Iñarrea, J. (2019). Plasmon–Phonon Coupling in Radiation-Induced Resistance Oscillations: Beating Pattern and Phase Reversal. physica status solidi (b), 256(6), 1800497.

10.1002/pssb.201800497

physica status solidi, 3 February 2019

Plasmon-phonon coupling in radiation-induced resistance oscillations: beating pattern and phase reversal.

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Received XXXX, revised XXXX, accepted XXXX

PACS 73.40.-c, 73.50.-h, 78.67.-n

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We present a theoretical study on the experimental observation of two different kinds of beating patterns in the microwave-induced resistance oscillations at very low magnetic field in a high mobility two dimensional electron system. In the first one there was no phase reversal through the beat, however the latest experiments present a clear phase reversal of π. Based on a model that considers that the beating pattern is produced as a result of the coupling between a system of electron Landau states harmonically driven by radiation and an acoustic phonon mode, we explain the contradictory results through a different intensity coupling in terms of amplitude. In the scenario of the non-phase-reversal case we study the dependence on radiation frequency and temperature and the possibility of observation of more than one beat lowering temperature and microwave power. We conclude that both results can be explained with the same theoretical model that in turn it is based on the microwave-driven electron orbit model that was previously developed to explain microwave-induced resistance oscillations.

1 Introduction Needless to say that microwave-induced magnetoresistance oscillations (MIRO) and zero resistance states (ZRS)[1,2] are two of the most striking and unexpected physical effects, discovered more than a decade ago, in the general field of the radiation-matter interaction. The MIRO effect turns up in very high mobility 2DES under illumination at low temperature ($T \sim 1K$) and low magnetic field (B) perpendicular to the 2D sample. The latter effect, ZRS, shows up when increasing radiation power (P), then, maxima and minima oscillations increase but the latter evolve into ZRS. They are so surprising and intriguing that to date there is not a clear consensus on the physical origin of such remarkable effects. After a lot of experiments [3–17] and theoretical models [18–31] we have to admit that the origin of both, specially the one of ZRS, remains unclear.

Since their discovery, MIRO and ZRS have been giving rise, in turn, to a quite a quite few of other novel effects that have challenged and surprised the researchers of the field. One of them is a remarkable beating pattern[32] at very low B and showing up simultaneously with MIRO. This subtle effect was not taken into account by the scientific community and not much attention was paid. Nevertheless, and unexpectedly as well, there has been recently presented new experimental results presenting a similar beating pattern on MIRO at very low B[33] too. It was recently proposed a possible physical explanation for such a beating pattern[34]: this microwave-induced beating patter indicates the presence of coupling between two oscillatory contributions, one is a system of electron Landau states harmonically driven by radiation and the other is a collective motion of the lattices ions, i.e., an acoustic

