# Measuring firms' default probabilities with imperfect information

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## Abstract

This dissertation aims to estimate default probabilities in an imperfect information setting. This is achieved through the implementation of a model that considers creditors cannot observe the firm asset values directly. Instead, they receive periodic information coming from accounting reports that can be imperfect. The dataset considered covers 18 non-financial companies that belong to the Euro STOXX 50, between 2010 and 2020. Evidence was found that probabilities of default overall behave monotonically when it comes to the degree of data inaccuracy. In periods marked by extreme events, such as the European sovereign debt crisis and the covid-19 pandemic, it is possible to observe a wider impact in the probabilities of default, as the assumption of the degree of accounting noise increases. Lastly, comparing the results to the default probabilities implied by Standard & Poor's credit ratings, it is possible to conclude that the model' results are underestimating the probabilities of default. On absolute terms, the difference between the model default probabilities and the ones implied by credit ratings, ranges between 0.46% and 0.75%.

**Keywords:** credit risk, structural models, reduced form approach, incomplete information, firm default probability.

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## Resumo

Esta tese tem como objetivo estimar as probabilidades de falência num cenário de informação imperfeita. Isto é conseguido através da implementação de um modelo que considera que os credores não conseguem observar diretamente os valores dos ativos da empresa. Em vez disso, os investidores recebem informações periódicas provenientes de relatórios contabilísticos que podem ser imperfeitos. O conjunto de dados considerado abrange 18 empresas não financeiras que pertencem ao Euro STOXX 50, entre 2010 e 2020. Os resultados obtidos por este estudo sugerem que as probabilidades de falência em geral comportam-se de forma monótona relativamente ao grau de imprecisão dos dados. Em períodos marcados por eventos extremos, tal como a crise europeia da dívida soberana e a pandemia causada pelo covid-19, é possível observar um impacto mais significativo nas probabilidades de falência à medida que se aumenta a hipótese relativa ao grau de ruído contabilístico. Finalmente, comparando os resultados com as probabilidades de falência implicadas pelas classificações de crédito elaboradas pela agência Standard & Poor's, é possível concluir que os resultados do modelo estão a subestimar as probabilidades de incumprimento, variando em termos absolutos, entre 0,46% a 0,75% das implicadas pelas classificações de crédito.



**Palavras-chave:** risco de crédito, modelos estruturais, abordagem de forma reduzida, informação incompleta, probabilidades de falência corporativa.


**Título:** Medição das probabilidades de falências das empresas com informação imperfeita


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## 1. Introduction

Credit risk is an unavoidable element of any business. As such, the measurement of whether a company is on the verge of failing to meet its debt obligations has always been the focus of many theorical and empirical studies. This importance is even more emphasized in certain industries, such as the financial industry, where many of its players' default is linked to several impactful externalities and whose main source of income comes from lending to firms. Consequently, making it essential to differentiate with accuracy the defaulters from the non-defaulters. The pandemic caused by the worldwide dispersion of Covid-19, reinforced this importance of measuring credit risk with precision. As it proved the need for methods capable of providing accurate assessments in times where uncertainty is rising and where there is limited visibility and access to reliable data.

Throughout the years, several models have been developed with the aim of measuring credit risk with accuracy. Some were attempts of improving the existing models, others aimed to break the current paradigm. Starting with the multivariate accounting credit scoring models, where the Altman (1968) Z-Score, a model capable of assessing the default likelihood using five ratios perceived as having predictability capability, had gain a great prominence. A few decades later, Ohlson (1980) as a way of achieving outputs of easier interpretation and reducing implementation challenges, proposed a logit model whose output can be interpreted as measure of default probability. Following Black and Scholes (1973) option pricing model, Merton (1974) introduced his model who views equity as a call option on the company assets, breaking this way the previous paradigm. Several extensions were made to Merton's (1974) model. One of the earliest extensions was made by Black and Cox (1976) when they introduced the possibility of the firm default in any period where it cannot meet its debt obligations. Further generalizations were then developed by Leland (1994), Leland and Toft (1996) and Goldstein, Ju, and Leland (2001) with the introduction of a default triggered by shareholders in conditions where they would no longer consider maintaining the control over the company as beneficial. More recently, Duffie and Lando (2001) removed the assumption that investors are capable of observing the firm's asset value, an assumption that does not hold in practice given that there is no market for it.

The main objective of this dissertation is to estimate default probabilities in an imperfect information setting. That is, it will be assumed that creditors cannot observe the assets' value directly, instead they receive imperfect information at discrete times, i.e., when the financial reports are disclosed. This goal is achieved with the implementation of the model proposed by Duffie and Lando (2001), where they derive the distribution of asset conditional on the informational setting creditors dispose. The model is applied to a sample of 18 non-financial companies present on the Euro STOXX 50 throughout the 2010-2020 period. Later on, the results are compared to the default probabilities implied by Standard & Poor's credit ratings, as the credit ratings from the three main credit ratings agencies, Moody's, Standard & Poor's and Fitch, are usually seen by the market as a benchmark when measuring credit risk.

As such, this analysis will address the following questions:

1. How are the probabilities of default affected by the quality of accounting information?
2. How well do the 5-year default probabilities estimated by the model compare with the 5-year default probabilities implied by credit ratings?

The remaining of the paper is structured as follows. Section 2 presents related literature on default predictability, namely, the evolution of models surrounding this thematic. Section 3 explains the two models used, the first considered as a "reference" (proposed by Goldstein, Ju and Leland, 2001), and another one that revokes the perfect information assumption (proposed by Duffie and Lando, 2001). Section 4 outlines the calibration strategy used. Section 5 outlines the data used and provides summary statistics. Section 6 illustrates the main results. Section 7 concludes.

## 2. Literature Review

Early studies on default predictability relied heavily on multivariate accounting credit scoring models, which gave a certain weight to accounting variables, believed to have predictability power. Depending on the approach, these models led to either a credit risk score or a probability of default measure.

One of the most renowned models based on discriminant analysis is the one developed by Altman (1968), commonly known as Altman Z-Score. This model results of an attempt to assess the, at the time, relevance of ratio analysis. The initial sample was divided in companies that filed for bankruptcy during 1946-1965 and companies without ties to bankruptcy at the time under analysis. A list of 22 variables with potential predictability capacity was then compiled, being subjected to several procedures, including the assessment of their relative predictability and the inter-correlation between the variables. This was done, in order to arrive at a more restricted variable list composed with the variables that jointly would have a bigger capacity in predicting default. In the end, this model uses five ratios, notably, working capital over total assets, retained earnings over total assets, earnings before interest and taxes over total assets, market value of equity over total debt (book value) and sales over total assets. Each of these ratios receives different weights. The default likelihood is assessed by comparing the output with several established thresholds, specifically, if the score is above 3, default is unlikely and below 1.8, there is a significant probability of said event to happen. In Altman's (1968) initial sample, the model had a proven accuracy of 94%.

Ohlson (1980) discusses some of the limitations of multivariate discriminant analysis, being the following considered the three primarily ones. First, discriminant analysis imposes some statistical requirements that reduce its practical application. Second, the model's outputs are of difficult interpretation. Lastly, as failed and non-failed firms are matched according to criteria such as size and industry, that results in a matching that is considered by the author as "somewhat arbitrary". Given that, the author proposes using a logistic regression, estimated through the maximum likelihood. It is important to note, that a logit model will have as an output a number between 0 and 1, that can be interpreted as a probability. Additionally, the data set is restricted to the period from 1970 to 1976, to industrial companies whose equity is traded on a US stock exchange or over the counter. For that estimation, nine independent variables were selected. The variables are the following, log(total assets /GNP price-level index), total liabilities (hereafter, TL) divided by total assets (hereafter, TA), working capital divided by TA, current liabilities divided by current assets, dummy variable that takes the value of one if TL exceeds TA and zero otherwise, net income divided by TA, and finally, operational cash flows divided by TL. The main criterion for this selection was simplicity, even though the first 6 variables had been frequently mentioned in the literature. Finally, one distinctive characteristic of this study is that contrary to previous studies that considered the financial statements as reported at the fiscal year date, it considers the date at the moment where financial statements were released to the public. This way the author avoids a problem related with "back-casting".

Later on, Altman and Saunders (1997), illustrate the three main limitations of traditional credit scoring models. First, as they are mainly based on book value accounting data, the output is backwards looking. Second, the assumption of linearity among explanatory variables does not hold in practice. Lastly, there is a very thin link between these models and an underlying theoretical model.

The studies around estimation of credit risk measures suffered a remarkable change with the introduction of Black and Scholes (1973) option pricing model. These authors introduced a model for valuing options that can be extended to all corporate liabilities. Their pricing model is based on the possibility of creating a risk-free portfolio by taking simultaneously a position in some option contract and in the underlying security. In other words, Black and Scholes argue that it is possible to replicate the payoffs of the market, thus making it possible to apply the risk-neutral approach when pricing option contracts.

Following the underlying intuition of pricing corporate bonds as contingent claims on assets, Merton (1974) introduced his model, that treats equity as a call option on assets where the strike price is the value of liabilities. Introducing this way, the structural default approach. This is, when the value of the firm is lower than the value of its liabilities it is no longer optimal for shareholders to keep the company, therefore choosing to default which will imply a transfer of the firm to debtholders. With this model it is possible to draw the probability of default of a firm, which would be the probability of not exercising the call option.

