NOTE ON THE SOLVABILITY OF THE FULLY FOURTH ORDER NONLINEAR BOUNDARY VALUE PROBLEMS

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1. Introduction

For the existence and uniqueness of solutions for the fourth order two point boundary value problems, many researchers have studied, see [1, 8, 11, 14, 15, 16, 21, 22, 23, 19]. The proof is carried out using the method of Leray?-Schauder degree theory [18] or the Schauder fixed point theorem on the base of using the method of lower and upper solution [3, 5, 6, 17] or Fourier analysis [13]. It should be noticed that the following boundary value problems for the fully nonlinear fourth order equation with boundary conditions were studied in several authors in recent years, see [12, 10, 7, 17, 4],

$$\begin{cases} u''''(t) = f(t, u(t), u'(t), u''(t), u'''(t)), \\ u(0) = u'(0) = u''(1) = u'''(1) = 0, \end{cases} \tag{1.1}$$

where f is a continuous nonlinear mapping of $[0,1] \times \mathbb{R}^4$ into $\mathbb{R}$. It should be emphasized that in all the above works there is an essential assumption that the function f satisfies a Nagumo-type condition. In [20], this condition be freed by them due to considering f only in a bounded domain. In paticular, authors in [20] show that some examples demonstrate the applicability of the proposed approach and iterative method. As the examples of boundary value problem (1.1) where f is a continuous nonlinear mapping of $[0,1] \times \mathbb{R}^3_+ \times \mathbb{R}_-$ into $\mathbb{R}_+$, they give nonlinear functons $f(t.u,y,v,z) = -\frac{3yz}{1152} + \frac{\sqrt{v^2}}{576} + \frac{t}{4} + \frac{9}{54}$, $\frac{u}{6}(u+y+v-z)+1$, $\frac{u+y}{1+u^2+y^2+v^2+z^2} + e^{-u^2}$, $?\frac{z}{24} + uv + \frac{y}{8} + \frac{1}{2}$, which satisfy Theorem 2.2, 2.3 in [20]. Moreover they give also the function $f(t.u,y,v,z) = u^2+|y|^{\frac{1}{2}}+|v|^{\frac{1}{2}}+|z|^{\frac{1}{2}}+\pi sin(\pi t)$, which does not satisfy Theorem 2.2, 2.3 in [20]. In this paper we proposed an one way of investigstion of method to obtain the positivity solution of boundary value problem (1.1). In particular we show that method of this paper is appricable for the function $f(t,u,y,v,z) = u^2 + |y|^{\frac{1}{2}} + |v|^{\frac{1}{2}} + |z|^{\frac{1}{2}} + \pi sin(\pi t)$ where f is a continuous nonlinear mapping of $[0,1] \times \mathbb{R}^3_+ \times \mathbb{R}_-$ into $\mathbb{R}_+$. The paper is organized as follows. In Section 2, we consider the existence and uniqueness of a solution of the problem (1.1) with its positivity. Section 3 is devoted to some examples, including ones to which the condition of theorems in [20] cannot be applied, for demonstrating the applicability of our approach. Finally, Section 4 is conclusion. Throughout this paper, we denote by $\mathbb{R}$ the set of all real numbers. A function $u \in C[0,1]$ is called a solution of problem (1.1) if $u'''' \in C[0,1]$, $u''' \in L^1[0,1]$, $u'' \in L^1[0,1]$, $u' \in L^1[0,1]$, and u satisfies the boundary conditions and equality in (1.1) a.e. on [0, 1]. The

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physical meaning of the derivatives of the deformation function $u(t)$ is as follows: u'''' is the load density stiffness, u''' is the shear force stiffness, u'' is the bending moment stiffness, and u' is the slope; see [1, 2, 8, 9].

