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# MODELLING DISTRIBUTION ROUTES IN CITY LOGISTICS BY APPLYING OPERATIONS RESEARCH METHODS 


#### Abstract

The article focuses on the up-to-date subject from the practical as well as scientific point of view. It specifically discusses a proposal of an approach concerning transport or distribution problems in the range of city logistics and investigates possibilities to use opted operations research methods in this particular area. Specific suggestions lie first and foremost in using selected tools of operations research (i.e. a set of methods concerning vehicle routing problem) to model multiple variants of distribution paths from a determined hub to multiple spokes in order to minimise the overall travelled distance in an urban area. As far as the very research goes, to define distribution paths to supply multiple logistics objects in the range of city logistics, ensuing methods are step by step used: Clarke-Wright algorithm, Mayer algorithm and the nearest neighbour algorithm. The article consists of a conceptual section, describing the relevant theory as well as data and methods used, the practical part and the section encompassing an assessment of the key findings, along with the discussion. A suitable combination of adequate operations research methods and their application to city logistics issues is where an innovative solution of this research lies.


## KEYWORDS

urban agglomeration; city logistics; urban distribution centre; operations research; vehicle routing problem.

## 1. INTRODUCTION

In the last two decades, logistics centres, above all those in big cities, have gained increased acceptance as fundamental for cooperation and coordination regarding the provision of city logistics activities at a high-quality level along with value-added services. Urban distribution centres (hereinafter referred to as UDC) can positively act towards
problematic freight transport in big cities. Freight transport is important for urban areas due to the following reasons [1]: goods are mainly transported by road; distances travelled are short and it is not practical to reload cargo; export of cargo produced in the city; export of waste out of the city; collection and delivery activities; warehouses operations located in city centres.

The European standard EN 14892:2005 regarding transport service, city logistics and the guidelines to define restricted entrance to centres of cities specifies several constraints in terms of entry in urban areas with big city centres and shopping zones, as well as other restricted areas, providing instructions for these associated parts and elements with a special emphasis on guidelines for freight-transport enterprises and local authorities to render transport planning and transport processes effective (optimal), precede any constraints and preserve the environment in such urban areas [2]. Emphasis is placed on several development trends causing changes in city logistics. Specifically, urban population growth, increasing e-commerce importance, rapid distribution in supply chains and increased focus on sustainability are the most topical. A whole series of city logistics policy measures have been introduced in urban agglomerations across the world. Moreover, a number of modelling instruments have been developed and implemented for the purposes of planning and assessing the aforementioned measures. Literature sources dealing with city logistics issues include, for instance, [3] and [4]. These publications provide an overview on the characteristics of the very concept of city logistics, primarily as looking for effective freight transport means in urban agglomerations when taking into consideration
negative transport effects on traffic flow, transport safety and environment. According to these authors, city logistics may be defined as an urban distribution, in particular of cargo, urban logistics, last mile logistics and/or cargo distribution in urban agglomerations.

Various city logistics concepts, measures and technologies to be implemented are at the heart of many published studies. For example, [5] presents a proposal related to planning and optimising a hub-and-spoke network within a certain urban area, whereas [6] and [7] discuss the issue of planning challenges at a city logistics scale, an integrated scheduling of logistics processes in a short-term horizon, as well as resource management involving a two-tier distribution structure.

The topics of studies and UDC as a city logistics initiative focused on minimisation of traffic associated with freight vehicles, emissions generated by lorries deployed in urban areas and air pollution from a local standpoint are addressed, for instance, in the publications [8] and [9]. In [10], van Duin el al. advise the municipality of the Hague whether the implementation of UDC is feasible and desirable based on identified circumstances leading to success or failure of UDC establishments in practice abroad. As for other examples, the authors Lindholm and Behrends in their paper [11] focus on a long-term system adjustment to traffic restrictions implemented in the particular city, whereas [12] assesses the efficiency of off-peak cargo deliveries in an urban area. They demonstrated that policies shifting cargo delivery in urban agglomerations from peak to off-peak periods can enhance cargo distribution effectiveness and mitigate negative effects of external surroundings.

Furthermore, challenges in transport network planning regarding freight transport in urban areas as well as urban travel modelling demand the use of mathematical programming methods, which may be found in [13, 14]. Unlike previous literature sources, Bu et al. apply preference methods for specifying suitable places for distribution reloading in city areas [15], and the case study [16] presents an evaluation of appropriate location in an urban centre for commodity distribution when applying a combined procedure consisting of two steps: spatial analysis, followed by multi-objective mixed-integer linear programming. Similarly, [17] and [18] utilise
a genetic algorithm linking various apparatuses in order to elaborate particular research relating to urban logistics centre location.

In regard to the imperative aspect of city logistics in the context of this article, a number of approaches concerning the concept of city logistics have been discussed in numerous publications. For example, the papers [19] and [20] deal with two-echelon capacitated vehicle routing problems (hereinafter referred to as VRP) and multi-echelon transport systems in a selected urban area. The paper [21] presents a set of crucial elements of urban economy affecting city distribution processes. Mason et al. designed a specific model related to an adaptable large-neighbourhood search to address a two-tier transport problem of cargo distribution in congested traffic flows in city centres in [22]. Lewczuk et al. in the research study on a transportation problem in an urban area [23] discuss routing freight vehicles within city logistics and propose a specific model established within a depiction of actual transport networks and addressed for certain data using twosteps heuristics.

The main objectives and contributions of this research are as follows:

- description and analysis of opted methods of operations research to deal with transport problems in city logistics;
- modelling the optimal interconnections of UDC at a city logistics network when applying certain operations research methods.


## 2. METHODS APPLIED

Distribution problems or tasks when using the operations research methods in most cases involve a simple travelling salesman problem or vehicle routing problem (sometimes referred to as multiple travelling salesman problem. VRP was first introduced by the Irish mathematician R.W. Hamilton and the British mathematician T. Kirkman in the $19^{\text {th }}$ century. However, this issue has been formulated as a mathematical problem since the 1930s, as stated in the literature [24].

VRP is defined by [25] as follows: the set $M$ is given, and for each two elements $x, y$, the number $d(x, y)$ is given, which is called distance between $x$ and $y$. The goal is to specify in which succession the salesman (or vehicle) should pass through the elements (nodes) of the $M$ set in order to go through each element just once, and then return to the point where the journey began while traveling the shortest
possible distance. In other words, we seek to rank the $M$ elements into a sequence $x_{1}, \ldots, x_{n}$, which contains each of the $M$ elements just once and the sum of $d\left(x_{1}, x_{2}\right)+d\left(x_{2}, x_{3}\right)+\ldots+d\left(x_{n-1}, x_{n}\right)+d\left(x_{n}, x_{1}\right)$ should be as small as possible.

This problem, thus, represents the type of distribution tasks where delivery of goods to customers is carried out by one or several routing (circuit) paths [26]. The objective here is to determine such sequence of visited places and possibly their inclusion into individual circuits so that each customer is visited (served) just once and the transport performance (optimisation criterion) is minimal. Customer requirements and technological limitations of vehicles must be respected. Depending on the number of circuits needed for the delivery of goods, single-circuit and multi-circuit tasks can be distinguished [27].

