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On Some Properties of the Surface Delta Interaction

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A method is discussed for solving the RPA equations without discarding any term in the case of separable interactions. It is specialized to the SDI for both spherical and deformed nuclei and applied in two simple examples.

1. Introduction

During the last years a great amount of work has been devoted to the task of understanding nuclear properties from a microscopic point of view. In particular, many details of the low-lying nuclear spectra have been successfully explained, within the framework of the quasi-particle random phase approximation (RPA), utilizing rather simple effective interactions. Among these, the pairing plus quadrupole (or octupole) force (PQF) employed in the pioneer paper of KISSLINGER and SORENSEN[1] has played an outstanding rol in subsequent developments of the theory. Because of its simplicity, it has been applied in a large number of nuclear structure calculations, being for a long time the only interaction used in cases involving a great number of single-particle levels (i. e., in studying core excitation in medium weight spherical nuclei or in RPA calculations in the deformed heavy region).

More recently an even simpler, and at the same time more realistic force, the Surface Delta Interaction (SDI)[2] has been applied to different nuclear structure problems[3–12] and in particular to the strongly

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deformed rare-earth and transuranic nuclei where only the PQF had been previously employed (for example SOLOVIEV[13] and BES[14]). The results reported in those calculations show that the SDI is as good an effective interaction as any other utilized before, with the advantage of being a one parameter force, while all other interactions contain a greater number of parameters (the PQF, for example, is a three parameter force).

It is worthwhile to recall that the SDI is able to reproduce at the same time both the strong seniority zero coupling of the pairing force and the field producing effects of the quadrupole or octupole force. In addition, it simulates the ^1S and ^3S phase shifts of the two nucleon scattering for the nuclear many body problem[15]. However, from the practical point of view of numerical calculations there remains a fact which makes the PQF preferable to the SDI in some special instances. For example, in order to calculate excited states in deformed heavy nuclei[6] or in studying core excitation[16], the RPA formalism leads to the diagonalization of huge matrices. This is not always an easy task, becoming sometimes an unmanageable numerical problem[7]. However, if the interaction is separable, one can avoid diagonalization because it is then possible to obtain the eigenvalues and eigenvectors from a secular equation of a rather simple form[17]. In order to enforce separability one has to discard particle-particle and exchange particle-hole terms, keeping only direct particle-hole ones[7]. This procedure constitutes a valid approximation in the PQF case, but not a very good one for the SDI although it still yields reasonable results in some cases (i.e., in the deformed heavy region[7, 8, 10]). In others, however, this approximation does not provide good results with the SDI. If one wants to compute core excitation effects along the lines of LOMBARD and CAMPI-BENET[16], the SDI fails when the above-referred terms of the interaction are neglected[18].

The purpose of the present work is to formulate a method which allows one to utilize the SDI within the framework of the RPA without

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discarding terms, and at the same time, avoiding the diagonalization of the RPA matrices. This formalism is described in Section 2, but for a more general kind of interaction. It is specialized for the SDI, both in the spherical and deformed case in Section 3 and illustrated by two simple examples in Section 4. Finally, the conclusions are stated in Section 5.

2. Theory

The starting point is given by the characteristic RPA relations for the eigenvector (ψ) [17].

$$\begin{aligned} \sum_{cd} P(abcdR)\psi_{cdR}+Q(abcdR)\phi_{cdR}&=\omega\psi_{abR} \\ \sum_{cd} Q(abcdR)\psi_{cdR}+P(abcdR)\phi_{cdR}&=-\omega\phi_{abR} \end{aligned} \tag{1}$$

where ω is the excitation energy of a state characterized by a set of quantum numbers R; P, and Q, being the usual RPA submatrices[3].

