Combinations of Normal and Non-normal Modal Logics for Normative Multi-Agent Systems

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Abstract. We provide technical details for both a fibring and an independent combination of a normal and a non-normal logics underlying a normative multi-agent system (*normative MAS*). Such combinations lead to different levels of expressiveness of the system. Based on the fibring, we give a possible structure for a combined model checker for the *MAS*. The independent combination provides: (i) an ontology of pairs (mental configuration-visible behavior), which can be understood as a structural account to supporting non-monotonicity; and (ii) a logics where to write and test the validity of a wider set of well-formed formulas.

Keywords modal logics, combination of logics, model checking, normative multi-agent systems.

1 Introduction

The notion of collective trust within a multi-modal setting is analyzed and defined in [17] using the normal modalities *Bel*, *Goal*, *Int*, *O*, *C-Bel*, and *C-Int* for modelling beliefs, goals, intentions, obligations, and collective beliefs and collective intentions respectively. Qualitatively distinct levels of trust are defined, and possible connections between different forms of group trust and the emergence of obligations within groups of agents are outlined in the theory. From the logical point of view, it is stated that the definitions provided do not add anything new to the base semantic techniques, which are essentially Kripke-styled uni-agent definitions extended to multi-agent definitions [3]. However, the notion of agency used amounts to a non-normal modal system and so requires some reframing of the whole system. Smith and Rotolo then reframe Dunin-Keplicz and Verbrugge's original semantics into an equivalent one based on multi-relational models [9]. The advantage of this semantics is to preserve the basic intuition and structure of Kripke models, and allows not to deviate from [3]'s account.

We make the following observations regarding some logical aspects of the theory in [17]:

Observation 1. Expressiveness of the system. The idea of direct and personal action to carry out a state of affairs is formalized by the well-known operator E [4, 9] (called *Does* in [17], name we will adopt from now on); a formula like $Does_i\ \mathscr{A}$ means that "agent i brings it about that $\mathscr{A}$". In this setting, the Does modalities are always applied to atomic propositional constants representing single behavioural actions, as in e.g. $Does_x\ PayBill$ (which is meant to stand for "agent x pays the bill"). In the theory under study, normal operators interact with the Does modalities in a restricted one-way manner: agents' actions always appear as innermost operators within well-formed formulas (*wffs*), as in e.g. $O(Does_x\ PayBill)$ (which is meant to stand for "it is obligatory that agent x pays the bill"). This means that no other modalities occur in the scope of a *Does*; we are not able to write sentences like e.g. $Does_i\ (Does_j\ PayBill)$ (meaning "agent i makes agent j pay the bill").

Observation 2. Semantics. The multi-relational model of the theory ([17], Definition 2), is presented as a natural embedding of Kripke semantics into a Scott-Montague semantics [9, 11]. Such embedding is assumed correct because Kripke semantics can be seen as a special case of Scott-Montague semantics (the completeness proof for *Does* is available in [4, 9]). Although the referred concrete embedding appears to be straightforward in [17], it is worth pointing out that it requires some detailed technical machinery for adapting Kripke-like completeness proofs to Scott-Montague proofs.

Observations 1 and 2 above motivated the analysis of the possibility of working towards concrete implementations for the outlined MAS; we also took as main inspiration the modal model checking approach from the work of Francheset et al. [7]. We are aware of the intrinsic problems regarding computational aspects of modal logics [8]. In our concrete case, we have a complex enough logics that puts together normal and non-normal modalities which lead us to a potentially huge multi-modal model not easily tractable from a computational point of view [10]. We initiated therefore the application of a divide-and-conquer strategy. As it is well known, there are no uniform solutions for putting together deductive engines; nonetheless we found that we can depict the multi-modal logic under study as the result of two different processes of combinations of logics: fibring and independent combination [6, 7]. The divide-an-conquer point of view helped us to see the main model as splitted into two component sub-models, each with its own peculiarities; both submodels are lately put together in such a way that they preserve their own completeness properties and we keep modularized their respective proof porcedures.

Independently of the MAS' logics we use throughout the paper, the accounts and techniques applied should be useful to approaches that aim at combining/decomposing normal and non-normal modal logics. Attempts of this kind should also take into account the description of the failed decomposition of two non-normal modal logics explained by Fajardo and Finger in [5].

A fibring is a particular combination of logics in which, intuitively, one logical machinery is placed on top of another base logical machinery. In this case, we see the normal logic placed on top of the non-normal logic. We believe that this way of looking at the theory in [17] is clean and may shed light on some implementation issues. For instance, we found a natural correspondence between the fibring an the interrelation of model checkers for both normal and non-normal formulas. Moreover, other nested fibrings become clearer to define and implement over the theory; for example, a further fibring with a temporal logic adds yet another useful modeling level to the system.