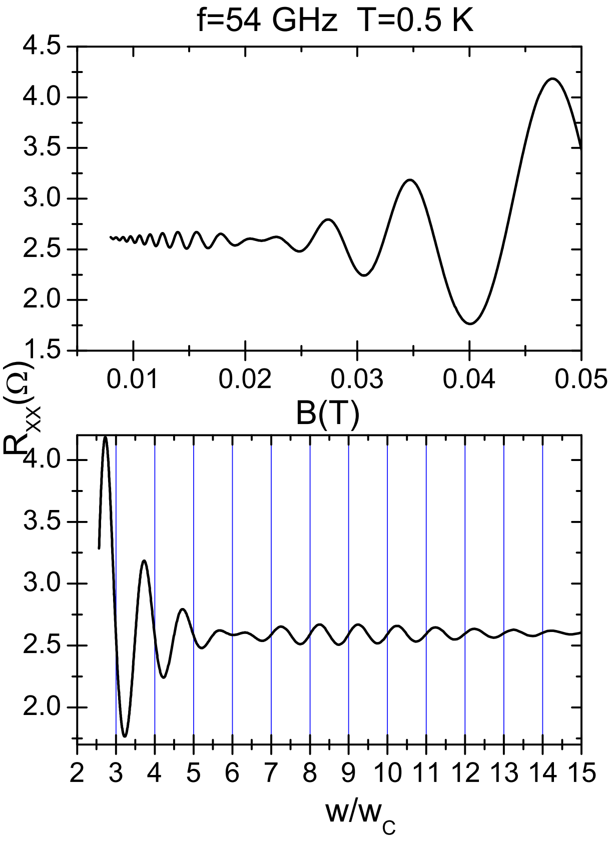

Figure 1 Calculated irradiated R_{xx} for a radiation frequency of 54 GHz according to a phase reversal of π scenario. In panel a) R_{xx} vs B and in b) R_{xx} vs w/w_c. We observe a clear beat at low B with a node around $B = 0.022T$ in panel a). In panel b) the vertical lines indicate a change in phase difference of π when crossing the node. This is due to the modulation effect of the slower function, (cosine) on the faster one (sine) in equation 7. This is a usual effect obtained from the interference of two harmonic functions of slightly different frequencies.

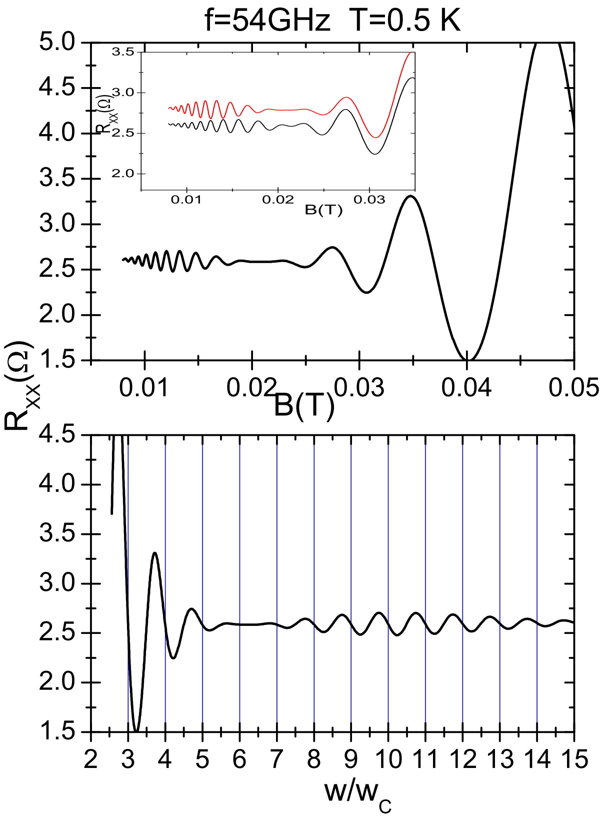

Figure 2 Similar quantities as in Fig. 1 but the calculated results stem the extension of the theory presented in this article to explain the contradictory experimental phase reversals: a partial coupling of the two contributing oscillatory modes involved. In the lower panel the vertical lines reveal the absence of phase reversal. In the inset of the upper panel we exhibit the two beats corresponding to the two experimental results. They are shifted for clarity. We observe that there is a phase shift of π (peaks and valleys of the beat are shifted in π) between the two beats, according to our theory.

phonon mode. *This is the most likely process for coupling, for two main reasons: the first is the low temperature of the experiments, about 1K or less, and then only acoustic phonons modes can be available. Optical phonons are further away in energy. The second reason is the frequency of radiation that is of the order of GHz, similar to the frequency of acoustic phonon modes. The rise of beating patterns (coupling) implies that only phonon modes with frequency around the one of radiation are efficient for coupling. Only acoustic phonons fulfil the requirement of the frequency. Another important point to highlight is that due to the broad dispersion of the acoustic phonon modes we are going to have available these modes in a wide range of frequencies, from a few GHz to a few hundreds GHz. Thus, we can have different beating patterns at different acoustic phonon modes with different frequencies. This would not be possible if were dealing with optical phonon modes that have a narrower range of frequencies.*