We now introduce two functions, f_{ab} and g_{ab} related to ψ_{ab} and ϕ_{ab} by

$$\psi_{abR}=f_{abR}+g_{abR}; \quad \phi_{abR}=f_{abR}-g_{abR} \tag{2}$$

and two new matrices U and V which in terms of P and Q are

$$\begin{aligned} U(abcdR)&=P(abcdR)+Q(abcdR) \\ V(abcdR)&=P(abcdR)-Q(abcdR). \end{aligned} \tag{3}$$

Using (2) and (3) we rewrite (1) as

$$\omega\, g_{abR}=\sum_{cd} U(abcdR)f_{cdR}; \quad \omega\, f_{abR}=\sum_{cd} V(abcdR)\, g_{cdR}. \tag{4}$$

Let us assume now that our interaction is of such a type that we can write both P and Q as a sum of separable terms. Then

$$\begin{aligned} U(abcdR)&=E_{ab}\delta_{ac}\delta_{bd}-\kappa\sum_{p=1}^{N} a_p(abR)\,a_p(cdR) \\ V(abcdR)&=E_{ab}\delta_{ac}\delta_{bd}-\kappa\sum_{q=1}^{M} b_q(abR)\,b_q(cdR) \end{aligned} \tag{5}$$

where E_{ab} are the so-called two quasi-particle energies and κ is a coupling constant. Both $a_p(ab)$ and $b_p(ab)$ must satisfy suitable symmetry relations. Straightforward substitution of (5) into (4) yields

$$\begin{aligned} \omega g_{abR}-E_{ab}f_{abR}&=-\kappa\sum_{p=1}^{N} a_p(abR)A_p=-\alpha_{abR} \\ \omega f_{abR}-E_{ab}g_{abR}&=-\kappa\sum_{q=1}^{M} b_q(abR)B_q=-\beta_{abR} \end{aligned} \tag{6}$$

where

$$A_p=\sum_{c\,d} a_p(c\,d\,R) f_{c\,d\,R}; \quad B_q=\sum_{c\,d} b_q(c\,d\,R)\, g_{c\,d\,R}. \tag{7}$$

It is now possible to express the eigenvectors ψ and ϕ in terms of the vectors α and β

$$\psi_{a\,b\,R}=\frac{1}{E_{a\,b}-\omega}(\alpha_{a\,b\,R}+\beta_{a\,b\,R}); \quad \phi_{a\,b\,R}=\frac{1}{E_{a\,b}+\omega}(\alpha_{a\,b\,R}-\beta_{a\,b\,R}). \tag{8}$$

In order to obtain a secular equation for the eigenvalue ω we eliminate f and g from the system (6),

$$f_{a\,b\,R}=D_{a\,b}(E_{a\,b}\alpha_{a\,b\,R}+\omega\beta_{a\,b\,R}), \tag{9a}$$

$$g_{a\,b\,R}=D_{a\,b}(E_{a\,b}\beta_{a\,b\,R}+\omega\alpha_{a\,b\,R}) \tag{9b}$$

with

$$D_{a\,b}=(E_{a\,b}^2-\omega^2)^{-1}. \tag{10}$$

Multiplying Eq. (9a) by $a_i(abR)$ and summing over all different configurations ab one can write the following expressions

$$A_i=\sum_{p=1}^{N}\Gamma_{i\,p}^{(1)}A_p+\sum_{q=1}^{M}\Gamma_{i\,q}^{(0)}B_q \quad (i=1\ldots N) \tag{11}$$

where

$$\Gamma_{i\,q}^{(0)}=\kappa\sum_{a\,b}D_{a\,b}\,a_i(a\,b\,R)\,b_q(a\,b\,R) \tag{12}$$

and

$$\Gamma_{i\,p}^{(1)}=\kappa\sum_{a\,b}E_{a\,b}D_{a\,b}\,a_i(a\,b\,R)\,a_p(a\,b\,R). \tag{13}$$

Analogously, from Eq. (9b) one obtains

$$B_j=\sum_{p=1}^{N}\Gamma_{p\,j}^{(0)}A_p+\sum_{q=1}^{M}\Gamma_{j\,q}^{(2)}B_q \quad (j=1\ldots M) \tag{14}$$

with

$$\Gamma_{j\,q}^{(2)}=\kappa\sum_{a\,b}E_{a\,b}D_{a\,b}\,b_j(a\,b\,R)\,b_q(a\,b\,R). \tag{15}$$

Introducing now the vector

$$X=\begin{pmatrix}A\\B\end{pmatrix} \tag{16}$$

and the symmetric $(N+M)$-matrix

$$\Gamma=\begin{pmatrix}\Gamma^{(1)} & \Gamma^{(0)}\\ \Gamma^{(0)+} & \Gamma^{(2)}\end{pmatrix} \tag{17}$$

we can write Eqs. (11) and (15) in matrix form

$$X=\Gamma(\omega)X. \tag{18}$$

Since the secular Eq. (18) will have a non-trivial solution only if

$$\det|\Gamma(\omega)-I|=0. \tag{19}$$

ω could be determined with the help of this relation. With ω known, Eq. (18) would yield the vector X up to a normalization factor by solving a related $(N+M-1)$-inhomogeneous subsystem, and this factor would afterwards be used to properly normalize the eigenvectors $\begin{pmatrix}\psi\\ \phi\end{pmatrix}$.

