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DIFFERENT TOPOGRAPHIC REDUCTION METHODS IN PRACTICAL GRAVIMETRIC GEOID DETERMINATION

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ABSTRACT

Three different topographic reduction methods in geoid determination were investigated. The first method is the classical Helmert second method of condensation yielding the geoid, the second is the Residual Terrain Model (RTM) method yielding the quasigeoid and the third is the Rudzki inversion method. The different types of indirect effects (indirect effect on gravity and indirect effect on geoid) in Helmert's method were also investigated. All three methods use the remove-restore technique and the EGM96 geopotential model as the reference gravity field. A mountainous area, ranging from 32°S to 42°S in latitude and 72°W to 68°W in longitude, was chosen as test area. The area was selected due to its high topography, with a maximum height of 6795 meters and a mean height of 1188 meters, and due to the existence of GPS/leveling points in three different networks. The topography in the test area is represented by a digital terrain model (DTM) with a grid spacing of 1 km x 1 km. Another test was carried out in a flat area with denser data coverage. The external accuracy of the three gravimetric geoids was evaluated by comparing them to undulations derived from GPS/leveling.
***Keywords*:** Helmert - RTM - Rudzki - geoid - quasigeoid - direct terrain effects - indirect effects.

RESUMEN

En el siguiente trabajo se investigan tres métodos diferentes de reducciones gravimétricas para la determinación práctica del geoide gravimétrico: el clásico segundo método de condensación de Helmert, el modelo residual de terreno y el método de inversión de Rudzki. Los tres métodos utilizan la técnica remover-restaurar y el modelo de geopotencial EGM96. Fueron seleccionadas dos áreas, una en una zona montañosa de alta topografía con una altura máxima de 6795 metros y una altura promedio de 1188 metros y otra en una zona plana con cobertura más densa. La topografía está representada por un modelo digital de terreno con un espaciamiento de grillado de un kilómetro por un kilómetro. La evaluación externa del geoide gravimétrico se realiza comparándolo con ondulaciones obtenidas a partir de puntos GPS sobre nivelación.
Palabras claves: Helmert - RTM-Rudzki - Geoide - Casi geoide- Efectos directos sobre el terreno - Efectos indirectos

INTRODUCTION

The use of Stokes's formula in gravimetric geoid determination requires that the gravity anomalies represent boundary values on the geoid. This means that the measured gravity (usually taken on the surface of the Earth) must be reduced to the geoid and there must be no masses outside the geoid (Heiskanen and Moritz, 1967).

There are several gravity reductions used in physical geodesy: the Bouguer reduction, isostatic reductions, the Rudzki inversion method, the second method of Helmert's condensation, etc. The Residual Terrain Model (Forsberg, 1984) is another type of reduction, which takes into account the high frequencies of the topography. RTM yields the quasigeoid, thus the correction between quasigeoid and geoid was applied to compare the results from the three methods with GPS/leveling data.

Theoretically, all reduction methods should lead us to the same geoid if the gravity reductions were applied consistently (Heiskanen and Moritz, 1967) even though each reduction treats the topography in a different way.

DATA SETS

Two areas were selected as test areas. One is an area covering part of the Mendoza and Neuquen provinces. This test area was bounded by latitudes 32° to 42° S and longitudes 68° and 72° W and it was selected due to the presence of GPS/leveling data, sparse gravity coverage coming from different sources, and rough topography.

The other test area is a flat area, with denser gravity data ranging from 34° to 38.5° S in latitude and 59° to 64.5° W in longitude.

Surface gravity measurements

The point gravity measurements, provided by different sources, were referenced to the International Standardisation Net 1971 (I.G.S.N.71). Most of the gravimetric data comes from the database of the Argentine Military Geographic Institute.

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A total of 1452 measured gravity points, with a mean data spacing of approximately 20 km are used in the mountainous area, and 4302 gravity points with a spacing of approximately 8 km are selected in the flat area. The distribution of the gravity points is shown in Figure 1 and Figure 3.

Gravity anomalies

Free-air gravity anomalies are calculated using the parameters of the Geodetic Reference System 1980 (GRS80). The point free-air gravity anomalies were calculated using the following expression (Torge, 1989):

$$\Delta g_{FA} = g + \delta g_{atm} - \gamma + 0.30877(1 - 0.00142\sin^2\varphi)h - 0.7510^{-7}h^2 \text{ [mGal]} \quad (1)$$

This formula uses the second order free-air reduction, applies atmospheric correction (δg_{atm}) and evaluates normal gravity with Somigliana's closed formula, using the parameters of the GRS80. The atmospheric correction is applied as follows (Torge, 1989):

$$\delta g_{atm} = 0.874 - 0.9910^{-4}h + 0.35610^{-8}h^2 \text{ [mGal]} \quad (2)$$

where the height h is in m.