But what is even more shocking is the different behavior in terms of the phase reversal of R_{xx} oscillations when crossing a beat according to both experimental results presented to date[32,33]. The former[32] does not show any phase reversal, however the latter[33] presents a phase reversal of π. In this article we try to explain this apparent contradictory behavior based on the radiation driven electron orbit model[18,19], considering that the electron Landau states-phonon coupling can be different depending on

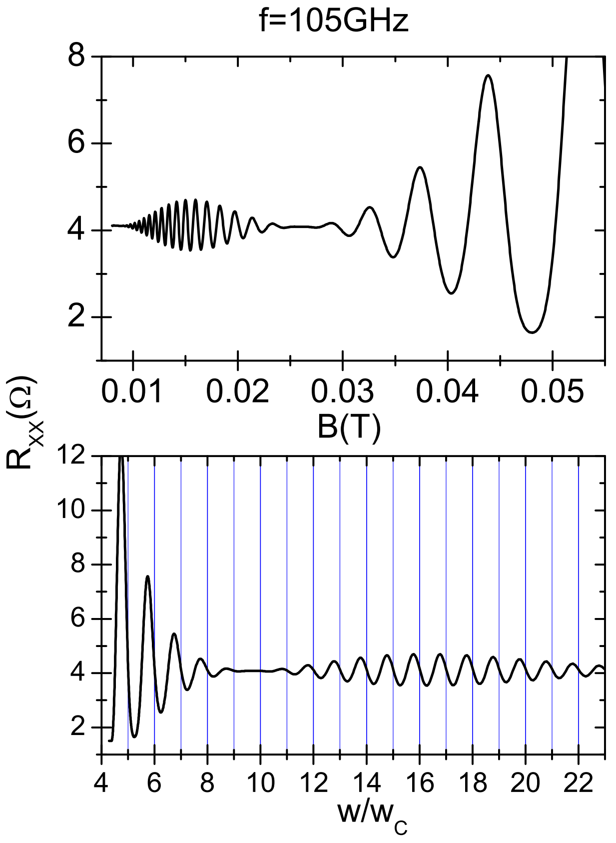

Figure 3 Same as in Fig. 2 (absence of phase reversal) for a radiation frequency of 105 GHz. In the upper panel irradiated R_{xx} vs B. We observe the node this time with much more oscillations, as expected due to higher radiation frequency, and the node at around $B = 0.25$T. In the lower panel, the vertical lines indicate the absence of phase reversal through the node.

the relative amplitudes of the oscillatory modes involved (driven-landau states mode and an acoustic phonon mode).

2 Theoretical model *The radiation-driven electron orbits model*[18,19,35–38] was developed to explain the striking effects of MIRO and ZRS. One of the main conclusion of this theory is that under radiation the Landau states spatially oscillate, with their guiding centers, at the radiation frequency according to

$$X(t) = X_0 + A \sin wt \tag{1}$$

where $X(t)$ is the time dependent Landau state guiding center position, X_0 is the same without radiation and

$$A = \frac{eE_o}{m^* \sqrt{(w_c^2 - w^2)^2 + \gamma^4}} \tag{2}$$

E_0 is the radiation electric field and w_c the cyclotron frequency. γ is a phenomenologically introduced damping factor for the electron scattering with the lattice ions. Based on this theory we presented in a previous contribution[34] a model to explain the physics of the beating pattern showing up in irradiated R_{xx} at very low magnetic fields. According to it, following the physics of plasmon-phonon coupling, we consider that the system of driven-Landau states, behaving like a "plasmon-like" mode, couples with an acoustic phonon mode. Thus, we were able to obtain a new expression for the position of the guiding center of the hybrid driven-Landau state mode[34]:

$$X(t) = X_0 + A \sin w_+ t + B \sin w_- t \tag{3}$$

where the new frequencies are[34]:

$$2w_\pm^2 = (w^2 + w_{ac}^2) \pm \sqrt{(w^2 - w_{ac}^2) + 16\lambda^2 w_{ac} w} \tag{4}$$

w is the frequency of both, radiation and the driven-Landau state mode. w_{ac} is the frequency of the acoustic phonon mode and λ the plasmon-phonon coupling constant.

