Ukrainian Journal
of Educational Studies and
Information Technology

ISSN: 2521-1234 online
Ukr. J. of Educ. Stud. and Inf. Technol.
10(1), 1 – 18
doi: 10.32919/uesit.2022.01.01

Vol. 10, Issue 1, 2022

Students' mathematical training for professional self-realization in a modern, inventive society

Mariia Astafieva, Dmytro Bodnenko, Oksana Lytvyn,
Volodymyr Proshkin, Oleksii Zhyltsov

Submitted:
2021 – 11 – 04
Accepted:
2022 – 01 – 27
Published online:
2022 – 03 – 30

Abstract
In the framework of their professional self-realization on the contemporary job market, the paper explores the issues with professionals' training for mathematics at universities. The structure of students' mathematical proficiency is taken into consideration, and its key elements are substantively presented (the ability to ask and answer questions in and with mathematics, the ability to deal with mathematical language and tools). The initial (ascertaining) step of the pedagogical experiment allowed for the establishment of the level of students' mathematical proficiency and formation in accordance with the predetermined structure. It has been demonstrated that students, both of mathematical and non-mathematical majors, lack the mathematical knowledge and abilities necessary to solve complicated issues, particularly those involving modeling in the context of future professional engagement. The following fundamental strategies for developing students' mathematical competence are supported: research-focused instruction, an interdisciplinary approach with a practical focus, and the use of digital learning environments and related resources. The implementation strategies for these techniques are described in the authors' optional course "Methods of formation of students' mathematical competence for the future successful career" for math specialist students. The research will continue with the introduction of the aforementioned discipline to the educational system, which will include the subsequent assessment of future teachers' readiness to employ the suggested methods in their professional work.
Keywords: higher school; mathematical preparation; mathematical competence; structure of mathematical competence; problems of Mathematics teaching; approaches to formation of mathematical competence.

INTRODUCTION

Today's innovative society needs professionals of a new type, able to act responsibly, actively and productively in the conditions of social change, to carry out work activities with a high degree of interdisciplinarity and adaptability, while being ready to learn throughout life and to do it quickly and effectively. It is proved that one of the key qualities of such a specialist is a mathematical competence, as indicated by a number of international and Ukrainian regulatory documents (Recommendation 2006/962/EC of the European Parliament and of the Council on basic competences for lifelong learning of 18 December 2006 (*Recommendation of the European Parliament*, 2006), The EU Framework Program for Renewed Key Competences (European Commission, 2018), the reform of the Ministry of Education of Ukraine "New Ukrainian School" (*The New Ukrainian School*, 2016), measures for the implementation of STEM education in Ukraine (MES of Ukraine, 2016), etc.).

Today's tasks (and the ones of the near future) require mathematical competence from specialists at both general (extra-curricular) and subject levels, which is caused by the expansion of the scope of mathematics, methods of mathematical modeling. A specialist in a particular subject area is often unable to formalize a task, select, and apply appropriate mathematical methods to solve it. Mathematician – to understand the essence of the real problem, to determine the characteristics of the process (phenomenon, object) being modeled, to assess the adequacy and effectiveness of the model developed. Therefore, their cooperation is needed, and, thus, qualitatively different mathematical preparation of the first and the second. These challenges of time have led to a change in the educational paradigm – the transition from a knowledge-based to a competent approach in teaching, the intensification of research and practice in this direction.

The issue of mathematical competence formation nowadays is the subject of research of many Ukrainian and foreign scientists. Here are just a few studies. S. Rakov (2005) highlights the issues of training mathematics teachers on the basis of a competent approach using digital technologies. B. Jaworski and J. Matthews (2011) explore the problems of teaching mathematics at engineering majors, offer ways to improve students' conceptual understanding of them, particularly through research and partnerships, so that they can flexibly use mathematics in engineering contexts (Jaworski, 2003; Jaworski, 2006). M. Niss, member of the PISA Expert Group on Mathematics, outlines the characteristic features of mathematical competence and argues that the key problems of mathematical preparation at all educational levels, from school to university, are the same and are addressed through mechanisms of competence-based learning (Niss, 1999). The authors of this article substantiate some approaches to the formation of critical thinking as an important component of students' mathematical competence (Astafieva, Bodnenko, & Proshkin, 2019a; 2019b), consider the problems of teaching mathematical modeling to students of different specialties, theoretical and methodological foundations of mathematical modeling (Goncharenko, 2011; Michelsen, 2015; Zulkarnaen, 2021).

However, despite numerous studies, there is a "paradox of relevance": modern society is interested in specialists in high-tech industries, and young people are reluctant to choose educational programs with a strong mathematical component or do not receive sufficient mathematical training at the university.

The results of the international project ERDF (European Foundation for Regional Development) "Cross-border network of adaptation of mathematical competences in socio-economic development (MatNet)" point to certain problems of university's mathematical training of specialists implemented in the period 2011 – 2013 by the Agrarian University of Latvia and the University of Šiauliai (Lithuania), during which the role of mathematics in the modern labour market in the context of modern education was studied (Zeidmane, 2013). In particular, surveys of more than 300 employers and employees in Latvia and of more than 200 – in Lithuania indicate that 22% of those surveyed consider mathematics to be a set of formulas to remember, and 20% say it is a meaningless game of numbers by the rules devised by mathematicians. Regarding the role of mathematics in professional activity, 52% of respondents said that mathematics is needed and widely used in their professional activity, 48% – that their profession does not require a deep knowledge of mathematics, it is enough to be able to perform arithmetic calculations and find percentages. 59% of respondents believe that mathematical thinking helps to solve real professional problems, and this indicator, despite the fact that it is not too high, indicates the great value for the professional activity of mathematical thinking, since one who does not possess such thinking could not evaluate its value. Significantly, more than 61% of respondents are sure that employees with good mathematical preparation achieve greater success in their profession and are highly valued by the employers.

