A basic model for the propagation of ideologies

Luis D. Mata-Boschini[1], Eduardo Salas-Jiménez[1], and Fabio Sanchez[2]

[1] *Universidad de Costa Rica, Escuela de Matemática, San José, Costa Rica*
[2] *Universidad de Costa Rica, Centro de Investigación en Matemática Pura y Aplicada-Escuela de Matemática, San José, Costa Rica*

Abstract

Ideas and ideologies move the world and are involved in almost every aspect of human life and society. This paper presents a mathematical model for the propagation of two different ideologies in a group of people that could convert or not to one of the ideologies. This model allowed us to analyze which relations between parameters influence the survival and dominance of an ideology. The basic reproductive number was computed and numerical simulations were performed to analyze different scenarios.

Keywords: Mathematical models, propagation of ideologies, epidemic models, social dynamics

1 Introduction

Since the beginning of society, different ideas on the same topic have flowed within groups. This can be seen in competing religions in the same geographical location or political parties looking for the presidency of a country. Because of this, it is essential to study the different factors that influence the dominance and persistence of an ideology throughout time. Sometimes, two ideologies can coexist, but the number of adepts is more significant for one of them.

The propagation of ideas and similar concepts have previously been studied with epidemiological models [1, 6, 3]. These have generally explored the propagation of one idea through time. Consider now that we want to model the propagation of two ideologies within the same group. Situations like bipartisan, such as Democrats against Republicans in the United States, the cold war with the fight between capitalism and communism, Catholicism against Protestantism in Christianity, and others are examples of this.

Although ideas and ideologies might be developing within a group, there can be a big part of these people that are skeptical about them or have not decided on their position yet. This can be seen in political election cases where high percentages of the population declare they are undecided in polls. This is exemplified with the 36% and 32% of undecided voters in the 2018 and 2022 Costa Rican presidential elections (see [4] and [5]). This also happens in most elections around the world, including the previous United States election between Donald Trump and Joe Biden [2].

We explore the situation with two competing ideologies in the same group of people to create an epidemiology-like model that studies the population behavior of their respective communities. We chose two ideologies specifically because three or more would make the model too complex to analyze easily, and one would not give enough information. This generic model may be used in the examples we have exposed previously, as well as many others.

In this project we pretend to analyze the behavior of the propagation of two ideologies in a group of people; also determine which conditions on parameter related to the credibility and diffusion of an idea influence more in the conversion of a part of the population, and through this, try to say how can an idea persist along time, and when the proportion of adepts of an idea could be greater than in the other ideologies.

The rest of the manuscript is organized as follows. In section 2, we describe the model and the parameters included in it. Afterward, we will find the equilibrium points for different cases and their basic reproductive number. In section 3, we create various numerical simulations for different conditions over the parameter. In section 4, we discuss the previously encountered results and give our conclusions on the project.

2 Mathematical Model for ideologies

We will work with two opposite ideologies, A and B, that exist together in the same group of people. The interaction between people of an ideology and the ideology-free individuals causes the movements within the population. Now we describe the model and the system of differential equations representing it.

2.1 Description of the model

This model studies the propagation of two opposite ideologies in a population, which is divided into four groups: people who believe in one of the ideologies (A or B), people who have never heard about any of these ideas, called susceptible (S), and a neutral group (E) that have been exposed to the ideas.

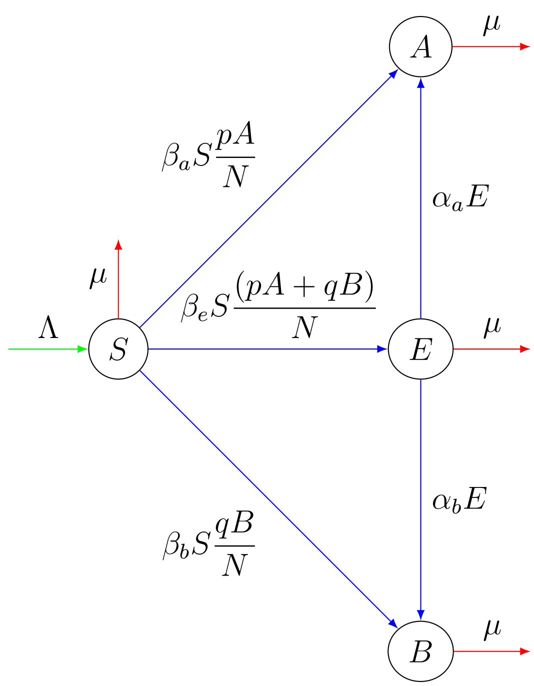

Figure 1: Model flowchart.

The susceptible population S initially does not know the ideas A nor B, but some of the people who support the ideologies try to convince them to adopt their position. This is a proportion $p, q \in [0,1]$ of the people in A and B, respectively. When the interaction occurs, some people in S become part of one of the ideologies or pass to the intermediate group E with other neutral people, with a corresponding conversion rate of β_a, β_b, or β_e. After neutral people had the first contact with the ideologies, they decide to believe in A or B at a rate of α_a or α_b, with $\alpha_a + \alpha_b \in [0,1]$. People only enter the system by group S, at a rate Λ. Also, there is a natural mortality rate of μ.

