High temperature observation and power modulation of radiation-induced resistance oscillations in the Terahertz band

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Abstract –We report on a theoretical work on magnetotransport under terahertz radiation with high mobility two-dimensional electron systems focussing on the radiation power and temperature dependence. On the one hand, we study the interaction between the obtained radiation-induced magnetoresistance oscillations (RIRO) and the Shubnikov-de Haas (SdHO) oscillations from the power dependence standpoint. We obtain strong modulation of the SdHO oscillations at sufficient terahertz radiation power. On the other hand from the temperature dependence standpoint we obtain an important result: the range of temperature where RIRO are observed can be largely extended by using terahertz radiation. In the terahertz region we still obtain RIRO up to $20K$ at a radiation frequency of $400GHz$. Since an increasing T gives rise in turn to more disorder in the sample, we would expect also the observation of RIRO with terahertz radiation when using low-mobility (high disorder) samples at low T.

Some of the most striking effects discovered in the last decade regarding radiation-matter coupling are the radiation-induced magnetoresistance (R_{xx}) oscillations (RIRO) [1,2] in two-dimensional electron systems (2DES). This effect shows up in high mobility 2DES when irradiated at low temperature ($T \sim 1K$) and under low magnetic fields (B) perpendicular to the 2DES. Peaks and valleys of RIRO increase with radiation power (P) but the latter turn into zero resistance states (ZRS) [1,2] at high enough P. Among the different features describing RIRO, three of them deserve to be highlighted: first, they are periodic in B^{-1}, second, the oscillations minima present a 1/4 cycle phase shift and third, the dependence of RIRO on radiation power follow s a sublinear relation (square root). After more than a decade of important experimental [3–26] and theoretical efforts [27–41], their physical origin still remains unclear. Among the different proposed theories, two theoretical models have been generally "accepted" to date to explain RIRO: the displacement [32] model and the inelastic [36] model.

From the pioneering work by Mani et al. [42], THz radiation has been used to study RIRO and ZRS in 2DES in quite a few works [20, 42–46]. THz radiation is a very

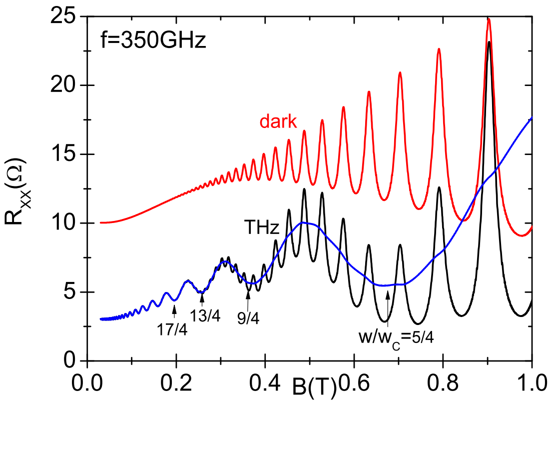

Fig. 1: Calculated irradiated magnetoresistance, R_{xx}, vs magnetic field B for a frequency of 350 GHz and the dark. The dark curve has been shifted up for clarity. It can be observed the interaction between RIRO and the Shubnikov-de Haas oscillations. The minima positions are given by: $\frac{w}{w_c} = \frac{5}{4}, \frac{9}{4}, \frac{13}{4}.... = \left(\frac{1}{4} + n\right)$ where $n = 1, 2, 3.....$

interesting tool not only from the basic physic point of view but also from the applications. For the latter we can cite for instance, novel sensors, application in imaging, communications and medicine, etc. From the RIRO standpoint, THz radiation is also a very suggesting band to be used for many reasons. The main is that THz offers the possibility of studying the interaction of RIRO and the Shubnikov de Haas oscillations (SdHO) in magnetotransport in 2DES. This interesting point is not possible when using MW frequencies because the most intense oscillations for both, RIRO and SdHO, show up in different B regions. However they coincide when using THz frequencies. Another important point is that using THz radiation is possible also to study the joint evolution of RIRO and SdHO to ZRS when increasing P at the appropriate B. Thus, THz experiments on RIRO and ZRS in 2DES could help to shed light on the physical origin of these striking phenomena, still under debate.

