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Simultaneous Transmission and Reflection Beamforming Design for RIS-Aided MU-MISO

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***Abstract*—In this paper, we study the beamforming design for the simultaneous transmission and reflection reconfigurable intelligent surface (STAR-RIS) assisted MU-MISO system, where the energy splitting (ES) mode is adopted for STAR-RIS. We aim to jointly design the beamforming strategy at the BS and the transmission-reflection coefficients (TRCs) at the STAR-RIS to minimize the total downlink transmit power of the base station (BS) subject to each user's SINR constraint. The formulated optimization problem is difficult to solve directly, and therefore we employ the iterative alternating optimization (AO) framework to obtain suboptimal solutions with promising performance. Specifically, we propose an AO-based solution to optimize each amplitude and phase of the TRCs at the STAR-RIS separately in a sequential manner, where two distinct approaches are introduced for the design of the amplitudes of STAR-RIS elements. Simulation results show that the proposed joint beamforming design at BS and passive design at STAR-RIS achieves a promising performance, and requires fewer iterations compared with the state-of-the-art.**

***Index Terms*—Reconfigurable Intelligent Surface (RIS), STAR-RIS, MU-MISO, beamforming, optimization.**

I. Introduction

In recent years, the concept of reconfigurable intelligent surface (RIS) and Holographic Multiple-Input Multiple-Output (HMIMO) have arouse worldwide interests from both industry and academia with the capability to reconfigure the wireless environment to a better state [1], [2]. Due to the unique benefits such as passive reflection of wireless signals, full-duplex operating mode, low cost, flexible deployments, etc., RIS is seen as a promising technique for the next generation of wireless communication systems [3]. The spatial degrees of freedom of the wireless communication systems can be enhanced by optimizing the orientation and location of RIS in the network [4], whose performance gains can be further improved by exploiting the deep reinforcement learning [5], [6]. The fundamental channel estimation problems have also been considered in RIS-assisted wireless scenarios, where the channels of the receiver-to-RIS and RIS-to-transmitter can be separately estimated [7] [8]. Meanwhile, channel estimation problems for both uplink and downlink in RIS-assisted wireless MISO systems are studied in [9].

Nevertheless, current works on RIS mainly focus on the reflecting-only case, where RIS can only reflect the incident signals. To overcome this shortcoming, a novel concept termed simultaneous transmission and reflection RIS (STAR-RIS) has recently been proposed in [10], [11]. Compared with conventional RISs, STAR-RIS is a new type of RIS which can transmit and reflect the signal simultaneously. Similar to conventional RISs, the transmission-reflection coefficients (TRCs) for STAR-RIS can be optimized independently via altering the electronic and magnetic parameters [10]. In [11], three operating modes for STAR-RISs are introduced: energy splitting (ES), mode switching (MS) and time switching (TS), where the advantages and shortcomings of each mode and their potential applications are discussed. [12] proposes to deploy STAR-RIS in non-orthogonal multiple access (NOMA) and orthogonal multiple access (OMA) communication systems, where STAR-RIS is employed to enhance the fundamental coverage range of the proposed communication networks of NOMA and OMA. [13] proposes a STAR-RIS assisted secrecy scheme to enhance the reliability of the MISO network, i.e. the secrecy achievable sum rate. [14] proposes general penalty-based semidefinite relaxation (SDR) approximation solutions to the power minimization problem for the three operation modes of STAR-RIS, where a multi-user MISO communication system is considered. Nevertheless, it should be mentioned that the above works have only considered the optimization on the phases of the STAR-RIS elements, while ignoring the impact of the amplitudes, whose effect is also crucial to the system performance.

In this paper, we investigate the beamforming design for the STAR-RIS-assisted MU-MISO system, where we allow the optimization on the amplitude of the TRCs as well. The ES model of STAR-RIS is adopted, where the energy of incident signal is split for users located in the transmission side and reflection side of STAR-RIS, respectively. We aim to jointly design the beamforming strategy at the BS and the transmission-reflection coefficients (TRC) at the STAR-RIS to minimize the total transmit power of the BS. Since the optimization on the amplitude and phase of TRCs is independent, we propose an alternating optimization (AO) design to update the phase and amplitude iteratively. We first design a beamforming scheme where the amplitudes of each

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STAR-RIS element can be updated independently. In order to further simplify the design procedure, we then assume that all the elements of STAR-RIS share a same amplitude value in transmission (T) mode or reflection (R) mode, based on which a feasible solution of the amplitudes of T-mode and R-mode is derived. Numerical results validate that the beamforming design with independent amplitude value of each STAR-RIS element achieves the best performance, while both proposed beamforming approaches outperform existing schemes that only optimize the phases of the STAR-RIS elements.

Notations: In this paper, a, $\mathbf{a}$ and $\mathbf{A}$ denote scalars, vectors and matrices, respectively. $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\mathrm{H}}$ denotes transpose and conjugate transpose, respectively. $|\cdot|$ denotes the absolute value of a real number or the modulus of a complex number, and $\|\cdot\|_2$ denotes ℓ_2 norm. $diag(\cdot)$ transforms a column vector into a diagonal matrix. $\mathrm{vec}(\cdot)$ extracts the diagonal elements of a square matrix and transforms them into a column vector. $\mathbf{v}(k)$ and $\mathbf{A}(k,m)$ denote the entry in the corresponding vector or matrix. $tr(\cdot)$ denotes the trace of a square matrix. $\mathbf{S} \succeq \mathbf{0}$ denotes matrix $\mathbf{S}$ is positive semidefinite. $\Re\{\cdot\}$ and $\Im\{\cdot\}$ respectively denote the real and imaginary part of a complex scalar, vector or matrix. $\mathbf{1}$ denotes a column vector with all entries being 1.

