



## Statistical Analysis of Industrial processed Cheese puffs

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**ABSTRACT:** This paper studied and fit a Multivariate linear regression model to the relationship between the response variables; Weight and Bulk density on one hand, and the predictor variables; Temperature, Moisture content before extrusion and Moisture content after extrusion on the other hand, of Cheese puffs product, manufactured by Zubix Company Limited, Anambra, Nigeria. A sample size of three hundred (300) cheese puffs packs were collected from a population of two-thousand, seventy-eight batches between August 2013 to June 2014, examined and used for analysis. A temperature of 186.67°C was discovered to be significantly related to the response variable. © JASEM

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Cheese puffs are a puffed corn snack, coated with a mixture of cheese or cheese-flavored powders. As a household name in Nigeria, Cheese puffs are commonly referred to as Cheese balls. They are either locally or industrially processed food in the form of confectionaries. Some common brand names include; Cheetos (U.S.), Cheez Doodles (Northeastern U.S.), Chee-Wees (New Orleans, South Central U.S.), Chizitos (Perú), Boliquesos (Perú), Cheezies (Canada), Twisties (Australia), Kurkure (India and Pakistan), Utz (U.S.) Wotsits (U.K.), Curl (Japan) and Chee.Toz (Iran) [1], etc. Cheese puffs were invented in the United States of America in the 1930s; there are two competing accounts of its origin [2]. According to one account, Edward Wilson and/or Clarence J. Schwabke of the Flakall Corporation of Beloit, Wisconsin (a producer of flaked, partially cooked animal feed) deep-fried and salted the puffed corn produced by their machines, and later added cheese. He applied for a patent in 1939 and the product, named *Korn Kurls*, was commercialized in 1946 by the Adams Corporation, formed by one of the founders of Flakall and his sons [3]. Adams was later bought by Beatrice Foods. Another account claims they were invented by the Elmer Candy Corporation of New Orleans, Louisiana some time during or prior to 1936 at which time the sales manager for Elmer's, Morel M. Elmer, Sr., decided to hold a contest in New Orleans to give this successful product a name. The winning name "CheeWees" is still being used today by the manufacturing company, Elmer's Fine Foods. The fictitious brand of cheese puffs called "Cheesy Poofs" appears regularly in the animated television series

South Park, and the Frito-Lay company made a limited run of the snack in August 2011 [4].

Cheese puffs are manufactured by extruding heated corn dough through a die that forms the particular shape. They may be ball-shaped, animal-shaped, curly ("cheese curls"), straight, or irregularly shaped. Some cheese puffs are puffy while others are crunchy.

The Enriched cornmeal and seasoning are the two main components of Cheese puffs. Cornmeal as one of the major or primary ingredient of cheese puffs is made by grinding dried maize or corn into coarse flour [5] [6]. Iron, Niacin, Thiamine, Riboflavin and Folic acid are vitamins and minerals which are added to the cornmeal to enrich the nutrient content [7]. Food fortification plays an important role in ensuring the health of the consumers, as the added micronutrients can replace the nutrients that are lost during the manufacturing process of the cornmeal flour [8]. Other recipes used in the production of cheese puffs are; Salt, Refined vegetable oil, Natural Cheese solid, Natural and Artificial flavor, Sunset yellow FCF, Corn maltodextrin etc. Fig. 1 (a) shown the industrially packaged cheese puffs in a polytene sachets. Cheese puffs can be found in local grocery and corner stores and are enjoyed by many [9]. It is usually sold to the public for consumption as shown in the bowl, while (b) shown the opened packets and the cheese puffs in a bowl.

Cheese puffs, particularly the ball-shaped type are usually consumed in Nigeria by both children and adult, and its average weight and Bulk density are

almost infinitesimal as can be observed in its light weight and volume per mass value. These parameters (average weight and Bulk density) which formed the amount of substance in the products, determines the quality of the products. However, the nature of this two parameters do not seems to affect its high consumption or sales either because of the taste or the inability of the consumers to make such judgments. Despite the high consumption of cheese puffs products in Nigeria and the world in general, there is currently no known research work on the analysis of any of its parameters, as most write-up or articles deals on its recipe and preparations. Therefore, this study whose main aim and objective is to analyzed the relationship between the average weight and bulk density, on one hand, the oven temperature and moisture content before and after extrusion on the other hand, will stand in the gap of research work to be considered and used as reference points.