These methods thus deal with the issue of supplying several nodes, with the route beginning at an origin point, and after accomplishing deliveries to individual nodes, the vehicle returns to the origin point. Each node/station/customer can be operated only once, and the order of stations is not determined. However, the major objective is to find the shortest route possible. VRP deals with the most economical delivery of products from suppliers to customers in order to meet their requirements (delivery period, and so on) [28].

The general mathematical model of the VRP (see Equation 1) formulated by Miller-Tucker-Zemlin is presented, for example, by [29] and [30]
to minimise:
$z=\sum_{i=1}^{n} \sum_{j=1}^{n} c_{i j} x_{i j}$
under conditions:
$\sum_{j=1}^{n} x_{i j}=1 \quad i=1,2, \ldots, n$
$\sum_{i=1}^{n} x_{i j}=1, \quad j=1,2, \ldots, n$
$u_{i}-u_{j}+n x_{i j} \leq n-1, \quad i=1,2, \ldots, n, \quad j=1,2, \ldots, n$,
$x_{i j} \in\{0,1\}, \quad i, j=1,2, \ldots, n$
where: $n$ represents the number of locations (including the origin point) to be visited; $c_{i j}$ denotes the distance travelled between the locations $i$ and $j$ in length or time units; $x_{i j}$ is a bivalent variable having a value of 1 if the vehicle moves from place $i$ to place $j$, and a value of 0 if the conditions are vice versa; the first and second condition ensure that each location is visited just once; the third condition is a set of Tucker conditions that prevents the
creation of sub-cycles; the fourth condition ensures that the variable $x_{i j}$ can only have the values 0 and 1 and be a bivalent variable.

In view of graph theory, VRP is a classic distribution problem on the sample transport network $G=(V, E)$. The vertex indicated as $V_{0}$ is the centre of the network and can also be denoted as a central or distribution point from which material is distributed to individual delivery points. These points (places requiring an operation) are in general referred to as vertices, $V_{i}, \ldots, V_{n}$, where $n$ denotes a number of vertices [31].

As for VRP, a requirement for a certain quantity of transported cargo in vertices is specified. Cargo is carried by vehicles wherein their maximum capacity is limited, and a vehicle's route starts and finishes at the same centre $V_{0}$. The task is basically to compile a certain number of circuit routes (routing paths) in order to meet requirements of each customer point, ensure only one operation for each place and achieve the smallest possible shipping cost.

As mentioned, depending on the number of circuits needed for the delivery of goods, there are sin-gle-circuit and multi-circuit VRP distribution tasks.

Single-circuit VRP methods can be divided into [28, 32]:

- methods for broader classes of distribution tasks (e.g. patching method, Hungarian method, Vogel approximation method);
- methods based on graph theory algorithms (e.g. greedy algorithm methods, nearest neighbour method, minimal skeleton method);
- methods for Euclidean VRP (e.g. convex hull methods, Arora method);
- methods improving solutions (e.g. r-opt method, Or-opt method).
Multi-circuit VRP methods encompass [27, 33]:
- delivery task - it is an extension of the VRP concept. Here, the goal is also to start at a certain point, serve a full set of customers and return to the origin point so that the circuit is as short as possible. In this case, the capacity limitation of a vehicle and the requirements of individual customers in terms of quantity of deliveries must also be taken into account (e.g. Clarke-Wright method, Mayer method, bin packing problem method);
- tasks with time windows - these have the same principle as in the previous case, but in addition, the time interval is defined in which delivery is
to be performed, which eliminates pick-up/delivery by one Hamiltonian cycle (e.g. insertion method, minimum spanning tree method);
- tasks with split delivery - for this type of tasks, one delivery point is visited multiple times by various vehicles. In practice, this task can occur because of a large order that cannot be delivered by a single vehicle due to capacity reasons, and thereby this delivery point is included in the list of multiple circuit routes;
- tasks with multiple depots - in these cases, two or more depots are available for a distribution task, wherein each operates a different region. Each vehicle can start its journey in another depot and, at the same time, it does not have to stop a journey in the point of origin;
- tasks with simultaneous pick-up and delivery these represent an extension of the basic delivery task with an option to carry out the collection and delivery of circuit routes at the same time.
To analyse the research problem in this article, information analysis methods, including their comparison, logical induction and deduction, were used. The individual findings were interlinked into sections by the synthesis method, the mutual compatibility of which gives the reader a comprehensive overview of the individual research issues addressed. Last but not least, selected operations research methods - multiple heuristic VRP methods - namely techniques of Clarke-Wright algorithm, Mayer algorithm and the nearest neighbour algorithm (hereinafter referred to as NNA), to define efficient distribution paths in order to supply UDC when minimising the distance travelled, were applied.

An appropriate combination of operations research instruments and their application to specific areas of logistics is where the novelty and innovative solution of this research lies; i.e. it fulfils the gap in literature by addressing distribution tasks in the range of city logistics.

The Clarke-Wright algorithm, as formulated in [34, 35], is a well-known classic heuristic technique for dealing with VRP issues. This method is above all used to address the multi-circuit distribution problems in which specific restrictive circumstances splitting up the entire journey into multiple legs are to be respected, in addition to the crucial factor of distance travelled. This tool is iterative, which means that the given solution is improved by degrees when executing a certain procedure (i.e. iter-
ation steps) until it is optimal. The principle of this instrument lies in designing the fundamental paths of the entire journey (circuit). Thereafter, they are aggregated together when carrying out iterations in order to be in line with the limiting stipulations of the permissibility of the final solution [36].

These limitations encompass, for instance, not exceeding a vehicle payload, maximum path distance, quantity of vertices visited, entire journey duration and so forth. The advantage/disadvantage of aggregating two paths into a single one is given by the savings generated by such an aggregation (i.e. grouping paths together). These savings are quantified by the so-called preference coefficient $z_{i j}$ in line with the equation $z_{i j}=\left(d_{0 i}+d_{0 j}-d_{i j}\right)$, where $z_{i j}$ entails a difference between a sum of paths' lengths ( $V_{0}-V_{i}-V_{0}$ ) and $\left(V_{0}-V_{j}-V_{0}\right)$ and aggregated path length $\left(V_{0}-V_{i}-V_{j}-V_{0}\right)$. In each iteration, the Clarke-Wright algorithm aggregates those two paths to obtain the highest preference coefficient; as long as this aggregation can be created with regard to the limiting stipulations of the solution [36].

The Clarke-Wright algorithm solutions can be formulated into seven successive stages. For detailed information, see, for instance, [37].

Since one of the aims of this article is to specify an operations research technique that allows separating the entire distribution journey into multiple legs during the supply of a large number of objects from the given hub, the Clarke-Wright algorithm, being a multi-circuit VRP method for tasks with limited capacity of vehicles deployed, seems to be an efficient and appropriate instrument.