II. System Model and Problem Formulation

A. System Model

As shown in Fig.1, we consider a STAR-RIS-aided MU-MISO communication system in the downlink, where the BS equipped with N_T transmit antennas serves K users located at either the transmission side (T) or the reflection side (R) of STAR-RIS, respectively, where the STAR-RIS consists of M reconfigurable elements. We focus on the ES mode of STAR-RIS, where the power of the incident signal is split into both transmission side and reflection side simultaneously. The baseband equivalent channel from BS to user k, BS to STAR-RIS, STAR-RIS to user k is denoted by $\mathbf{h}_{d,k} \in \mathbb{C}^{N_t \times 1}$, $\mathbf{H}_t \in \mathbb{C}^{M \times N_t}$, and $\mathbf{h}_{r,k} \in \mathbb{C}^{M \times 1}$, respectively, where $k \in \mathcal{K} = \{1, 2, \cdots, K_0, K_0+1, \cdots, K\}$, $\mathcal{T} = \{1, 2, \cdots, K_0\} \subseteq \mathcal{K}$ and $\mathcal{R} = \{K_0+1, \cdots, K\} \subseteq \mathcal{K}$ collect K_T and K_R users at the T region and R region, respectively, where $K_T = K_0$ and $K_R = K - K_0$. $\mathbf{\Phi}_\chi = diag\left(\left[\beta_{\chi,1}e^{j\theta_{\chi,1}}, \beta_{\chi,2}e^{j\theta_{\chi,2}}, \cdots, \beta_{\chi,M}e^{j\theta_{\chi,M}}\right]\right)^{\mathrm{T}}$ denotes the TRC matrices, where $\chi \in \{T, R\}$. We introduce $\boldsymbol{\beta}_\chi = [\beta_{\chi,1}, \beta_{\chi,2}, \cdots, \beta_{\chi,M}]^{\mathrm{T}}$, and $\mathbf{\Theta}_\chi = \left[e^{j\theta_{\chi,1}}, e^{j\theta_{\chi,2}}, \cdots, e^{j\theta_{\chi,M}}\right]^{\mathrm{T}}$ to represent the amplitude and phase vectors of STAR-RIS in T and R region, respectively. Then, the received signal at user k in the T or R region is given by

$$y_k = \left(\mathbf{h}_{r,k}^{\mathrm{T}}\mathbf{\Phi}_\chi\mathbf{H}_t + \mathbf{h}_{d,k}^{\mathrm{T}}\right)\mathbf{w}_k s_k + \sum_{j\in\mathcal{K}, j\neq k}\left(\mathbf{h}_{r,k}^{\mathrm{T}}\mathbf{\Phi}_\chi\mathbf{H}_t + \mathbf{h}_{d,k}^{\mathrm{T}}\right)\mathbf{w}_j s_j + n_k, k \in \mathcal{K}, \quad (1)$$

where $\mathbf{w}_k \in \mathbb{C}^{N_t \times 1}$ denotes the transmit beamforming vector of BS, s_k denotes the transmit signals satisfying $\mathbb{E}\left(s_k s_k^H\right) = 1$. $n_k \sim \mathcal{CN}(0, \sigma_k^2)$ denotes the additive Gaussian white noise at user k's receiver. Without loss of generality, we express user k's equivalent channel as $\mathbf{h}_k^{\mathrm{T}} = \mathbf{h}_{r,k}^{\mathrm{T}}\mathbf{\Phi}_\chi\mathbf{H}_t + \mathbf{h}_{d,k}^{\mathrm{T}}$. Thus, the SINR of user k can be given by

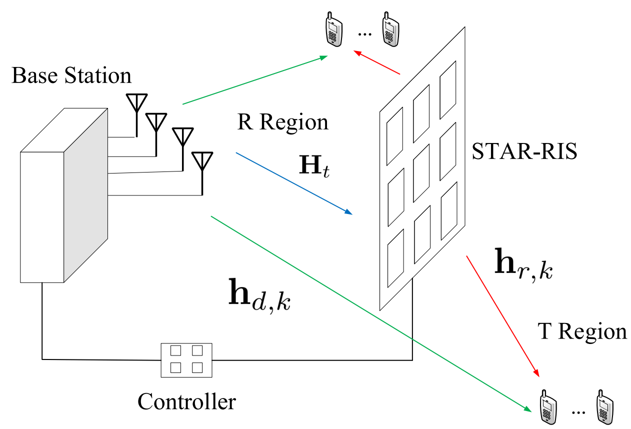

Fig. 1: A STAR-RIS aided MISO wireless network.

$$\mathrm{SINR}_k = \frac{\left|\mathbf{h}_k^{\mathrm{T}}\mathbf{w}_k\right|^2}{\sum_{j\in\mathcal{K}, j\neq k}\left|\mathbf{h}_k^{\mathrm{T}}\mathbf{w}_j\right|^2 + \sigma_k^2}, \ \forall k. \quad (2)$$

B. Problem Formulation

We aim to minimize the total power consumption of the BS by jointly optimizing the beamforming vectors $\mathbf{w}_k$ and the TRCs $\mathbf{\Phi}_\chi$ subject to the received SINR target of each user [15]. Accordingly, the optimization problem can be formulated as

$$\mathcal{P}_1: \quad \min_{\mathbf{w}_k, \mathbf{\Phi}_\chi} \sum_{k\in\mathcal{K}} \|\mathbf{w}_k\|_2^2, \quad (3a)$$

$$\text{s.t.} \quad \mathbf{C1}: \frac{\left|\mathbf{h}_k^{\mathrm{T}}\mathbf{w}_k\right|^2}{\sum_{j\in\mathcal{K}, j\neq k}\left|\mathbf{h}_k^{\mathrm{T}}\mathbf{w}_j\right|^2 + \sigma_k^2} \geq \gamma_k, \forall k \in \mathcal{K}, \quad (3b)$$

$$\mathbf{C2}: \beta_{T,i}^2 + \beta_{R,i}^2 \leq 1, i = 1, 2, \cdots, M, \quad (3c)$$

where γ_k denotes user k's SINR threshold, $\beta_{T,i}$ and $\beta_{R,i}$ denote the amplitudes of the i-th element of STAR-RIS for T region and R region, respectively. (3c) denotes the STAR-RIS's passive feature, where the STAR-RIS cannot amplify the incident signal and the conservation law of energy must be satisfied, i.e. the energy of transmission and reflection signal should be no more than that of the incident signal, i.e., $\beta_{T,i}^2 + \beta_{R,i}^2 \leq 1$.