This paper used the Multivariate linear regression analysis which models the relationship between “m” responses and a set of predictor variables, where each response is assumed to follow its own regression model [10]. It is an extension of the multiple linear regressions.

**MATERIALS AND METHODS**

The Multivariate linear regression analysis was applied to the 300 data shown in appendix I, collected from Zubix Company Limited, Anambra, Nigeria. The multivariate modeled the relationship between the "p" responses  $X_1, X_2, X_3, \dots \dots X_p$  and a set of "q" predictor variable  $Y_1, Y_2, Y_2, \dots \dots Y_q$ . Each of the "p" responses is assumed to follow own regression model. Statistical software such as R version 3.0.1 (2013-05-16) and Minitab (2006) were used in carrying out the analysis.

The response variables used are;

$$X_1 = \text{Weight (grame)}, X_2 = \text{Bulk density (grame/litre)}$$

While the predictor variables are;

$$Y_1 = \text{Temperature}(^{\circ}\text{C}), Y_2 = \text{Moisture before Extrusion}, Y_3 = \text{Moisture after Extrusion}$$

Therefore,

$$\begin{matrix} X_1 = \beta_{01} + \beta_{11}Y_1 + \beta_{21}Y_2 + \dots \dots \dots + \beta_{q1}Y_q \\ X_2 = \beta_{02} + \beta_{12}Y_1 + \beta_{22}Y_2 + \dots \dots \dots + \beta_{q2}Y_q \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ X_p = \beta_{0p} + \beta_{1p}Y_1 + \beta_{2p}Y_2 + \dots \dots \dots + \beta_{qp}Y_q \end{matrix}$$

Where the error term is;

$$E(\varepsilon) = E \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \cdot \\ \cdot \\ \varepsilon_p \end{pmatrix} = 0$$

And  $Var(\varepsilon) = \Sigma$

Let  $[Y_{j0}, Y_{j1}, Y_{j2} \dots \dots \dots Y_{jq}]$  be the  $j^{th}$  trial for the predictor variable;

and

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$$X_j = \begin{pmatrix} X_{j1} \\ X_{j2} \\ \vdots \\ X_{jp} \end{pmatrix}, \quad \varepsilon_j = \begin{pmatrix} \varepsilon_{j1} \\ \varepsilon_{j2} \\ \vdots \\ \varepsilon_{jp} \end{pmatrix} \text{ be the response and errors for the } j^{th} \text{ trial}$$

Therefore,

$$Y_{(n \times (q+1))} = \begin{pmatrix} Y_{10}, Y_{11}, \dots, Y_{1q} \\ Y_{20}, Y_{21}, \dots, Y_{2q} \\ \vdots \\ Y_{n0}, Y_{n1}, \dots, Y_{nq} \end{pmatrix}, \quad X_{(n \times p)} = \begin{pmatrix} X_{11}, X_{12}, \dots, X_{1p} \\ X_{21}, X_{22}, \dots, X_{2p} \\ \vdots \\ X_{n1}, X_{n2}, \dots, X_{np} \end{pmatrix} = [X_{(1)}, X_{(2)}, \dots, X_{(p)}]$$

$$\beta_{((q+1) \times p)} = \begin{pmatrix} \beta_{01}, \beta_{02}, \dots, \beta_{0p} \\ \beta_{11}, \beta_{12}, \dots, \beta_{1p} \\ \vdots \\ \beta_{q1}, \beta_{q2}, \dots, \beta_{qp} \end{pmatrix} = [\beta_{(1)}, \beta_{(2)}, \dots, \beta_{(p)}]$$

Where,

" $\beta$ " is the  $((q + 1) \times p)$  matrix of parameter of regression, "X" is the  $(n \times p)$  matrix of the response variable, and " $\varepsilon$ " is the  $(n \times p)$  matrix of errors or residuals.

$$\varepsilon_{(n \times p)} = \begin{pmatrix} \varepsilon_{11}, \varepsilon_{12}, \dots, \varepsilon_{1p} \\ \varepsilon_{21}, \varepsilon_{22}, \dots, \varepsilon_{2p} \\ \vdots \\ \varepsilon_{n1}, \varepsilon_{n2}, \dots, \varepsilon_{np} \end{pmatrix} = [\varepsilon_{(1)}, \varepsilon_{(2)}, \dots, \varepsilon_{(p)}] = \begin{pmatrix} \varepsilon_{(1)} \\ \varepsilon_{(2)} \\ \vdots \\ \varepsilon_{(p)} \end{pmatrix}$$