With the expression of $X(t)$ (Eq. 3) and according to the radiation-driven electron orbit model, we obtain for the average distance advanced by the electron in a scattering event[34]

$$\begin{aligned} \Delta X(t) &= \Delta X_0 - e^{-\frac{\gamma}{2}\tau} A[\sin w_+ \tau + \sin w_- \tau] \\ &= \Delta X_0 - 2Ae^{-\frac{\gamma}{2}\tau} \sin w\tau \cos \lambda\tau \end{aligned} \tag{5}$$

where we have used the well-known expression:

$$\sin(\theta_1) + \sin(\theta_2) = 2 \sin\left[\frac{\theta_1 + \theta_2}{2}\right] \cos\left[\frac{\theta_1 - \theta_2}{2}\right] \tag{6}$$

and we have considered that $A \simeq B$ and that $w_\pm \simeq w \pm \lambda$. The time, τ, according to the radiation driven electron orbit model[39,40], is the "flight time", the time it takes the electron to jump due to scattering from one orbit to another and its value is given by $\tau = \frac{2\pi}{w_c}$. Finally and following the radiation-driven electron orbit model we end up with an expression for R_{xx}[34,39,40]:

$$R_{xx} \propto \Delta X_0 - 2Ae^{-\pi\frac{\gamma}{w_c}} \sin\left(2\pi \frac{w}{w_c}\right) \cos\left(2\pi \frac{\lambda}{w_c}\right) \tag{7}$$

This is the obtained expression of R_{xx} for a phase reversal scenario[34].

Now we extend the previous model to explain the controversy consisting in the different behaviors that the experimental results reveal on the issue of phase reversal through the beat. The first experiment[32] on the beat did not show any phase reversal but in the latter[33] there was a very clear phase reversal of π when crossing the beat. From the theoretical standpoint, according to our model there must be always such a phase reversal whenever one beat or node is crossed. In principle our first approach[34] could not explain the absence of phase reversal in R_{xx} according to

Eq. 7. Nevertheless, an absence of phase reversal can be obtained from our theory in a potential scenario of partial coupling between the driven Landau states mode and the acoustic phonon mode. Thus, if we consider that, in terms of amplitude, around half of the driven Landau states mode is coupled with the acoustic mode, we can obtain an expression for the x coordinate of the guiding center of the Landau state:

$$X = Xo + \frac{A}{2}\sin(wt) + \left[\frac{A}{2}\sin(w_+t) + \frac{A}{2}\sin(w_-t)\right] \tag{8}$$

And following the same algebra as before[34] we can get to:

$$\begin{aligned} \Delta X &= \Delta X_0 - e^{-\frac{\gamma}{2}\tau}\left\{\frac{A}{2}\sin(w\tau) - \frac{A}{2}\sin(w\tau)\cos(\lambda\tau)\right\} \\ &= \Delta Xo - e^{-\frac{\gamma}{2}\tau}\frac{A}{2}\sin(w\tau)[1+\cos(\lambda\tau)] \end{aligned} \tag{9}$$

Then the magnetoresistance is given now by:

$$R_{xx} \sim -e^{-\pi\frac{\gamma}{w_c}}\frac{A}{2}\sin\left(2\pi\frac{w}{w_c}\right)\left[1+\cos\left(2\pi\frac{\lambda}{w_c}\right)\right] \tag{10}$$

Thus, according to this expression for R_{xx}, we would not obtain phase reversal through the node and it would be constant as in the experiment[32]. This expression is the same as the one suggested by Mani et al. in their experiment[32].

