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# A 'Dynamic Kibble' Mass Balance for the Undergraduate Physics Teaching Laboratory

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## Abstract

Originally envisaged in 1975 to realise the SI unit of electrical current, the Ampere, the Kibble balance has since developed into a powerhouse of modern scientific measurement. By combining theoretical simplicity with precision of measurement, it has enabled the redefinition Planck's constant, and subsequently a practical method of defining the kilogram in terms of fundamental constants. This article introduces a novel version of this classic apparatus, the 'Dynamic Kibble' Balance. Dynamic in this case because the magnet velocity is now 3 orders of magnitude higher than the original, but the same theory applies. The apparatus is simple in approach, robust, easy to set up and capable of a high level of precision using only electrical measurements (plus length and time). The importance of this measurement to metrology re-enforces the link between what is measured in the laboratory via calibration, and measurement standards and traceability. Using the apparatus and measurements described in this paper, the mass of the magnet assembly was measured as  $19.4 \pm 0.3$  g, which lies within one SEM of the known value. This paper describes an uncomplicated method with a clear focus on the key physics and theory required. This experiment is intended for use in a first-year undergraduate physics laboratory. Further potential for both more advanced theory demonstration and experimental work is discussed.

## 1. Introduction

This article is primarily of interest to educators who are interested in introductory level undergraduate Physics teaching, but will be of interest to a wider community who might wish to use the ideas for projects at a more advanced level or for pre-university projects. The apparatus described here has been designed specifically for use in a first-year undergraduate physics laboratory. This investigation is one of a set of experiments that cover mechanics, electricity and magnetism, semiconductors, optics, and acoustics. Each weekly experiment is 3 hours in duration and students are expected to plan and research the experiment before the session. Scripts, videos and helpful information are provided for them to be able to do this, and they are encouraged to view and ask questions beforehand in the laboratory or by email.

The year-long 20 credit level 1 laboratory module commences with a series of workshops that allow students to get to grips with analysis of measurement errors, simple linear fitting, planning, and recording of data. These highly scaffolded workshops, similar to the 'laborials' developed at the University of Calgary [1], are carried out in a low risk, supportive environment, with the aim to reduce any anxieties students may have when commencing an introductory experimental module and give students confidence to tackle the longer experiments, like the one described here.

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3 Students are expected to plan their experiment prior to attending the weekly laboratory activity.  
4 They carry out some background reading on the concepts and theory and demonstrate their  
5 understanding in an informal viva near the start of the experimental session, which also gives  
6 immediate feedback on the experimenter's plans for the experiment. Because of the round-robin  
7 nature of the sequence of experiments conducted, some of the concepts and theory will not be  
8 familiar from pre-university physics. In this experiment, the theory presented uses well known  
9 concepts such as force on a current carrying wire in a uniform field and introduces simple vector  
10 notation concepts such as 'divergence'. A series of leading questions form part of the script for this  
11 experiment, one of which asks the student to predict the form of the EMF generated for the falling  
12 bar-magnet pair. They will be familiar with the concept of a single bar magnet moving through a loop  
13 already. Students will see the use of theory and other ideas which they will be learning soon, as part  
14 of their course of study, although they don't have to understand these fully to complete the  
15 experiment. There are opportunities during the experiment for the experimenter to make their own  
16 informed decisions about how best to make measurements, for example how to measure the  
17 velocity of the magnet assembly, which gives scope to develop problem solving skills necessary for  
18 contemporary careers.[2] It would also be possible for this experiment and theory to be utilised at a  
19 more advanced level (in demonstrations or projects), where for example, analysis employing the  
20 magnetic vector potential could be used to derive the induced electric fields and total EMF due to a  
21 moving magnet. Although hardware and results are shown from an implementation of this  
22 experiment which is suitable for a first year undergraduate, we discuss the prospects for using data  
23 acquisition and further processing which could be carried out in more advanced laboratories or for  
24 projects.

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41 To give historical and physics context for the experiment: the Kibble balance was constructed to re-  
42 define the SI unit of current, the Ampere.[3] First introduced in 1975 by Bryan Kibble, a metrologist  
43 at the National Physical Laboratory (UK), the apparatus balances the force on a current carrying coil  
44 with the weight of a mass the user is seeking to find. Equating electrical and mechanical power in  
45 this way was not a new idea, as the Watt balance had previously been used. Prior to Kibble's work,  
46 however, the geometric factor of the balance was found using measured positions of current  
47 carrying wires in nested coils, the calculation of which was particularly irksome.[4] By a significant  
48 margin, this was also the largest contributor to uncertainty in the subsequent measurement. Kibble  
49 implemented an additional calibration step by employing a new 'velocity' mode to measure this  
50 geometric factor and therefore remove the geometrical factors from the analysis altogether. He  
51 greatly simplified the method, which allowed for several orders of magnitude improvement in its  
52 precision.[5] In 1980, the quantum Hall effect was explained by von Klitzing. [6] This discovery, in  
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3 combination with the Josephson effect which had been explained back in 1962, heralded a change in  
4 the purpose of Kibble's original design.[7] It could now be used to measure Planck's constant,  $h$ , to  
5 a previously unmatched level of precision. [8] It was increasingly clear that a new standard for the  
6 kilogram was needed utilising a redefinition of the kilogram in terms of physical constants.[9]