As for employers' and employees' assessment of mathematics teaching at university, only 50% of respondents consider that teaching mathematics at university was interesting and useful, and 62% confessed that they did not understand mathematics and tried to learn by heart the theorems and formulas. More than 65% said that mathematical knowledge helped them to understand and learn the specific disciplines of their subject area in one way or another, and 76% are convinced that mathematics develops logical thinking.

There is no reason to believe that if such a study was conducted in Ukraine, the results would be fundamentally different. Therefore, there is a need to overcome the existing contradiction between modern market inquiries, the needs of a person in successful professional self-realization and the real state of mathematical preparation of future specialists.

This encourages further research by scholars and investigations by practitioners. One of the directions and evidences of such activation is purposeful scientific researches, international projects on mathematical preparation of students with the purpose of enabling their professional self-realization in the modern innovative society, studying and creative introduction of positive foreign pedagogical experience, which is realized at the Department of Computer Science and Mathematics of Borys Grinchenko Kyiv University, both within the complex scientific topic of the department and through international projects on the problems of teaching and learning mathematics at high school.

The aim of the article is to draw up and develop approaches to the formation of mathematical competence of youth for a successful career in an innovative society, using the European experience.

Research methods

The achievement of the aim of the research was facilitated by the use of a set of appropriate methods: analysis of scientific literature in order to establish the state of elaboration of the problem under study, determination of the categorical and conceptual apparatus of the research; synthesis, generalization, systematization, modeling for theoretical substantiation of approaches of formation of mathematical competence of youth for successful career in an innovative society; empirical: diagnostic (talk, testing), statistical (Pearson criteria) to assess the level of mathematical competence.

The research was performed within the framework of the complex scientific topic of the Department of Computer Science and Mathematics of Borys Grinchenko Kyiv University "Theoretical and practical aspects of the use of mathematical methods and information technologies in education and science", DR No. 0116U004625. The experimental base of the research is Borys Grinchenko Kyiv University.

Results and discussion

Based on the ideas of a competent approach from the project "KOM: Competencies and the Learning of Mathematics", implemented in Denmark (Niss, 2004), and taking into account the results of our own experience of forming students' mathematical competence, we are trying to answer the key question: "How to teach mathematics to students to receive the best possible mathematical preparation at the university in order to help them in their future professional activity?"

The Danish project KOM, led by Mogens Niss, professor of mathematics at Roskilde University, was implemented to create a platform for reforming mathematical education in Denmark, from school to university. The ideas and results of this project have become widespread in European countries.

The summary documents of the KOM project define a person's mathematical competence as the ability to formulate, apply, and interpret mathematics in a variety of contexts (both mathematical and non-mathematical). In other words, it is knowledge, understanding, in particular, also understanding of situations in which mathematics, its methods play or may play a certain role, and the compulsory ability to act in the context of knowledge, competencies, skills, formed ways of thinking and values. The Project's recommendations highlight 8 components of mathematical competence, which are divided into two groups (Table 1) (Niss, 2004).

Let's describe each of the components briefly.

1. Mathematical thinking, which involves:

- awareness of the types of questions specific to mathematics, the ability to ask such questions and understand what type of answer can be expected;

Table 1. **Structure of mathematical competence**

The ability to ask and answer questions in and with mathematics	The ability to deal with mathematical language and tools
1. Mathematical thinking 2. Mathematical reasoning 3. Posing and solving mathematical problems 4. Mathematical modeling	5. Representing of mathematical entities 6. Handling mathematical symbols and formalism 7. Communication in, with and about mathematics 8. Making use of aids and tools

- understanding the scope and limitations of mathematical notions, concepts, methods;
- ability to distinguish different types of mathematical expressions;
- ability to abstract and generalize.

2. The ability to give a reason mathematically means:

- ability to trace and evaluate the chain of arguments;
- understanding what mathematical proof is and the ability to distinguish strict proof from heuristic reasoning;
- ability to reveal the main idea in argumentation (proof);
- possession of deductive method of proof.

3. Posing and solving mathematical problems involves not only identifying problems and formulating different types of “pure” and applied problems and solving either their own or formulated by others problems, but also the ability to search for and find different approaches and ways of solving them, evaluate the pros and cons of each.

4. Mathematical modeling involves:

- ability to analyze the properties of existing models, to assess their compliance with the criteria of correctness and adequacy;
- translation and interpretation of elements of mathematical model in terms of real model;
- ability to create mathematical models in a given context.

5. Representation of mathematical entities implies:

- interpretation of mathematical essence in different ways (analytical, graphic, visual, dynamic, schematic, tabular, symbolic, verbal-descriptive, etc.);
- understanding the strengths and weaknesses of each interpretation;
- ability to choose the most appropriate interpretation, according to the aim.

6. Handling mathematical symbols and formalism is:

- understanding, deciphering and interpreting symbolic mathematical expressions;
- understanding of the nature and rules of formal mathematical systems;
- translation of mathematical texts written in natural language into formal language.

