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Application of Newton Identities in Solving Selective Harmonic Elimination Problem With Algebraic Algorithms

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***Abstract*— Algebraic algorithms are powerful methods in solving the selective harmonic elimination (SHE) problem, which can find all exact solutions without the requirements of choosing initial values. However, the huge computational burden and long solving time limit the solving capability of algebraic algorithms. This article presents a novel Newton's identifies-based method to simplify the SHE equations including the order reduction and the variable elimination, thereby reducing the computational burden and the solving time of algebraic algorithms or in other words improving the solving capability of the algebraic algorithms. Compared with existing simplification methods, the proposed method significantly improves the efficiency of solving SHE equations. With the proposed method, the degree of reduction is no longer the bottleneck of solving the SHE equations by using algebraic algorithms. By using the proposed method, the SHE equations with ten switching angles are completely solved with the algebraic algorithm for the first time. The simulation and experimental results indicate that the proposed method is effective and correct.**

***Index Terms*— Elementary symmetric polynomials (ESPs), Newton's identities, power sums, selective harmonic elimination (SHE).**

I. Introduction

THE power electronic converters usually utilize very low switching frequency in high-power applications because of limitations of the switching losses and the electromagnetic interface (EMI) issue [1]–[7]. With such a low switching frequency, the selective harmonic elimination (SHE) technology has been the best modulation strategy due to its outstanding output harmonic performance in the medium- and high-power applications, such as motor drives, grid-connected converters, and active rectifiers [8]–[13]. The switching angles should be obtained first by solving a group of strong nonlinear transcendental equations namely the SHE equations in practical applications. However, how to solve the SHE equations is quite a complicated problem because of the complexity and the multisolution feature of the SHE equations.

There are three classes of methods to solve the SHE equations: numerical algorithms [14], intelligent optimization algorithms [15]–[23], and algebraic algorithms [24]–[35]. Numerical algorithms are the most traditional methods to solve the SHE equations. They can provide high accuracy results with fast convergence. However, they strongly rely on the guess of initial values which is quite an issue especially for multilevel converters because there has no systematic method to find feasible initial values. Moreover, numerical algorithms cannot handle the multisolution feature of the SHE equations, and usually, they can provide only one solution or just a part of the complete solutions. Even though a modified numerical method [14] was proposed aiming to obtain the complete solution of the SHE equations, the obtained results cannot be mathematically proved to be complete solutions of the SHE equations. Based on numerical computation technology, the development of intelligent optimization algorithms provides new strategies to solve the SHE equations. Intelligent optimization algorithms, to some extent, overcome the initial values issue, because the initial values of intelligent optimization algorithms can be selected randomly. However, intelligent optimization algorithms lack the support of mathematical theory and are sensitive to input parameters, so the precision of solutions is difficult to be guaranteed. Therefore, most intelligent optimization algorithms focus on the research of convergence speed and fitness function value, and some satisfactory results have been obtained [16], [22], [23]. Nevertheless, they also cannot deal with the multisolution feature of the SHE equations because of the local optimum problem. In a summary, both the numerical algorithms and the intelligent optimization algorithms cannot obtain complete solutions to the SHE equations.

Algebraic algorithms are introduced to solve the SHE equations because of their outstanding characteristics [27]–[29].

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They are implemented based on the algebraic theory, and all computations inside are carried out with symbols, not specific values, and therefore, they do not need to be given any specific initial values, and the complete solutions can be obtained by only one solving procedure. Moreover, they can give a direct and clear conclusion about the complete solutions of the SHE equations, for example, whether the SHE equations have solutions or not, and the conditions for the SHE equations to have solutions. So as to say, the algebraic algorithms seem to be the most powerful tool to solve the SHE equations compared with numerical and intelligent optimization algorithms. However, the algebraic algorithms face a big problem to solve the SHE equations. The algebraic methods require huge computation sources, for instance, the huge computer burden and the large random access memory (RAM) space. In addition, the time for solving is usually very long to solve the SHE equations with more switching angles. Thus, the solving capability of the algebraic algorithms is limited.

To improve the solving capability of the algebraic algorithms, our previous study [34] and other publications [31], [32] found that the SHE equations can be transformed into the symmetric polynomial system, and it can be further simplified into a lower-order and less-variables polynomial system. With the simplification, the computation burden and the solving time can be reduced, thus the solving capability of algebraic algorithms can be pushed to a higher level.

Two attempts have gained success in simplification of the SHE equations. The power sums-based method was proposed to simplify the SHE equations [30], in which the SHE equations are transformed into the form of power sums polynomials and then solved by the resultant elimination method. With the power sum simplification method, the solving capability of the resultant elimination method is improved from three to five switching angles. However, this simplification faces a problem that the simplification is coupled with the solving procedure. It introduces one extra polynomial system, which is, to some extent, almost the same complicated as the simplified SHE equations, thus, the solving procedure has to be called twice and as a result, the solving time becomes longer. Likewise, the elementary symmetric polynomial (ESP) is also introduced to simplify the SHE equations. The results published in [31], [32], and [34] show that the ESP simplification can significantly improve the solving capability of the Groebner basis method and the solving capability is pushed to nine switching angles. However, the efficiency of this simplification method is not so high enough to deal with the SHE equations with more switching angles, for example, to simplify the SHE equations with eight switching angles, it needs more than 34 h in a common workstation to finish the simplification. Furthermore, if the number of switching angles is larger than 9, it fails to finish the computation. Consequently, even though these two methods have been successfully used to simplify the SHE equations, the simplification method is still a bottleneck of the algebraic algorithms to solve the SHE equations.

To improve the performance of the algebraic algorithms for solving the SHE equations, this article presents a novel simplification method based on Newton's identities for the SHE equations, including the degree reduction and variable elimination. Compared with the existing two simplification methods, the proposed method is much simple and more effective, so it can dramatically improve the speed and the capability of the simplification of the SHE equations. If only the simplification procedure is considered, the proposed method can deal with the SHE equations with more than 50 switching angles, and arguably, the bottleneck of the simplification of the SHE equations can be eliminated with the proposed method. Obviously, the solving capability of the existing algebraic algorithms can be further improved with the proposed method. For example, with the proposed method, the solving capability of the Groebner basis method can be improved to 10. To our best knowledge, this is the first time to obtain the complete solutions of the SHE equations with ten switching angles, whatever kind of solving method is used. Furthermore, the proposed method can also be used for the numerical and the intelligent optimization algorithms, if the SHE equations are also needed to be simplified. Because this topic is out of the focus of this article, the details will not be discussed in this article.

This article is organized as follows. Section II describes the unified mathematical model of the SHE problem. Section III first provides basic concepts and principles of Newton's identities and then presents the proposed simplification algorithm. In Section IV, some computational results of nine and ten switching angles are analyzed to identify the correctness of the proposed method. Besides, the proposed method is compared and evaluated with the existing simplification method in Section V. Furthermore, in Section VI, the experiments of motor-drive applications and inverters are carried out to verify the effectiveness and correctness of the proposed method. Finally, this article is concluded in Section VII.

II. Mathematical Model of the SHE Problem

The SHE is based on the principle of Fourier expansion. According to basic concepts of mathematics, any periodic signal can be expanded into a Fourier series. The output PWM waveform of converters is commonly periodic, and obviously it can be expanded into the Fourier series, in which the amplitude of fundamental and harmonic components is represented with a sum of trigonometric functions with the switching angles as variables. If a group of switching angles, such that the fundamental component of the output waveform is equal to the desired value while the amplitudes of the selected harmonics are all equal to zero, can be obtained, and the output voltage waveform of the converter can be constructed with the obtained switching angles, then the output of the converter will not contain the selective harmonics. According to the conclusion in [35], the switching angles can be obtained by solving such a group of equations expressed as (1), namely the SHE equations, in which $m = \pi U/4V_{\mathrm{dc}}$ is the modulation index and n is the number of the switching angles and also the number of the equations. U is the desired amplitude of the fundamental component, V_{dc} is the voltage of the dc source, $\alpha_i (i = 1, 2, \ldots, n)$ are the switching angles in a quarter period, and k is the order of the eliminated harmonics

$$\begin{cases} \sum_{i=1}^{n} \cos(\alpha_i) = m \\ \sum_{i=1}^{n} \cos(k\alpha_i) = 0, \quad k = 5, 7, 11, \ldots \end{cases} \tag{1}$$

It should be pointed out that the SHE equations expressed in (1) is a unified model. Although it looks like the traditional SHE model for the multilevel staircase waveform, they are completely different in essence. The traditional model is limited by the switching pattern, that is, the combination of transition states on each switching angle, so there has an inequality constraint of switching angles for the traditional model. However, the unified model removes the inequality constraint of switching angles and includes all possible switching patterns, which significantly increase the solution space. Thus, this model can be used for converters with any topology, for example, two-level, three-level, or multilevel converters. For more details, refer to the literature [35].