7  
8 Although the kilogram is now defined precisely in terms of Planck's constant, this is of little use in a  
9 standards laboratory. The Kibble balance presented the perfect solution, enabling a physical mass to  
10 be defined in terms of Planck's constant, and to the equivalent precision. The X-ray crystal density  
11 method is the only other technique that can measure mass at the kilogram level with a relative  
12 uncertainty of 1 part in  $10^8$ . [10] The importance of the Kibble balance to the science of metrology  
13 and standards cannot be overstated. An appreciation of its workings seems invaluable to any  
14 undergraduate physicist, giving an insight into the importance of metrology, experimental method  
15 and the physics of electromagnetism.

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17 Determining the EMF using the 'velocity' mode of a traditional Kibble balance is not elementary. The  
18 EMF generated is of order millivolts which is not easy to measure in the undergraduate teaching  
19 laboratory. It also requires a velocity measurement of the coil moving through the magnetic field of  
20 the order of about a millimetre per second. This can be done to high precision in a professional  
21 laboratory using interferometry [5], and so presents no obstruction to obtaining a precise  
22 measurement of the mass. This is an additional complication and outside the scope of a simple first-  
23 year laboratory experiment. We therefore present the 'dynamic Kibble' balance, where the  
24 velocities involved are 3 orders of magnitude higher, of the order  $1 \text{ ms}^{-1}$ . These velocities can be  
25 measured more easily using the breaking of a beam from an infra-red LED, together with a  
26 phototransistor detector connected to an oscilloscope. In addition, the peak EMF generated is of the  
27 order of a few volts, again leading to an easy to measure signal on the oscilloscope. In this version,  
28 the magnet 'shuttle' is free to drop at various velocities and the coil is fixed. A number of small-scale  
29 versions of the Kibble Balance can be found in the literature. One of the most impressive is the  
30 demonstration version made by National Institute of Standards and Technology (NIST) using a  
31 LEGO™ based balance, where they report precision of about 1%.[11] However, this is still a device  
32 requiring careful use, with a finely balanced pivot and fragile components, unsuitable for daily use.  
33 Other implementations use a balance arm (or wheel) with significant friction. In some there is a  
34 requirement to have wires connected to a moving coil giving rise to friction or drag.[12]

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36 The apparatus described here allows the student to conduct a relatively simple experiment and  
37 subsequently to perform simple analysis of the measurements, in the process gaining a significant  
38 appreciation for the beautifully simple physics behind one of the most influential measuring  
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3 techniques developed over the last half century. Through this experiment they can gain an insight  
4 into the science of metrology and standards, and how they are linked to the equipment in the  
5 laboratory. They do this here by using their own electrical measurements to weigh a mass, without  
6 calibration against a standard weight or dependence on a physical material property (as in Hooke's  
7 Law). The apparatus also allows for more advanced level experimental techniques and  
8 measurements, leading to further insights into electromagnetism theory.  
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## 15 2. Theory

### 16 2.1 Balance Point

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19 The determination of the balance point is to measure the current which generates a force on the  
20 magnets such that it just ceases to support the weight of the magnet. When this condition occurs,  
21 the centre of the magnet pair is centred within the coil. Consider the geometry depicted in figure 1a.  
22 Two, opposing, axially-symmetric cylindrical magnets are held apart in a shuttle frame such that a  
23 magnetic field gradient,  $G$ , in the  $z$  component of the magnetic field,  $\mathbf{B}$ , is produced at the plane  $z =$   
24  $0$  between the poles such that  $dB_z/dz = G$ . As a consequence, there is a radial field,  $B_r(r)$ , which  
25 is radially symmetric with zero  $B_\phi$  components. If  $G$  is the gradient of the  $z$  component of the  
26 magnetic field,  $\mathbf{B}$ , then the requirement of  $\nabla \cdot \mathbf{B} = 0$  and the radial symmetry of the field requires  
27 that  $B_r(r) = -rG/2$ , as can be shown in cylindrical coordinates. If the magnets are at rest and  
28 positioned symmetrically in the  $z$  direction around  $z = 0$ , then the radial field is largest in the  $z = 0$   
29 plane. This is the balance position where the total force on a current carrying loop at  $z = 0$  is,  
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$$40 \quad \mathbf{F} = \int_{\text{Vol}} \mathbf{J} \times \mathbf{B} d^3x, \quad (1)$$

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43 where  $\mathbf{J}$  is the current density, integrated over the volume. If the current,  $I$ , flows in a circular loop  
44 of radius,  $r$ , as defined by wire paths then this integral reduces to,  
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$$50 \quad F_c = 2\pi r N I B_r(r) = mg, \quad (2)$$

