Nu' at time-t1 is a LEGAL-RELATION-LR at time-t1, AND
                 (B) a CONDITIONAL-LEGAL-RELATION-CLR at time-t1 (that IF 'v' at
                     time-t2 THEN 'w' at time-t2) is a LEGAL-RELATION at time-t1,
                     AND
                 (C) a POWER at time-t1 (of person-x that will be exercised by
                     person-x doing v at time-t2 to create 'w' at time-t2) is a
                     LEGAL-RELATION at time-t1.
          (3) IF a candidate does not qualify as a LEGAL-RELATION-LR at time-t1 by
              (1) or (2), THEN the candidate is NOT a LEGAL-RELATION-LR at time-t1.
POd:      DEFINITION OF POWER
          PO(v(t2),LR(t2),y,t1) =df K --NLR(t1)
                                      --SwKK -BD2(w(t2),y,t1)
                                             -I .D2(w(t2),y) .v(t2)
                                             -I :v(t2)
                                                :K .LR(t2) .NPO(v(t3),LR(t3),y,t2)
          'Person-y has POWER at time-t1 to create LEGAL-RELATION-LR at time-t2 by
          doing something that will be legally characterized as event-v at time-t2
          (exercising that POWER)' is equal to by definition
          '1. LEGAL-RELATION-LR is NOT so at time-t1, AND
           2. there is a w such that
              A. it is naturally possible at time-t1 for person-y to do w at
                 time-t2, AND
              B. IF   1. person-y does w at time-t2,
                 THEN 2. event-v (y's POWER is exercised) occurs at time-t2, AND
              C. IF   1. event-v occurs at time-t2,
                 THEN 2. LEGAL-RELATION-LR is created at time-t2, AND
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3. IT IS NOT SO THAT person-y has POWER at time-t2 to create the LEGAL-RELATION-LR at time-t3 by doing something that will be legally characterized as event-v at time-t3 (exercising the POWER involved, if there were such POWER), that is: person-y has DISABILITY at time-t2 to create the LEGAL-RELATION-LR at time-t3 by doing something that will be legally characterized as event-v at time-t3 (exercising the POWER that is the negation of such DISABILITY)'.

DId: DEFINITION OF DISABILITY DI(v(t2),LR(t2),y,t1) =df NPO(v(t2),LR(t2),y,t1)
'Person-y has DISABILITY at time-t1 to create LEGAL-RELATION-LR at time-t2 by doing something that will be legally characterized as event-v at time-t2 (exercising the POWER that is the negation of the DISABILITY, if there were such POWER)' is equal to by definition 'IT IS NOT SO THAT person-y has POWER at time-t1 to create LEGAL-RELATION-LR at time-t2 by doing something that will be legally characterized as event-v at time-t2'.

LId: DEFINITION OF LIABILITY LI(v(t2),y,LR(t2),t1) =df PO(v(t2),LR(t2),y,t1)
'The LEGAL-RELATION-LR has LIABILITY at time-t1 of being created at time-t2 by person-y doing something that will be legally characterized as event-v at time-t2 (exercising the POWER associated with that LIABILITY)' is equal to by definition 'Person-y has POWER at time-t1 to create LEGAL-RELATION-LR at time-t2 by doing something that will be legally characterized as event-v at time-t2'.

IMd: DEFINITION OF IMMUNITY IM(v(t2),y,LR(t2),t1) =df NPO(v(t2),LR(t2),y,t1)
'The LEGAL-RELATION-LR has IMMUNITY at time-t1 of being created at time-t2 by person-y doing something that will be legally characterized as event-v at time-t2 (exercising the POWER that is the negation of the DISABILITY, if there were such POWER)' is equal to by definition 'IT IS NOT SO THAT person-y has POWER at time-t1 to create LEGAL-RELATION-LR at time-t2 by doing something that will be legally characterized as event-v at time-t2'.

CLRd: DEFINITION OF CONDITIONAL LEGAL RELATION

```
CLR(v(t2),LR(t2),t1)  =df  K --NLR(t1)
                             --SwKK -B(w(t2),t1)
                                    -I .w(t2) .v(t2)
                                    -I :v(t2)
                                       :K .LR(t2) .NCLR(v(t3),LR(t3),t2)
```

'There is a CONDITIONAL-LEGAL-RELATION-CLR at time-t1 that LEGAL-RELATION-LR will be created at time-t2 by the fulfillment of condition-v at time-t2' is equal to by definition

```
'1. LEGAL-RELATION-LR is NOT so at time-t1, AND
 2. there is an event-w such that
    A. it is naturally possible at time-t1 for event-w to occur at time-t2,
       AND
    B. IF   1. event-w occurs at time-t2,
       THEN 2. condition-v is fulfilled at time-t2, AND
    C. IF   1. condition-v is fulfilled at time-t2,
       THEN 2. LEGAL-RELATION-LR is created at time-t2, AND
            3. IT IS NOT SO THAT there is a CONDITIONAL-LEGAL-RELATION-CLR
               at time-t2 that LEGAL-RELATION-LR will be created at time-t3
               by the fulfillment of condition-v at time-t3'.
```