The Mayer algorithm, described as a heuristic approach in [38], is intended to address the multi-circuit transport problems with capacity limitations and a complete network of delivery paths. It is used to compile circuit routes with a selection of minimal elements, which can be used for assigning pick-up and delivery plans [39]. Addressing distribution tasks using this method is conducted in two consecutive stages. In the first stage, all the sites to be supplied are divided into groups representing individual circuit routes so that the sum of requirements within each group does not exceed the limitations of the capacity of vehicles deployed to do their work. In the second stage, it is necessary to determine such succession of supply operations (deliveries) to individual sites as to minimise the distance travelled within each circuit (by implementing some other suitable VRP methods).

When addressing problems using the Mayer algorithm, just like in the case of the Clarke-Wright algorithm, the procedure is based on a symmetric matrix of distances among all the sites involved, where they are sorted by the distance to be travelled so that the origin point (i.e. the starting location) is listed as the last site, and other sites are sorted by their distances from that origin point. In the matrix, the furthest site is listed as first, followed by less distant sites to the origin point, up to the least distant site in the matrix [40, 41].

The procedure for addressing the first stage of the Mayer algorithm is presented, for example, in [42]. This can be summarised into several successive stages, as seen in [43].

Based on the above description and following the summarised advantages of the Mayer algorithm, it can be stated that this technique seems to be an effective and suitable instrument for the distribution problem scenarios in this work.

The nearest neighbour algorithm is an operations research tool that can be applied to distribution problems wherein only one operator picks up or distributes cargo to defined objects [44]. After all the predetermined spokes are served, the vehicle gets back to the starting location. Each and every object can be operated only once. The crucial objective of this approach is to aid in seeking a solution that determines the most efficient order of supply of each object being regarded. The aim of the task is to cut down the travelled distance or the overall transport costs [45]. This classic heuristic instrument is an unsophisticated method not requiring any intricate computations. Its input sources of data comprise above all a matrix of distances respecting travelled kilometres between each operated object which are sought gradually. In line with formulations defined in the research study [46], this method is one of the efficient tools that can be applied to deal with VRPs. Its essence lies in specifying a starting location from which the optimal link to the next location needs to be detected. Such approach is repeated until all relevant objects are served. When all the locations are interlinked, the procedure will get back to the starting point. This method could be summarised in multiple steps, as seen in [47].

Following the previously mentioned statements, the NNA seems to be an appropriate tool to be implemented in the application section of this research, i.e. to determine the most efficient distribu-
tion paths in supplying predetermined objects (i.e. UDC in our case) while decreasing the overall travelled distance.

## 3. DESIGN OF MODELS

This section is aimed at an endeavour to optimise findings achieved when addressing the issue of supply of selected objects (in our case, UDC), focusing exclusively on execution of distribution journeys from a starting location to multiple objects using specific mathematical methods with a special emphasis on a certain territory within city logistics.

The researched regional public logistics centre (hereinafter $V_{0}$ ) is suggested to be located in a fictitious place with dense roads and busy freight transport, in which an industrial site out of the city centre might be located as well. Afterwards, the considered objects (urban distribution centres; hereinafter $V_{i}$ ) will be periodically operated by predetermined cargo consignments when deploying lorries with low capacity suited to distribute in the range of urban areas. These objects $V_{i}$, including the starting location $V_{0}$ as well as travelled distances between all of them given in km , are summarised in the following Table 1 - entry matrix of distances $D$. This table represents a $17 \times 17$ matrix, where each number denotes a distance travelled between two corresponding nodes given in km .

The purpose is to define the most efficient delivery sequence improving the cargo availability for each object, which means to determine the optimal distribution paths from $V_{0}$ to $V_{i}$. In order to observe the city logistics aspects, the task postulates are as follows: (a) distribution is performed by way of circuit paths; thus, it all concerns a capacity limited VRP; (b) in total, three cargo delivery lorries are available to ensure distribution (A with a payload of $3,400 \mathrm{~kg}$, B with a payload of $2,500 \mathrm{~kg}$ and C with a payload of $3,000 \mathrm{~kg}$ ) - lorry capacity limitation (i.e. available lorry payload) represents a major limiting stipulation of the transport problem addressed. The time restriction of the distribution process is set by the maximum uninterrupted period of driving, and it is defined in the EC Regulation 561/2006 as 270 minutes/journey (including periods of auxiliary activities as well as periods of all reloading operations in this case) [48]; (c) the criterion of optimisation is interpreted by the overall lorry transport performance (given in travelled distance in km ) directly associated with the transport costs; (d) the quantity of each consignment to be delivered is

Table 1 - Entry matrix of distances D (traveled distances given in km between individual objects)

| $i / j$ | $V_{0}$ | $V_{1}$ | $V_{2}$ | $V_{3}$ | $V_{4}$ | $V_{5}$ | $V_{6}$ | $V_{7}$ | $V_{8}$ | $V_{9}$ | $V_{10}$ | $V_{11}$ | $V_{12}$ | $V_{13}$ | $V_{14}$ | $V_{15}$ | $V_{16}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $V_{0}$ | 0 | 1.2 | 1.1 | 2.3 | 3.6 | 1.0 | 3.2 | 1.8 | 5.9 | 2.2 | 4.7 | 1.6 | 1.0 | 3.4 | 3.3 | 2.9 | 5.3 |
| $V_{1}$ | 1.2 | 0 | 1.3 | 1.0 | 3.3 | 1.8 | 2.0 | 1.0 | 3.2 | 1.3 | 3.5 | 0.65 | 2.1 | 3.1 | 2.0 | 1.7 | 4.1 |
| $V_{2}$ | 1.1 | 1.3 | 0 | 2.0 | 4.7 | 0.45 | 3.0 | 2.3 | 4.2 | 2.6 | 4.5 | 1.6 | 2.3 | 4.4 | 3.0 | 0.35 | 4.6 |
| $V_{3}$ | 2.3 | 1.0 | 2.0 | 0 | 3.3 | 2.5 | 1.3 | 1.1 | 2.5 | 1.4 | 2.8 | 0.8 | 2.2 | 3.2 | 1.3 | 2.4 | 4.7 |
| $V_{4}$ | 3.6 | 3.3 | 4.7 | 3.3 | 0 | 3.1 | 3.2 | 0.7 | 3.2 | 0.45 | 3.1 | 1.4 | 1.9 | 1.5 | 3.2 | 3.3 | 5.7 |
| $V_{5}$ | 1.0 | 1.8 | 0.45 | 2.5 | 3.1 | 0 | 3.4 | 2.4 | 4.6 | 2.8 | 4.9 | 2.1 | 2.7 | 5.0 | 3.4 | 3.5 | 5.1 |
| $V_{6}$ | 3.2 | 2.0 | 3.0 | 1.3 | 3.2 | 3.4 | 0 | 2.0 | 1.4 | 2.3 | 1.6 | 1.8 | 3.1 | 3.9 | 0.08 | 3.3 | 3.8 |
| $V_{7}$ | 1.8 | 1.0 | 2.3 | 1.1 | 0.7 | 2.4 | 2.0 | 0 | 3.5 | 0.35 | 3.5 | 0.75 | 1.2 | 2.1 | 2.4 | 2.6 | 5.1 |
| $V_{8}$ | 5.9 | 3.2 | 4.2 | 2.5 | 3.2 | 4.6 | 1.4 | 3.5 | 0 | 4.0 | 0.45 | 3.0 | 4.5 | 3.7 | 1.4 | 4.7 | 5.0 |
| $V_{9}$ | 2.2 | 1.3 | 2.6 | 1.4 | 0.45 | 2.8 | 2.3 | 0.35 | 4.0 | 0 | 3.2 | 1.1 | 1.6 | 1.8 | 3.3 | 3.0 | 5.8 |
| $V_{10}$ | 4.7 | 3.5 | 4.5 | 2.8 | 3.1 | 4.9 | 1.6 | 3.5 | 0.45 | 3.2 | 0 | 3.3 | 4.3 | 3.6 | 1.7 | 4.9 | 5.3 |
| $V_{11}$ | 1.6 | 0.65 | 1.6 | 0.8 | 1.4 | 2.1 | 1.8 | 0.75 | 3.0 | 1.1 | 3.3 | 0 | 2.0 | 2.9 | 2.0 | 2.2 | 4.7 |
| $V_{12}$ | 1.0 | 2.1 | 2.3 | 2.2 | 1.9 | 2.7 | 3.1 | 1.2 | 4.5 | 1.6 | 4.3 | 2.0 | 0 | 2.9 | 3.6 | 3.3 | 5.7 |
| $V_{13}$ | 3.4 | 3.1 | 4.4 | 3.2 | 1.5 | 5.0 | 3.9 | 2.1 | 3.7 | 1.8 | 3.6 | 2.9 | 2.9 | 0 | 4.0 | 4.7 | 7.7 |
| $V_{14}$ | 3.3 | 2.0 | 3.0 | 1.3 | 3.2 | 3.4 | 0.08 | 2.4 | 1.4 | 3.3 | 1.7 | 2.0 | 3.6 | 4.0 | 0 | 3.4 | 3.8 |
| $V_{15}$ | 2.9 | 1.7 | 0.35 | 2.4 | 3.3 | 3.5 | 3.3 | 2.6 | 4.7 | 3.0 | 4.9 | 2.2 | 3.3 | 4.7 | 3.4 | 0 | 4.9 |
| $V_{16}$ | 5.3 | 4.1 | 4.6 | 4.7 | 5.7 | 5.1 | 3.8 | 5.1 | 5.0 | 5.8 | 5.3 | 4.7 | 5.7 | 7.7 | 3.8 | 4.9 | 0 |