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52 where  $N$  is the number of turns of the loop. In turn, the reaction force at balance on the magnet  
53 shuttle of mass,  $m$ , is,  $F_M = F_c = mg$ . Note the similarity of equation (2) to the familiar form,  
54  $F = BIl$ , where  $l$  is the length of wire in a uniform magnetic field. Although this analysis is  
55 simplified, it is still true that the form of the magnetic field integrated over the wire path is the same  
56 both for balance and EMF generated and will therefore cancel out.  
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## 2.2 Generated EMF

The magnets now travel through the loop coil and the EMF is measured as shown in figure 1b. As the velocity is only a few metres per second, the quasi-static approximation can be assumed in the analysis. There is no need to accommodate any relativistic delay between the shuttle position and the magnetic field perceived at the coil. Expressing Faraday's law in its integral form (using Stokes' Theorem), the closed loop integral of the electric field,  $\mathbf{E}$ , is given by,

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = - \frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{A} , \quad (3)$$

where the surface area is defined by the vector,  $\mathbf{A}$ , bounded by the loop of length  $2\pi r$ . [13] If the  $z$ -component of magnetic field has the form of a gradient,  $G$ , then the time derivative of the total flux is given by,

$$\frac{dB_z}{dt} \pi r^2 = \frac{dB_z}{dz} \frac{dz}{dt} \pi r^2 = G v_z \pi r^2, \quad (4)$$

where  $v_z$  is the velocity of the shuttle. Hence the total peak EMF (which occurs at the same point as the balance condition),  $V$ , induced around the  $N$  turns of the coil is,

$$V = - N G v_z \pi r^2. \quad (5)$$

The expression for the EMF now becomes,

$$V = 2N B_r(r) v_z \pi r. \quad (6)$$

Again, notice the similarity to the familiar  $V = Blv$  formula. Combining equations (2) and (6) yields,

$$mg = I \frac{V}{v_z}. \quad (7)$$

If the ratio of EMF to velocity and the balance current are measured, then the mass can be calculated using the known value of  $g$ .

From Faraday's law in equation (3), the time accumulated summation of the EMF is then a measure of,  $\mathbf{B}(t) \cdot \mathbf{A}$ , and therefore the total integrated  $B_z$  over the loop area with time. This temporal summation plotted as a function of axial position of the magnet pair,  $B_z(z)$ , should be independent of the velocity of the shuttle as it depends on the spatial arrangement of the magnets and their

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3 fields only. After the magnets have both passed through the coil then the total integral should be  
4 zero.  
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### 8 **3. Methods**

#### 9 **3.1 Apparatus**

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11 The apparatus for this experiment comprised: A magnet shuttle; coil and detector assembly;  
12 controller and interface unit; oscilloscope and ammeter. A further voltmeter may be used to monitor  
13 the coil temperature.  
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19 The coil was wound on a nylon former, as shown in figure 2a, having nominally 200 turns of  
20 insulated copper wire of 0.224 mm diameter. This resulted in a net resistance of approximately 7  
21 ohms and a current of approximately 250 mA was needed to balance the magnet shuttle. This means  
22 there is a net power dissipation of less than half a watt in normal operation, and therefore the coil is  
23 unlikely to get hot. However, with higher currents potentially being set (inadvertently if a bench  
24 power supply is used) a temperature sensor (MCP9701, Minichip) is bonded to the coil using non-  
25 corrosive silicone rubber to provide a temperature measurement. The former shown in figure 2a  
26 also contains twin infra-red matched pair 860 nm emitters and detectors both above and below the  
27 coil. These are Osram SFH4346 and SFH309 FA-4 devices respectively, and although now obsolete,  
28 equivalent alternative 3 mm diameter devices are available. The authors originally used a  
29 transparent acrylic former so the experimenter could see the shuttle balance more easily. But this  
30 should be avoided as IR light travels around the wall of the former between LED and detector,  
31 making detection of an object in the beam difficult. An exit tube and an extended vertical fall tube  
32 allow the shuttle to fall from different heights determined by the experimenter to give a range of  
33 velocities. A piece of sponge rubber or similar is placed at the foot of the tube to arrest the fall of the  
34 shuttle. An additional bar magnet is useful in order to position the shuttle within the tube at the  
35 desired height before the bar magnet is pulled away and the shuttle drops. A scale can be provided  
36 to help determine the start points, and although the precise height is not important, it does provide  
37 a useful way of remembering what height to use next.  
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51 The magnet shuttle comprises four 6 mm disk magnets arranged in two opposing pairs. A spacer is  
52 used to define the gap between the poles. Threaded inserts are then used at either end to push the  
53 magnets in to rest against the spacer. Sizes and dimensions given are not critical and different  
54 configurations can be used, as the actual magnetic field magnitude and geometry do not need to be  
55 known precisely.  
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3 The controller unit works in two separate modes: current and EMF. The mode is selected by a switch  
4 which connects the coil either to a controllable power supply or routes the coil signal directly to the  
5 oscilloscope for measurement.  
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9 In current mode, a 0 – 500 mA constant current amplifier is employed, having a set point determined  
10 by a potentiometer control. The block diagram shown in figure 3 depicts the functions of the  
11 controller circuitry. An external ammeter is used to measure the current delivered to the coil. The  
12 amplifier is based on a Texas Instruments OPA344 op-amp, which allows a single rail power supply of  
13 5V to be used. The inputs of the op-amp allow operation with inputs down to zero volts – allowing  
14 the user to set zero current. A TIP32C PNP power transistor is used in a series pass configuration,  
15 and together with a 1 ohm current sense resistor and 7 ohm coil load, gives a maximum possible  
16 current of 500 mA. In addition to the temperature sensor reading voltmeter, the voltage is  
17 compared to a set-point. A comparator can be set to a suitable temperature and shut down the  
18 current supply, allowing the coil to cool.  
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27 It is not necessary to have a dedicated current amplifier if a bench supply is available. The  
28 experimenter can determine the polarity of current required and check the shuttle is balancing at  
29 the mid-point. Bench supplies may not give a level of control required, and potentially could deliver  
30 too much current, resulting in over-heating of the coil. For this equipment designed for a first-year  
31 undergraduate laboratory, having the circuit pre-wired to give the right current polarity and control  
32 required would be an advantage.  
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38 In EMF mode, the signals from the photo-transistors are buffered by 74AC14 Schmitt input inverters.  
39 Visible LEDs on the controller show the experimenter when the shuttle is positioned correctly, or  
40 flash when falling. The individual OPTO 1 and OPTO 2 outputs are available to view on an  
41 oscilloscope. These allow the experimenter to think about what is happening during fall and  
42 acceleration of the shuttle. The oscilloscope can be used to determine relative timings. These two  
43 signals are 'added' together (more correctly it is an average) at a SUM port. This is to encode both  
44 signals onto a single channel of the oscilloscope. Most labs will only have access to two-channel  
45 'scopes and the EMF measurement also requires a channel. For a more advanced laboratory these  
46 signals could be fed to an analogue acquisition system, with velocities and peak EMF determined in  
47 software. [14]  
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55 Figure 4 shows a photograph of the equipment ensemble described. Detail of the coil/detector  
56 assembly is shown together with the magnet shuttle and scale. Wiring to the opto-emitters and  
57 detectors is visible, together with the bonded temperature sensor on the coil. On the oscilloscope an  
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3 EMF trace can be seen, together with the signal from the SUM port. The shuttle was dropped from a  
4 high point for this photograph. Hence there is no appreciable asymmetry in the EMF trace and  
5 optical timing waveforms as the shuttle is approaching its terminal velocity (within a close-fitting  
6 tube).  
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### 10 **3.2 Experimental**