However, Eq. (19) might possess several solutions in the physically interesting region $0\leqq\omega\leqq(E_{ab})_{\min}$ and not too much can a priori be said about them. In addition, in the neighborhood of the two-quasi-particle energies the matrix elements of Γ diverge and the calculation of the determinant becomes numerically unstable.

In the case where the two quasi-particle states are not degenerate, however, we can prove that the eigenvalue λ of the equation

$$\Gamma(\omega)X=\lambda(\omega)X \tag{20}$$

is a monotonous function of ω for $\lambda_{\min}\leqq\lambda(\omega)\leqq\infty$ in the interval $0\leqq\omega\leqq(E_{ab})_{\min}$, with $\lambda_{\min}>0$. Physical values of ω must satisfy the condition

$$\lambda(\omega)=1. \tag{21}$$

Then, if $\lambda_{\min}\leqq 1$ there will always be a solution.

These results arise as a consequence of the following properties of the matrix Γ [19]:

i) Γ is a positive definite matrix for $0\leqq\omega\leqq(E_{ab})_{\min}$. Indeed, we have for the quadratic form $X^{\dagger}\Gamma X$

$$X^{\dagger}\Gamma X=\sum_{ab}D_{ab}[E_{ab}(\alpha^2_{abR}+\beta^2_{abR})+2\omega\alpha_{abR}\beta_{abR}]\geqq 0.$$

The eigenvalues of Eq. (20) are then positive in this range.

ii) The eigenvalues of Γ are monotonous increasing functions of ω. For any physical solution ($\omega>0$) the leading terms in the wave function satisfy the relation

$$f_{abR}g_{abR}\geqq 0 \tag{22}$$

for all ab.

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19. A similar analysis has been made for the scattering problem by WEINBERG, S.: Phys. Rev. **131**, 440 (1963).

On the other hand, due to the fact that an eigenvector is orthogonal to its derivative, we can write

$$\frac{d\lambda}{d\omega}=X^{\dagger}\frac{d\Gamma}{d\omega}X=2\sum_{ab}D_{ab}^{2}[\omega E_{ab}(\alpha_{abR}^{2}+\beta_{abR}^{2})$$
$$+(E_{ab}^{2}+\omega^{2})\alpha_{abR}\beta_{abR}]=2\sum_{ab}f_{ab}g_{ab}\geqq 0.$$

iii) If all two-quasi-particle energies E_{ab} are different, then for each of them there is only one eigenvalue which diverges when $\omega\to E_{ab}$. Let us consider the expression

$$\lim_{\omega\to E_{ab}-0}(E_{ab}-\omega)\Gamma=\tfrac{1}{2}\begin{pmatrix} a_i(abR)\,a_j(abR)\ldots a_i(abR)\,b_j(abR)\ldots \\ \vdots \qquad\qquad \vdots \\ a_i(abR)\,b_j(abR)\ldots b_i(abR)\,b_j(abR)\ldots \\ \vdots \qquad\qquad \vdots \end{pmatrix}.$$

This is a separable matrix, and as such has only one non-zero eigenvalue, whose value is equal to the trace

$$\lambda=\tfrac{1}{2}\sum_{i=1}^{N}[a_i(abR)]^2+\tfrac{1}{2}\sum_{j=1}^{M}[b_j(abR)]^2>0.$$

The obvious procedure to find the solutions of Eq. (18) is by means of an iterative process in which Eq. (20) is diagonalized until the desired eigenvalue $\lambda=1$ is obtained.

3. Application to the SDI

a) Spherical Case. In this case the quantum number a of Eq. (1) denotes the set n, l, j and the quantum number R represents the total angular momentum J and the parity Π.

