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NUCLEAR PHYSICS

Effects of Antineutrino mass on β⁻-Decay Rates Calculated Within the Gross Theory of Beta Decay

M. R. Azevedo[1] · R. C. Ferreira[1] · A. J. Dimarco[2] · C. A. Barbero[3,4] · A. R. Samana[2] · D. N. Possidonio[1,2]

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Abstract

We analyze the effect of the antineutrino mass over the β^--decay rates calculated within the scheme of the Gross Theory of Beta Decay (GTBD). We give a non-null value to the mass of the antineutrino participating in β^--decay, $(A, Z) \rightarrow (A, Z+1) + e^- + \bar{\nu}_e$, which is usually neglected because we know it is small compared with electron mass. We have slightly modified the GTDB by inserting the antineutrino mass in the formalism. We have adopted a Gaussian energy distribution function with the axial-vector weak coupling constant $g_A = 1$, as well as a new set of the adjustable parameter σ_N related to the standard deviation for the Gamow-Teller resonance, updated experimental mass defects, and also an improved approximation for the Fermi function. Our sample consists of a set of 94 nuclei of interest in the pre-supernova phase, which have experimental data in terrestrial conditions available in the Letter of Nuclide. We have compared the calculation without the inclusion of the antineutrino mass with that adopting a really overestimated value of 50 keV for it to illustrate the effect on the decay rates. We have shown that they are improved only by approximately one per thousand in this case. We conclude that the effect of the antineutrino mass on decay rates is not relevant.

Keywords Gross Theory · β^--decay · Antineutrino mass

1 Introduction

Since the emergence of nuclear physics, one of the major challenges in this area has been to develop theoretical models that can reproduce as accurately as possible the experimental results of the various naturally occurring reactions, particularly those of the β-decay. Between the most diverse nuclear models, we chose the Gross Theory of Beta Decay (GTDB) in this research, which presents a great advantage over the others specially in astrophysical applications: its simplicity for the computational work since it facilitates the calculation of the rates in a systematic way for a large number of nuclei involved in a given phenomenon. The GTBD is a nuclear model that takes into account rough approximations of the final energy states of the decaying nucleon, and it is essentially a parametric model for nuclear disintegration rates, which combines arguments of independent particle associated to the Fermi gas model. Thus, the GTBD is a microscopic model which associates two other major nuclear models with statistical arguments in a phenomenological way. This is done when the β amplitude function of the independent particle model is convulsed with the density levels of the Fermi gas model corrected to take into account the pairing and shell effects. The contributions in the final part of the resonance (Gamow-Teller) are included in a parametric way.

The original version was proposed by Takahashi and Yamada in the end of the sixties and includes only the contribution of allowed transitions [1]. Successive versions of the GTBD were emerging over the years with the aim of improving the agreement with the experimental data. Between that improvements we remark the following: (i) the addition of forbidden transitions [2]; (ii) the inclusion of the pairing effect and the observation of the non-energy weighted sum rule in the Fermi transition [3]; (iii)

✉ C. A. Barbero
cab@fisica.unlp.edu.ar

1 Departamento de Ciências Exatas e Naturais, Universidade Estadual do Sudoeste da Bahia, BR 415 Km 03 Itapetinga BA, Brazil

2 Departamento de Ciencias Exatas e Tecnologicas, Universidade Estadual de Santa Cruz, Ilhéus BA, Brazil

3 Departamento de Física, Universidad Nacional de La Plata, C. C. 67, 1900 La Plata, Argentina

4 Instituto de Física La Plata, CONICET, 1900 La Plata, Argentina

the modification of the single-particle energy function in the Gamow-Teller transition, which has a peak with a long tail on each side [4]; (iv) application to neutrino-nucleus reactions, employing a more realistic description of the energetics of the Gamow-Teller resonances for nuclei with $A < 70$, of astrophysical interest [5]; (v) analysis about which of the Fermi functions available in the literature represents better the Coulombian interaction between nucleus and electron both in β^--decay and electron capture, also for nuclei in the mass region $A < 70$ [6]; (vi) analysis of the influence of the axial-vector coupling constant and the energy distribution function on β^--decay rates [7].