Black and Cox (1976) generalized Merton's (1974) model by studying the effects of covenants on the value of the firm's securities. In this new scenario, there are considered other circumstances, besides the one previously considered by Merton (1974), that will make the firm' shareholders trigger default. That is, previously it was considered that the firm only could default when the outstanding debt reached its maturity, in here it is considered that default can occur in any point in time in which the firm cannot meet its obligations.

More recently, Leland (1994) Leland and Toft (1996) and Goldstein, Ju, and Leland (2001) consider the possibility of endogenous default. In these models, the level of asset value for which default is triggered (henceforth defined as default barrier) is defined by shareholders in a way, such that it maximizes their claim. Similar to all previous referred models, these papers consider the firm asset value to follow a diffusion process and assume the firm' activity to not be affected by its financial structure. Leland (1994) assumes nominal debt is constant and has infinite maturity. Leland and Toft (1996) consider that debt must be rolled over, and as result distress costs become relevant to define the default barrier. The former one implies that when debt reaches its maturity a new amount of debt with the same coupon and principal is issued. Goldstein, Ju and Leland (2001) introduce the option of the firm issuing further debt in the future. As a result of this option, it is optimal for the management to initially issue a lower level of debt, and for investors to demand a higher yield spread on the firm's bonds given the higher risk. Furthermore, the cash flows to government are treated the same way as cash flows to shareholders and debtholders, as such all of them have a claim on the firm's payouts.

So far, all the referred structural models assume that investors observe the firm's asset value, which has tremendous implications for the default time. In these models, even though default time is assumed to be a random variable that corresponds to the first time the asset value hits the default barrier, as the person modelling the model can observe the asset's value process, the arrival of this event ends up being predictable (Jarrow and Protter, 2004). This assumption does not hold in practice, as investors do not observe the firm's asset value because there is no market for it. In addition, the information provided in the financial statements or accounting reports is frequently not accurate or provided with a delay, turning asset value estimation even harder. As an alternative to structural models, Jarrow and Turnbull (1992) propose to consider default as the result of an intensity process. In other words, it is assumed that default follows a Cox process, making the stopping time being formulated in a way that is totally inaccessible. These models are usually referred as reduced-form models. Additionally, the recovery rate is exogenously supplied, as the recovery rate process is frequently continuously observed. Lando (1998) implements a generalization of Jarrow, Lando, and Turnbull' (1997) model to allow for changes between ratings, and the intensities capable of controlling the probability of default per unit of time, that is default intensity, to be dependent on state variables. Later on, Jarrow and

Yu (2001) introduce an intensity-based credit risk model where the firm default probability depends on the firm's counterparties' probability of default. Jarrow and Protter (2004), reinforce this distinction between structural models and reduced-form models. In their paper, they claim that in structural models, the modeler is assumed to have complete information regarding the firm's assets and liabilities. Contrasting with reduced form models, where the modeler is assumed to have less information than the firm's managers. Thus, making the main distinction between those two types of models, one related with informational assumptions. Moreover, Jarrow and Protter (2004), claim that structural models can be transformed in reduced-form models by aligning them with the informational assumptions of a reduced-form model. Following this line of thought, Duffie and Lando (2001) introduced a structural model that is consistent with the default intensity assumption present on the previously mentioned reduced-form models. In their model it is considered that shareholders have perfect information, that is, they are able to observe with accuracy the market value of the firm's asset. In contrast, all other players in the market, including creditors, only have access to imperfect information from time to time. Additionally, debtholders are aware that when the firm's asset value reaches the default barrier, shareholders will choose to default but they cannot observe how far the asset value is from the asset barrier, making default an unforeseen event.

## 3. Model

My choice of model in this dissertation is the model proposed by Duffie and Lando (2001). In their paper, they start by reviewing what they refer as "a standard model" that overall is very similar to the EBIT-based model proposed by Goldstein, Ju, and Leland (2001) except that, agents are effectively risk neutral. In this model, which is taken as reference, the asset value is continuously observed by all firm stakeholders. The first subsection of this chapter presents this model. In contrast to Duffie and Lando (2001), I derive the model formulas following a partial differential equation approach similar to Goldstein, Ju, and Leland (2001). Throughout this subsection, differences between the two models are pointed out. Subsequently, Duffie and Lando (2001) analyse the impact of considering that only shareholders can observe the asset value without error, while the remaining market agents receive imperfect information at discrete times. As result, in order to provide a formula for the default probability, one has to first derive the distribution of the firm's assets conditional on the observation of the asset value with error. This section is divided in three parts. The first presents the reference model, the second introduces the hypothesis of imperfect information and the last part presents a numerical illustration of the model.

### 3.1. Reference Model

Consider a company that holds a perpetual project, whose dynamics are given by

$$\frac{dV}{V} = \mu_p dt + \sigma dz \tag{1}$$

where $\mu_p$ and $\sigma$ both are constants. Here, $\mu_p$ represents the drift of the project, while $\sigma$ represents the volatility of the project' returns. $dz$ is a variation of the Brownian motion also called the Wiener Process. It is assumed that at each moment in time, this project generates a payout of $k$*V. The authors assume that investors are risk neutral. As such, the expected present value of all cash flows generated by the project can be assessed by discounting all future cash flows at the risk-free rate without changing the probability measure:

$$E_t\left(\int_t^{\infty} kV_s e^{-rs} ds\right) = \frac{kV_t}{r-\mu_p}, \tag{2}$$

where k is the project ratio and r is the risk-free rate. Equation (2) differs from the one given by Goldstein, Ju and Leland (2001) where the risk-neutral pricing technique is used but agents are still risk averse. In that case, the asset risk premium (computed as the volatility times the market price of risk) is added to the denominator.

The authors consider an otherwise identical firm that decides to take on debt in order to maximize shareholders' claim. This company will issue a perpetual bond with a constant coupon, C, that will keep on being paid as long as the firm remains solvent, but when the payout level is not sufficient to meet the interest payments (that is, $kV$<C) equity holders have the right to infuse payments to avoid bankruptcy. As a result of this leverage, when the firm's assets reach a certain threshold, it is no longer optimal for the firm to continue its operations, so its shareholders will choose to default. This threshold is called default barrier, $V_B$.

Given the previously mentioned assumptions, and using the standard replicating strategy argument of Black and Scholes, it is possible to show that any claim to the project must satisfy the following partial differential equation (PDE):

$$(r-k)VF_V + \frac{\sigma^2}{2}V^2F_{VV} + F_t + P = rF, \tag{3}$$

where F is the valuation function of the claim and P is the payout flow specific to that claim. Duffie and Lando (2001) do not use the replicating strategy argument because they assume from the beginning that agents are risk neutral. This argument can be used however to extend the model to the case where agents are not risk neutral.

Notice that the total return from holding the project is given by the sum of its payout, $k$ , and the capital gain, $\mu_p$. In Duffie and Lando (2001), since they are assuming a risk neutral economy, one must have that the total return must equal the risk-free rate (i.e., $k + \mu_p = r$ ) and thus $r - k = \mu_p$. Replacing,

$$\mu_p VF_V + \frac{\sigma^2}{2}V^2F_{VV} + F_t + P = rF, \tag{4}$$

I proceeded with this change in order to obtain the same formulas as Duffie and Lando (2001). However, I opted to use r-k in the empirical application of the model.

Since all claims we are interested are time-independent, the above PDE can be reduced to a second order ordinary differential equation (ODE):

$$0 = \mu_p V F_V + \frac{\sigma^2}{2} V^2 F_{VV} + P - rF \tag{5}$$

The solution for equation (5), is usually found as the sum of the general solution to the homogenous equation and a particular solution, which is specific to the contract one wants to price. The general solution to the homogeneous equation is given by

$$F_{GS} = A_1 V^{-y} + A_2 V^{-x}, \tag{6}$$

where,

$$x = \frac{1}{\sigma^2}\left[\left(\mu_p - \frac{\sigma^2}{2}\right) + \sqrt{\left(\mu_p - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right] \tag{7}$$

$$y = \frac{1}{\sigma^2}\left[\left(\mu_p - \frac{\sigma^2}{2}\right) - \sqrt{\left(\mu_p - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right] \tag{8}$$

and $A_1$ and $A_2$ are constants that are determined by boundary conditions specific to the contract one wishes to price. Additionally, x is positive, while y is negative. Thus, as V increases, the first term also increases.

The general solution $F_{GS}$ does not consider any intertemporal cash flows, but they are accounted for by the particular solution. Finding the particular solution often comes out from the economic intuition behind the problem.