The model described above is illustrated in Figure 1 and gives us the following system of differential equations, whose parameters are summarized in Table 1.

$$S'(t) = \Lambda - (\beta_a + \beta_e)S\frac{pA}{N} - (\beta_b + \beta_e)S\frac{qB}{N} - \mu S \quad (1)$$

$$E'(t) = \beta_e S\frac{(pA + qB)}{N} - (\alpha_a + \alpha_b)E - \mu E \quad (2)$$

$$A'(t) = \beta_a S\frac{pA}{N} + \alpha_a E - \mu A \quad (3)$$

$$B'(t) = \beta_b S\frac{qB}{N} + \alpha_b E - \mu B \quad (4)$$

Table 1: Parameters involved in the model's system.

Parameter	Definition
Λ	birth rate
μ	mortality rate
p	proportion of adepts of group A who promote that ideology
q	proportion of adepts of group B who promote that ideology
β_a	conversion rate from susceptibles to A
β_b	conversion rate from susceptibles to B
β_e	pre-conversion rate from susceptibles to neutrals (influenced by A and B)
α_a	conversion rate from neutrals to ideology A
α_b	conversion rate from neutrals to ideology B

With the explicit system of differential equations, we proceed to find the equilibrium points of the system.

2.2 Equilibrium points

When at equilibrium, we have that $S'(t) = E'(t) = A'(t) = B'(t) = 0$, hence, the system is as follows:

$$0 = \Lambda - (\beta_a + \beta_e)S\frac{pA}{N} - (\beta_b + \beta_e)S\frac{qB}{N} - \mu S \quad (5)$$

$$0 = \beta_e S\frac{(pA + qB)}{N} - (\alpha_a + \alpha_b)E - \mu E \quad (6)$$

$$0 = \beta_a S\frac{pA}{N} + \alpha_a E - \mu A \quad (7)$$

$$0 = \beta_b S\frac{qB}{N} + \alpha_b E - \mu B \quad (8)$$

As we have a symmetric system with two opposite ideologies, we need to study the ideology-free equilibrium point in which $A = B = 0$, but also the equivalent cases of a single ideology: $A = 0 \neq B$ and $A \neq 0 = B$. Due to the system's nonlinearities, we do not analyze the endemic point where $A \neq 0 \neq B$.

In the calculations of the points (S^*, E^*, A^*, B^*), we omit the asterisk in the variables.

2.2.1 Ideology free

When there are no ideologies in the system, we have that $A = B = 0$. Then the equations become:

$$0 = \Lambda - \mu S \tag{9}$$

$$0 = -(\alpha_a + \alpha_b)E - \mu E \tag{10}$$

$$0 = \alpha_a E \tag{11}$$

$$0 = \alpha_b E \tag{12}$$

Therefore, the ideology free equilibrium point is $(S^*, E^*, A^*, B^*) = (N, 0, 0, 0)$.

2.2.2 Single ideology

Note that the case $A = 0 \neq B$ is equivalent to $A \neq 0 = B$ by replacing A and B with their respective parameters. For this observation, we will analyze the second situation without loss of generality. The system in this section will look as follows:

$$0 = \Lambda - (\beta_a + \beta_e)S\frac{pA}{N} - \mu S \tag{13}$$

$$0 = \beta_e S\frac{pA}{N} - (\alpha_a + \alpha_b)E - \mu E \tag{14}$$

$$0 = \beta_a S\frac{pA}{N} + \alpha_a E - \mu A \tag{15}$$

$$0 = \alpha_b E \tag{16}$$

The equation (16) distinguishes the cases $E = 0$ and $E \neq 0 = \alpha_b$. If we assume that $E = 0$, since $A \neq 0$ then we have that $\beta_a pS = \mu N > 0$ and $\beta_e pS = 0$, so $pS \neq 0$ and therefore $\beta_e = 0$. Also we can see that $S = \frac{\mu N}{\beta_a p} = \frac{N}{R_a^{(1)}}$, with $R_a^{(1)} = \frac{\beta_a p}{\mu}$ and $R_b^{(1)} = \frac{\beta_b q}{\mu}$. Since $S < N$, then a condition to have positive populations is that $1 < R_a^{(1)}$, and then $A = \frac{\Lambda}{\mu} - S = N\left(1 - \frac{1}{R_a^{(1)}}\right)$, hence this single ideology point is:

$$(S^*, E^*, A^*, B^*) = \left(\frac{N}{R_a^{(1)}}, 0, N\left(1 - \frac{1}{R_a^{(1)}}\right), 0\right)$$