Taking the expressions for P and Q given in Ref. [3] one sees that $N=M=2$. In addition

$$a_1(abJ)=\cos(a-b)\,h'(abJ)$$
$$a_2(abJ)=2(-)^{j_a+l_a-\frac{1}{2}}\sin(a-b)\,h'(abJ)\,\phi(abJ) \tag{23}$$
$$b_1(abJ)=\cos(a+b)\,h'(abJ);\qquad b_2(abJ)=\sin(a+b)\,h'(abJ)$$

with

$$h'(abJ)=(1+\delta_{ab})^{-\frac{1}{2}}h(abJ). \tag{24}$$

The quantities $h(abJ)$ and $\phi(abJ)$ are defined in Ref. [3] (Eqs. (3b) and (3c) respectively).

We denote the coefficients of the Bogoliubov-Valatin transformation by

$$\cos a\equiv u_a,\qquad \sin a=v_a. \tag{25}$$

The RPA problem is thus reduced to the repeated diagonalization of a (4×4)-matrix. This procedure will be illustrated by two simple examples in Section 4.

b) Deformed Case. From the expressions for the matrices P and Q given in Ref. [6] one obtains:

$$\begin{aligned} U_{\alpha\beta\gamma\delta} &= E_{\alpha\beta}\,\delta_{\alpha\gamma}\,\delta_{\beta\delta} + \tfrac{1}{2}\cos(\alpha+\beta)\cos(\gamma+\delta)\,V^A_{\alpha\beta\gamma\delta} \\ &\quad + \tfrac{1}{2}\sin(\alpha+\beta)\sin(\gamma+\delta)\left[V^A_{\alpha\bar{\delta}\bar{\beta}\gamma} - V^A_{\alpha\bar{\gamma}\bar{\beta}\delta}\right], \end{aligned} \tag{26}$$

$$\begin{aligned} V_{\alpha\beta\gamma\delta} &= E_{\alpha\beta}\,\delta_{\alpha\gamma}\,\delta_{\beta\delta} + \tfrac{1}{2}\cos(\alpha-\beta)\cos(\gamma-\delta)\,V^A_{\alpha\beta\gamma\delta} \\ &\quad + \tfrac{1}{2}\sin(\alpha-\beta)\sin(\gamma-\delta)\left[V^A_{\alpha\bar{\delta}\bar{\beta}\gamma} + V^A_{\alpha\bar{\gamma}\bar{\beta}\delta}\right]. \end{aligned} \tag{27}$$

Here $V^A_{\alpha\beta\gamma\delta}$ denotes the antisymmetrized matrix element

$$V^A_{\alpha\beta\gamma\delta} = \langle\alpha(1)\,\beta(2)|\,V_{12}\,|\gamma(1)\,\delta(2)\rangle - \langle\alpha(1)\,\beta(2)|\,V_{12}\,|\delta(1)\,\gamma(2)\rangle.$$

The basis functions $|\alpha\rangle$, $|\beta\rangle$, $|\gamma\rangle$ and $|\delta\rangle$ are solutions of a deformed potential. Usually, one takes them to be Nilsson functions. The bar indicates the time reversed state[6]

$$|\bar{\alpha}\rangle = T\,|\alpha\rangle = (-)^{\frac{1}{2}-\Omega}\,|-\alpha\rangle \tag{28}$$

with a negative projection Ω of the angular momentum along the intrinsic axes. The coefficients u_α and v_α are assumed to be independent of the sign Ω_α, but

$$T\,|\bar{\alpha}\rangle = T^2\,|\alpha\rangle = -\,|\alpha\rangle$$

which is not the usual convention[6].

For the SDI we have[6]

$$V^A_{\alpha\beta\gamma\delta} = -\kappa\sum_{\lambda K} M^{\lambda K}_{\alpha\beta}\,M^{\lambda K}_{\gamma\delta}, \tag{29}$$

λ is the multipole order and $K=\Omega_\alpha\pm\Omega_\beta$

$$\begin{aligned} M^{\lambda K}_{\alpha\beta} &= \sum_{(\text{all } l\Lambda)} a^\alpha_{l\Lambda}\,a^\beta_{l\Lambda}\,C(l_a\,l_b\,\lambda|\Lambda_a\,\Lambda_b\,K)\left[\frac{(2l'_a+1)(2l_b+1)}{2\lambda+1}\right]^{\frac{1}{2}} \\ &\quad\cdot C(l_a\,l_b\,\lambda|000)\,\delta_{\Sigma_a,\,-\Sigma_b}(-)^{\frac{1}{2}-\Sigma_a} \\ &= N^{\lambda K}_{\alpha\beta}\,\delta_{\Sigma_a,\,-\Sigma_b}(-)^{\frac{1}{2}-\Sigma_a}. \end{aligned} \tag{30}$$

The symbols $a_{l\Lambda}$ represent the normalized Nilsson[20] coefficients, l is the orbital angular momentum, Λ its projection and Σ the z component of the spin along the intrinsic axis.