Following Goldstein, Ju and Leland (2001), I proceed by defining $p_B(V)$ as the present value of a claim that pays \$1 contingent on the company value reaching the default barrier. As $p_B(V)$ is not subject to any intertemporal cash flows (i.e., it is in line with $F_{GS}$), from equation (6) we can define $p_B(V)$ as

$$p_B(V) = A_1 V^{-y} + A_2 V^{-x} \,.$$

*(9)*

The authors proceed by considering the following boundary conditions

$$\lim_{V \to \infty} p_B(V) = 0, \qquad \lim_{V \to V_B} p_B(V) = 1$$

As company value goes towards infinity, $p_B(V)$ becomes zero, since it gets further away of reaching the default barrier, situation which would lead to the claim paying \$1. On the other hand, as V goes towards the default barrier, naturally, the value of this claim becomes one. Taking these conditions into consideration, we obtain the following

$$p_B(V) = \left(\frac{V}{V_B}\right)^{-x}$$

*(10)*

While the company has not reached a situation of insolvency, i.e., V has not reached $V_B$, the payout kV will be shared by the shareholders, government, and debtholders. These claims are received through dividends, taxes, and coupon payments, respectively. The authors defined the sums of these claims as $V_{solv}$ . The PDE associated with this security is

$$0 = \mu_p V F_V + \frac{\sigma^2}{2} V^2 F_{VV} + kV - rF$$

*(11)*

In contrast to $p_B$, $V_{solv}$ has intertemporal cash flows. It can be shown however that $\frac{kV}{(r-\mu_p)}$ is a particular solution to the above equation. Replacing,

$$0 = \mu_p * V \frac{k}{r - \mu_p} + kV - \frac{rkV}{r - \mu_p}$$

$$\Leftrightarrow 0 = \frac{\mu_p \times Vk}{r - \mu_p} + \frac{kV(r - \mu_p)}{r - \mu_p} - \frac{rkV}{r - \mu_p}$$

$$\Leftrightarrow 0 = \frac{\mu_p \times Vk}{r - \mu_p} + \frac{kVr - kV\mu_p}{r - \mu_p} - \frac{rkV}{r - \mu_p}$$

$$\Leftrightarrow 0 = 0$$

*(12)*

As a result,

$$V_{solv} = \frac{kV}{(r - \mu_p)} + A_1 V^{-y} + A_2 V^{-x}.$$

*(13)*

If V goes towards infinity, $V_{solv}$ approaches $\mathrm{V}/(r - \mu_p)$. On the other hand, if V is equal to $V_B$, $V_{solv}$ becomes zero. This constraint makes it possible to determine $A_1$ and $A_2$. As a result, $V_{solv}$ can be written as

$$V_{solv} = \frac{k}{(r - \mu_p)} [V - V_B p_B(V)]$$

*(14)*

Following the same approach as for $V_{solv}$ , the value of the claim to the interest payments, $V_{int}$ is given by

$$V_{int} = (1 - \theta) \frac{C}{r} [1 - p_B(V)]$$

*(15)*

where C is the coupon.

The value of the shareholders, government and debtholders' claim is, respectively

$$E_{solv}(V) = V_{solv} - V_{int}$$

*(16)*

The optimal default barrier, $V_B$, is found invoking the smooth-pasting condition

$$\left.\frac{\partial E}{\partial V}\right|_{V=V_B} = 0$$

The optimal default barrier, $V_B$*, is given by

$$V_B^* = \lambda \frac{(1-\theta)C}{r},$$

*(17)*

where,

$$\lambda = \left(\frac{x}{x+1} \times \frac{(r-\mu_p)}{k}\right)$$

*(18)*

While the payout (i.e., k*V) already takes taxes into consideration, the coupon does not, as such it will need to be multiplied by $(1-\theta)$, where $\theta$ represents the corporate tax rate.

Please note, that in contrast with Goldstein, Ju, and Leland (2001) model, the reference model that is being implemented in this thesis, does not consider taxes over dividends, as this model's tax rate corresponds to the corporate tax rate.

The default probability, under measure Q, is given by,

$PD^Q(t^* \leq T) =$

$$= N\left(-\frac{\ln\left(\frac{V}{V_B}\right) + \left(\mu_p - 0.5\sigma^2\right)T}{\sigma\sqrt{T}}\right) + \left(\frac{V}{V_B}\right)^{-\frac{2(\mu_p - 0.5\sigma^2)}{\sigma^2}} N\left(-\frac{\ln\left(\frac{V}{V_B}\right) - \left(\mu_p - 0.5\sigma^2\right)T}{\sigma\sqrt{T}}\right)$$

*(19)*

### 3.2.Introduction of Accounting Noise

In this new setting, Duffie and Lando (2001) assume that after the issuing of the debt by the company, debtholders are not fully informed about the company' financial situation, while equity holders see no changes to their informational set. That is, creditors cannot observe with accuracy the value of the firm. Instead, they receive imperfect accounting data on the firm's asset value, from time to time. Formally, in log terms

$$\log(\overline{V_t}) = \log(V_t) + U_t, \tag{20}$$

where $\overline{V_t}$ is the project value debtholders observe, where log refers to the napier logarithm and $U_t$ is the noise term. The latter is considered to be normally distributed and independent of $\log(V_t)$. From now on, I will call $Y_t = \log(\overline{V_t})$ and $Z_t = \log(V_t)$.

Despite not having perfect information, they are aware that managers will choose to default as soon as $V_t$ reaches $V_B$ but they simply cannot observe how far $V_t$ is from $V_B$, making default an unpredictable event. Debtholders observe whether the firm has defaulted, though.

Hence, the information available in the secondary market at time t is

$$\mathcal{H}_t = \sigma\left(\left\{Y_{t1}, \dots, Y_{tn}, 1_{\{\tau \leq s\}}: 0 \leq s \leq t\right\}\right) \tag{21}$$

for the largest n such that $t_n \leq t$, where $\tau = \tau(V_B)$.

That is, the debtholders will continue to observe $Y_t$ at discrete times, until the firm has been liquidated (i.e., the moment where $\tau \leq s$).

For simplicity, Duffie and Lando (2001) assume that equity is not traded in public markets and that equity owners are precluded from trading in public debt markets. However, in order to implement the model, this assumption was excluded, as for simplicity, it was assumed that $Y_t$ would correspond to the market value of equity. Lastly, while the authors created extensions that include multiple observation times, in this thesis, I will only consider the simple case where debtholders have observed a single noisy observation at time $t = t_1$.

For the remaining of the section, the main goal will be to compute the probability of default from debtholders' point of view (i.e., taking into consideration the distribution of assets conditional on the information debtholders have and not the true distribution of assets). This is done in 4 steps. The first three steps correspond to the derivation of the distribution of assets conditional on what debtholders observe.

**Step 1:**

The authors proceed to compute the probability of the minimum value of Z between 0 and *t* being higher than zero, conditional on the current value of Z and the initial value of Z, i.e., the probability that the firm survives until t. This can be defined as follows

$$\psi(z_0, x, \sigma\sqrt{t}) = \Pr\left(\min(Z_s\colon 0 \le s \le t) > 0 \middle| Z_0 = z_0 \; and \; Z_t = x\right), \tag{22}$$

where $z_0$ is the value Z takes at time 0, and x is the value Z takes at time *t*. We will consider that bond holders know $z_0$ (one can think $z_0$ as the value coming from the last noise-free report) but don't know x (=$Z_t$). Additionally, notice that the probability in equation (22) does not depend on the drift of Z, $m = \mu_p - \frac{\sigma^2}{2}$, as it becomes irrelevant when we condition for x.

Duffie and Lando (2001) show that this probability is given by,

$$\psi(z_0, x, \sigma\sqrt{t}) = 1 - \exp\left(-\frac{2z_0 x}{\sigma^2 t}\right) \tag{23}$$

**Step 2:**

It is important to note that the probability given by equation (23) is not the one perceived by debtholders because debtholders cannot observe Z with accuracy. The default probability formula we are interested needs to take into consideration the conditional distribution of assets. As such, the authors also derive the probability density of Z taking a certain value *x* and default not having occurred yet, conditional on the observation of $Y_t$ (i.e., conditional on the current debtholders perception on the asset value). That is,

$$b(x|Y_t, z_0, t)dx = \Pr\left(\tau > t \; and \; Z_t \in dx | Y_t\right) \tag{24}$$

for $x > \underline{v}$ and where $\underline{v}$ is the log of the default barrier and $\tau = inf\{t\colon z_t \le \underline{v}\}$.

Using the definition of $\psi$ and Bayes' rule,

$$b(x|Y_t, z_0, t) = \frac{\phi_U(Y_t - x)\phi_z(x)}{\phi_Y(Y_t)}\psi\left(z_0 - \underline{\upsilon}, x - \underline{\upsilon}, \sigma\sqrt{t}\right) \tag{25}$$

where $\phi_U$ $\phi_z$ and $\phi_Y$ are the densities of $U_t$, $Z_t$ and $Y_t$ respectively.

These densities are Normal with the respective means,

$$E(U_t) = -\frac{a^2}{2} = \bar{\mu} \tag{26}$$

$$E(Z_t) = mt + z_0 \tag{27}$$

$$E(Y_t) = mt + z_0 + \bar{\mu} \tag{28}$$

And with the respective variances,

$$var(U_t) = a^2 \tag{29}$$

$$var(Z_t) = \sigma^2 t \tag{30}$$

$$var(Y_t) = a^2 + \sigma^2 t \tag{31}$$

where the standard deviation of $U_t$, a, can be interpreted as a measure of the degree of accounting noise.