7. Communication in, with and about mathematics is:

- understanding of the texts of mathematical content, represented in written, visual, oral or other form;

- ability to present certain mathematical results in different forms and at different levels of theoretical and technical accuracy.

8. The use of aids and tools, in particular, digital ones means:

- awareness of the existence and properties of different techniques and tools for mathematical activity, understanding of their capabilities and limitations;
- the ability to reflectively use such techniques and tools.

In order to find out the real state of the students' mathematical competence formation, a testing (ascertaining stage of pedagogical experiment) was conducted at the Faculty of Information Technology and Management of Borys Grinchenko Kyiv University in September – October 2019. A total of 39 students of different years of study of specialties "Finance", "Economics", "Management", "Computer Science", which provides the study of mathematical disciplines ("Higher Mathematics", "Probability Theory and Mathematical Statistics", "Econometrics", etc.), as well as 20 students of different years of study of the specialty "Mathematics" took part in the study. Such groups division was caused the need of determination the level of readiness mathematical competency by students of different specialties.

According to Pearson's criterion χ^2, this division of students into two groups is comparable to equable as $\chi^2_{emp} < \chi^2_{cr}$, where $\chi^2_{emp} = 5.492$ is the empirical value of the criterion, $\chi^2_{cr} = 6.635$ is the critical value of the criterion. In addition, 8 teachers of mathematical disciplines the Department of Computer Science and Mathematics of Borys Grinchenko Kyiv University were involved in the study. For the determination of students' mathematical competency paying attention to its components (see Table 1) the tasks were prepared. The results of testing are shown in Table 2.

Table 2. **Formation of mathematical competence of students**

No	Components of mathematical competence	Gr.	Level, %		
			Low	Medium	High
1.	Mathematical thinking	A	40.0	45.0	15.0
		B	41.0	48.7	10.3
2.	Mathematical reasoning	A	35.0	50.0	15.0
		B	30.8	56.4	12.8
3.	Posing and solving mathematical problems	A	40.0	50.0	10.0
		B	35.9	56.4	7.7
4.	Mathematical modeling	A	45.0	45.0	10.0
		B	33.4	58.9	7.7
5.	Representing of mathematical entities	A	25.0	60.0	15.0
		B	38.4	51.3	10.3
6.	Handling mathematical symbols and formalism	A	40.0	50.0	10.0
		B	25.6	66.7	7.7
7.	Communication in, with and about mathematics	A	35.0	50.0	15.0
		B	38.4	51.3	10.3
8.	Making use of aids and tools	A	35.0	55.0	10.0
		B	35.9	56.4	7.7

Table 2 includes the notation: A – a group of students of specialty "Mathematics", B – a group of students of non-mathematical specialties. As the results of the experiment show, the percentage of mathematics students with a high level of development of mathematical competence is slightly higher than the percentage of the same students in other specialties. This excess is not statistically significant, after all

$$\chi^2_{emp} = 0.266, \chi^2_{cr} = \begin{cases} 14.067, p \leq 0.05 \\ 18.475, p \leq 0.01 \end{cases} \quad (1)$$

Exactly students with high level of mathematical competence, as practice shows, are able to correctly carry out logical reasoning, competently build the proof of mathematical facts, demonstrate the ability to use fundamental mathematical regularities in solving theoretical and applied mathematical tasks and problems, recognize mathematical structures in other (non-mathematical) theories, etc.

Mathematics students are better at solving mathematical problems, but just like other students are not good enough at using mathematics to solve applied "life" problems (students are practically unable to comprehensively solve the problem, to reveal its scientific essence in the professional field, not capable in analyzing, verifying, evaluating the completeness and reliability of information in the course of professional activity, produce new ideas, creative approaches to their implementation, etc.). Therefore, math does not serve as a basis for students to develop a successful career.

As a result of conducting a testing and on the basis of discussions with expert teachers, we outlined the problematic field of research, the ground of which lies in insufficient level of mathematical competence of students, and also confirmed the factors that hinder the successful (effective) teaching of mathematics at the university:

- insufficient basic mathematical knowledge, which makes it impossible to study the proper mathematics courses at the university (the critical state of school mathematics education in Ukraine is attested by the results of the External Independent Evaluations (EIEs) of recent years and the recently released PISA–2018 report (OECD, 2019));
- superficial and formal mastering of mathematics, which in turn causes a weak ability to apply it in solving vital and professional problems;
- low interest of students to study mathematics at university;
- poor general education and research skills (or total lack of them) at a large proportion of students.

Considering that, as our research has shown, the levels of formation of most components of mathematical competence are approximately the same for students of different specialties, we have come to the conclusion that the way to improve the situation is not due to changes in the content of mathematical training, programs of mathematical disciplines (each specialty has its own program), but due to changing teaching approaches, improving teaching methods. The traditional technological approach to education (defining a mandatory standard minimum of knowledge, skills, acquaintance students with them, standardized testing of the ability to reproduce this knowledge), which prevails today in higher education, not only Ukrainian, has proved to be not very effective and inappropriate to the time requirements. Since mathematical competence means the ability to act, it is obvious that its formation is possible only in the course of active performance.

Therefore, to improve the motivation for learning, to deepen the conceptual understanding of mathematics and the ability to apply it, to develop general educational and research skills, which will inevitably lead to an increase in the actual level of mathematical knowledge and mathematical competence in general, active approaches to learning are required. The most effective of them are:

- research-oriented training;
- interdisciplinary approach and practical orientation;
- use of virtual learning environments and appropriate tools.