It can be seen that the SHE equations (1) contain only cosine functions with switching angles α_i as variables. In order to apply the algebraic algorithms, the SHE equations should be first transformed into an algebraic polynomial system, and this procedure can be carried out with the application of the first-kind Chebyshev polynomial: $T_k(\cos(\alpha_i)) = \cos(k\alpha_i)$, where T_k represents a polynomial expression. Furthermore, if $x_i = \cos(\alpha_i)$, the Chebyshev polynomial can be rewritten as $T_k(x_i) = \cos(k\alpha_i)$. Based on the Chebyshev polynomial, all cosine function terms in the SHE equations (1) can be transformed into polynomial equations (3) by using the recursions (2). Finally, the SHE equations (1) is transformed into the polynomial system (3), based on which the algebraic algorithms can be applied to solve the system. As mentioned in Section I, this system is quite complicated, so it is very hard to solve the polynomial system directly. The proposed simplification method aims to reduce the order and the amount of variables of the polynomial system to make it being much easier to be solved

$$\begin{cases} T_1(x) = x, \quad T_2(x) = 2x^2 - 1 \\ \vdots \\ T_k(x) = 2xT_{k-1}(x) - T_{k-2}(x) \end{cases} \tag{2}$$

$$\begin{cases} f_1(x) = \sum_{i=1}^{n} x_i - m = 0 \\ f_5(x) = \sum_{i=1}^{n} \left(5x_i - 20x_i^3 + 16x_i^5\right) = 0 \\ f_7(x) = \sum_{i=1}^{n} \left(-7x_i + 56x_i^3 - 112x_i^5 + 64x_i^7\right) = 0. \\ \qquad \vdots \end{cases} \tag{3}$$

III. Solving SHE Problem With the Proposed Algorithm

This section proposes a novel simplification method, which has a simple process and only involves multiplications and additions. This simplification method can equivalently transform (3) into a lower-order algebraic polynomial system, and no solution is lost in this process. Combined with other algebraic methods, the proposed method can solve the highest number of switching angles so far. This section will give the principle of the proposed method and the detailed steps of the whole solving process.

A. Newton's Identities

In the algebraic theory [36], Newton's identifies gives the relationship between the power sums polynomials and the ESPs. Let x_i, $1 \le i \le n$ are variables, the kth power sums polynomials are defined as

$$p_k = \sum_{i=1}^{n} x_i^k.$$

And the ESP is defined as

$$\begin{cases} e_0 = 1 \\ e_1 = x_1 + x_2 + \cdots + x_n \\ e_2 = \sum_{1 \le i < j \le n} x_i x_j \\ \vdots \\ e_n = x_1 x_2 \cdots x_n \\ e_k = 0, \quad \text{for } k > n. \end{cases} \tag{4}$$

Then, Newton's identities can be stated as follows, and it is valid for all $n \ge k \ge 1$

$$ke_k = \sum_{i=1}^{k} (-1)^{i-1} e_{k-i} p_i. \tag{5}$$

Also, when $k > n \ge 1$, Newton's identities should be stated as

$$0 = \sum_{i=k-n}^{k} (-1)^{i-1} e_{k-i} p_i. \tag{6}$$

Therefore, according to the definitions of Newton identities, power sum polynomials can be recursively expressed in terms of ESPs, and p_k can be concretely rewritten as

$$\begin{cases} p_1 = e_1 \\ p_2 = e_1 p_1 - 2e_2 \\ p_3 = e_1 p_2 - e_2 p_1 + 3e_3 \\ \vdots \\ p_n = e_1 p_{n-1} - e_2 p_{n-2} + \cdots + (-1)^{n-1}(n)e_n \\ \vdots \\ p_k = e_1 p_{k-1} + e_2 p_{k-2} + \cdots + (-1)^{k-1} e_n p_{k-n}. \end{cases} \tag{7}$$

B. Degree Reduction With Newton's Identities

The algebraic polynomial system (3) can be simplified based on the principle of Newton's identities. As the subsequent derivation is related to the number of switching angles, for convenience, the case described following is with ten switching angles. First, by substituting the power sum polynomials to the SHE equations (3), the following equation can be obtained. It can be seen that (8) is undetermined since the number of unknown variables is more than the number of equations, so, it cannot be solved directly:

$$\begin{cases} p_1 - m = 0 \\ 5p_1 - 20p_3 + 16p_5 = 0 \\ -7p_1 + 56p_3 - 112p_5 + 64p_7 = 0 \\ \vdots \\ 29p_1 - 4060p_3 + \cdots + 268435356p_{29} = 0. \end{cases} \tag{8}$$

Based on the principle of Newton identities, the number of variables of (8) can be reduced to the same number as equations. According to the Newton's identities, all the power sums whose degree higher than n can be rewritten in the low-order power sums $p_1 \sim p_n$, which could make the system balanced. Therefore, the second step is to rewritten ESPs $e_1 \sim e_{10}$ in terms of power sums polynomials $p_1 \sim p_{10}$ according to (5)

$$\begin{cases} e_1 = p_1 \\ e_2 = \dfrac{p_1^2}{2} - \dfrac{p_2}{2} \\ e_3 = \dfrac{p_1^3}{6} - \dfrac{p_1 p_2}{2} + \dfrac{p_3}{3} \\ e_4 = \dfrac{p_1^4}{24} - \dfrac{p_1^2 p_2}{4} + \dfrac{p_2^2}{8} + \dfrac{p_1 p_3}{3} - \dfrac{p_4}{4} \\ \vdots \\ e_{10} = \dfrac{p_1^{10}}{3628800} - \dfrac{p_2 p_1^8}{80640} + \cdots + \dfrac{p_4 p_6}{24} - \dfrac{p_{10}}{10} \end{cases} . \tag{9}$$

The third step is to transform $p_{11} \sim p_{29}$ into $p_1 \sim p_{10}$ by taking the ESPs (9) into the Newton identities (7). The results are expressed as follows:

$$\begin{cases} p_{11} = \dfrac{-p_1^{11}}{3628800} + \dfrac{11p_2 p_1^9}{725760} - \cdots - \dfrac{11p_3 p_7}{21} + \dfrac{11p_1 p_{10}}{10} \\ p_{12} = \dfrac{-p_2 p_1^{10}}{3628800} + \dfrac{p_3 p_1^9}{362880} + \cdots - \dfrac{p_2 p_4 p_6}{4} + \dfrac{3p_2 p_{10}}{5} \\ p_{13} = \dfrac{-p_3 p_1^{10}}{3628800} + \dfrac{p_4 p_1^9}{362880} + \cdots - \dfrac{13p_3 p_4 p_6}{72} + \dfrac{13p_3 p_{10}}{30} \\ \vdots \\ p_{29} = \dfrac{-p_1^{10}}{3628800} + \dfrac{p_1^8 p_2}{80640} + \cdots + \dfrac{p_1 p_3 p_6}{18} - \dfrac{p_1 p_3 p_7}{21}. \end{cases} \tag{10}$$

Finally, by substituting the high-order power sums polynomials in (8), that is, $p_{11}, p_{13}, p_{17}, p_{19}, p_{23}, p_{25}, p_{29}$, with their expression in (10), the polynomial system with degree reduction are obtained as follows:

$$\begin{cases} p_1 - m = 0 \\ 5p_1 - 20p_3 + 16p_5 = 0 \\ -7p_1 + 56p_3 - 112p_5 + 64p_7 = 0 \\ -4p_1^{11} + 220p_1^9 p_2 - \cdots - 6160p_2 p_9 = 0 \\ \vdots \\ 20512\ p_1^{29} \cdots - 6916756105728000000p_2^3 p_4^2 p_9 = 0. \end{cases} \tag{11}$$