With the aim of obtaining even more precise results when compared with the experimental data, we search other ways to improve the agreement between theory and experiments. The antineutrino mass was discarded from previous calculations due to the fact that its value is much smaller than the electron mass (see, for example, Refs. [1–14] and references therein). The general assumption is that neutrino masses are not relevant in the calculation of decay rates. Do we really need to perform the full calculation to make sure that this is so? In order to answer this question, and being that there not exist in the literature any calculation including the antineutrino mass and analyzing quantitatively its effect over the decay rates, we will include here the antineutrino mass and analyze its contribution to the β^--decay rates for a group of 94 elements in the mass range $46 < A < 70$, which are of astrophysical interest in the stellar pre-supernova evolution phase.[1] All the experiments are in agreement with few eVs for the upper limit on the neutrino mass. In fact, the most accepted and strong limit in RPP-2016 from PDG based on β-decay of ^{3}H is ~ 2 eV and the new results from KATRIN experiment report 1.1 eV at 90% of confidence level, reflecting the actual sensitivity [16]. On the other hand, slightly older works suggested less restrictive limits (due to the experimental capabilities of that age) for the neutrino mass, such as the work of Ref. [17] in which it could reach a maximum value 50 keV. With the aim to verify if the quantitative analysis confirms the common sense about the irrelevance of the neutrino mass on the decay rates, and to illustrate the maximum effect of a nonzero neutrino mass in its calculation, we will assume here this overestimated value of 50 keV.

The paper is organized as follows. The formalism is summarized in Section 2, where we describe the model for the evaluation of the decay rates, with some details of the GTBD including the antineutrino mass for β^--decay together with a description of the fitting method. In Section 3, we present and discuss our results. Final remarks are drawn in Section 4.

[1] The GTBD has been suitable for the description of neutrino-nucleus cross sections in the low energy range ($E_\nu < 250$ MeV) with E_ν being the energy of the incident neutrino [15].

2 Formalism

2.1 Beta Decay Rate

The transition probability will be evaluated from the Fermi's golden rule (in natural units $m_e = c = \hbar = 1$)

$$\lambda_\beta = 2\pi |U_{fi}|^2 \rho(E_f), \tag{1}$$

where U_{fi} is the matrix element for the transition and $\rho(E_f)$ is the energy density of final states. For β^--decay, $(A, Z) = (A, Z{+}1){+}e^-{+}\bar{\nu}_e$, between the initial state (i) of the parent nucleus (A, Z) and final one (f) of the daughter nucleus $(A, Z + 1)$, we have

$$U_{fi} = \int \phi_e^* \phi_{\bar{\nu}_e}^* \psi_f^* O_\beta \Omega_\beta \psi_i d^3r d^3r_e d^3r_{\bar{\nu}_e}. \tag{2}$$

Here $\phi_e(r_e) = \frac{1}{\sqrt{V}} e^{ip_e \cdot r_e}$ and $\phi_{\bar{\nu}_e}(r_{\bar{\nu}_e}) = \frac{1}{\sqrt{V}} e^{ip_{\bar{\nu}_e} \cdot r_{\bar{\nu}_e}}$ are the electron and antineutrino plane wave functions in a volume V, with momentum p_e and $p_{\bar{\nu}_e}$, respectively. The operator responsible of the electroweak interaction can be written within a local approximation as [17] $O_\beta = G_F \delta(r - r_e)\delta(r - r_{\bar{\nu}_e})$, with $G_F = (3.034545 \pm 0.00006) \times 10^{-12}$ being the Fermi weak coupling constant [18]. Additionally, ψ_i (ψ_f) is the parent (daughter) nucleus wave function and Ω_β is the operator for the nuclear transition. Next, using the fact that the length of the leptonic functions is of the order 10^{-11} cm [19], one order of magnitude greater than the nuclear size, we can affirm that the functions of the electron and antineutrino present little variation inside the nucleus and therefore we can approximate them by their value at $r = 0$. Thus, we can write

$$\lambda_\beta = \frac{2\pi G_F^2}{V^2} |M|^2 \rho(E_e, E_{\bar{\nu}_e}), \tag{3}$$

where we have defined the nuclear matrix element (NME)

$$M = \int \psi_f^*(r) \Omega_\beta \psi_i(r) d^3r. \tag{4}$$

The density of final energy levels of the electron and antineutrino will be calculated using the Fermi gas model [17]:

$$\rho(E_e, E_{\bar{\nu}_e}) = \frac{V^2 p_e^2 dp_e p_{\bar{\nu}_e}^2 dp_{\bar{\nu}_e}}{4\pi^4}. \tag{5}$$