On the other hand, when $E \neq 0 = \alpha_b$, the system is as follows:

$$0 = \Lambda - (\beta_a + \beta_e)S\frac{pA}{N} - \mu S \tag{17}$$

$$0 = \beta_e S\frac{pA}{N} - \alpha_a E - \mu E \tag{18}$$

$$0 = \beta_a S\frac{pA}{N} + \alpha_a E - \mu A \tag{19}$$

and the system has the only solution:

$$\begin{cases} S^* &= \dfrac{\Lambda\alpha_a + \Lambda\mu}{(\alpha_a\beta_a + \alpha_a\beta_e + \beta_a\mu)p} \\ \\ E^* &= -\dfrac{\Lambda\alpha_a\beta_e\mu + \Lambda\beta_e\mu^2 - \left(\Lambda\alpha_a\beta_a\beta_e + \Lambda\alpha_a\beta_e^2 + \Lambda\beta_a\beta_e\mu\right)p}{(\alpha_a^2\beta_a^2 + 2\,\alpha_a^2\beta_a\beta_e + \alpha_a^2\beta_e^2 + (\beta_a^2 + \beta_a\beta_e)\mu^2 + (2\,\alpha_a\beta_a^2 + 3\,\alpha_a\beta_a\beta_e + \alpha_a\beta_e^2)\mu)p} \\ \\ A^* &= -\dfrac{\Lambda\alpha_a\mu + \Lambda\mu^2 - (\Lambda\alpha_a\beta_a + \Lambda\alpha_a\beta_e + \Lambda\beta_a\mu)p}{((\beta_a + \beta_e)\mu^2 + (\alpha_a\beta_a + \alpha_a\beta_e)\mu)p} \\ \\ B^* &= 0 \end{cases}$$

If we define $\varphi := \varphi_a = \alpha_a\beta_a + \alpha_a\beta_e + \beta_a\mu$, the equilibrium point can be simplified as:

$$\begin{cases} S^* &= \dfrac{\mu(\alpha_a + \mu)}{\varphi p}N \\ \\ E^* &= \dfrac{\beta_e\mu\left(\varphi p - \mu(\alpha_a + \mu)\right)}{(\alpha_a + \mu)(\beta_a + \beta_e)\varphi p}N \\ \\ A^* &= \dfrac{(\varphi p - \mu(\alpha_a + \mu))}{(\alpha_a + \mu)(\beta_a + \beta_e)p}N \\ \\ B^* &= 0 \end{cases}$$

It is clear that since $0 \leq S^*, E^*, A^* \leq N$, we need $R_a := \dfrac{\varphi_a p}{\alpha_a\mu + \mu^2}$ to be greater than one. With this definition, the equilibrium point can be expressed as:

$$\begin{cases} S^* &= \dfrac{1}{R_a}N \\ \\ E^* &= \dfrac{\beta_e\mu\left(1 - \dfrac{1}{R_a}\right)}{(\alpha_a + \mu)(\beta_a + \beta_e)(\varphi p)^2}N \\ \\ A^* &= \dfrac{1 - \dfrac{1}{R_a}}{(\alpha_a + \mu)(\beta_a + \beta_e)\varphi p^2}N \\ \\ B^* &= 0 \end{cases}$$

Symmetrically, we can define $\varphi_b = \alpha_b\beta_b + \alpha_b\beta_e + \beta_b\mu$ and $R_b = \dfrac{\varphi_b q}{\alpha_b\mu + \mu^2}$. Computing the basic reproduction number proved to be difficult, but with simulations we see that R_a and R_b are good approximations. Further more, the other single ideology point when $B = E = 0$ depends on the number $R_a^{(1)}$, that coincide with R_a if $\alpha_a = 0$, and that makes sense because there's no flow from E to A.

2.3 Computation of the basic reproduction numbers

The Jacobian matrix associated to the system is:

$$J(S,E,A,B) = \begin{bmatrix} -(\beta_a+\beta_e)\frac{pA}{N} - (\beta_b+\beta_e)\frac{qB}{N} - \mu & 0 & -(\beta_a+\beta_e)\frac{pS}{N} & -(\beta_b+\beta_e)\frac{qS}{N} \\ \beta_e\frac{(pA+qB)}{N} & -(\alpha_a+\alpha_b)-\mu & \beta_e\frac{pS}{N} & \beta_e\frac{qS}{N} \\ \beta_a\frac{pA}{N} & \alpha_a & \beta_a\frac{pS}{N}-\mu & 0 \\ \beta_b\frac{qB}{N} & \alpha_b & 0 & \beta_b\frac{qS}{N}-\mu \end{bmatrix}$$

When replacing the ideology free point we have:

$$J(N,0,0,0) = \begin{bmatrix} -\mu & 0 & -(\beta_a+\beta_e)p & -(\beta_b+\beta_e)q \\ 0 & -(\alpha_a+\alpha_b)-\mu & \beta_e p & \beta_e q \\ 0 & \alpha_a & \beta_a p-\mu & 0 \\ 0 & \alpha_b & 0 & \beta_b q-\mu \end{bmatrix}$$

But besides the negative eigenvalue $-\mu$, the other eigenvalues are not easy to compute. Then, we tried with the single ideology equilibrium points. In the first case we have that $E=0$ and $\beta_e=0$, so:

$$J\left(\frac{1}{R_a^{(1)}}N, 0, \left(1-\frac{1}{R_a^{(1)}}\right)N, 0\right) = \begin{bmatrix} -\beta_a p & 0 & \underline{-\mu} & -\frac{\beta_b q}{R_a^{(1)}} \\ 0 & \underline{-(\alpha_a+\alpha_b)-\mu} & 0 & 0 \\ \underline{\beta_a p-\mu} & \alpha_a & 0 & 0 \\ 0 & \alpha_b & 0 & \underline{\frac{\beta_b q}{R_a^{(1)}}-\mu} \end{bmatrix}$$

However, in this situation the stability of the system depends on the condition that the eigenvalues are negative. Is trivial that $-\mu$ and $-(\alpha_a+\alpha_b)-\mu$ are negative, but we also need that $R_b^{(1)} < R_a^{(1)} < 1$.