3 Results In Fig. 1 we present irradiated magnetoresistance, R_{xx}, for a radiation frequency of 54 GHz in a scenario of phase reversal of π. In Fig. 1a, we exhibit R_{xx} vs B and in Fig. 1b, R_{xx} vs w/w_c. Thus, in the upper panel, we observe the well-known MIRO for high values of B and a clear beat at very low B with a node around $B = 0.022T$. In the lower panel the vertical lines for integer values of the abscissa indicate a phase change of π in the R_{xx} oscillations when crossing the node, as in the latter experiment[33]. Eq. 10 easily explains this effect as shown in a previous article[34]; the change of phase in π is due to the modulation effect of the slower function, which is the cosine function, on the faster one, i.e., the sine function. This is a general situation in Physics: whenever two harmonic functions with slightly different frequencies interfere we obtain the rise of beats and the change of phase in π when crossing each node.

In Fig. 2 we exhibit the same as in Fig. 1 but the calculated results stem from Eq. 10. In other words, this figure is based on the extension of the theory presented in this article to explain the contradictory experimental phase reversals: a partial coupling of the two contributing oscillatory modes, (radiation-driven Landau states and an acoustic phonon mode), could give rise to the absence of the phase reversal through the beat. In Fig. 2b the vertical lines reveal the absence of phase reversal at both sides of the node, as expected. In Fig. 2a in the inset we exhibit together the two beats corresponding to the two experimental results. They are shifted for clarity. We observe that there is a phase shift of π (peaks and valleys of the beat are shifted in π) according to our theory. While Fig. 1 is in qualitatively agrement with the latter beating pattern experiment[33], the calculated results in Fig 2 are in qualitatively agreement with Mani's results[32].

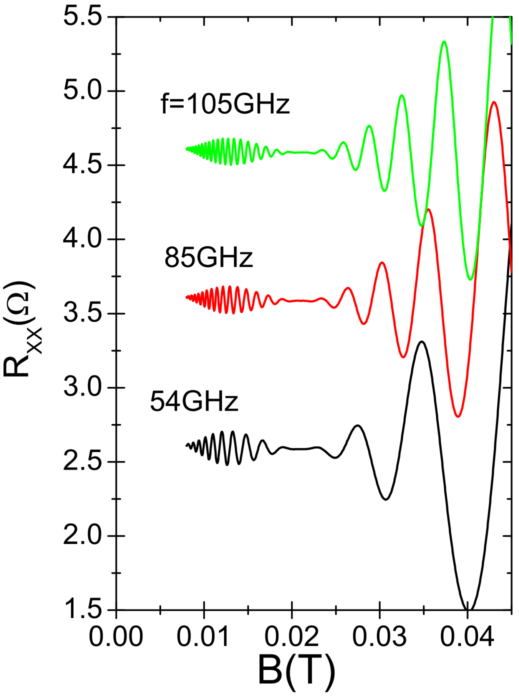

Figure 4 Dependence of the beating pattern on the radiation frequency. Three frequencies are exhibited 54 GHz, 85 GHz and 105 GHz. The node position in terms of magnetic field does not change irrespective of the radiation frequency. The node position only depends on the cosine functions where the radiation frequency does not show up.

In Fig. 3 we exhibit the same as in Fig. 2 (absence of phase reversal) but for a frequency of 105 GHz. In the upper panel we represented irradiated R_{xx} vs B. We observe the node this time with much more oscillations, as expected due to higher radiation frequency, and the node at around $B = 0.25$T. Even without the help of the lower panel the shape of the node itself suggest that there is no phase reversal in this case. In the lower panel the vertical lines at both sides of the node reveal that the phase is the same. Thus, we can conclude that the radiation frequency does not play any role on the phase of the beat, as expected, according to our theory.

