

# Geoid modelling in the area of Fagnano Lake, Tierra del Fuego (Argentina): insights from mean lake-level observations and reduced gravity data

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**Abstract** This paper evaluates a combined geoid model in the area of Fagnano Lake located in Tierra del Fuego, Argentina. The model includes GPS/levelling points, gravity data and GPS buoy observations on the lake. The GPS buoy information allowed to determine a mean lake level (MLL) surface which was used to extend the geoid model to an area with restricted access by land (Del Cogliano et al. 2007). An approach to optimize the selection and distribution of the MLL data is developed in order to use them as input in the Equivalent Source Technique, and to combine them with different types of observations.

Furthermore, the global geopotential model EGM2008 is validated in the remote lake area. This model does not include observed gravity data in this region. Its behaviour is compared to the results of our observations along the levelling lines. Differences of several decimetres are found when EGM2008 geoid undulations are compared to geoid undulations derived from observations in this region. In the areas where EGM2008 relies on real gravimetric observations, differences between model and observations are only of a few centimetres.

The comparison of the EGM2008 model with our observations reveals the effect of missing or not representative gravity information may have on the estimation of geoid undulations in high mountain regions. The data on which EGM2008 is based in the Fagnano Lake area does not result from gravimetric observations on land. Therefore, we consider them as not representative and we show that their effect can be significant in such areas.

**Keywords** Geoid · Mean lake level · Fagnano Lake · Tierra del Fuego

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## 1 Introduction

Since 1998, several attempts have been made to estimate a geoid model for the province of Tierra del Fuego, Argentina. One of the most recent models consisted in a geoid model that combined gravity observations with levelling and GPS (Global Positioning System) observations (Gomez 2010).

The estimated precision of this model was 5 cm for 65 % of the region and 10 cm for the southern part of the island.

This model was generated using the Equivalent Source Technique (EST, Dampney 1969) and the best available observations. The method was programmed by Gomez (2010), following the approach proposed by Guspí et al. (2004) with slight adjustments. These adjustments consisted in precision formulae for the gravity modelling and the possibility of including more than one gravity field observables.

In the present study, a new geoid model is presented and analyzed which includes in addition MLL data derived from GPS buoy observations on the Fagnano Lake surface. This lake has an extension of approximately 100 km in east-west direction. It is located in the southern part of the main island of Tierra del Fuego, extending over the national territory of Argentina and the Republic of Chile. Our MLL data covers the Argentine portion of the lake surface (approximately 90 km length). This large area is characterized by the absence of roads and on-land access and, consequently, by the absence of any gravity or levelling data. Thus, our MLL measurements represent the only geodetic information available to validate geoid models in the south-western part of Tierra del Fuego. At the same time, this area is surrounded by the Andean mountains and characterized by steep topographic gradients.

Based on our research we derived guidelines for the optimum density, distribution and treatment of the MLL observations for their use in the EST aiming at a precision similar to that obtained in areas covered by gravity and GPS/levelling data.

In the western part of Fagnano Lake, the EGM2008 geopotential model includes so-called “fill-in gravity”. This data is not generated from real gravimetric observations on land. In this work we show that this kind of gravity can increase errors in geoid undulation estimations.

## 2 Data included

Figure 1 shows the distribution of 58 GPS/levelling marks, the raw gravity data (512 gravity anomalies) and 81 MLL observations on the Fagnano Lake which gave origin to the geoid model analyzed here. The distance between levelling marks is approximately 10 km.