Integrating equation (25) for all possible values that $Z_t$ can take at time t (above the barrier) one obtains the probability that the process has not hit the barrier, up to current time t, conditional on debtholders perception of the current asset value

$$P(\tau > t|Y_t) = \int_{\underline{\upsilon}}^{\infty} b(x|Y_t, z_0, t)dx$$

*(32)*

**Step 3:**

Using equations (25) and (32) as well as the Bayes' rule, the density of $Z_t$ conditional on the observation of $Y_t$ and default not having occurred (i.e., $\tau > t$) is given by:

$$g(x|y, z_0, t) = \frac{b(x|y, z_0, t)}{\int_{\underline{\upsilon}}^{+\infty} b(x|y, z_0, t)dz}.$$

*(33)*

Letting $\tilde{y} = y - \underline{\upsilon} - \bar{\mu}$ , $\tilde{x} = x - \underline{\upsilon}$, and $\tilde{z}_0 = z_0 - \underline{\upsilon}$, we can re-write equation (33) as follows,

$$g(x|y, z_0, t) = \frac{\sqrt{\frac{\beta_0}{\pi}}\, e^{-J(\tilde{y}, \tilde{x}, \tilde{z}_0)}\left[1 - \exp\left(\frac{-2\tilde{z}_0\tilde{x}}{\sigma^2 t}\right)\right]}{\exp\left(\frac{\beta_1^2}{4\beta_0} - \beta_3\right)\Phi\left(\frac{\beta_1}{\sqrt{2\beta_0}}\right) - \exp\left(\frac{\beta_2^2}{4\beta_0} - \beta_3\right)\Phi\left(-\frac{\beta_2}{\sqrt{2\beta_0}}\right)}$$

*(34)*

where

$$J(\tilde{y}, \tilde{x}, \tilde{z}_0) = \frac{(\tilde{y} - \tilde{x})^2}{2a^2} + \frac{(\tilde{z}_0 + mt - \tilde{x})^2}{2\sigma^2 t}$$

*(35)*

For

$$\beta_0 = \frac{a^2 + \sigma^2 t}{2a^2\sigma^2 t}$$

*(36)*

$$\beta_1 = \frac{\tilde{y}}{a^2} + \frac{\widetilde{z_0} + mt}{\sigma^2 t}$$

(37)

$$\beta_2 = -\beta_1 + 2\frac{\widetilde{z_0}}{\sigma^2 t}$$

(38)

$$\beta_3 = \frac{1}{2}\left(\frac{\tilde{y}^2}{a^2} + \frac{(\widetilde{z_0} + mt)^2}{\sigma^2 t}\right)$$

(39)

and where Φ is the standard-normal cumulative distribution function.

From equation (34) one can easily obtain the distribution of assets conditional on the debtholder's perception of the current market value of assets and the initial accounting report. For this, it is enough to multiply equation (34) by the derivative of the x. As $x = \log{(V_t)}$, this corresponds to $\frac{1}{e^x}$ . That is,

$$g(V_t|y, z_0, t) = g(x|y, z_0, t)\frac{1}{e^x}$$

(40)

A more detailed explanation can be found in Appendix A.

**Step 4:**

Finally, debtholders perception on the probability of the firm not defaulting up to some future time s (in other words, surviving until time s) at a given time t is given by:

$$p(t,s) = \int_{\underline{v}}^{\infty}\left(1 - \pi\left(s - t, x - \underline{v}\right)\right) g\left(x\middle|Y_{t,z_0,t}\right)dx,$$

(41)

where $\pi(t,x)$ denotes the probability of first passage of a Brownian motion with drift m and volatility parameter $\sigma$ from an initial condition x > 0 to a level below 0 before time *t*. This integral does not have a closed form, so it has to be calculated through a numerical integration.

From expression (11) on page 14 of Harrison (1985), and substituting y by $(x - \underline{v})$, $\mu_p$ by m and *t* by (s − t), we have the following

$$\pi(s-t, x-\underline{v}) = 1 - \Phi\left(\frac{(x-\underline{v}) + \left(\mu_p - \frac{\sigma^2}{2}\right) \times (s-t)}{\sigma\sqrt{(s-t)}}\right) + \exp\left(-\frac{2\left(\mu_p - \frac{\sigma^2}{2}\right)(x-\underline{v})}{\sigma^2}\right)\Phi\left(\frac{-(x-\underline{v}) + \left(\mu_p - \frac{\sigma^2}{2}\right) \times (s-t)}{\sigma\sqrt{(s-t)}}\right) \tag{42}$$

The default probability between t and s > t, for this setting, is given by

$$1 - p(t, s). \tag{43}$$

For both the standard model and the model that revokes the perfect information assumption, the following formula for the distance to default applies,

$$DD = \frac{log\left(\frac{V}{V_B}\right) + \left(\mu_P - k - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \tag{44}$$

### 3.3.Numerical Illustration

In this subsection, I replicated three figures presented in Duffie and Lando (2001) as they are useful to illustrate the model. Duffie and Lando (2001) consider the following parameters as their base case:

$$\theta = 0.35;\ \ \sigma = 0.05;\ \ r = \ 0.06;\ \ k = 0.05;\ \ \mu_p = 0.01125;\ \ C = 8$$

Figure 1 illustrates how the conditional distribution of assets for a given company varies for different levels of accounting precision. Duffie and Lando assume that an accurate asset report of $\hat{V}(t-1) = V(t-1) = 86.3$ was received a year ago. Moreover, the current asset value observed by creditors, $\hat{V}$, has an outcome equal to the last accurate report, 86.3.

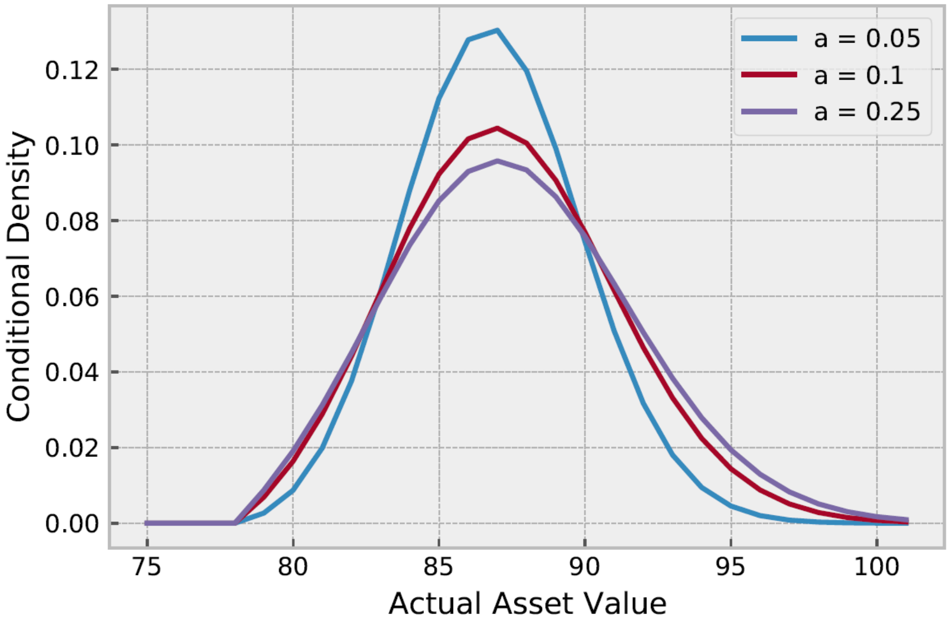


***Figure 1 - Conditional density for varying accounting precision.***

From Figure 1, we can assess that as we increase the degree of accounting noise, the tails of the conditional distribution will become heavier, that is the likelihood that the actual asset value will take extreme values is increasing. In addition, the likelihood, that the actual asset value takes a similar value to the current asset value creditors observe, that is 86.3, is decreasing. Surprisingly, the difference between the distributions with a degree of accounting noise of 5% and 10% is much more significative than when we compare the distributions with a degree of accounting noise equal to 10% and 25%.

Figure 2 presents how the conditional asset density is affected by the lagged asset report. Similarly, to figure 1, for the all the three scenarios, the last accurate report was provided a year ago, and the current asset value $\hat{V}$, is equal to the asset value reported in the previous year report (that is to $V_0$). For the accounting noise, it was assumed Duffie and Lando (2001), standard case, that is, 10%.

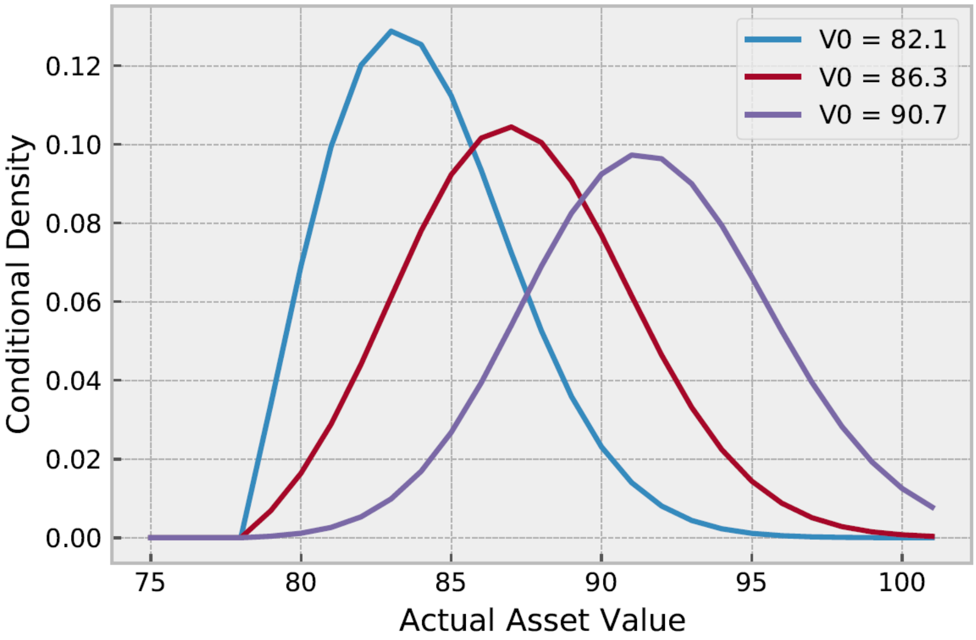


***Figure 2 - Conditional asset density, varying previous year asset value.***

From Figure 2, we can observe that as the asset value of the previous year increases and simultaneously the current value that creditors observe also increases (as $\hat{V}(t) = V(t-1)$), the uncertainty around the actual asset value is also increasing.