11 Two parameters need to be measured for determination of mass: The ratio of induced EMF to  
12 magnet shuttle velocity; and the balance point current for a given shuttle mass. The former only  
13 needs to be measured once, but (non-ferrous) masses can be added to the shuttle to allow for  
14 determination of their mass if required.  
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20 The velocities can be determined in a number of different ways, of increasing complexity. How this is  
21 done should be an exercise for the student during planning. In the lab, using the equipment, their  
22 chosen method can be enacted. As the shuttle is accelerating, then determining the velocity at the  
23 mid-point is not as simple as might be envisaged at first. From the known timing points and shuttle  
24 length the coil entry and exit velocities can be determined, and an average taken (which has been  
25 used in this paper). A slightly more advanced approach takes the acceleration into account, knowing  
26 the vertical spacing of the detectors, but is still an interpolated approximation. Most laboratory  
27 digital oscilloscopes allow the experimenter to set cursors to measure time intervals. Determining  
28 the error in these measurements is a necessary part of this measurement process. It is also possible  
29 to determine the acceleration from a position-time quadratic fit, which is then used to compute the  
30 required velocity. This is usually too time consuming for the manual approach in the laboratory, and  
31 is an exercise best conducted using analogue acquisition and numerical calculation as a more  
32 advanced experiment or project.  
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43 Once a method of determining timings and velocities has been determined by practice, the shuttle  
44 can be dropped from a range of heights and the peak EMF recorded for each velocity. The students  
45 may be expected to determine what start-point heights would result in a roughly equal horizontal  
46 spacing of velocity points on their graph. They may notice that this is not the case in practice.  
47 Students should notice how the asymmetry of the detected EMF varies with velocity.  
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52 In current mode, the balance points can be determined by positioning the shuttle in approximately  
53 the right position and setting the current to a higher strength than is required for balance. A plastic  
54 tool with a bend in it can be supplied which allows the shuttle to be raised from the bottom  
55 upwards. This is easier than dropping in from the top and expecting the current to catch the shuttle.  
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59 This process would require a high current. Again, the bar magnet placed externally to the coil former  
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3 can be used to roughly position the shuttle. With the dimensions given, it is convenient that both  
4 opto detectors show as 'on' at the mid-point, which is a good guide to correct positioning. The  
5 current is slowly and carefully reduced until the balance point is reached and the magnet drops out  
6 of the coil. The current value for several repeats can be taken. A small additional mass may be  
7 weighed using this method if it is attached to the shuttle and balance point taken. The shuttle mass  
8 can be subtracted to give the additional mass on its own. This is a useful part of the experiment as it  
9 is close in principle to the Kibble Balance principle of measuring a reference mass using electrical  
10 measurements only.  
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#### 19 4. Results