III. Proposed Solution via Iterative Alternating Opmitization

Due to the tight coupling of $\mathbf{w}_k$ and $\mathbf{\Phi}_\chi$, problem $\mathcal{P}_1$ is hard to solve directly [15]. Thus, we propose an AO-based iterative algorithm to obtain a feasible solution, i.e., we alternately update the amplitude and phase of STAR-RIS. More specifically, we decompose the original problem $\mathcal{P}_1$ into three subproblems related to $\mathbf{w}_k$, $\mathbf{\Theta}_\chi$ and $\boldsymbol{\beta}_\chi$, which are solved in an iterative manner as shown below.

A. Optimization on $\mathbf{w}_k$

For given $\mathbf{\Phi}_\chi$, problem $\mathcal{P}_1$ reduces to an optimization problem on $\mathbf{w}_k$ only:

$$\mathcal{P}_2: \quad \min_{\mathbf{w}_k} \sum_{k\in\mathcal{K}} \|\mathbf{w}_k\|_2^2, \tag{4a}$$

$$\text{s.t.} \quad \mathbf{C1}: \quad \frac{\left|\mathbf{h}_k^{\mathrm{T}}\mathbf{w}_k\right|^2}{\sum_{j\in\mathcal{K},j\neq k}\left|\mathbf{h}_k^{\mathrm{T}}\mathbf{w}_j\right|^2+\sigma_k^2} \geq \gamma_k, \forall k\in\mathcal{K}. \tag{4b}$$

Problem $\mathcal{P}_2$ is a standard power minimization problem which can be solved by fixed-point iteration or SDR method [16]. In what follows, we focus on the optimization on $\boldsymbol{\theta}_\chi$ and $\boldsymbol{\beta}_\chi$.

B. Optimization on $\mathbf{\Theta}_\chi$

For the optimization on $\mathbf{\Phi}\chi$ for given $\mathbf{w}_k$, we first perform a decomposition of $\mathbf{\Phi}\chi$ into

$$\mathbf{\Phi}_\chi = diag\left(\boldsymbol{\beta}_\chi\right)\cdot\mathbf{\Theta}_\chi, \tag{5}$$

where $\mathbf{\Theta}_\chi = diag\left(\left[e^{j\theta_{\chi,1}}, e^{j\theta_{\chi,2}}, \cdots, e^{j\theta_{\chi,M}}\right]^T\right)$. Generally, the phases of the T region and R region in a STAR-RIS system are independent and adjustable [10]. In order to simplify the problem, we assume the T region and R region share the same phase, i.e. $\mathbf{\Theta}_T = \mathbf{\Theta}_R = \mathbf{\Theta}$.

For the optimization on $\mathbf{\Theta}$ with a given $\boldsymbol{\beta}_\chi$, we employ the SDR method in [15]. Let

$$\mathbf{a}_{k,n} = diag\left(\boldsymbol{\beta}_T\right)\cdot diag\left(\mathbf{h}_{r,k}^{\mathrm{T}}\right)\mathbf{H}_t\mathbf{w}_n, k\in\mathcal{T}, \forall n\in\mathcal{K}, \tag{6a}$$

$$\mathbf{a}_{k,n} = diag\left(\boldsymbol{\beta}_R\right)\cdot diag\left(\mathbf{h}_{r,k}^{\mathrm{T}}\right)\mathbf{H}_t\mathbf{w}_n, k\in\mathcal{R}, \forall n\in\mathcal{K}, \tag{6b}$$

$$b_{k,n} = \mathbf{h}_{d,k}^{\mathrm{T}}\mathbf{w}_n, \forall k,n\in\mathcal{K}. \tag{6c}$$

Moreover, by introducing

$$\mathbf{v} = \left[e^{j\theta_1}, e^{j\theta_2}, \cdots, e^{j\theta_M}\right]^{\mathrm{T}}, \tag{7}$$

and

$$\mathbf{v}(i) = e^{j\theta_i}, i = 1, 2, \cdots, M, \tag{8}$$

such that $\mathbf{h}_{r,k}^{\mathrm{T}}\mathbf{\Phi}_\chi\mathbf{H}_t\mathbf{w}_n = \mathbf{v}^{\mathrm{T}}\mathbf{a}_{k,n}$, the optimization on $\mathbf{\Theta}$ is equivalent to finding $\mathbf{v}$:

$$\mathcal{P}_3: \quad Find \quad \mathbf{v}, \tag{9a}$$

$$\text{s.t.} \quad \mathbf{C1}: \quad \frac{\left|\mathbf{v}^{\mathrm{T}}\mathbf{a}_{k,n}+b_{k,k}\right|^2}{\sum\limits_{n\in\mathcal{K},n\neq k}\left|\mathbf{v}^{\mathrm{T}}\mathbf{a}_{k,n}+b_{k,n}\right|^2+\sigma_k^2} \geq \gamma_k, \forall k\in\mathcal{K}, \tag{9b}$$

$$\mathbf{C2}: \quad |\mathbf{v}(i)| = 1, i = 1, 2, \cdots, M. \tag{9c}$$

Note that problem $\mathcal{P}_3$ is nonconvex due to the constant-modulus constraint (10c). Considering that the constraint (10b) can be transformed into a quadratic form, we then employ the SDR technique to solve problem $\mathcal{P}_3$ efficiently. To proceed, we introduce an auxiliary variable t and form a new vector $\overline{\mathbf{v}} = \left[\mathbf{v}^{\mathrm{T}} \quad \mathrm{t}\right]^{\mathrm{T}}$. Thus, we can transform problem $\mathcal{P}_3$ into a semidefinite programming (SDP) form [15]:

$$\mathcal{P}_4: \quad \max_{\mathbf{V},\alpha_k} \quad \sum_{k\in\mathcal{K}} \alpha_k, \tag{10a}$$

$$\begin{aligned} \text{s.t.} \quad \mathbf{C1}: \quad & tr\left(\mathbf{R}_{k,k}\mathbf{V}\right)+|b_{k,k}|^2 \geq \\ & \gamma_k \sum_{n\in\mathcal{K},k\neq n}\left(tr\left(\mathbf{R}_{k,n}\mathbf{V}\right)+|b_{k,n}|^2\right)+\gamma_k\sigma_k^2+\alpha_k, \\ & k\in\mathcal{K}, \end{aligned} \tag{10b}$$

$$\mathbf{C2}: \quad |\mathbf{V}(i,i)| = 1, i = 1, 2, \cdots, M+1, \tag{10c}$$

$$\mathbf{C3}: \quad \text{rank}\left(\mathbf{V}\right) = 1, \tag{10d}$$

where

$$\mathbf{R}_{k,n} = \begin{bmatrix} \mathbf{a}_{k,n}\mathbf{a}_{k,n}^{\mathrm{T}} & \mathbf{a}_{k,n}^{\mathrm{T}} \\ \mathbf{a}_{k,n} & 0 \end{bmatrix}, \tag{11}$$

$\mathbf{V} = \overline{\mathbf{v}}\overline{\mathbf{v}}^{\mathrm{H}}$ and α_k, $k\in\mathcal{K}$ denote nonnegative auxiliary variables introduced into the users' SINR constraints to maximize each user's achievable SINR as well as to obtain a better solution of the total transmit power.

