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FRAGMENTATION AND CLUSTERING IN VERTICALLY LINKED INDUSTRIES*

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ABSTRACT. This paper models the location of two vertically related firms in a low labor cost country and in a country with a large market. The upstream industry is more labor intensive than the downstream industry. We find that spatial fragmentation occurs for low values of the input-output coefficient and intermediate values of the transport rate, particularly if the countries are very asymmetric in size. Otherwise, we obtain agglomeration either in the low cost country (when the transport rate is low) or in the large market (when the transport rate is high). Multiple agglomerated equilibria arise when the transport cost of the intermediate good is significant.

1. INTRODUCTION

A crucial issue in economic geography is the decision taken by firms either to fragment their operations (manufacturing, distribution, R&D) in space or to integrate them in the same location. Amiti (2005) dealt with this issue at the regional level. She remarked cases such as the textile industries that relocate labor intensive manufacturing processes from high wage developed countries to low wage developing countries, while keeping in the former countries the design and marketing stages. This process is determined by the decline in trade costs related with the progress in transport and communications.

According to Rossi-Hansberg et al. (2005), the same process occurs in metropolitan areas. With the growth of the city, the firms tend to locate their headquarters in the urban center and settle the production plants in the suburbs. The outcome of this fragmentation is the increase of the ratio managers/nonmanagers in total employment in the urban centers with relation to suburbs. A few examples are evident. For instance, the Federal Reserve Bank of New York moved its cash and check-processing center to nearby East

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Rutherford, NJ. *The Washington Post* moved its printing operations away from its headquarters in downtown Washington to close to Springfield, VA. The tire manufacturer Michelin, with headquarters in Greenville, SC, shifted its rubber manufacturing to nearby Anderson County.

However, spatial fragmentation of firm operations is not ubiquitous. We can observe that vertically related firms with different labor intensities tend often to agglomerate inside the same country. An example mentioned by Amiti (2005) is the rubber-based components for motor vehicles and aerospace. Although they are intensive in natural rubber and labor, the components' manufacturers are not usually located in countries that are abundant in rubber and labor, such as Malaysia, Indonesia, and Thailand. Instead, they are usually located in European countries and in the United States, close to final assembly producers.

On the other hand, it is easily noted that this kind of agglomeration of vertically related firms has been growing in the last decades, so that the proximity of buyers and sellers tends to outweigh labor cost considerations. A special example is the aircraft and related parts industries in Southern California. But this trend is typical of almost all engineering industries such as aerospace, electronics, pharmaceuticals, and cars. Particularly, in the car industry, component suppliers and assembly plants tend to co-locate.

In this paper, we model the decision by vertically related firms either to colocate or to disperse in space. We assume that there exist two countries, Home (H) and Foreign (F). Country F has lower wages than country H ($w_f < w_h$), but country H has a higher purchasing power (modeled through the number of consumers, $n_h > n_f$) than country F.

There are two vertically linked firms: the downstream firm D, which produces a consumer good to be sold in both countries, and the upstream firm U, which provides an intermediate good to the downstream firm. The unit transport costs of both the intermediate good and the final good vary in proportion. For the sake of simplicity, this is modeled by assuming that both transport costs are equal. Moreover, the value for this transport cost is zero within each country and t ($t > 0$) between countries. The upstream firm uses some amount of labor c_U to produce a unit of the intermediate good and the downstream firm uses α units ($0 < \alpha < 1$) of the intermediate good and some amount of labor c_D to produce one unit of the final good. Hence, α measures the intensity of vertical linkages between the two firms.

The main results of the paper can be described by Figure 1, where the equilibria of locations of firms (D, U) is plotted in the space of parameters (α, t).

Figure 1 does not show the equilibrium of locations for all values of (α, t) but only for the following regions: t close to zero; α close to zero; and α and t arbitrarily high. This follows from the fact that very general assumptions were taken, namely concerning the shape of the final demand function. Still, several conclusions are evident in Figure 1. The fall in transport costs t to very low levels always leads to the agglomeration of upstream and downstream firms in the low labor cost country, since the choice of locations is then driven by production

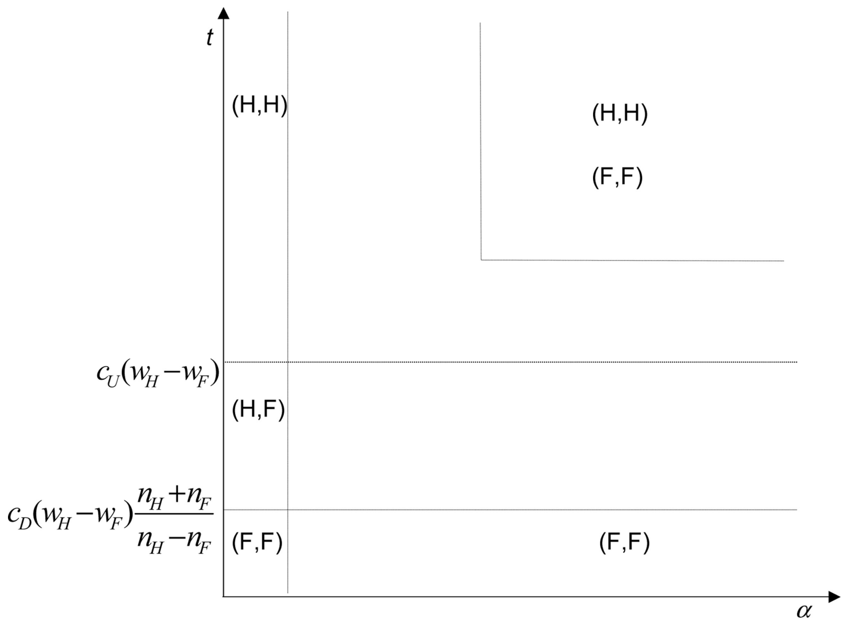

FIGURE 1: Theoretical Results of Locational Equilibria.

costs. However, this process exhibits two distinct patterns depending on the intensity of vertical linkages α. If α is low, the decrease of trade costs may determine a transition from the agglomeration from the large, high labor cost country H to spatial fragmentation, where the upstream firm U locates locates in the small, low labor cost country F and the downstream unit D stays in the large, high labor cost market H. Further reduction of t leads eventually to agglomeration in the small, low labor cost country F. If α is high, there are multiple agglomeration equilibria for high values of the transport cost t, since in this case the transport cost of the intermediate good is high, and for each firm to cluster in either country is better than selecting an isolated location. Again, for very low values of the transport cost t, agglomeration takes place in the low labor cost country.