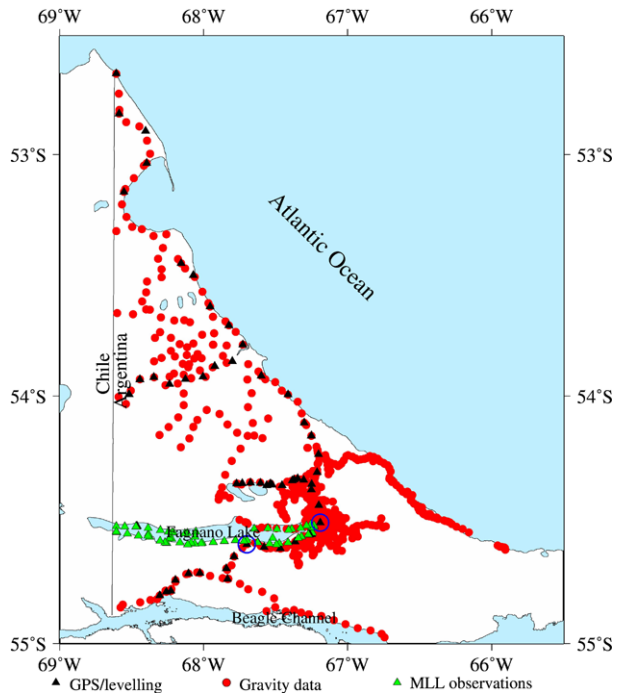
The geodetic coordinates of the original data referred initially to different reference frames. In order to unify the geodetic reference frames, all data were converted to TDF08 (Mendoza 2008), which is aligned to IGS05.

The altimetric datum was defined by the mean sea level at the Ushuaia tide gauge.

Between 2003 and 2005, several measurements were carried out with a GPS buoy on the Fagnano Lake. During those years, three pressure tide gauges were operated on the lake bed (Richter et al. 2010). Those observations allowed Del Cogliano et al. (2007) to determine the mean lake level surface which represents a closely enough approximation of an equipotential surface. The MLL was linked to the levelling lines by two points located at the south-eastern shore (Fig. 1), yielding a difference of 26.44 m between the MLL and the levelling origin.

All gravimetric data were completely referred to the International Gravity Standardization Net 1971 (IGSN 71) and the distance between each gravimetric point was 7 km approximately.

**Fig. 1** Map of the data distribution in Tierra del Fuego (Argentina). 58 GPS/levelling points (*black triangles*), 512 gravimetric observations (*red circles*) and 81 MLL observations (*green triangles*) are shown. *Two circles* indicate GPS/levelling points at which the link between the mean lake level surface and the origin of levelling was established



### 3 Data selection for the geoid modelling process

As already mentioned, the EST was applied at the computation stage. Point masses or equivalent sources generated by this technique are located below each observation site at a depth related to the data distribution. The masses are estimated after a least squares adjustment of the included observations. For more detail, see Guspí et al. (2004) or Dampney (1969).

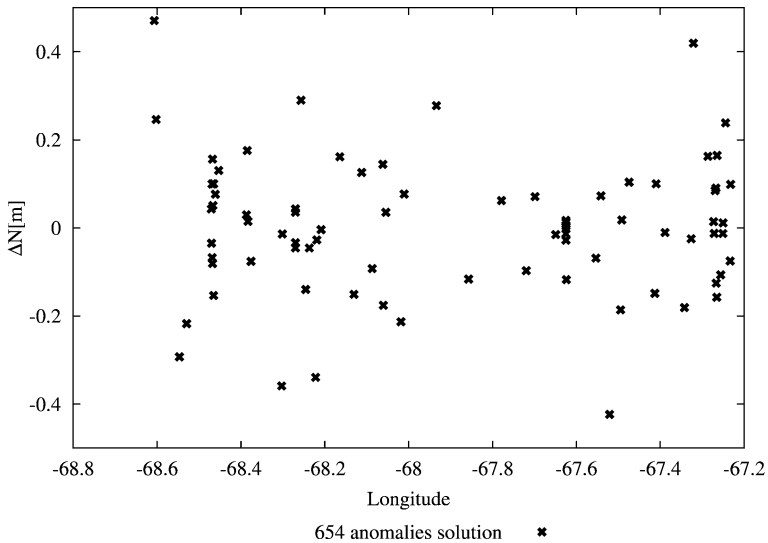
Gravity data were reduced by means of the remove-restore procedure and a grid of 654 gravity anomalies was obtained. They were reduced by short wavelength effects by means of a digital elevation model (DEM) based on SRTM3 data (Farr et al. 2007). EIGEN-GL04c (Förste et al. 2008) was used to remove the long wavelengths and the gravimetric reduction scheme was the Helmert's second method of condensation.