CRId: DEFINITION OF CONDITIONAL RIGHT

```
CRI(v(t2),RI(w(t4),x,y,t2),t1)  =df
          K --NRI(w(t4),x,y,t1)
            --SwKK -B(w(t2),t1)
                   -I .w(t2) .v(t2)
                   -I :v(t2)
                      :K .RI(w(t4),x,y,t2) .NCRI(v(t3),RI(w(t4),x,y,t3),t2)
```

'There is a CONDITIONAL-RIGHT-CRI at time-t1 that y's RIGHT at time-t2 that x do w at time-t4 will be created at time-t2 by the fulfillment of condition-v at time-t2' is equal to by definition

```
'1. IT IS NOT SO THAT y has a RIGHT at time-t1 that x do w at time-t4,
    AND
```

```
      2. there is an event-w such that
         A. it is naturally possible at time-t1 for event-w to occur at
            time-t2, AND
         B. IF   1. event-w occurs at time-t2,
            THEN 2. condition-v is fulfilled at time-t2, AND
         C. IF   1. condition-v is fulfilled at time-t2,
            THEN 2. y has a RIGHT at time-t2 that x do w at time-t4, AND
                 3. IT IS NOT SO THAT there is a CONDITIONAL-RIGHT-CRI at
                    time-t2 that y's RIGHT at time-t3 that x do w at time-t4
                    will be created by the fulfillment of condition-v at
                    time-t3'.
```

CDUd, CPRd, and CNOd: DEFINITIONS OF CONDITIONAL DUTY, CONDITIONAL PRIVILEGE, AND CONDITIONAL NORIGHT

In a similar manner to the way that the defined concept of RIGHT replaces the occurrences of LEGAL-RELATION-LR in CONDITIONAL-LEGAL-RELATION-CLR to obtain the above definition of CONDITIONAL-RIGHT-CRI, the defined concepts of DUTY, PRIVILEGE, AND NORIGHT replace the occurrences of LEGAL-RELATION-LR in the definition of CONDITIONAL-LEGAL-RELATION-CLR to obtain the definitions of CONDITIONAL-DUTY-CDU, CONDITIONAL-PRIVILEGE-CPR, and CONDITIONAL-NORIGHT-CNO.

CPOd: DEFINITION OF CONDITIONAL POWER

```
CPO(v(t2),PO(w(t4),LR(t4),y,t2),t1)  =df
                    K --NPO(w(t4),LR(t4),y,t1)
                       --SuKK -B(u(t2),t1)
                              -I .u(t2) .v(t2)
                              -I :v(t2)
                                 :K .PO(w(t4),LR(t4),y,t2)
                                    .NCPO(v(t3),PO(w(t4),LR(t4),y,t3),t2)
```

'There is a CONDITIONAL-POWER-CPO at time-t1 that y's POWER at time-t2 to create LEGAL-RELATION-LR at time-t4 by doing w at time-t4 will be created by the fulfillment of condition-v at time-t2' is equal to by definition

```
'1. IT IS NOT SO THAT y has POWER at time-t1 to create LEGAL-RELATION-LR
    at time-t4 by doing w at time-t4, AND
 2. there is an event-u such that
    A. it is naturally possible at time-t1 for event-u to occur at
       time-t2, AND
    B. IF   1. event-u occurs at time-t2,
       THEN 2. condition-v is fulfilled at time-t2, AND
    C. IF   1. condition-v is fulfilled at time-t2,
       THEN 2. y has POWER at time-t2 to create LEGAL-RELATION-LR at
               time-t4 by doing w at time-t4, AND
            3. IT IS NOT SO THAT there is a CONDITIONAL-POWER-CPO at
               time-t2 that y's POWER at time-t3 to create
               LEGAL-RELATION-LR at time-t4 by doing w at time-t4 will be
               created by the fulfillment of condition-v at time-t3'.
```

CLId, CDId, and CIMd: DEFINITIONS OF CONDITIONAL LIABILITY, CONDITIONAL DISABILITY, AND CONDITIONAL IMMUNITY

In a similar manner to the way that the defined concept of POWER replaces the occurrences of LEGAL-RELATION-LR in CONDITIONAL-LEGAL-RELATION-CLR to obtain the above definition of CONDITIONAL-POWER-CPO, the defined concepts of LIABILITY, DISABILITY, AND IMMUNITY replace the occurrences of LEGAL-RELATION-LR in the definition of CONDITIONAL-LEGAL-RELATION-CLR to obtain the definitions of CONDITIONAL-LIABILITY-CLI, CONDITIONAL-DISABILITY-CDI, and CONDITIONAL-IMMUNITY-CIM.