20 With care and a few trial runs, the minimum balance current can be set repeatedly and reliably to  
21 within  $\pm 1$  mA. For the apparatus described the average of 6 repeat measurements is  $222.2 \pm 0.4$  mA.  
22 During this measurement cycle, where the current was continuously set at approximately this value,  
23 the temperature rise of the coil above ambient was no more than 15 Celsius. For the magnet shuttle  
24 drop part of the experiment, a typical EMF trace is shown in figure 5 (solid line), together with the  
25 periods of the 'detected' signals from the upper and lower detectors. The EMF trace has been  
26 acquired from the digital oscilloscope. The timing points from the start and stop points (measured  
27 using timing cursors), taken together with the measured length of the shuttle of  $45.90 \pm 0.02$  mm,  
28 gives the average velocity at the mid-point. The peak EMF (which occurs at approximately the mid-  
29 point of the upper and lower detectors overlap of 'on' periods) is measured using the oscilloscope  
30 cursors. The peak EMF is plotted as a function of shuttle velocity in figure 6. An un-weighted least-  
31 squares fit of the gradient allows the calculation of the magnet shuttle mass using equation (7). The  
32 calculation based on equation (7) gives a shuttle mass of  $19.4 \pm 0.3$  g. The mass of the shuttle as  
33 weighed on laboratory scales (taking care that the magnetic field does not affect the scales) is  $19.46$   
34  $\pm 0.01$  g.  
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46 A small piece of blu-tack (Bostik) stuck to the top of the shuttle is a convenient way of attaching a  
47 small mass. This was measured by repeating the balance point part of the experiment to determine  
48 a new current value. The ratio of EMF to velocity is not repeated as there is no reason for this to  
49 change. The piece of blu-tack mass was determined to be  $1.5 \pm 0.4$  g and the weight measured using  
50 the scales was  $1.60 \pm 0.01$  g.  
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## 5. Discussion and Conclusions

The experiment described in this paper would take place in a single laboratory session. Apart from the planning of the experiment, all the measurements and analysis would fit into a 3 hour period. For the planning of the experiment, the student would be expected to assimilate the theory and principles of the experiment. For example, what EMF trace would be expected? A typical GCE A-level specification (UK) would generally expect a student to know what EMF may be expected for a single dipole magnet falling through a coil. They can build on this to determine what an opposing dipole pair would do. They would be expected to determine how the velocity at the mid-point might be determined from the timings they measure. They would plan how the analysis might be carried out, and final values calculated.

The apparatus described is designed to be easy to use and has little in the way of setting up which can go wrong. The current supply has protections e.g. maximum allowed current and over temperature cut-out. The apparatus could be used with a bench power supply which would be connected to the coil by the student. This would show how polarity matters, as with enough current the shuttle may be balanced with one end outside the coil. In a more advanced laboratory this apparatus might also be used in conjunction with analogue acquisition hardware. A suitable device might be the National Instruments USB 6216 NiDAQ used with LabView (National Instruments), python or matlab (Mathworks Inc). Whilst such a device has the capability of measuring the opto-detector timings using the built-in hardware counters, the simplest method is to use three analogue input channels to acquire these plus the coil EMF. The USB 6216 would then allow a high sampling frequency of 100 KHz on all three channels giving a timing resolution of 10  $\mu$ s. The acquisition can be triggered from the top opto-detector with a suitable pre-trigger acquisition period. This will allow for a sensible amount of data to be acquired for each drop of the magnet. The detector timing points might then be extracted from edge detection, and true acceleration and mid-point velocity computed from a quadratic fit to the distance travelled with time. This would allow a more precise measurement of the mid-point velocity. It was observed that, at lower velocities, the simple linear interpolation assumption tends to underestimate the velocity by a few percent. . The acceleration measurements could be plotted against velocity using an appropriate graph and the terminal velocity of the shuttle in the tube calculated. Although a 1 m drop is not quite long enough to achieve terminal velocity, an extrapolation of the fitted function would demonstrate this clearly. The acquired EMF voltage may be filtered using an appropriate filter of suitable bandwidth, noting that the bandwidth requirements increase with velocity. Students would need to determine the bandwidth to set by checking that the filtered waveform has not been distorted by the filtering process at the highest velocities. The same filter settings should then be used throughout. The peak