This reduction scheme involved the interpolation of gravity data on a regular grid that covered the Argentine portion of Tierra del Fuego. It should be noted that this required gravity anomaly information also to the north-west of Fagnano Lake, where no observed gravity was available. Through the text, these gravity anomalies are referred to as reduced gravity anomalies due to the fact that they are a product of interpolation and gravimetric reductions.

By means of the EST, these gravity anomalies were combined with geoid undulations which were obtained from 58 levelling points and 81 MLL observations. The EIGEN-GL04c geoid undulations were removed from the latter so as to extend the remove-restore process to the whole modelling. All the subtracted effects were restored at the end of the computation process in the obtained geoid undulations.

The errors assumed for the data were 5 cm RMS (Root Mean Square) for MLL data, 2 cm for GPS/levelling data and 2 mgals for the grid of reduced gravity anomalies.

The first approximation of the geoid by means of the EST did not include the MLL information and it yielded a precision near 5 cm on land. In the following we seek an optimum



**Fig. 2** Geoid undulation differences obtained after the cross validation procedure on the set of 81 MLL observations located on the lake

selection and distribution of MLL observations which allow us to incorporate additional data within the lake area while keeping the precision achieved on land.

In a first step, all MLL data were included in the geoid model and after its evaluation, the undulation differences that are shown in Fig. 2 were obtained by means of a cross validation process (Fotopoulos 2003).

As shown in Fig. 2, differences between observations and model exceed 40 cm after the cross validation process in the lake. The RMS of the differences amounts to 35 cm. The same approach yielded 5 cm RMS for a geoid model based only on GPS/levelling and gravity observations on land.

What is remarkable in Fig. 2 is that big differences occur near the eastern shore, located near the levelling lines. One of the problems consists in the heterogeneous density of MLL data which, in combination with the variability of such data, does not allow the method to fit the entire set of observations well. The GPS buoy observations are made from aboard a boat and, due to the often adverse weather and navigation conditions, in some places an agglomeration of buoy observation sites could not be avoided.

The uncertainty of each MLL value comprises the measurement uncertainties of the GPS buoy and tide gauge observations, but also residual uncertainties of the model of spatio-temporal lake-level variations used to reduce the instantaneous buoy observations to the mean lake level. This model (Richter et al. 2010) accounts for the major driving mechanisms (water volume fluctuations, response to atmospheric forcing, seiches and lake tides). Nevertheless, especially winds and related hydrodynamic processes may produce local, short-term lake-level variations at the position and time of the buoy observations not completely reflected by the model which may result in slight discrepancies between neighbouring MLL measurements.

According to the EST proposed here, given a buoy observation, the depth of the mass below each observation site is proportional to the distance of the nearest observation of the same kind. Therefore, the variability and distribution of the data forced us to adopt some selection criteria before introducing MLL data in the EST, because for each observation the

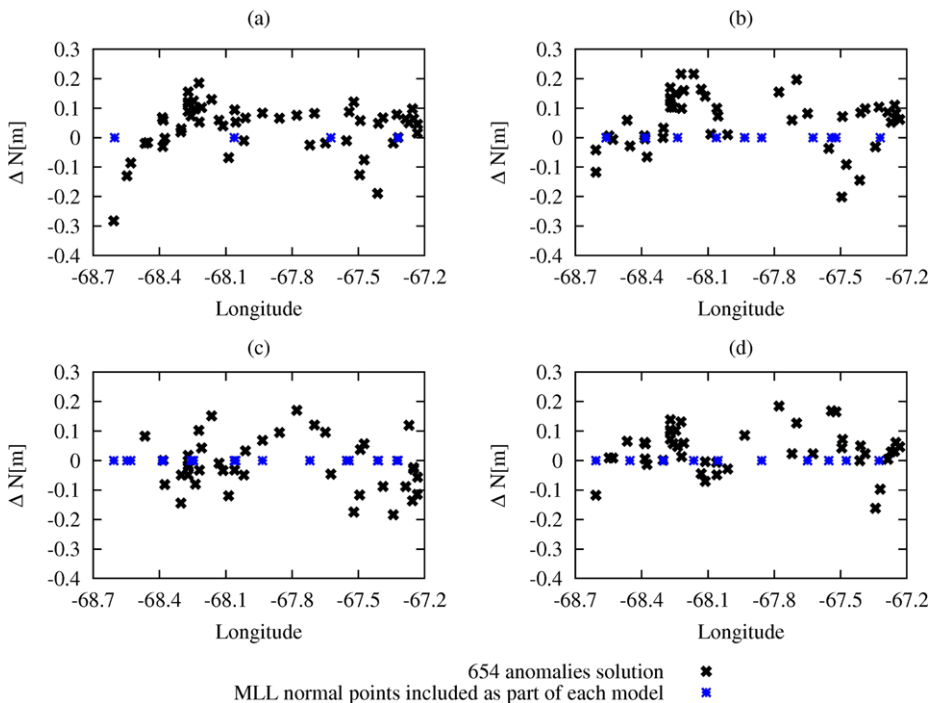
error of measurement and the depth of the mass to be adjusted (related to the data distribution), had to be considered.