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20. NILSSON, S. G.: Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **29**, No. 16 (1955).

The antisymmetrized particle-particle matrix element (29) is already given as a sum of separable terms. This is not the case for the particle-hole matrix elements[6]. However, both the sum and the difference of antisymmetrized particle-hole matrix elements which appear in Eqs. (26) and (27) can also be written as a sum of separable terms.

Let us consider only one multipole order for the moment. Taking into account the property[6] $M^{\lambda K}_{\alpha\beta} = -M^{\lambda K}_{\beta\alpha}$, one has:

$$V^{A}_{\alpha\bar{\delta}\bar{\beta}\gamma} + V^{A}_{\alpha\bar{\gamma}\bar{\beta}\delta} = -(V_{\alpha\bar{\delta}\gamma\bar{\beta}} + V_{\alpha\bar{\gamma}\delta\bar{\beta}}), \tag{31a}$$

$$V^{A}_{\alpha\bar{\delta}\bar{\beta}\gamma} - V^{A}_{\alpha\bar{\gamma}\bar{\beta}\delta} = -(V_{\alpha\bar{\delta}\gamma\bar{\beta}} - V_{\alpha\bar{\gamma}\delta\bar{\beta}}) + 2V_{\alpha\bar{\delta}\bar{\beta}\gamma} \tag{31b}$$

where $V_{\alpha\beta\gamma\delta}$ denotes a non-antisymmetrized matrix element. The first term on the right side of Eq. (31b) is already separable, being proportional to the antisymmetrized particle-particle matrix element[6].

Now, the spatial part of the expressions in brackets in Eqs. (31a) and (31b) are readily seen to be separable[6]. Indeed one obtains

$$-(V_{\alpha\bar{\delta}\gamma\bar{\beta}} \pm V_{\alpha\bar{\gamma}\delta\bar{\beta}}) = N^{\lambda K}_{\alpha\beta} N^{\lambda K}_{\gamma\delta} (\delta_{\Sigma_a \Sigma_c} \delta_{\Sigma_b \Sigma_d} \pm \delta_{\Sigma_a \Sigma_d} \delta_{\Sigma_b \Sigma_c}).$$

The spin part is not separable as it is written above, but we can recast it in separable form, as it is done for the difference in [6].

$$\delta_{\Sigma_a \Sigma_c} \delta_{\Sigma_b \Sigma_d} \mp \delta_{\Sigma_a \Sigma_d} \delta_{\Sigma_b \Sigma_c} = [1 \pm (-)^S] C(\tfrac{1}{2}\tfrac{1}{2} S | \Sigma_a \Sigma_b \Sigma_a + \Sigma_b) \cdot C(\tfrac{1}{2}\tfrac{1}{2} S | \Sigma_c \Sigma_d \Sigma_c + \Sigma_d).$$

Defining now

$$M^{(+)\lambda K}_{\alpha\beta} = \sqrt{2} N^{\lambda K}_{\alpha\beta} \delta_{\Sigma_a, \Sigma_b}; \quad M^{(-)\lambda K}_{\alpha\beta} = N^{\lambda K}_{\alpha\beta} \delta_{\Sigma_a, -\Sigma_b} \tag{32}$$

one has

$$\begin{aligned} a^{\lambda K}_1(\alpha\beta) &= \frac{1}{\sqrt{2}} \cos(\alpha+\beta) M^{\lambda K}_{\alpha\beta}; \quad a^{\lambda K}_2(\alpha\beta) = \frac{1}{\sqrt{2}} \sin(\alpha+\beta) M^{\lambda K}_{\alpha\beta} \\ b^{\lambda K}_1(\alpha\beta) &= \frac{1}{\sqrt{2}} \cos(\alpha-\beta) M^{\lambda K}_{\alpha\beta}; \quad b^{\lambda K}_2(\alpha\beta) = \frac{1}{\sqrt{2}} \sin(\alpha-\beta) M^{(+)\lambda K}_{\alpha\beta} \\ b^{\lambda K}_3(\alpha\beta) &= \frac{1}{\sqrt{2}} \sin(\alpha-\beta) M^{(-)\lambda K}_{\alpha\beta}. \end{aligned} \tag{33}$$