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3 height may then be determined from the maximum value. Use of a data acquisition method like this  
4 would improve the estimate of peak EMFs and velocities to better than 1%.  
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7 An additional part of the experimental analysis might be to estimate the magnitude of the radial  
8 component of the magnetic field from the shuttle at a given radial distance at  $z = 0$ . This estimate  
9 could then be compared to a measurement using a laboratory Hall probe and meter.  
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13 The graph of figure 5 also shows the integrated EMF (dashed line, normalised). This can be carried  
14 out using an oscilloscope function if available, or if the EMF signal is acquired using an ADC. In either  
15 case, care should be taken to ensure no zero offset before the integration. The trace effectively  
16 shows the total  $B_z$  across the plane area defined by the coil loop, which goes to zero at  $z = 0$  as  
17 expected. This trace should be independent of velocity and is defined by equation (3).  
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22 The mass of the magnet shuttle was determined as  $19.4 \pm 0.3$  g compared to a mass of the shuttle as  
23 weighed on laboratory scales of  $19.46 \pm 0.01$  g. We would expect students to discuss the dominant  
24 sources of random and systematic errors in the measurements, plus suggest how they may be  
25 reduced. The apparatus and experiment described do show how mass may be defined in terms of  
26 measurable electrical quantities, length and time – without calibration of the method by a known  
27 mass. In principle all these measured values can be determined from fundamental units – as in the  
28 NIST or NPL versions of the Kibble Balance. However, in the undergraduate laboratory, this level of  
29 accuracy is not feasible or even desirable (even if it could be afforded).  
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36 An affordable version of the Kibble Balance which uses identical principles has been described which  
37 is simple to construct and operate, and with three orders of magnitude higher magnet velocities  
38 allows easy measurement of both speed and EMF. There is no need for precise and low-friction  
39 balance arms or wheels in this implementation of a novel mass balance. The experiment is accessible  
40 and understandable for undergraduate students who may not have been exposed to more advanced  
41 electromagnetism concepts before carrying out the experiment. Analysis is straightforward in its  
42 simplest form, but with scope for students to take ideas further if time and more advanced analytic  
43 skills are available to them.  
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## 51 **7. Acknowledgements**

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## Figures and Captions

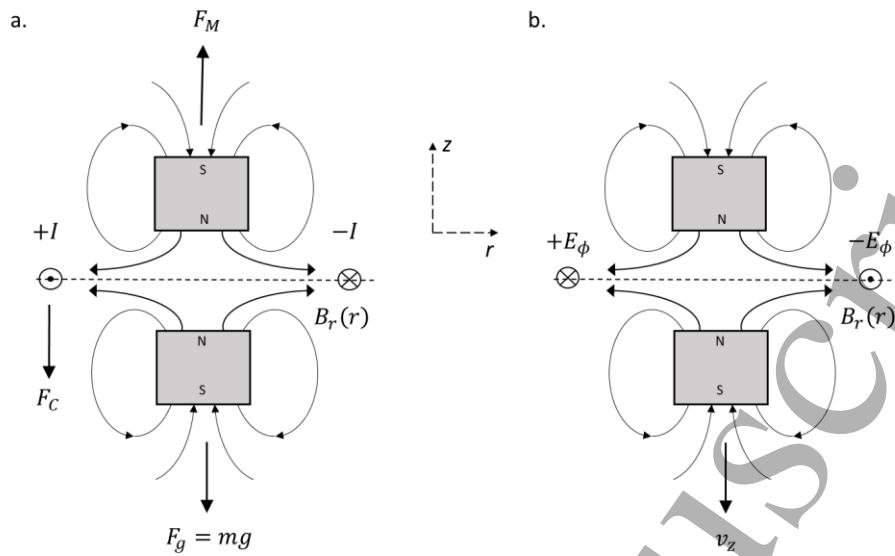


Figure 1: Two cylindrical bar magnets are fixed in a frame so that their magnetic fields are in opposition and a purely radial magnetic field is generated in the  $z = 0$  plane. A planar loop of wire is depicted at  $z = 0$  having radius,  $r$ . (a) Shows a current,  $I$ , flowing in loop which provides a force in the vertical direction which counteracts the gravitational force on the magnets' mass,  $m$ , at the balance point. (b) The magnets fall with velocity,  $v_z$ , through the loop and the EMF induced in the loop is recorded.

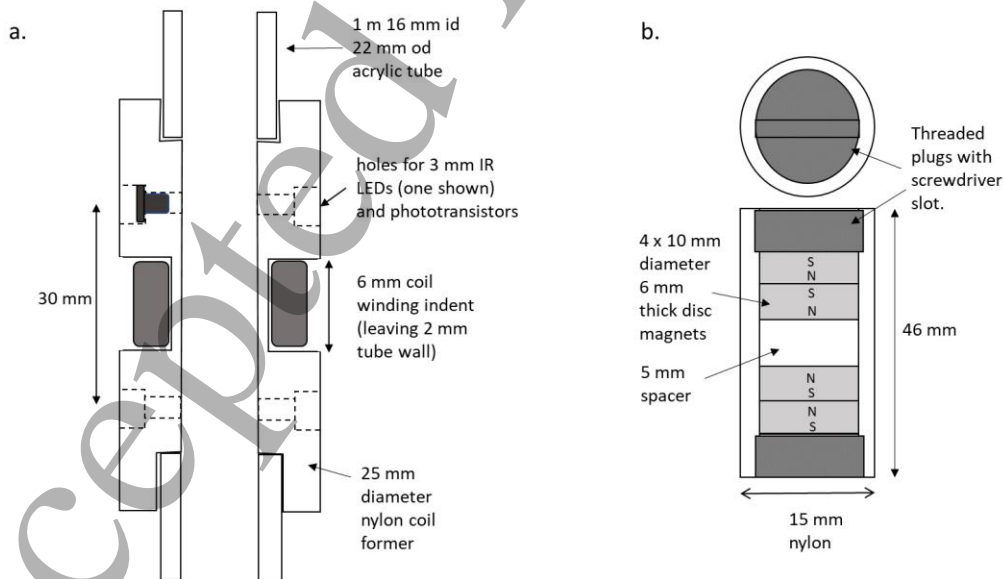


Figure 2: (a) The coil is wound on a nylon former counter bored to accommodate an acrylic tube for the magnet shuttle to pass through the coil. Diametrically opposed infra-red emitters and detectors