We would prove our choice.

By research-oriented training, we mean a set of pedagogical techniques and tools for the development of an individual's research competence, which implies the ability to identify a problem, formulate a research problem and find ways to solve it. Research-oriented learning, as opposed to reproductive learning, is aimed at developing students' experience of self-seeking and constructing new knowledge, ways of action, their application in different situations and conditions, developing the ability to creative activity, certain values.

Obviously, the real practice of teaching has never been pure reproduction. It is equally impossible to imagine a study based on research only. We are talking about creating such conditions for a student under which he / she would take not just an active position in the educational process, but an initiative position as well, not only master the proposed material (this also requires activity), but also master the methods of the world's comprehension, making an active dialogue with him / her, seek for the answers and ask himself/herself new questions. A number of significant differences between reproductive (informative) and research-oriented teaching of mathematics are summarized by us in Table 3.

Here are some examples that illustrate these alternative approaches.

Example 1. Formation in students of the concept of continuous at the point of function of one variable ("Mathematical analysis", first year of study).

Reproductive (informative) approach. The teacher reports the definition of a continuous function at a point: "A function $f(x)$ is called continuous at a point x_0 if its limits at a point x_0 is equal to a value at this point, i.e. $lim_{x\to x_0} f(x) = f(x_0)$.

A research-oriented approach. Case task. The teacher shows the image (Fig. 1), indicates that only in Fig. 1a) a graph of a continuous at x_0 point of a function is shown, and proposes to find out the characteristic signs of a continuous function at a point and suggest its definition.

Table 3. **Comparison of reproductive and research-oriented approaches to mathematics teaching**

No	Reproductive (informative) training	Research-oriented learning
1.	The teacher informs the students the theoretical material provided by the program (definition of concepts, formulation and proving of theorems, the essence of certain methods, problems, ideas and ways of solving them)	The teacher "brings" students to concepts, facts, ideas, creating situations in which they, on the basis of observations, independently establish the characteristic features of mathematical structures, express hypotheses, participate in the proving
2.	Mathematics emerges as a unitary and complete system of consistent, non-questionable, authoritative information	The presentation of a certain theory is followed by consideration of alternative points of view, different approaches, analysis of their advantages and disadvantages
3.	The main purpose of the practical classes is the formation of standard manipulative abilities and skills, as well as the ability to act on a model, algorithm, instruction; exercises and tasks that illustrate lecture theory prevail	The share of reproductive exercises and tasks is negligible; higher-level tasks that create the conditions for students to come up with ideas, offer alternative approaches and ways to solve, discuss, persuade and oppose, evaluate, interpret, and so on prevail
4.	Emphasis on individual learning activities (each solves the task independently)	Promotion and organization of collective research work

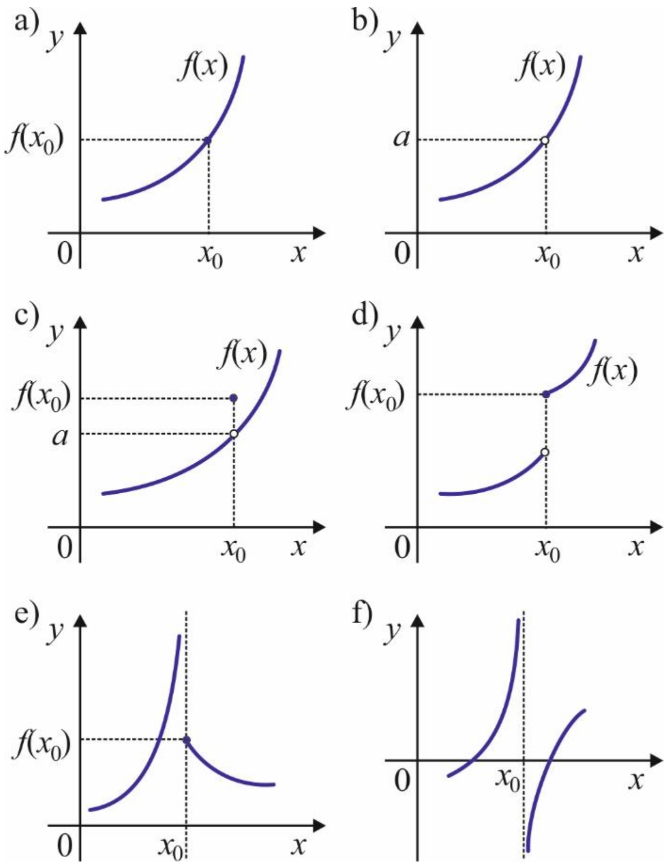


Figure 1. **Continuity of function**

The benefits of the second scenario to form not only a deeply conscious notion of continuous function at the point, but also the ability to observe, analyze, based on previously acquired knowledge (in this case, the boundaries of the function), to identify essential features, to generalize, etc. – are obvious.

Example 2. Formation of students' notion of non-native elements in extended Euclidean space as a model of projective space ("Projective Geometry and Image Methods", second year of study).

Reproductive (informative) approach. The teacher introduces the concept of projective space and formulates the basic axioms of projective geometry. Give examples of models of projective space.