In this letter we present a theoretical work on magnetotransport in high mobility 2DES under terahertz (THz) radiation from the P and T dependence standpoints in regards of a recent experimental work by Hermann et al. [44]. In this experiment it is suggested that it is possible to obtain observable RIRO at higher temperatures than in the MW case when using Thz radiation. Thus, according to them, RIRO could be obtained at temperatures around 20 K. The theory of this letter is based on the radiation-driven electron orbit model [27–29]. With Thz radiation and from the P dependence point of view, we study the observed disappearance of the SdHO oscillations simultaneously with the vanishing resistance at the zero resistance region [8, 42, 43] and the strong modulation of the SdHO oscillations at sufficient THz radiation power [8,20,45,46]. We also study and recover the sublinear law dependence (in fact a square root law) of RIRO on radiation power, confirming that it is universal and then independent of the frequency. From the T dependence point of view we obtain the novel and striking result that RIRO can be observed at much higher T when using THz radiation than in the case of MW. For the former we still obtain RIRO up to around $20K$, for a THz frequency of $400GHz$. For the latter, RIRO can be considered totally wiped out when reaching about $5K$, for a MW frequency of $50GHz$. One of the main effects of a rising T is to increase scattering, and in turn, disorder in the sample. Then, we would expect the same effect with THz radiation when using low-mobility (high disorder) samples at low T. Accordingly, by using THz radiation we could observe RIRO in a wider range of materials without the restriction of such high mobility requirements. This has been recently pointed out and experimentally observed by Herrmann et al., [44].

The theoretical contribution of this letter is based on the *radiation-driven electron orbits model* [27–29]. This theory was proposed to study the magnetoresistance of a 2DES subjected to MW at low B and T [27,28,47–49]. According to the model, the time-dependent Schrödinger equation of an electron in the presence of a constant magnetic field

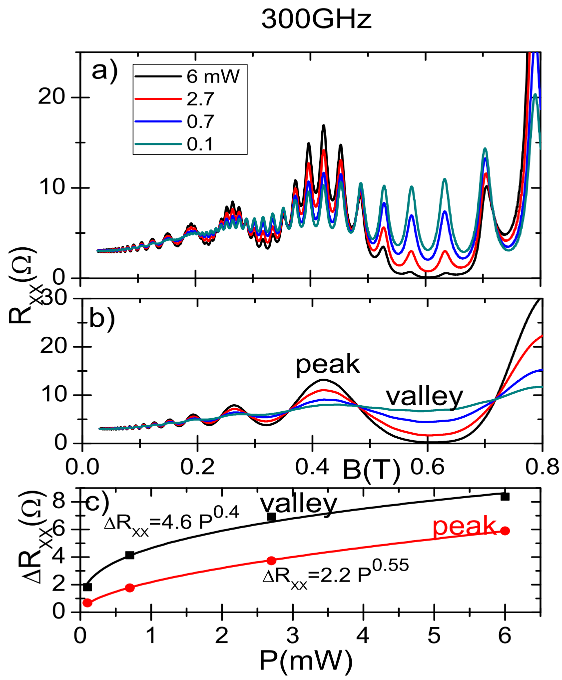

Fig. 2: Radiation power dependence of the calculated irradiated R_{xx} versus B, for increasing radiation power from to 0.1 mW to 6 mW. Fig. 2a, total R_{xx} vs B. In 2b, the same as in 2a but now R_{xx} without Shubnikov-de Haas oscillations. Fig. 2c exhibits ΔR_{xx}, i.e., the difference of irradiated R_{xx} minus the dark one for the labelled peak and valley of 2b, vs P. It is also exhibited the two sublinear fits corresponding to the two sets of values.

and radiation can be exactly solved and the solution for the total wave function or Landau State (LS) [27,28,37,47–49]reads: $\Psi_n(x,t) \propto \phi_n(x - X(0) - x_{cl}(t), t)$, where ϕ_n is the solution for the Schrödinger equation of the unforced quantum harmonic oscillator. Thus, the obtained wave function representing the LS is the same as the one of the standard quantum harmonic oscillator where the guiding center of the LS, $X(0)$ without radiation, is displaced by $x_{cl}(t)$. $x_{cl}(t)$ is the classical solution of a forced damped harmonic oscillator [27–29],

$$\begin{aligned} x_{cl}(t) &= \frac{eE_o}{m^*\sqrt{(w_c^2 - w^2)^2 + \gamma^4}} \cos(wt - \beta) \\ &= A\cos(wt - \beta) \qquad (1) \end{aligned}$$