Geopotential model

The reference gravity field is computed from the EGM96 geopotential model (Lemoine *et al.*,1998) complete to degree and order 360.

From the contribution of the EGM96 geopotential model, a reference gravity anomaly (Δg_{GM}) and a reference geoidal undulation (N_{GM} can be calculated. The gravity anomaly estimated at a position (ϕ_p, λ_p) is expressed in spherical approximation (Heiskanen and Moritz, 1967) as:

$$\Delta g_{GM} = R \sum_{n=2}^{n=n_{max}} (n-1) \sum_{m=0}^{n} (\overline{C}_{n,m}\cos m\lambda_P + \overline{S}_{n,m}\sin m\lambda_P)\overline{P}_{n,m}(sen\phi_P) \quad (3)$$

and the reference geoidal undulation as:

$$N_{GM} = R \sum_{n=2}^{n=n_{max}} \sum_{m=0}^{n} (\overline{C}_{n,m}\cos m\lambda_P + \overline{S}_{n,m}\sin m\lambda_P)\overline{P}_{n,m}(sen\phi_P) \quad (4)$$

where R is the mean radius of the Earth, $\overline{C}_{n,m}$ and $\overline{S}_{n,m}$ are the fully normalised spherical harmonic coefficients of the disturbing potential, $\overline{P}_{n,m}$ are the fully normalized associated Legendre functions, and n_{max} denotes the maximum degree and order of expansion of the geopotential solution.

Digital Elevation Model

The global digital elevation model GTOPO30 with a horizontal grid spacing of 30 arc seconds (approximately 1 kilometre) is used to represent the topography in the test area. Detailed information on the characteristics of GTOPO30 including the data distribution format, the data sources, production methods, accuracy, and hints for users, is found in the GTOPO30 documentation file. (http://edcdaac.usgs.gov/gtopo30/gtopo30.html).

The statistics of the topographic data in the test area

Table 1. Statistics of the GTOPO30. Unit: [m]

DEM	Max	Min	Mean	Standard Deviation
GTOPO30 (rough area)	6795	0	1183.530	880.990
GTOPO30 (flat area)	1617	0	136.109	115.393

are given are Table 1.

GPS/leveling data

A total of 166 GPS/leveling points in three different networks were used for comparison with the gravimetric geoid in the rough area, and a total of 119 points were used in the flat area. The distributions of the GPS points can be seen in Figure 2 and Figure 4, respectively. There are no GPS/leveling points above

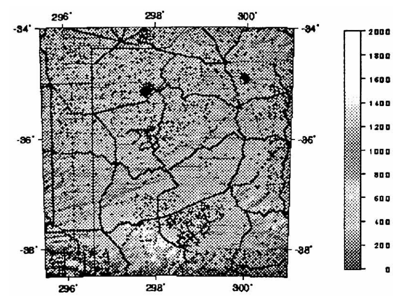

Figure 1. Distribution of the gravity pointsin the flat area on elevation map (m)

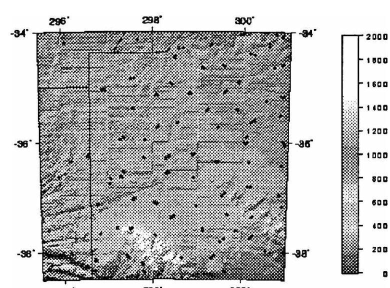

Figure 2. Distribution of GPS/leveling points in the flat area on elevation map (m)

the elevation of 1885 m.

COMPUTATIONAL FORMULAS FOR TERRAIN EFFECTS

The three methods compared in this investigation use the remove-restore technique for the determination of the gravimetric geoid/quasigeoid. Each method handles the topography in a different way.

The geoid or quasigeoid is calculated from Stokes's formula with gravity anomalies as input. Before applying this formula, gravity anomalies must be reduced, in the remove step, by

$$\Delta g = \Delta g_{FA} - \Delta g_{GM} - \Delta g_T \tag{5}$$

where Δg_{FA} is the free-air gravity anomaly, Δ_{GM} is the reference gravity anomaly computed from the geopotential model and Δg_T is the terrain effect, which depends on the reduction method used.