A research-oriented approach. Teachers and students analyze Euclid's fifth postulate in Hilbert's formulation: "Let *l* be an arbitrary straight line and *A* be a point that does not belong to it; then in the plane defined by point *A* and line *l*, it is possible to draw not more than one line that passes through *A* and does not intersect *l*". Replacing it with negation of that statement, students come to the following conclusions: 1) it's possible to draw at least two straight lines that pass through *A* and do not intersect *l* (Lobachevsky's geometry); 2) it's not possible to draw any that passes through *A* and does not intersect *l*, and further – arbitrary lines on a plane always have a common point.

The example illustrates understandable for students and logical introduction to projective geometry. After explaining the concept of a non-peculiar element, the teacher emphasizes that the extended Euclidean space is only one of the models of projective space. Finding other models of projective spaces is also an interesting research challenge that we recommend to tackle with students. This scenario allows students to integrate different approaches to reflecting the world in geometry. Possibilities of projective geometry for the development of abstract and figurative thinking were also successfully used.

Research-oriented learning (in it's any organizational model) aims at gaining new knowledge by students (about objects, phenomena, processes, relationships, methods, actions) through the direct experience of purposefully designed experience.

Research-oriented learning can be implemented as a creative combination of different methods. We consider the most basic of them are the following three: problem-based learning, inquiry-based learning and project-based learning. Each of them has its own peculiarities. For instance, the aim of problem-based learning is *application of theoretical knowledge*, the aim of inquiry-based learning – *gaining of a new (for a student) knowledge*, the aim of project-based learning – *project implementation*.

Despite the peculiarities and differences of each of the methods (which we will not dwell on in detail), let us note their common features:

1. transfer of the initiative in the organization of his educational knowledge to a student himself; the teacher in this process is not the main source of knowledge and directives, but a manager, coordinator and partner;
2. reflective activity on both intellectual and emotional levels;
3. collective search and research work; synergy;
4. enhancement of intrinsic motivation for learning;

5. skills development, development of research procedures are a separate, independent didactic goal.

Therefore, only a research-oriented approach to learning develops cognitive abilities and provides a deep understanding of the essence of processes, cause and effect, and other relationships. With the constant improvement and widespread use in all subject areas of computer mathematical programs, these are the qualities of a specialist, rather than a simple, even very broad, set of certain knowledge and technical (operational) skills that cannot be formed once in a lifetime, will be more and more demanded in the labor market.

An important and necessary condition for successful and effective mastering of mathematical disciplines by students is a clear answer to the question "why study it?". For non-mathematics students, this should be in the context of seeing mathematical knowledge as a compulsory basis for the study of professional training disciplines in future and professional activity in various subject areas. For mathematicians, on the contrary, there must be an understanding of how a mathematician can become an indispensable specialist in the fields where it is required to have a sophisticated mathematical apparatus to analyze processes and implement the latest technologies in production, science, economics, etc.

We see the implementation of cross-curricular links traditionally through a system of interdisciplinary and applied problems and the use of mathematical modeling (Zadorozhnaya & Krasyuk, 2017; Ferri & Mousoulides, 2017). And also through some specific forms of classes in the so-called Centers of competences and within the framework of student's teaching (vocational) practices.

Let us dwell on the latter. The main purpose of creation of the Centers of competences at the University is to introduce new forms and methods of teaching that will meet the request of the parties concerned to change higher education towards greater practical orientation, the development of independence of educational recipients, the formation, in addition to professional, social competencies. The centers provide solutions not only to practical tasks but also to problems that are not always well known and require knowledge and skills acquired during the learning of different disciplines.

For example, within the discipline "Computational geometry and computer graphics", mathematics students study the geometric foundations of computer graphics using the Autodesk 3ds MAX suite, using knowledge of analytical, differential and projective geometries, and image theory. Studying of logical programming by computer science students and building of expert systems requires knowledge of mathematical logic and mechanisms of logical inference (methods of logical inference).

An interesting experience is to conduct specialty training for students in the first year of study of the specialty Mathematics and Computer Science in the hackathon format.

Hackathon as a technology competition is an example of project-based learning where teams work on an idea, design, product prototype, and present solutions within a proposed topic. The event ends by the presentation of its project by each team to the jury. During the hackathon, student teams create different devices of the Internet of things. The work is very intensive: workshops for participants, distance courses, selfstudy of the issue and, in fact, the hackathon. The company representatives evaluate the development. This form of work allows students to master a professional competencies (finding and developing solutions with rapid prototyping technologies; creating circuits with electronics, encoders and actuators; design and construction of mathematical and computer models of the device, their calculation and implementation in the form of a bulk model using different materials; building and programming devices with embedded systems and the Internet of things) and general (soft skills), as well, (systematic approach to problem solving; creativity and creation; team collaboration and management; self-presentation, effective presentation and protection of the created product; entrepreneurship, self-management, stress resistance and others).

One of the components of mathematical competence is the use of digital technologies: virtual learning environments, web-oriented educational portals, computer mathematics systems (CMS), cloud services for communication and collaboration, tools for presentation and multimedia materials, services for monitoring activity etc. In addition, their introduction into the educational process enhances the effectiveness of research-oriented and interdisciplinary and practically oriented approaches to learning.