where e is the magnitude of the electron charge and E_0 is the amplitude of the radiation electric field. γ is a phenomenologically-introduced damping factor for the interaction of electrons with the lattice ions giving rise to the emission of acoustic phonons. β is the phase difference between the radiation-driven guiding center and the driving radiation itself. In the presence of radiation, the electronic orbit center coordinates change and are given according to our model by $X(t) = X(0) + x_{cl}(t)$. This means that due to the radiation field all the electronic orbit centers in the sample harmonically oscillate at the radiation frequency in the x direction through x_{cl}. Applying initial conditions, at $t = 0$, $X(t) = X(0)$ and then $\beta = \pi/2$. As a result the expression for the time dependent guiding center is now: $X(t) = X(0) + A \sin wt$. In the presence of charged impurities, electrons in their Landau orbits suffer scattering and jump between radiation-driven LS. Thus, scattering gives rise to an average advanced distance that will be reflected in the longitudinal conducivity, σ_{xx} . After some algebra [52] the advanced distance due to scattering in the presence of radiation reads, $\Delta X = \Delta X(0) - A \sin\left(2\pi \frac{w}{w_c}\right)$, where $\Delta X(0)$ is the advanced distance in the dark.

To calculate σ_{xx} in the 2DES we use the Boltzmann transport theory. With this theory and within the relaxation time approximation, σ_{xx} is given by [50, 51]:

$$\sigma_{xx} = 2e^2 \int_0^\infty dE \rho_i(E)(\Delta X)^2 W_I \left(-\frac{df(E)}{dE}\right) \quad (2)$$

being E the energy and $\rho_i(E)$ the density of initial LS. W_I is the remote charged impurity scattering rate, given, according to the Fermi's Golden Rule, by $W_I = \frac{2\pi}{\hbar} | < \Psi_f|V_s|\Psi_i > |^2 \delta(E_f - E_i)$, where E_i and E_f are the energies of the initial and final LS. Ψ_i and Ψ_f are the wave functions corresponding to the initial and final LS respectively. V_s is the scattering potential for charged impurities [50]. After some algebra we get to an expression for σ_{xx} [51–53]:

$$\sigma_{xx} = \frac{2e^2 m^*}{\pi \hbar^2}(\Delta X)^2 W_I \left[1 + \frac{2X_s}{\sinh(X_s)} e^{-\frac{\pi \Gamma}{\hbar w_c}} \cos\left(\frac{2\pi E_F}{\hbar w_c}\right)\right] \quad (3)$$

where $X_s = \frac{2\pi^2 k_B T}{\hbar w_c}$, Γ is the Landau level width and E_F the Fermi energy. To find the expression for R_{xx} we use the well-known tensorial relation $R_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2} \simeq \frac{\sigma_{xx}}{\sigma_{xy}^2}$, where $\sigma_{xy} \simeq \frac{n_e e}{B}$, n_e being the electron density, and $\sigma_{xx} \ll \sigma_{xy}$. Finally, the expression of R_{xx} reads:

$$R_{xx} \propto \left[\Delta X(0) - A \sin\left(2\pi \frac{w}{w_c}\right)\right]^2 \times \left[1 + \frac{2X_s}{\sinh(X_s)} e^{-\frac{\pi \Gamma}{\hbar w_c}} \cos\left(\frac{2\pi E_F}{\hbar w_c}\right)\right] \quad (4)$$

With this expression we want to stand out the terms that are going to be responsible of the interference between RIRO's, first bracket, and the SdHO, second bracket. On the other hand, it is important to point out the presence of the amplitude A in the above expression to understand the influence of P and T on R_{xx} and RIRO. The dependence on P will be given by the radiation electric field E_0 and on T by the damping factor γ.

Fig. 1 exhibits irradiated R_{xx} vs B for dark and THz radiation of 350 GHz. The dark curve has been shifted up for clarity. For the THz curve we represent the total R_{xx} (black curve online) and the averaged out R_{xx} (without SdHO, blue curve online), in order to stand out only the effect of RIRO. By doing this, we can see intense RIRO in the THz regime that clearly fulfill the periodicity in B^{-1} and the 1/4-cycle phase shift of the oscillations minima, $(w/w_c = 5/4, 9/4, 13/4...)$. Besides, it is interesting to observe with the THz regime, how the radiation-induced oscillations overlap with the more rapidly varying with the magnetic field SdHO giving rise to a strong modulation of the latter. This modulation is explained, according to our model, by the interference effect between the harmonic terms showing up in Eq. 4. Thus, this effect is mainly dependent on the radiation frequency and the Fermi energy or electron density.