Figure 1 also shows that fragmentation is more likely to arise if countries have very different sizes, since the coefficient $\frac{n_H+n_F}{n_H-n_F}$ is a measure of the degree of symmetry of the countries. In fact, if the combined population of both countries is kept constant while the difference in size increases, the firms will probably select different locations. Alternatively, when total population increases, but the difference between countries remains constant, firms tend to agglomerate for a wider range of parameter values. This contrasts with Rossi-Hansberg et al. (2005), where spatial fragmentation in a city follows directly from the increase in total urban population.

In Section 2, we present the assumptions of the model. In Section 3, the main propositions on the equilibrium of locations are enunciated and proved. In section 4, we explore the connections of this paper with the previous literature in this field. Section 5 contains the concluding remarks.

2. THE MODEL

The model is based on the following assumptions. There are two countries called Home (H) and Foreign (F). The distance between any two locations within each country is 0, while the distance between a location in H and a location in F is normalized to 1. Country F has lower wages than H, so that $w_H > w_F \geq 0$. On the other hand, the purchasing power in country H exceeds the purchasing power in country F, this being expressed by the fact that the number of consumers in H (n_H) is higher than the number of consumers in F (n_F), i.e., $n_H > n_F$. Consumers in each country have identical demand functions $f(p)$, where p represents the delivery price, and satisfies the following assumptions:

1. f is continuous and differentiable;
2. f is monotonically decreasing, i.e., $f'(p) < 0$;
3. The maximum price, $\bar{p} = f^{-1}(0)$, is finite;
4. f satisfies $-\frac{f''(p)p}{f'(p)} < 2$, yielding a concave total revenue function.

There are two vertically linked firms: the downstream firm (D), producing a consumer good to be sold in both countries, and the upstream firm (U), providing an intermediate good to the downstream firm. The production technology is the following: U transforms $c_u (c_u \geq 0)$ units of labor into one unit of the intermediate good and D uses α units of the intermediate good together with $c_d (c_d \geq 0)$ units of labor to produce one unit of the consumer good. The parameter α, satisfying $0 \leq \alpha < 1$, thus represents the magnitude of vertical linkages between the two firms. Each firm incurs the transport cost of its own product; t denotes the transport cost per unit of distance of both the intermediate good and the consumer good. The assumption that the transport costs are equal rests on the fact that they usually vary in proportion. When D locates in country X_D and U locates in country X_U, with $X_D, X_U \in \{H, F\}$, firm D sets discriminatory prices $p_H^{X_D}$ and $p_F^{X_D}$ in each country, while firm U sets a delivery price k^{X_U} for the intermediate good.

Given these assumptions, firm D's profit function is

$$\Pi_D^{(X_D,X_U)} = n_H \cdot f\left(p_H^{X_D}\right)\left[p_H^{X_D} - \alpha \cdot k^{X_U} - c_D \cdot w_{X_D} - t \cdot d(X_D, H)\right] + n_F \cdot f\left(p_F^{X_D}\right)\left[p_F^{X_D} - \alpha \cdot k^{X_U} - c_D \cdot w_{X_D} - t \cdot d(X_D, F)\right],$$

and firm U's profit function is given by

$$\Pi_U^{(X_D,X_U)} = \alpha\left[n_H \cdot f\left(p_H^{X_D}\right) + n_F \cdot f\left(p_F^{X_D}\right)\right]\left[k^{X_U} - c_U \cdot w_{X_U} - t \cdot d(X_D, X_U)\right],$$

where $d(X, Y)$ represents the distance between locations X and Y, with $X, Y \in \{H, F\}$.

Firms D and U play a noncooperative game that is composed of three stages. In the first stage, firms simultaneously choose where to establish themselves.

Given the adopted locations X_U and X_D, in the second stage, firm U sets k^{X_U}, the price of the intermediate good and, in the third stage, D sets $p_H^{X_D}$ and $p_F^{X_D}$, the prices for the final good in countries H and F, respectively.

3. EQUILIBRIUM

The purpose of this paper is to determine equilibrium locations for both firms, depending on the value of transport costs and the intensity of vertical linkages. The solution concept we use here is subgame perfect equilibrium, so that we will solve the game by backward induction. We start by determining the prices set by each firm in order to maximize profits for each possible combination of elected locations X_U, X_D. We then compare the profits obtained for each combination of locations and conclude on equilibrium locations.

For illustration, when the chosen locations are (H, F),[1] firm D's problem becomes:

$$\underset{p_H^H, p_F^H}{Max} \; n_H \cdot f(p_H^H)[p_H^H - \alpha \cdot k^F - c_D \cdot w_H] + n_F \cdot f(p_F^H)[p_F^H - \alpha \cdot k^F - c_D \cdot w_H - t].$$

The first-order conditions are:

$$\begin{aligned} &f'(p_H^{*H})[p_H^{*H} - \alpha \cdot k^F - c_D \cdot w_H] + f(p_H^{*H}) = 0 \\ &f'(p_F^{*H})[p_F^{*H} - \alpha \cdot k^F - c_D \cdot w_H - t] + f(p_F^{*H}) = 0, \end{aligned} \tag{1}$$

where p_H^{*H} and p_F^{*H} represent the prices that maximize firm D's profits. Firm U's problem is given by:

$$\underset{k^F}{Max} \; \alpha[n_H \cdot f(p_H^H) + n_F \cdot f(p_F^H)][k^F - c_U \cdot w_F - t]. \tag{2}$$

After substituting for $f(p_H^{*H})$ and $f(p_F^{*H})$ in (2) using (1), the first-order condition gives the optimum price for the intermediate good:

$$k^{*F} = \frac{\alpha \cdot c_U \cdot w_F - c_D \cdot w_H + \alpha \cdot t}{2\alpha} + \frac{1}{2\alpha} \frac{n_H \cdot f'(p_H^{*H}) \cdot p_H^{*H} + n_F \cdot f'(p_F^{*H}) \cdot (p_F^{*H} - t)}{n_H \cdot f'(p_H^{*H}) + n_F \cdot f'(p_F^{*H})}. \tag{3}$$

By substituting (3) in (1), this system of equations implicitly defines the values for p_H^{*H} and p_F^{*H}. For particular functional forms of f, it is possible to arrive at explicit expressions for the optimum prices and maximum profits for both firms in locations (H, F), $\Pi_D^{*(H,F)}$ and $\Pi_U^{*(H,F)}$, are simply obtained by substituting these prices in each firm's profit expression. The same process should be followed for the remaining combinations of chosen locations.[2] Finally,

[1] Every location is written in the form (X_D, X_U).