The last condition contradicts the existence of this single ideology point, because we saw before that $R_a^{(1)}$ must be greater than one. This implies that the point may exist, but we can't ensure the stability at that point.

For the other two symmetric situations we tried some methods to find the basic reproduction numbers, but it is always needed to compute the eigenvalues of a 4×4 matrix, that is almost impossible with the amount of parameters involved.

3 Numerical Simulations

The preceding section make us think that the numbers R_a and R_b have some influence in the behavior of the ideas among the population. Then we define the *pseudo-basic reproduction number* $R_0 = \max\{R_a, R_b\}$, and then run some numerical simulations based on the relation between the conversion rates $(\beta_a, \beta_b, \beta_e)$, the parameters α_a and α_b, and the situations $R_0 > 1$ and $R_0 < 1$.

3.1 $\beta_e < \beta_a < \beta_b$ and $1 < R_0$

In the first simulation we have that $R_a = 0.993$ and $R_b = 1.516$, so $1 < R_0 = 1.516$. This happens with the parameters in the caption of Figure 2, and we begin with the respective initial conditions shown there. The numerical simulation under this conditions gives us the next graph about the number of people in each group during 6 centuries.

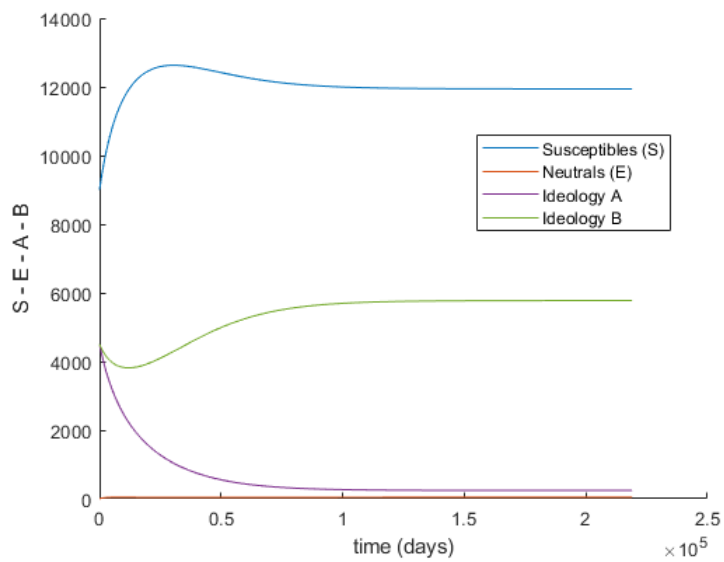

Figure 2: Parameters: $\mu = 0.00015$, $p = 0.05$, $q = 0.05$, $\alpha_a = 0.0003$, $\alpha_b = 0.0001$, $\beta_a = 0.0029$, $\beta_b = 0.0045$, $\beta_e = 0.00012$. Initial conditions: $S = 9,000$, $E = 0$, $A = 4,500$, $B = 4,500$, $N = 18,000$. Number of people in each group over time, when $1 < R_0 = R_b$ and both ideologies persist over time, and ideology B has many more adepts than ideology A.

3.2 $\beta_e < \beta_b < \beta_a$ and $1 < R_0$

The second simulation has that $R_a = 19.9$, $R_b = 12.6$, and hence $1 < R_0 = 19.9$. The initial conditions and parameters involved in this simulation are resumed in the caption of the graph, and we used a time of 600 years.

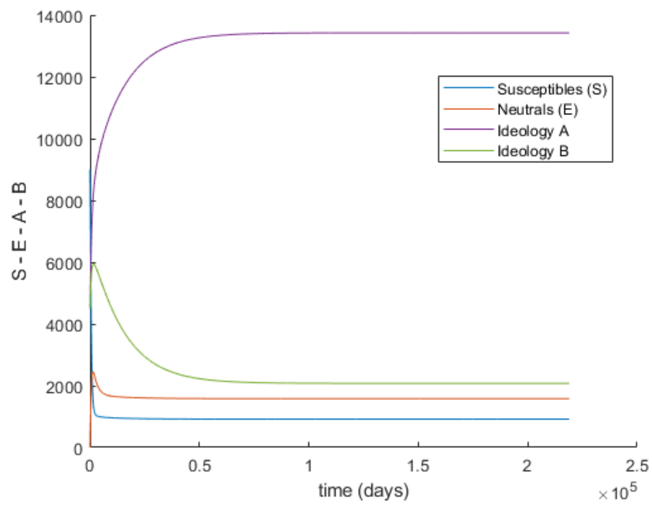

Figure 3: Parameters: $\mu = 0.00015$, $p = 0.05$, $q = 0.05$, $\alpha_a = 0.0003$, $\alpha_b = 0.0001$, $\beta_a = 0.0045$, $\beta_b = 0.0029$, $\beta_e = 0.00022$. Initial conditions: $S = 9,000$, $E = 0$, $A = 4,500$, $B = 4,500$, $N = 18,000$. Number of people in each group over time, where $1 < R_0 = R_a$ and ideology A predominates in the full population.