Figure 3 illustrates the outcomes of the conditional default probability for the base-cases, for various time horizons and various levels of accounting noise.

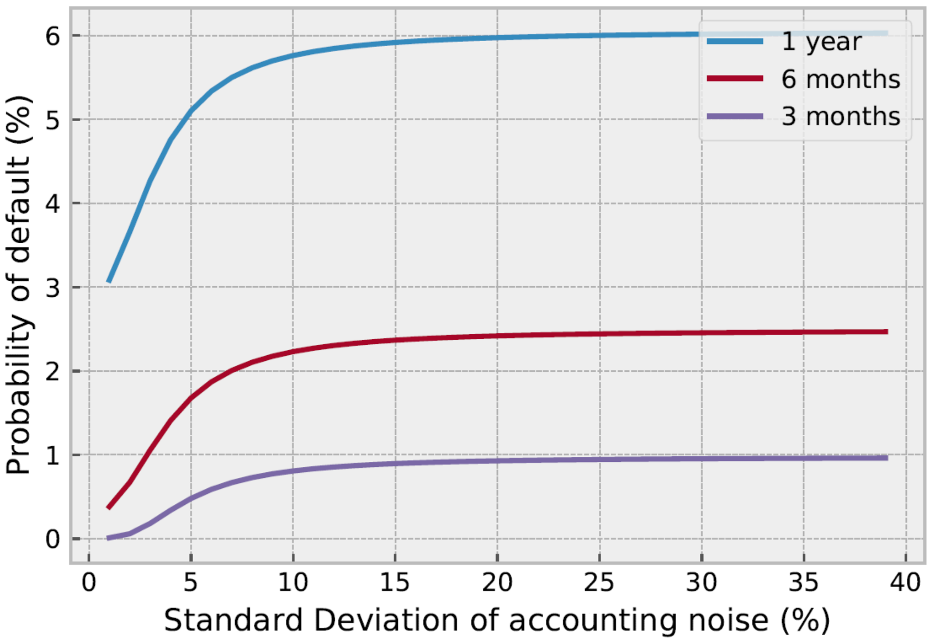


***Figure 3 - Default probability, varying accounting precision.***

From Figure 3, we can observe that as the degree of accounting noise increases, the probability of default also increases. In addition, similarly, to Figure 1, there is a bigger difference when comparing the probabilities of default for a degree of accounting noise of 5% and 10%, and as the degree of accounting noise starts taking values above 10%, the difference in the probability of default becomes marginal. Lastly, as we increase the time horizon, the probability of default also increases.

These three figures correspond to figure 2, 3 and 4, respectively, in Duffie and Lando (2001).

## 4. Calibration Strategy

In order to estimate the probability of default perceived by debt holders, I had to calibrate the asset value perceived by debt holders (corresponding to the value of Y), the asset volatility (or as it has been mentioned, the volatility, σ) and the project ratio (k). These three variables were calibrated using the iterative approach, that is explained in detail in this section.

### Iterative Approach

To calculate the asset volatility, I employed a similar approach to the one proposed by Crosbie and Bohn (2003) and Vassalou and Xing (2004):

1. Define a level of convergence. Same as Vassalou and Xing, the level of convergence was set to be 10E-4.
2. Set an initial value for σ and k. The initial value of σ was set at 20%[1]. The initial estimate of k was set at 10%.
3. For each week, estimate the value of assets, by using the equity valuation formula, equation (16), with the equity value corresponding to the market value of equity of that week.
4. Compute a new guess for σ by:
    a. Computing the log returns based on asset prices for each week.
    b. Calculating the annualized standard deviation of the log returns.
5. Repeat steps 3 to 5 until the estimated σ from two consecutive iterations converges.

For the calibration of k, a similar approach can be undertaken, by following the same approach as my colleagues Lukas Weisel, and Simen Madsen. Their approach is as follows:

1. Same as the previous approach, the tolerance level of convergence was set to 10E-4.
2. Set an initial estimate of k. The initial estimate of k was set at 10%[2].
3. Implement the Vassalou and Xing (2004) algorithm in order to find σ.
4. Estimate the value of k as the average of EBIAT[3] divided by the asset values obtained in step 3. That is,

[1] Changing the value of initial estimate of the asset volatility does not change its final value.
[2] Changing the value of initial estimate of k will not change the final value of k.
[3] Duffie and Lando (2001) only state that the kV corresponds to the firm's cash flow rate. Nevertheless, as this value is not being multiplied by any tax rate, it is plausible to assume that the state variable already considers taxes. As such, this model' state variable is assumed to be the Earnings Before Interest and After Taxes (EBIAT).

$$k = \frac{\sum_{t=1}^{T} \frac{EBIAT_t}{V_t}}{T}$$

*(45)*

5. Repeat steps 3 and 4 until the value of k obtained from two consecutive iterations is below the tolerance level of convergence.

Lastly, we can use the obtained variables to find the asset value perceived by the creditors.

## 5. Data

This section presents the data used to implement the model proposed by Duffie and Lando (2001). Section 4.1 describes the dataset. Section 4.2 explains how each model parameter was calibrated.

### 5.1. Data Set

The companies considered in this dissertation were chosen from the Euro STOXX 50 Index. The time period considered spans from 2010 to 2020. The list of the constituents and their respective accounting data was retrieved from Refinitiv Datastream. In order to make it comparable over the entire timespan, companies that were not present in the index in all periods under analysis, were excluded. Additionally, financial companies (i.e., companies with the NAICS Sector code of 52) and companies defined as "NonClassifiable Establishments" (i.e., companies with NAICS Sector code of 99) were also excluded. The reason for the prior exclusion, is due to the financial structure, financial companies typically present. That is, the high leverage that is typical for these companies does not have the same meaning as for non-financial companies, where this high leverage would likely indicate distress. All these data restrictions resulted in a sample of 18 companies.

In the second phase of the data treatment, all companies where the average EBIAT was negative were eliminated from the sample. This restriction is related with the fact that the geometric Brownian motion never takes negative values. In addition, companies with zero-interest expense in at least one of the periods were also eliminated. This restriction follows from the fact that without interest expenses there is no endogenous default barrier. None of these data constraints lead to any exclusion, as such, the final sample is composed of the previously mentioned 18 companies.

The sector and regional distribution of the final sample is outlined in Table 1.

**Table 1: Sample Description**

| **Panel A: Companies by Sector** | Total | % |
|---|---|---|
| Construction | 1 | 5.56 |
| Information | 2 | 11.11 |
| Manufacturing | 13 | 72.22 |
| Utilities | 2 | 11.11 |
| **Total** | 18 | 100.00 |

| **Panel B: Companies by Country** | Total | % |
|---|---|---|
| Belgium | 1 | 5.56 |
| Germany | 6 | 33.33 |
| Spain | 1 | 5.56 |
| France | 7 | 38.89 |
| Italy | 2 | 11.11 |
| Netherlands | 1 | 5.56 |
| **Total** | 18 | 100.00 |

Numbers might not add up due to rounding.

A detailed description of all firms can be found in Appendix B.

## 5.2. Model Inputs

### 5.2.1. EBIAT and the coupon

The company specific EBIT and interest expenses with debt were retrieved from Refintiv DataStream. The EBIAT specific of each firm was obtained by multiplying the EBITs by $(1-\theta)$. The interest expenses on debt (i.e., the coupon) is assumed to be constant, while the EBIAT will vary throughout time. Given that these two variables are gathered from firms' annual financial statements, these are yearly values. In order to adapt these variables to weekly data, I used interpolation. The interpolation process consisted of two steps. The initial step was to construct start of the year values for each year as the yearly EBIAT or interest expenses on debt. Finally, the weekly values were constructed using a linear interpolation, that is, by drawing a straight line between the start of the year values.

### 5.2.2. Interest rate

For the period under analysis, i.e., from 2010 to 2020, as a proxy of the interest rate used in the model, a time series of the 30-year Government German Bonds was used. For that, the yields were downloaded from Refintiv DataStream. Contrarily to Goldstein, Ju, and Leland

(2001), the model being implemented does not take into consideration the interest tax rate, as the only tax rate considered is the corporate tax rate. As such, the interest rate considered will be the before-tax interest rate.

### 5.2.3. Corporate tax rate

Given that one of the underlying assumptions of the model is a constant tax rate and that each country within the Euro Zone has its own fiscal policy, the corporate tax rate was set to be 35%, as it is commonly assumed in the literature.

### 5.2.4. Asset volatility

Asset volatility is one of the three variables obtained from the iterative approach described in Section 4.1. In summary, this approach is as follows, after setting an initial guess of 20% for each company, for every week of the period under analysis, the asset values will be computed using this estimate. Following this, the log returns of these new asset values will be obtained. The new estimate of standard deviation will correspond to the annualized standard deviation of the log returns. This process will be repeated, replacing the previous estimate with the new one, until two iterations converge. Please note, the asset volatility is kept constant for each company throughout the entire timespan.
The asset volatilities for the entire sample can be found in Appendix C.