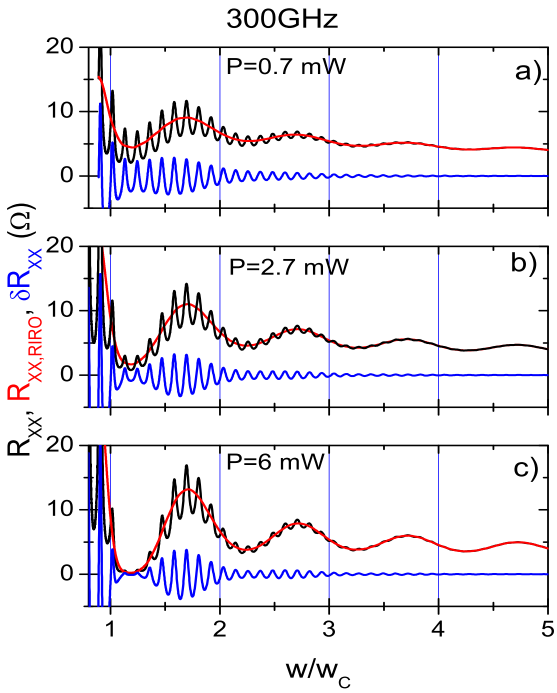

Fig. 3: Calculated irradiated R_{xx}, R_{xx} without SdHO, ($R_{xx,RIRO}$), and the difference of both, δR_{xx} vs w/w_c for three different radiation powers: in panel a) $P = 0.7mW$, in panel b) $P = 2.7mW$ and in panel c) $P = 6.0mW$.

In Fig. 2, we present the P dependence of the THz irradiated R_{xx} versus B for increasing P, from to 0.1 mW to 6 mW. In Fig. 2a, we exhibit the complete R_{xx} whereas in 2b, we plot R_{xx} without SdHO. In Fig. 2c we plot ΔR_{xx} that is the difference of irradiated R_{xx} minus the dark one for the labelled peak and valley of 2b, vs P. For the latter panel, as expected, we obtain a sublinear dependence of ΔR_{xx} on P. This can be straightforward explained according to our model since the radiation electric field E_0 shows up in the numerator of the amplitude of RIRO and, on the other hand, $\sqrt{P} \propto E_0$. Thus, the exponent of the sublinear expression is close to 0.5. Another interesting effect can be observed in Fig. 2a around

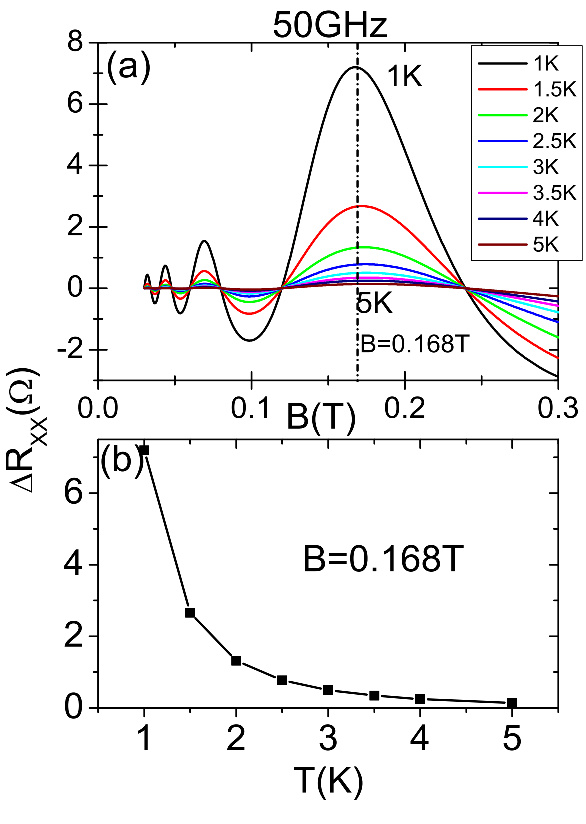

Fig. 4: Calculated irradiated magnetoresistance for different T from $1K$ to $5K$ and a MW frequency of 50 GHz. In panel a) ΔR_{xx}, the difference between R_{xx} under radiation and dark vs B. In panel b) ΔR_{xx} vs T for $B = 0.168$ T. It can be observed in both panels that the amplitude of RIRO dramatically decreases when increasing T. When T is about $5K$, the oscillations can hardly be observed.ΔR_{xx}