predetermined for each UDC (see Tables 3-5); (e) uniform mean lorry velocity is set at $40 \mathrm{~km} / \mathrm{h}$; (f) comparatively flat height-profile of roads in the territory under investigation is kept; (g) cargo delivery is required during night-time periods; thus, no extra waiting intervals occur given that signalling devices at junctions are switched off.

### 3.1 Model of distribution paths - original state

For the purpose of the research conducted, a single distribution day was chosen to demonstrate an example delivery process. In this scenario, a proposal of optimised distribution paths is discussed in section 3.2 when applying the Clarke-Wright algorithm, and then in section 3.3 using the Mayer algorithm (along with the NNA) for addressing VRP. Their application in terms of designing distribution paths per day is discussed as well.

A basic prerequisite for the design of distribution routes using the specific mathematical method is to create a symmetric matrix of distances for the addressed model example (see Table I). In addition to a matrix of distances, a matrix expressing jour-
ney times between each logistics object (UDC) is required as well. Such a matrix will be used to count overall journey times per day for each delivery path when applying the Clarke-Wright and Mayer algorithms. To this end, an overall journey time comprises a total period consumed while distributing shipments to clients, inclusive of cargo unloading period at clients, which is 15 min , as well as a period spent at the starting location $V_{0}$ needed for all technological operations, which is 60 min . To create a matrix of distances, mapa.cz as a simple online route-planner was utilised to find out the journey times in minutes between each object. Also, the distance matrix was created on the basis of a constant mean velocity of lorries during distribution, which is $40 \mathrm{~km} / \mathrm{h}$. A $17 \times 17$ entry matrix of times was created, however, it is not included here due to limited extent.

Figure 1 shows a graphical depiction of all the original circuit distribution paths during delivery. In this figure, the individual circuits are colour-differentiated. A path for lorry A is marked by blue, a path for lorry B is highlighted in green and a path for lorry C is marked by red.


Figure 1 - Original distribution paths of the model example

Table 2 - Attributes of the original distribution paths with deploying the lorry $A$

| Path | Distance <br> $[\mathrm{km}]$ | Demand of <br> client $[\mathrm{kg}]$ | Time of <br> distribution <br> $[\mathrm{min}]$ | Handling <br> period <br> $[\mathrm{min}]$ |
| :---: | :---: | :---: | :---: | :---: |
| $V_{0}$ | - | - | - | - |
| $V_{1}$ | 1.2 | 160 kg | 2.5 | 15 |
| $V_{2}$ | 1.3 | 600 kg | 3.0 | 15 |
| $V_{3}$ | 2.0 | 180 kg | 4.0 | 15 |
| $V_{4}$ | 3.3 | 100 kg | 7.0 | 15 |
| $V_{5}$ | 3.1 | 140 kg | 6.0 | 15 |
| $V_{7}$ | 2.4 | 200 kg | 5.0 | 15 |
| $V_{6}$ | 2.0 | 360 kg | 4.0 | 15 |
| $V_{8}$ | 1.4 | 100 kg | 3.0 | 15 |
| $V_{9}$ | 4.0 | 360 kg | 8.0 | 15 |
| $V_{0}$ | 2.2 | - | 4.5 | - |
| Total | 26 | 2,200 | 53 | 135 |

All the attributes relating to the original distribution path in which the lorry A (blue path) was deployed are summarised in Table 2. This table presents a sequence of operation of individual UDCs $V_{i}$, overall travelled distances of the distribution path as well as travelled kilometres between corresponding objects, client demands for cargo, times of distribution drive among pertinent logistics objects and handling periods at individual objects.

From the table above, the overall journey time was determined to be 248 min (including ancillary works of 60 min ). Based on the data detected, it can be stated that the model path meets all the limiting

Table 3 - Attributes of the original distribution paths with deploying the lorry $B$

| Path | Distance <br> $[\mathrm{km}]$ | Demand of <br> client $[\mathrm{kg}]$ | Time of <br> distribu- <br> tion [min] | Handling <br> period <br> $[\mathrm{min}]$ |
| :---: | :---: | :---: | :---: | :---: |
| $V_{0}$ | - | - | - | - |
| $V_{12}$ | 1.0 | 840 | 2.0 | 15 |
| $V_{10}$ | 4.3 | 360 | 8.5 | 15 |
| $V_{11}$ | 3.3 | 840 | 6.5 | 15 |
| $V_{0}$ | 1.6 | - | 3.0 | - |
| Total | 10.2 | 2,040 | 20 | 45 |

stipulations. Table 3 lists individual parameters of the second path, for which the lorry B (green route) was deployed. The table includes the same types of data as the previous path.

Based on Table 3, the overall journey time was set to 125 minutes after adding all the necessary values. Following the data obtained, it can be declared that even this model path meets all the limiting stipulations. Table 4 shows the characteristics of the third distribution path (red colour), for which the lorry C was assigned.