In order to obtain masses sensitive to the geoid signal, MLL normal points (i.e. mean values of MLL observations) were derived in those places where there was an agglomeration of GPS buoy observations. The formation of MLL normal points reduced their number from 81 to 54. Only after this data reduction step it was possible to combine the MLL data with land observations.

#### 4 Optimization of the distribution of MLL data

In order to determine the best distribution of MLL data, different models were derived. They differed in the number and position of the MLL observations included. All of them were combined with the grid of 654 reduced gravity anomalies and 58 GPS/levelling observations. These models were validated with the remaining MLL observations not included in the respective model.

When four MLL normal points were introduced in the model, distributed in a longitudinal direction, the results were unsatisfactory in the western part (Fig. 3a). In that place, the



**Fig. 3** The *four panels* depict the differences obtained after the comparison of 4 geoid models with observed geoid undulations at the sites of the remaining MLL observation, which were not introduced in the model. The four geoid models are based on the same set of 654 reduced gravity anomalies and the information of 58 GPS/levelling points, but they include different subsets of MLL normal points considered as reference: (a) 4 MLL normal points; (b) 10 MLL normal points; (c) 18 MLL normal points; (d) 10 MLL normal points as reference not also distributed along the east-west axis as case (b), but considering the north-south direction

strong gradient of the geoid could not be represented by only one normal point every 24 km. The estimated RMS was 10 cm at the remaining buoy observation sites.

Another case consisted in 10 MLL observations, separated by 11 km along the east-west axis of Fagnano Lake. As shown in Fig. 3b, the result is not satisfactory, either. For the remaining 44 mean lake level observations the RMS amounts to 10 cm.

When the distance between MLL data was further reduced, producing as a result 18 reference points (Fig. 3c) distributed in the same direction as before, the results did also not improve significantly. The estimated RMS was 9 cm.

It is observed that the distributions proposed so far did not allow the model to fit the observations, because they did not represent the north-south geoid gradient prevailing in that area.

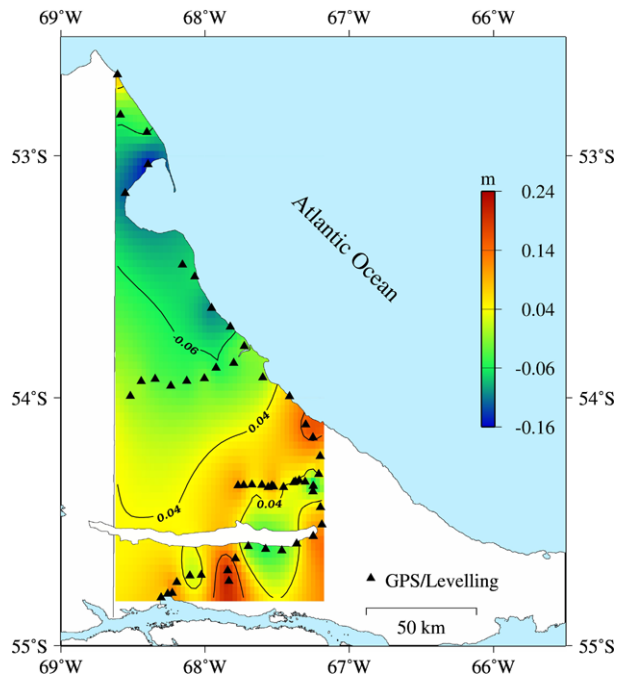
Satisfactory results (RMS: 7 cm) were obtained when 10 MLL observations were used (11 km separation), not only aligned in east-west direction, but also in north-south (Fig. 3d). It should be noted that they were located at a distance similar to that existent between GPS/levelling points (7–10 km).