3.3 $\beta_a < \beta_b < \beta_e$ and $1 < R_0$

In this case we have that $R_a = 7.6825$, $R_b = 8.2016$, and $1 < R_0 = 8.2016$. The parameters and initial conditions used are shown in the caption of Figure 4. The fact that β_e is bigger than β_a and β_b may influence the dominance of ideology A because $\alpha_b < \alpha_a$, as can be seen in Figure 4. We expand on this in section 4.

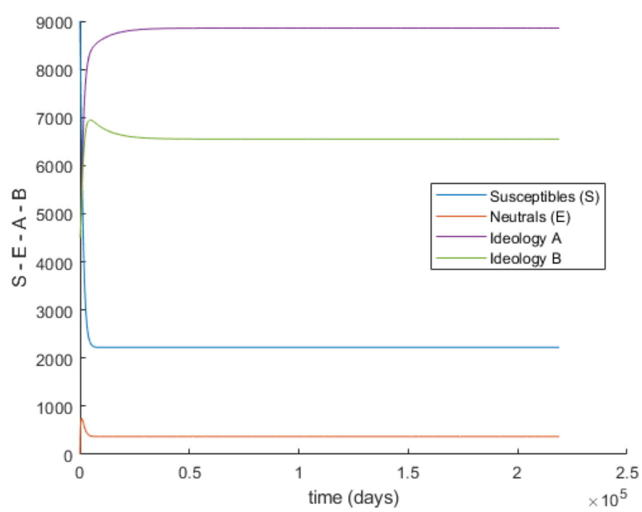

Figure 4: Parameters: $\mu = 0.00015$, $p = 0.10$, $q = 0.10$, $\alpha_a = 0.003$, $\alpha_b = 0.002$, $\beta_a = 0.002$, $\beta_b = 0.003$, $\beta_e = 0.010$. Initial conditions: $S = 9,000$, $E = 0$, $A = 4,500$, $B = 4,500$, $N = 18,000$. Number of people in each group over time, where ideology A predominates even if R_b is bigger than R_a.

3.4 $\beta_a < \beta_b < \beta_e$ and $R_0 < 1$

All the previous simulations have that $1 < R_0$ and the system gets stability in an endemic point, and the ideologies doesn't disappear. We want to know if the condition that $R_0 < 1$ could implies that ideologies eventually die out.

In this simulation, the parameters used implies that $R_a = 0.668$ and $R_b = 0.714$, so we have that $R_0 = 0.714 < 1$. The results of the simulation reflect that the ideologies die out, as can be seen in Figure 5.

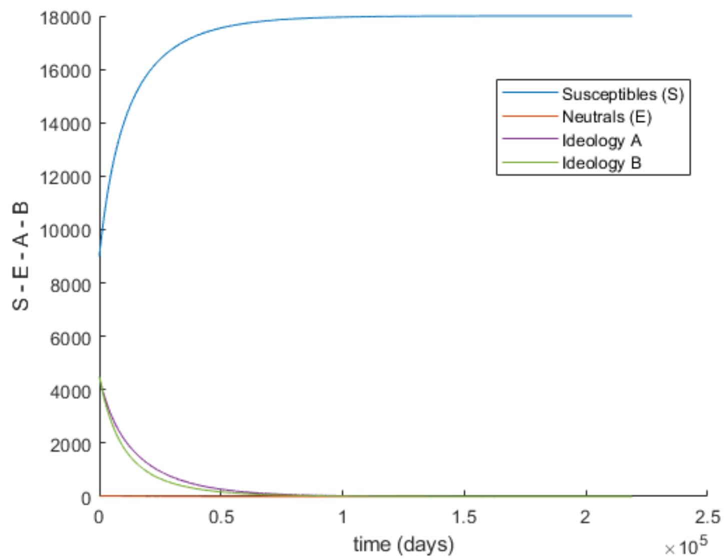

Figure 5: Parameters: $\mu = 0.00015$, $p = 0.078$, $q = 0.078$, $\alpha_a = 0.009$, $\alpha_b = 0.005$, $\beta_a = 0.0004$, $\beta_b = 0.0005$, $\beta_e = 0.0009$. Initial conditions: $S = 9,000$, $E = 0$, $A = 4,500$, $B = 4,500$, $N = 18,000$. Number of people in each group over time. Here, both ideologies die out and R_0 is less than 1.

3.5 $1 < R_a < R_b$ and $\alpha_b < \alpha_a$

When the parameter β_e was bigger than β_a and β_b, we saw that the relation between α_a and α_b gain importance in the domination of one of the ideologies. This guide us to analyze this with some simulations.