The gravimetric geoid is obtained, in the restore step, by

$$N = N_{\Delta g} + N_{GM} + N_{ind} \tag{6}$$

where N_{GM} is the reference geoidal undulation implied by the geopotential model, N_{ind} is the indirect effect on the geoid and depends on the reduction method used, and $N_{\Delta g}$ Srepresents residual geoid computed with the residual gravity anomalies given in Eq. (5).

Stokes's formula with the rigorous spherical kernel was evaluated by the one-dimensional Fast Fourier Transform algorithm (Haagmans *et al.*, 1993).
The indirect effect on the geoid is

$$N_{ind} = \frac{\Delta W}{\gamma} \tag{7}$$

ΔW is the change of the potential at the geoid due to the terrain reduction applied.
The indirect effect on gravity, which reduces gravity anomalies from the geoid to the cogeoid is expressed by

$$\delta\Delta g = 0.3086 N_{ind}\ [\text{mGal}] \tag{8}$$

The RTM method estimates the quasigeoid. The reduced gravity anomalies refer to the surface of the topography. The final quasigeoid is obtained by

$$\zeta = \zeta_{\Delta g} + \zeta_{GM} + \zeta_T \tag{9}$$

In order to compare the results of this method with the other two reduction methods and to make comparisons with the GPS/leveling derived geoid, the quasigeoid is converted to geoid using the quasigeoid-geoid separation given in Heiskanen and Moritz (1967) as

$$N \approx \zeta + \frac{\Delta g_B}{\bar{\gamma}} h \tag{10}$$

where Δg_B is the Bouguer anomaly and γ is the mean normal gravity (9.81m/seg^2)

Helmert's second method of condensation

Helmert's second method of condensation condenses the topographic masses on a layer on the geoid. Before applying Stokes's formula, the indirect effect on gravity has been considered.

The terrain effect on gravity gT is given by

$$\Delta g_T = -\delta\Delta_g - c_P \tag{11}$$

$\delta\Delta_g$ is the indirect effect on gravity (Sideris and She, 1995)

$$\delta\Delta_g = \frac{2\pi G\rho h^2}{\gamma} \tag{12}$$

and c_p is the classical terrain correction given by the following equation:

$$c_P(i,j) = -G\iint_E\int_{h(i,j)}^{h(x,y)} \frac{\rho(x,y,z)((h(i,j)-z)}{r^3(x_i-x, y_j-y, h(i,j)-z)}\,dxdydz \tag{13}$$

where (x,y,z) is the topographical density at the running point, here is assumed constant at 2.67 gr/cm3, G is the gravitational constant and E denotes the integration area.

The indirect effect on the geoid, up the second order is in planar approximation (Wichiencharoen, 1982)

$$N_{ind} = -\frac{\pi G\rho h^2(i,j)}{\gamma} - \frac{G\rho}{6\gamma}\iint_E \frac{h^3(x,y) - h^3(i,j)}{r_0^3}\,dxdy \tag{14}$$

where r_0 is the planar distance between computation point and data point.

Rudzki inversion method

The Rudzki inversion method is a reduction where the indirect effect is zero. Rudzki shifts all the topographic masses inside the geoid.

$$\Delta g_T = A_T - A_C \tag{15}$$

where A_T is the attraction of the topographic masses, A_C is the attraction of the inverted masses, with the density of the topographic masses being equal to the density of the inverted masses and the thickness of the inverted masses is equal to the height of the topography

$$A_T = -G\iint_E\int_0^{h(x,y)} \frac{\rho(x,y,z)(h(i,j)-z)}{r^3(x_i-x, y_j-y, h(i,j)-z)}\,dxdydz \tag{16}$$

$$A_c = -G \iint_E \int_{h(x,y)-h(i,j)}^{h(i,j)} \frac{\rho(x,y,z)(h_{(i,j)}-z)}{r^3(x_i-x, y_j-y, h_{(i,j)}-z)} dxdydz \quad (17)$$

RTM method

The RTM reduction method takes into account the high frequencies of the topography. The effect of the topography above a long wavelength topographic surface is first removed and later restored.

The RTM gravity terrain effect, Δg_T is given by the approximate expression (Forsberg, 1984)

$$\Delta g_T = \Delta g_{RTM} \approx 2\pi G\rho(h-h_{ref}) - c_P \quad (18)$$

where h is the topographic height given by a digital terrain model, h_{ref} is the height of a smooth mean reference surface and c_p is the classical terrain correction.