### 5.2.5. Project ratio

Similarly, to the asset volatility, the project ratio was extracted from the iterative approach outlined in Section 4.1. In summary, this approach is as follows, after setting an initial guess of 10% for each company, the new asset values will be computed. Following this, a new estimate for the project ratio will be computed using Equation (45). This process will be repeated, replacing the previous estimate with the new one, until two iterations converge. Please note, the project ratio is kept constant for each company throughout the entire timespan.
The project ratios for the entire sample can be found in Appendix C.

### 5.2.6. Project value (V)

The project value was also found through the iterative approach, following the estimation of the asset volatility and project ratio. To start, the market value of equity, for each company during the timeframe under analysis, was downloaded from Refinitiv DataStream on a weekly basis. Following that, the parameters in equation (16) were replaced with the data previously retrieved and that non-linear equation was solved having the project value as the incognita.

### 5.2.7. Degree of accounting noise

Several degrees of accounting noise were considered, namely, 5%, 10% (the standard case of Duffie and Lando (2001)), and 25%, as there isn't empirical evidence at the time of the writing that there is a reasonable level of accounting noise. Adding to that, this value would also presumably vary with the nature of the company.

### 5.2.8. Last noise-free report ($Z_0$)

Similarly, to Duffie and Lando (2001), the last noise-free report was assumed to have been provided one year ago, i.e., in $(t-1)$.

## 6. Results

This section is divided in two parts. The first part discusses the values obtained for the default probabilities[4] assuming perfect information (i.e., a = 0) and assuming a degree of accounting noise of 5%, 10% and 25%. The second part compares the obtained results with the ones implied by the credit ratings given by Standard & Poor's Global Ratings.

### 6.1. Estimated default probabilities

The default probabilities were computed at the 5-year range, under the neutral measure. This choice results from the Wiener process not presenting a significant variation in short periods, potentially leading to extreme results (i.e., either very high or very low default probabilities). All calculations were done on a weekly frequency. This choice results from balancing the benefits of having more data points and the fact that the normal distribution tends to be more representative of log returns for lower frequencies. The latter led me to exclude using daily data.

Figure 4 presents the average default probability for the 18 companies at each point in time with varying degrees of accounting noise. In this figure, four degrees of accounting noise are considered. That is for a=0 it is assumed perfect information, as such equation (19) was used. For the remaining lines, it is assumed that the noise volatility equals 5%, 10% and 25%. These are computed using equation (43) which requires numerical integration. Please note, that for some cases where the degree of accounting noise was 5%, 10% and 25%[5], the integral was not able to converge, as such for those cases, it was assumed that the default probability corresponds to the average of the two nearest converging data points.

[4] While the time period considered for this dissertation is the one from 2010 to 2020, the default probabilities were only computed for the 2011-2020 period, as it was needed at least one year of data given that the last noise free report was assumed to be provided at t-1.
[5] For this degree of accounting noise, this was only verified for one company and for a small number of cases.

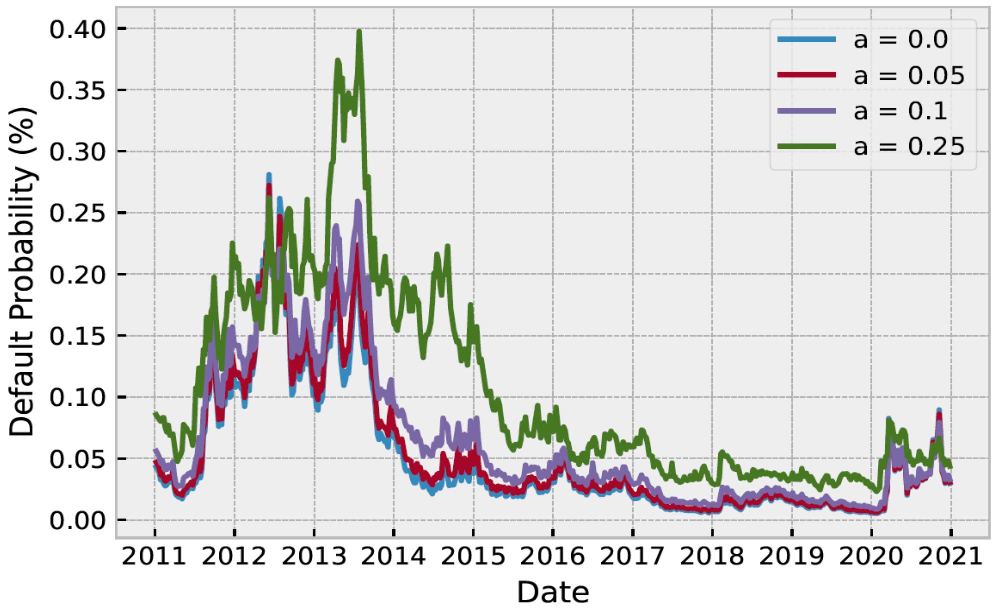


***Figure 4 – Sample probability of default, varying the accounting precision***

From Figure 4 we can observe that from mid-2011 to late 2013, there is a period of increased default likelihood. This was followed by a period, from 2014 to 2015, where the default probabilities continuously decreased. From 2016 onwards, the probabilities of default stabilized at relatively low levels, until early 2020 where the pandemic caused by the worldwide dissemination of the Covid-19 emerged. In this last period, there was a significant disruption in global supply chains, affecting companies' financial position and contributing to an increase in default probabilities. The increase in default probabilities during the first period should be related with the European sovereign debt crisis that drastically affected some of the most indebted Eurozone economies. Though only 3 companies in the sample (two based in Italy and one based in Spain) belong to the group of the most affected countries (i.e., countries that experienced more problems associated with the credit risk of their financial institutions and their sovereign debt), this crisis had effects in the entire currency union, as it led to concerns about a possible break-up of the eurozone.

From Figure 4, we can conclude that overall, the 5-year default probabilities increase as we increase the degree of accounting noise. In other words, the probabilities of default in general behave monotonically when it comes to noise.

Additionally, we observe that by increasing the accounting noise, especially in the case a=25%, the peaks in 2013 are amplified.

Figure 5 explores the impact of the accounting noise for different levels of default probability. This is done at the firm level for the case a=0.25. In particular, Figure 5 presents the difference between the default probabilities assuming an accounting noise degree of 25% and perfect information (i.e., PD(a = 25%) – PD(a = 0.0%)). Based on this figure, we can conclude that there is a positive correlation for probabilities of default up to 0.15%. For default probabilities between 0.15% and 0.25% we start to observe an overall negative correlation[6].

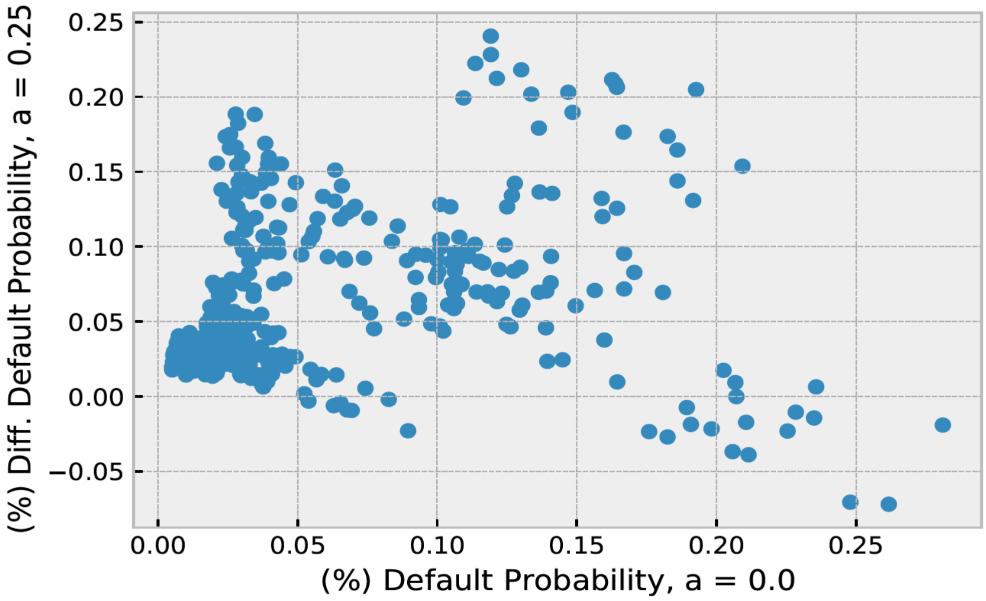


***Figure 5 – Relationship between probabilities of default with a =25% and 0.0%***

Figure 6 presents the difference between the default probabilities assuming an accounting noise degree of 25%, 10% and 5%, and the default probabilities with a degree of accounting noise of 0%.