The energy conservation in β^--decay gives:

$$E(A, Z) = E(A, Z + 1) + E_e + E_{\bar{\nu}_e}, \tag{6}$$

where $E(A, Z)$ and $E(A, Z + 1)$ are the energies of the initial and final nucleus, respectively (A and Z are the mass and atomic numbers, respectively), E_e is the electron energy, and $E_{\bar{\nu}_e}$ the antineutrino one. The maximum available energy for the decay is

$$E_\beta^{\max} \equiv E(A,Z) - E(A,Z+1) = E_e + E_{\bar{\nu}_e}. \tag{7}$$

Otherwise, the Q-value of the reaction is defined as the maximum kinetic energy available for the decay, $Q = T_{after} - T_{before}$, with T_{after} and T_{before} being the total kinetic energies "after" and "before" the reaction, respectively. They can be written as

$$\begin{aligned} T_{before} &= E(A,Z) - m_P; \\ T_{after} &= E(A,Z+1) - m_D + E_e - 1 + E_{\bar{\nu}_e} - m_{\bar{\nu}_e}, \end{aligned} \tag{8}$$

where m_P and m_D are the parent and daughter nucleus masses, respectively, and $m_{\bar{\nu}_e} = 50$ keV $= 0.098$ is the antineutrino mass (in natural units). After using energy conservation, we obtain

$$Q = m_P - m_D - 1 - m_{\bar{\nu}_e}. \tag{9}$$

We will calculate the Q-values using the experimental data of the mass defects found in the Letter of Nuclide [20], including in this way shell effects. The maximum kinetic energy and the maximum total energy available for the reaction can be related as follows:

$$E_{\beta max} = Q + 1 + m_{\bar{\nu}_e}. \tag{10}$$

Next, using $p_e = \sqrt{E_e^2 - 1}$ and $p_{\bar{\nu}_e} = \sqrt{E_{\bar{\nu}_e}^2 - m_{\bar{\nu}_e}^2}$, from (3), (5), (7), and (10), we get

$$\begin{aligned} \lambda_\beta = \frac{G_F^2}{2\pi^3} &\int_{E_e^{min}}^{E_e^{\max}} |M(E)|^2 E_e \sqrt{E_e^2 - 1} F(Z,E_e) dE_e \\ &\times \int_{E_{\bar{\nu}_e}^{min}}^{E_{\bar{\nu}_e}^{\max}} dE_{\bar{\nu}_e} \sqrt{(E+1-E_e+m_{\bar{\nu}_e})^2 - m_{\bar{\nu}_e}^2} \\ &\times (E+1-E_e+m_{\bar{\nu}_e}), \end{aligned} \tag{11}$$

where we have replaced Q by E and we have introduced by hand the well-known Fermi function $F(Z, E_e)$ to take into account the Coulombian interaction between the daughter nucleus and the electron.[2] Using $E_{\bar{\nu}_e} = E + 1 + m_{\bar{\nu}_e} - E_e$, we have

$$\lambda_\beta = \frac{G_F^2}{2\pi^3} \int_0^Q |M(E)|^2 f(E) dE, \tag{12}$$

[2]Here we adopt the proposal of Aufderheide et al. [9], which has been proven to best represent the experimental results for the elements of our interest.

where we have defined

$$\begin{aligned} f(E) = \int_1^{E+1} &\sqrt{(E+1-E_e+m_{\bar{\nu}_e})^2 - m_{\bar{\nu}_e}^2} \\ &\times (E+1-E_e+m_{\bar{\nu}_e}) E_e \sqrt{E_e^2 - 1} F(Z,E_e) dE_e. \end{aligned} \tag{13}$$

It is important to remark that this equation reduces to (2) from [7] when the antineutrino mass is neglected, as it should be.