Notice that if we drop the rank-1 constraint, problem $\mathcal{P}_4$ becomes a convex problem that can readily be solved [17]. Subsequently, Gaussian randomization approach is employed to obtain an approximate solution to problem $\mathcal{P}_4$ [15]. Finally, we extract $\mathbf{v}$ from $\overline{\mathbf{v}}$ and obtain $\mathbf{\Theta} = diag(\mathbf{v})$.

C. Optimization on $\boldsymbol{\beta}_\chi$

In this subsection, we focus on optimizing the amplitude $\boldsymbol{\beta}_\chi$ based on given $\mathbf{w}_k$ and $\mathbf{\Theta}$. Since the total energy of the incident signal is split into both T region and R region, $\boldsymbol{\beta}_T$ and $\boldsymbol{\beta}_R$ should satisfy $\beta_{T,i}^2+\beta_{R,i}^2 = 1$, $i\in 1, 2, \cdots, M$, where $\beta_{\chi,i} = \boldsymbol{\beta}_\chi(i)$. Accordingly, the SINR constraint of the user k in T region for given $\mathbf{w}_k$ and $\mathbf{\Theta}$ can be simplified as

$$\frac{\left|\mathbf{h}_{r,k}^{\mathrm{T}}diag\left(\boldsymbol{\beta}_T\right)\mathbf{\Theta}\mathbf{H}_t\mathbf{w}_k+\mathbf{h}_{d,k}^{\mathrm{T}}\mathbf{w}_k\right|^2}{\sum\limits_{n\in\mathcal{K},k\neq n}\left|\mathbf{h}_{r,k}^{\mathrm{T}}diag\left(\boldsymbol{\beta}_T\right)\mathbf{\Theta}\mathbf{H}_t\mathbf{w}_n+\mathbf{h}_{d,k}^{\mathrm{T}}\mathbf{w}_n\right|^2+\sigma_k^2} \geq \gamma_k, k\in\mathcal{T}, \tag{12}$$

and the SINR constraints of users in R region can be similarly constructed. Thus, the optimization on $\boldsymbol{\beta}_T$ and $\boldsymbol{\beta}_R$ is given by:

$$\mathcal{P}_5: \quad Find \quad \boldsymbol{\beta}_T, \boldsymbol{\beta}_R, \tag{13a}$$

$$\begin{aligned} \text{s.t.} \quad \mathbf{C1}: \; & \frac{\left|\mathbf{h}_{r,k}^{\mathrm{T}}diag\left(\boldsymbol{\beta}_T\right)\mathbf{\Theta}\mathbf{H}_t\mathbf{w}_k+\mathbf{h}_{d,k}^{\mathrm{T}}\mathbf{w}_k\right|^2}{\sum\limits_{n\in\mathcal{K},k\neq n}\left|\mathbf{h}_{r,k}^{\mathrm{T}}diag\left(\boldsymbol{\beta}_T\right)\mathbf{\Theta}\mathbf{H}_t\mathbf{w}_n+\mathbf{h}_{d,k}^{\mathrm{T}}\mathbf{w}_n\right|^2+\sigma_k^2} \\ & \geq \gamma_k, k\in\mathcal{T}, \end{aligned} \tag{13b}$$

$$\begin{aligned} \mathbf{C2}: \; & \frac{\left|\mathbf{h}_{r,k}^{\mathrm{T}}diag\left(\boldsymbol{\beta}_R\right)\mathbf{\Theta}\mathbf{H}_t\mathbf{w}_k+\mathbf{h}_{d,k}^{\mathrm{T}}\mathbf{w}_k\right|^2}{\sum\limits_{n\in\mathcal{K},k\neq n}\left|\mathbf{h}_{r,k}^{\mathrm{T}}diag\left(\boldsymbol{\beta}_R\right)\mathbf{\Theta}\mathbf{H}_t\mathbf{w}_n+\mathbf{h}_{d,k}^{\mathrm{T}}\mathbf{w}_n\right|^2+\sigma_k^2} \\ & \geq \gamma_k, k\in\mathcal{R}, \end{aligned} \tag{13c}$$

$$\mathbf{C3}: \quad \text{vec}\left(\boldsymbol{\beta}_T\boldsymbol{\beta}_T^{\mathrm{T}}+\boldsymbol{\beta}_R\boldsymbol{\beta}_R^{\mathrm{T}}\right) = \mathbf{1}, \tag{13d}$$

Problem $\mathcal{P}_5$ is a non-convex quadratically-constrained quadratic programming (QCQP) problem due to the non-convex constraint (14b) and (14c). In the remaining part of

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this subsection, we derive two schemes to obtain the solution to $\mathcal{P}_5$.