The independent combination allows us to put together two distinct models. Looking at the original model in [17] as an independent combination amounts to consider it as two sub-structures, one normal, one non-normal, that can be put together to build an ontology for representing pairs of states-of-affairs where to model and test *mental aspects - conducts*. This ontology appears useful to capture scenarios where what agents have in mind and what they do is relevant. It is clear that agents act differently in different times or under different circumstances, e.g. when believing in or knowing different or new things. The independent combination may be seen therefore as putting together two perspectives: (i) a traditional modal cognitive account of agents, and (ii) a structural approach to non-monotonicity. The relation between mental states and non-monotonicity has already been studied by Meyer and van der Hoek, and Thomason [14, 19]. Our approach can be seen as a way of introducing non-monotonicity from the modeling point of view. As an example, suppose the following facts: one, at a given point, may have no religion and therefore eats all types of animal meat. Later on time, one embraces some cult that forbids meat eating. Possible pairs of situations such as (no religion, meat eating), (no religion, vegetarian), (this religion, meat eating), (this religion, vegetarian) provide a basis for analyzing e.g. agents' commitment to the cult at different points.

The rest of the paper is organized as follows. Section 2 describes the logical framework used to talk about agents, their internal states, their visible behavior, and norms. Section 3 reorganizes the multi-relational model in [17] as a fibring, which amounts to place the normal logics on top of the non-normal logics. For doing this, we first obtain two restrictions of the original logics. Based on the fibring, the sketch for an appropriate model checker is also outlined. Section 4 presents an independent combination of the normal and the non-normal counterparts of the base logics. This combination leads to an ontology of pairs of state-of-affairs which allows a structural basis for non-monotonic inferences and to more expressiveness. For example, it is possible to write and test in the new ontology *wffs* such as $Does_i\ (Does_j\ (Goal\ \mathscr{A}))$. Finally, Section 5 exposes some remarks on the technical work presented.

2 Logical Framework

In [17] some forms of collective trust are presented, and a perspective on how these forms of trust can be logically related to the emergence of obligations within groups of agents are suggested. The starting point is the definition of individual trust proposed by Castelfranchi and Falcone in [15]. It is argued that the basic ingredients of trust can be captured within a modal approach; this approach, which is also widely accepted when collective attitudes are considered, proved useful in identifying trust settings in multi-agent systems, each corresponding to a different degree of group confidence. Finally, it is discussed -with special attention to the legal domain- the relation between collective trust (which is a form of trust based on mutual belief and strong delegation of tasks) and the emergence of normative conventions within groups of agents. The paper provides evidence that minimal adjustments are required for existing frameworks such as those in [3] to deal with a theory of common trust and norms; it is shown that multi-modal frameworks for MAS can be easily extended to cover highly structured scenarios involving trust.

For working within the framework described, we need a language to talk about the internal configuration of agents, obligations, and actions. We briefly outline our standard language in this section.

We use a multi-modal approach for dealing with agents' attitudes, but integrated by adding agency operators [17]. Hence, we work with a finite set of agents $A = \{x, y, z, ...\}$ and a countable set of atomic propositional sentences usually denoted by $P = \{p, q, r, ...\}$. Complex expressions are formed syntactically from these in the usual inductive way using $\perp$ ($false$) and $\top$ ($true$), standard Boolean connectives, and the unary modalities we describe next.

We use $Goal_x\ \mathscr{A}$ to mean that "agent x has goal $\mathscr{A}$", where $\mathscr{A}$ is a proposition. Propositions reflect particular state-of-affairs, as in [3]. $Int_x\ \mathscr{A}$ is meant to stand for "agent x has the intention to make $\mathscr{A}$ true". Intentions within the area of Cooperative Problem Solving (CPS) are viewed as inspiration for goal-directed activities. The doxastic modality $Bel_x\ \mathscr{A}$ represents that "agent x has the belief that $\mathscr{A}$". The $Does_x\ \mathscr{A}$ operator is to be understood in the same sense given in Elgesem's account to represent successful agency i.e. "x indeed brings about that $\mathscr{A}$" [4]. To simplify technicalities, the logic in [17] assumes that in expressions like $Does_x\ \mathscr{A}$ no modal operators occur in the scope of the $Does$; therefore $\mathscr{A}$ denotes any behavioral action concerning a conduct, such as withdrawal, inform, purchase, payment, etc. We will assume the same restriction for Section 3, and we will eliminate it for Section 4 for regaining expressiveness.

As classically established, $Goal$ is a K_n operator, while Int and Bel are, respectively, KD_n and $KD45_n$. O is the deontic operator with standard KD semantics [13]. The logic of $Does$, instead, is non-normal and it amounts to the following schemata [4, 9]: $Does_x\ \mathscr{A} \rightarrow \mathscr{A}$, $(Does_x\ \mathscr{A} \wedge Does_x\ \mathscr{B}) \rightarrow Does_x(\mathscr{A} \wedge \mathscr{B})$, and $\neg Does_x \top$.

3 The Fibring

Before reorganising the logic in [17] as a fibring, we recall some background knowledge.