TABLE I
COMPARISON OF THE DEGREE OF $f_{29}(x)$ AND $f_{29}(p)$

	x_1/p_1	x_2/p_2	x_3/p_3	x_4/p_4	x_5/p_5
$f_{29}(x)$	29	29	29	29	29
$f_{29}(p)$	0	14	9	6	5
	x_6/p_6	x_7/p_7	x_8/p_8	x_9/p_9	x_{10}/p_{10}
$f_{29}(x)$	29	29	29	29	29
$f_{29}(p)$	4	4	3	3	2

Algorithm 1 Proposed Algorithm

1: Algebraic polynomial system $f(x_1, x_2, \ldots, x_n)$
2: Substitute $x_1, x_2, \ldots, x_n$ with $p_1, p_2, \ldots, p_n, \ldots, p_k$, $f(x_1, \ldots, x_n)$ is transformed into $f(p_1, \ldots, p_n, \ldots, p_k)$.
3: **if** The number of p larger than n **then**
4: Eliminate $p_{n+1}, \ldots, p_k$ according to (7).
5: **end if**
6: Eliminate all the ESPs $e_1, e_2, \ldots, e_n$ according to (7), get the final reduced polynomial system $f(p_1, p_2, \ldots, p_n)$.
7: Solve the Groebner basis of the reduced polynomial system, get the results of $p_1, p_2, \ldots, p_n$.
8: Solve $e_1, e_2, \ldots, e_n$ from the results of $p_1, p_2, \ldots, p_n$ according to (7).
9: Using the coefficients $e_1, e_2, \ldots, e_n$, construct the univariate higher-order equation with variables $x_1, x_2, \ldots, x_n$, according to (4).
10: Solve the univariate higher-order equation, and use the inverse triangle transformation to get the final switching angles.

In (11), as the modulation index m will be preset and $p_1 = m$, the number of variables is decreased from 10 to 9 and their degree is greatly reduced. Table I gives the degree of comparison between the original algebraic SHE equations and the degree-reduced SHE equations. At this point, the stage of simplifying SHE equations has been completed. In the process of simplification, every step is equivalent transformation. Thus, although the degree of the polynomial system has been significantly reduced, the solutions of the simplified polynomial system are exactly the same as the original polynomial system.

C. Solving Final Results by Algebraic Algorithm

The reduced polynomial system (11) can be solved by using algebraic algorithms, such as the resultant elimination method, the Wu's method, and the Groebner basis method. According to the published literature [27], the Groebner basis method has the best computation ability, so the Groebner basis method is chosen here to solve (11). As the implementation of the Groebner basis method is beyond the subject of this article, the detailed principle of this method is omitted here and it can be found in [27]. In fact, some commercial symbolic computing software, such as *Maple* and *Mathematica*, provide the computing command of Groebner basis. Here, the command *Basis* in *Maple* is used to compute Groebner basis of (11), and the results are shown as follows:

$$\begin{cases} a_{162}p_2^{162} + a_{161}p_2^{161} + \cdots + a_1 p_2 + a_0 = 0 \\ b_1 p_3 + f_1(p_2) = 0 \\ b_2 p_4 + f_2(p_2) = 0 \\ \vdots \\ b_8 p_{10} + f_8(p_2) = 0 \end{cases} \quad (12)$$

where $a_0, a_1, \ldots, a_{162}$ and $b_1, b_2, \ldots, b_8$ are all big integers, and $f_1, f_2, \ldots, f_8$ are all univariate polynomials in p_2, which are too large to be listed here. It can be seen from (12) that the first equation is a univariate high-order polynomial equation in p_2. Although the degree of the first equation is very high, how to solve a univariate higher-order polynomial equation is well studied in the algebraic field. Therefore, it is easy to find all solutions of p_2 in some mathematical software, such as *Maple*, *Mathematica*, and *MATLAB*. Here, we compute the first equation in (12) by using the command *fsolve* in *Maple*. Once the solutions of p_2 are solved, the other eight equations are converted to univariate linear equations, so all solutions of $p_3, p_4, \ldots, p_{10}$ can be easily obtained.

After finding all solutions of $p_2, p_3, \ldots, p_{10}$, the last step is to solve the results of $x_1, x_2, \ldots, x_{10}$. Actually, the results of x can be solved from the univariate polynomial $F(x)$ with the ESPs as coefficients. Suppose $F(x)$ as a univariate polynomial equation defined on the real number field with roots $x_1, x_2, \ldots, x_{10}$, which can be written as

$$F(x) = (x - x_1)(x - x_2)\cdots(x - x_{10}). \quad (13)$$

If (13) is expanded, it can be seen that the coefficients of $F(x)$ have the same form of ESP (4). Therefore, once the solutions of $e_1, e_2, \ldots, e_{10}$ are obtained, the results of $x_1, x_2, \ldots, x_{10}$ can be solved from ESPs (4) by constructing the univariate polynomial equation with coefficients $e_1, e_2, \ldots, e_{10}$ as follows:

$$f(x) = x^{10} - e_1 x^9 + e_2 x^8 - e_3 x^7 + \cdots - e_9 x + e_{10}. \quad (14)$$

The solutions of $e_2, e_3, \ldots, e_{10}$ can be easily solved by using (9). Then, the final solutions for the algebraic form of SHE equations (3) can be easily obtained by solving (14). Finally, according to $\arccos(x_i) = \alpha_i$, switching angles $\alpha_1, \alpha_2, \ldots, \alpha_{10}$ can be obtained. In order to make the algorithm easier to understand, the whole solving process has been given in Algorithm 1.

IV. Computation Results

Based on a workstation with XEON E3-1230 CPU and 16-GB RAM, and the symbolic computing software *Maple21*, some results for the SHE equations with nine and ten switching angles are obtained by combining the proposed method and the Groebner basis method.

A. SHE Equations With Ten Switching Angles

For the case of ten switching angles described in Section III, when the modulation index $m = 0.8$, there are 69 groups of solutions in total, which are all listed in Table II. The arrows on the right sides of the angles indicate the transition states of the PWM waveforms at each switching angle. According to the levels of the generated waveforms, the 69 groups of solutions can be further classified into three-level, five-level, seven-level, and nine-level waveforms, and each of them has three, 50, 14, and two groups of solutions, respectively. This is the first time that all the possible solutions for the SHE equations with ten switching angles are given.

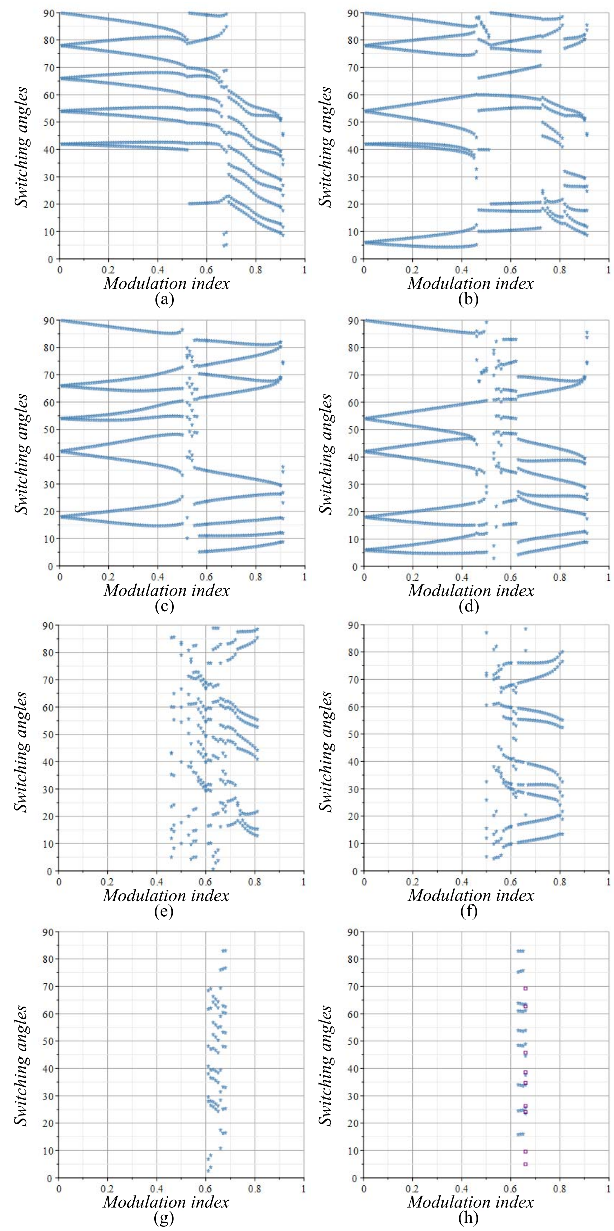

Fig. 1. Solution trajectories of three-level SHE problem. (a)–(g) One group of solution and two groups of solutions in (h).