In Fig. 4 we exhibit the dependence of the beating pattern on the frequency for an absence of phase reversal scenario. We present three cases: for lower frequency of 54 GHz, for an intermediate of 85 GHz and for a higher one of 105 GHz. The three curves are shifted for clarity. As in

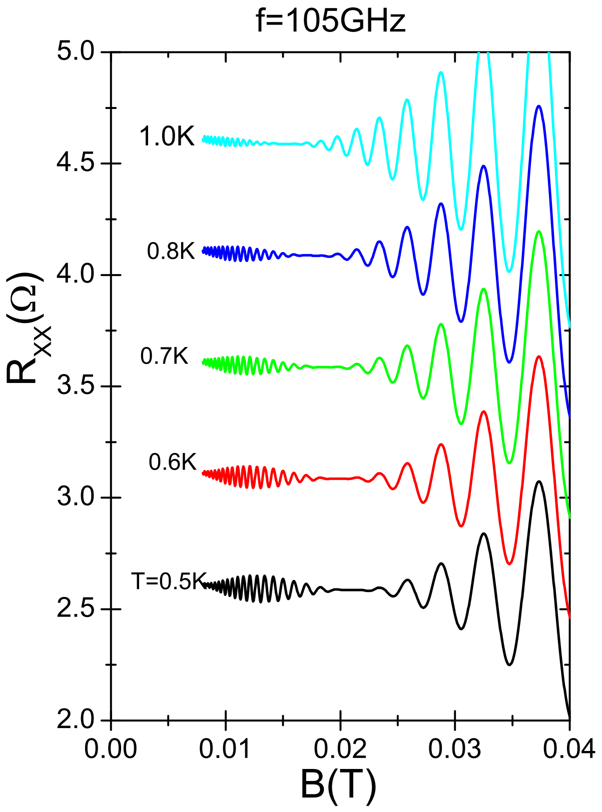

Figure 5 Dependence of the beating pattern on temperature for the case of non-phase reversal. We exhibit irradiated R_{xx} vs B for a frequency of 105 GHz and a temperature ranging from 0.5 K to 1.0 K. We clearly observe that the node moves to lower B for increasing temperature. In the same way the beat itself gets less and less intense.

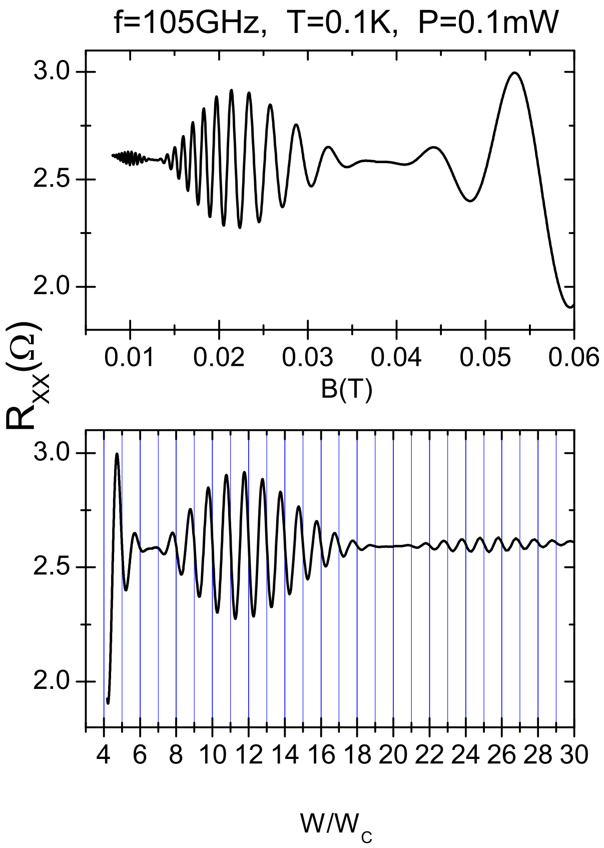

Figure 6 Irradiated magnetoresisance vs magnetic field for a radiation frequency of 105 GHz and low values of power and temperature, $P = 0.1mW$ and $T = 0.1K$. In both panels we observe the appearance of an extra beat and node. In the lower panel the vertical lines indicate that the phase reversal keeps constant for all the range of B irrespective of the beats and nodes.

