THE IMPACT OF MATHEMATICAL WRITING ON MATHEMATICAL PROBLEM SOLVING SKILLS AND METACOGNITION AMONG THE FIRST YEAR UNIVERSITY STUDENTS

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by

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KESAN PENULISAN MATEMATIK TERHADAP KEMAHIRAN PENYELESAIAN MASALAH MATEMATIK DAN METAKOGNISI DI KALANGAN PELAJAR UNIVERSITI TAHUN SATU

ABSTRAK

Kajian ini bertujuan mengkaji penulisan matematik sebagai satu kaedah untuk meningkatkan kemahiran dan metakognisi dalam proses penyelesaian masalah Matematik di kalangan 30 orang pelajar universiti dari Kursus Asas Kejuruteraan. Pelajar tersebut telah menyertai satu siri sesi penulisan matematik untuk menyelesaikan 15 masalah matematik dalam tempoh masa lima minggu. Data kualitatif dan kuantitatif telah dikumpul termasuk: respon bertulis pelajar dalam buku kerja penulisan matematik; prestasi pelajar dalam penilaian penyelesaian masalah; laporan kendiri terhadap kemajuan metakognisi sendiri; serta persepsi pelajar mengenai penulisan matematik. Analisi kajian kes bersifat penerokaan telah digunakan untuk menganalisa data tersebut. Kajian ini berdasarkan sampel secara bertujuan yang terdiri daripada tiga puluh orang pelajar universiti dari Kursus Asas Kejuruteraan sebagai subjek kes itu. Hasil kajian menunjukkan bahawa usaha pelajar dalam penulisan matematik memberi kesan terhadap setiap peringkat penyelesaian masalah. Ini bermakna semakin banyak pelajar bertulis, semakin banyak mereka terlibat dalam proses penyelesaian masalah semakin mahir kemahiran penyelesaian masalah pelajar. Penulisan matematik membantu pelajar untuk berfikir dan mencapai kefahaman yang lebih mendalam sepanjang proses penyelesaian masalah. Ini mencetus keaktifan minda pelajar dalam pemahaman, perancangan, perlaksanaan dan pertimbangan, dimana ini turut meninggalkan kesan yang mendalam terhadap keupayaan metakognisi di kalangan pelajar. Hasil kajian juga menunjukkan bahawa

kebanyakan pelajar memperolehi peningkatan dalam keupayaan metakognisi mereka, namun pelajar lain pula menyedari kekurangan keupayaan mereka dalam menyelesaikan masalah matematik. Maka mereka mengambil tindakan untuk menyelaras semula tahap metakognisi yang dikenalpasti semasa di peringkat awal kajian. Selain itu, penulisan matematik mendorong perubahan sikap pelajar dalam penyelesaian masalah matematik dimana pelajar belajar untuk menggabungkan ayat dan cara kerja pengiraan untuk memulakan proses pemahaman dan merumuskan pelan ke dalam bahagian konstituen. Teks bertulis pelajar turut menunjukkan kesan-kesan penyemakan semula untuk membetulkan teks bertulis tersebut untuk menyatukan proses penyelasaian masalah apabila perlu. Maka, kajian ini memberi implikasi bahawa proses penulisan matematik berpotensi menjadi satu pendekatan dalam penyelesaian masalah yang mempunyai ciri-ciri yang menggalakkan dan dapat menyelaraskan perkembangan dalam penyelesaian masalah Matematik, komunikasi dan metakognisi.

THE IMPACT OF MATHEMATICAL WRITING ON MATHEMATICAL PROBLEM SOLVING SKILLS AND METACOGNITION AMONG THE FIRST YEAR UNIVERSITY STUDENTS

ABSTRACT

This study focuses on mathematical writing as a mode of developing mathematical problem solving skills and metacognition among thirty Foundation in Engineering students. These students participated in a series of mathematical writing sessions to solve 15 mathematical problems for five weeks. Qualitative and quantitative data were collected including the students‘ written responses in mathematical writing workbook, their performance in a problem solving assessment, their self-report about metacognition progression as well as their perceptions about mathematical writing. An exploratory case study analysis was applied to examine these data. This study was based on a purposive sample of thirty Foundation in Engineering students as the subject of the case. The findings showed that students' effort in mathematical writing impacted the problem solving actions taken at each problem solving phase. In other words, the more they write the more they engage in problem solving actions which imply the emerging of problem solving skills. Mathematical writing assists students to think along the path of problem solving that activate their mind to understand, plan, execute and reflect which has also created a profound impact on students' metacognitive abilities. The findings revealed that majority of the students felt the growth of their metacognitive ability while a few students became aware of their limitation in mathematical problem solving that they reweighed their initial potency in metacognition. Moreover, mathematical writing has induced a change in students' approach in mathematical problem solving, such as they learned to combine

words and formulas to initiate their understanding and to itemize their plan into its constituent parts. Their written text also revealed the traces of revising where they discarded and edited the text whenever necessary, so as to consolidate the problem solving process. Hence, this study implies that the potential of mathematical writing as a problem solving approach has a considerable amount of features that sustain and synchronize the development of mathematical problem solving, communication and metacognition.