Then, the multivariate linear regression model is;  $X = Y\beta + \varepsilon$

Where;

$$E[\varepsilon_{(i)}] = 0 \text{ and } Cov(\varepsilon_i, \varepsilon_k) = \sigma_{ik}I$$

Therefore, the covariance matrix;

$$\Sigma = \begin{pmatrix} \sigma_{11}, \sigma_{12}, \dots, \sigma_{1p} \\ \sigma_{21}, \sigma_{22}, \dots, \sigma_{2p} \\ \vdots \\ \sigma_{p1}, \sigma_{p2}, \dots, \sigma_{pp} \end{pmatrix}$$

The ordinary least square estimate given by;

$$\hat{\beta}_{(i)} = (Y'Y)^{-1}Y'X_{(i)} \quad \text{for parameter } \beta = [b_{(1)}, b_{(2)}, \dots, b_{(p)}]$$

with matrix of error;  $X - Y\beta$

Then the error sum of square and product is,

$$(X - Y\beta)'(X - Y\beta) = \begin{bmatrix} (X_{(1)} - Yb_{(1)})' (X_{(1)} - Yb_{(1)}) \cdot \cdot \cdot \cdot \cdot (X_{(1)} - Yb_{(1)})' (X_{(p)} - Yb_{(p)}) \\ \vdots \\ (X_{(1)} - Yb_{(1)})' (X_{(1)} - Yb_{(1)}) \cdot \cdot \cdot \cdot \cdot (X_{(1)} - Yb_{(1)})' (X_{(p)} - Yb_{(p)}) \end{bmatrix}$$

Where  $b_{(i)} = \hat{\beta}_{(i)}$  minimizes the  $i^{th}$  diagonal sum of square  $(X_{(i)} - Yb_{(i)})' (X_{(i)} - Yb_{(i)})$

therefore,  $tr[(X_{(i)} - Yb_{(i)})' (X_{(i)} - Yb_{(i)})]$

Then matrix of predicted and residuals values formed are;

$$\hat{X} = Y\hat{\beta} = Y(Y'Y)^{-1}Y'X, \quad \hat{\varepsilon} = X - \hat{X} = [1 - Y(Y'Y)^{-1}Y']X \quad \text{respectively}$$

The hypothesis that the responses do not depend on predictor variables  $Y_{u+1}, Y_{u+2}, \dots, Y_q$  is,

$$H_0 : \beta_{(2)} = 0 \quad \text{where } \beta = \frac{\beta_{(1)}}{\beta_{(2)}}$$

Therefore the general model can be written as;

$$E[X] = Y\beta = [Y_{(1)} \ Y_{(2)}] \begin{bmatrix} \beta_{(1)} \\ \beta_{(2)} \end{bmatrix} = Y_{(1)}\beta_{(1)} + Y_{(2)}\beta_{(2)}$$

And the likelihood ratio for the hypothesis is given as;  $H_0 : \beta_{(2)} = 0$

If the ratio of generalized variance is given by;

$$\Lambda = \frac{\text{Max}_{\beta, \Sigma} \mathcal{L}(\hat{\beta}_{(1)}, \hat{\Sigma}_{(1)})}{\text{Max}_{\beta, \Sigma} \mathcal{L}(\hat{\beta}, \hat{\Sigma})} = \frac{\mathcal{L}(\hat{\beta}_{(1)}, \hat{\Sigma}_{(1)})}{\mathcal{L}(\hat{\beta}, \hat{\Sigma})} \quad \text{and} \quad \Lambda^{\frac{2}{n}} = \frac{|\hat{\Sigma}|}{|\hat{\Sigma}_{(1)}|}$$

The multivariate regression model with full rank  $(Y) = q + 1, n \geq q + 1 + p$  normally distributed with error  $\varepsilon$ , and the null hypothesis is true,

$$- \left[ n - q - 1 - \frac{1}{2} (p - q + w + 1) \right] \ln \left[ \frac{|\hat{\Sigma}|}{|\hat{\Sigma}_{(1)}|} \right] \sim \chi_{q(r-w)}^2$$

If the confidence interval for the predicted mean value of  $X_0$  associated with  $Y_0$  Model, with