are mounted above and below the coil to measure the velocity of the shuttle as it falls. (b) Shows construction of the shuttle containing four 6 mm thick, 10 mm diameter, cylindrical magnets arranged in two pairs. Screw-in end caps hold the magnets in position.

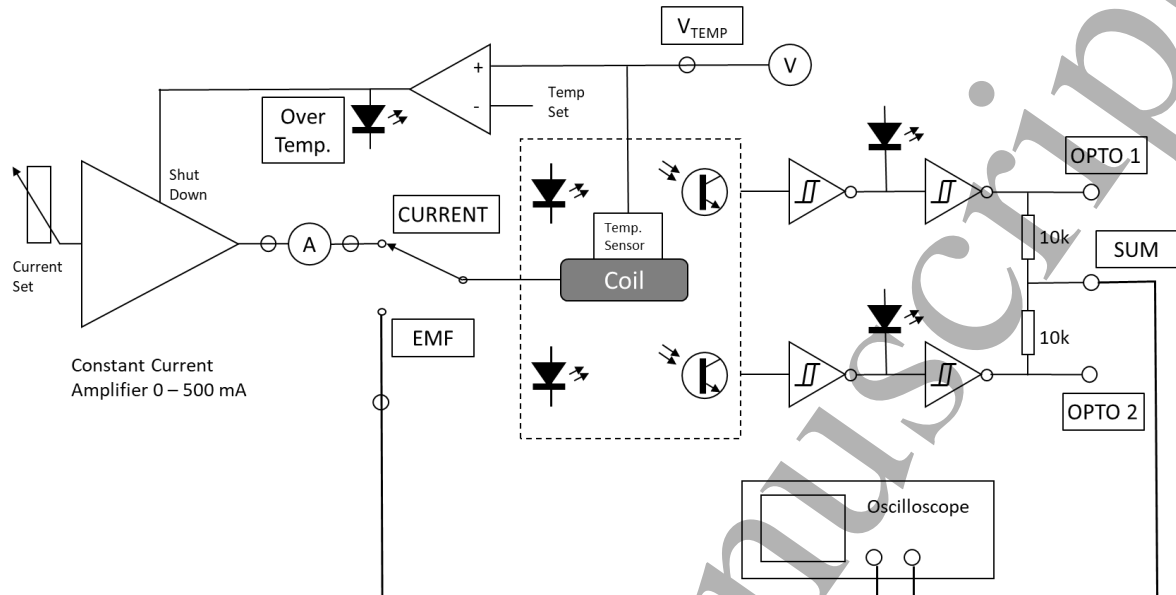


Figure 3: Functional block diagram of the experimental apparatus. The enclosed dotted components are mounted on the former shown in Fig. 2. The signals from the phototransistors are buffered before being passed to the oscilloscope. A switch connects the coil to either the oscilloscope (in EMF mode) or to a constant-current supply. A temperature sensor is bonded to the coil which can cut off the current in case of over temperature being detected.

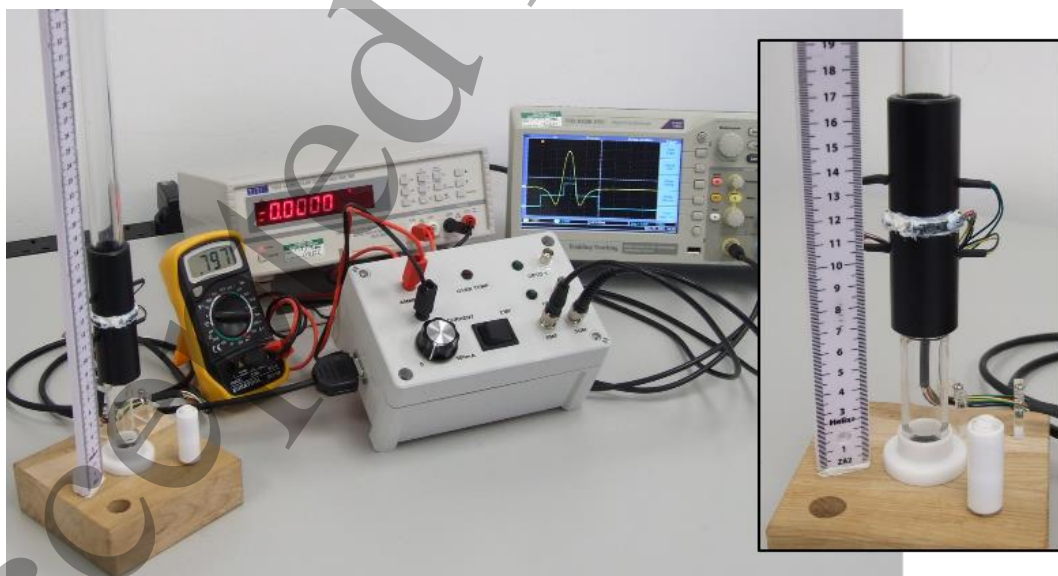


Figure 4: A photograph of the equipment assembled for use in the lab. The coil, detectors, tube and ruler scale are shown on the left. The hand-held meter displays the temperature sensor voltage. A