1) Independent Optimization on Each $\beta_{\chi,m}$: We first study the scheme where each $\beta_{\chi,m}$ can be optimized independently. Note that the constraints (13b) and (13c) in $\mathcal{P}_5$ can be transformed as well. We let

$$\hat{\mathbf{a}}_{k,n} = diag\left(\mathbf{h}_{r,k}^{\mathrm{T}}\right)\mathbf{\Theta}\mathbf{H}_t\mathbf{w}_n, \tag{14}$$

$$\hat{\mathbf{R}}_{k,n} = \begin{bmatrix} \hat{\mathbf{a}}_{k,n}\hat{\mathbf{a}}_{k,n}^{\mathrm{T}} & \hat{\mathbf{a}}_{k,n}^{\mathrm{T}} \\ \hat{\mathbf{a}}_{k,n} & 0 \end{bmatrix}, \tag{15}$$

and

$$\hat{\boldsymbol{\beta}}_T = \left[\boldsymbol{\beta}_T^{\mathrm{T}}, t_T\right]^{\mathrm{T}}, \tag{16a}$$
$$\hat{\boldsymbol{\beta}}_R = \left[\boldsymbol{\beta}_R^{\mathrm{T}}, t_R\right]^{\mathrm{T}}, \tag{16b}$$

where t_T and t_R are auxiliary variables introduced to form new amplitude vectors. Recalling $b_{k,n} = \mathbf{h}_{d,k}^{\mathrm{T}}\mathbf{w}_n$ in (6c), then we can transform problem $\mathcal{P}_5$ into an SDP form similar to the transformation of $\mathcal{P}_3$:

$$\mathcal{P}_6: \quad \max_{\hat{\boldsymbol{B}}_T,\hat{\boldsymbol{B}}_R,\hat{\alpha}_k} \quad \sum_{k\in\mathcal{K}}\hat{\alpha}_k, \tag{17a}$$

$$\text{s.t. } \mathbf{C1}: \; tr\left(\hat{\mathbf{R}}_{k,k}\hat{\boldsymbol{B}}_T\right) + |b_{k,k}|^2 \geq \gamma_k \sum_{n\in\mathcal{K},k\neq n}\left[tr\left(\mathbf{R}_{k,n}\hat{\boldsymbol{B}}_T\right) + |b_{k,n}|^2\right] + \gamma_k\sigma_k^2 + \hat{\alpha}_k, \; k\in\mathcal{T}, \tag{17b}$$

$$\mathbf{C2}: \; tr\left(\hat{\mathbf{R}}_{k,k}\hat{\boldsymbol{B}}_R\right) + |b_{k,k}|^2 \geq \gamma_k \sum_{n\in\mathcal{K},k\neq n}\left[tr\left(\mathbf{R}_{k,n}\hat{\boldsymbol{B}}_R\right) + |b_{k,n}|^2\right] + \gamma_k\sigma_k^2 + \hat{\alpha}_k, \; k\in\mathcal{R}, \tag{17c}$$

$$\mathbf{C3}: \; \mathrm{vec}\left(\hat{\boldsymbol{B}}_T + \hat{\boldsymbol{B}}_R\right) = \mathbf{1}, \tag{17d}$$

$$\mathbf{C4}: \; \mathrm{rank}\left(\hat{\mathbf{B}}_T\right) = 1, \mathrm{rank}\left(\hat{\mathbf{B}}_R\right) = 1, \tag{17e}$$

where $\hat{\mathbf{B}}_\chi = \hat{\boldsymbol{\beta}}_\chi\hat{\boldsymbol{\beta}}_\chi^T$. Notice that if we drop the rank-1 constraint, problem $\mathcal{P}_6$ becomes a convex problem that can readily be solved [17]. Then, we derive feasible $\boldsymbol{\beta}_\chi$ from the solution to $\mathcal{P}_6$, i.e., $\hat{\boldsymbol{B}}_\chi$ by extracting the square roots of the first M diagonal entries in $\hat{\boldsymbol{B}}_\chi$:

$$\boldsymbol{\beta}_\chi = \left[\sqrt{\hat{\boldsymbol{B}}_\chi(1,1)}, \sqrt{\hat{\boldsymbol{B}}_\chi(2,2)}\cdots, \sqrt{\hat{\boldsymbol{B}}_\chi(M,M)}\right]^{\mathrm{T}}. \tag{18}$$

2) A Low-Complexity Solution: The independent optimization on the amplitudes of each STAR-RIS elements increases the computational complexity of the proposed beamforming design dramatically. To obtain a low-complexity solution, we propose a beamforming approach where each element of STAR-RIS shares the same β_T and β_R values. Accordingly, the amplitude vectors $\boldsymbol{\beta}_\chi$ can be simplified into two scalars, i.e. β_T and β_R, respectively, and the SINR constraint of the user k in T region for given $\mathbf{w}_k$ and $\mathbf{\Theta}$ can be simplified as

$$\frac{\left|\beta_T\mathbf{h}_{r,k}^{\mathrm{T}}\mathbf{\Theta}\mathbf{H}_t\mathbf{w}_k + \mathbf{h}_{d,k}^{\mathrm{T}}\mathbf{w}_k\right|^2}{\sum\limits_{n\in\mathcal{K},k\neq n}\left|\beta_T\mathbf{h}_{r,k}^{\mathrm{T}}\mathbf{\Theta}\mathbf{H}_t\mathbf{w}_n + \mathbf{h}_{d,k}^{\mathrm{T}}\mathbf{w}_n\right|^2 + \sigma_k^2} \geq \gamma_k, k\in\mathcal{T}. \tag{19}$$

The SINR constraints of users in R region can be similarly constructed, which is omitted for brevity. For convenience of subsequent derivations, we further introduce

$$c_{k,n} = \mathbf{h}_{r,k}^T\mathbf{\Theta}\mathbf{H}_t\mathbf{w}_n. \tag{20}$$

Recalling $b_{k,n} = \mathbf{h}_{d,k}^T\mathbf{w}_n$ in (6c), we then formulate the optimization on β_χ, given by:

$$\mathcal{P}_7: \quad Find \quad \beta_T, \beta_R, \tag{21a}$$

$$\text{s.t. } \mathbf{C1}: \; |\beta_T c_{k,k} + b_{k,k}|^2 \geq \sum_{n\in\mathcal{K},k\neq n}\gamma_k|\beta_T c_{k,n} + b_{k,n}|^2 + \gamma_k\sigma_k^2, k\in\mathcal{T}, \tag{21b}$$

$$\mathbf{C2}: \; |\beta_R c_{k,k} + b_{k,k}|^2 \geq \sum_{n\in\mathcal{K},k\neq n}\gamma_k|\beta_R c_{k,n} + b_{k,n}|^2 + \gamma_k\sigma_k^2, k\in\mathcal{R}, \tag{21c}$$