2. Main Result

In this paper we consider the boundary value problem (1.1). For $u \in C[0,1]$, we set

$$\varphi(t) = f(t, u(t), u'(t), u''(t), u'''(t)).$$

The problem becomes

$$\begin{cases} u''''(t) = \varphi(t), 0 < t < 1, \\ u(0) = u'(0) = u''(1) = u'''(1) = 0. \end{cases}$$

It has a unique solution

$$u(t) = \int_0^1 G(t,s)\varphi(s)ds,$$

where

$$G(t,s) = \begin{cases} \frac{1}{6}\left(-s^3 + 3ts^2\right), \text{ if } 0 \le s \le t, \\ \frac{1}{6}\left(-t^3 + 3st^2\right), \text{ if } t \le s < 1. \end{cases}$$

It is easy to see that

$$u'(t) = \int_0^1 G_1(t,s)\varphi(s)ds,$$

$$G_1(t,s) = \frac{\partial G(t,s)}{\partial t} = \begin{cases} \frac{1}{2}s^2, \text{ if } 0 \le s \le t, \\ \frac{1}{2}\left(-t^2 + 2st\right), \text{ if } t \le s < 1, . \end{cases}$$

Also we have

$$u''(t) = \int_0^1 G_2(t,s)\varphi(s)ds,$$

and

$$u'''(t) = \int_0^1 G_3(t,s)\varphi(s)ds,$$

where

$$G_2(t,s) = \frac{\partial G_1(t,s)}{\partial t} = \begin{cases} 0, \text{ if } 0 \le s \le t, \\ s - t, \text{ if } t \le s < 1, \end{cases}$$

and

$$-G_3(t,s) = \frac{\partial G_2(t,s)}{\partial t} = \begin{cases} 0, \text{ if } 0 \le s \le t, \\ 1, \text{ if } t \le s < 1. \end{cases}$$

It is easy verify that these functions are nonnegative and

$$\begin{aligned} &\max_{0\le t\le 1}\int_0^1 G(t,s)ds = \frac{1}{8}, \max_{0\le t\le 1}\int_0^1 G_1(t,s)ds = \frac{1}{6}, \\ &\max_{0\le t\le 1}\int_0^1 G_2(t,s)ds = \frac{1}{2}, \max_{0\le t\le 1}\int_0^1 -G_3(t,s)ds = 1. \end{aligned} \tag{2.1}$$

We give the following settings. For $\varphi \in C^4[0,1]$, we set

$$y(t) = u'(t), v(t) = u''(t), z(t) = u'''(t). \tag{2.2}$$

Clearly, the solutions u, y, v and z of the above problems depend on φ, that is, $u(t) = u_\varphi(t)$, $y(t) = y_\varphi(t)$, $v(t) = v_\varphi(t)$, $z(t) = z_\varphi(t)$. Therefore, for φ we have the equation

$$\varphi = A\varphi,$$

where A is a nonlinear operator defined by

$$(A\varphi)(x) = f(x, u(t), y(t), v(t), z(t)). \tag{2.3}$$

We shall prove that under some conditions A is contractive operator. Let $M > 0$ and t_0 with $0 \le t_0 < \frac{1}{2}$. For the space of continuous functions $C[0,1]$, define the norm $\|\varphi\| = \max_{0\le t\le 1} |\varphi|$ for $\varphi \in C[0,1]$ and consider the integral value $\int_{t_0}^1 \varphi(s)ds$. We assume that $\varphi(t) > 0$ for any $t \in [0,1]$. We also assume that for $M > 0$ and t_0, there exists $D = D(M, t_0)$ such that $\frac{D}{M} \le \int_{t_0}^1 \varphi(s)ds \le \|\varphi\| \le M$, in particular if $t_0 = 0$, then we assume that $D = 0$.

By [12], $\frac{s^2t^2}{3} \le G(t,s)$ for any $s, t \in [0,1]$ we have

$$\begin{aligned} \frac{t_0^4}{3}\frac{D}{M} &\le \frac{t_0^4}{3}\int_{t_0}^1 \varphi(s)ds \le \frac{t_0^4}{3}\int_{t_0}^1 \varphi(s)ds \\ &\le \int_0^1 G(t,s)\varphi(s)ds = u(t) \le \int_0^1 G(t,s)ds\|\varphi(s)\| \le \frac{1}{8}\|\varphi(s)\| \le \frac{M}{8} \end{aligned} \tag{2.4}$$

for any $t \in [0,1]$.

By the definiton of $G_1(s,t)$, we have

$$\begin{aligned} \frac{t_0^2}{2}\frac{D}{M} &\le \frac{t_0^2}{2}\int_{t_0}^1 \varphi(s)ds \le \int_{t_0}^1 \frac{s^2}{2}\varphi(s)ds \\ &\le \int_0^1 G_1(t,s)\varphi(s)ds = y(t) \\ &\le \int_0^1 G_1(t,s)ds\|\varphi(s)\| \le \frac{1}{6}\|\varphi(s)\| \le \frac{M}{6} \end{aligned} \tag{2.5}$$

for any $t \in [0,1]$.