[2] The expressions for these cases are presented in Appendix A.

by comparing the maximum profits obtained in the several locations, we are able to determine which countries are chosen by each firm in a subgame perfect equilibrium for every possible values of t and α

Nevertheless, in the general setup described above, without imposing further assumptions on the demand function, no results for arbitrary values of t and α can be obtained. In the following set of results, we characterize subgame perfect equilibrium locations for arbitrarily small values of t, for arbitrarily small values of α, and equilibrium locations for t increasing without limit and α bounded away from zero.[3]

Firstly, for low values of the transport costs, production costs are the driving force in the choice of locations. In fact, both firms choose to settle in country F in every subgame perfect equilibrium.

PROPOSITION 1: *Let $t > 0$ be arbitrarily small. Then, choosing F is a dominant strategy for both firms in the first stage of the game and every subgame perfect equilibrium has both firms choosing to settle in country F.*

Proof: Let us start by assuming that $t = 0$. Given X_U, when established in country F, firm D's profit function is given by $\Pi_D^{(F,X_U)} = n_H \cdot f(p_H^F)\cdot$

$\cdot[p_H^F - \alpha \cdot k^{X_U} - c_D \cdot w_F] + n_F \cdot f(p_F^F)[p_F^F - \alpha \cdot k^{X_U} - c_D \cdot w_F]$. Since the first-order conditions for the prices of the final good in each country are identical, we have $p_H^{*F} = p_F^{*F}$. Let $p^{*F} = p_H^{*F} = p_F^{*F}$; clearly, p^{*F} satisfies $f'(p^{*F})(p^{*F} - \alpha \cdot k^{X_U} - c_D \cdot w_F) + f(p^{*F}) = 0$. Similarly, given X_U, when settled in H, firm D's profits are $\Pi_D^{(H,X_U)} = n_H \cdot f\big(p_H^H\big)(p_H^H - \alpha \cdot k^{X_U} - c_D \cdot w_H) + n_F \cdot f\big(p_F^H\big)(p_F^H - \alpha \cdot k^{X_U} - c_D \cdot w_H)$ and, following the same reasoning, we have $p_H^{*H} = p_F^{*H}$. Let $p^{*H} = p_H^{*H} = p_F^{*H}$; p^{*H} satisfies $f'(p^{*H})(p^{*H} - \alpha \cdot k^{X_U} - c_D \cdot w_H) + f(p^{*H}) = 0$. Now let X_D be the location chosen by firm D. Firm U's profit functions are $\Pi_U^{(X_U,F)} = \alpha(n_H + n_F) \cdot f(p^{X_D})(k^F - c_U \cdot w_F)$ and $\Pi_U^{(X_U,H)} = \alpha(n_H + n_F) \cdot f(p^{X_D})(k^H - c_U \cdot w_H)$, when it adopts countries F and H, respectively.

We will now show that choosing F is a dominant strategy for firm D in the first stage of the game. To start, let U locate in H. When D chooses H, we have $k^{*H} = \frac{\alpha \cdot c_U \cdot w_H - c_D \cdot w_H}{2\alpha} + \frac{p^H}{2\alpha}$ and firm D's profit becomes $\Pi_D^{(H,H)} = (n_H + n_F) \cdot f(p^H)(\frac{p^H}{2} - \beta_1)$, with $\beta_1 = \frac{1}{2}(c_D \cdot w_H + \alpha \cdot c_U \cdot w_H)$. Similarly, when D chooses F, we have $k^{*H} = \frac{\alpha \cdot c_U \cdot w_H - c_D \cdot w_H}{2\alpha} + \frac{p^F}{2\alpha}$ and firm D's profit becomes $\Pi_D^{(F,H)} = (n_H + n_F) \cdot f(p^F)(\frac{p^F}{2} - \beta_2)$, with $\beta_2 = \frac{1}{2}(c_D \cdot w_F + \alpha \cdot c_U \cdot w_H)$. Note that $\beta_1 > \beta_2$. Hence, $\Pi_D^{*(H,H)} - \Pi_D^{*(F,H)}$ may be linearly approximated by $\frac{d\Pi_D^{*(F,H)}}{d\beta}(\beta_1 - \beta_2)$; since, by the envelope theorem, $\frac{d\Pi_D^{*(F,H)}}{d\beta} = \frac{\partial \Pi_D^{*(F,H)}}{\partial \beta} = -(n_H + n_F) \cdot f(p^{*F}) < 0$, we have $\frac{d\Pi_D^{*(F,H)}}{d\beta}(\beta_1 - \beta_2) < 0$. It follows that D's best reply when U selects H is F. Now assume U locates in F. When D chooses H, we have $k^{*F} = \frac{\alpha \cdot c_U \cdot w_F - c_D \cdot w_H}{2\alpha} + \frac{p^H}{2\alpha}$ and firm D's profit becomes $\Pi_D^{(H,F)} = (n_H + n_F) \cdot f(p^H)(\frac{p^H}{2} - \beta_3)$, with

[3] Note that these results are summarized in Figure 1, provided that Corollary 1 below holds.

$\beta_3 = \frac{1}{2}(c_D \cdot w_H + \alpha \cdot c_U \cdot w_F)$. Equivalently, when D chooses F, we have $k^{*F} = \frac{\alpha \cdot c_U \cdot w_F - c_D \cdot w_F}{2\alpha} + \frac{p^F}{2\alpha}$ and firm D's profit becomes $\Pi_D^{(F,F)} = (n_H + n_F) \cdot f(p^H)(\frac{p^H}{2} - \beta_4)$, with $\beta_4 = \frac{1}{2}(c_D \cdot w_F + \alpha \cdot c_U \cdot w_F)$. Note that $\beta_3 > \beta_4$. Now $\Pi_D^{*(H,F)} - \Pi_D^{*(F,F)}$ may be linearly approximated by $\frac{d\Pi_D^{*(F,F)}}{d\beta}(\beta_3 - \beta_4)$; by the envelope theorem, $\frac{d\Pi_D^{*(F,H)}}{d\beta} = \frac{\partial \Pi_D^{*(F,H)}}{\partial \beta} = -(n_H + n_F) \cdot f(p^{*H}) < 0$ and $\frac{d\Pi_D^{*(F,H)}}{d\beta}(\beta_1 - \beta_2) < 0$. It follows that D's best reply when U selects F is F. Hence, F is a dominant strategy for D.