CHAPTER 1

INTRODUCTION

1.0 Introduction

Students' exposure to mathematical learning begins from their primary education and problem solving is reckoned as the leading skill that students should develop when learning mathematics. The mathematics curriculum at the pre-tertiary education in Malaysia has been systematically structured to provide opportunities for students to develop mathematical knowledge and problem solving skills throughout their academic years (Zanzali, 2000). When students proceed to university level, a great demand on their achievement in problem solving skill is further emphasized in the learning outcomes of every degree programmes and they are aligned with one of the eight domains of learning outcomes, i.e., problem solving and scientific skills, in the Malaysian Qualification Framework (MQF) of Higher Education (Malaysian Qualification Agency, 2005). Nevertheless, a series of continuous actions has been carried out over the years that the system went through a significance evolution and eventually, the Malaysia Education Blueprint 2015-2025 (Higher Education) or MEB(HE) was launched in 2015. It is a 10-year transition period to revamp the higher education system in Malaysia. One of the notable goals of the blueprint is the student-centred education that emphasizes individual students' achievement, not only in the realm of knowledge and skills but also in the sense of individual's ethics and morality. Six primary attributes are introduced to assist individual students to harmonize between both knowledge and skills as well as ethics and morality. One of the primary attributes is the development of thinking skills that includes problem solving skills. This shows that developing students' problem solving skills is still a continual mission in the Malaysian education system.

Communication in mathematics is another integral sector in the teaching and learning of the Mathematics Curriculum in Malaysia since the last few decades (Ministry of Education Malaysia, 2003a, 2003b, 2006a, 2006b). The curriculum strongly stresses the implementation of effective communication to engage students in active mathematical conversation. In fact, mathematical writing appears to be one of the means of communication to achieve its goal (Ministry of Education Malaysia, 2003a, 2003b, 2006a, 2006b). The curriculum is designed such a way that students are encouraged to communicate mathematics in a written structure so that the writing reflects students' mathematical understanding (Pugalee, 2001; Steele, 2005). Apart from this, many researchers asserted that the act of mathematical writing eventually helps students to develop metacognitive consciousness (Kazemi, Fadaee, & Bayat, 2010, Knox, 2017; Lester, Garofalo, & Kroll, 1989; Mayer, 1998; Pugalee, 2001). Metacognition is simply defined as the ability to think within the cognitive process (Kayashima, Inaba, & Mizoguchi, 2004; Mayer, 1998; Ozsoy & Ataman, 2009; Schraw & Dennison, 1994). It is all about being mindful of one' own thinking and learning process when solving a mathematics problem. It is the impelling mechanism behind successful problem solving (Garofalo & Lester, 1985; Knox, 2017; Pugalee, 2001). In other words, the absence of metacognition in the problem solving process merely arouses students' low level of problem solving skills that cater for solving routine mathematics problems (Mayer, 1998). In a way, mathematical writing is synchronizing the process of mathematical problem solving and metacognition where it gets students to engage in metacognition during mathematical problem solving. Hence, this study intends to extend and amplify the hidden potential of mathematical writing by adopting it as a practicable tool to facilitate the instigation of students' mathematical problem solving skills and metacognition.

1.1 Background of the Study

Helping students to become successful problem solvers is an endless expedition in the education system in Malaysia. The MEB(HE) (Ministry of Higher Education, 2015) continues to aggrandize and transform the existing education system to increase its quality in the aspect of thinking skills among the students from basic through to higher education. In fact, since the mid of 80's, the Mathematics Curriculum at School in Malaysia has inaugurated Polya's (2014) problem solving model as a basic approach to instigate students' problem solving skills (Zanzali, 2000). The curriculum attempts to make problem solving as the center of attention in the teaching and learning of mathematics (Zanzali, 2000). Since then, many studies have been performed to investigate the adoption of problem solving strategy in the teaching and learning of mathematics in a mathematics classroom (Ahmad, Salim, & Zainuddin, 2008; Saleh, 2009; Tambychik & Meerah, 2010; Yunus & Ali, 2009). However, it is still not clear to what extent the school teachers in Malaysia have enforced the problem solving strategy to teach mathematics. Saleh worked together with Aziz (2012) to examine closely the teaching practices in Malaysian secondary schools, whether teachers have adjusted to a more effective teaching strategy or continued to adore the conventional teaching methods. Their findings deduced that majority of the existing teachers stay focused on traditional teaching style, i.e., teacher centered methods of instruction. As a matter of fact, an overview of the following six years of research on the teaching and learning of mathematics in Malaysia, many researchers continue to address teacher centered instruction and students' prior knowledge in mathematics as part of issues behind the students' poor performance in mathematical problem solving (Chew, Idris, & Leong, 2014; Hasan, Abdul, & Selamat, 2018; Lohgheswary, Nopiah, Aziz, & Zakaria, 2018; Rodzalan & Saat, 2015; Salim, Ahmad,

Waini, & Miswan, 2017; Ujang et al., 2017; Zakaria & Salleh, 2015; Zulkpli, Mohamed, & Abdullah, 2017). In other words, the quality of teaching and learning of mathematics has remained unchanged, i.e., teacher centered approach, even though various educational workshops and trainings have been administrated by Ministry of Education to enhance the teaching profession for the past two decades (Ghazali, 2017). In some respects, the teacher-centered approach has predominantly cultured the students' dependent learning style where the students rely on rote memorization and spoon-feeding way of learning throughout their education experiences (Rodzalan & Saat, 2015). Hence, the passive learning environment has driven students to appreciate the culture of rote learning and made them reluctant to learn and think at the same time.