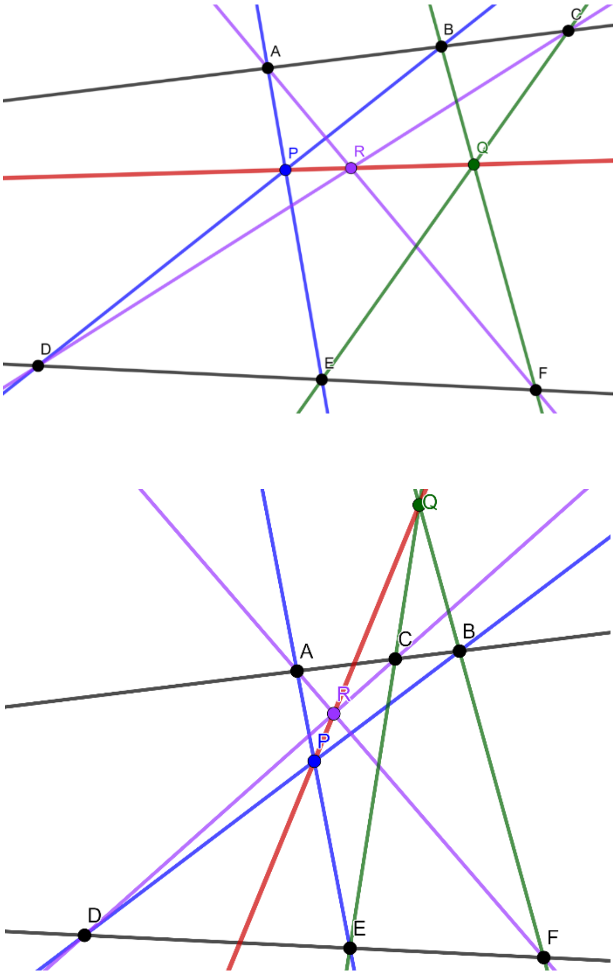
For example, to find and visualize solutions of mathematical problems, properties of geometric objects, illustrations of regularities, relations, correlations, transformations, we most often use CMS GeoGebra.

Here is an example of a mathematical experiment in using the GeoGebra system to study Pappus's projective theorem: let A, B, C be three points on one line, D, E, F – be three points on another line. Let the three straight lines AE, BF, CD intersect the three straight lines DB, EC, FA, respectively, at points P, Q, R. Then the points P, Q, R lie on one straight line (Fig. 2).

Students, by constructing the described configuration in GeoGebra and changing the position of its elements, formulate themselves the hypothesis that the points P, Q, R belong to one straight line (Figs. 2a, b, red straight line), and remain on it regardless of the position of given points at straight lines or given straight lines.

One can continue the study by considering the two straight lines as a degenerate second-order curve. Then, considering the non-degenerate oval curve with students, we come to the formulation of Pascal's theorem.

The approaches of formation of mathematical competence of students outlined by us require, in turn, modernization of the content, forms and methods of preparation of future teachers of mathematical disciplines at higher education establishments.



a)

b)

Figure 2. **Pappus configuration at different elements positions**

As an example, we take an optional discipline for Master's students of specialty "Mathematics" developed by us – "Methods of formation of students' mathematical competence for the future successful career".

The main idea that we plan to implement in the course of teaching this discipline is to familiarize students with the latest methods of teaching mathematical disciplines at the university, which based also on using a positive European experience.

The main blocks of theoretical knowledge:

1. Transformation of mathematical education to ensure a successful career for the youth in an innovative society.

2. Research-oriented learning is the basis of a competence approach and means of achieving quality mathematical education.

3. Implementation of cross-curricular relations in the study of mathematical subjects for the formation of conceptual and systematic mathematical knowledge.

4. The role of practical problems in different subject areas and mathematical modeling in the formation of professional qualities of future specialists.

5. The use of network services and cloud-based learning technologies in the process of learning mathematics.

6. STEM-education and its implementation in Ukraine. Use of STEM-technologies for formation of key and subject competencies.

A lecture was and remains the main form of theoretical studies. The didactic purpose of the lecture is to introduce students to the scientific problem (task), to reveal the main issues of the topic, to solve the given problem, to focus on the most difficult (problematic, contradictory) moments, to prepare students for further selfstudy work. In order to achieve this goal, it is necessary to achieve educational interaction "lecturer – student", aimed at mastering of educational material. Properly organized lecture makes passive learning impossible. In such a way, the basis of the students' actions for the further formation of knowledge is laid, and this happens when students directly communicate with the teacher. Therefore, by virtue of influencing the student's personality, the lecture cannot be replaced by any of the newest learning tools or sources of information with which he or she is able to work independently.

Active and fruitful interaction in the system "teacher – training material – student" can be significantly enhanced by the use of modern digital technologies. In our view, an effective use of this is a multimedia lecture. It does not simply allow the computer to transfer part of the features of the lecturer. It provides students with a better understanding of the material through a combination of verbal and visual perception. In addition, a multimedia lecture enhances learning motivation, making the lessons emotionally engaging and interesting. An example of one frame of a multimedia lecture on "Formation of mathematical competence in students of non-mathematical specialties" of our optional course is shown in Fig. 3.