In what follows, we prove that selecting F is also dominant for firm U in the first stage of the game. Let D choose H. When U locates in H, we have $\Pi_U^{(H,H)} = (n_H + n_F) \cdot f(p^H)(\frac{p^H}{2} - \beta_1)$, and when U locates in F, $\Pi_U^{(H,F)} = (n_H + n_F) \cdot f(p^H)(\frac{p^H}{2} - \beta_3)$. Following the reasoning above, as $\beta_1 > \beta_3$, U's best reply to H is choosing F. Conversely, when D locates in F, we have $\Pi_U^{(F,H)} = (n_H + n_F) \cdot f(p^F)(\frac{p^F}{2} - \beta_2)$ or $\Pi_U^{(F,F)} = (n_H + n_F) \cdot f(p^F)(\frac{p^F}{2} - \beta_4)$, depending on whether U settles in H or in F. Since $\beta_2 > \beta_4$, U's best reply to F is choosing F. Hence, choosing F is a dominant strategy for U and firms choose (F, F) in every subgame perfect equilibrium.

Finally, when $t > 0$ and arbitrarily small, by continuity of the profit functions, selecting F remains a dominant strategy for both firms and every subgame perfect equilibrium yields the locations (F, F).

The following results refer to the case in which vertical linkages are weak. In this scenario, depending on parameter values, there may be subgame perfect equilibria where firms choose different locations. In fact, for intermediate values of t, firm D may settle in H seeking high demand while firm U privileges low production costs and chooses F.

LEMMA 1: *Let* $\alpha = 0$ *and let* $\theta(t) = \frac{n_H \cdot f(p_H^{*H}) + n_F \cdot f(p_F^{*H})}{n_H \cdot f(p_H^{*H}) - n_F \cdot f(p_F^{*H})}$. *Then,* $\theta(t)$ *satisfies the following properties:*

1. $\theta(0) = \frac{n_H + n_F}{n_H - n_F}$
2. $\lim_{t \to \infty} \theta(t) = 1$
3. $\theta(t)$ *is a continuous and monotonically decreasing function of t, for all* $t \in [0, +\infty)$.

Proof: First, note that, as shown above, when $t = 0$, we have $p_H^{*H} = p_F^{*H}$ and $\theta(0) = \frac{n_H + n_F}{n_H - n_F}$. Observe that $\frac{n_H + n_F}{n_H - n_F} > 1$. On the other hand, as $t \to \infty$, using (1) above and the fact that $\alpha = 0$, it is easy to see that the price p_H^{*H} is constant, while $p_F^{*H} \to \bar{p}$. Hence, $f(p_F^{*H}) \to 0$ and $\lim_{t \to \infty} \theta(t) = 1$. Moreover, since p_H^{*H} does not depend on t, we have $\theta'(t) = \frac{2n_H n_F f(p_H^{*H}) f'(p_F^{*H}) \frac{dp_F^{*H}}{dt}}{[n_H \cdot f(p_H^{*H}) - n_F \cdot f(p_F^{*H})]^2}$ and clearly $\theta'(t) < 0$. The result follows.

LEMMA 2: *Let* $\alpha = 0$ *and let* $\theta(t) = \frac{n_H \cdot f(p_H^{*H}) + n_F \cdot f(p_F^{*H})}{n_H \cdot f(p_H^{*H}) - n_F \cdot f(p_F^{*H})}$. *Then, firm D chooses to settle in country H if and only if transport costs are sufficiently high, i.e., if and only if* $t \geq c_D \cdot (w_H - w_F) \cdot \theta(t)$.

Proof: Given X_U, firm *D*'s profit when located in *H* is given by $\Pi_D^{(H,X_U)} = n_H \cdot f(p_H^H)[p_H^H - c_D \cdot w_H] + n_F \cdot f(p_F^H)[p_F^H - c_D \cdot w_H - t]$ and, when established in *F*, $\Pi_D^{(F,X_U)} = n_H \cdot f(p_H^F)[p_H^F - c_D \cdot w_F - t] + n_F \cdot f(p_F^F)[p_F^F - c_D \cdot w_F]$. These profit functions may be written generically as $\Pi_D = n_H \cdot f(p_H)[p_H - \gamma_H] + n_F \cdot f(p_F)[p_F - \gamma_F]$, where, depending on whether we consider $\Pi_D^{(H,X_U)}$ or $\Pi_D^{(F,X_U)}$, *(i)* p_H and p_F stand for p_H^H and p_F^H or p_H^F and p_F^F, respectively, *(ii)* γ_H equals $\gamma_H^H = c_D \cdot w_H$ or $\gamma_H^F = c_D \cdot w_F + t$, and *(iii)* γ_F equals $\gamma_F^H = c_D \cdot w_H + t$ or $\gamma_F^F = c_D \cdot w_F$. Now consider maximal profits $\Pi_D^{*(H,X_U)}$ and $\Pi_D^{*(F,X_U)}$. The difference $\Pi_D^{*(H,X_U)} - \Pi_D^{*(F,X_U)}$ may be linearly approximated by $\frac{d\Pi_D^*}{d\gamma_H}(\gamma_H^H - \gamma_H^F) + \frac{d\Pi_D^*}{d\gamma_F}(\gamma_F^H - \gamma_F^F)$. Using the envelope theorem, $\frac{d\Pi_D^*}{d\gamma_H} = \frac{\partial \Pi_D^*}{\partial \gamma_H} = -n_H f(p_H^*)$ and $\frac{d\Pi_D^*}{d\gamma_F} = \frac{\partial \Pi_D^*}{\partial \gamma_F} = -n_F f(p_F^*)$, so that $d\Pi_D^* = -n_H f(p_H^*)[c_D \cdot w_H - c_D \cdot w_F - t] - n_F f(p_F^*)[c_D \cdot w_H + t - c_D \cdot w_F]$. It is easy to see that $d\Pi_D^* \geq 0$ if and only if $\frac{t}{\theta(t)} \geq c_D \cdot (w_H - w_F)$.