With respect to the above fact, the transition period from secondary school to university is becoming a critical period experienced by the students. Many mathematics educators have noticed that the majority of first year university students came with low prior knowledge and skills in mathematics which were incompatible with the students' excellent performance in the national examination, i.e., SPM (Sijil Pelajaran Malaysia) examination (Salim et al., 2017). Some educators at the tertiary level began to query about the reliability of students' mathematics performance in the SPM examination (Hasan et al., 2018). This has brought up another issue where students with the mindset of rote thinking might experience loss and unable to cope with the university syllabus which requires application of knowledge and ability to solve problems. As a result, students might not reach the right kinds of attributes required to be skillful workers after they graduate (Yunus et al., 2005). The worst possible thing that could happen is the unemployment rate among the Malaysian graduates might be increased (Rodzalan & Saat, 2015). Hence, it is very important to ensure that students are exposed to skills training

development especially problem solving skills, right from the beginning. Thus, there is a need to rethink the mathematical teaching and learning strategies that can be merged with the problem solving approach to foster the students' problem solving skills and at the same time can cultivate their metacognitive abilities. It is hoped that this study would help the first year university students to develop their problem-solving skills and metacognition early in their curriculum and continue to improve and upgrade their skills throughout their course in the higher education.

1.2 Problem Statement

1.2.1 Mathematical Problem Solving among Malaysian Students

Malaysian students have spent their eleven years of quality time in school to develop their skills and knowledge in subjects such as science, mathematics and languages. Nevertheless, their proficiency levels in Mathematics, especially in application and reasoning, remain challenging to meet the level of competence against the international standards such the Programme for International Student Assessment (PISA) and the Trends in International Mathematics and Science Study (TIMSS) (Ministry of Education Malaysia, 2013). According to the PISA 2012, almost 60% of the 15-year-old Malaysian students who participated in PISA achieved a very low level of problem solving skills. This means that these students lacked in problem solving skills to tackle real life problems. Only about one percent of the Malaysian students aged 15 could solve the most complex problems. The report was in line with a study done by Kiong, Yong, and Hoe (2007) about the mathematical problem solving of the Form Four students. Their findings showed that the overall performance of the students declined drastically as the level of difficulty of problems increased. The students were only able to solve routine mathematics problems that only demanded basic mathematics skills. In fact, this problem continues to

persist, as reported in the research of Nasir and his team (2013) about the problem solving skills on a group of 113 Form Four Malaysian students in solving algebraic problems and differentiation problems. They concluded that the students still lacked in central skills to solve application problems. They tended to merely memorize and apply mathematical rules and definitions without fully understanding the underlying concepts. It is an undeniable fact that the rote learning pattern among students has been unchanged over the years (Hasan et al., 2018). This is very alarming as it has created a serious challenge for the Malaysia government to expand a workforce of problem solvers and inventors to compete globally. The prime concern now is regarding those students who have completed their secondary education and are going to proceed to pre-university education, whether they are able to cope with the amount of mathematics at the tertiary level. Singh (2009) investigated the conception of 127 college students and their heuristic action in mathematical problem solving. 98.5% of these participants had obtained an A grade in the national examination for Mathematics. The data indicated that these students' ability to conceptualize and use heuristic approaches to mathematics problem solving were weak. The study also concluded that the grade obtained in the national examination did not reflect the students' mathematical knowledge in mathematical problem solving. Mathematical knowledge gap encountered by first year university students seem to be a recurrent issue in the Malaysia context. Salim, Ahmad, Waini, and Miswan (2017) conducted a comparison on students' performance in mathematics between their SPM and first year university examination. They deduced that majority of the students came with insufficient knowledge and skills in mathematics during the admission to university. With regards to the issue, some of the mathematics educators at the tertiary level began to wonder the way mathematics was taught in schools.

In fact, many studies have reported that the first-year university students face difficulty in the application of elementary mathematical concepts to a given problem (Basitere & Ivala, 2015; Bowles, Dobson, Fisher, & McPhail, 2011; Briggs, Clark, & Hall, 2012; Yusof & Rahman, 2001). The long duration of exposure to rote learning styles and examination-oriented education system at school has somehow shaped the students' mindset about mathematical problem solving (Lim, 2009; Saleh & Aziz, 2012). Numerous studies have indicated that Malaysian university students struggle to think critically and become inefficient in solving real world application problems (Ghadi, Bakar, Alwi, & Talib, 2013; Lim, Lee, Yap, & Ling, 2016; Rodzalan & Saat, 2015). In a way, Malaysian students fail to engage in deeper level of problem solving thinking since they continue to adopt memorization and rote techniques to solve university level mathematics problem (Bayat & Tarmizi, 2010; Salim et al., 2017; Zakaria & Yusoff, 2009).