2.2 Gross Theory of Beta Decay

The NME, represented by $M(E)$ in (12), is the term that differentiates the Gross Theory of Beta Decay (GTBD) from other models. The β^--decay rates receive contribution from different types of transitions like the allowed Fermi (F) and Gamow-Teller (GT) ones, the first forbidden transitions, and the second forbidden ones [21]. Neglecting the contribution of forbidden transitions, the total decay rate within the GTBD can be written as

$$\begin{aligned} \lambda_\beta = \frac{G_F^2}{2\pi^3} &\int_{-Q}^0 \left[g_V^2 |M_F(E)|^2 + 3 g_A^2 |M_{GT}(E)|^2 \right] \\ &\times f(-E) dE. \end{aligned} \tag{14}$$

Here $g_V = 1$ and $g_A = 1$ are, respectively, the vector and axial-vector effective coupling constants [7], and $-E > 0$ appears because we integrate over the final energy states.

The NME can be evaluated by using the sum rule as described in Ref. [22]. From (4), it reads $|M_\Omega(E)|^2 = |\langle \psi_f | \Omega | \psi_i \rangle|^2 \rho(E)$, where $\Omega \equiv 1$ and $\Omega \equiv \boldsymbol{\sigma}$ are the F and GT nuclear operators, respectively, and $\rho(E)$ is the final level energy density. Within the sum rule, the β^--decay operator is a sum of independent particle operators [1]. Assuming the nucleons as independent particles, the energy E can be considered as the difference between the energies of the independent nucleon decay in daughter and parent nucleus. Therefore, the NME can be expressed as

$$|M_\Omega(E)|^2 = \int_{\epsilon_0(E)}^{\epsilon_1} D_\Omega(E,\epsilon) \frac{dN_1}{d\epsilon} W(E,\epsilon) d\epsilon. \tag{15}$$

where ϵ_1 is the energy of the highest occupied state and $\epsilon_0(E) = max(\epsilon_{min}, \epsilon_1 - Q - E)$ with ϵ_{min} being the lowest single-particle energy of the parent nucleus. Pauli's principle is considered in the lower limit of the integral, and in the term $W(E, \epsilon)$, which measures the probability of occupation of the final states (vacancy level). Equation (15) is valid for the special case of a step surface, where $W(E, \epsilon) = 1$, because $\epsilon + E > 1 - Q$. In other cases, the term $W(E,\epsilon)$ vanishes because $\epsilon + E \leq 1 - Q$. [3] Within this approximation, the NME reads

$$|M_\Omega(E)|^2 = \int_{\epsilon_0(E)}^{\epsilon_1} D_\Omega(E,\epsilon)\frac{dN_1}{d\epsilon}d\epsilon. \tag{16}$$

Following the original version of the GTBD [1], the Fermi gas model was used to estimate the density of independent nucleon levels, $\frac{dN_1}{d\epsilon}$, as

$$\frac{dN_1}{d\epsilon} = N_1\left[1 - \left(1 - \frac{Q+E}{\epsilon_F}\right)^{\frac{3}{2}}\right], \tag{17}$$

where N_1 is the number of neutrons of the parent nuclei and ϵ_F is the nucleon Fermi energy given by

$$\epsilon_F = \frac{76.52}{\frac{M_n^*}{M_n}}\frac{1}{r_0^2}\left(\frac{N_1}{A}\right)^{\frac{2}{3}} \text{ MeV}, \tag{18}$$

being M_n^* and M_n the effective and bare nucleon masses, respectively, and r_0 the nuclear radius. We used the relations $r_0 = 1.25(1+0.65A^{-2/3})$, and for $\frac{M_n^*}{M_n} = 0.6+0.4A^{-1/3}$ [4].

Finally, within the GTBD, the β^--decay rate reads

$$\lambda_\beta = \frac{G_F^2}{2\pi^3}\int_{-Q}^{0}\left[g_V^2 D_F(E,\epsilon) + 3g_A^2 D_{GT}(E,\epsilon)\right] \times N_1\left[1 - \left(1 - \frac{Q+E}{\epsilon_F}\right)^{\frac{3}{2}}\right] f(-E)dE. \tag{19}$$

