Dynamics of a diffusive vaccination model with therapeutic impact and non-linear incidence rates

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In this study, we propose a more general diffusive spatially dependent vaccination model for infectious disease. In our diffusive vaccination model, we consider both therapeutic impact and three different nonlinear incidence rate functions. Also, in this model, the susceptible, vaccinated, and infectious compartments are considered to be functions of both time and location, where the set of locations (equivalently, spatial habitats) is a subset of $\mathbb{R}^n$ with a smooth boundary. We compute the basic reproduction number, $\mathcal{R}_0$. At $\mathcal{R}_0 = 1$, our model exhibits a bifurcation: if $\mathcal{R}_0 \leq 1$, the disease-free equilibrium is globally asymptotically stable, whereas the endemic equilibrium is globally asymptotically stable when $\mathcal{R}_0 > 1$. Also, the existence of a unique solution of the model and uniform persistence results are studied. Finally, using the finite difference scheme, a number of numerical examples verify our analytical results. Our results indicate that the global dynamics of the model are completely determined by the basic reproduction number.

Keywords— spatially dependent model, vaccination model, nonlinear incidence, basic reproduction number, local stability, global stability, uniform persistence