1.2.2 Metacognition Issue

It is no more a surprising fact that Malaysian students' performance in mathematical problem solving skills is inferior. The examination orientation nature of the curriculum in Malaysia education impose the conventional way of teaching where teachers focus only on contents and skills that will be evaluated in the public examination (Lim, 2009; Saleh, 2009). Students tend to "regurgitate information" according to the examination format and ignore the necessity of understanding "what is being put down in their answers" (Nordin, 2009, cited in Lim, 2009). As a result, the learning culture at school has more emphasized upon the mathematical result, rather than the process of constructing idea (Singh, 2009). In fact, students may stop thinking about mathematical reasoning and continue to value the algorithm procedures as the prime entity of learning.

In other words, students may lose their most powerful thinking tool which is metacognition and continue to be disengaged from learning and possess low problem solving skills. They are physically contented with routine problems which is common learning practice in classroom (Bayat & Tarmizi, 2010; Ismail, 2013).

Many studies have been concerned with enhancing students' mathematical problem solving skills and in fact, the treatment to this issue is to increase the metacognition awareness among the students. Recent research in mathematics education indicates that more and more attention has been given to metacognition (Al Shabibi & Alkharusi, 2018; Bayat & Meamer, 2016; Chimuma & DeLoach Johnson, 2016; Duque Jr & Tan, 2018; Ozsoy & Ataman, 2009; Smith, 2012; Telaumbanua & Surya, 2017). These studies have justified the claim that metacognition enhances problem solving abilities among learners. Thus, teaching students how to metacognitively solve mathematical problems is a key to the success of any mathematics curriculum and the efforts to achieve the goal has been increased. Many Malaysian researchers start to venture into studies related to metacognition in the teaching and learning of mathematics. They strongly believed that metacognition is the strength behind successful mathematical problem solving which leads to the improvement of students' mathematics achievement (Bayat & Meamer, 2016; Hassan & Rahman, 2017; Kazemi et al., 2010; Mohamed & Tan, 2005). In fact, students' abilities to master the skills of mathematical problem solving are always connected to metacognition abilities. According to Schoenfeld (1985), students are not actually weak in solving problems but deficient in the skills to manage strategies that help to solve specific problem. For example, Abdullah, Rahman, and Hamzah (2017) conducted a study to identify the metacognitive ability of a group of 304 Form Four students and examine the effect of such abilities to solve non-routine mathematics

problems. The findings concluded that the students' abilities to solve non-routine mathematics problems were weak and the metacognitive skills that they demonstrated were rated as medium. In other words, their difficulties in mathematical problem solving relate to their inferiority in metacognition. Thus, students should be psychologically prepared particularly at the younger age to escalate their metacognition ability especially in mathematical problem solving.

Mathematics is a key subject for students in all engineering disciplines at the tertiary level. By learning mathematics, students are expected to develop the mathematical skills especially in problem solving that are transferable to all engineering disciplines. Hence, helping students to be a good problem solver becomes an imperative action in the first year of teaching mathematics. The incorporation of mathematical writing in the teaching and learning of mathematics serves as an initial step to assist students to "talk and think about" mathematics. As claimed by many researchers, the writing mode drives the students' momentum to manifest their metacognitive ability in a gradual manner and the writing responses attempt to validate the students' metacognitive experience while solving a mathematics problem (Kazemi et al., 2010; Knox, 2017; Lester et al., 1989; Mayer, 1998; Pugalee, 2001). For the past 25 years, Mathematics Curriculum in Malaysia has already acknowledged mathematical writing as a practical tool to encourage students to communicate mathematically (Ministry of Education Malaysia, 2003a, 2003b, 2006a, 2006b). However, despite its importance, relatively little is known about its potential since there is little or no research concerning the practices of mathematical writing from preschool to tertiary education in Malaysia. Hence, there is an urgent need for evidence to validate the profound impact of mathematical writing as an approach to facilitate students' metacognitive thinking through problem solving in all education levels.

This study intends to conduct an empirical investigation on using mathematical writing as a stimulant tool to enforce mathematical problem solving skills and metacognition among the first year university students. It is attempted to determine whether mathematical writing is able to make a great impact on students' mathematical problem solving skills and strengthen their metacognitive abilities.

1.3 Purpose of the Study

This study aims to focus on the impact of mathematical writing as a mode of developing mathematical problem solving skills among the first year university students. It attempts to merge the writing mechanism into the teaching and learning of mathematics to cultivate students' metacognition abilities.

1.4 Research Objectives and Research Questions

The objectives of this study are:

I. to explore the impact of mathematical writing on mathematical problem solving skills among the first year university students

II. to examine the impact of mathematical writing on metacognition among the first year university students.

III. to identify how the mechanism of mathematical writing can change students' approach when solving a mathematical problem.