$\hat{\beta}'Y_0 \sim Np(\hat{\beta}'Y_0, Y_0'(Y'Y)^{-1}Y_0\Sigma)$  and  $n\hat{\Sigma} \sim W_n - r - 1(\Sigma)$ . Then the  $100(1 - \alpha)\%$  confidence intervals for the mean value of  $X_i$  is;

$$Y_0'\hat{\beta}_{(i)} \pm \sqrt{\frac{p(n-q-1)}{n-q-p} F_{p, n-q-p}(\alpha)} * \sqrt{Y_0'(Y'Y)^{-1}Y_0 \frac{n}{n-q-1} \hat{\sigma}_{ii}} \quad i = 1, 2, \dots, \dots, \dots, p$$

And the  $100(1 - \alpha)\%$  prediction interval for the  $X_0$  is given as;

$$Y_0'\hat{\beta}_{(i)} \pm \sqrt{\frac{p(n-q-1)}{n-q-p} F_{p, n-q-p}(\alpha)} * \sqrt{1 + Y_0'(Y'Y)^{-1}Y_0 \frac{n}{n-q-1} \hat{\sigma}_{ii}} \quad i = 1, 2, \dots, \dots, \dots, p$$

Where  $\hat{\beta}_{(i)}$  and  $\hat{\sigma}_{ii}$  is the  $i^{th}$  column of  $\hat{\beta}$  and  $\hat{\Sigma}$  respectively.

The statistical hypotheses for this study are;

$H_0$  : All parameters (average weight, Bulk density, oven temperature, and moisture content before and after extrusion) are not significant. Vs  $H_1$  : At least a parameter is significant.

**RESULTS AND DISCUSSION**

If the predictor and response variables are given by “Y” and “X” respectively, then the formed matrices are given as;

$$Y'Y = \begin{bmatrix} 300 & 56586 & 4536 & 846 \\ 56586 & 11076031 & 857142 & 159491 \\ 4536 & 857142 & 71084 & 12794 \\ 846 & 159491 & 12794 & 2427 \end{bmatrix}$$

and,

$$Y'X = \begin{bmatrix} 4262 & 12179 \\ 803410 & 2298686 \\ 64381 & 184173 \\ 12007 & 34339 \end{bmatrix}, \quad (Y'Y)^{-1} = \begin{bmatrix} 0.389637 & -0.000472 & -0.005894 & -0.073758 \\ -0.000472 & 0.000002 & -0.000001 & 0.000009 \\ -0.005894 & -0.000001 & 0.000403 & 0.000031 \\ -0.073758 & 0.000009 & 0.000031 & 0.025386 \end{bmatrix}$$

$$\hat{\beta} = Y(Y'Y)^{-1}Y'X = \begin{bmatrix} 15.8796 & 41.3836 \\ -0.0011 & 0.0033 \\ -0.0240 & 0.0001 \\ -0.3896 & -0.4994 \end{bmatrix} \quad \hat{X} = Y \begin{bmatrix} 15.8796 & 41.3836 \\ -0.0011 & 0.0033 \\ -0.0240 & 0.0001 \\ -0.3896 & -0.4994 \end{bmatrix} + \varepsilon$$

$$\hat{X}_{(1)} = 15.8796 - 0.0011Y_1 + 0.0240Y_2 - 0.3896Y_3,$$

$$\hat{X}_{(2)} = 41.3836 + 0.0033Y_1 + 0.0001Y_2 - 0.4994Y_3$$

Therefore,

$$\hat{\beta}_1 = \begin{bmatrix} 15.8796 \\ -0.0011 \\ -0.0240 \\ -0.3896 \end{bmatrix}, \quad \hat{\beta}_2 = \begin{bmatrix} 41.3836 \\ 0.0033 \\ 0.0001 \\ -0.4994 \end{bmatrix}$$

The Regression Parameter test of Significance,

$$n\hat{\Sigma} = \begin{bmatrix} 571.362 & 6.806 \\ 6.806 & 1417.661 \end{bmatrix} \quad \hat{\Sigma} = \begin{bmatrix} 1.90454 & 0.02269 \\ 0.02269 & 4.72554 \end{bmatrix}$$

$$H_0 : \beta_{(2)} = 0$$

Where

$$\beta = \frac{\beta_{(1)}}{\beta_{(2)}} \quad \beta = \begin{bmatrix} 15.8796 & 41.3836 \\ -0.0011 & 0.0033 \\ -0.0240 & 0.0001 \\ -0.3896 & -0.4994 \end{bmatrix}$$