From Table 4, after counting all the necessary values, the overall journey time was determined as 163 minutes. On the basis of the data obtained, it can be stated that even this model original path meets all the limiting stipulations. At this stage, it is desirable to assess all meaningful aspects concerning original distribution paths executed by the corresponding lorries. Table 5 summarises all the data relevant to this assessment.

Table 4 - Attributes of the original distribution paths with deploying the lorry $C$

| Path | Distance <br> $[\mathrm{km}]$ | Demand of <br> client $[\mathrm{kg}]$ | Time of <br> distribution <br> $[\mathrm{min}]$ | Handling <br> period <br> $[\mathrm{min}]$ |
| :---: | :---: | :---: | :---: | :---: |
| $V_{0}$ | - | - | - | - |
| $V_{15}$ | 2.9 | 120 | 6.0 | 15 |
| $V_{14}$ | 3.4 | 400 | 7.0 | 15 |
| $V_{16}$ | 3.8 | 700 | 7.5 | 15 |
| $V_{13}$ | 7.7 | 140 | 15.5 | 15 |
| $V_{0}$ | 3.4 | - | 7.0 | - |
| Total | 21.2 | 1,360 | 43 | 60 |

Table 5 - Assessment of all the original distribution paths

| Value of the criterion of optimisation | 57.4 km |
| :--- | :---: |
| Quantity of distribution paths | 3 |
| Quantity of lorries deployed | 3 |
| Lorry A payload utilisation | $64.71 \%$ |
| Lorry B payload utilisation | $81.60 \%$ |
| Lorry C payload utilisation | $45.33 \%$ |
| Lorries mean payload utilisation | $63.88 \%$ |

### 3.2 Model of optimised paths using the Clarke-Wright algorithm

This section presents the optimised distribution paths for the model example when applying the Clarke-Wright algorithm. This is done according to individual steps (section 2). The first 2 stages are focused on creating an entry matrix of distances $D$ (see Table 1). To this end, the next phase of formulating an elementary solution of the assignment can be approached directly, which means to determine the fundamental paths from the starting location $\left(V_{0}\right)$ to each UDC $\left(V_{i}\right)$ and return journey. As for fundamental paths, it is necessary to define the criterion of optimisation and limiting stipulations of the addressed task as well [46]. The fundamental paths of the model example and their relating attributes are shown in Table 6. These parameters include: $l_{i}$ - length of the path section (km); $l$-overall transport performance; $q_{i}-$ client demand quantity (kg); $t_{i j}$ - time of distribution drive (min); $t_{v}$ - handling period at individual UDCs (min); $t_{c}-$ an overall journey time (min).

After determining the fundamental paths, a fol-low-up phase of the Clarke-Wright algorithm is the formulation of a preference coefficient matrix denoted as $Z=\left\{z_{i j}\right\}$ in accordance with the formula $z_{i j}=\left(d_{0 i}+d_{0 j}-d_{i j}\right)$, expressing a difference between

Table 6 - Fundamental paths of the model example and their attributes

| Paths | $l_{i}[\mathrm{~km}]$ | $q_{i}[\mathrm{~kg}]$ | $t_{i j}[\mathrm{~min}]$ | $t_{v}[\mathrm{~min}]$ | $t_{c}[\mathrm{~min}]$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $V_{0}-V_{1}-V_{0}$ | 2.4 | 160 | 5 | 15 | 80 |
| $V_{0}-V_{2}-V_{0}$ | 2.2 | 600 | 4 | 15 | 79 |
| $V_{0}-V_{3}-V_{0}$ | 4.6 | 180 | 9 | 15 | 84 |
| $V_{0}-V_{4}-V_{0}$ | 7.2 | 100 | 15 | 15 | 90 |
| $V_{0}-V_{5}-V_{0}$ | 2.0 | 140 | 4 | 15 | 79 |
| $V_{0}-V_{6}-V_{0}$ | 6.4 | 360 | 13 | 15 | 88 |
| $V_{0}-V_{7}-V_{0}$ | 3.6 | 200 | 8 | 15 | 83 |
| $V_{0}-V_{8}-V_{0}$ | 11.8 | 100 | 24 | 15 | 99 |
| $V_{0}-V_{9}-V_{0}$ | 4.4 | 360 | 9 | 15 | 84 |
| $V_{0}-V_{10}-V_{0}$ | 9.4 | 360 | 19 | 15 | 94 |
| $V_{0}-V_{11}-V_{0}$ | 3.2 | 840 | 6 | 15 | 81 |
| $V_{0}-V_{12}-V_{0}$ | 2.0 | 840 | 4 | 15 | 79 |
| $V_{0}-V_{13}-V_{0}$ | 6.8 | 140 | 14 | 15 | 89 |
| $V_{0}-V_{14}-V_{0}$ | 6.6 | 400 | 13 | 15 | 88 |
| $V_{0}-V_{15}-V_{0}$ | 5.8 | 120 | 12 | 15 | 87 |
| $V_{0}-V_{16}-V_{0}$ | 10.6 | 700 | 22 | 15 | 97 |

the sum of lengths of two fundamental paths and a length of a path formed by aggregating them. The resulting $Z$ matrix is given in Table 7 .

Now, we can proceed to the proposal of delivery paths itself. This is conducted as an iteration process, in which fundamental paths are gradually aggregated together on the basis of individual values of preference coefficients listed in the $Z$ matrix. At the same time, the permissibility of aggregating pertinent paths is verified continuously in individual iterations, which means to check whether an aggregated path is in line with all the limiting stipulations, and whether an aggregation does not merge 2 marginal locations of the path. Hence, the very first phase of the procedure is to search for the highest value of all the coefficients in a matrix, which is in our case represented by a joint of UDCs $V_{8}$ and $V_{10}$ with the highest value of 10.15 . Thus, the possibility to connect these two locations needs to be investigated. In total, 23 iterations were performed; nevertheless, due to the considerable range of this process, only the findings are shown in this article (see Table 8).

Following the table above, in total, two optimised distribution paths were created to operate individual clients (in our case, UDC) by using the Clarke-Wright algorithm. In order to provide for the first path, it is needed to deploy the lorry A given the weight of cargo carried and as well as the lorry capacity, and as far as the second path goes, it is meaningful to use the delivery lorry B, for which a maximum payload will be utilised effectively.

A graphical representation of both distribution paths is shown in Figure 2. The circuit path for the lorry A is highlighted in blue and the lorry B path is marked in red.

### 3.3 Model of optimised paths using the Mayer algorithm and NNA

In this chapter, the original distribution paths for the model example are optimised using the Mayer algorithm whereby individual delivery sequences will be determined by the technique of NNA. This will be done in accordance with the procedure specified in section 2 .