Now we will prove that there exists a unique threshold t^* above which firm *D* elects location *H*. By Lemma 1, *1*, for $t = 0$, we have $\theta(0) = \frac{n_H + n_F}{n_H - n_F}$ and $0 = \frac{t}{\theta(t)} < c_D \cdot (w_H - w_F)$, since $w_H > w_F$ by assumption; furthermore, when $t \to \infty$, we have $\lim_{t\to\infty}\theta(t) = 1$, by Lemma 1, *2*, and $\frac{t}{\theta(t)} > c_D \cdot (w_H - w_F)$. Since $\theta(t)$ is a decreasing function of t (Lemma 1, *3*), $\frac{t}{\theta(t)}$ increases with t. As a consequence, there exists a unique t^*, satisfying $\frac{t^*}{\theta(t^*)} = c_D \cdot (w_H - w_F)$, above which *D* selects location *H*.

LEMMA 3: *Let* $\alpha > 0$ *be arbitrarily small. Then, when firm D opts for locating in country F, firm U's best reply is to locate in F. In addition, when firm D chooses to locate in country H, firm U's best reply is to locate in F for low values of the transport cost, i.e., when* $t \leq c_U(w_H - w_F)$*, and to elect H otherwise.*

Proof: Let *D* elect country *F*. Then, *U*'s profit function is $\Pi_U^{(F,H)} = \alpha[n_H \cdot f(p_H^F) + n_F \cdot f(p_F^F)](k^H - \gamma^H)$ or $\Pi_U^{(F,F)} = \alpha[n_H \cdot f(p_H^F) + n_F \cdot f(p_F^F)](k^F - \gamma^F)$, with $\gamma^H = c_U w_H + t$ and $\gamma^F = c_U w_F$, depending on whether it locates in *H* or in *F*. Now consider maximal profits $\Pi_U^{*(F,F)}$ and $\Pi_U^{*(F,H)}$. The difference $\Pi_U^{*(F,F)} - \Pi_U^{*(F,H)}$ may be linearly approximated by $\frac{d\Pi_U^*}{d\gamma^H}(\gamma^F - \gamma^H)$. Using the envelope theorem, $\frac{d\Pi_U^*}{d\gamma^H} = \frac{\partial \Pi_U^*}{\partial \gamma^H} = -\alpha[n_H f(p_H^{*F}) + n_F f(p_F^{*F})]$ and $\frac{d\Pi_U^*}{d\gamma^H}(\gamma^F - \gamma^H) = -\alpha[n_H f(p_H^{*F}) + n_F f(p_F^{*F})](c_U w_F - c_U w_H - t) > 0$, since $w_H > w_F$ and $t \geq 0$. Hence, we have $\Pi_U^{*(F,F)} - \Pi_U^{*(F,H)} > 0$ for every value of t. Now let *D* select country *H*. In this case, *U*'s profit function is $\Pi_U^{(H,H)} = \alpha[n_H \cdot f(p_H^H) + n_F \cdot f(p_F^H)](k^H - \gamma^H)$ or $\Pi_U^{(H,F)} = \alpha[n_H \cdot f(p_H^H) + n_F \cdot f(p_F^H)](k^F - \gamma^F)$, with $\gamma^H = c_U w_H$ and $\gamma^F = c_U w_F + t$, depending on whether *U* locates in *H* or in *F*. Now the difference $\Pi_U^{*(H,F)} - \Pi_U^{*(H,H)}$ may be linearly approximated by $\frac{d\Pi_U^*}{d\gamma^H}(\gamma^F - \gamma^H)$. By the envelope theorem, $\frac{d\Pi_U^*}{d\gamma^H}(\gamma^F - \gamma^H) = -\alpha[n_H f(p_H^{*H}) + n_F f(p_F^{*H})](c_U w_F + t - c_U w_H)$. It follows that $\Pi_U^{*(H,F)} - \Pi_U^{*(H,H)} \geq 0$ if and only if $t \leq c_U(w_H - w_F)$.

PROPOSITION 2: *Let* $\alpha > 0$ *be arbitrarily small. Then, in a subgame perfect equilibrium, both firms elect H when* $t \geq \max\{c_D \cdot (w_H - w_F) \cdot$

$\frac{n_H \cdot f(p_H^{*H})+n_F \cdot f(p_F^{*H})}{n_H \cdot f(p_H^{*H})-n_F \cdot f(p_F^{*H})}, c_U \cdot (w_H - w_F)\}$, *both firms choose F when* $t \leq \min\{c_D \cdot (w_H - w_F) \cdot \frac{n_H \cdot f(p_H^{*H})+n_F \cdot f(p_F^{*H})}{n_H \cdot f(p_H^{*H})-n_F \cdot f(p_F^{*H})}, c_U \cdot (w_H - w_F)\}$, *and firm D elects H while U chooses F when* $c_D \cdot (w_H - w_F) \cdot \frac{n_H \cdot f(p_H^{*H})+n_F \cdot f(p_F^{*H})}{n_H \cdot f(p_H^{*H})-n_F \cdot f(p_F^{*H})} \leq t \leq c_U \cdot (w_H - w_F)$.

Proof: By Lemma 2, firm D elects country H if and only if transport costs obey $\frac{t}{\theta(t)} \geq c_D \cdot (w_H - w_F)$. By continuity, the same result applies for $\alpha > 0$ arbitrarily small. On the other hand, by Lemma 3, firm U's best reply to F is to choose F and its best reply to H is F if and only if $t \leq c_U(w_H - w_F)$. The conclusion follows.

The following corollary is obvious.

COROLLARY 1: *A fragmentation equilibrium where firm U chooses country F and firm D selects country H emerges if and only if the labor intensities in the industries are sufficiently different, i.e., if and only if* $c_D \cdot \theta(t) < c_U$.

Finally, when both t and α are high, firms D and F cluster in order to avoid $\alpha\, t$, the transport cost of the intermediate good.

PROPOSITION 3: *Let t increase without limit and let* α *assume a value bounded away from zero. Then, firm D and firm U always choose the same locations in a subgame perfect equilibrium.*

Proof: We will show that, for the values of t and α defined above, both firms have zero profits when they elect different countries and positive profits otherwise. Let us start by considering the case (H, H). The following conditions define optimum prices:

$$\begin{aligned} f'(p_H^{*H})\left[p_H^{*H} - \alpha \cdot k^{*H} - c_D \cdot w_H\right] + f(p_H^{*H}) &= 0 \\ f'(p_F^{*H})\left[p_F^{*H} - \alpha \cdot k^{*H} - c_D \cdot w_H - t\right] + f(p_F^{*H}) &= 0, \end{aligned} \tag{4}$$

$$k^{*H} = \frac{\alpha \cdot c_U \cdot w_H - c_D \cdot w_H}{2\alpha} + \frac{1}{2\alpha}\frac{n_H \cdot f'(p_H^{*H}) \cdot p_H^{*H} + n_F \cdot f'(p_F^{*H}) \cdot (p_F^{*H} - t)}{n_H \cdot f'(p_H^{*H}) + n_F \cdot f'(p_F^{*H})}. \tag{5}$$