B. SHE Equations With Nine Switching Angles

Fig. 1 is the part of the solutions for three-level PWM with nine switching angles. It can be seen that there have four groups of solutions under most modulation indices, and when $m = 0.66$, there are nine groups of solutions. The results indicate that there are many groups of solutions under

TABLE II
SIXTY-NINE GROUPS OF SWITCHING ANGLES FOR $m = 4/5$

		α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9	α_{10}
	1	12.89° ↓	18.44° ↑	21.47° ↓	53.10° ↑	55.77° ↓	74.27° ↑	76.68° ↓	80.54° ↑	82.70° ↓	87.67° ↑
three-level	2	13.45° ↓	15.78° ↑	21.33° ↓	42.06° ↑	45.06° ↓	52.92° ↑	55.68° ↓	82.64° ↑	84.46° ↓	88.00° ↑
	3	12.55° ↓	17.52° ↑	23.28° ↓	37.44° ↑	39.91° ↓	53.34° ↑	55.91° ↓	73.65° ↑	75.87° ↓	87.71° ↑
	4	38.53° ↓	41.54° ↑	47.09° ↓	53.07° ↑	55.75° ↓	73.41° ↓	75.75° ↑	82.09° ↓	85.49° ↑	88.56° ↓
	5	21.47° ↓	41.54° ↑	47.10° ↓	53.07° ↑	55.75° ↓	73.38° ↓	75.68° ↑	83.33° ↑	85.20° ↓	88.22° ↑
	6	12.89° ↓	38.53° ↓	41.56° ↑	53.09° ↑	55.77° ↓	73.14° ↑	75.97° ↓	77.42° ↑	83.27° ↓	87.74° ↑
	7	18.46° ↑	21.48° ↓	47.09° ↓	53.06° ↑	55.74° ↓	73.48° ↓	76.02° ↑	78.97° ↓	84.96° ↑	88.34° ↓
	8	4.12° ↓	38.56° ↓	41.61° ↑	47.17° ↓	53.21° ↑	64.18° ↑	73.78° ↓	76.32° ↑	83.07° ↓	87.67° ↑
	9	12.93° ↓	21.49° ↓	41.52° ↑	53.02° ↑	55.72° ↓	69.67° ↑	69.79° ↓	75.21° ↑	84.70° ↑	88.20° ↓
	10	4.34° ↓	21.50° ↓	41.50° ↑	47.04° ↓	52.96° ↑	64.30° ↑	73.13° ↓	75.34° ↑	84.68° ↑	88.19° ↓
	11	13.22° ↓	21.57° ↓	41.35° ↑	51.54° ↑	52.37° ↓	54.29° ↑	56.16° ↓	75.44° ↑	84.76° ↑	88.22° ↓
	12	3.98° ↓	18.33° ↑	21.41° ↓	47.24° ↓	53.36° ↑	64.10° ↑	74.30° ↓	78.37° ↑	81.33° ↓	87.45° ↑
	13	7.39° ↓	21.53° ↑	36.58° ↓	44.23° ↓	48.96° ↑	61.59° ↑	63.00° ↓	69.03° ↑	77.94° ↓	87.35° ↓
	14	6.10° ↓	9.22° ↑	37.79° ↓	40.25° ↑	45.70° ↓	63.98° ↑	66.54° ↓	78.04° ↓	81.55° ↑	87.28° ↓
	15	15.56° ↑	21.76° ↓	40.75° ↑	43.54° ↓	47.60° ↓	53.31° ↑	55.87° ↓	73.29° ↓	85.03° ↑	88.35° ↓
	16	8.52° ↓	28.11° ↓	33.87° ↑	42.48° ↓	47.17° ↑	59.62° ↑	60.99° ↓	69.92° ↑	78.37° ↓	82.18° ↑
	17	4.27° ↓	12.91° ↓	18.47° ↑	38.52° ↓	53.04° ↑	64.25° ↑	74.56° ↑	77.26° ↓	83.46° ↓	87.81° ↑
	18	22.34° ↓	25.79° ↑	34.72° ↓	40.91° ↑	46.86° ↓	52.90° ↑	55.66° ↓	73.52° ↓	75.92° ↑	88.63° ↓
	19	13.49° ↓	15.88° ↑	38.68° ↓	42.11° ↑	45.04° ↓	52.91° ↑	55.67° ↓	82.31° ↓	85.85° ↑	88.75° ↓
	20	22.34° ↓	28.07° ↓	30.78° ↑	40.79° ↑	46.79° ↓	52.84° ↑	55.63° ↓	73.54° ↓	75.97° ↑	85.58° ↑
	21	7.02° ↑	12.95° ↓	18.50° ↑	21.50° ↓	55.68° ↓	67.09° ↓	75.09° ↑	78.37° ↓	84.62° ↑	88.15° ↓
	22	4.38° ↓	12.98° ↓	18.52° ↑	21.51° ↓	52.92° ↑	64.31° ↑	75.07° ↑	78.34° ↓	84.59° ↑	88.14° ↓
	23	20.14° ↑	23.94° ↓	28.72° ↑	33.63° ↓	46.49° ↓	52.63° ↑	55.51° ↓	73.60° ↓	75.92° ↑	79.49° ↓
	24	2.66° ↓	5.53° ↑	12.38° ↓	38.74° ↓	41.96° ↑	64.42° ↑	67.26° ↓	75.22° ↑	82.75° ↓	87.55° ↑
	25	8.85° ↓	17.74° ↓	26.80° ↑	30.07° ↓	46.82° ↑	58.68° ↑	60.05° ↓	70.08° ↑	80.73° ↑	84.64° ↓
five-level	26	5.91° ↓	8.94° ↑	19.53° ↑	22.08° ↓	45.91° ↓	63.99° ↑	66.53° ↓	77.15° ↓	83.62° ↑	87.58° ↓
	27	13.47° ↓	23.70° ↓	28.56° ↑	33.48° ↓	39.98° ↑	52.65° ↑	55.53° ↓	75.44° ↑	77.43° ↓	78.28° ↑
	28	13.10° ↓	16.45° ↓	17.63° ↑	38.65° ↓	42.69° ↑	53.00° ↑	55.72° ↓	74.63° ↑	83.27° ↓	87.75° ↑
	29	4.19° ↓	6.88° ↑	12.89° ↓	21.47° ↓	41.55° ↑	64.34° ↑	67.12° ↓	75.15° ↑	84.69° ↑	88.20° ↓
	30	12.38° ↓	17.41° ↑	27.39° ↑	31.88° ↓	39.55° ↓	53.50° ↑	56.01° ↓	73.42° ↑	75.75° ↓	83.10° ↓
	31	6.82° ↓	10.32° ↑	15.23° ↓	20.86° ↑	37.05° ↓	63.53° ↑	65.82° ↓	73.07° ↑	76.07° ↓	87.20° ↓
	32	8.19° ↓	12.51° ↑	17.36° ↓	27.95° ↓	34.02° ↑	61.06° ↑	62.58° ↓	70.66° ↑	73.85° ↓	81.60° ↑
	33	3.66° ↓	16.34° ↑	23.03° ↓	38.22° ↑	41.25° ↓	48.01° ↓	53.91° ↑	63.78° ↑	73.82° ↓	87.61° ↑
	34	4.39° ↓	23.43° ↓	28.37° ↑	33.38° ↓	40.14° ↑	46.60° ↓	52.76° ↑	64.43° ↑	73.89° ↓	76.53° ↑
	35	10.18° ↑	10.37° ↓	12.80° ↓	21.46° ↓	41.57° ↑	52.89° ↑	55.67° ↓	75.14° ↑	84.68° ↑	88.19° ↓
	36	4.28° ↓	13.76° ↓	16.30° ↑	38.77° ↓	42.57° ↑	45.25° ↓	52.94° ↑	64.31° ↑	83.07° ↓	87.68° ↑
	37	3.19° ↓	16.13° ↑	27.01° ↑	31.73° ↓	40.53° ↓	48.50° ↓	54.55° ↑	63.37° ↑	73.71° ↓	82.67° ↓
	38	7.04° ↑	13.05° ↓	15.27° ↑	21.46° ↓	41.60° ↑	44.88° ↓	55.67° ↓	67.10° ↓	84.65° ↑	88.18° ↓
	39	4.37° ↓	13.11° ↓	15.31° ↑	21.46° ↓	41.61° ↑	44.90° ↓	52.91° ↑	64.33° ↑	84.64° ↑	88.17° ↓
	40	6.97° ↑	12.86° ↓	21.16° ↓	24.32° ↑	35.51° ↓	41.75° ↑	55.70° ↓	67.05° ↓	75.01° ↑	88.07° ↓
	41	6.90° ↑	12.78° ↓	21.07° ↓	28.18° ↓	32.17° ↑	41.91° ↑	55.74° ↓	66.99° ↓	74.89° ↑	84.28° ↑
	42	7.39° ↑	13.52° ↓	20.17° ↑	24.02° ↓	28.77° ↑	33.67° ↓	55.50° ↓	67.40° ↓	75.80° ↑	79.43° ↓
	43	6.11° ↓	9.06° ↑	18.03° ↓	21.58° ↑	36.79° ↓	43.45° ↑	46.45° ↓	63.77° ↑	66.19° ↓	87.33° ↓
	44	4.51° ↓	13.53° ↓	20.19° ↑	24.04° ↓	28.79° ↑	33.67° ↓	52.60° ↑	64.50° ↑	75.80° ↑	79.42° ↓
	45	8.88° ↓	18.55° ↓	22.41° ↑	28.38° ↓	34.42° ↑	38.71° ↓	46.52° ↑	59.05° ↑	60.45° ↑	70.27° ↓
	46	6.00° ↓	9.18° ↑	20.95° ↑	26.56° ↓	31.56° ↑	35.75° ↓	45.49° ↓	64.11° ↑	66.75° ↓	77.69° ↓
	47	13.61° ↓	16.09° ↑	22.63° ↓	26.24° ↑	34.55° ↓	40.99° ↑	44.46° ↓	52.71° ↑	55.56° ↓	88.81° ↓
	48	13.64° ↓	16.12° ↑	22.59° ↓	27.83° ↑	30.34° ↓	40.83° ↑	44.36° ↓	52.67° ↑	55.54° ↓	85.79° ↓
	49	3.91° ↓	6.77° ↑	13.18° ↓	23.23° ↓	28.25° ↑	33.37° ↓	40.36° ↑	64.54° ↑	67.47° ↓	75.95° ↑
	50	13.61° ↓	15.96° ↑	20.06° ↑	23.91° ↓	28.71° ↑	33.63° ↓	44.18° ↓	52.59° ↑	55.50° ↓	79.45° ↓
	51	13.59° ↓	18.52° ↓	19.15° ↑	23.79° ↓	28.65° ↑	33.60° ↓	40.53° ↑	52.59° ↑	55.49° ↓	75.83° ↑
	52	7.47° ↑	13.92° ↓	16.58° ↑	23.49° ↓	28.47° ↑	33.51° ↓	40.47° ↑	44.12° ↓	55.48° ↓	67.44° ↓
	53	4.50° ↓	13.89° ↓	16.55° ↑	23.48° ↓	28.46° ↑	33.50° ↓	40.46° ↑	44.12° ↓	52.56° ↑	64.52° ↑
	54	18.45° ↑	38.53° ↓	47.11° ↓	53.08° ↑	55.77° ↓	73.16° ↓	74.97° ↑	77.47° ↑	83.39° ↓	87.78° ↑
	55	6.84° ↑	38.55° ↓	41.59° ↑	47.15° ↓	55.83° ↓	66.77° ↓	73.84° ↓	76.38° ↑	83.09° ↓	87.68° ↑
	56	7.00° ↑	21.49° ↓	41.51° ↑	47.06° ↓	55.69° ↓	67.05° ↓	73.08° ↓	75.30° ↑	84.69° ↑	88.19° ↓
	57	7.02° ↑	12.95° ↓	38.50° ↓	41.50° ↑	55.68° ↓	67.10° ↓	75.14° ↑	81.31° ↓	84.46° ↑	88.10° ↓
	58	4.39° ↓	12.98° ↓	38.49° ↓	41.47° ↑	52.91° ↑	64.32° ↑	75.13° ↑	81.23° ↓	84.36° ↑	88.06° ↓
	59	6.75° ↑	18.37° ↑	21.43° ↓	47.19° ↓	55.91° ↓	66.62° ↓	74.40° ↓	78.48° ↑	81.41° ↓	87.47° ↑
seven-level	60	6.94° ↑	12.91° ↓	18.46° ↑	38.53° ↓	55.74° ↓	66.97° ↓	74.56° ↑	77.27° ↓	83.45° ↓	87.81° ↑
	61	5.62° ↓	8.53° ↑	19.22° ↑	38.10° ↓	46.21° ↓	63.98° ↑	66.50° ↓	76.54° ↓	84.47° ↓	88.26° ↑
	62	8.58° ↓	16.81° ↓	24.51° ↓	37.06° ↑	47.54° ↑	58.44° ↑	59.76° ↓	69.59° ↑	79.54° ↑	88.12° ↑
	63	13.20° ↓	15.02° ↑	18.25° ↑	38.59° ↓	45.38° ↓	53.00° ↑	55.73° ↓	77.32° ↓	83.34° ↓	87.77° ↑
	64	6.22° ↑	16.42° ↑	23.06° ↓	38.17° ↑	41.18° ↓	47.94° ↓	56.21° ↓	66.10° ↓	73.91° ↓	87.62° ↑
	65	7.28° ↑	23.45° ↓	28.38° ↑	33.38° ↓	40.12° ↑	46.59° ↓	55.58° ↓	67.23° ↓	73.94° ↓	76.58° ↑
	66	7.10° ↑	13.80° ↓	16.34° ↑	38.77° ↓	42.58° ↑	45.26° ↓	55.69° ↓	67.06° ↓	83.08° ↓	87.69° ↑
	67	4.35° ↓	12.86° ↓	21.02° ↓	24.15° ↑	35.60° ↑	41.84° ↓	52.99° ↑	64.27° ↑	74.94° ↑	88.01° ↓
nine-level	68	5.67° ↑	16.23° ↑	27.04° ↑	31.72° ↓	40.47° ↓	48.40° ↓	56.60° ↓	65.51° ↓	73.81° ↓	82.71° ↓
	69	4.31° ↓	12.71° ↓	20.86° ↓	28.17° ↓	32.35° ↑	42.11° ↑	53.08° ↑	64.22° ↑	74.74° ↑	84.08° ↑