In this simulation we what an endemic point, because the parameters make that $R_a = 1.075, R_b = 1.747$, and $1 < R_0 = 1.747$. It's important to note that $\alpha_a = 0.009$ and $\alpha_b = 0.002$, and α_a is more than four times α_b. This simulation is illustrated in Figure 6, and a dominance of the ideology A is notorious, even though it started with only one person, while ideology B began with $2,000$ people.

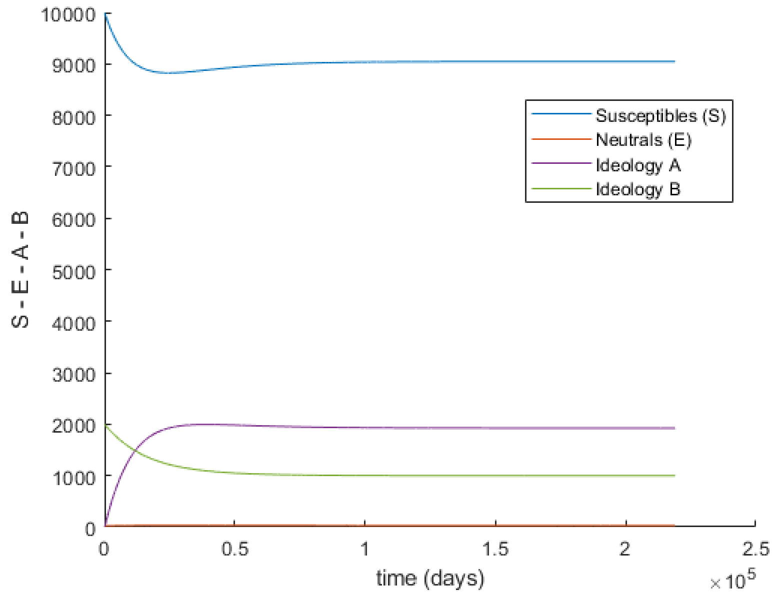

Figure 6: Parameters: $\mu = 0.00015$, $p = 0.078$, $q = 0.078$, $\alpha_a = 0.009$, $\alpha_b = 0.002$, $\beta_a = 0.0001$, $\beta_b = 0.0015$, $\beta_e = 0.0020$. Initial conditions: $S = 9,999$, $E = 0$, $A = 1$, $B = 2,000$, $N = 12,000$. Number of people in each group over time. It's clear that ideology A begins with less adepts, but increase and get bigger than the number of people in ideology B.

3.6 $R_a < 1 < R_b$ and $\alpha_b < \alpha_a$

This simulation illustrates that an ideology may dominate with $R_a < 1$. The flows from group E to both ideologies affects the growth of population in ideology A when $\alpha_a = 0.0095$ is bigger than $\alpha_b = 0.0012$ as in 3.5, but in this case we reduce the convincing power of A over susceptible people.

Here, $R_a = 0.137927 < 1 < 21.8519 = R_0 = R_b$, but it's not enough to make the ideology B dominate in the population. Figure 7 shows the results of this simulation, with an evident growth in the number of people who support ideology A.

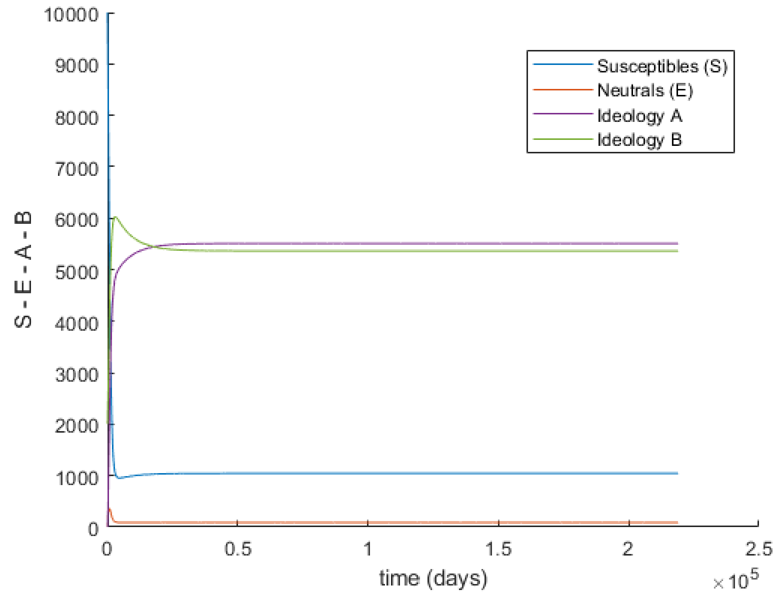

Figure 7: Parameters: $\mu = 0.00015$, $p = 0.01$, $q = 1.00$, $\alpha_a = 0.0095$, $\alpha_b = 0.0012$, $\beta_a = 0.0001$, $\beta_b = 0.0015$, $\beta_e = 0.0020$. Initial conditions: $S = 9,999$, $E = 0$, $A = 1$, $B = 2,000$, $N = 12,000$. Number of people in each group over time. Even if R_a is small, the fact that $\alpha_a > \alpha_b$ influence the behavior of the system, in which a big portion of the people finally gets converted to ideology A.