Now consider the increasing and unbounded sequence of values for the transport cost $t_1, t_2, t_3, \ldots$. Clearly, there is a corresponding sequence of optimum prices for the intermediate good $k_1^{*H}, k_2^{*H}, k_3^{*H}, \ldots$ determined by (4) and (5). This sequence is bounded from above since *(i)* the first term in (5) is a constant and *(ii)* the second term equals $\frac{p_{av}}{2\alpha}$, where p_{av} is a weighted average of the optimum prices in markets H and F net of transport costs, which is approximately constant. As a consequence, the limits of p_H^{*H} and p_F^{*H} when t increases depend exclusively on the direct effect of the transport cost over p_H^{*H} and p_F^{*H}. Using (4), we can see that t does not affect p_H^{*H} directly, so that $p_H^{*H} < \bar{p}$ and

$f(p_H^{*H}) > 0$. Also, the assumption $-\frac{f''(p)p}{f'(p)} < 2$ guarantees that $\frac{dp_F^{*H}}{dt} > 0$, so that p_F^{*H} increases with t until reaching $\bar{p}$ and $f(p_F^{*H})$ approaches 0 as t increases without limit. From $f(p_H^{*H}) > 0$ it follows that firm D's profit is positive and that firm U's demand, which depends on D's demand, and profit are also positive.

The proof for the case in which firms choose locations (F, F) follows the same steps, the only difference being that, when t increases without limit, p_H^{*F} approaches $\bar{p}$ and p_F^{*F} remains unaffected. Consequently, $f(p_F^{*F}) > 0$ ensures that firm D's profit is non-negative and that firm U's demand and profit is also non-negative.

Now consider the case (F, H). The first-order conditions are:

$$\begin{aligned} f'\left(p_H^{*F}\right)\left[p_H^{*F} - \alpha \cdot k^{*H} - c_D \cdot w_F - t\right] + f\left(p_H^{*F}\right) = 0 \\ f'\left(p_F^{*F}\right)\left[p_F^{*F} - \alpha \cdot k^{*H} - c_D \cdot w_F\right] + f\left(p_F^{*F}\right) = 0, \end{aligned} \tag{6}$$

$$k^{*H} = \frac{\alpha \cdot c_U \cdot w_H - c_D \cdot w_F + \alpha \cdot t}{2\alpha} + \frac{1}{2\alpha} \frac{n_H \cdot f'\left(p_H^{*F}\right) \cdot \left(p_H^{*F} - t\right) + n_F \cdot f'\left(p_F^{*F}\right) \cdot p_F^{*F}}{n_H \cdot f'\left(p_H^{*F}\right) + n_F \cdot f'\left(p_F^{*F}\right)}. \tag{7}$$

In this case, when t increases without limit, *(i)* $k \to \infty$ once we assume α is bounded away from 0, *(ii)* $p_H^{*F} \to \bar{p}$ and $p_F^{*F} \to \bar{p}$ since $\frac{dp_H^{*F}}{dk} > 0$ and $\frac{dp_F^{*F}}{dk} > 0$, and *(iii)* demand and profits of both firms are 0. The same reasoning applies to the case in which firms choose locations (H, F).

To conclude, as t increase without limit and with α bounded away from 0, we have $\Pi_D^{*(F,H)}$, $\Pi_U^{*(F,H)}$, $\Pi_D^{*(H,F)}$, and $\Pi_U^{*(H,F)}$ approaching 0, while $\Pi_D^{*(H,H)}$, $\Pi_U^{*(H,H)}$, $\Pi_D^{*(F,F)}$, and $\Pi_U^{*(F,F)}$ remain positive. Hence, every subgame perfect equilibrium yields either (H, H) or (F, F).

This result was firstly shown by Fujita (1981) and fits with the intuition: if the transport cost of the intermediate good is high, because both t and α are high, then each pattern of locations where this cost is avoided by the firms is an equilibrium and the situation corresponds to a coordination game.

In order to obtain a complete characterization of equilibrium locations, we now focus on a particular case. The demand function is linear and given by $f(p) = 1 - p$. We also let $w_H = 1$, $w_F = 0$, $n_H = 1$, $n_F = 0$, $c_D = 0.1$, and $c_U = 0.2$ and determine equilibria for all possible values of α and t. The following figure illustrates the results.[4]

4. CONNECTIONS WITH PREVIOUS LITERATURE

In the previous literature, it has been widely acknowledged that, when choosing where to locate, firms face a trade-off between the location where access to consumers is maximized (usually the central point of the market) and

[4]All computations are contained in Appendix B.

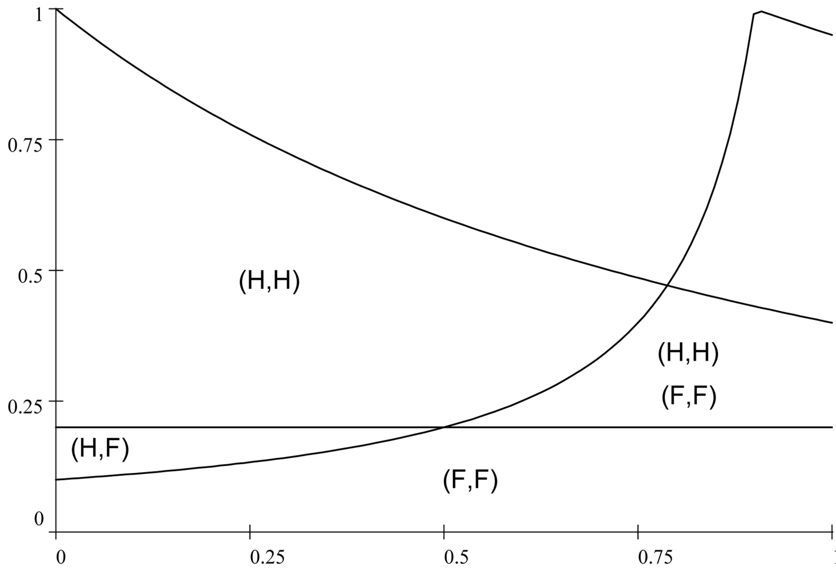

FIGURE 2: A Numerical Example of Industry Location.

the location where the firm's production costs are minimized. Mayer (2000) dealt with this problem in the context of a duopoly in a bounded linear space, where consumers are uniformly distributed and where, by contrast, the distribution of unit production costs is nonuniform. He concluded that with a globally convex distribution of production costs, there will be an agglomerated equilibrium of locations that is intermediate between the central point of the market and the minimum production cost point. He added that the equilibrium locations of firms will be closer to the minimum production cost point than to the central point, because a deviation from the former point would cause a loss to *all* consumers, whereas a deviation from the central point would harm *some* consumers, while benefiting others. He also concluded that a fall in the unit transport costs of the consumer good would shift equilibrium locations toward the point of minimum production costs.