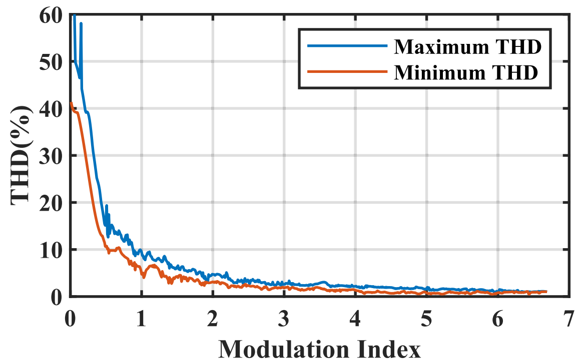

Fig. 2. Comparison of the lowest THD and the maximum THD in full modulation index range.

most modulation indices, which can be used for the optimal modulation of converters to improve the THD performance. In order to show the THD performance of different solutions, the solutions for SHE equations with nine switching angles are solved for m in $[0, 6.7]$ with an increment step of $\Delta m = 0.01$. Then, the THD performance of the obtained solutions is shown in Fig. 2, in which the blue line represents the maximum THD value and the orange line indicates the lowest THD value of the obtained solutions. It can be seen that in the same modulation index, different solutions will lead to different THD performance. Therefore, solving all the solutions can increase the possibility to find valid solutions for specific modulation indices. Furthermore, it can be seen that the continuity of these solutions is difficult to determine. Thus, it is very hard to solve the trajectories by some numerical methods [21], [22] or fit them with simple piecewise linear functions for multilevel converters. Therefore, these solution trajectories can not only prove the correctness and completeness of the proposed method, but also provide comprehensive solutions for multilevel converters.