IV. to investigate the students' perceptions of mathematical problem solving using mathematical writing.

Thus, this study is designed to seek answers to the following research questions:

1) To what extent is the impact of mathematical writing on mathematical problem solving skills among the first year university students?

2) To what extent is the impact of mathematical writing on metacognition among the first year university students?
3) How does the mathematical writing change students' approach when solving a mathematical problem?
4) What are the students' perception of mathematical problem solving using mathematical writing?

1.5 Significance of the Study

It is undeniable that the lack of problem solving skills amongst Malaysian undergraduates are partly due to their rote learning experience in schools and the demand from their parents, teachers and peers to excel academically because of the examination-based education system (Ahmad, 1998; Maesin, Mansor, Shafie, & Nayan, 2009). Students' performance at the primary and secondary school levels is constantly assessed by how many A's they achieve in their examinations. In the process, they fail to develop an inquisitive mind and analytical skills as most of their time they attend tuition classes, extra classes, and examination workshops to better prepare themselves for the upcoming examinations. As a result, these students retain the rote learning mindset and studying pattern when they enter the university (Salim et al., 2017; Wahid & Shahrill, 2014). The findings of this study will be used as an antidote to rote learning that help the first year university students to experience a non-rote teaching and learning that is, use writing to think, communicate and learn. Mathematical writing is often seen as having a minor role in mathematical thinking. This study will expand the potential role of mathematical writing as a facilitator tool that drives students to constitute, organize, perform and revise various knowledge and skills which will increase their thinking abilities. Moreover, the activities in this study will guide students to have a clear understanding about a truly

problem solving process which they often overlook in the past and gradually cultivate and keep upgrading their problem solving skills to become highly skilled graduates.

The development of students' problem solving abilities highly depends on the teaching and learning process and the contextual learning environment during their academic years. The quality of teaching and learning is relied on the teaching methodology and the time allocation for students actively engage in learning activities within and outside the classroom (Reynolds & Muijs, 1999; Papanastasiou, 2008). Thus, teaching mathematics is a challenging task to most teachers as they are the ones who give great impact on developing students' mathematical problem solving skills. The findings of this study will provide evidence of mathematical writing as an effective instrument to teach mathematical problem solving and assist students to construct their problem solving skills. It will help some educators to start adjusting to alternative teaching practices rather than continue to adore the conventional teaching methods. A shift from conventional teaching practices may also prompt educators to change their pedagogical approach that will be credited to their own professional development.

Malaysia Qualification Agency, known as MQA comes up with the Malaysian Qualifications Framework (MQF) that serves as a quality assurance mechanism to measure the qualification levels, learning outcomes and credit systems that are equivalent to international good practices. Problem solving is one of the generic skills that portray a special emphasis in one of the eight MQF learning outcome domains. It is a prime skill which students should nurture when they learn mathematics in any disciplines of higher education. It is documented in the mathematics course outline as the intended learning outcomes so that instructors and students should be aware of what they would experience in the process of teaching and learning. Assessment activities then would provide concrete

evidence of how the students have achieved the intended learning outcomes (MOHE, 2013). Many educational institutions have undergone extreme pressure to equip students with more than just good academic performance and provide a testimony of what students' have achieved (Abdullah, 2015; Mohamad et al., 2013). The findings of this study will offer an alternative instructional methodology to the higher education provider (HEP) in cultivating and giving evidence of students' problem solving skill throughout the full duration of their course. The results will reveal the instructional and assessment alignment, and identify where and how the observed students' attributes are taught, practiced and assessed within the course. The students' responses of the study can be used as a collection of evidence to show how the learning outcomes of a course are in line with the educational objectives of a programme in a higher learning arena. Furthermore, the findings will also benefit HEP who is responsible for conducting and awarding the higher education qualification where the results may be used as part of the students' learning evidence in the institution accreditation that is recognized by the MQA.

Nowadays, employers are keen to retain good employees who have a balanced combination of a good academic achievement and generic skills such as communication skills, problem solving skills, interpersonal skills and flexibility in adapting to different workspace. However, Malaysian employers have reached to a common consensus that most of the Malaysian graduates are efficient in their areas of specialization but still lacking in generic skills unfortunately (Hamid, Islam, & Hazilah, 2014; Singh & Singh, 2008; Yusof & Jamaluddin, 2017). As a result, they have to struggle to find employment after graduating. Many researchers believe that the development of generic skills should be integrated into the education curriculum to ensure that students graduate from these institutions with skills needed by employers (Chung, Cheong, Leong, & Hill, 2018; Omar,

Bakar, & Rashid, 2012; Shafie & Nayan, 2010). The Malaysia government is greatly aware of the unemployment issue among graduates and hence, the Minister of Higher Education demands that all institution of higher learning in Malaysia must integrate the generic skill elements into the undergraduate syllabus (MOHE, 2015). In fact, the greatest skill that students can experience during the educational process is learning how to think, which might generate other intellectual skills (Yoong, Don, & Foroutan, 2017). Mathematical writing is a way of thinking (Pugalee, 2001; Fung, 2010; Martin, 2015) and therefore, the finding of this study may upgrade students' competency not only in problem solving but other soft skills as well, such as communication skills and critical thinking.