The energy distribution function $D_\Omega(E,\epsilon)$ measures the probability that a nucleon with single-particle energy ϵ undergoes a β-transition. As in Takanahashi et al. [1], we neglect the ϵ-dependence, i.e., it is assumed that all nucleons have the same decay probability, independent of their energies, $D_\Omega(E,\epsilon) \equiv D_\Omega(E)$. We assume here a Gaussian form [1, 7]

$$D_\Omega(E) = \frac{1}{\sqrt{2\pi}\sigma_\Omega}e^{\frac{-(E-E_\Omega)^2}{2\sigma_\Omega^2}}, \tag{20}$$

where E_Ω is the resonance energy and σ_Ω the standard deviation. Following the original work from Ref. [1], we assume the nuclei as a uniform charged sphere with radius $1.2 \times A^{\frac{1}{3}}$ fm, which allows to consider the Coulombian (c) displacement of independent particle such as

$$E_F = E_c = \pm(1.44Z_1A^{-\frac{1}{3}} - 0.7825)MeV, \tag{21}$$

$$\sigma_F = \sigma_c = 0.157Z_1A^{-\frac{1}{3}}, \tag{22}$$

where Z_1 is the proton number of the parent nuclei for β^--decay. For the GT resonance, we use the approximation [5, 24]

$$E_{GT} = E_F + \delta, \tag{23}$$

with

$$\delta = 26A^{-\frac{1}{3}} - \frac{18.5(N-Z)}{A}\text{MeV}, \tag{24}$$

and

$$\sigma_{GT} = \sqrt{\sigma_F^2 + \sigma_N^2}, \tag{25}$$

with σ_N being a setting parameter which comes from the energy propagation produced by the forces dependent of the nuclear spin. Calling $\tau_{1/2}^{cal}$ to the theoretical value for the half-life calculated with our GTBD, and bearing in mind that intend to reproduce its experimental value, $\tau_{1/2}^{cal}$, as correctly as possible, we are going to follow the procedure of Ref. [15] to determine σ_N through the minimization of the function

$$\chi^2 = \sum_{n=1}^{N_0}\left[\frac{\log\left(\tau_{1/2}^{cal}(n)/\tau_{1/2}^{\exp}(n)\right)}{\Delta\log\left(\tau_{1/2}^{\exp}(n)\right)}\right]^2, \tag{26}$$

where N_0 is the number of experimental β^--decaying nuclei, fulfilling the following conditions: (i) the branching ratio of the allowed transitions exceeds $\sim 50\%$ of the total β^--decay branching ratio and (ii) the ground-state Q-value is $\geq 10A^{-1/3}$ MeV,

$$\Delta\log\left(\tau_{1/2}^{\exp}(n)\right) = \log\left[\tau_{1/2}^{\exp}(n) + \delta\tau_{1/2}^{\exp}(n)\right] - \log\left[\tau_{1/2}^{\exp}(n)\right], \tag{27}$$

and $\delta\tau_{1/2}^{\exp}(n)$ is the experimental error. This χ^2-function reinforces the contributions of data with small experimental errors. Moreover, we perform different fittings for even N-even Z ($N_0 = 17$), odd N- odd Z ($N_0 = 28$), odd N-even Z ($N_0 = 29$), and even N-odd Z ($N_0 = 20$) nuclei. Needless to say that for $\tau_{1/2}^{\exp}$, we use recent data from [20],

[3] We follow here this approach of stepped function described in (15)–(23) from [1]. On the other hand, an improved version of the $W(E,\epsilon)$ function is given in Eqs. (51)–(77) from [1]. In Ref. [23], the authors show that the results for the β-decay are only slightly modified when using this non-staggered function.

instead of those that were available when the GTBD was formulated [1].

3 Results and Discussions

We have adopted the GTBD version from Ref. [7]. We have calculated the β^--decay rates using the (13) and (19) which allow to include the antineutrino mass effects as described in the previous section. We have made use of the new relevant modifications regarding the calculation of these rates, namely for the Q-values defined in (9), we used the updated experimental data present in the Letter of Nuclide [20], the Fermi function has been calculated following Aufderheide et al. [9], we have used the Gaussian distribution function, and for the weak coupling constant, we have used $g_A = 1$ [7]. We have considered a set of 94 isotopes of the families of iron, cobalt, nickel, manganese, chromium, copper, titanium, and scandium, including all isotopes of these families that decay spontaneously via β^--decay. These nuclei were selected taking into account the mass, which ranges from $46 < A < 70$, and the abundance of these isotopes in the pre-supernova phase, which is of interest in future research.

As a first step, we start exhibiting the results for the adjusted parameter σ_N. We have performed two simulations: the first one (GTBD1) adopting a null mass for the antineutrino, which leads to the results previously published in Ref. [7], and the second one (GTBD2) includes the antineutrino mass effects. The results for σ_N obtained in this way are shown in Table 1.