3.7 Variation in the initial conditions without changing the parameters

All the simulations that have been realized tried to exposed the sensibility of the system when the parameters vary. The initial conditions in the number of people in each group seems not to affect the proportion of people that support an ideology at the end of time. The present simulation will show it explicitly.

In this simulation we have fixed parameter that are summarized in the caption of Figure 8, and here $R_a = 1.8852$, $R_b = 2.3372$, and $1 < R_0 = 2.3372$. We run the simulation with four different cases about the initial conditions, that are explicit in Table 2, and then the four graphs are in Figure 8.

Table 2: Initial conditions used each case of the numerical simulation 3.7.

Group	Case #1	Case #2	Case #3	Case #4
S	9,998	199,998	10,000	15,000
E	0	0	0	0
A	1	1	3,000	15,000
B	1	1	3,000	15,000
N	10,000	200,000	16,000	45,000

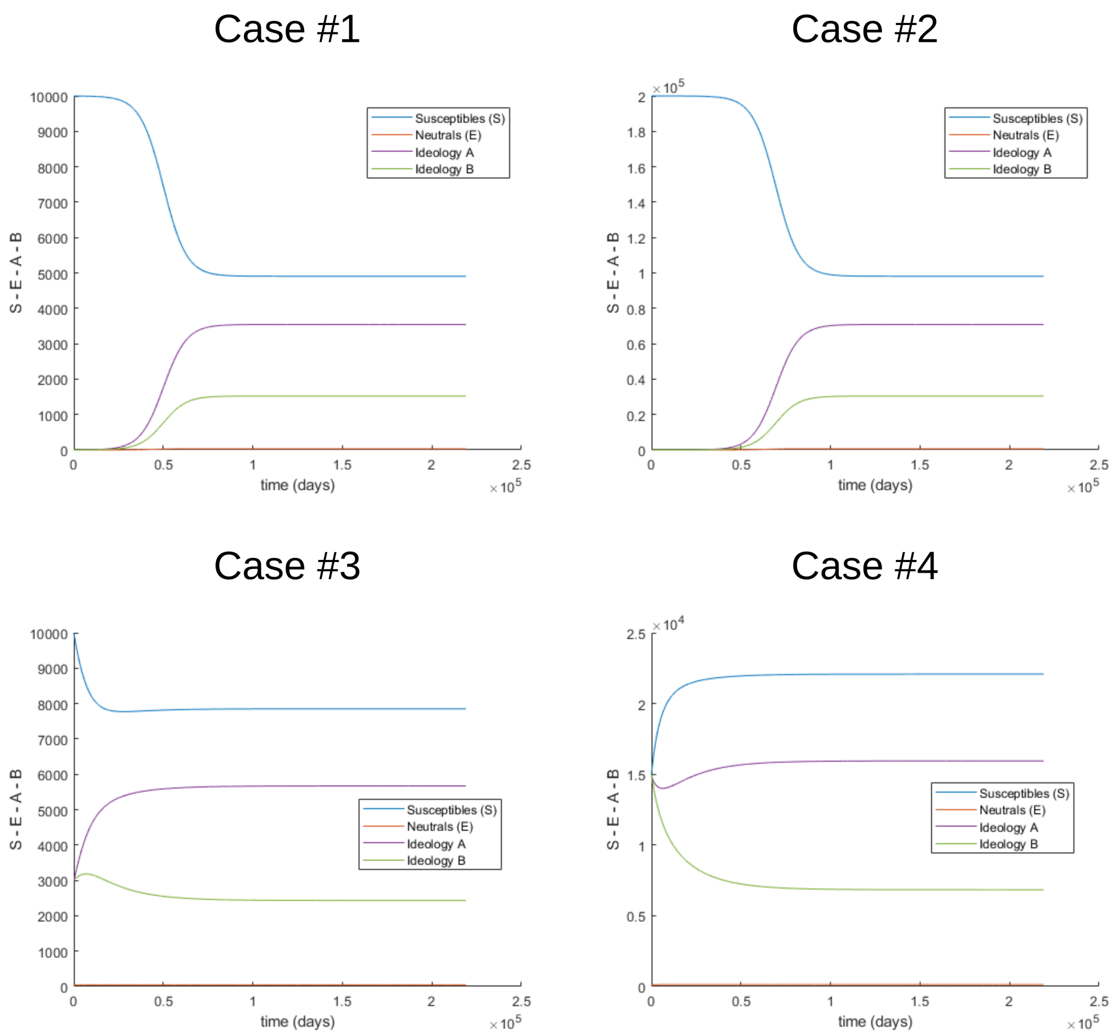

Figure 8: Parameters: $\mu = 0.00015$, $p = 0.08$, $q = 0.08$, $\alpha_a = 0.009$, $\alpha_b = 0.002$, $\beta_a = 0.0010$, $\beta_b = 0.0015$, $\beta_e = 0.0009$. Number of people in each group over time in the four cases analysed. In all the cases, the system stabilizes and have similar equilibrium point, proportional to the full population in each case.

4 Discussion

We started this project by analyzing the equilibrium points of our system. We found in 2.2.1 that the system has an ideology-free equilibrium at $(S^*, E^*, A^*, B^*) = (N, 0, 0, 0)$.