According to Hansson and Gärdenfors [11], a generalization of the traditional Kripke semantics is as follows. Instead of a collection of worlds connected to a given world w through a relation R, consider a set of collections of worlds connected to w. These collections are the *neighbourhoods* of w. Formally, a Scott-Montague frame is an ordered pair $\langle W, N \rangle$ where W is a set of worlds and N is a function assigning to each w in W a set of subsets of W (the neighbourhoods of w). A Scott-Montague model is a triple $\langle W, N, V \rangle$ where $\langle W, N \rangle$ is a Scott-Montague frame and V is a valuation function defined as for Kripke frames, except for $\Box\mathscr{A}$: it is true at w iff the set of elements of W where $\mathscr{A}$ is true is one of the sets in $N(w)$; i.e. is a neighbourhood of w.

Let us bring in the structure for the *MAS* in [17]. It is a multi-relational frame of the form:

$$\mathfrak{F} = \langle A, W, \{B_i\}_{i\in A}, \{G_i\}_{i\in A}, \{I_i\}_{i\in A}, \{D_i\}_{i\in A}\rangle$$

where:

- A is the finite set of agents;
- W is a set of situations, or points, or possible worlds;
- $\{B_i\}_{i\in A}$ is a set of accessibility relations wrt *Bel*, which are transitive, euclidean and serial;
- $\{G_i\}_{i\in A}$ is a set of accessibility relations wrt *Goal*, (standard K_n semantics);
- $\{I_i\}_{i\in A}$ is a set of accessibility relations wrt *Int*, which are serial; and
- $\{D_i\}_{i\in A}$ is a family of sets of accessibility relations D_i wrt *Does*, which are pointwise closed under intersection, reflexive and serial [9].

A model based on $\mathfrak{F}$ is in its turn of the form $< \mathfrak{F}, V >$, where V is the corresponding valuation function ([17], Definition 2).

Put this way, it is easy to identify two overlapping "nets" of relations over the same set W. The first net (or multi-graph) corresponds to "wires" for normal operators, the second net corresponds to the accessibility relations for the $Does_i$ modalities.

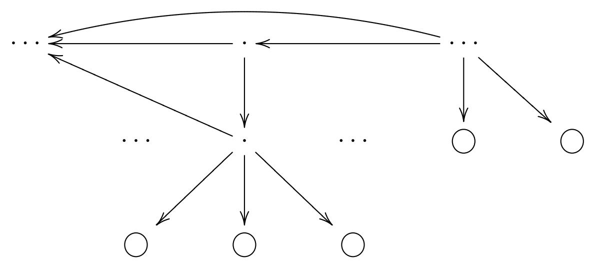
This definition does not include the O modality. O was incorporated later in $\mathfrak{F}$ for dealing with the deontic connotation of an operator of the theory ([17], Section 4). We will keep it apart in what follows to keep the set of modalities manageable. We come back to O later with the purpose to showing further possibilities for combining logics (Section 5).

Following, we can assert two facts based on Definition 4.24 and Theorem 4.22 in [2] (which respectively settle how to construct a canonical model for a normal logic, and state that a normal modal logic is strongly complete with respect to its canonical model). First, that the modal similarity type built up from the normal modalities above has a canonical model; second, that this logic is complete w.r.t. its canonical model. Let us call **N** to the logic with signature (*Bel*, *Int*, *Goal*) above (the normal modalities), hence **N** is a normal multi-modal multi-agent logics, which is complete (this proof is available in [1], we also provide it in the Appendix).

Taking into account *Observation 1* and what stated regarding **N**, and according to the definition of fibring given by Finger and Gabbay [6], (see also [7]) the logic in [17] can be seen as a combination of logics where the normal modal machinery is placed on top of the non-normal logic. The non-normal equipment is in its turn multi-modal, as there is one $Does_i$ modality for each agent i.

Let us develop this insight.

Consider $\mathfrak{F}$ as splitted into an outer normal multi-modal frame, and inner Scott-Montague frames. Graphically:



Provided this rearrangement, the intuition behind the valuation of wffs within the system is the following. When we evaluate normal operators (e.g. we parse a *wff*) we navigate through the outer Kripke model. When a $Does_i$ formula at any given point w is to be evaluated, we navigate through a Scott-Montague model.

The following subsection presents the technical aspects of the fibring.

3.1 Fibring: Syntax and Semantics

Take the logic in [17]. Call **N** to the restriction of $\mathfrak{F}$ to its normal part, and call Does to the restriction of $\mathfrak{F}$ to its non-normal part. We can safely assume that Does is a propositional logic [4, 9]. According to the methodology in [7], we partition the set of formulas in Does into two subsets: Boolean formulas, BDoes, and monolithic formulas, MDoes. A formula $\mathscr{A}$ belongs to BDoes if its outermost operator is a Boolean connective (e.g. $Does_x\ \mathscr{A} \wedge Does_x\ \mathscr{B}$); otherwise it belongs to MDoes (e.g. $Does_x\ \mathscr{A}$). It is clear that there is no intersection among the set of modalities of **N** and Does. Call **N**(Does) to the fibring of Does by means of **N**.