$$\mathbf{C3}: \; \beta_T^2 + \beta_R^2 \leq 1, \tag{21d}$$

$$\mathbf{C4}: \; \beta_T, \beta_R \in [0,1]. \tag{21e}$$

Thus, the optimization on $\boldsymbol{\beta}_\chi$ is equivalent to finding two scalars β_T and β_R. Note that $\mathcal{P}_7$ is a QCQP problem as well, which enables us to transform problem $\mathcal{P}_7$ into an SDP form. However, more than two auxiliary variables are required, which would further increase the computational complexity. To obtain a low-cost solution, we expand the square terms in (22b) and (22c) and then form K quadratic functions on β_T and β_R, respectively. Thus, problem $\mathcal{P}_7$ can be solved by finding appropriate β_T and β_R which satisfies constraint (22b) and (22c), respectively. To be more specific, we introduce

$$m_{2,k} = \left(|c_{k,k}|^2 - \gamma_k|c_{k,n}|^2\right), \tag{22}$$

$$m_{1,k} = 2\left[\left(\Re\{c_{k,k}\}\Re\{b_{k,k}\} + \Im\{c_{k,k}\}\Im\{b_{k,k}\}\right) - \gamma_k\left(\Re\{c_{k,n}\}\Re\{b_{k,n}\} + \Im\{c_{k,n}\}\Im\{b_{k,n}\}\right)\right], \tag{23}$$

$$m_{0,k} = |b_{k,k}|^2 - \gamma_k\left(|b_{k,n}|^2 + \sigma_k^2\right), \tag{24}$$

where $m_{2,k}$, $m_{1,k}$, $m_{0,k}$ denote the coefficient for the quadratic term, one-order term and constant term in the k-th expression, respectively. Thus, constraints (22b) and (22c) can be expressed as

$$m_{2,k}\beta_T^2 + m_{1,k}\beta_T + m_{0,k} \geq 0, k\in\mathcal{T}, \tag{25a}$$

$$m_{2,k}\beta_R^2 + m_{1,k}\beta_R + m_{0,k} \geq 0, k\in\mathcal{R}. \tag{25b}$$

Note that (26a) and (26b) are quadratic inequalities with respect to β_T and β_R, respectively. We then solve K_T inequalities in (26b) and obtain K_T regions satisfying constraint (26b). Due to the feature of quadratic function, the feasible region of

β_T depends on the value of the coefficient for quadratic term, i.e.

$$[\beta_{T,k-}, \beta_{T,k+}], \text{if} \quad m_{2,k} < 0, \tag{26a}$$

$$(-\infty, \beta_{T,k-}] \cup [\beta_{T,k+}, \infty), \text{if} \quad m_{2,k} > 0, \tag{26b}$$

where $\beta_{T,k-}$ and $\beta_{T,k+}$ denote the values of β_T in the k-th inequality when strict equality is achieved. We compare region $[0,1]$ with K_T regions obtained from (27a) and then obtain the overlapping region $[\beta_{T,-}, \beta_{T,+}]$ from the above (K_T+1) regions, where $\beta_{T,-}$ and $\beta_{T,+}$ denote the overlapping region's endpoint, respectively. Similarly, we solve the K_R inequalities in (27b) and obtain a feasible region $[\beta_{R,-}, \beta_{R,+}]$ similarly to the procedure above. We then take the value of the overlapping regions' left endpoints as the feasible solutions to β_T and β_R, i.e., $\beta_T = \beta_{T,-}$ and $\beta_R = \beta_{R,-}$.

Finally, we verify whether the value of $\beta_T^2 + \beta_R^2$, β_T and β_R safisfy constraint (22e), and implement a normalization procedure on the amplitude if $\beta_T^2 + \beta_R^2 \le 1$. Conversely, if $\beta_T^2 + \beta_R^2 > 1$, problem $\mathcal{P}_7$ becomes infeasible, and we take the value of β_T and β_R in the previous iteration as a feasible solution. If β_T and β_R have no feasible solutions in the first iteration, we take the initial $\mathbf{w}_k$ calculated by the initial $\boldsymbol{\theta}_\chi$ and $\boldsymbol{\beta}_\chi$, which is used to calculate the required transmit power.

The overall beamforming procedure including the above 3 steps is summarized in Algorithm 1 below.

Algorithm 1 Alternating Optimization for Solving $\mathcal{P}_1$

Input: $\mathbf{h}_{d,k}, \mathbf{h}_{r,k}, \mathbf{H}_t, \sigma_k^2, \gamma_k$.

Initialization: $\boldsymbol{\theta}^1$, $\boldsymbol{\beta}_T^1 = \boldsymbol{\beta}_R^1 = \frac{1}{\sqrt{2}} \cdot \mathbf{1}$ and the number of iteration $n = 1$.

Repeat:

1.Calculate the equivalent channel $\mathbf{h}_k$ with given $\mathbf{h}_{d,k}, \mathbf{h}_{r,k}, \mathbf{H}_t, \boldsymbol{\theta}^n, \boldsymbol{\beta}^n$.

2.Solve $\mathcal{P}_2$ to obtain $\mathbf{w}_k^n$.

3.Solve $\mathcal{P}_4$ to obtain $\boldsymbol{\theta}^{n+1}$.

4.Solve $\mathcal{P}_5$ or $\mathcal{P}_7$ to update amplitude vectors.

If: $\left(\hat{\beta}_T^{n+1}\right)^2 + \left(\hat{\beta}_R^{n+1}\right)^2 \le 1$

$\beta_T^{n+1} = \frac{\hat{\beta}_T^{n+1}}{\sqrt{\left(\hat{\beta}_T^{n+1}\right)^2 + \left(\hat{\beta}_R^{n+1}\right)^2}}$,

$\beta_R^{n+1} = \frac{\hat{\beta}_R^{n+1}}{\sqrt{\left(\hat{\beta}_T^{n+1}\right)^2 + \left(\hat{\beta}_R^{n+1}\right)^2}}$, $\boldsymbol{\beta}_T^{n+1} = \beta_T^{n+1} \cdot \mathbf{1}$,

and $\boldsymbol{\beta}_R^{n+1} = \beta_R^{n+1} \cdot \mathbf{1}$.

Else: $\boldsymbol{\beta}_T^{n+1} = \boldsymbol{\beta}_T^n$ and $\boldsymbol{\beta}_R^{n+1} = \boldsymbol{\beta}_R^n$.