We then explore the single ideology scenario. Here equation 16 splits the calculations into the cases $E = 0$ and $\alpha_b = 0$. They both give us equilibrium points for the system. When $\alpha_b = 0$ we obtain a condition for the existence of the solution given by $R_a > 1$, where $R_a =$. Analogous to this, we find R_b, which determines the existence of the single ideology solution where only B is present. Therefore, as we said before, we consider $R_0 = \max\{R_a, R_b\}$ to be the *pseudo-basic reproduction number*.

The stability of these points was studied in 2.3, finding that the single ideology case where $E = 0$ gives an unstable solution. Since its stability condition contradicts the existence of the solution, all other cases gave inconclusive information since the eigenvalues needed to calculate the stability were hard to calculate and analyze within the context.

In the multiple ideology case, analytics do not yield useful results, which is the reason why we turn to simulation to represent our problem in a better way. We run several simulations for the system where we vary the relation between the conversion rates $(\beta_a, \beta_b, \beta_e)$, the parameters α_a and α_b, and the presence of the condition $R_0 > 1$.

From the first two simulations (3.1 and 3.2), we see that having $R_0 > 1$ seems to be related to the presence of an endemic equilibrium point. We also realize that when $\beta_e < \min\{\beta_a, \beta_b\}$, the majority of the people who become adepts come directly from the Susceptible class (S) since the movement to the Neutral group (E) is smaller than the direct one. We notice this because we have no growth in population for the ideology with the lower R_x, for $x \in \{a, b\}$. Therefore the initial contact of the ideologies with the individuals seems to be more relevant to their prevalence in this case.

With the following simulations (3.3 and 3.4), we reinforce our belief that $R_0 > 1$ is the condition for the presence of an endemic equilibrium point. Since it 3.4, $R_0 < 1$ causes the system to go to an ideology-free equilibrium. Also, in 3.3, we can see that both ideologies grow in size from their original total of 4,500 members. This can be related to the transitions between E, and both ideologies are linear, meaning that the people in E get relocated to A or B after a certain time. This entails that α_a and α_b have a greater influence over the sizes of their respective ideologies when $\beta_e > \max\{\beta_a, \beta_b\}$.

Continuing with 3.5 we see that even tho $R_a < R_b$, A turns out to be the most popular ideology. This happens because α_a is considerably greater than α_b. After all, as we said before, the transitions to A and B from E are lineal, causing indirect movement to be more effective than the direct transition into each ideology, and the speed of the indirect case is given mostly by parameters α_a and α_b. We find that the influence of α_a and α_b can be more important of that of R_a and R_b given that $\beta_e > \max\{\beta_a, \beta_b\}$.

Furthermore, checking the sixth simulation (3.6), we see that even making $R_a < 1$ can be insufficient to make B the most popular ideology since the people who reach E mostly go towards A, causing it to win out over B. Showing even further, that α_a and α_b are the general deciders of the final outcome for the ideologies when $\beta_e > \max\{\beta_a, \beta_b\}$.

Finally, in the last simulation 3.7 we modify the initial values for a given set of parameters. We notice that under the same parameters, all four graphics behave similarly, suggesting that the system has a solution independent of the system's initial conditions. This would mean that even if an ideology started with one single person and that person was trying to convince 200,000 people. They would achieve it as long as the credibility of their ideology does not change. The difference in results can be ignored when looking at the proportion of each group's population.

Acknowledgments

The author is solely responsible for the views and opinions expressed in this research.

References

[1] L.M.A. Bettencourt, A. Cintrón-Arias, D.I. Kaiser, C. Castillo-Chávez. The power of a good idea: Quantitative modeling of the spread of ideas from epidemiological models. *Elsevier, Physica A*, 364:513–536, 2006.

[2] S. Cabral. Elecciones en Estados Unidos: los votantes a los que no les gusta ni Trump ni Biden. *BBC News, Washington*, Sept. 13th 2020. Url: `www.bbc.com/mundo/noticias-internacional-54142056`

[3] M. Maleki, M. Arani, E. Buchholz, E. Mead, N. Agarwal. Applying an Epidemiological Model to Evaluate the Propagation of Misinformation and Legitimate COVID-19-Related Information on Twitter. *Social, Cultural and Behavioral Modeling*, 14:23–34, 2021.

[4] A. Méndez. Situación inédita: 36% de indecisos definirán resultados electorales. *Universidad de Costa Rica*, Feb. 1st 2018. Url: `www.ucr.ac.cr/noticias/2018/02/01/situacion-inedita-36--de-indecisos-definiran-resultados-electorales.html`

[5] A. Murillo. Encuesta CIEP-UCR confirma punteros y 32% de indecisos al cierre de campaña. *Semanario Universidad*, Feb. 1st 2022. Url: `www.semanariouniversidad.com/pais/encuesta-ciep-uce-confirma-punteros-y-31-de-indecisos-al-cierre-de-la-campana/`

[6] L. Wang, B.C. Wood. An epidemiological approach to model the viral propagation of memes. *Elsevier, Applied Mathematical Modelling*, 35:5442–5447, 2011.