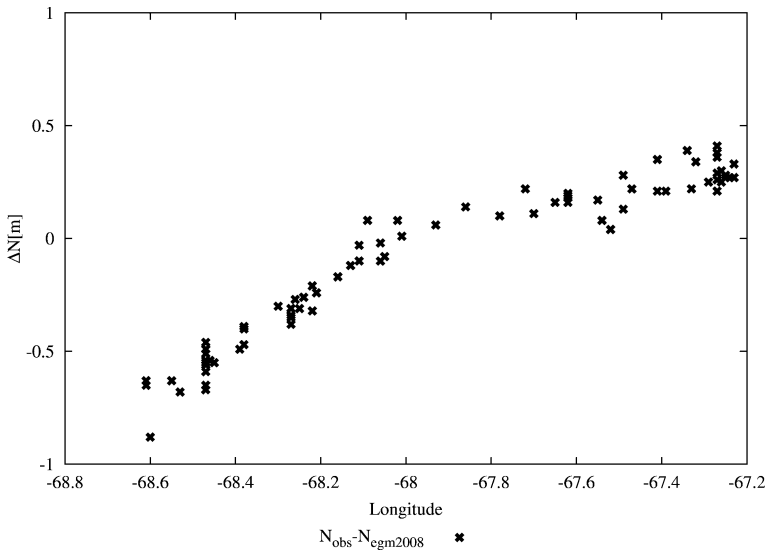
Taking into account that the RMS of the geoid model was estimated in 5 cm for the land part of Tierra del Fuego, it is possible to affirm that after the inclusion of the MLL data, a 6 cm RMS geoid model was reached for almost the entire province.

## 5 Regional validation of the EGM2008 model

In 2008, Pavlis et al. published a global geopotential model with a 9 km resolution. A comparison of the model with our data revealed a different behavior over land on one hand, and over the lake surface on the other hand. In the western part of the lake the performance of the global model was not as good as expected.

**Fig. 4** EGM2008 evaluation at GPS/levelling points (*black triangles*). The *colour scales* represents geoid undulation differences; contour interval: 10 cm





**Fig. 5** Comparison of the EGM2008 geopotential model with MLL observations. The differences between model and observations are shown along the direction of the main axis of Fagnano Lake

Figure 4 illustrates the good agreement between land observations and the EGM2008 as published in Pavlis et al. (2008).

The comparison of the EGM2008 with MLL observations along the main axis of the lake is shown in Fig. 5. The geopotential model fits the observations in the central-eastern part of the lake well, but large discrepancies reaching 80 cm are obtained in the western part.

As far as it is known from Pavlis et al. (2008), in the central-western part of Tierra del Fuego there is no terrestrial gravimetric information included in the model but, what is called a “fill in”. This fact suggests that due to the absence of observed gravity, the “fill in” data based on a  $5' \times 5'$  DEM, a satellite-only geopotential model (Pavlis et al. 2006) and the old EGM96 (Lemoine et al. 1998) might not be enough to represent the gravity field appropriately in that region.