Table 7 - Preference coefficient matrix Z (relating to Clarke-Wright algorithm)

| $i / j$ | $V_{1}$ | $V_{2}$ | $V_{3}$ | $V_{4}$ | $V_{5}$ | $V_{6}$ | $V_{7}$ | $V_{8}$ | $V_{9}$ | $V_{10}$ | $V_{11}$ | $V_{12}$ | $V_{13}$ | $V_{14}$ | $V_{15}$ | $V_{16}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $V_{1}$ | 0 | 1.0 | 2.5 | 1.5 | 0.4 | 2.4 | 2.0 | 3.9 | 2.1 | 2.4 | 2.15 | 0.1 | 1.5 | 2.5 | 2.4 | 2.4 |
| $V_{2}$ |  | 0 | 1.4 | 0.0 | 1.65 | 1.3 | 0.6 | 2.8 | 0.7 | 1.3 | 1.1 | -0.2 | 0.1 | 0.4 | 3.65 | 1.8 |
| $V_{3}$ |  |  | 0 | 2.6 | 0.8 | 4.2 | 3.0 | 5.7 | 3.1 | 4.2 | 3.1 | 1.1 | 2.5 | 4.3 | 2.8 | 2.9 |
| $V_{4}$ |  |  |  | 0 | 1.5 | 3.6 | 4.7 | 6.3 | 5.35 | 5.2 | 3.8 | 2.7 | 5.5 | 3.7 | 3.2 | 3.2 |
| $V_{5}$ |  |  |  |  | 0 | 0.8 | 0.4 | 2.3 | 0.4 | 0.8 | 0.5 | -0.7 | -0.6 | 0.9 | 0.4 | 1.2 |
| $V_{6}$ |  |  |  |  |  | 0 | 3.0 | 7.7 | 3.1 | 6.3 | 3.0 | 1.1 | 2.7 | 6.42 | 2.8 | 4.7 |
| $V_{7}$ |  |  |  |  |  |  | 0 | 4.2 | 3.65 | 3.0 | 2.65 | 1.6 | 3.1 | 2.7 | 2.1 | 2.0 |
| $V_{8}$ |  |  |  |  |  |  |  | 0 | 4.1 | 10.15 | 4.5 | 2.4 | 5.6 | 7.8 | 4.1 | 6.2 |
| $V_{9}$ |  |  |  |  |  |  |  |  | 0 | 3.7 | 2.7 | 1.6 | 3.8 | 2.2 | 2.1 | 1.7 |
| $V_{10}$ |  |  |  |  |  |  |  |  |  | 0 | 3.0 | 1.4 | 4.5 | 6.3 | 2.7 | 4.7 |
| $V_{11}$ |  |  |  |  |  |  |  |  |  |  | 0 | 0.6 | 2.1 | 2.9 | 2.3 | 2.2 |
| $V_{12}$ |  |  |  |  |  |  |  |  |  |  |  | 0 | 1.5 | 0.7 | 0.6 | 0.6 |
| $V_{13}$ |  |  |  |  |  |  |  |  |  |  |  |  | 0 | 2.7 | 1.6 | 1.0 |
| $V_{14}$ |  |  |  |  |  |  |  |  |  |  |  |  |  | 0 | 2.8 | 4.8 |
| $V_{15}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 0 | 3.3 |
| $V_{16}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 0 |

Table 8 - Optimised distribution paths for the lorry $A$ and $B$ when applying Clarke-Wright algorithm

| Paths | Lorry | $l[\mathrm{~km}]$ | $q[\mathrm{~kg}]$ | $t_{i j}[\mathrm{~min}]$ | $t_{v}[\mathrm{~min}]$ | $t_{c}[\mathrm{~min}]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $V_{0}-V_{1}-V_{3}-V_{6}-V_{8}-V_{10}-V_{14}-V_{16}-V_{15}-V_{2}-V_{5}-V_{0}$ | A | 17.55 | 3,120 | 36.5 | 150 | 246.5 |
| $V_{0}-V_{11}-V_{7}-V_{9}-V_{4}-V_{13}-V_{12}-V_{0}$ | B | 8.55 | 2,480 | 17.5 | 90 | 167.5 |



Figure 2 - Optimised distribution paths for both lorries when applying Clarke-Wright algorithm

This method must be implemented in 2 elementary consecutive phases. In phase 1 , the UDCs will be divided into individual circuit paths, and as for the phase 2 , the succession of deliveries to each object will be defined for each designed path so that the distance travelled is as short as possible [29]. First of all, we assign UDCs to delivery paths. To this end, a table of rates based on the entry matrix of distances given in Table 1 needs to be compiled (see Table 9). In this table, the distances travelled among the UDCs are expressed in the same way as the matrix of distances. However, the UDCs are sorted in descending order by the distance travelled from the starting location $V_{0}$ which is not included in the table per se. The table also comprises a column indicating client demand from the corresponding line.

Subsequently, we may continue to the process of assigning the objects (UDCs) to individual distribution paths. It is done through iterations according to steps 2-9 (see section 2 ). When executing iterations, it is required to monitor whether any of the limiting stipulations are violated by incorporating UDC into the specific delivery path. In order to continuously observe the time constraints of the task, the default succession of supply of clients within the circuit paths will also be defined in each iteration. Now, the very iteration process can be initiated. In total, 13 iterations for the distribution path no. 1 and 4 iterations for the distribution path no. 2 were executed, and again,
due to the considerable range, only the outcomes from the conducted iterations are listed as follows (see Table 10 ).

As mentioned, it is reasonable to supplement the very assignment of objects to individual distribution paths when using the Mayer algorithm by a different technique to deal with the single-circuit transport problem. By using such method, the order of individual locations to be served during single-circuit route is modified. While delivering, the succession of visited nodes needs to be put into order so that the distance travelled is as short as possible. For this purpose, the NNA will be used in the following section [49].

To define the supply succession to individual clients, this heuristic technique searching for the shortest possible single-circuit distribution path will be used. This algorithm is simple and does not need complicated calculations. A matrix of distances between individual vertices for the designed paths is used as a data source, which is gradually analysed [50]. At the beginning of the procedure, the starting location is defined. The closest unused vertex is identified in a matrix of distances for the starting location, and this link is listed in the continuous solution of the distribution path. Step by step, this procedure is applied to all the vertices, and after finding the last vertex, it is connected to the starting point [45]. Thereby, the entire path is completed.

Table 9 - Table of rates of Mayer algorithm (distances between objects in km along with a demand of client in kg )