By the definiton of $G_2(s,t)$, since $t_0 < \frac{1}{2}$ and $\varphi(t) > 0$ for any $t \in [0,1]$, we have

$$\begin{aligned} \frac{t_0}{2}\frac{D}{M} &\le \frac{t_0}{2}\int_{t_0}^1 \varphi(s)ds \le \int_{t_0}^1 (s-t_0)\varphi(s)ds \\ &\le \int_0^1 G_2(t,s)\varphi(s)ds = v(t) \\ &\le \int_0^1 G_2(t,s)ds\|\varphi(s)\| \le \frac{1}{2}\|\varphi(s)\| \le \frac{M}{2} \end{aligned} \tag{2.6}$$

for any $t \in [0,1]$.

Also by the definiton of $G_3(s,t)$,

$$\frac{D}{M} \le \int_{t_0}^1 \varphi(s)ds \le \int_0^1 -G_3(t,s)ds\|\varphi(s)\| \le M,$$

then we have

$$(2.7)\qquad \begin{aligned}-M \le -\int_0^1 G_3(t,s)ds\|\varphi(s)\| \le -\int_0^1 G_3(t,s)\varphi(s)ds = z(t)\\ \le \int_{t_0}^1 -\varphi(s)ds \le -\frac{D}{M}\end{aligned}$$

for any $t \in [0,1]$. Under these settings, we can define

$$SD_M^+ = \left\{ (t,u,y,v,z) \;\middle|\; \begin{array}{l} 0 \le t \le 1,\ \frac{t_0^4}{2}\frac{D}{M} \le u \le \frac{1}{8}M,\ \frac{t_0^2}{2}\frac{D}{M} \le y \le \frac{1}{6}M, \\ t_0\frac{D}{M} \le v \le \frac{1}{2}M, -M \le z \le -\frac{D}{M} \end{array} \right\},$$

and we denote

$$SB_M = \left\{ \varphi \in C[0,1] \mid 0 < \varphi(t) \le M \text{ for any } t \in [0,1], \frac{D}{M} \le \int_{t_0}^1 \varphi(t)dt \right\}.$$

Remark 1. *If $t_0 = 0$, then $D = 0$ so SD_M^+ and SB_M coincide the following D_M^+ and S_M in* [20]*, respectively;*

$$D_M^+ = \left\{ (t,u,y,v,z) \;\middle|\; \begin{array}{l} 0 \le t \le 1, 0 \le u \le \frac{1}{8}M,\ 0 \le y \le \frac{1}{6}M, \\ 0 \le v \le \frac{1}{2}M, -M \le z \le 0 \end{array} \right\}$$

and

$$S_M = \{\varphi \in C[0,1] \mid 0 < \varphi(t) \le M \text{ for any } t \in [0,1]\}.$$

In this case our setting coinside the one in [20].

Under these settings, we have the following lemma.

Lemma 1. *Assume that there exists $M > 0$, t_0 with $0 \le t_0 < \frac{1}{2}$ and $D = D(M,t_0) > 0$ such that*

$$(2.8)\qquad 0 \le f(t,u,y,v,z) \le M,$$

$$(2.9)\qquad \frac{D}{M} \le \int_{t_0}^1 f(t,u,y,v,z)dt,$$

for any $(t,u,y,v,z) \in SD_M^+$ and also there exist $C_1, C_2, C_3, C_4 \ge 0$ such that

$$(2.10)\qquad |f(t,x_1,x_2,x_3,x_4,) - f(t,y_1,y_2,y_3,y_4)| \le \sum_{i=1}^4 C_i|x_i - y_i|$$

for any $(t,x_1,x_2,x_3,x_4), (t,y_1,y_2,y_3,y_4) \in SD_M^+$. Then the operator A defined by (2.3) maps the SB_M into itself. Moreover, if

$$(2.11)\qquad q := \frac{C_1}{8} + \frac{C_2}{6} + \frac{C_3}{2} + C_4 < 1,$$

then A is contractive operator in SB_M.