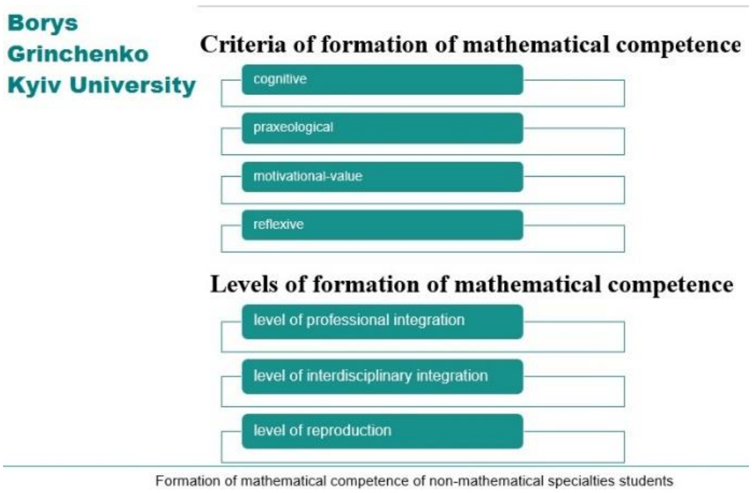


Figure 3. **Fragment of the lecture "Formation of mathematical competence of non-mathematical specialties students"**

The seminars are provided in an interactive form. These include, in particular, discussion seminars, research seminars, business games, analytical seminars, interdisciplinary seminars. For example, the course on the topic "Transforming Mathematics Education to Ensure a Successful Youth Career in an Innovative Society" is planned to take the form of a discussion seminar, where future teachers will learn how to make discussion tolerantly: express their thoughts accurately and convincingly, listen and hear the position, different from their own, actively defend their point of view, oppose reasoningly.

Seminar-research on the topic: "Formation of mathematical competence in students of mathematical specialty" provides within the limits of research training work in groups, collective search for the solution of professional tasks and problems. The main purpose of the lesson is to form students' concept of the content of the subject mathematical competence, to understand its importance for the professional activity of mathematician; development of practical skills of work of the teacher for formation of mathematical competence at students of mathematical specialties. Students are offered the following tasks:

1. Investigate the specifics of mathematical competence in mathematics, what structural components distinguish it from the mathematical competence of specialists in other spheres.

2. On the basis of comparative analysis of Ukrainian and European practices, to find out the content, forms and methods, technologies of mathematical education of students of mathematical specialties, optimal for the formation of mathematical competence in them.

3. Develop a fragment of a practical lesson on a given topic (prepare a case; plan a business game or other type of training or task) in mathematical discipline for bachelors in Mathematics.

Of particular interest to students are the lessons in the form of business games. So it is planned to hold the project "Project Method as STEM Technology".

At the analytical seminars, students have the opportunity to try themselves in four professional roles:

- participant – performer of specific actions in a certain mathematical field;
- expert – a person with a high level of knowledge in a narrow field of mathematics, conducts examination and is responsible for the accuracy and completeness of analysis, validity of conclusions in accordance with the tasks of examination;
- moderator – the person who coordinates the group work participants;
- seminar facilitator for the overall management of the seminar.

Taking into account the principle of interdisciplinarity in teaching as one of the leading in the conditions of modern higher education, we plan interdisciplinary seminars, which are jointly held by different teachers. For example, such seminar is "Networking Services, Computer Mathematics Systems, and Cloud-Based Mathematics Learning Technologies." To prepare for this seminar, students are offered a homework assignment – to prepare a project "Using Computer Mathematics (Cloud Service) in Professional Activity".

CONCLUSIONS AND PROSPECTS FOR FURTHER RESEARCH

1. An analysis of the scientific literature and the results of European research projects has shown that students' mathematical competence is the key to their successful professional self-realization and the demand in the modern labour market in an innovative society. The most important components of mathematical competence, which are conventionally divided into two groups, are considered and substantively characterized:

I. The ability to ask and answer questions in and with mathematics,

II. The ability to deal with mathematical language and tools.

2. As a result of conducting the first (ascertaining) stage of pedagogical experiment at the Borys Grinchenko Kyiv University (September, October 2019), a problematic field of study is identified, which consists in the insufficient level of students' mathematical competence formation, and also the factors that inhibit the successful study of mathematics are confirmed: insufficient school mathematical preparation for studying university courses in mathematics; superficial and formal learning of mathematics, which does not allow to apply it effectively in solving vital and professional problems; low interest of students in teaching mathematics; poor general education and research skills in a large proportion of students.

3. Effective approaches to the teaching of mathematics at the university are substantiated: research-oriented teaching; interdisciplinary approach and practical orientation; use of digital learning environments and related tools. In particular, examples of the difference between reproductive (informative) and research-oriented teaching of mathematics are described and illustrated. A form of realization of cross-curricular communication as a students' learning practice, namely hackathon, is demonstrated. Examples of the use of digital technologies for mathematical experiment are given.

4. Outlined approaches to teaching mathematics are the main content of the developed elective discipline for the Master's program students of the specialty "Mathematics" – "Methods of formation of students' mathematical competence for the future successful career". The implementation of this discipline during the second (formation) stage of pedagogical experiment, followed by an assessment of the willingness and ability of future teachers to apply these approaches in their professional activities will be the subject of further scientific research.

REFERENCES

Astafieva, M. M., Bodnenko, D. M., & Proshkin, V. V. (2019a). Cloud-oriented Training Technologies as a Means of Forming the XXI Century Skills of Future Mathematics Teachers. *CEUR Workshop Proceedings*, *2387*, 507–512. Retrieved from http://ceur-ws.org/Vol-2387/20190507.pdf.