We show in Fig. 1 our results for the logarithm of the ratio between the calculated and the experimental β^--decay half lives. We have added two horizontal lines to more easily visualize the nuclei whose half lives differ by less than an order of magnitude from the experimental results, it means, those closer to the data. In the figure, we show the results obtained within the GTBD2, which should be compared with the corresponding ones calculated with GTBD1 presented in Fig. 2 from [7] (remember that we use $g_A = 1$).[4] These results show that the 75.5% (88.3%) of our calculated half lives within GTBD2 are in good agreement with the experimental data, because they differ by less than one (two) order of magnitude with the data. These percentages are very similar to those obtained in Ref. [7] within GTBD1.

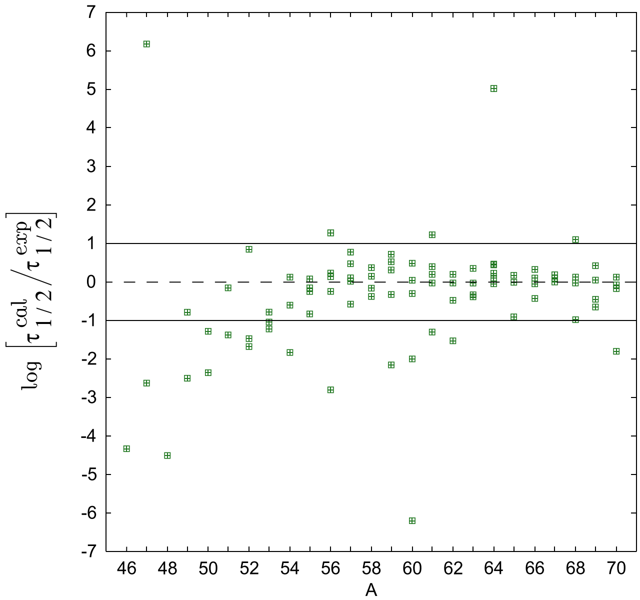

Fig. 1 β^--decay rates calculated using the GTBD2 model with $m_{\bar{\nu}_e} = 50$ keV $= 0.098$. Experimental data from Letter of Nuclide [20]

Table 1 Adjusted parameter σ_N (in units of MeV) using the GTBD from [7] with $m_{\bar{\nu}_e} = 0$ (GTBD1) and $m_{\bar{\nu}_e} = 0.098$ (GTBD2)

Model	$Z - N$ (parity)	σ_N
GTBD1 ($m_{\bar{\nu}_e} = 0$)	Even - even	7.70
	Even - odd	6.60
	Odd - even	7.48
	Odd - odd	8.14
GTBD2 ($m_{\bar{\nu}_e} = 0.098$)	Even - even	3.95
	Even - odd	3.85
	Odd - even	4.08
	Odd - odd	4.20

In Fig. 2, we compare the results obtained by using GTBD1 and GTBD2 for the cobalt family. We use the cobalt isotopes as an example, but all the nuclei of the sample show the same behavior. We observed that the inclusion of an antineutrino mass of 50 keV improved only slightly the results of some isotopes, such as ^{60}Co, ^{62}Co, ^{63}Co, ^{65}Co, ^{67}Co, ^{68}Co, ^{69}Co, and ^{70}Co, at about one-thousandth with respect to the value calculated within GTBD1, which becomes almost imperceptible in Fig. 2. The other isotopes such as ^{61}Co, ^{64}Co, and ^{66}Co had departures from the experimental data in relation to the GTBD1, which is due to the value of the fitting parameter σ_N used. In fact, it has been determined minimizing the χ^2 function from (26), and leads to a Gaussian width (for the E_{GT} function) which is far from the experimental value for some nuclei of our sample. This problem could be solved by obtaining a σ_N for each nucleus through the experimental data.

[4] A comparison of the results obtained using GTBD1 with the corresponding ones calculated in other previous evaluations was presented, for example, in Refs. [6, 7, 15, 22].