precision ammeter is being used to obtain 1 mA resolution of the current in the coil. The controller containing the rest of the electronics depicted in Fig. 3 is shown together with an oscilloscope which displays a typical EMF measurement. Inset is the coil and detector assembly detail with the magnet shuttle in the foreground.

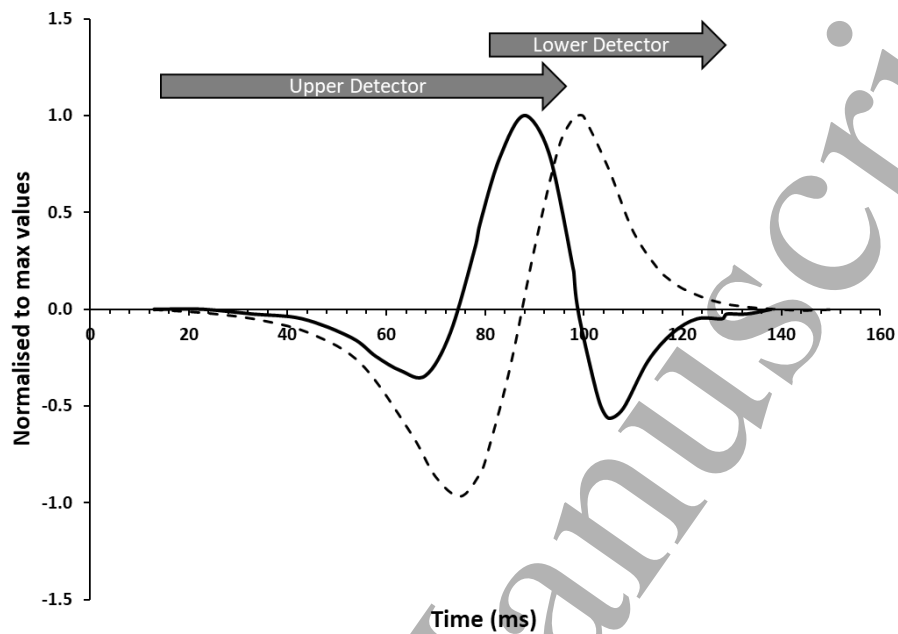


Figure 5: Graph showing a typical EMF measurement (solid line) and which depicts a slower velocity. In this case the asymmetry of the induced EMF can be seen as the shuttle is accelerating. The graph is normalised so that the integral of the EMF (dashed line) can be shown with similar amplitude. The arrows depict when the upper and lower detector pairs have their beam broken by the shuttle. The difference in timing caused by the acceleration can be observed. The mid-point velocity can be determined from the four start and end timing points.

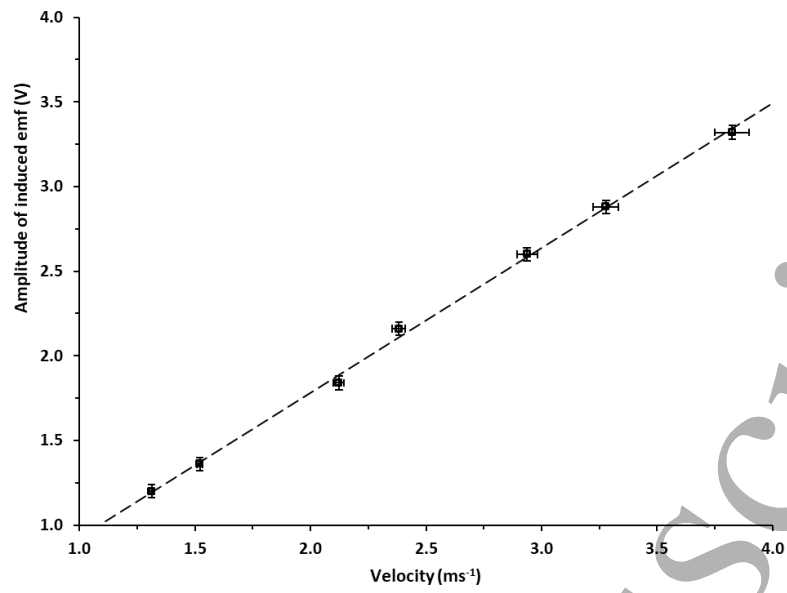


Figure 6: A graph of peak amplitude of EMF detected occurring at the mid-point ( $z = 0$  plane) against velocity at the mid-point determined from the four timing points and known dimensions of the shuttle and detector.

END