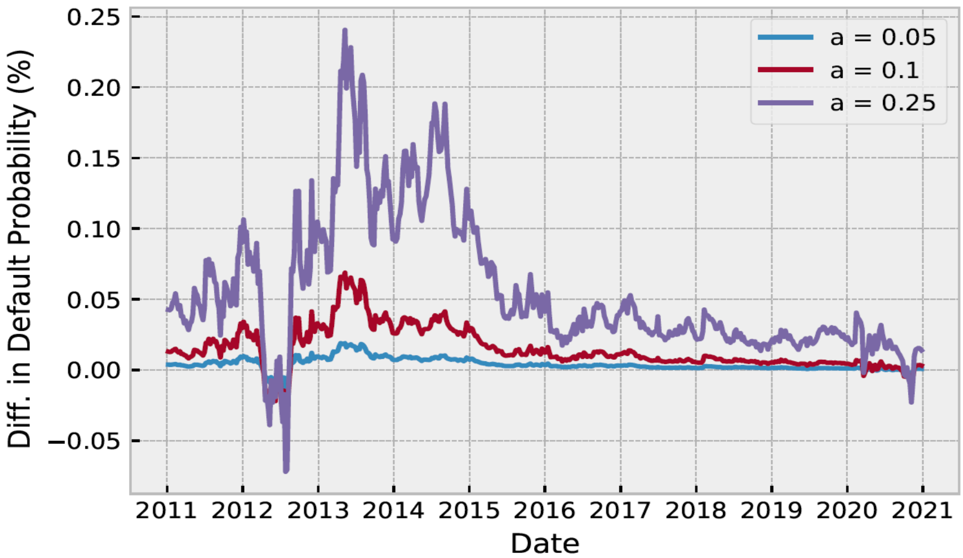


***Figure 6 – Relationship between the probabilities of default***

[6] This is due to some the estimated values of two companies.

From Figure 6, we can also observe that contrasting with the previously referred behaviour, in some periods marked by peaks, the difference between the PDs with an accounting noise of 25% and 0% actually present a lower absolute variation than the remaining ones.

## 6.2.Distance to default

Since Merton (1974) proposed his model, the distance to default has become one of the most well-known credit risk metrics. This metric provides the number of standard deviations the market value of assets needs to decrease in order to reach the default barrier, moment where default is triggered. Figure 7 presents the average of the "Merton equivalent" 5-year distance to default, throughout the time period under analysis. Not surprisingly, DDs in Figure 7 present the opposite behaviour to the PDs in Figure 4, although the default probabilities do present more abrupt movements as they increase non-linearly as they get closer to the default barrier.


***Figure 7 – Sample distance to default***

## 6.3.Comparison with credit ratings

Credit ratings are a measure of borrower's creditworthiness. While credit ratings reputation has been affected by the 2007-2008 Financial Crisis, overall, they are still seen by the market as the most credible assessment of borrowers' riskiness. Thus, they can be interpreted as a benchmark when evaluating a firm's probability of default.

The credit rating industry is currently dominated by three credit agencies. They are Standard & Poor's Global Ratings (S&P), Moody's, and Fitch Group. For the purpose of this dissertation, the 5-year obtained probabilities of default were compared with the S&P credit ratings for long term debt (corresponding to the S&P Long-term Issuer Rating [SPI]), due to the lack of data availability of Moody's and Fitch ratings for some of the sample companies.

Since the 1970s, credit ratings agencies have switched to an "issuer-pay" model. In this model, it is the debtor who pays the rating agencies for the assessment of their one creditworthiness, which is later on publicly made available. As a result, in some situations companies do not see the benefits of requesting this assessment, which leads to a lack of accessible data. In the sample analysed in this dissertation, this issue was verified in the case of L'Oréal, which was thus not included in this section. In addition, the credit rating of SAP SE only started being available in 2014, as such it was also excluded. Subsequently, all the remaining credit ratings for the time period of 2011 to 2020, were downloaded from the Thomson Reuters Eikon database and converted into probabilities of default using an historical transition matrix (that is set out in Appendix D). Figure 8 shows the evolution of the percentage of companies with a certain credit rating throughout the 2011-2020 period.

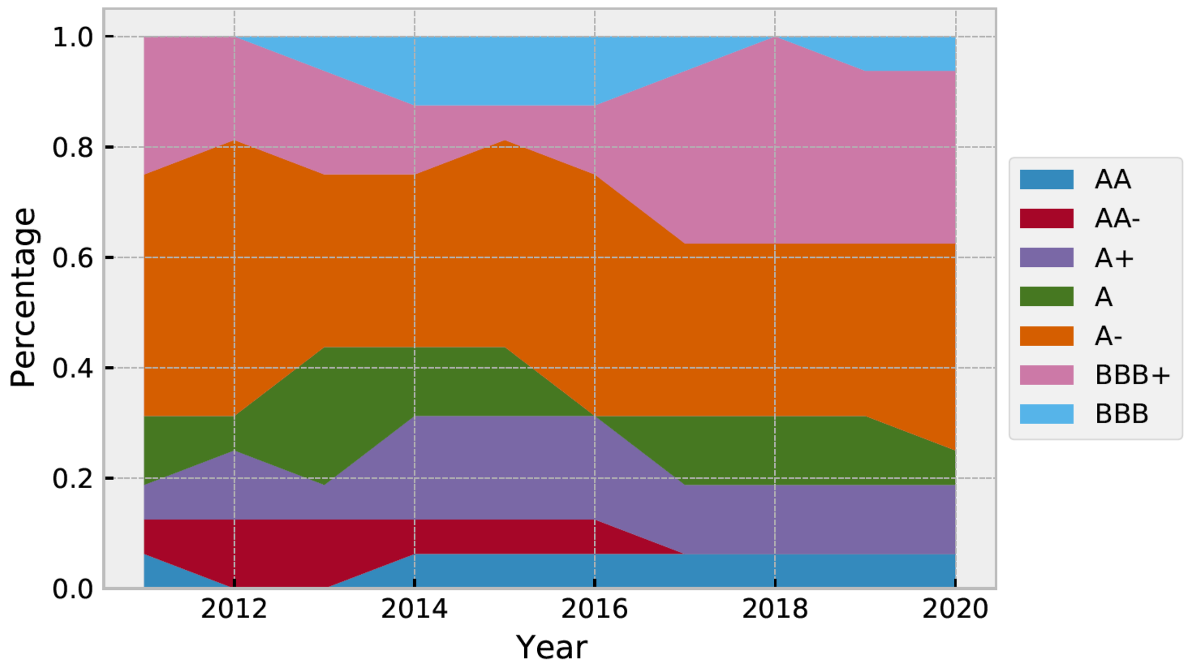


***Figure 8 – Distribution of credit ratings per year***

From Figure 8, we can assess that all individual ratings are investment grade. The quality of the long-term debt issued by these companies presented marginal variations in the 2011-2020 period, being the A- rating the predominant throughout most periods. While there is a significant proportion of upper medium grades (from A- to A+), there is a reduced number of high grades (from AA- to AA+). Additionally, the lower rating present in this sample is BBB.

Figure 9 presents the distance between the default probabilities assuming the previously mentioned 4 different degrees of accounting noise, 0%, 5%, 10% and 25%, with the ones implied by S&P credit ratings.

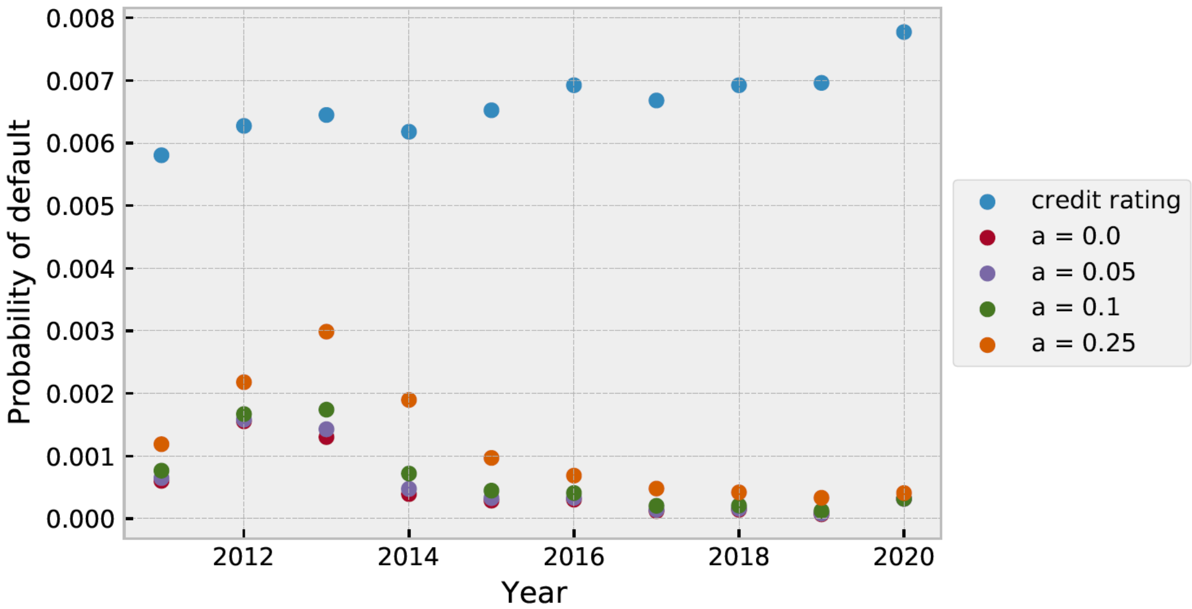


***Figure 9 – Credit Rating implied default probabilities and model comparison***

From Figure 9, we can conclude that the model' results are underestimating the probabilities of default, although the absolute difference is reduced (varying from 0.46% to 0.75%). As presented in Figure 4, as we increase the assumption of the degree of accounting noise, the estimated probabilities increase. Figure 10 presents the distance between the distance to default obtained assuming the previously referred 4 different degrees of accounting noise, with the ones implied by S&P credit ratings.