| $i / j$ | $V_{8}$ | $V_{16}$ | $V_{10}$ | $V_{4}$ | $V_{13}$ | $V_{14}$ | $V_{6}$ | $V_{15}$ | $V_{3}$ | $V_{9}$ | $V_{7}$ | $V_{11}$ | $V_{1}$ | $V_{2}$ | $V_{12}$ | $V_{5}$ | Demand $[\mathrm{kg}]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $V_{8}$ | 0 | 5.0 | 0.45 | 3.2 | 3.7 | 1.4 | 1.4 | 4.7 | 2.5 | 4.0 | 3.5 | 3.0 | 3.2 | 4.2 | 4.5 | 4.6 | 100 |
| $V_{16}$ | 5.0 | 0 | 5.3 | 5.7 | 7.7 | 3.8 | 3.8 | 4.9 | 4.7 | 5.8 | 5.1 | 4.7 | 4.1 | 4.6 | 5.7 | 5.1 | 700 |
| $V_{10}$ | 0.45 | 5.3 | 0 | 3.1 | 3.6 | 1.7 | 1.6 | 4.9 | 2.8 | 3.2 | 3.5 | 3.3 | 3.5 | 4.5 | 4.3 | 4.9 | 360 |
| $V_{4}$ | 3.2 | 5.7 | 3.1 | 0 | 1.5 | 3.2 | 3.2 | 3.3 | 3.3 | 0.45 | 0.7 | 1.4 | 3.3 | 4.7 | 1.9 | 3.1 | 100 |
| $V_{13}$ | 3.7 | 7.7 | 3.6 | 1.5 | 0 | 4.0 | 3.9 | 4.7 | 3.2 | 1.8 | 2.1 | 2.9 | 3.1 | 4.4 | 2.9 | 5.0 | 140 |
| $V_{14}$ | 1.4 | 3.8 | 1.7 | 3.2 | 4.0 | 0 | 0.08 | 3.4 | 1.3 | 3.3 | 2.4 | 2.0 | 2.0 | 3.0 | 3.6 | 3.4 | 400 |
| $V_{6}$ | 1.4 | 3.8 | 1.6 | 3.2 | 3.9 | 0.08 | 0 | 3.3 | 1.3 | 2.3 | 2.0 | 1.8 | 2.0 | 3.0 | 3.1 | 3.4 | 360 |
| $V_{15}$ | 4.7 | 4.9 | 4.9 | 3.3 | 4.7 | 3.4 | 3.3 | 0 | 2.4 | 3.0 | 2.6 | 2.2 | 1.7 | 0.35 | 3.3 | 3.5 | 120 |
| $V_{3}$ | 2.5 | 4.7 | 2.8 | 3.3 | 3.2 | 1.3 | 1.3 | 2.4 | 0 | 1.4 | 1.1 | 0.8 | 1.0 | 2.0 | 2.2 | 2.5 | 180 |
| $V_{9}$ | 4.0 | 5.8 | 3.2 | 0.45 | 1.8 | 3.3 | 2.3 | 3.0 | 1.4 | 0 | 0.35 | 1.1 | 1.3 | 2.6 | 1.6 | 2.8 | 360 |
| $V_{7}$ | 3.5 | 5.1 | 3.5 | 0.7 | 2.1 | 2.4 | 2.0 | 2.6 | 1.1 | 0.35 | 0 | 0.75 | 1.0 | 2.3 | 1.2 | 2.4 | 200 |
| $V_{11}$ | 3.0 | 4.7 | 3.3 | 1.4 | 2.9 | 2.0 | 1.8 | 2.2 | 0.8 | 1.1 | 0.75 | 0 | 0.65 | 1.6 | 2.0 | 2.1 | 840 |
| $V_{1}$ | 3.2 | 4.1 | 3.5 | 3.3 | 3.1 | 2.0 | 2.0 | 1.7 | 1.0 | 1.3 | 1.0 | 0.65 | 0 | 1.3 | 2.1 | 1.8 | 160 |
| $V_{2}$ | 4.2 | 4.6 | 4.5 | 4.7 | 4.4 | 3.0 | 3.0 | 0.35 | 2.0 | 2.6 | 2.3 | 1.6 | 1.3 | 0 | 2.3 | 0.45 | 600 |
| $V_{12}$ | 4.5 | 5.7 | 4.3 | 1.9 | 2.9 | 3.6 | 3.1 | 3.3 | 2.2 | 1.6 | 1.2 | 2.0 | 2.1 | 2.3 | 0 | 2.7 | 840 |
| $V_{5}$ | 4.6 | 5.1 | 4.9 | 3.1 | 5.0 | 3.4 | 3.4 | 3.5 | 2.5 | 2.8 | 2.4 | 2.1 | 1.8 | 0.45 | 2.7 | 0 | 140 |

Table 10 - Assignment of UDCs to distribution paths for both lorries when applying Mayer algorithm

| Paths | Lorry | $q_{i}[\mathrm{~kg}]$ | $t_{i j}[\mathrm{~min}]$ | $\mathrm{t}_{v}[\mathrm{~min}]$ | $t_{c}[\mathrm{~min}]$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $V_{0}-V_{8}-V_{10}-V_{6}-V_{14}-V_{3}-V_{11}-V_{1}-V_{7}-V_{9}-V_{4}-V_{13}-V_{0}$ | A | 3,200 | 37 | 165 | 262 |
| $V_{0}-V_{16}-V_{15}-V_{2}-V_{5}-V_{12}-V_{0}$ | B | 2,400 | 30.5 | 75 | 165.5 |



Figure 3 - Optimised distribution paths for both lorries by applying Mayer and the nearest neighbour algorithms

Thus, after compiling paths (see Table 10), individual steps are carried out according to the procedure (see section 2). However, given the substantial scope of the whole procedure, only the outcomes are indicated below.

Following the investigation using the NNA, the optimal distribution path no. 1 when deploying the lorry A is as follows: $V_{0}-V_{3}-V_{11}-V_{1}-V_{7}-V_{9}-$ $V_{4}-V_{13}-V_{10}-V_{8}-V_{6}-V_{14}-V_{0}$ or $V_{0}-V_{7}-V_{9}$ $-V_{4}-V_{11}-V_{1}-V_{3}-V_{6}-V_{14}-V_{8}-V_{10}-V_{13}-V_{0}$, wherein for both variants, the criterion of optimisation is 15.88 km . The optimal distribution path no. 2 when deploying the lorry B is as follows: $V_{0}-V_{12}$ $-V_{5}-V_{2}-V_{15}-V_{16}-V_{0}$, wherein the total distance travelled is 14.70 km . A graphical illustration of both paths is depicted in Figure 3, in which the route no. 1 for the lorry A is marked in blue and the delivery path no. 2 for the lorry B is highlighted in red.

Based on proper analysis of individual paths, the final evaluation as well as discussion of the entire distribution process designed by applying the Clarke-Wright and consequently Mayer algorithms along with the NNA can be carried out (see the following section).

## 4. FINAL EVALUATION AND DISCUSSION

### 4.1 Evaluation of the limiting stipulations

First of all, it is necessary to assess whether the design of distribution paths by using opted methods complies with all the predetermined limiting stipulations. This evaluation was already carried out in individual designs in section 3. Thus, all the values of two specified limitations (i.e. lorry payload and overall journey time) achieved when modelling several cases of distribution paths using each method were taken into account. Following the conducted modelling process, all the stipulations were observed in both scenarios.

### 4.2 Evaluation of the optimisation criterion

Here, it is of highest priority to evaluate whether the designed scenarios of distribution paths by using all the methods results in a reduction of the total transport performance required for execution of the proposed distribution compared to the original state. The overall distance travelled is therefore considered as the chosen optimisation criterion. Based on its value decrease in comparison with the delivery initial status, the efficiency of the designed distribution process by applying the opted operations research techniques can be assessed. To this end, Table 11 is compiled to illustrate the overall transport performance obtained by each method.

In the table above, by using all the methods chosen for modelling the distribution paths, the transport performance is reduced in comparison with the original status. The best outcome is achieved by applying the Clarke-Wright algorithm, which reduces the overall original transport performance by 31.3 km . The efficiency of this technique may be attributed primarily to the fact that it is a method appropriate to address multi-circuit distribution problems with various types of limiting stipulations, including those set out in this publication. Unlike the Mayer algorithm, its potential may be thus fully utilised when designing delivery paths.