Taking now into account all multipole orders one can finally write

$$\begin{aligned} U_{\alpha\beta\gamma\delta} &= E_{\alpha\beta} \delta_{\alpha\gamma} \delta_{\beta\delta} - \kappa \sum_{i=1}^{2} \sum_{\lambda K} a^{\lambda K}_i(\alpha\beta)\, a^{\lambda K}_i(\gamma\delta) \\ V_{\alpha\beta\gamma\delta} &= E_{\alpha\beta} \delta_{\alpha\gamma} \delta_{\beta\delta} - \kappa \sum_{j=1}^{3} \sum_{\lambda K} b^{\lambda K}_j(\alpha\beta)\, b^{\lambda K}_j(\gamma\delta). \end{aligned} \tag{34}$$

4. Simple Examples

In order to obtain some physical insight into the theory developed above we shall now consider two simple examples, namely, the degenerate case and symmetric two-shell model.

a) The Degenerate Case. In this case we find

$$E_i = G\Omega; \quad (U_i)^2 = 1 - \frac{\Delta}{2\Omega}; \quad (V_i)^2 = \frac{\Delta}{2\Omega}$$

for all i, and

$$2\Omega = \sum_j (2j+1).$$

The matrix Γ is constructed straight forwardly and one finds

$$\Gamma = \frac{\kappa \sum_{ab} [h(abJ)]^2}{(2E)^2 - \omega^2} \begin{pmatrix} 2E & \omega(u^2-v^2) & 2uv\omega \\ \omega(u^2-v^2) & 2E(u^2-v^2)^2 & 2E(u^2-v^2)2uv \\ 2uv\omega & 2E(u^2-v^2)2uv & 2E(2uv)^2 \end{pmatrix}. \quad (35)$$

The eigenvalues of this matrix are:

$$\lambda_{1;2} = \frac{\kappa \sum_{ab} [h(abJ)]^2}{2E \mp \omega}; \quad \lambda_3 = 0.$$

Only the first attains the value 1, the corresponding ω being

$$\omega = 2E - \kappa \sum_{ab} [h(abJ)]^2. \quad (36)$$

From the eigenvector belonging to λ_1, the eigenfunctions ψ, ϕ can be found to be

$$\psi_{ab} = \frac{h(abJ)}{\left[\sum_{ab} h(abJ)^2\right]^{\frac{1}{2}}}; \quad \phi_{ab} = 0. \quad (37)$$

Eqs. (36) and (37) are the well-known solutions of the TD equations for the degenerate case[21], which for an SDI become identical to the corresponding ones in the RPA treatment[3]. Of course, only the collective state is present.

b) The Symmetric Two Shell Model[22]. This consists of two levels of equal angular momentum j and opposite parity. Let ε be the energy of the first level and $l = j - \frac{1}{2}$ its orbital angular momentum.

For the second level the corresponding quantities are-ε and $l = j + \frac{1}{2}$.

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21. ARVIEU, R., and S. A. MOSZKOWSKI: Phys. Rev. **145**, 830 (1966).
22. HOGAASEN-FELDMAN, J.: Nuclear Phys. **28**, 258 (1961).

In this simple case we can solve everything in closed form. Let X denote the expression $\frac{\varepsilon}{\kappa\Omega}$, where $2\Omega=(2j+1)$. It is then easily seen that one obtains:

$$\begin{aligned} v_1=u_2&=\left(\frac{1+X}{2}\right)^{\frac{1}{2}}; \qquad u_1=v_2=\left(\frac{1-X}{2}\right)^{\frac{1}{2}} \\ E=E_1=E_2&=\kappa\Omega; \qquad \Delta=\kappa\Omega(1-X^2)^{\frac{1}{2}}. \end{aligned} \tag{38}$$

Of course, E denotes the energy of a quasi-particle while Δ is half the energy gap.