5. Update $n = n + 1$.

Until: The fractional decrease of the objective value is lower than a threshold $\varepsilon > 0$ or a maximum iteration number is achieved.

D. Complexity Analysis

The analysis of computational complexity of Algorithm 1 is summarized as follows. According to [18], computational complexity of conventional power minimization problem $\mathcal{P}_2$ is $\mathcal{O}\left(KN_T^{3.5}\right)$. Problem $\mathcal{P}_4$ is a standard SDR problem with complexity of $\mathcal{O}\left(KM^{3.5}\right)$. Problem $\mathcal{P}_6$ is an SDP problem with complexity of $\mathcal{O}\left(KM^{3.5}\right)$. The procedure of solving $\mathcal{P}_7$ has a complexity of $\mathcal{O}\left(K\right)$. Thus, the total computational complexity of Algorithm 1 is $\mathcal{O}\left(KN_T^{3.5} + KM^{3.5} + K\right)$, while the total computational complexity of optimization scheme by solving problem $\mathcal{P}_6$ to obtain amplitude vectors $\boldsymbol{\beta}_\chi$ is $\mathcal{O}\left(KN_T^{3.5} + 2KM^{3.5}\right)$.

IV. Numerical Results

In this section, numerical results are given to illustrate the effectiveness of our proposed schemes. We consider a three-dimensional (3D) simulation setup similar to [14], where the BS and the STAR-RIS are located at $(0,0,0)$ and $(0,50,0)$, respectively. K users are all located on the circle centered at the STAR-RIS with a radius of $2\sqrt{2}$ meters, half of which are located on the T region and half on R region. The STAR-RIS is assumed to be equipped with a uniform planar array (UPA) composed of $M = M_y M_z$ elements, where $M_y = 4$ and we increase M_z linearly with M. Throughout our simulations, Rician fading is adopted, and the channel vector $\mathbf{h}_{d,k}$ is modeled as

$$\mathbf{h}_{d,k} = C_0 \left(\frac{d_{\mathbf{h}_{d,k}}}{d_0}\right)^{-\alpha} \left(\sqrt{\frac{\kappa}{\kappa+1}} \mathbf{h}_{d,k}^{\text{LoS}} + \sqrt{\frac{1}{\kappa+1}} \mathbf{h}_{d,k}^{\text{NLoS}}\right), \tag{27}$$

where C_0, $d_{\mathbf{h}_{d,k}}$, d_0 and α denote the fading of reference distance, distance of channel, reference distance and large-scale fading factor, respectively. κ denotes the Rician factor. $\mathbf{h}_{d,k}^{LoS}$, $\mathbf{h}_{d,k}^{NLoS}$ denote the line-of-sight (LoS) and non-line-of-sight (NLoS, i.e. Rayleigh fading), respectively. The other channels are similarly modeled. We set $C_0 = -30$dB, $d_0 = 1$ meter, $\alpha = 2.2$ and $\kappa = 3$dB for all channels, respectively. We set the noise of the receivers as $\sigma_k^2 = -80$dBm. We compare our proposed scheme of the STAR-RIS aided communication system with two other baseline schemes proposed in [14]: 1) STAR-RIS with uniform amplitude and optimizing the phase only. 2) conventional RIS with half of the elements serving users in the transmitting region and half serving users in the reflecting region. The proposed AO-based scheme has been proved for its convergence in [15] under the AO framework. For a fair comparison, we also compare our proposed schemes with the penalty-based SDR approximation scheme for ES model proposed in [14].

In Fig.2, we study the convergence of the total transmit power versus the number of iterations in each scheme. We fix $N_T = 8$, $M = 40$, $K = 4$, $K_0 = 2$ and SINR threshold $= 20$ dB. As can be seen, both proposed schemes ('STAR-RIS, independent/same optimized amplitude') that allow the optimization on amplitudes converge to a lower transmit power requirement, validating the effectiveness of optimizing amplitudes of STAR-RIS. Conventional RIS with merely phase optimization requires a higher power consumption because of the limited capability in affecting the wireless environment. In addition, compared with conventional reflecting-only RISs that can only serve users in the R region, STAR-RIS aided wireless communication systems can enhance the link quality

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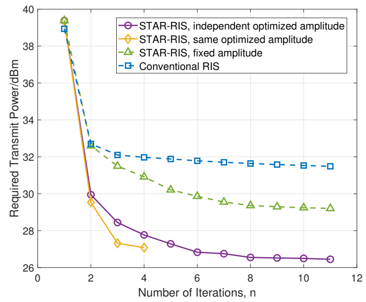

Fig. 2 Convergence, M = 40, γ_k = 20dB

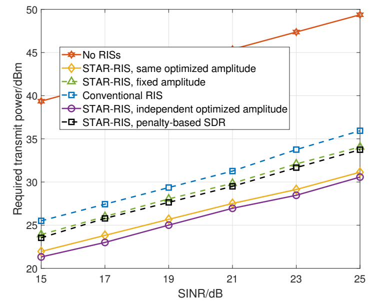

Fig. 3 Transmit power v.s. SINR threshold γ_k, M = 40

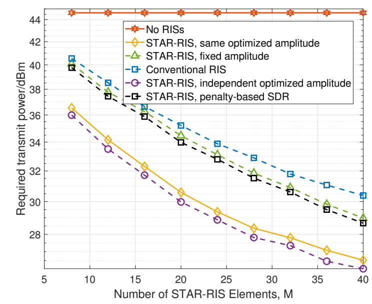

Fig. 4 Transmit power v.s. number of RIS elements M, γ_k = 20dB

Numerical results of the proposed STAR-RIS precoding, $N_T = 8$, $K = 4$

for users in both the T Region and R Region. The independent optimization on amplitudes leads to a better performance but requires more iterations than the proposed low-complexity solution.

In Fig.3, we study the total required transmit power versus the users' targeted SINR threshold. We set $N_T = 8$, $K = 4$, $K_0 = 2$ and $M = 40$. Our proposed optimization scheme obtains a desirable result compared with conventional RIS and STAR-RIS that does not optimize the amplitude ('STAR-RIS, fixed amplitude') and the penalty-based SDR scheme ('STAR-RIS, penalty-based SDR') for ES mode in [14], which implies the effectiveness of optimizing the amplitude.