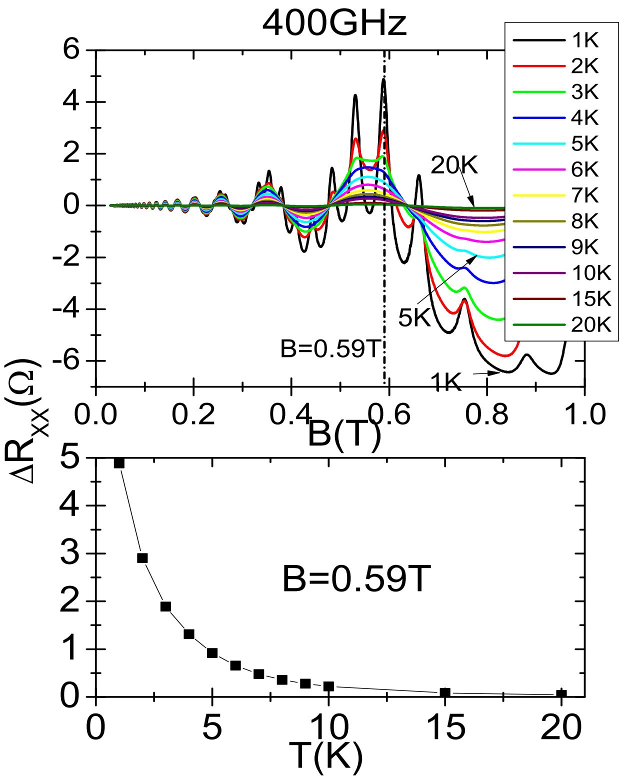

Fig. 5: Same as in Fig 4, but for the THz band and a frequency of $f = 400GHz$ with a power to frequency ratio similar to the MW case. We notice that radiation-induced oscillations can still be observed at much higher T than the case of MW band. The results in panel b) have been obtained for $B = 0.59$ T.

$B = 0.6$ T. In this region we obtain the evolution of SdHO as a function of increasing P. Interestingly, as in experiment [42], the SdHO vanish as R_{xx} tends to zero. In other words, we obtain the suppression of SdHO in the region of radiation-induced zero resistance states. According to our model this is because this region corresponds to a situation where the advanced distance $\Delta X \to 0$, making smaller and smaller the obtained R_{xx}, including resistance background and SdHO. Thus, both simultaneously decrease in agreement with the experimental results. [42].

In Fig. 3 we present calculated results of irradiated R_{xx}, R_{xx} without SdHO ($R_{xx,RIRO}$), and the difference of both, δR_{xx}, vs w/w_c for three different radiation powers: in panel a) $P = 0.7mW$, in panel b) $P = 2.7mW$ and in panel c) $P = 6.0mW$. The remarkable result is that SdHO turn out to be strongly modulated by the presence of radiation, being the modulation harmonic and periodic in $1/B$ and completely in phase with RIRO. It is also noteworthy that δR_{xx} shows an intense interference effect with the appearance of beats of increasing intensity for increas-

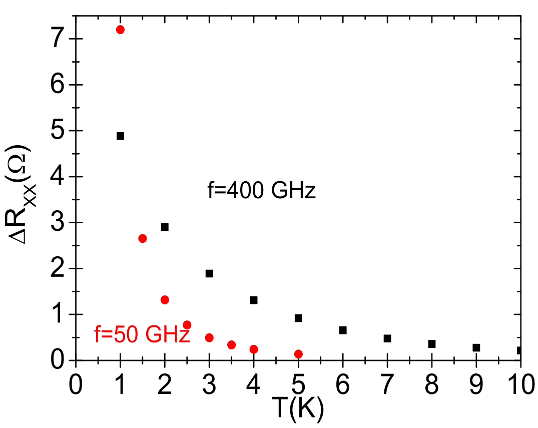

Fig. 6: ΔR_{xx} vs T for both frequencies of 50 and 400 GHz. Both set of results are exhibited together to contrast them quantitatively.

ing power (see Figs 3a, 3b and 3c). The coincidence in phase and period is not trivial and reveals deep physical consequences. Thus, according to our model (Eq. 4), the presence in R_{xx} of the ΔX term is the main responsible of the effect. The reason is that ΔX is harmonically dependent on w/w_c getting this dependence across to the the SdHO term. In the end, both contributions end up sharing period and phase as obtained in experiments. In physical terms and as we explain above, the average advanced distance of the scattered electron between radiation-driven LS, strongly modulates the influence of the initial density of LS on R_{xx}. And this effect can be totally and clearly observed in the THz band and not in the MW due to the coincidence of SdHO and RIRO versus B that takes in the THZ band.