$$Y = [Y_{(1)}/Y_{(2)}]$$

Where the dimension of both  $Y_{(1)}$  and  $Y_{(2)}$  is given as (500 X 2)

$$n\hat{\Sigma}_{(1)} = (X - Y_1\hat{\beta}_1)'(X - Y_1\hat{\beta}_1)$$

$$= \begin{bmatrix} 578.72 & 14.43 \\ 14.43 & 1427.49 \end{bmatrix}$$

$$\hat{\Sigma}_{(1)} = \begin{bmatrix} 1.9290 & 0.0481 \\ 0.0481 & 4.7583 \end{bmatrix} \quad n(\hat{\Sigma}_{(1)} - \hat{\Sigma}) = \begin{bmatrix} 7.35744 & -7.62435 \\ 7.62435 & 9.82716 \end{bmatrix}$$

$$|\hat{\Sigma}| = 8.9994, \text{ and } |\hat{\Sigma}_{(1)}| = 9.1767$$

$$\Lambda^{\frac{2}{n}} = \frac{8.9994}{9.1767} = 0.9807$$

$$= -[500 - 3 - 1 - \frac{1}{2}(2 - 3 + 1 + 1)] \ln 0.9807 = 5.7553, \chi_{4, 0.05}^2 = 9.4900$$

Therefore,

$$5.7553 < 9.4900$$

Thus, the oven temperature affects the weight and bulk density of Zubix International company significantly. In order words, there is a joint relationship between average weight and bulk density on one hand, and the oven temperature on the other hand. If the proposed model in predicting the values of the response variable, is given as;

$$\hat{X} = Y\hat{\beta}, [X_{j1}/X_{j2}] = [Y_1][\hat{\beta}_1]$$

Then the 16<sup>th</sup> trial is given as,

$$[X_{16(1)}/X_{16(2)}] = [1 \ 204.924] \begin{bmatrix} 15.8796 & 41.3836 \\ -0.0011 & 0.0033 \end{bmatrix}$$

$$[X_{16(1)}/X_{16(2)}] = [15.6541 \ 42.0598]$$

The implication of this is that when the oven temperature is fix at 204.9240 °C, then the average weight and bulk density are predicted to be 15.6541 grammes and 42.0598 grammes per litre respectively. This shows that the predictive model performs well given the predicted values as 15.6541 and 42.0598. Therefore, for the 16<sup>th</sup> trial, the confidence interval for the predicted mean value of the Average weight is given as;

$$X_{16,1} = [15.65418363 \pm \sqrt{\frac{2(300-3-1)}{300-3-2} * F_{2,295} (0.05)} * \sqrt{41993.7455 * \frac{300}{296} * 1.90454}]$$

$$\therefore X_{16,1} = 15.65418363 \pm 698.5742927 = 714.2284 \geq X_{16,1} \geq -682.9201$$

While the bulk density is given as;

$$X_{16,2} = [42.0598492 \pm \sqrt{\frac{2(300)-3-1}{300-3-2} * F_{2,295} (0.05)} * \sqrt{41993.7455 * \frac{300}{296} * 4.72554}]$$

$$\therefore X_{16,2} = 42.0598492 \pm 2699.937712 = 2741.9975 \geq X_{16,2} \geq -2657.8778$$

Similarly, the 16<sup>th</sup> trial for 95% prediction intervals for values of  $X_0$  is given as;

$$X_{16,1} = [15.65418363 \pm \sqrt{\frac{592}{295} * 300} * \sqrt{1 + 41993.7455 * \frac{300}{296} * 1.90454}]$$

Therefore,

$$\begin{aligned} X_{16,1} &= 15.65418363 \pm 1100.384215 \\ &= 1116.0383 \geq X_{16,1} \geq -1084.7300 \end{aligned}$$

while the bulk density is given as;

$$X_{16,2} = [42.0598492 \pm \sqrt{\frac{592}{295} * 300} * \sqrt{1 + 41993.7455 * \frac{300}{296} * 4.72554}]$$

$$X_{16,2} = 42.0598492 \pm 1100.38399$$

$$\Rightarrow 1142.0398 \geq X_{16,2} \geq -1058.3241$$

*Conclusion:* In this study, a test of significance revealed that only the oven temperature is significance when a multivariate linear regression model was used in modeling the relationship between

average weight and Bulk density of the Cheese balls on one hand, and oven temperature moisture content before extrusion and moisture content after extrusion on the other hand.

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