In Fig.4, we study the total transmitting power versus the elements of STAR-RIS. We set $N_T = 8$, $K = 4$, $K_0 = 2$ and each user's SINR threshold = 20dB. Simulation results show that our proposed alternate optimization scheme can achieve better performance compared with two baseline schemes and the penalty-based SDR scheme for ES mode in [14] with an increasing M due to the exploitation of the amplitude.

V. Conclusion

In this paper, we propose an AO-based beamforming design to minimize the total transmit power for a STAR-RIS-aided MU-MISO communication system. The proposed AO-based beamforming iteratively updates the beamforming vectors at the BS, the phases of each STAR-RIS element, and the amplitude of each STAR-RIS element, where SDR-based solutions are obtained. Simulation results show that joint beamforming design of BS and the STAR-RIS can achieve a better performance compared with conventional RIS, validating the effectiveness of optimizing the amplitudes of STAR-RIS.

References

[1] Q. Wu and R. Zhang, "Towards Smart and Reconfigurable Environment: Intelligent Reflecting Surface Aided Wireless Network," *IEEE Communications Magazine*, vol. 58, no. 1, pp. 106–112, Jan. 2020.

[2] C. Huang, S. Hu, G. C. Alexandropoulos, A. Zappone, C. Yuen, R. Zhang, M. D. Renzo, and M. Debbah, "Holographic MIMO Surfaces for 6G Wireless Networks: Opportunities, Challenges, and Trends," *IEEE Wireless Communications*, vol. 27, no. 5, pp. 118–125, October 2020.

[3] Q. Wu, S. Zhang, B. Zheng, C. You, and R. Zhang, "Intelligent Reflecting Surface-Aided Wireless Communications: A Tutorial," *IEEE Transactions on Communications*, vol. 69, no. 5, pp. 3313–3351, May. 2021.

[4] Y. Cheng, W. Peng, C. Huang, G. C. Alexandropoulos, C. Yuen, and M. Debbah, "RIS-Aided Wireless Communications: Extra Degrees of Freedom via Rotation and Location Optimization," *IEEE Transactions on Wireless Communications*, vol. 21, no. 8, pp. 6656–6671, Aug 2022.

[5] C. Huang, R. Mo, and C. Yuen, "Reconfigurable Intelligent Surface Assisted Multiuser MISO Systems Exploiting Deep Reinforcement Learning," *IEEE Journal on Selected Areas in Communications*, vol. 38, no. 8, pp. 1839–1850, Aug 2020.

[6] C. Huang, Z. Yang, G. C. Alexandropoulos, K. Xiong, L. Wei, C. Yuen, Z. Zhang, and M. Debbah, "Multi-Hop RIS-Empowered Terahertz Communications: A DRL-Based Hybrid Beamforming Design," *IEEE Journal on Selected Areas in Communications*, vol. 39, no. 6, pp. 1663–1677, June 2021.

[7] S. Buzzi, C. D'Andrea, A. Zappone, M. Fresia, Y.-P. Zhang, and S. Feng, "RIS Configuration, Beamformer Design, and Power Control in Single-Cell and Multi-Cell Wireless Networks," *IEEE Transactions on Cognitive Communications and Networking*, vol. 7, no. 2, pp. 398–411, June 2021.

[8] A. Zappone, M. Di Renzo, X. Xi, and M. Debbah, "On the Optimal Number of Reflecting Elements for Reconfigurable Intelligent Surfaces," *IEEE Wireless Communications Letters*, vol. 10, no. 3, pp. 464–468, March 2021.

[9] L. Wei, C. Huang, G. C. Alexandropoulos, C. Yuen, Z. Zhang, and M. Debbah, "Channel Estimation for RIS-Empowered Multi-User MISO Wireless Communications," *IEEE Transactions on Communications*, vol. 69, no. 6, pp. 4144–4157, June 2021.

[10] J. Xu, Y. Liu, X. Mu, and O. A. Dobre, "STAR-RISs: Simultaneous Transmitting and Reflecting Reconfigurable Intelligent Surfaces," *IEEE Communications Letters*, vol. 25, no. 9, pp. 3134–3138, Sept. 2021.

[11] Y. Liu, X. Mu, J. Xu, R. Schober, Y. Hao, H. V. Poor, and L. Hanzo, "STAR: Simultaneous Transmission And Reflection for 360 ° Coverage by Intelligent Surfaces," *arXiv Preprint arXiv:2103.09104*, 2021.

[12] C. Wu, Y. Liu, X. Mu, X. Gu, and O. A. Dobre, "Coverage Characterization of STAR-RIS Networks: NOMA and OMA," *IEEE Communications Letters*, vol. 25, no. 9, pp. 3036–3040, Sept. 2021.

[13] H. Niu, Z. Chu, F. Zhou, and Z. Zhu, "Simultaneous Transmission and Reflection Reconfigurable Intelligent Surface Assisted Secrecy MISO Networks," *IEEE Communications Letters*, vol. 25, no. 11, pp. 3498–3502, Nov. 2021.

[14] X. Mu, Y. Liu, L. Guo, J. Lin, and R. Schober, "Simultaneously Transmitting And Reflecting (STAR) RIS Aided Wireless Communications," *IEEE Transactions on Wireless Communications*, pp. 1–16, Oct. 2021.

[15] Q. Wu and R. Zhang, "Intelligent Reflecting Surface Enhanced Wireless Network via Joint Active and Passive Beamforming," *IEEE Transactions on Wireless Communications*, vol. 18, no. 11, pp. 5394–5409, Nov. 2019.

[16] M. Bengtsson and B. Ottersten, "Optimal and Suboptimal Transmit Beamforming," *Handbook of Antennas in Wireless Communications*, pp. 18–51, Jan. 2001.

[17] M. Grant and S. Boyd, "CVX: Matlab Software for Disciplined Convex Programming," http://cvxr.com/cvx, 2014.

[18] Z. Luo, W. Ma, A. M. So, Y. Ye, and S. Zhang, "Semidefinite Relaxation of Quadratic Optimization Problems," *IEEE Signal Processing Magazine*, vol. 27, no. 3, pp. 20–34, May. 2010.