V. Evaluation of the Proposed Method

As described in Section III, the procedure of solving SHE equations with algebraic algorithms can be divided into two steps: degree reduction with the simplification method, and solving final results with the algebraic method. Therefore, the performance of the proposed method is evaluated in terms of two aspects: the improvement of the simplification effect and the improvement of the whole solving procedure.

A. Evaluation of the Proposed Method in Terms of the Simplification Process

To evaluate the simplification effect of the proposed method, the simplification efficiency and ability are compared with the commonly used ESP-based method. Table III shows the comparison of the executing time for the proposed method and the commonly used method, which are calculated on a desktop computer with XEON E3-1230 CPU and 16-GB RAM. It can be seen that the computation time consumed by the ESP-based method increases dramatically when the number of switching angles is more than six. When the

TABLE III

Comparison of the Executing Time Between the Symmetric Polynomials Method and the Proposed Method (Unit: Second)

Switching points	Symmetric polynomials	Proposed method
$N = 5$	0.969	0.490
$N = 6$	60.782	0.689
$N = 7$	2062.385	2.624
$N = 8$	122503.216	3.559
$N = 9$	N/A	10.182
$N = 10$	N/A	50.434

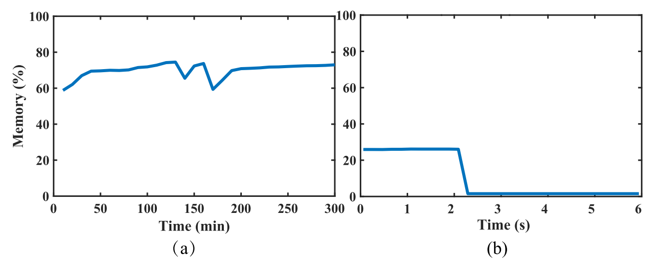

Fig. 3. Comparison of the computer memory between the symmetric polynomials method and the proposed method. (a) Computer memory used to solve eight switching angles with the symmetric polynomial method. (b) Computer memory used to solve eight switching angles with the proposed method.

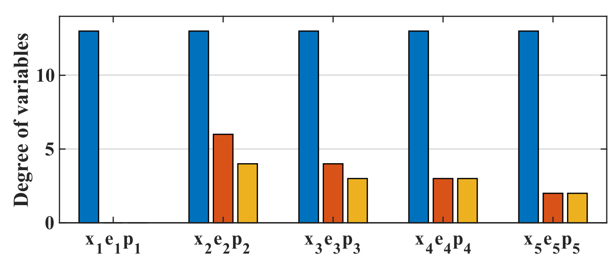

Fig. 4. Degree of comparison between x, e, and p with five switching angles.

number of switching angles is eight, the proposed method performs 40 800 times faster than the ESP-based method. More importantly, the ESP-based method totally fails to give the final results due to the huge computing burden when the number of switching angles exceeds eight. Besides, Fig. 3 indicates the computer memory occupied by running the two simplification methods. Fig. 3(a) only shows a 5-h process of the ESP method, and actually, it needs nearly 35 h to complete the whole process. It can be seen that the computer memory occupied by the ESP-based method is almost three times as large as the proposed method.

In addition to simplification efficiency, the simplification ability of the proposed method has been improved. As shown in Figs. 4 and 5, the degree of comparisons for five and seven switching angles are given, in which the blue, orange, and yellow represent the original degree of the SHE equations, the reduced degree achieved by the ESP-based method, and the proposed method, respectively. It can be seen that the degree of the SHE equations simplified by the proposed method is lower than that of the ESP-based method, which can further reduce the computing burden of the subsequence solving procedure.

TABLE IV

COMPARISON OF COMPUTATIONAL ABILITY AND EFFICIENCY BETWEEN THE PROPOSED METHOD AND OTHER METHODS (UNIT: SECOND)

Algorithms	**Running Time**						**Software**
	$N = 5$	$N = 6$	$N = 7$	$N = 8$	$N = 9$	$N = 10$	
Groebner basis method	2.269	N/A	N/A	N/A	N/A	N/A	*Maple*
Symmetric polynomials + Groebner basis	1.574	63.282	2079.264	122677.981	N/A	N/A	*Maple*
Proposed method + Groebner basis	0.935	3.391	6.251	46.399	365.249	8862.571	*Maple*

TABLE V

COMPARISON OF THE MAXIMUM SOLVABLE NUMBER OF THE PREVIOUS ALGEBRAIC METHODS AND THE PROPOSED METHOD

	Algorithms	**Literatures**	**Proposed year**	**Maximum solvable number**
Simplification Methods	Symmetric polynomial method	[36]	2005	9
	Proposed method	\	\	>50
Complete Algorithms	Resultant method	[32]	2002	3
	Wu's method	[33]	2005	4
	Resultant+Symmetric polynomial method	[36]	2005	5
	Wu's+Symmetric polynomial method	[28]	2007	5
	Groebner basis method	[34]	2015	5
	Groebner+Symmetric polynomial method	[29]	2016	9
	Groebner+Proposed method	\	\	10

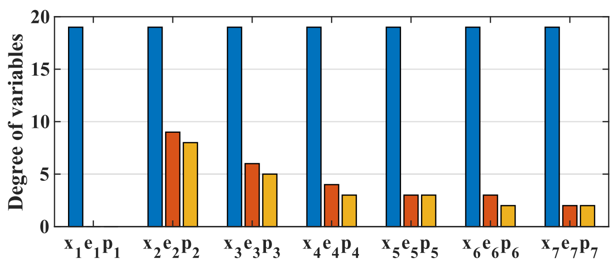

Fig. 5. Degree of comparison between x, e, and p with seven switching angles.

Therefore, the simplification ability of the proposed method is better than the ESP-based method.

B. Evaluation of the Proposed Method in Terms of the Whole Solving Procedure

The proposed method can not only improve the effect and efficiency of the simplification process, but also improve the computation ability and efficiency of the whole solving procedure. Table IV shows the executing time of the whole solving procedure. It can be seen that without using any simplification method, the Groebner basis method can only solve five switching angles [27]. With the ESP simplification method, the Groebner basis can solve eight switching angles in our computer, but the solving time is very long. However, by using the proposed simplification method, the executing time of the whole procedure is significantly reduced, and the switching angles can be solved to ten.

More importantly, the proposed method breaks the upper limit of the solving ability of the previous algebraic algorithms. Table V describes the development process of algebraic algorithms since the resultant elimination method was proposed in 2002. It can be seen that there are only three switching angles that can be solved by algebraic algorithms at the beginning. Over the past two decades, the solvable number of switching angles has increased very slowly. This is the first time that all solutions of the SHE equations with nine switching angles within the full modulation index range are given, and the number of solvable switching angles by algebraic algorithms is increased to ten. It should be pointed out that one solution of SHE equations with nine switching angles was given in [34], but in the limited conditions of our computer, only eight switching angles can be solved by the ESP-based method. Therefore, in Table III, the executing time of nine switching angles represents not applicable.

C. Limitation of the Proposed Method

Since the principle of Newton's identities is based on the relation between power sum symmetric polynomials and ESPs, the proposed method requires the SHE equations should be symmetric. It means that the output waveform of converters must be quarter-symmetric and the amplitude of dc voltage should be equal so that the SHE equations can be transformed into a symmetric polynomial system.