The RTM geoid effect is expressed in linear approximation as:

$$\zeta_T = \zeta_{RTM} = \frac{G\rho}{\gamma} \iint_E \int_{href}^{h} \frac{1}{r_0} dxdydz = \frac{G\rho(h-h_{ref})}{\gamma} \iint_E \frac{1}{r_0} dxdy \quad (19)$$

where r_0 is the planar distance

NUMERICAL EXAMPLES

Terrain corrections (c_p) were calculated by FFT for each gravity point from the GTOPO30 Digital Elevation Model (DEM) using the Tc2DFTPL program (Li, 1993). The statistics of terrain corrections in gravity stations can be seen in Table 2. For the rough area, we used up to the third order term of a mass prism model and for the flat area we used up to second order term of a mass line model.

Table 2. Statistics of terrain corrections c_p in gravity stations. Unit: [mGal} in both test areas

	Max	Min	Mean	Standard Deviation
c_P (rough area)	35.39	0.07	2.85	4.12
c_P (flat area)	1.51	0.00	0.02	0.04

The indirect effect on gravity due to Helmert's second method of condensation was considered before applying Stokes's formula. The statistics of this effect, together with the indirect effect on the geoid in the rough area, can be seen in Table 3. In the mountainous area, the computation of the indirect effect on the geoid should be done up to at least second order term. Figure 5 and Figure 6 show the indirect effect on the geoid and on gravity for Helmert's method. The maximum indirect effects are correlated with the topography. In the flat area, the topographic indirect effect on the geoid does not add any significant contribution to the gravimetric geoid undulations. The computation was done only consider the first term in Eq. (14) and the indirect effect on gravity was neglected.

The TC program was used to compute the RTM effects (Forsberg, 1984) and it was modified to

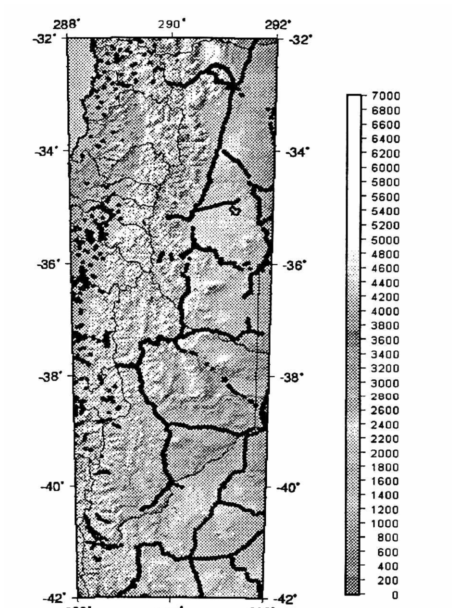

Figure 3. Distribution of gravity pointsin the rough area on elevation map Color scale Unit:[m]

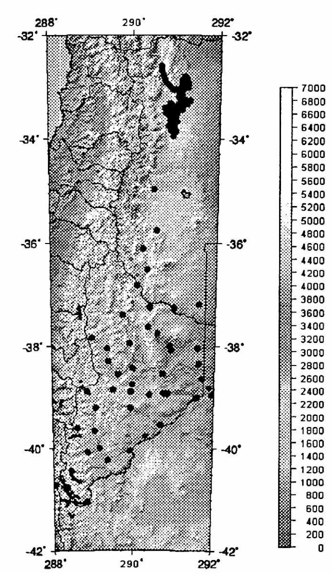

Figure 4. Distribution of GPS/leveling points in the rough area on elevation map Color scale Unit:[m]

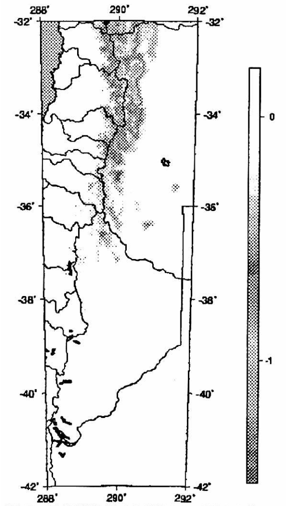

Figure 5. Indirect effect on gravity due to the Helmert's condens ation method in the rough area Unit: [mGal]

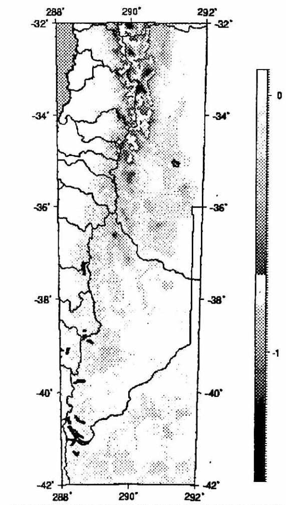

Figure 6. Indirect effect on the geoid due to the Helmert's condensation method in the rough area Unit: [m]