N(Does): Syntax. Let $\mathscr{L}_{\text{Does}}$ denote the language of the logic of agency (as in Section 2, with no normal modalities and without their syntax formation rules), and $\mathscr{L}_{\mathbf{N}}$ denote the language of **N** (as in Section 2, without the *Does* modality and its syntax formation rule). The language $\mathscr{L}_{\mathbf{N}(\text{Does})}$ of **N**(Does) -over the set of proposition letters P- is obtained by replacing the formation rule of sentences in $\mathscr{L}_{\mathbf{N}}$ that says "every proposition letter in P is a formula" by the formation rule:

$$every\ monolithic\ formula\ in\ \mathscr{L}_{Does}\ is\ a\ formula$$

As pointed out in [6], this replacement can be matched with a process called "fuzzling" or layering: formulas in the base system can be substituted for atoms of the top system.

To formally outline the semantics for the fibring, we need a reframing of models based on $\mathfrak{F}$ in terms of the restricted models.

A model for **N**(Does) has the structure:

$$\mathfrak{M} = \langle A, W, \{B_i\}_{i\in A}, \{G_i\}_{i\in A}, \{I_i\}_{i\in A}, V', \{d_i\}_{i\in A}\rangle \quad (1)$$

where:

- A is a finite set of agents;
- W is a set of points, or possible worlds;
- $\{B_i\}_{i\in A}$ is a set of accessibility relations wrt *Bel*, which are transitive, euclidean and serial;
- $\{G_i\}_{i\in A}$ is a set of accessibility relations wrt *Goal*;
- $\{I_i\}_{i\in A}$ is a set of accessibility relations wrt *Int*, which are serial;
- V' is the valuation function V restricted to the normal operators, defined as follows:
    1. standard Boolean conditions;
    2. $V'(w, Bel_i\ \mathscr{A}) = 1$ iff $\forall v \in W$(if wB_iv then $V'(v, \mathscr{A}) = 1$);
    3. $V'(w, Goal_i\ \mathscr{A}) = 1$ iff $\forall v \in W$(if wG_iv then $V'(v, \mathscr{A}) = 1$);
    4. $V'(w, Int_i\ \mathscr{A}) = 1$ iff $\forall v \in W$(if wI_iv then $V'(v, \mathscr{A}) = 1$); and
- each d_i is a total function mapping, for each world w in W, for each agent i, a neighborhood model of the form:

$$\eta = \langle W, D_i, \mathtt{v}\rangle \quad (2)$$

where:
- W is the (same, original) set of worlds,
- D_i is a family of sets of accessibility relations $\mathtt{D}_i$ wrt agency regarding agent i, which are pointwise closed under intersection, reflexive and serial [9].
- $\mathtt{v}$ is V restricted to the non-normal operators. That is, the valuation function for agency that says that $Does_i\ \mathscr{A}$ holds in w if and only if the set of worlds where $\mathscr{A}$ is true is one of the neighborhoods of w. Formally:
    1. standard Boolean conditions;
    2. $\mathtt{v}(w, Does_i\ \mathscr{A}) = 1$ *iff* $\exists \mathtt{D}_i \in D_i$ *such that* $\forall u(w\mathtt{D}_iu$ *iff* $\mathtt{v}(u, \mathscr{A}) = 1)$.

Let us call $\mathcal{K}\mathscr{L}_{\text{Does}}$ to the set of models for $\mathscr{L}_{\text{Does}}$, then $d_i\colon W \to \mathcal{K}\mathscr{L}_{\text{Does}}$.

N(Does): Semantics. Given a model $\mathfrak{M}$, given $w \in W$, given V' valuation function in $\mathfrak{M}$, and given functions d_i (the fibring itself), the semantics for **N**(Does) is obtained by replacing the clause for **N** that says

$$\mathfrak{M}, w \models p \text{ } \textit{iff } p \in V'(w), \textit{whenever } p \in P$$

,

by the clause:

$$\mathfrak{M}, w \models \mathscr{A} \text{ } \textit{iff } d_i(w) \models \mathscr{A}, \textit{whenever } \mathscr{A} \in \text{MDoes}.$$

Note here that $\mathscr{A}$ has the form $Does_i\ \mathscr{B}$, as $\mathscr{A}$ is a monolithic formula.

Once a formula has entered the "*Does* component" it can not come back to the top level [7]. Subsequently, as pointed out in *Observation 1*, in the present layout we can not test the validity of statements such as $Does_i(Goal_j\ \mathscr{A})$ (which can be seen as capturing a form of persuasion: "agent i makes agent j have $\mathscr{A}$ as a goal"). We address a possible solution to this drawback in Section 4.

Regarding the notion of fibring used, note that we combine the logics in a rather plain way: there are no bridging axioms nor intricate interactions among modal operators. Therefore, soundness and completeness results are applicable as follows. Fix a finite number of agents to prevent possible infiniteness of the system. For the normal operators, apply the results in [1](see Appendix); for the logics of agency, apply [9]. By proceeding this way, we profit from the separate proofs and properties of each logic, and avoid the embeddings pointed out in *Observation 2*.

The logic of Does has the finite model property, i.e. we do not have to bother about arbitrary infinite models, for we can always find an equivalent finite one [9]. This opens the door for decidability results and, from a computational point of view, for the design of model checkers.