the phase reversal case[34] we obtain that the B-position of the node is immune to the radiation frequency. This in agreement with previous experimental results[32]. As in any interference event involving two harmonic functions of similar frequencies, the node position depends on the cosine function where w does not show up. Thus, the node position is immune to w. Nevertheless, any variation of the coupling constant λ that shows up in the cosine will clearly affect the node position and even the number of beats that can be obtained.

In Fig. 5 we present the calculated results of the dependence of the beating pattern of R_{xx} on temperature (T) for a radiation frequency of 105 GHz and T from 0.5K to 1.0K. These results correspond to non-phase reversal scenario. The R_{xx} curves for different temperatures are shifted for clarity. As in the phase reversal case the node position is not constant with B and moves to lower B when temperature goes up getting less and less intense. The displacement of the node and the intensity variation indicate that a changing temperature affects the coupling and, in turn, λ. Thus, we have to conclude that temperature has a key influence in the coupling constant. In other words, an increasing temperature tends to destroy the coupling between the oscillating modes. Similar behaviour is obtained (not shown) when we study the influence of radiation power on the coupling; an increasing radiation power moves the node in the same direction as temperature and the beat intensity goes down. Thus, both properties, T and P, affect the beat in the same way: a decrease of any of them makes the coupling stronger and λ bigger. And, on the other hand, an increase gives rise to a progressive destruction of the beat.

The physical explanation is absolutely similar to the one of the phase reversal case[34]. Scattering between the electrons and the lattice ions in the system, via a higher T or P, is responsible of destroying the coupling. The phenomenological equation consisting in adding a first order (linear) correction to λ in the variation of T and P[34] has been also applied on this case of non-phase reversal and the corresponding results are exhibited in Fig. 5.

In a previous work[34] it was predicted that more beats and nodes could be obtained lowering sufficiently T and P. Now, we wonder if for a case of non-phase reversal we would obtain similar results. In Fig. 6 we present the calculated results of irradiated magnetoresisance vs magnetic field for simultaneous low values of T and P. According to our model, low values of T and P would lead to a more robust coupling between modes that would be reflected in a more intense beating pattern. Thus, we would expect also the appearance of more nodes and beats. This latter possibility is presented in Fig. 6 for a radiation frequency of 105 GHz, $T = 0.1K$ and $P = 0.1mW$. In the upper panel we exhibit irradiated R_{xx} vs B and in the lower one the same vs w/w_c. For both panels we clearly observe the appearance of an extra beat and node. In the lower panel the vertical lines show that the phase keeps constant along the whole range of B irrespective of the beats and nodes, as expected. The scenario of multiple modes and beats has not been experimentally observed yet.

4 Conclusions

We have presented a theoretical study on the observation of a beating pattern in the radiation-induced resistance oscillations at very low magnetic field offering a theoretical explanation on the contradictory results about the beating pattern phase reversal that show up in different experiments. Based on a model that considers that the beating pattern is produced by the coupling between Landau states harmonically driven by radiation and a phonon mode, we explain the contradictory results through a different intensity coupling in terms of amplitude of the oscillatory modes involved. In the scenario of the non-phase-reversal we have studied the dependence of the beating pattern on radiation frequency and temperature. The possibility of observation of more than one beat lowering temperature and microwave power is also analyzed.

This work is supported by the MINECO (Spain) under grant MAT2017-86717-P and ITN Grant 234970 (EU). Grupo de Matematicas aplicadas a la Materia Condensada, (UC3M), Unidad Asociada al CSIC.

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