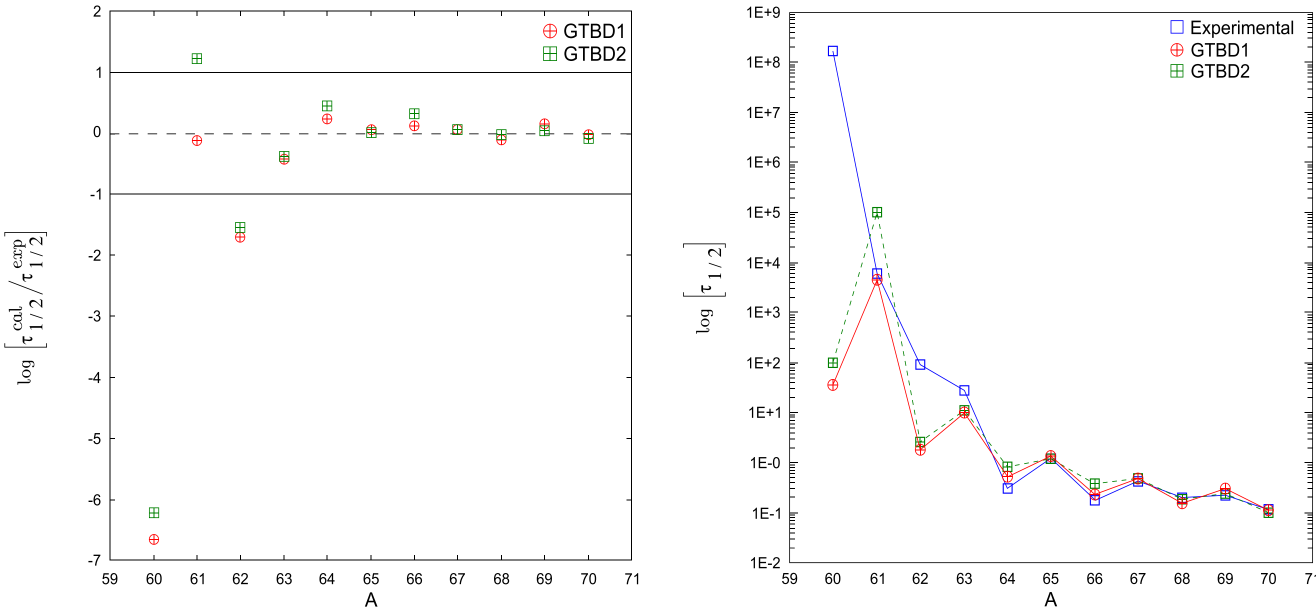

Fig. 2 Comparison of β^--decay half lives for the cobalt family. Left panel shows the results for the logarithm of the ratio between the calculated half-life ($\tau_{1/2}^{cal}$) using both the GTBD1 with $m_{\bar{\nu}_e} = 0$ and the GTBD2 with $m_{\bar{\nu}_e} = 50\,\mathrm{keV} = 0.098$, and the experimental data ($\tau_{1/2}^{\exp}$) taken from Letter of Nuclide [20]. Right panel exhibits the same theoretical and experimental values for the logarithm of the half-life (in units of *sec*)

4 Concluding Remarks

We have adopted the Gross Theory as a nuclear model for calculating the β^--decay rates and we have tried to introduce improvements in the model with the aim of obtaining a better agreement with the experimental data presented in the Letter of Nuclide. In this work, we have included the effect of the antineutrino mass in the rates of β^--decay, which has been neglected in other calculations due to the fact that the value of its mass is much smaller than the mass of the electron. We recalculated the fitting parameters σ_N and compared the results for the half lives in the models without and with the inclusion of antineutrino mass, GTBD1 and GTBD2, respectively. The GTBD2 presented results with slight improvements, with some nuclei tending to deviate from the experimental results. We observed that this fact is due to the adopted fitting parameters. Assuming an overestimated value of $m_\nu = 50$ keV within the GTBD model, we have obtained an average variation of thousandths of difference with the calculation made using zero neutrino mass. That difference will be even smaller if we use real values for the antineutrino mass, of the order of eVs. We conclude that the inclusion of antineutrino mass for β^--decay rates through the Gross Theory did not bring significant contributions that justify its inclusion. Our conclusion remains valid even within the framework of other models different from the GTBD, since what differentiates one nuclear model from another is the evaluation of the nuclear matrix element which is not affected by inclusion of the antineutrino mass in the model (see (11)).

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