The damping parameter γ describes the interaction of the 2D electrons in the driven-LS with the lattice ions. Accordingly, part of the energy absorbed from radiation is continuously released to the lattice in the form of acoustic phonons. The final result is a quenching effect on the amplitude of the driven-oscillations of the LS. The more interaction between electrons and lattice (acoustic phonon emission) the more quenched is the amplitude. Thus, γ is proportional to the probability rate of electron-acoustic phonon interaction: $\gamma \propto \frac{1}{\tau_{ac}}$. The electron-acoustic phonon scattering rate can be calculated using the Fermi's Golden rule obtaining an expression [50, 54] that reads:

$$\frac{1}{\tau_{ac}} = \frac{m^* \Xi_{ac}^2 k_B T}{\hbar^3 \rho u_l^2 < z >} \tag{5}$$

where Ξ_{ac} is the acoustic deformation potential, ρ the mass density, u_l the sound velocity, $< z >$ is the effective layer thickness and k_B the Boltzmann constant. Finally, we find that γ depends linearly on T through the probability rate $\frac{1}{\tau_{ac}}$, $\gamma \propto T$ Then, the higher T the bigger γ and the more intense is the damping of the swinging driven motion of LS. In other words, when increasing T we obtain a decrease in the amplitude of the oscillations as a result of energy being increasingly drained from the 2D electron system to the lattice. Now is possible to calculate R_{xx} as a function of T [30] for both the MW and the THz band and contrast results.

In Fig.4 we exhibit the T dependence of irradiated magnetoresistance vs B for MW radiation. In panel a) ΔR_{xx}, the difference between R_{xx} under radiation and dark vs B. In panel b) ΔR_{xx} vs T. Both panels correspond to a frequency of the MW band of $f = 50GHz$ where $f = 2\pi w$. The set of T values varies from $1K$ to $5K$. As in experiments [1, 2] the RIRO amplitude rapidly decreases when T increases and when reaching $5K$, RIRO can hardly be observed.

In Fig. 5 we exhibit the same as in Fig 4, but for the THz band and a frequency of $f = 400GHz$ with a power to frequency ratio similar to the MW case. As in Fig. 4, the RIRO amplitude decreases as T increases. Yet, now we have to go further in T to make oscillations disappear. Thus, when reaching $20K$ the oscillations no longer exist. In Fig. 6 we exhibit together the two sets of results (only up to 10 K) to easily compare, even quantitatively the different behavior in terms of T. This is a remarkable result regarding the THz band. Other frequencies of the same band have been used (not shown) concluding that the observation of RIRO can be extended to much higher T increasing the frequency of radiation. This is a theoretical prediction that has not been explicitly confirmed by experiments yet, but it has been suggested by Hermmann et al., [44].

The explanation of this high-temperature observation can be readily obtained considering the expression of the RIRO amplitude A. Thus, the key point is the relative value of the denominator parameters, i.e., the frequency and γ terms; when the latter is preponderant, RIRO are being wiped out. Nevertheless, when considering the other way around, i.e., the frequency term more important, RIRO are obtained. Then, if we increase the radiation frequency from the MW up to the THz band keeping approximately constant the power to frequency ratio, we have an extended margin to increase γ and still observe RIRO. In other words, if in the THz scenario we want to completely quench RIRO, we have to increase T much higher than in MW. And this is what we obtain in our simulations where we have considered, with the experimental [1] parameters we have at hand, a numerical value for $\gamma \simeq 5. \times 10^{11} s^{-1}$.

The result we have obtained in terms of T still admits and interesting approach. The reason is that when increasing T the main effect on the sample is a more intense motion of ions in the lattice that in turn causes an increasing disorder in the sample. The key point is that we can have a similar scenario with a lower mobility sample at low T, i.e., samples with an important "built-in" disorder. Thus, we would expect to observe RIRO in this low quality samples but using THz radiation. And we can think of low-mobility GaAS or different semiconductors platforms where RIRO can not be observed with MW, and now by switching to THz may turn up. Accordingly, by using THz radiation we could observe RIRO in a wider range of materials without the restriction of such high mobility requirements. This has been recently pointed out and experimentally observed by Herrmann et al., [44].

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