In the field of research, the findings in this study will add a substitute contribution to the field of mathematics education, namely fostering problem solving skills and metacognition using mathematical writing especially in the higher education practices. It will provide insights to educators or researchers to further explore the mathematics teaching and learning through mathematical writing and adopting applicable assessment practices that improve the students' understanding about mathematical problem solving. Numerous national and international documents have designated problem solving as the central focus of mathematics curriculum yet many educational reports attempt to discuss the mathematics achievement rather than the development of problem solving learning (English & Sriraman, 2010). The findings of this study may reduce the ambiguity of problem solving investigation and fill the gap in the literature that more concern about skills and concept development through problem-solving. Moreover, this study could contribute to the increasing global need of using metacognition in other fields, besides mathematics.

1.6 Limitation of the Study

Using mathematical writing as a teaching technique is not widely utilized among the Malaysian educators although it is specified in the Mathematics curriculum in Malaysia, as an essentiality to implement written communication in the teaching and learning of mathematics. Significant changes in pedagogy are demanded to push through the conventional teaching barrier that normally restrains the student-centered teaching strategies (Ramli, Shafie, & Tarmizi, 2013). Teachers are required to change in terms of the instructional process and play as a facilitator rather than just an information provider. The challenge of bringing the language and mathematics instruction together is also another tough task as many mathematics teachers tend to think of mathematics as a subject that does not require a strong command of language (Seto & Meel, 2006). Thus, the implementation of mathematical writing in a classroom requires a great deal of time, efforts and determinations. Some researchers experience teachers' unwillingness and reluctance to participate in the writing pedagogy practices due to additional and time-consuming work, lack of recognition from the stakeholders and uncertainty about the new pedagogy practices (Van Thienen, 2002; Varank & Tozoglu, 2006). Thus, in this study the researcher will take the role as the facilitator to implement mathematical writing as a pedagogical tool in the teaching and learning of mathematics. Consequently, researcher bias would be a limitation in this case study research. Particularly when the researcher is the main instrument for data collection, data interpretation and data analysis, the subjectivity issues may arise. Stringent measures will be extensively utilized during the research process in order to keep the research honest and minimize its biases.

Thirty participants involve in this study are engineering foundation students at one of the higher institutions. The students are school leavers who have completed their SPM

examination and it is the first time they experience higher education learning environment. The learning culture at school may have engendered the students' opinions about using writing to learn mathematics. Students' writing practices only take place during language lessons at school. Several researchers on language studies claimed that most students faced difficulty to cope with writing because of their inability to think critically and thus, most students perceived that writing was difficult (Bakar, Awal, & Jalaluddin, 2011; Shah, Mahmud, Din, Yusof, & Pardi, 2011). Owing to their insufficient writing experience at school, students may find it difficult to demonstrate their problem solving in a written form and thus, they may find themselves copying their peers' answers. This may also cause a weakness to this study. However, it is worth noting that students' problem solving work will not be taken as part of the grading assessment and thus, participation of the students is strictly on voluntary basis.

It is also important to note that the sample size of this study may be another limitation to explain the significant impact of mathematical writing on students' problem solving skills and metacognition. Many researchers have addressed the challenges of determining an appropriate sample size as a representative distribution of a population (Hackshaw, 2008; Kar & Ramalingam, 2013; Schanzenbach, 2012). Hence, a sample size of thirty may only reveal the impact of mathematical writing within the scope of this study that may be not generalizable to a large segment of population. In fact, thirty participants in this study are the total number of students enrolled into the Foundation Engineering program in the year of 2014. Hence, the results of this study may not applicable to other groups of students with different learning contexts and levels of education. Further limitation may arise due to the time allocation to achieve the purpose of this study. The

effectiveness of the intervention may require longer time to evaluate the impact of mathematical writing on students' problem solving skills and metacognition.

1.7 Definition of Terms

It is important to define the keywords used in this study in order to keep the consistency and remove any ambiguity description. The followings clearly describe the terms that will be frequently used in this study.

Mathematical Writing combines words and mathematical formulae or images (Seo, 2015). The mechanism helps learners to express mathematical ideas and explanations on paper. It requires writing sentences in addition to the equations and formulas whereby learners demonstrate their mathematical ideas visibly and comprehensibly to others as well. In other words, the mechanism extends the oral conversation to a "thinking aloud on paper" process.

Mathematical writing workbook is the homework practical worksheets that demanded clear and precise description of the Polya's (2014) problem solving model**,** i.e, understanding the problem, planning, performing the plan and confirmation of the answer.

Metacognitive or metacognition is simply defined as the awareness of cognitive process (Kayashima et al., 2004). It is all about being mindful of one' own thinking and learning process when solving a mathematics problem. This study concerns the students' learning experience from one mode of thinking to the other and construction of meaningful mathematical ideas or explanation through mathematical writing as the assumption of this study is that the approach of writing to problem solving helps the students to be aware and recognize their metacognitive abilities.