Proof. Let $\varphi \in B_M$ with $\varphi(t) > 0$ for any $t \in [0,1]$. By (2.4), (2.5), (2.6) and (2.7),

$$
\begin{aligned}
&\frac{t_0^4}{3}\frac{D}{M} \le u(t) \le \int_0^1 G(t,s)ds\|\varphi(s)\| \le \frac{1}{8}\|\varphi(s)\| \le \frac{M}{8},\\
&\frac{t_0^2}{2}\frac{D}{M} \le y(t) \le \int_0^1 G_1(t,s)ds\|\varphi(s)\| \le \frac{1}{6}\|\varphi(s)\| \le \frac{M}{6},\\
&\frac{t_0}{2}\frac{D}{M} \le v(t) \le \int_0^1 G_2(t,s)ds\|\varphi(s)\| \le \frac{1}{2}\|\varphi(s)\| \le \frac{M}{2},\\
&-M \le z(t) \le \int_{t_0}^1 -\varphi(s)ds \le -\frac{D}{M}.
\end{aligned}
\tag{2.12}
$$

Therefore $(t,u,y,v,z) \in SD_M^+$ for $t \in [0,1]$. In view of (2.3) and the condition (2.8), we have $A\varphi \in SB_M$, i.e. the operator maps SB_M into itself. Now, let $\varphi_1, \varphi_2 \in SB_M$ and u_1, u_2 be the solutions of the problem (1.1) for φ_1, φ_2 respectively. We also denote $y_i = u_i'$, $v_i = u_i''$, $z_i = u_i'''(i = 1,2)$. Then, as induced above $(t, u_i, y_i, v_i, z_i) \in SB_M$ $(i = 1,2)$ for $t \in [0,1]$. Due to the estimates (2.12) we have

$$
\begin{aligned}
&\|u_2 - u_1\| \le \frac{1}{8}\|\varphi_2 - \varphi_1\|,\ \|y_2 - y_1\| \le \frac{1}{6}\|\varphi_2 - \varphi_1\|,\\
&\|v_2 - v_1\| \le \frac{1}{2}\|\varphi_2 - \varphi_1\|,\ \|z_2 - z_1\| \le \|\varphi_2 - \varphi_1\|.
\end{aligned}
\tag{2.13}
$$

Now from (2.3) and (2.10) it follows that

$$
\begin{aligned}
\|A\varphi_2 - A\varphi_1\| &= \|f(t,u_2,y_2,v_2,z_2) - f(t,u_1,y_1,v_1,z_1)\|\\
&\le C_0\|u_2 - u_1\| + C_1\|y_2 - y_1\| + C_2\|v_2 - v_1\| + C_3\|z_2 - z_1\|.
\end{aligned}
$$

Using the estimate (2.13) we obtain

$$
\|A\varphi_2 - A\varphi_1\| \le \left(\frac{C_1}{8} + \frac{C_2}{6} + \frac{C_3}{2} + C_4\right)\|\varphi_2 - \varphi_1\|.
$$

Therefore, A is a contractive operator in SB_M provided the condition (2.11) is satisfied. The lemma is proved. □

Theorem 2. *Under the assumptions of Lemma 1, the problem (1.1) has a unique nonnegative solution u and there hold the estimates*

$$
\frac{t_0^4}{3}\frac{D}{M} \le \|u\| \le \frac{M}{8},\ \frac{t_0^2}{2}\frac{D}{M} \le \|u'\| \le \frac{M}{6},\ \frac{t_0}{2}\frac{D}{M} \le \|u''\| \le \frac{M}{2},\ \frac{D}{M} \le \|u'''\| \le M.
$$

Proof. The proof is same as that of Theorem 2.3 in [20]. □

Remark 2. *By the Remark 1, Theorem 2 includs the Theorem 2.3 in* [20].

3. Examples

We consider the folllowing example of nonlinear function f for demonstrating the applicability of the obtained theoretical result, which does not satisfies the conditions in Theorem 2.2 in [20] and Theorem 3.1, 3.2 in [12].