Table 3. Statistics of indirect effects due to the Helmert's condensation method

	Max	Min	Mean	Standard Deviation
On gravity [mGal] (rough area)	0.20	0.00	0.02	0.03
On geoid [m] (rough area)	0.000	-1.162	0.110	0.225
On gravity [mGal] (flat area)	0.00	0.00	0.00	0.00
On geoid [m] (flat area)	-0.001	-0.004	-0.002	-0.001

compute the Rudzki inversion method (Bajracharya *et al.*, 2001). The statistics of the gravity anomalies calculated with the three topographic gravity reductions are presented in Table 4. RTM anomalies are the smoothest gravity anomalies with a standard deviation of 29.5 mGal. Faye anomalies have the maximum standard deviation compared to the other reduction methods. The removal of the reference field (EGM96) does not improve the statistics of RTM reduced gravity anomalies but it does improve the statistics for Helmert and Rudzki reduced gravity anomalies.

The systematic datum differences between the gravimetric geoid and the GPS/leveling geoid and the long wavelengths errors of the geoid were removed by a four-parameter transformation model (Heiskanen and Moritz, 1967). The statistics of the absolute differences between the GPS/leveling-derived geoid and the gravimetric geoids computed using different methods of handling the topography are summarised in Table 5. The numbers in parentheses refer to the results after the least-squares fitting of the four-parameter transformation model has been applied

Table 4. Statistics of the gravity anomalies calculated with the three topographic reductions. Unit [mGal]

		Max	Min	Mean	Standard Deviation
Faye anomalies (rough area)	$\Delta g_{FA}+C_P$	173.59	-124.76	7.05	40.31
	$\Delta g_{FA}-\Delta g_{EGM96}+C_P$	192.02	-192.21	-17.20	37.53
Rudzki anomalies (rough area)	$\Delta g_{FA}-\Delta g_{RU}$	127.09	-122.73	11.36	38.30
	$\Delta g_{FA}-\Delta g_{RU}-\Delta g_{EGM96}$	140.65	-131.77	-12.89	28.77
RTM anomalies (rough area)	$\Delta g_{FA}-\Delta g_{RTM}$	155.40	-71.57	31.28	29.53
	$\Delta g_{FA}-\Delta g_{RTM}-\Delta g_{EGM96}$	100.97	-175.03	7.03	35.85
Faye anomalies (flat area)	$\Delta g_{FA}+C_P$	96.84	-28.18	10.01	11.86
	$\Delta g_{FA}-\Delta g_{EGM96}+C_P$	94.99	-41.19	1.82	7.54
Rudzki anomalies (flat area)	$\Delta g_{FA}-\Delta g_{RU}$	96.91	-27.72	10.01	11.86
	$\Delta g_{FA}-\Delta g_{RU}-\Delta g_{EGM96}$	95.07	-40.73	1.81	7.54
RTM anomalies (flat area)	$\Delta g_{FA}-\Delta g_{RTM}$	97.43	-33.34	10.13	11.61
	$\Delta g_{FA}-\Delta g_{RTM}-\Delta g_{EGM96}$	95.58	-46.36	1.94	7.83

to the original differences. Before applying the four-parameter transformation model, two GPS on benchmark points with large gross errors in either the GPS or the leveling data were removed.

In the mountainous area, gravimetric geoid computed with Rudzki inversion topographic reduction shows the smallest differences from GPS/leveling before fit and RTM geoid shows the highest differences compared to Helmert and Rudzki methods. These large differences could be due to discrepancies between the model elevation and the actual elevation at the station.

CONCLUSIONS

Three different gravity reduction methods have been presented. They treat the topography in a very different way. Helmert's second method of condensation and the RTM method are the most used reduction techniques for the determination of a gravimetric geoid. In this paper, we applied the Rudzki inversion method as well, which is not very often used, even though it has the advantage of no indirect effects.

In the mountainous area, the gravimetric geoid computed with the Rudzki inversion method gave better results compared with the GPS/leveling-derived geoid before and after fit and was the only method that improved the gravimetric geoid considerably compared to the EGM96 results. The gravimetric data needs to be improved in the area of the Andes in order to see further improvements in the geoid. In the flat area, the three reduction methods gave identical results as expected. In the future it is planned to test two other gravity reduction methods, namely the Airy-Heiskanen and Pratt-Hayford topographic-isostatic reductions.

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