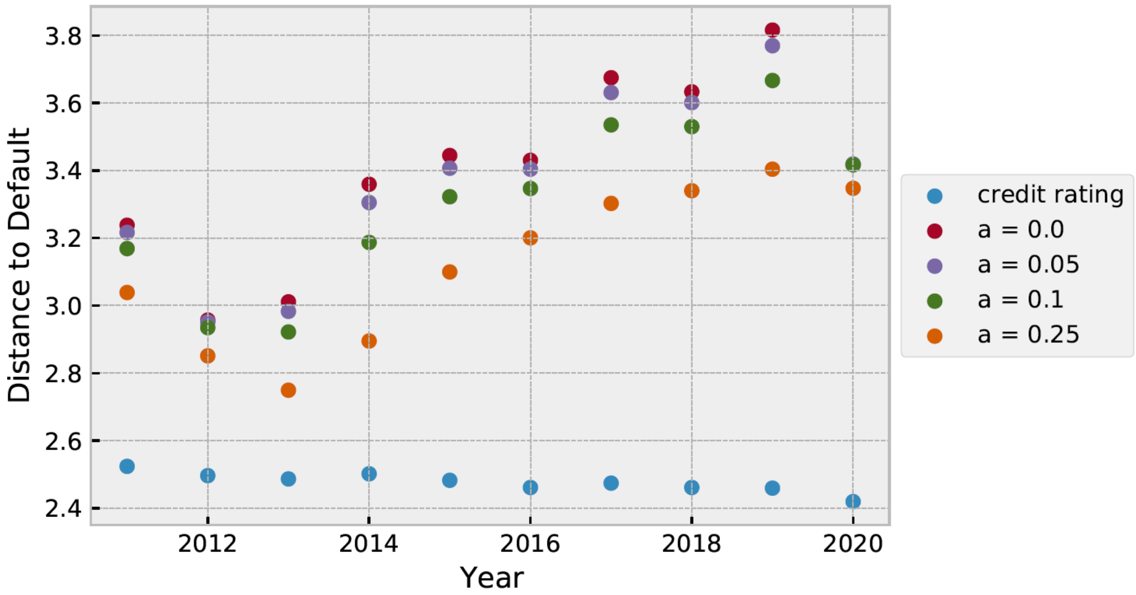


***Figure 10 – Credit Rating implied distance to default and model comparison***

Similarly, to Figure 9, from Figure 10 we can conclude the model results are underestimating the credit measures implied by the credit ratings.

## 7. Conclusion

This dissertation aimed to study how default probabilities are affected by the quality of accounting information using a structural contingent claims model. As such, Duffie and Lando (2001) model was implemented and used to estimate 5-year default probabilities assuming four degrees of accounting noise, i.e., 0%, 5%, 10% and 25%.

It was concluded that default probabilities overall behave monotonically when it comes to the degree of accounting noise. That is, the probabilities of default increase as we increase the assumption of the degree of accounting noise.

My default probability estimates capture the most relevant extreme events that marked the period under analysis, notably, the European sovereign debt crisis and covid-19 pandemic. Those events tend to have a more drastic effect when considering a noise volatility of 25%. Even so, this is not always the case. There are certain peaks, such as the one in late 2012 and late 2020, where the behaviour observed is the opposite.

When estimating the "Merton equivalent" 5-year distance to default, the same conclusions are drawn, even though the default probabilities do present more abrupt movements when compared to the distance to default estimates.

Further, the results were compared with the ones implied by S&P credit ratings. Independently of the accounting noise level, the model implied default probabilities and credit rating implied credit risk measures were found to be relatively close. Even so, the model underestimates slightly the probabilities of default implied by credit ratings with the absolute difference ranging between 0.46% and 0.75%.

Notwithstanding, this study has some limitations. The first set of limitations derives from the underlying assumptions of the model, while the second one derives from the methods used to implement this model.

From the first set of limitations, we have that as the implemented model does not allow for negative EBIATs, we are this way excluding companies in early stages of development or companies operating on negative EBIATs. Further, the assumption of a geometric Brownian motion does not allow for jumps, which is widely observed in real life; the model assumes the debt to be perpetual, which is not the case for most companies; and the assumption of a constant tax rate, might not be well adjusted to most companies' reality. Lastly, the model requires an assumption for the degree of accounting noise and an assumption for the last noise-free report, where there is a lack of efficient methods to access such assumptions.

For the assumptions stemming from the implementation of the model, we have five main ones. First, the sample used is overall small with an over-representation of the Manufacturing sector. Further, with the exclusion of companies that were not present in the Euro STOXX 50 Index throughout the entire period under analysis, we might be excluding companies that went bankrupt, thus leading to a survivorship bias. In addition, different methods could have been used as an alternative to interpolation or to the iterative approach, possibly leading to different estimates for the model inputs. Finally, the method used to calculate the integral presents some numerical limitations, that were previously referred.

## 8. Appendices

### Appendix A – Proof of equation (40)

Let V be a random variable whose probability density function is f($V$).

By definition:

$$P(a \leq V \leq b) = \int_a^b f(V)dV$$

Any function of a random variable is itself a random variable. Let $x(V) = \log(V)$ and $V(x) = e^x$, where log refers to the napier logarithm. If $a \leq V \leq b$ then one must have that $a \leq e^x \leq b$, $\log(a) \leq x \leq \log(b)$, and $P(\log(a) \leq x \leq \log(b)) = P(a \leq V \leq b)$.

From the transformation theorem it is know that

$$\int_a^b f(V)dV = \int_{\log(a)}^{\log(b)} f(V(x))\frac{dV}{dx}dx,$$

where the term in the integral, which is expressed exclusively in terms of x, is now the density function of x. Since $\frac{dV}{dx} = e^{x}$ ,

$$P(a \leq V \leq b) = P(\log(a) \leq x \leq \log(b)) = \int_{\log(a)}^{\log(b)} f(e^{x})e^{x}dx$$

In the main text the density of x is denoted as g(x). One must have that

$$g(x)=f(e^{x})e^{x}$$

and thus

$$f(e^{x}) = \frac{1}{e^x}g(x)$$

The density of each possible value of V can thus be computed by first computing $x(V) = \log(V)$, evaluating its density using function $g(x)$ and then multiplying by $\frac{1}{e^x}$.

## Appendix B – Detailed sample description

**Table 2: Sample**

| Company Name | Sector | Country |
|---|---|---|
| KONINKLIJKE PHILIPS NV | Manufacturing | Netherlands |
| BASF SE | Manufacturing | Germany |
| DANONE SA | Manufacturing | France |
| MERCEDES-BENZ GROUP AG | Manufacturing | Germany |
| TOTALENERGIES SE | Manufacturing | France |
| ENI SPA | Manufacturing | Italy |
| BAYER AG | Manufacturing | Germany |
| L'OREAL SA | Manufacturing | France |
| L'AIR LIQUIDE SA | Manufacturing | France |
| SANOFI | Manufacturing | France |
| ENEL SPA | Utilities | Italy |
| DEUTSCHE TELEKOM AG | Information | Germany |
| IBERDROLA SA | Utilities | Spain |
| SAP SE | Information | Germany |
| SCHNEIDER ELECTRIC SE | Manufacturing | France |
| VINCI SA | Construction | France |
| BAYERISCHE MOTOREN WERKE AG | Manufacturing | Germany |
| ANHEUSER-BUSCH INBEV | Manufacturing | Belgium |

## Appendix C – Sample Asset Volatilities and Project Ratios

**Table 3: Asset volatility and project ratio**

| Company Name | Asset Volatility (%) | Project Ratio (%) |
|---|---|---|
| KONINKLIJKE PHILIPS NV | 23.8 | 4.6 |
| BASF SE | 23.0 | 6.7 |
| DANONE SA | 20.2 | 4.1 |
| MERCEDES-BENZ GROUP AG | 20.0 | 3.8 |
| TOTALENERGIES SE | 25.8 | 4.4 |
| ENI SPA | 17.1 | 6.2 |
| BAYER AG | 17.0 | 4.0 |
| L'OREAL SA | 17.3 | 4.5 |
| L'AIR LIQUIDE SA | 20.1 | 5.2 |
| SANOFI | 22.2 | 6.8 |
| ENEL SPA | 17.2 | 3.7 |
| DEUTSCHE TELEKOM AG | 22.6 | 2.7 |
| IBERDROLA SA | 23.9 | 5.1 |
| SAP SE | 24.1 | 1.4 |
| SCHNEIDER ELECTRIC SE | 26.7 | 11.1 |
| VINCI SA | 29.8 | 9.8 |
| BAYERISCHE MOTOREN WERKE AG | 14.9 | 3.5 |
| ANHEUSER-BUSCH INBEV | 23.5 | 3.5 |

**Appendix D – Conversion matrix**

**Table 4: Credit Ratings Conversion Matrix**[7]

| Rating | 5-Year Probabilities of Default (%) |
|---|---|
| AAA | 0.15 |
| AA+ | 0.32 |
| AA | 0.35 |
| AA- | 0.36 |
| A+ | 0.43 |
| A | 0.48 |
| A- | 0.54 |
| BBB+ | 0.97 |
| BBB | 1.36 |
| BBB- | 2.77 |
| BB+ | 3.69 |
| BB | 6.17 |
| BB- | 9.27 |
| B+ | 14.15 |
| B | 17.09 |
| B- | 25.43 |
| CCC | 46.06 |

[7] Data from S&P 2018 annual corporate default study and rating transition report.

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