VI. EXPERIMENTAL VERIFICATION

Two experimental case studies are carried out to verify the proposed method. The first experimental study is established on a seven-level cascaded H-bridge (CHB) converter, in which the IRFP250N MOSFETs are used as switching devices, the ADum1400 is used as the isolator, and the STM32F407 are used as the controller to generate the SHEPWM driven single. The dc power supply of every H-bridge is set to 30 V. The first, fourth, and 54th solutions shown in Table II are implemented in the microcontroller to verify the correctness of solutions solved by the proposed method. Because this experimental case aims to validate the correctness of solutions, an open-loop experiment is carried out here. The output PWM waveform is recorded, and the related FFT results are also given in Fig. 6. It can be seen that the aimed fifth, seventh, 11th, 13th, 17th,

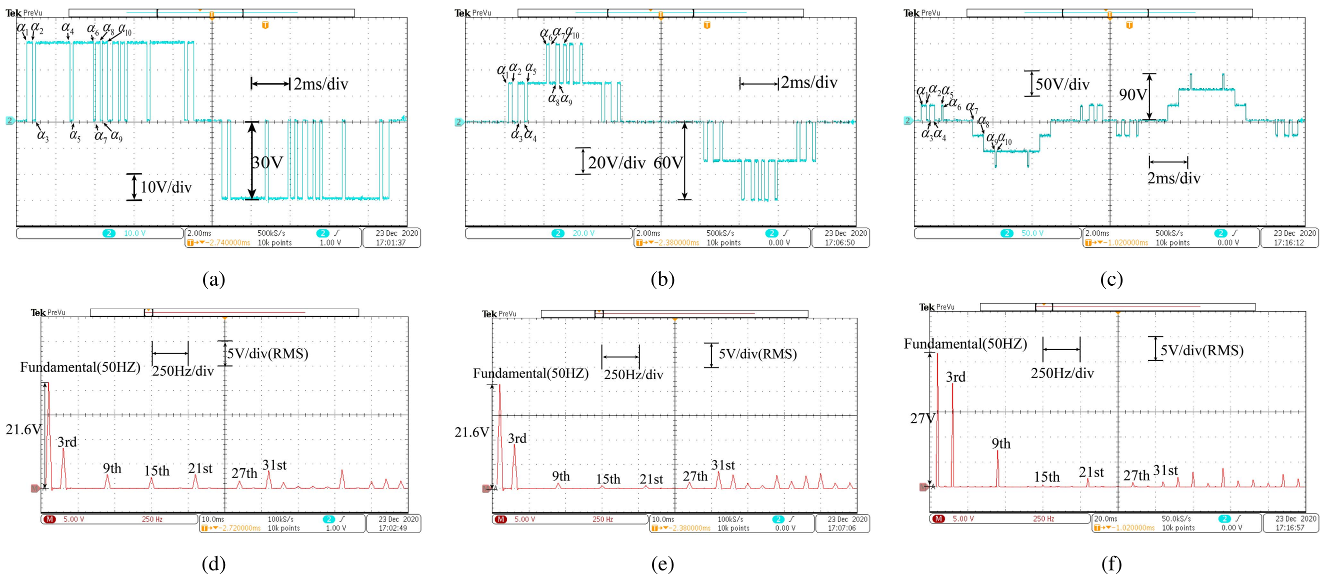

Fig. 6. Output voltage and the related FFT analysis of the experiment carried on the seven-level CHB converter. (a) Three-level phase voltage of the CHB with the first solution in Table II. (b) Five-level phase voltage of the CHB with the fourth solution in Table II. (c) Seven-level phase voltage of the CHB with the 54th solution in Table II. (d) FFT analysis result of the output voltage with the first solution in Table II. (e) FFT analysis result of the output voltage with the fourth solution in Table II. (f) FFT analysis result of the output voltage with the 54th solution in Table II.

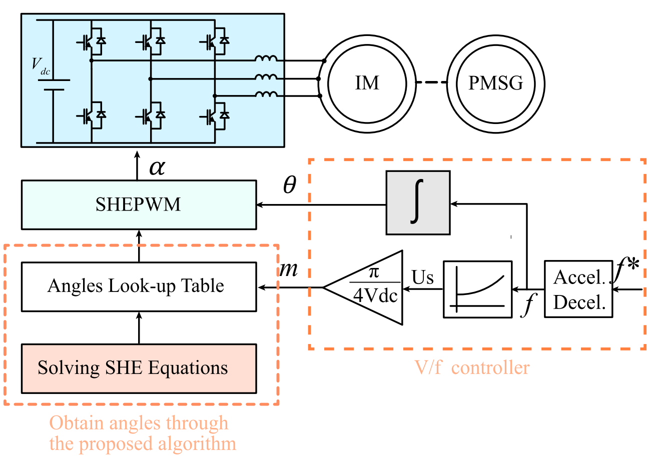

Fig. 7. Schematic of SHEPWM-based controller of experimental platform.

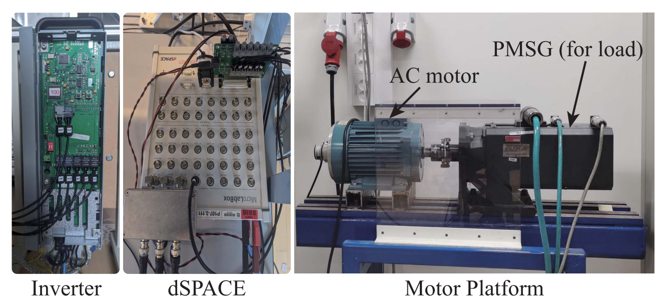

Fig. 8. Photograph of the asynchronous motor experimental platform.

TABLE VI
PARAMETERS OF THE MOTOR EXPERIMENTAL PLATFORM

Parameters	Values
Type of the motor	Y801-4
Power Filter	Type-L 10kW/4.2mH
Inverter	FC-051P15KT4
DC Power Supply	750V
Controller	dSPACE DS1006
Switching frequency (for 50 Hz fundamental)	900 Hz
Switching frequency (for 40 Hz fundamental)	720 Hz
Switching frequency (for 30 Hz fundamental)	540 Hz
Load parameters	0 - 14 Nm

19th, 23rd, 25th, and 29th harmonics are precisely eliminated, which validates the correctness of the solved switching angles.

The second experiment case study aims to present the performance of solutions solved by the proposed method in motor-driven applications. The schematic of the controller is given in Fig. 7. This experiment is established on an asynchronous motor experimental platform as shown in Fig. 8, and the parameters of the experimental platform are shown in Table VI. First, the reference frequency of the asynchronous motor is given to the V/f controller. To maintain the stability of torque and magnetic flux of the motor, the ratio of voltage and frequency is always kept constant, based on which the output voltage can be solved. Then, according to the output voltage of the asynchronous motor, the modulation index can be obtained, and the switching angles can be chosen from the lookup table. Based on the switching angles α and the input phase angle θ, the SHEPWM Block will output the PWM signal to control the three-phase inverter and then drive the asynchronous motor. Besides, the PMSG is used as a controllable load machine. The reference value of output torque of the PMSG can be given through the in-built control system implemented in the dSPACE controller. Besides, the L-type filter is used to reduce high-frequency noise, because the L-type filter does not

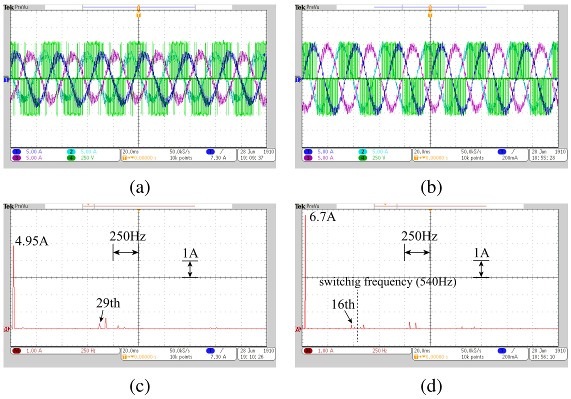

Fig. 9. Harmonic performance of the steady-state motor current when the fundamental frequency is 30 Hz. (a) Steady-state current and line-to-line voltage of the motor with SHEPWM. (b) Steady-state current and line-to-line voltage of the motor with SPWM. (c) FFT analysis of the steady-state current in (a). (d) FFT analysis of the steady-state current in (b).