3.2 Model Checking

A model checker is a program that solves the model checking problem. The global model checking problem for $\mathbf{N}$(Does) consists in checking whether, given a *wff* φ, and given $\mathfrak{M}$ model for $\mathbf{N}$(Does); there exists a $w \in W$ such that $\mathfrak{M}, w \models \varphi$. We follow the modal model checker construction in [7]. Let φ be a formula and let $\text{MM}\mathscr{L}_{\text{Does}}(\varphi)$ be the set of *maximal monolithic subformulas of* φ belonging to $\mathscr{L}_{\text{Does}}$. Let φ' be the $\mathbf{N}$-formula obtained by replacing every subformula $\alpha \in \text{MM}\mathscr{L}_{\text{Does}}(\varphi)$ by a new proposition letter p_α. Below are the sketches of the model-checkers needed to solve the modal checking problem for $\mathbf{N}$(Does):

Function $MC_{\mathbf{N}(\text{Does})}((A, W, B_i, G_i, I_i, V', \{d_i\}), \varphi)$
    input: a fibred model $\mathfrak{M}$ and a formula $\varphi \in \mathscr{L}_{\mathbf{N}(\text{Does})}$
    compute $\text{MM}\mathscr{L}_{\text{Does}}(\varphi)$
    for every $\alpha \in \text{MM}\mathscr{L}_{\text{Does}}(\varphi)$
        $i :=$ *identify the agent involved in* α
        for every $w \in W$
            if$(MC_{\text{Does}}(d_i(w), \alpha) = true)$ *then*
                $V'(w) := V'(w) \bigcup \{p_\alpha\}$ /**fuzzling**/

    build up φ' /* *systematically replace variables generated above* */
    return $MC_{\mathbf{N}}((A, W, B_i, G_i, I_i, V', \{d_i\}), \varphi')$;/**calls to the normal checker**/

Function $MC_{\text{Does}}(d_i(w), \alpha)$
    input: a Scott-Montague model of structure η and a maximal monolithic sub-formula α.
    while there are neighbourhoods unchecked in $d_i(w)$

```
        n_k = set n_i ∈ d_i(w)    /*n_k iterates on the set of neighbourhoods*/
        for every w ∈ n_k
            if α ∉ v(w) then return false
    return true
```

```
FunctionMC_N((A, W, B_i, G_i, I_i, V', d_i), φ')
    input: a model 𝔐 = (A, W, B_i, G_i, I_i, V', d_i) and a formula φ'
    for  every w ∈ W
        if check((A, w, B_i, G_i, I_i, V'), φ')
                return w
    returnfalse
```

```
Function check((A, w, B_i, G_i, I_i, V'), α)
    case on the form of α
        α = p_α' :
                if p_α' ∉ V'(w)
                    return false
        α = ¬α' :
                if check((A, w, B_i, G_i, I_i, V'), α')
                    return false
        α = α_1 ∧ α_2 :
                if not check((A, w, B_i, G_i, I_i, V'), α_1) or not check((A, w, B_i, G_i, I_i, V'), α_2)
                    return false
        α = α_1 ∨ α_2 :
                if not check((A, w, B_i, G_i, I_i, V'), α_1) and not check((A, w, B_i, G_i, I_i, V'), α_2)
                    return false
        α = Bel_i(α') :
                for each v such that wB_iv
                    if not check((A, v, B_i, G_i, I_i, V'), α')
                        return false
        α = Goal_i(α') :
                for each v such that wG_iv
                    if not check((A, v, B_i, G_i, I_i, V'), α')
                        return false
        α = Int_i(α') :
                for each v such that wI_iv
                    if not check((A, v, B_i, G_i, I_i, V'), α')
                        return false
        others : return false
    return true
```

The procedures should be understood as follows. Given a fibred model and a formula φ, $MC_{\mathbf{N}(\text{Does})}$ first computes the set $\text{MM}\mathscr{L}_{\text{Does}}(\varphi)$ of maximal monolithic sub-formulas of φ. For each of these, the checker identifies which agent is carrying out the action. Then, the checker establishes the worlds where that action has been carried out successfully. For doing this, the MC_{Does} checker is called with the Scott-Montague model $d_i(w)$ as parameter (recall d_i has structure η). MC_{Does} is nothing but pseudo-code for the valuation function $\mathtt{v}$, it tests whether there is a neighborhood of w where α holds. If so, the new letter p_α is added to $V'(w)$ to register such successful agency. Finally, before calling the normal model checker $MC_{\mathbf{N}}$, the new formula φ' is built without the *Does* modalities; these have been replaced in the former fuzzling.

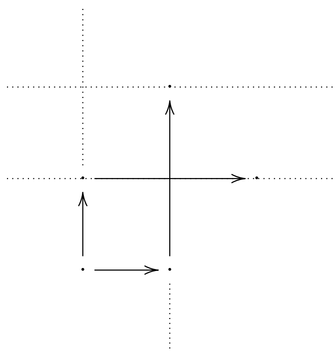
4 Independent Combination of Mental Aspects and Visible Actions

$Does_i(Goal_j\ \mathscr{A})$ is a formula in which the normal modality appears within the scope of a non-normal *Does*. Note that, according to our *Observation 1*, we can not express this formula in the original system. An independent combination between a basic temporal and a simple deontic logic for MAS has been recently depicted in [18]. That combination puts together two normal modal logics: a temporal one and a deontic one.