If we restrict ourselves to the 2^+ states, the secular determinant (19) reduces to $(h=h(ab^2))$

$$\begin{vmatrix} \frac{2Eh^2\kappa}{(2E)^2-\omega^2}-1 & 0 & \frac{h(1-X^2)^{\frac{1}{2}}\kappa}{(2E)^2-\omega^2} \\ 0 & \frac{2EX^2h^2\kappa}{(2E)^2-\omega^2}-1 & 0 \\ \frac{h^2(1-X^2)^{\frac{1}{2}}\kappa}{(2E)^2-\omega^2} & 0 & \frac{2E(1-X^2)h^2\kappa}{(2E)^2-\omega^2}-1 \end{vmatrix}. \tag{39}$$

and possesses only two roots for ω, which are

$$\omega_1^2=2E(2E-\kappa X^2h^2); \qquad \omega_2^2=(2E-h^2\kappa)[2E-\kappa h^2(1-X^2)]. \tag{40}$$

The eigenvalues of matrix Γ will now be:

$$\begin{aligned} \lambda_{1,2}&=\frac{\kappa h^2}{(2E)^2-\omega^2}[E(2-X^2)\pm\sqrt{E^2X^4+\omega^2(1-X^2)}] \\ \lambda_3&=\frac{2E\kappa h^2X^2}{(2E)^2-\omega^2}. \end{aligned} \tag{41}$$

Now both the first and third eigenvalues diverge and property iii) is violated. This is because the two-quasi-particles energies are degenerate. The ψ and ϕ eigenvectors can be calculated for both states now, and one finds

$$\begin{aligned} \psi_{111}&=-\psi_{221}=\frac{1}{2\sqrt{\omega_1}}\{\omega_1+2E-\tfrac{1}{2}\kappa h^2X^2\}^{\frac{1}{2}} \\ \phi_{111}&=-\phi_{221}=\frac{1}{2\sqrt{\omega_1}}\{-\omega_1+2E-\tfrac{1}{2}\kappa h^2X^2\}^{\frac{1}{2}} \\ \psi_{112}&=-\psi_{222}=\frac{1}{2\sqrt{\omega_2}}\{\omega_2+2E-\tfrac{1}{2}\kappa h^2(2-X^2)\}^{\frac{1}{2}} \\ \phi_{112}&=\phi_{222}=\frac{1}{2\sqrt{\omega_2}}\{-\omega_2+2E-\tfrac{1}{2}\kappa h^2(2-X^2)\}^{\frac{1}{2}}. \end{aligned} \tag{42}$$

It is instructive to study now the same problem along conventional lines, the RPA matrices will be now:

$$P=\begin{pmatrix} 2E-\kappa h^2/2 & -\kappa h^2(1-X^2)/2 \\ -\kappa h^2(1-X^2)/2 & 2E-\kappa h^2/2 \end{pmatrix}, \tag{43a}$$

$$Q=\begin{pmatrix} 0 & -\kappa h^2 X^2/2 \\ -\kappa h^2 X^2/2 & 0 \end{pmatrix}. \tag{43b}$$

The eigenvalues of the RPA matrix

$$\begin{pmatrix} P & Q \\ -Q & -P \end{pmatrix}$$

are again given by (40), as expected. The eigenvectors are also given by Eq. (42).

It is interesting to compare the solution (40) with the Tamm-Damcoff approximations and with the solution obtained by taking only the particle-hole part of the interaction. We obtain respectively

$$\omega_1^{2\,\mathrm{TD}}=2E-\tfrac{1}{2}\kappa h^2 X^2; \quad \omega_2^{2\,\mathrm{TD}}=2E-\tfrac{1}{2}\kappa h^2(2-X^2) \tag{44}$$

and

$$[\omega^{P-H}]^2=2E[2E-\kappa h^2(1-X^2)]. \tag{45}$$

We see that Eq. (45) does not tend to the correct value in the degenerate case ($X\to 0$), and it misses the first solution (40).

5. Conclusion

A method has been developed to solve in a very simple way the RPA equations without diagonalizing a huge matrix. It can be applied only to separable interactions, like the SDI, but it takes into account all terms of the interaction and all multipole orders.

The method has been illustrated by two simple examples. It is hoped it will provide a more convenient tool to investigate the nuclear structure of heavy deformed nuclei, where only approximations which discard a lot of terms of the interactions have been so far applied.

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