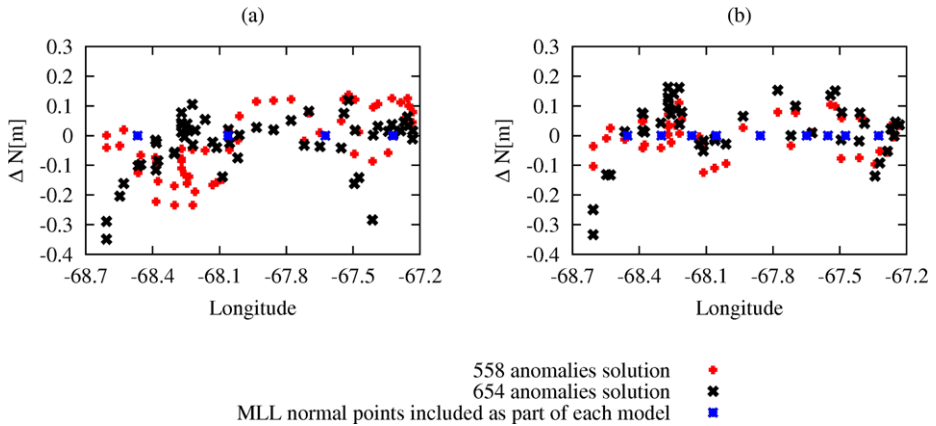
## 6 Reduced gravity anomalies in areas lacking gravity data

As was shown in Fig. 3d, there is a good agreement between the local geoid model and the observations.

Considering the absence of observed gravity to the north-west of the lake (Fig. 1), part of the 654 reduced gravity anomalies can be considered similar to the “fill in” mentioned for the EGM2008, in that area.

In the next step, the impact of reduced (interpolated) gravity anomalies on the geoid estimation in the area of Fagnano Lake was analyzed. For this purpose, two model validations were carried out and compared, similar to those made in the previous sections, but this time, reduced gravity anomalies located to the north-west of the lake were excluded. Then, 558 gravity anomalies of the 654 were used.

In order to emphasize the effect of gravity data, most of the MLL data from the western part of the lake were excluded from both models.



**Fig. 6** Both panels depict the differences obtained after the comparison of the combined geoid models with the remaining values of MLL data, which were not included in the model. Both were based on the information of 58 GPS/levelling points and a certain number of reduced gravity anomalies and MLL normal points. Two solutions are shown in each graphic: one which includes 654 reduced gravity anomalies (*black*) and other with 558 reduced gravity anomalies. In (a) 4 MLL normal points served as reference while in (b) the number of considered MLL observations was 9, not also distributed along the east-west axis, but considering the north-south direction as well

In Fig. 6 two types of solutions overlap: one which includes the 654 reduced gravity anomalies and the other which just contains 558 of them. They also differ in the number of MLL normal points which were introduced in each model: 4 and 9, respectively. The rest of the information used as input to generate the model was the same as in the previous sections.

The differences obtained for those solutions, which did not include “artificial” reduced gravity to the north-west of the lake, were always the smallest. The RMS fell from 10 cm (654 solution) to 9 cm (558 solution) in the 4 MLL observation configuration shown in Fig. 6a. In the case of 9 MLL observations, the RMS ranged from 11 cm to 6 cm (Fig. 6b). These nine MLL observations were a subset of the optimum configuration shown in Fig. 3d, excluding the westernmost MLL measurement.

The most remarkable change took place on the western side, where differences reached 40 cm when “artificial” reduced gravity anomalies were used. Hence, this result might be associated to the 80 cm differences found when comparing observations and EGM2008 in the area of the lake.

## 7 Final remarks

The use of MLL data in combination with gravity and GPS/levelling information in the EST involves the use of normal points. Considering the way in which measurements with the GPS buoy are performed, their direct use in the EST generates point masses very close to the surface and to each other. With such an arrangement of masses, they become very sensitive to undesired high frequencies related to the measurement itself and its uncertainties.

The generation of normal points in a suitable distributed way allowed us the use of the MLL observations to determine sensitive sources to the natural frequencies of the geoid undulation. The benefit of this strategy, especially in areas not accessible by land, has been demonstrated.



Based on these methodological insights, a combined geoid model of Tierra del Fuego was obtained with an accuracy of 5 cm north of Fagnano Lake and 7 cm to the south of the lake

The impact of using reduced “artificial” gravity anomalies close to rough topographic zones, like the Andes Mountains, was estimated in many decimetres. The global EGM2008 model does not include observed terrestrial gravity information, neither in the central-western part of Tierra del Fuego nor in the Fagnano Lake area. Differences from observations of up to 80 cm in geoid undulation have been detected in that part of the lake, probably as a consequence of the “fill-in” gravity.

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