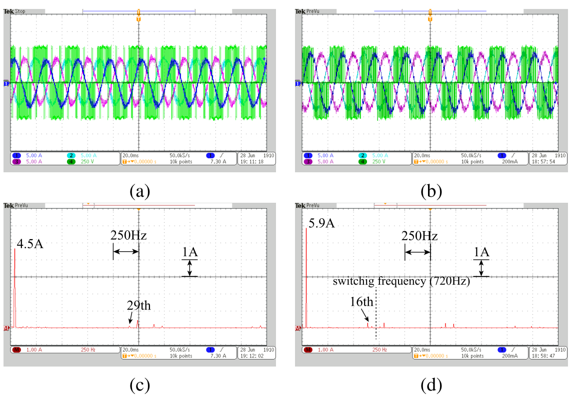

Fig. 10. Harmonic performance of the steady-state motor current when the fundamental frequency is 40 Hz. (a) Steady-state current and line-to-line voltage of the motor with SHEPWM. (b) Steady-state current and line-to-line voltage of the motor with SPWM. (c) FFT analysis of the steady-state current of the motor with SHEPWM. (d) FFT analysis of the steady-state current of the motor with SPWM.

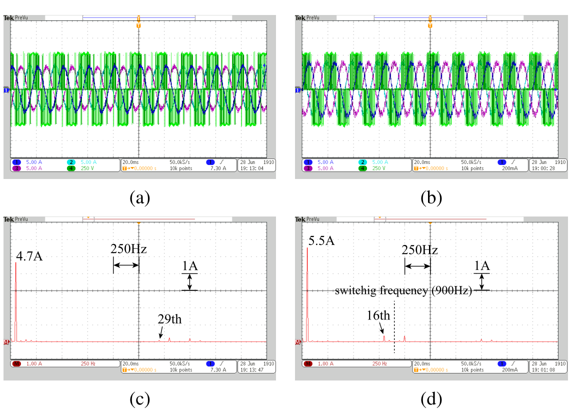

Fig. 11. Harmonic performance of the steady-state motor current when the fundamental frequency is 50 Hz. (a) Steady-state current and line-to-line voltage of the motor with SHEPWM. (b) Steady-state current and line-to-line voltage of the motor with SPWM. (c) FFT analysis of the steady-state current of the motor with SHEPWM. (d) FFT analysis of the steady-state current of the motor with SPWM.

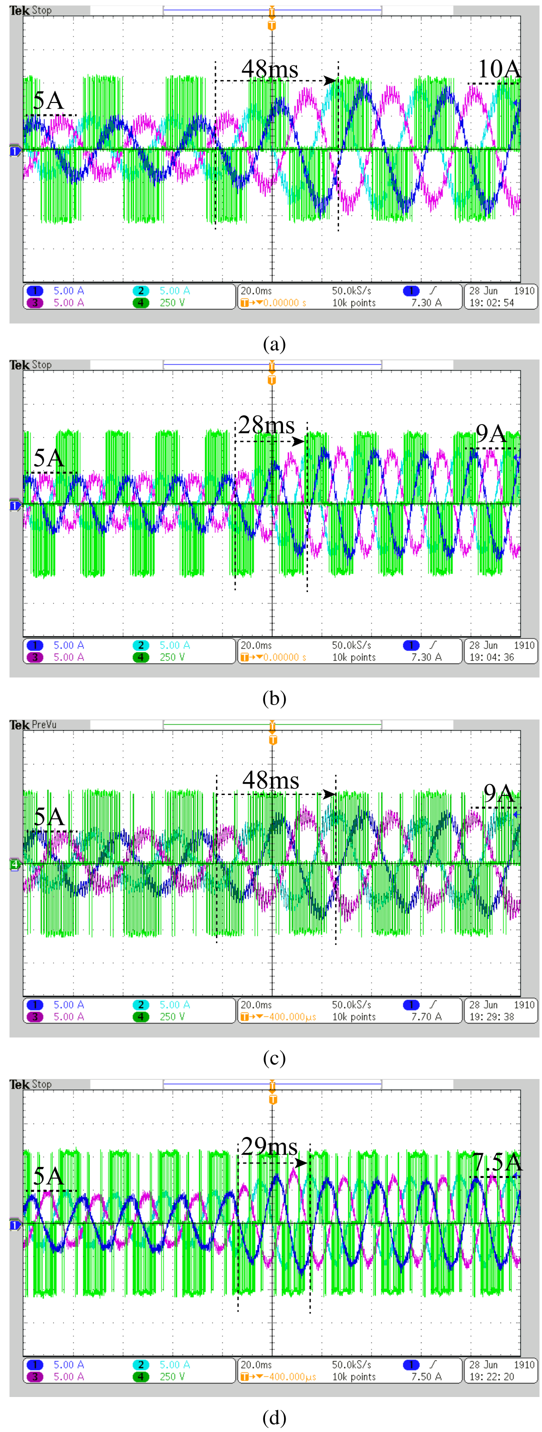

Fig. 12. Comparison between SPWM and SHEPWM when the load step changes. (a) Dynamic performance of current and line-to-line voltage of the motor with SPWM when the load increases to 50% and the fundamental frequency is 30 Hz. (b) Dynamic performance of current and line-to-line voltage of the motor with SPWM when the load increases to 50% and the fundamental frequency is 50 Hz. (c) Dynamic performance of current and line-to-line voltage of the motor with SHEPWM when the load increases to 50% and the fundamental frequency is 30 Hz. (d) Dynamic performance of current and line-to-line voltage of the motor with SHEPWM when the load increases to 50% and the fundamental frequency is 50 Hz.

eliminate any order of harmonic, and it can keep the original effect of the proposed method for eliminating harmonics. Two subcases are carried out in this case to present the harmonic performance of the solved switching angles and the dynamic performance of the proposed scheme. The first subcase aims to show the steady-state current performance of the solved angles, and the results are shown in Fig. 9–11. The second subcase is used to verify the dynamic performance of the solved switching angles, and the experimental results are shown in Fig. 12.

The three-phase steady-state output currents, line-to-line voltage, and the corresponding FFT analysis based on the solved switching angles are given with the 30-, 40-, and 50-Hz fundamental frequency in Fig. 9–11. To compare the harmonic performance between SHEPWM and SPWM, the steady-state output currents, line-to-line voltage, and FFT analysis based on SPWM are also given. In Figs. 9–11, it can be seen that, for SHEPWM, the first uneliminated harmonic is the 29th, but for SPWM, the first uneliminated harmonic is 16th. Moreover, at the bandwidth from 3–29th harmonics, the SPWM has some other harmonics, but the SHEPWM has almost no harmonics. This phenomenon shows that SHEPWM has a better performance in low-switching frequency applications.

The experimental results of the dynamic performance of the proposed method are shown in Fig. 12. The load of the ac motor has a step change from 50% to 100% (14 Nm). The time period of the dynamic process is marked in the figure. Besides, to compare the performance between SHEPWM and SPWM, the experimental results of SPWM with the same switching frequency are shown in Fig. 12. The experimental results show that the solved switching angles almost have the same performance as the SPWM with the same switching frequency, no matter at the 30-Hz fundamental frequency or the 50-Hz fundamental frequency. Based on the experimental results of the two subcases, the correctness and effectiveness of the proposed method can be well verified.

VII. Conclusion

This article proposes a general degree reduction method to simplify the SHE equations based on Newton's identities and the power sums, which makes the degree reduction no longer the bottleneck of solving the SHE equations with algebraic algorithms. The major contributions of the proposed method are summarized as follows.

1) By combining the proposed simplification method with the Groebner basis method, the SHE equations with ten switching angles can be solved by the algebraic algorithm for the first time, which means that the harmonics below 30th can be accurately eliminated.
2) Compared with the commonly used simplification method, the simplification efficiency of the proposed method has been significantly improved. For example, the proposed method performs almost 40 800 times faster when the number of switching angles is eight.
3) The reduced degree of the simplified SHE equations is slightly lower than the ESP method.
4) Experimental results verify that the switching angles solved by this article can obtain good harmonic performance in low switching frequency.

This article is aimed at the solving algorithm for SHE equations and takes motor drive as an example to verify the practicality of the proposed method. In future work, more applications will be explored to use this method, such as grid-connected converters, active rectifiers, STATCOM systems, and so on.

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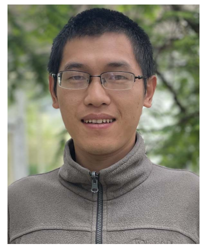

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