Usually, differences in production costs across locations follow from the fact that the supply of an input is localized, so that the firm not only has to pay for the price of the input at its source but it is also responsible for its transport over the distance between the firm's location and the input site. Mayer (2000) explicitly considers this cause of spatial heterogeneity in production costs. However, the localized input is often an intermediate good produced by upstream firms. Hence, the location of the input is endogenous and interdependent with the location of the consumer good firms. Hwang and Mai (1989) model this interdependence through a two-stage game involving two players that are successive monopolists. In the first stage, the upstream and the downstream firms simultaneously select locations in an interval whose left boundary is a "port," through which a raw material is imported, and whose right boundary is a "market" in which all consumers locate. In the second stage, the firms set mill prices for the intermediate good and for the consumer good. Subgame perfect equilibrium locations are derived for the firms, depending on the unit transport costs of the three goods (the raw material, the intermediate good, and the final good) and on the input-output coefficients. This model suffers from the limitation that the source of the primary input is, by assumption, distinct from the location of the consumers.

Amiti (2005) overcomes this limitation in the sense that she presents a general equilibrium model with two countries (Home and Foreign) that are both locations of consumers and firms. There are two vertically related industries, Upstream and Downstream, which operate under monopolistic Dixit-Stiglitz competition. The industries use two primary factors, labor and capital, in different proportions: upstream firms are capital-intensive, while downstream firms are labor-intensive. The countries differ in terms of their factor endowments, so that Home is abundant in labor while Foreign is abundant in capital. Besides primary factors, each downstream firm uses a composite intermediate good made by the products of each upstream firm, as in Ethier (1982).

Apart from the case of autarky, where upstream and downstream firms divide evenly between the two countries in order to serve the local consumers, there are two possibilities. If transport costs are intermediate, all the firms (upstream and downstream) agglomerate in one country, and the downstream industry supplies the other country in manufactured goods through exports. Agglomeration occurs in the Foreign (capital-abundant) country if the transport costs of the intermediate good are low enough in relation to the transport costs of the final good. Agglomeration takes place in the Home (labor-abundant) country if the transport costs of the intermediate good are high enough and the existence of multiple locational equilibria is possible. Finally, if both types of transport cost are low enough, the upstream and downstream firms locate in different countries, according to comparative advantage, and a fragmented equilibrium emerges. However, Amiti (2005) does not shed enough light on the basic trade-off that firms incur between production costs (which are mainly felt by upstream firms) and market access (which is mainly felt by downstream firms). The reason is that she focuses on the allocation of each production stage to the country that is abundant in the factor (capital or labor) used more intensively by that production stage. Moreover, she assumes that the demand function is isoelastic thus making a specific analysis of consumer and firm behavior.

Rossi-Hansberg et al. (2005) deal with the fragmentation of firms in headquarters and plant in a metropolitan area. They conclude that fragmentation takes place as an effect of the growth of the urban area in terms of total population. Other factors of fragmentation such as the decline in transport costs or the intensity of vertical linkages are disregarded.

5. CONCLUDING REMARKS

We conclude that spatial fragmentation of vertically linked industries occurs for low values of input-output coefficients and intermediate values of the transport cost, provided that the labor intensity of upstream and downstream industries differ enough. Otherwise, clustering of industries arises. The agglomeration takes place in the low labor cost country if transport costs are low and it occurs in the large market otherwise. Locational equilibria are unique everywhere except in the case where the transport cost of the intermediate good is high (i.e., if both the transport rate and the input-output coefficient are high), when agglomeration can happen in either country. Spatial fragmentation becomes more likely if the countries are strongly asymmetric in size.

These locational results are not completely new, but here they are derived with a minimal specification of the demand function, i.e., with a consumer demand function that is not necessarily isoelastic. However, the model remains a partial equilibrium approach. Its generalization in order to encompass general equilibrium remains a task for the future.

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APPENDIX A

In this section, we present both firms' problems and the expressions used to derive optimal prices for each combination of locations.

Case (H,H)

Firm *D*'s problem:

$$\underset{p_H^H, p_F^H}{Max} \; n_H \cdot f\left(p_H^H\right)\left[p_H^H - \alpha \cdot k^H - c_D \cdot w_H\right] + n_F \cdot f\left(p_F^H\right)\left[p_F^H - \alpha \cdot k^H - c_D \cdot w_H - t\right].$$

The first-order conditions are:

$$f'\left(p_H^{*H}\right)\left[p_H^{*H} - \alpha \cdot k^H - c_D \cdot w_H\right] + f\left(p_H^{*H}\right) = 0$$

$$f'\left(p_F^{*H}\right)\left[p_F^{*H} - \alpha \cdot k^H - c_D \cdot w_H - t\right] + f\left(p_F^{*H}\right) = 0,$$

Firm *U*'s problem:

$$\underset{k^H}{Max}\ \alpha[n_H \cdot f(p_H^H) + n_F \cdot f(p_F^H)][k^H - c_U \cdot w_H],$$

and the optimum price for the intermediate good is:

$$k^{*H} = \frac{\alpha \cdot c_U \cdot w_F - c_D \cdot w_H}{2\alpha} + \frac{1}{2\alpha}\frac{n_H \cdot f'(p_H^{*H}) \cdot p_H^{*H} + n_F \cdot f'(p_F^{*H}) \cdot (p_F^{*H} - t)}{n_H \cdot f'(p_H^{*H}) + n_F \cdot f'(p_F^{*H})}.$$

Case (F,H)

Firm *D*'s problem:

$$\underset{p_H^F, p_F^F}{Max}\ n_H \cdot f(p_H^F)[p_H^F - \alpha \cdot k^H - c_D \cdot w_F - t] + n_F \cdot f(p_F^F)[p_F^F - \alpha \cdot k^H - c_D \cdot w_F].$$