Example 3. We consider the following boundary value problem:

$$
\begin{cases}
u''''(t) = u^2 + |u'(t)|^{1/2} + |u''(t)|^{1/2} + |u'''(t)|^{1/2} + \pi\sin(\pi t), 0 < t < 1,\\
u(0) = u'(0) = u''(1) = u'''(1) = 0.
\end{cases}
$$

In this example

$$f(t,u,y,v,z) = u^2 + |y|^{1/2} + |v|^{1/2} + |z|^{1/2} + \pi\sin(\pi t),$$

which maps $[0,1]\times\mathbb{R}_+^3\times\mathbb{R}_- \to \mathbb{R}_+$. In this case, by (2.12), if $14.435 < M < 35.718$, then $f(t,u,y,v,z) \le M$. Thus (2.8) is satisfied. We take t_0 with $0 \le ?t_0 < \frac{1}{2}$. Since f is continuous and positive, we take $L(M,t_0)$ with $0 < L(M,t_0) \le M$ and $(M - L(M,t_0)) \le \int_{t_0}^1 f(t,u,y,v,z)$. If we take $D = M(M - L(M,t_0))$, then we have $\frac{D}{M} < \int_{t_0}^1 f(t,u,y,v,z)$. In particular if $t_0 = 0$, then we take $L(M,t_0) = M$, that is $D = 0$. We take $C_1 = |f_u|, C_2 = |f_y|, C_3 = |f_v|$, and $C_4 = |f_z|$. Then we have

$$\begin{aligned}
q &= \frac{1}{8}C_1 + \frac{1}{6}C_2 + \frac{1}{2}C_3 + C_4 \\
&= \frac{1}{8}|f_u| + \frac{1}{6}|f_y| + \frac{1}{2}|f_v| + |f_z| \\
&\le \frac{1}{8} * 2 * |u| + \frac{1}{6} * \frac{1}{2} * \frac{1}{|y|^{1/2}} + \frac{1}{2} * \frac{1}{2} * \frac{1}{|v|^{1/2}} + \frac{1}{2} * \frac{1}{|z|^{1/2}} \\
&\le \frac{1}{4} * \frac{1}{8}M + \frac{1}{12} * \left(\left(2 * \left(\frac{1}{t_0}\right)^2\right)\left(\frac{M}{M(M - L(M,t_0))}\right)\right)^{1/2} \\
&+ \frac{1}{4} * \left(\left(2 * \frac{1}{t_0}\right) * \frac{M}{M(M - L(M,t_0))}\right)^{1/2} \\
&+ \frac{1}{2} * \left(\frac{M}{M(M - L(M,t_0))}\right)^{1/2} \\
&\le \frac{M}{32} + \frac{1}{12} * \left(\frac{2}{t_0^2(M - L(M,t_0))}\right)^{1/2} + \frac{1}{4} * \left(\frac{2}{t_0(M - L(M,t_0))}\right)^{1/2} + \frac{1}{2} * \left(\frac{1}{M - L(M,t_0)}\right)^{1/2}.
\end{aligned}$$

Moreover if we take $M = 16$, $t_0 = \frac{1}{3}$, and $L(M,t_0) = 7$, in this case $D = 16 * 9$, then we have

$$\frac{D}{M} = 9 \le \int_{1/3}^1 f(t,u,y,v,z)dt < 16.$$

In fact if $f(t,u,y,v,z) = 15$, then we have $\int_{t_0}^1 f(t,u,y,v,z)dt = 10 > 9 = \frac{D}{M}$. Thus (2.9) is satisfied. Moreover by the mean value theorem, we have (2.10). Finally we have

$$\begin{aligned}
q &= \frac{1}{8}|f_u| + \frac{1}{6}|f_y| + \frac{1}{2}|f_z| + |f_w| \\
&\le \frac{16}{32} + \frac{1}{12} * \left(\frac{2 * 3^2}{9}\right)^{1/2} + \frac{1}{4} * \left(\frac{2 * 3}{9}\right)^{1/2} + \frac{1}{2} * \left(\frac{1}{9}\right)^{1/2} \\
&\approx 0.988642 < 1.
\end{aligned}$$

4. Conclusion

In this paper we have proposed a method for investigating the solvability of solution of the cantilever beam equation with fully nonlinear term. In particuler for the contraction mapping principle to an operator equation for the function $f(t,u(t),u'(t),u''(t),u'''(t)) = u^2 + |u'(t)|^{1/2} + |u''(t)|^{1/2} + |u'''(t)|^{1/2} + \pi\sin(\pi t)$ in [20], we have proposed a one way of investigation of method to apply to this function f. The proposed approach can be used for some other nonlinear boundary value problems for ordinary and partial differential equations.

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