Mathematical problem solving skills refers to a hierarchy of skills associate with a sequence of problem solving activities (Mcguire, 2001; Schoenfeld, 2016; Stanic &

Kilpatrick, 1989). Mathematical problem solving focuses the process of how to solve problems, rather than the product. Thus, it is a series of mathematical cognition processes that intertwine between four phases of problem solving, i.e., understanding the problem, planning, performing the plan and confirming the answer (Polya, 2014).

The approach of writing to problem solving refers to a procedure that uses writing as a responsive instrument to keep track and record students' problem solving process, right from the moment students face the problem until they finally obtain a correct answer. The problem solving process adapts Polya's (2014) problem solving model that consists of four phases, i.e., understanding the problem, planning, performing the plan and confirming the answer.

Problem solving actions are the steps of efforts and activities taken as individual proceeds from the initial state of understanding the problem to the final state of confirming the answer.

Problem task is defined as an assignment that one needs to accomplish within a time frame. The level of efforts and struggles to execute a task depends on the strength and ability of a problem solver (Schoenfeld, 1985). Thus, a task is characterized to be a problem depending on the individual's knowledge and experience (Xenofontos & Andrews, 2014; Yeo, 2007).

CHAPTER 2

LITERATURE REVIEW

2.0 Introduction

There has been a lot of debate among the experts over the years about the definitions of problem solving, but with no specific definition emerging as the accepted one especially in the mathematical field (Zawojewski, 2013). The problem solving skills that a student should demonstrate when solving a mathematical problem are vaguely outlined and sometimes seem to be overlapping. Thus, this chapter will first review the characteristics of mathematical problem solving with respect to the types of knowledge that supports the problem solving process. In section 2.2, it reviews the role of metacognition in mathematical problem solving. It also discovers the critical elements that lie within the metacognitive strategy in stimulating students' potential to think and learn. It is an undeniable fact that the formation of students' problem solving skills depends on a pedagogical approach. Over time many researchers have emphasized on metacognitive strategy as an explicit teaching model to train and encourage students to think metacognitively (Michalsky, Mevarech, & Haibi, 2009; Ozsoy &Ataman, 2009; Yang & Lee, 2013). As a matter of fact, writing inspires various metacognitive processes and the mental process of writing can be treated as a form of problem solving. Hence, the relationship between writing and problem solving is discussed in section 2.3 while section 2.4 explores the role of mathematical writing as a pedagogical approach to activate the metacognitive experiences. It is rather important to review and explore different kinds of writing activities that correspond to activating specific types of mathematical problem solving skills and this is illustrated in section 2.5. Lastly, the chapter attempts to formulate theoretical and conceptual framework of the study in section 2.6 and 2.7 respectively. The

frameworks capture the potential of mathematical writing as a learning pathway to cultivate students' metacognition and enhance their skills in mathematical problem solving.

2.1 Mathematical Problem Solving

The meaning of problem solving varies ranging from working routine exercises to doing mathematics as a challenging task for which the method of solution is not known immediately (Schoenfeld, 2016). Nevertheless, the actions of problem solving only take place when an individual is committed to complete a task but does not hold enough knowledge or experience to reach an ultimate solution (Dougherty & Fantaske, 1996). In other words, a specific task may not be a problem to a skilled problem solver who can solve the problem smoothly but may become a problem task to a novice who has limited access to the solution (Zawojewski, 2013). The theme of problem solving also involves a series of constructive procedures that suit and consolidate with the problem situation in order to achieve the best solution (Dougherty & Fantaske, 1996; Mcguire 2001; Zawojewski, 2013). Thus, problem solving in nature associates with two parts: a problem task which is generally defined with respect to the competence of a problem solver and the process of seeking the path of a solution or a set of solutions (Zawojewski, 2013).

Krulik and Rudnik (1989) described problem solving as "*the means by which an individual uses previously acquired knowledge, skills and understanding to satisfy the demands of an unfamiliar situation. The student must synthesize what he or she has learned, and apply it to a new and different situation.*" (p. 5). In other words, problem solving is a complex cognitive process in which the thinking of a problem solver begins with the awareness of an unfamiliar situation (Dostal, 2015; Rahman & Ahmar, 2016). The phrase implies that one must have some specific background or domain knowledge

of the problem before solving it. He or she must be able to integrate the past and newly learned knowledge and skills from different areas of mathematics, and apply to the new problem. The inadequate knowledge and learned skills will affect the problem solving performance. Hence, a problem solver always find himself or herself engages in cognitive processing to understand and resolve problem situation where a method of solution is not known immediately.