Our aim now is to combine the normal and the non-normal counterparts of $\mathfrak{F}$ to get a new system where we can write and test the validity of formulas with arbitrarily interleaved cognitive and agency modalities.

For doing this, let us take a look to $\mathfrak{F}$ again. Consider it once more as splitted into two separate substructures: one gathering the normal logics, and another one gathering the logics of agency. Again, there are two overlapping "nets" of relations identifiable over the same set W. The former is a Kripke-styled cognitive ontology where goals, beliefs, intentions are interpreted i.e. it captures internal (mental) motivational and informational aspects of agents (also the deontological aspects of the system, but recall that we keep this apart as a matter of presentation) the latter is a Scott-Montague structure which captures the external, visible, behavioral side of agents.

Now to the combination. First, duplicate and subindicate the elements in W to get one set of situations W_N, and another set W_D. Now build an ontology $W_N \times W_D$ of pairs (w_N, w_D) representing the intuition *this mental configuration - this conduct.* Classically:



Combination. **Syntax**. Let $\mathscr{L}_{\mathbf{N}}$ denote the language of **N** (the base logic restricted to the normal operators), and $\mathscr{L}_{\text{Does}}$ denote the language of the logic of agency. The language $\mathscr{L}_{\mathbf{N}\times\text{Does}}$ is obtained by taking the union of the formation rules for the combination of $\mathscr{L}_{\mathbf{N}}$and $\mathscr{L}_{\text{Does}}$. Unlike the case of $\mathscr{L}_{\mathbf{N}(\text{Does})}$, $Does_i(Goal_j\ \mathscr{A})$ and $Goal_j(Does_i\ \mathscr{A})$ are both formulas of $\mathscr{L}_{\mathbf{N}\times\text{Does}}$.

Combination. **Semantics**. Assume that we have two structures: (A, W_N, $\{B_i\}$, $\{G_i\}$, $\{I_i\}$, V',) and $(A, W_D, \{D_i\}, \mathtt{v})$, where to respectively test the validity of the normal modalities and the non-normal (*Does*) modalities. The former is a Kripke model; the latter a Scott-Montague model. Interpret $\mathscr{L}_{\mathbf{N}\times\text{Does}}$ formulas over a combined model

$$\mathfrak{C} = (A, W_N \times W_D, \{B_i\}_{i\in A}, \{G_i\}_{i\in A}, \{I_i\}_{i\in A}, \{D_i\}_{i\in A}, \mathtt{V}), \tag{3}$$

where:

- A is the set of agents;
- $W_N \times W_D$ is a set of pairs of situations;
- $\{B_i\}_{i\in A}, \{G_i\}_{i\in A}, \{I_i\}_{i\in A}$ are the accessibility relations for the normal operators (with semantics as in Section 3);
- $\{D_i\}_{i\in A}$ are the accessibility relations for the agency operators; and
- $\mathtt{V} : W_N \times W_D \to Pow(P)$ is a function assigning to each pair in $W_N \times W_D$ the set of proposition letters in P which are true.

The definition of a formula in $\mathscr{L}_{\mathbf{N}\times\text{Does}}$ being satisfied in a model $\mathfrak{C}$ at state (w_N, w_D) amounts to:

$$\begin{aligned}
\mathfrak{C},(w_N,w_D) &\models Bel_i\ \mathscr{A}\ \textit{iff}\ \ \forall v_N \in W_N(\ \textit{if}\ w_N B v_N\ \textit{then}\ \mathfrak{C},(v_N,w_D) \models \mathscr{A}).\\
\mathfrak{C},(w_N,w_D) &\models Goal_i\ \mathscr{A}\ \textit{iff}\ \ \forall v_N \in W_N(\ \textit{if}\ w_N G v_N\ \textit{then}\ \mathfrak{C},(v_N,w_D) \models \mathscr{A}).\\
\mathfrak{C},(w_N,w_D) &\models Int_i\ \mathscr{A}\ \textit{iff}\ \ \forall v_N \in W_N(\ \textit{if}\ w_N I v_N\ \textit{then}\ \mathfrak{C},(v_N,w_D) \models \mathscr{A}).\\
\mathfrak{C},(w_N,w_D) &\models Does_i\ \mathscr{A}\ \textit{iff there exists a neighborhood}\ n\ \textit{of}\ w_D\ \textit{such that}\\
&\qquad \forall v \in n\ (\mathfrak{C},(w_N,v) \models \mathscr{A}).
\end{aligned} \tag{4}$$

A scan through the combined structure is done according to which operator is being tested. Normal operators move along the first component (w_N), and non-normal operators move along the second component of the current world (w_D).