### 4.3 Evaluation of the lorry capacity utilisation

The capacity utilisation of lorries deployed in distribution needs to be evaluated as well. To this end, utilisation of a lorry payloads separately as well as the average payload utilisation of all the lorries together was assessed. To clearly evaluate this parameter, Table 12 is compiled, showing the percentage utilisation of lorry payload during distribution designed by each method and the original state.

Table 11 - Comparison of the total distance travelled

| Criterion of optimisation | Original state | Clarke-Wright | Mayer algorithm and NNA |
| :---: | :---: | :---: | :---: |
| Quantity of lorries | 3 | 2 | 2 |
| Travelled distance $[\mathrm{km}]$ | 57.4 | 26.1 | 30.58 |
| A | 26 | 17.55 | 15.88 |
| B | 10.2 | 8.55 | 14.70 |
| C | 21.2 | - | - |

Table 12 - Comparison of the lorries' capacity utilisation

| Investigated parameter | Original state | Clarke-Wright | Mayer and NNA | Max. value |
| :---: | :---: | :---: | :---: | :---: |
| Quantity of lorries | 3 | 2 | 2 | 3 |
| Lorry payload utilisation | $63.88 \%$ | $95.48 \%$ | $95.06 \%$ | $100 \%$ |
| A | $2,200 \mathrm{~kg}(64.71 \%)$ | $3,120 \mathrm{~kg}(91.76 \%)$ | $3,200 \mathrm{~kg}(94.12 \%)$ | $3,400 \mathrm{~kg}$ |
| B | $2,040 \mathrm{~kg}(81.60 \%)$ | $2,480 \mathrm{~kg}(99.20 \%)$ | $2,400 \mathrm{~kg}(96.00 \%)$ | $2,500 \mathrm{~kg}$ |
| C | $1,360 \mathrm{~kg}(45.33 \%)$ | - | - | $3,000 \mathrm{~kg}$ |

In the table above, we can see at first glance that the modelling process of distribution paths using the selected operations research tools in all the cases resulted in a substantial increase in the capacity utilisation of the lorries A and B , as well as the average capacity utilisation compared to the original state. The increase in average capacity utilisation is mainly due to the fact that the lorry C was not deployed during the optimisation process. The best average payload utilisation of deployed lorries was achieved with the Clarke-Wright algorithm reaching a value of $95.48 \%$ (i.e. an increase of $31.6 \%$ compared to the original status), which is only slightly more compared to when implementing the Mayer technique ( $95.06 \%$ ).

## 5. CONCLUSION

This research work designed various delivery scenarios by applying adequate mathematical methods for planning distribution paths in the context of a transport problem, enabling to minimise the overall distance travelled when supplying urban distribution centres. Specifically, vehicle routing problem techniques were implemented as follows: ClarkeWright algorithm, Mayer algorithm in combination with the nearest neighbour algorithm. Thereafter, the findings were compared with the initial status of distribution paths.

The partial benefits of the manuscript lie above all in (a) a description of specific attributes in relation to the concept of city logistics and elaboration of an in-depth analysis of existing literature in the topic, as well as (b) a summary of several known methods of operations research focused on solving transport problems (VRP) in the range of city logistics.

The major contribution of this research publication consists in modelling the transport problem presented via a specific approach of distribution process in city logistics when starting from an initial logistics object (designated as $V_{0}$ ) in order to determine the optimal distribution paths for operating
multiple distribution centres in urban area. The key objective of this research lies in an effort towards minimising the distance travelled, denoted as the optimisation criterion when applying several VRP techniques.

Based on key findings from the review of literature dealing with issue of distribution task optimisation in relation to the concept of city logistics (see section 1), it can be stated that there are as of yet no scientific papers discussing a similar subject while comparing the application of multiple VRP methods. It is precisely the suitable and effective combination of adequate operations research methods and their application to city logistics issues where the novelty of this work lies; it fills the gap in the literature in terms of addressing the distribution tasks in city logistics. Moreover, the obtained results may be applied even to needs of state government or regional government, logistics service providers, developers, decision-makers and experts in the research topic, as well as other entities that can use the acquired knowledge for their needs.

Moreover, such types of transport problems can be successfully implemented to further analogical tasks to a greater or lesser extent and ought to be investigated even more thoroughly. Further research can therefore aim especially at the ensuing topics.

Introduction of efficient telematics applications or other information systems ought to be regarded as another suitable recommendation in terms of distribution tasks. These technologies are essential in the process of logistics services provision and transport process management, as well as their connection to both internal and external environments. The importance of this subject is given by the fact that an efficient logistics system will involve a large number of interdependent entities, whose cooperation must be coordinated by means of sophisticated technical apparatuses. To do this, it is necessary to propose the concept of telematics interconnection of online information related to several transport modes and types of logistics services (i.e. their
optimal deployment, utilisation of their capacities with regard to transport infrastructure capacity, entry prices - fuel, tolls, charges for infrastructure with respect to environment and so forth).

In regard to the economic advantages/disadvantages of the given designs, models and methods, it would be reasonable to deal with the economic aspects as well. Given the complexity of these topics as well as the limited range of this research, it was not possible to focus on this aspect. Since there is no formulated universal approach to assess the economic efficiency of building and allocating a network of public logistics centres in connection with city logistics issues yet, this topic should also be addressed in the future in detail.

In addition, it ought to be emphasised that all the models designed are globally applicable and can be utilised in various transport-related areas.

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## MODELOVÁNÍ DISTRIBUČNÍCH TRAS V CITY LOGISTICE APLIKOVÁNÍM METOD OPERAČNÍHO VÝZKUMU

## ABSTRAKT

Článek se zaměřuje na aktuální téma z praktického $i$ vědeckého hlediska. Konkrétné se zabývá návrhem přistupu $k$ dopravnimu či distribučnímu problému $v$ oblasti městské logistiky a zkoumá možnosti využití zvolených metod operačního výzkumu v této oblasti. Konkrétní návrhy spočivají především v použití vybraných nástrojů operačniho výzkumu (tj. souboru metod týkajících se okružního dopravniho problému) k modelování více variant distribučnich tras ze stanoveného uzlu do vice lokalit, aby se minimalizovala celková ujetá vzdálenost v městské oblasti. Pokud jde o samotný výzkum, pro definování distribučních cest pro zásobování více logistických objektů v rámci městské logistiky jsou krok za krokem použity následující metody: Clarke-Wrightův algoritmus, Mayerův algoritmus a algoritmus nejbližšiho
souseda. Předkládaný článek je rozdělen na koncepčni část, která zahrnuje relevantní teorii i použitá data a metody, samotnou praktickou část a sekci obsahujicí zhodnocení kličových zjištění spolu s diskusí. Inovativní řešení této výzkumné práce spočívá ve vhodné kombinaci adekvátnich metod operačního výzkumu a jejich aplikace na problematiku městské logistiky.

## KLÍČOVÁ SLOVA

městská aglomerace; městská logistika; městské distribuční centrum; operační výzkum; okružní dopravní problém.

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