Astafieva, M. M., Bodnenko, D. M., & Proshkin, V. V. (2019b). Using computer oriented geometry means in the process of critical thinking formation of future mathematics teachers. *Information Technologies and Learning Tools*, *71*(3), 102–121. DOI: https://doi.org/10.33407/itlt.v71i3.2449.

European Commission. (2018). *Proposal for a council recommendation on key competences for lifelong learning*. Retrieved from https://eur-lex.europa.eu/resource.html?uri=cellar:395443f6-fb6d-11e7-b8f5-01aa75ed71a1.0001.02/DOC_1&format=PDF.

Ferri, R. B. & Mousoulides, N. (2017). Mathematical modelling as a prototype for interdisciplinary mathematics education? – Theoretical reflections. In T. Dooley & G. Gueudet (Eds.), *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education (CERME10, February 1–5, 2017)* (pp. 900–907). Dublin, Ireland: DCU Institute of Education and ERME. Retrieved from http://www.mathematik.tu-dortmund.de/ieem/erme_temp/CERME10_Proceedings_final.pdf.

Goncharenko, Ya. (2011). Economical and mathematical methods and models in the system of training students of economic majors. *Didactics of mathematics: Problems and Investigations*, *36*, 48–53.

Jaworski, B. (2003). Research practice into/influencing mathematics teaching and learning development: Towards a theoretical framework based on co-learning partnerships. *Educational studies in mathematics*, *54*, 249–282. DOI: https://doi.org/10.1023/B:EDUC.0000006160.91028.f0.

Jaworski, B. (2006). Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education*, *9*, 187-211. DOI: https://doi.org/10.1007/s10857-005-1223-z.

Jaworski, B. & Matthews, J. (2011). Developing teaching of mathematics to first year engineering students. *Teaching Mathematics and Its Applications*, *30*(4), 178–185. DOI: https://doi.org/10.1093/teamat/hrr020.

MES of Ukraine. (2016). *Action Plan for the Implementation of STEM Education in Ukraine for 2016–2018*. Retrieved from https://drive.google.com/file/d/0B3m2TqBM0APKQmc4LUd2MmVFckk/view.

Michelsen, C. (2015). Mathematical modeling is also physics interdisciplinary teaching between mathematics and physics in Danish upper secondary education. *Physics Education*, *50*(4), 489–494. DOI: http://doi.org/10.1088/0031-9120/50/4/489.

Niss, M. A. (1999). Kompetencer og uddannelsesbeskrivelse (Competencies and educational description). *Uddannelse*, *9*, 21–29.

Niss, M. A. (2004). The Danish "KOM" Project and possible consequences for teacher education. In R. Strässer, G. Brandell, B. Grevholm, & O. Helenius (Eds.), *Educating for the Future: Proceedings of an International Symposium on Mathematics Teacher Education: Preparation of Mathematics Teachers for the Future*. Malmö University, Sweden, 5–7 May 2003 (pp. 179–190). Kungl. Vetenskapsakademien. KVA rapport.

OECD. (2019). *PISA 2018 Results (Volume I): What Students Know and Can Do*. Paris: OECD Publishing. DOI: https://doi.org/10.1787/5f07c754-en.

Rakov, S. (2005). *Matematychna osvita: kompetentnisnyi pidkhid z vykorystanniam IKT (Mathematical Education: A Competent Approach Using ICT)*. Kharkiv: Fact.

Recommendation of the European Parliament and of the Council of 18 December 2006 on key competences for lifelong learning (2006/962/EC). (2006). *Official Journal of the European Union*, L394. Retrieved from https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=uriserv:OJ.L_.2006.394.01.0010.01.ENG.

The New Ukrainian School. Conceptual Principles of Secondary School Reform. (2016). Retrieved from https://mon.gov.ua/storage/app/media/zagalna%20serednya/nova-ukrainska-shkola-compressed.pdf.

Zadorozhnia, T. M. & Krasiuk, Yu. M. (2017). Implementation of interdisciplinary communications through the system of applied tasks. *Physical and Mathematical Education*, *3*(13), 57–61.

Zeidmane, A. (2013). Development of Mathematics Competences in Higher Education Institutions. *International Journal of Engineering Pedagogy (iJEP)*, *3*(S2), 11–17. DOI: https://doi.org/10.3991/ijep.v3iS2.2394.

Zulkarnaen, R. (2021). Why is mathematical modeling so difficult for students? *AIP Conference Proceedings*, 060026 (2018). DOI: https://doi.org/10.1063/1.5062790.

About the authors:

Mariia Astafieva, Borys Grinchenko Kyiv University, Kyiv, Ukraine. ORCID: https://orcid.org/0000-0002-2198-4614. m.astafieva@kubg.edu.ua

Dmytro Bodnenko, Borys Grinchenko Kyiv University, Kyiv, Ukraine. ORCID: https://orcid.org/0000-0001-9303-6587. d.bodnenko@kubg.edu.ua

Oksana Lytvyn, Borys Grinchenko Kyiv University, Kyiv, Ukraine. ORCID: https://orcid.org/0000-0002-5118-1003. o.lytvyn@kubg.edu.ua

Volodymyr Proshkin, Borys Grinchenko Kyiv University, Kyiv, Ukraine. ORCID: https://orcid.org/0000-0002-9785-0612. v.proshkin@kubg.edu.ua

Oleksii Zhyltsov, Borys Grinchenko Kyiv University, Kyiv, Ukraine. ORCID: https://orcid.org/0000-0002-7253-5990. o.zhyltsov@kubg.edu.ua