Example. Persuasion. The formula $Does_i(Goal_j\ \mathscr{A})$ can be seen as a form of persuasion, meaning that agent i makes agent j have $\mathscr{A}$ as goal. How do we test the validity of such a formula in a world (w_N, w_D)? The movements along the multi-graph are determined by $\mathfrak{C}, (w_N, w_D) \models Does_i\ (Goal_j\ \mathscr{A})$ iff $\exists$ neighbourhood n_i of w_D such that $\forall v_k \in n_i\ (\mathfrak{C}, (w_N, v_k) \models Goal_j\ \mathscr{A})$, which amounts to test $\forall v_k \in n_i($ iff $\forall u_N \in W_N($ if $w_N G_j u_N$ then $\mathfrak{C}, (u_N, v_k) \models \mathscr{A}))$.

5 Final Remarks

The Logics and the Code. In this work, the technical manipulation of the logics involved is a more complicated task than the later construction of the model checkers for the fibring in (Subsection 3.2). That imperative pseudo-code is nothing but the iteration over a specific domain with the purpose of verifying satisfiability. As shown in subsections 1.1 and 1.3 of [2], there is no mathematical distinction between a modal and a corresponding first order model -both are relational structures. We naturally used this correspondence to build the pseudo-code. Let us apply the concepts of subsection 2.4 to sketch the validity of this ideas. In particular, Definition 2.45 and Proposition 2.47 are of interest here. The notion of Standard Translation in Definition 2.45 explains how to convert modal formulas into first-order formulas. Proposition 2.47 [2] provides a first-order reformulation of the modal satisfaction definition.

The O Modality. We dealt with some of the modalities underlying the trust theory in [17]. In that work, a deontic connotation for the concept of collective trust is developed (Section 4). Lawful support to collective trust is guaranteed in the theory with the schema: $(C\text{-}trust^G_y\ \mathscr{A}) \to O^G(Does_y\ \mathscr{A})$, which is devised with a view to reflect the lawful force of trust, relativized to groups. The schema is to be understood as a standard of (good faith) behavior that can be identified with reference to social or group norms, to correctness, or reasonableness: if the group trusts in agent y with respect to $\mathscr{A}$, agent y is obliged to carry out $\mathscr{A}$.

For capturing -in systems as presented here- this deontic connotation of C-$trust$, and for dealing with lawful concepts in general, we must consider deontic modalities such as O and O^G. O is the deontic operator for generic obligations, meaning "it is obligatory that" [16, 13], and O^G is a relativized obligation operator which is meant to stand for "it is obligatory in the interest of G that" (see e.g. [12]). O has classical KD semantics while the relativized operators have the accepted KD_n semantics. Therefore, correspondingly, extend the $\mathbf{N}$-$frame$ in Section 1 with the set of accessibility relations $\mathcal{O}$ and $\{\mathcal{O}^i\}_{i\in A}$ w.r.t. general and relativized obligations. It should be clear that this extension needs the extension of the signature, of the formation rules, and of the valuation function, but also requires a further completeness proof for the renewed $\mathbf{N}$ (we leave this proof apart in this paper; nonetheless recall that applicable techniques are as in the Appendix).

Further fibrings. Theoretically speaking, the very idea of reasoning about time should extend the MASs in this work and other MASs of similar structure consistently. From a computational standpoint, a minimal functionality for the manipulation of time from a basic modal (and linear) perspective involves few technical adjustments for either combination presented in this paper. For example, intuitively, a basic temporalization amounts to place the temporal machinery on top of the fibred MAS (Section 3.1), just in the same spirit we placed the normal machinery on top of the non-normal one. Consider the model $(T, <, g, t_0)$. The outer frame $(T, <)$ corresponds to the temporal evolution of the system; t_0 in T is the initial point in time. The system evolves through time in the sense that new groups and generic/individual beliefs, intentions, trust relations, obligations are settled while some others become obsolete. In its turn, g is the total function that brings in a model $\mathfrak{M}$ for each point in time.

A Completeness proof for N

In this appendix, a completeness proof is given for the restriction **N**. The method used is often applied in modal logic for proving completeness with respect to finite models; is in turn inspired by the completeness proofs of mutual intentions shown by Dunnin-Keplicz and Verbrugge in [3]. In fact, we adapt that to **N**, and apply Definition 4.24 and Theorem 4.22 described in [2]; these respectively settle how to construct a canonical model for a normal logic, and state that a normal modal logic is strongly complete with respect to its canonical model.

We have to prove that, supposing that $\mathbf{N} \nvdash \varphi$, there is a model $\mathfrak{M}_{\mathbf{N}}$ and a $w \in \mathfrak{M}_{\mathbf{N}}$ such that $\mathfrak{M}_{\mathbf{N}}, w \not\models \varphi$. The proof has four steps:

Step 1. Closure. Construct a finite set of formulas Φ called the closure of φ. Φ contains φ and all its sub-formulas, plus certain other formulas that are needed in Step 4 below to show than an appropriate valuation falsifying φ at a certain world can be defined. The set Φ is also closed under single negations.