Problem solving itself is abstract that cannot be physically seen. Many experts have developed models to show the interactive cycling process in problem solving and describe knowledge and skills required at each process of problem solving (Tambychik, Meerah, & Aziz, 2010). These models produce cognitive activities in phases that guide students from the minute they face the problem until they finally obtain a correct answer. Mayer (1982) described mathematical problem solving in two phases, namely problem representation and problem solution. He emphasized that the problem representation is built upon a problem solver's understanding of the problem situation that influences the solution procedures. Mayer (2002) stressed that a problem solver needs to be equipped with four intertwined cognitive skills, i.e., translating, integrating, planning and executing in the mathematical problem solving and each skill depends on respective knowledge to perform. For example, when a student reads a problem, he or she requires the translating skills to interpret words, sentences, ideas, or terms into cognitive expressions that make use of symbolic, verbal or pictorial manners. In other words, students need an enormous resource of semantic and linguistic knowledge to complete the process of translating. Semantic knowledge refers to knowledge of facts about objects and events while linguistic knowledge refers to knowledge of the language that is used to present the problem. Thus, a strong retention of the previous mathematical knowledge is the core of problem solving

process that strengthen one problem solving ability to apply the skills learned in one situation to a different but similar situation (Carson, 2007). In fact, most researchers have concluded that knowledge of context is the most critical feature of skill in problem solving (Kirkley, 2003).

2.1.1 Types of Knowledge to Support Mathematical Problem Solving

The knowledge that is needed to solve a problem is multiple and can be characterized into a variety of features and qualities (De Jong & Ferguson-Hessler, 1996). It is composed of many principles, examples, technical details, generalizations, heuristics and other pieces of relevant information (Solaz-Portoles & Lapez, 2008). Many attempts have been made to give a systematic description of knowledge. Some attempt to describe the knowledge development through the cognitive perspective whereas others express knowledge as a range of understanding and information that serve as a fundamental of the instructional design framework. Nevertheless, some characterize knowledge depending on its functions that satisfy the nature of the problem task. This means that task performance forms the basis for the identification of relevant aspect of knowledge (Gott, 1989).

Mathematical problem is normally classified into well-structured problem and ill-structured problem or sometimes routine and non-routine problem. Well-structured problem normally produces a convergent answer that requires a finite number of information or constrained knowledge based on the contents covered in mathematics textbook (Simon, 1978). Such problem frequently involves routine application to reach a final solution (Schoenfeld, 2016). Ill-structured problem, on the other hand, involves descriptions and goals which are vaguely defined that they require extensive knowledge and experience in the problem solving process. It may possess multiple valid solutions or

no solution at all that requires the solver's personal opinions and reasoning on the problem solving strategies (Hong, 1998).

The primary knowledge needed to solve a well-structured problem is domain-specific knowledge (Glaser, 1989) which includes declarative knowledge and procedural knowledge. Declarative knowledge, also known as conceptual knowledge is the "know what" context that includes the basic concepts, facts and principles within a certain domain (Kirkley, 2003; Solaz-Portoles & Lapez, 2008). Procedural knowledge plays the role of "knowing how" that contains the actions, rules or procedure for solving a mathematical problem. It supports the process of executing the problem solving operations (Dougherty & Fantaske, 1996). However, Mayer (2002) explained the difference between a procedure and a strategy. According to him, a procedure involves routine actions that achieve a specific solution whereas a strategy is a dynamic approach to a problem that depends on the circumstances and sometimes does not guarantee a solution.

In solving a well-structured problem, learners may possess the ability to classify groups of problems into categories in terms of patterns and structures, and choose the appropriate method that leads to a solution (Birney, Fogarty, & Plank, 2005; Steele, 2005). This is known as schematic knowledge that contains declarative knowledge and procedural knowledge where learners can solve the problem immediately due to the recognition of similar problem-solving situation (Steele, 2005). The repetition of related problems helps learners to construct a schematic network that may apply to new related problem situations (Piaget 1925, cited in Steele 2005).

The effective use of skills in solving well-structured problems extends one's capabilities in solving ill-structured problems (Schloeglmann, 2004). The ill-structured problem still requires the domain-specific knowledge that assists a solver in the selection

of procedures and additional information in finding a solution path (Roberts, 1991). A well-developed domain specific knowledge is a key factor in solving the ill-structured problem. The substantial content knowledge in particular domain upgrades one's problem solving ability to reason and explain about a problem (De Jong & Ferguson-Hessler, 1996). However, the domains-specific knowledge alone is insufficient to accommodate an ill-structured problem. It must be integrated appropriately to fit into the problem situation and this requires structural knowledge or conditional knowledge that engages the integration of declarative and procedural knowledge into useful knowledge structures. It is the type of knowledge which captures the important domain concepts and shows the interrelation between the concepts. In addition to this, learners should also be equipped with the strategic knowledge when solving an ill-structured problem. Strategic knowledge helps the learner to develop a course of action plans for solving problems and determine whether the course of problem solving process is working and modify it if it is not working (Mayer 2002; Solaz-Portoles & Lapez, 2008).

Thus, the knowledge needed to solve different types of problems varies considerably. Lee (2002) found that the success in problem solving performance depends on two important features, i.e., retrieval and mathematical knowledge. According to her findings, a successful problems solver has the ability to retrieve one's knowledge and connect it to a new situation. With this ability, a successful solver generates many ideas that may adapt to the problem situation and the process is repeated until an ultimate solution is obtained. In other words, a lack of information, or the content knowledge constitutes a major barrier in mathematical problem-solving.