The first-order conditions are:

$$f'(p_H^{*F})[p_H^{*F} - \alpha \cdot k^H - c_D \cdot w_F - t] + f(p_H^{*F}) = 0$$
$$f'(p_F^{*F})[p_F^{*F} - \alpha \cdot k^H - c_D \cdot w_F] + f(p_F^{*F}) = 0,$$

Firm *U*'s problem:

$$\underset{k^H}{Max}\ \alpha[n_H \cdot f(p_H^F) + n_F \cdot f(p_F^F)][k^H - c_U \cdot w_H - t],$$

and the optimum price for the intermediate good is:

$$k^{*H} = \frac{\alpha \cdot c_U \cdot w_H - c_D \cdot w_F + \alpha t}{2\alpha} + \frac{1}{2\alpha}\frac{n_H \cdot f'(p_H^{*F}) \cdot (p_H^{*F} - t) + n_F \cdot f'(p_F^{*F}) \cdot p_F^{*F}}{n_H \cdot f'(p_H^{*F}) + n_F \cdot f'(p_F^{*F})}.$$

Case (F,F)

Firm *D*'s problem:

$$\underset{p_H^F, p_F^F}{Max}\ n_H \cdot f(p_H^F)\left[p_H^F - \alpha \cdot k^F - c_D \cdot w_F - t\right] + n_F \cdot f(p_F^F)\left[p_F^F - \alpha \cdot k^F - c_D \cdot w_F\right].$$

The first-order conditions:

$$f'(p_H^{*F})\left[p_H^{*F} - \alpha \cdot k^F - c_D \cdot w_F - t\right] + f(p_H^{*F}) = 0$$
$$f'(p_F^{*F})\left[p_F^{*F} - \alpha \cdot k^F - c_D \cdot w_F\right] + f(p_F^{*F}) = 0,$$

Firm *U*'s problem:

$$\underset{k^H}{Max}\ \alpha\left[n_H \cdot f\left(p_H^F\right) + n_F \cdot f\left(p_F^F\right)\right][k^F - c_U \cdot w_F],$$

and the optimum price for the intermediate good is:

$$k^{*F} = \frac{\alpha \cdot c_U \cdot w_F - c_D \cdot w_F}{2\alpha} + \frac{1}{2\alpha}\frac{n_H \cdot f'\left(p_H^{*F}\right) \cdot \left(p_H^{*F} - t\right) + n_F \cdot f'\left(p_F^{*F}\right) \cdot p_F^{*F}}{n_H \cdot f'\left(p_H^{*F}\right) + n_F \cdot f'\left(p_F^{*F}\right)}.$$

APPENDIX B

In this section, we explore the particular case described in Section 4. Demand is linear and given by $f(p) = 1 - p$. We also let $w_H = 1$, $w_F = 0$, $n_H = 1$, $n_F = 0$, $c_D = 0.1$, and $c_U = 0.2$.

To start, note that the number of consumers in country F is zero, so that D sets a single price for the final good, p^{X_D}. When firms locate in (X_D, X_U), the payoff functions are as follows: $\Pi_D^{(X_D,X_U)} = (1 - p^{X_D})(p^{X_D} - \alpha k^{X_U} - 0.1 \cdot w_{X_D} - t \cdot d(X_D, H))$ and $\Pi_U^{(X_D,X_U)} = \alpha(1 - p^{X_D})(k^{X_U} - 0.2 \cdot w_{X_U} - t \cdot d(X_D, X_U))$. When firms locate in (H, H), equilibrium prices are $p^{*H} = 0.05\alpha + 0.775$ and $k^{*H} = \frac{0.45}{\alpha} + 0.1$; in order to ensure that demand is positive, we impose $p^{*H} < 1$, which becomes $\alpha < 0.45$. If firms are settled in (F, H), equilibrium prices are $p^{*F} = \frac{1}{4}t + 0.05\alpha + \frac{1}{4}t\alpha + \frac{3}{4}$ and $k^{*H} = \frac{1}{2}t + \frac{1}{2\alpha} - \frac{1}{2\alpha}t + 0.1$; the condition $p^{*F} < 1$ leads to $t < \frac{1-0.2\alpha}{\alpha+1}$. The locations (H, F) yield equilibrium prices $p^{*H} = \frac{1}{4}t\alpha + 0.225\alpha^2 + 0.55$ and $k^{*F} = \frac{1}{2}t + 0.45\alpha$; imposing $p^{*H} < 1$ gives $t < \frac{0.9}{\alpha}$. Finally, when firms locate in (F, F), equilibrium prices are $p^{*F} = \frac{1}{4}t + \frac{3}{4}$ and $k^{*F} = \frac{1}{2\alpha} - \frac{1}{2\alpha}t$; the condition $p^{*F} < 1$ becomes $t < 1$. Note that $t < \frac{1-0.2\alpha}{\alpha+1}$ is the most stringent condition on the parameters and $t = \frac{1-0.2\alpha}{\alpha+1}$ is included in Figure 2 above, limiting the space of parameters. Maximum profits for both firms, as a function of t and α, are represented in the following matrix:

	Firm U: H	Firm U: F
Firm D H	$\frac{1}{1600}\left(2.0\alpha - 9.0\right)^2$, $0.005\alpha^2 - 0.045\alpha + 0.101\,25$	$\frac{1}{1600}\left(10.0\alpha t - 9.0\right)^2$, $\frac{1}{800}\left(10.0\alpha t - 9.0\right)^2$
F	$\left(0.05\alpha + \frac{1}{4}t + \frac{1}{4}\alpha t - \frac{1}{4}\right)^2$, $\frac{1}{200}\left(\alpha + 5.0t + 5.0\alpha t - 5.0\right)^2$.	$\frac{1}{16}\left(t-1\right)^2$, $\frac{1}{8}t^2 - \frac{1}{4}t + \frac{1}{8}$.

By comparing the above expressions, we obtain each firm's best replies: when firm U chooses H, firm D's best reply is H when

$$t \geq \frac{1}{50}(a + 1.0)^{-2} \cdot \big(40.0a - 10.0a^2 - \sqrt{3150.0a + 325.0a^2 - 700.0a^3 + 100.0a^4 + 2025.0} + 50.0\big),$$

and it is F otherwise; when firm U chooses F, firm D's best reply is H when

$$t \geq \frac{1}{\alpha^2 - 1}\left(0.9\alpha + \frac{1}{2}\sqrt{-7.2\alpha + 4\alpha^2 + 3.24} - 1\right).$$

As for firm U, its best reply to F is always F and its best reply to H is H as long as $t \geq 0.2$.