The closure of φ with respect to **N** is the minimal set Φ of **N**-*formulas* such that, for every agent, the following hold:

1. $\varphi \in \Phi$.
2. If $\psi \in \Phi$ and χ is a sub-formula of ψ, then $\chi \in \Phi$;
3. If $\psi \in \Phi$ and Φ itself is not a negation, then $\neg\psi \in \Phi$;
4. If $M\text{-}INT_G(\psi) \in \Phi$ then $E\text{-}INT_G(\psi \wedge M\text{-}INT_G(\psi)) \in \Phi$;
5. If $E\text{-}INT_G(\psi) \in \Phi$ then $INT(i, \psi) \in \Phi$ for all $i \in G$;
6. $\neg INT(i, \bot) \in \Phi$ for all $i \leq m$;
7. If $C\text{-}BEL_G(\psi) \in \Phi$ then $E\text{-}BEL_G(\psi \wedge C\text{-}BEL_G(\psi)) \in \Phi$;
8. If $E\text{-}BEL_G(\psi) \in \Phi$ then $BEL(i, \psi) \in \Phi$ for all $i \in G$;
9. $\neg BEL(i, \bot) \in \Phi$ for all $i \leq m$;

10. $\neg GOAL(i, \bot) \in \Phi$ for all $i \leq m$.

It should be clean that for every formula ϕ, Φ is a *finite* set of formulas (recall that the language in [17] includes: $M\text{-}INT$, $E\text{-}INT$, $C\text{-}BEL$, $E\text{-}BEL$) [3].

Step 2. Canonical model. To construct a canonical model we need to define the worlds and relations between them. Each of these worlds are *maximally* **N**-*consistent* sets. To build this sets, we apply the Lindenbaum Lemma (which is proved in Lemma 4.17 [2]) over Φ *step 1*, as follows:

Let Φ be the closure of ϕ with respect to **N**. If $\Gamma \subseteq \Phi$ is **N**-*consistent*, then there is a set $\Gamma' \supseteq \Gamma$ which is maximally **N**-*consistent* in Φ.

Step 3. Build a canonical model using Definition 4.24 [2]. This model will turn out to contain a world where $\neg\psi$ holds.
Let $\mathfrak{M}_\varphi =< S_\phi, \pi, I_1, ..., I_m, B_1, ..., B_m, G_1, ..., G_m >$ be the Kripke model defined as follows:

- As domain of states, one state s_Γ is defined for each *maximally* **N**-*consistent* $\Gamma \subseteq \Phi$. Note that, because Φ is finite, there are only finitely many states. Formally, we defined $CON_\Phi = \{\Gamma | \Gamma \text{ is } maximally\ \mathbf{N}\text{-}consistent \text{ in } \Phi\}$ and $S_\varphi = \{s_\Gamma | \Gamma \in CON_\Phi\}$.
- To make a truth assignment π, we want to conform to the propositional atoms that are contained in the maximally consistent sets corresponding to each world. Thus we define $\pi(s_\Gamma)(p) = 1$ if and only if $p \in \Gamma$. Note that this makes all propositional atoms that do not occur in φ false in every world of the model.
- The corresponding relations are defined as follows:

$$I_i = \{(s_\Gamma, s_\triangle) | \psi \in \triangle \ for\ all\ \psi\ such\ that\ INT_i(\psi) \in \Gamma\}$$

$$B_i = \{(s_\Gamma, s_\triangle) | \psi \in \triangle \ for\ all\ \psi\ such\ that\ BEL_i(\psi) \in \Gamma\}$$

$$G_i = \{(s_\Gamma, s_\triangle) | \psi \in \triangle \ for\ all\ \psi\ such\ that\ GOAL_i(\psi) \in \Gamma\}$$

It will turn out that with this definition we get $\mathfrak{M}_\varphi, s_\Gamma \models p$ iff $p \in \Gamma$ for propositional atoms p.

Step 4. Completeness of **N**. If $\mathbf{N} \nvdash \varphi$ then there is a model $\mathfrak{M}$ and a w such that $\mathfrak{M}, w \not\models \varphi$. Proof: Suppose $\mathbf{N} \nvdash \varphi$. Take $\mathfrak{M}_\varphi$ as in step 3. Note that there is a formula χ logically equivalent to $\neg\varphi$ that is an element of Φ; if φ does not start with a negation, χ is the formula $\neg\varphi$ itself. Now, using the Lindenbaum Lemma, there is a maximally consistent set $\Gamma \subseteq \Phi$ such that $\chi \in \Gamma$. By the Finite Truth Lemma, if $\Gamma \in CON_\Phi$ then for all $\psi \in \Phi$ it holds that $\mathfrak{M}_\varphi, s_\Gamma \models \psi$ iff $\psi \in \Gamma$. Thus, this implies that $\mathfrak{M}_\varphi, s_\Gamma \models \chi$, thus $\mathfrak{M}_\varphi, s_\Gamma \not\models \varphi$. Details of the Finite Truth Lemma proof are left to the reader (see [3] and [2], Lemma 4.21).

[3] The collective operators $M\text{-}INT$, $E\text{-}INT$, $C\text{-}BEL$, $E\text{-}BEL$ are defined using the uni-agent modalities INT and BEL [3].

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