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Some Improved Class of Ratio Estimators for Finite Population Variance with the Use of Known Parameters

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ABSTRACT

In this paper, we proposed some improved class of ratio estimators for finite population variance with the use of known parameters. The proposed estimators are obtained by transforming both the sample variances of the study and auxiliary variables, as well as the use of known parameters. The Mean Square Error of the proposed estimators have been obtained and the conditions for their efficiency over some existing variance estimators have been established. The present family of finite variance estimator, having obtaining the optimal values of the constants, exhibit significant improvement over the estimators considered in the study. The empirical study is also conducted to support the theoretical results and the results revealed that the suggested estimators are more efficient.

Keywords: Population Variance, Mean Square Error, Ratio Estimator, Efficiency.

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INTRODUCTION

Background

In sample survey, there are many approaches used in estimating some certain parameters of the study variable when there is availability of an auxiliary variable which is correlated with study variable positively or negatively. The ratio method of estimation is used when the correlation between the study variable and the auxiliary variable is positive, and the product method of estimation is used when there is negative correlation between them. However the regression method of estimation is employed when the correlation between the study variable and auxiliary variable is either positive or negative. Many researches employed both the ratio and regression methods of estimation because of their efficiency, although the regression method of estimation is more efficient than the ratio method of estimation. With this, other researcher employed their combination.

The variation of produce or yields in the manufacturing industries and pharmaceutical laboratories are sometime a matter of concern to researchers Ahmed et al. (2003). The use of auxiliary information, being constant with unit (for example population mean, population standard deviation, population median, and so on) or unit free constant (for example Coefficient of variation, kurtosis, and so on), can enhance the efficiency at the estimation stage. In recent past, this concept has been utilized by several researchers to improve the efficiency of ratio and product type estimators for estimating population mean as well as population variance of study variable.

Many other researchers including Kadilar and Cingi (2007), Gupta and Shabbir (2008), Singh and Vishwakarma (2008), Singh and Singh et al. (2011), Singh et al. (2011), Sanaullah, et al. (2012), Subramani and Kumaranpandiyan (2012), Yadav and Kadilar (2013), Singh and Solanki (2013) and Solanki and Singh (2013) have significantly contributed to the improved of both ratio, product, mean and variance estimators in sampling survey.

Let $\Omega = (1, 2, 3, ..., N)$ be a population of size N and Y, X be two real valued functions having values $(Y_i, X_i) \in \mathbf{R}^+ > 0$ on the i^{th} unit of $U(1 \le i \le N)$. We assume positive correlation $\rho > 0$ between the study variable Y and auxiliary variable X. Let S_y^2 and S_x^2 be the finite population variance of Y and X respectively and s_y^2 and s_x^2 be respective sample variance based on the random sample of size n drawn without replacement.

Objectives

In this study, some improved class of ratio estimators for the estimation of finite population variance with the use of known parameters has been suggested with the objective to produce efficient estimators and their properties have been established. Parameters that are known in this study includes; population coefficient of variation, population kurtosis, population variation of the auxiliary variable X, and population size.

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LITERATURE REVIEW

Singh et al. (2007) defined the general family of estimators for estimating finite population variance S_y^2 of the study variable Y as

$$\eta = s_y^2 \left[\frac{aS_x^2 + b}{\alpha\left(as_x^2 + b\right) + (1-\alpha)\left(aS_x^2 + b\right)} \right] \tag{1}$$

Where a and b are constants based on auxiliary variable X like coefficient of skewness, kurtosis and correlation coefficient and so on α is the constant that minimizes the mean square error (MSE) of the estimator. Table 1 shows some members of η – family for different values of a, b and α .

Table 1: Some Member of η – family for different values of a, b and α

Estimators	a	b	α
$\eta_0 = s_y^2$	1	0	0
Sample variance			
$\eta_1 = s_y^2\left(\frac{S_x^2}{s_x^2}\right)$	1	0	1
Isaki (1983)			
$\eta_2 = s_y^2\left(\frac{S_x^2 - C_x}{s_x^2 - C_x}\right)$	1	C_x	1
Kadilar and Cingi (2006)			
$\eta_3 = s_y^2\left(\frac{S_x^2 - \beta_2(x)}{s_x^2 - \beta_2(x)}\right)$	1	$\beta_2(x)$	1
$\eta_4 = s_y^2\left(\frac{S_x^2\beta_2(x) - C_x}{s_x^2\beta_2(x) - C_x}\right)$	$\beta_2(x)$	C_x	1
$\eta_5 = s_y^2\left(\frac{S_x^2 C_x - \beta_2(x)}{s_x^2 C_x - \beta_2(x)}\right)$	C_x	$\beta_2(x)$	1
$\eta_6 = s_y^2\left(\frac{S_x^2 + \beta_2(x)}{s + \beta_2(x)}\right)$	1	$-\beta_2(x)$	1
Upadhyaya and Singh (1999)			

The MSEs/Variance of the estimators in table 1 are given below:

$$Var(\eta_0) = \gamma S_y^4(\psi_{40} - 1) \tag{2}$$

$$MSE(\eta_i)=\begin{cases} S_y^4\gamma\left[(\psi_{40}-1)+(\psi_{04}-1)-2(\psi_{22}-1)\right], i=1 \\ S_y^4\gamma\left[(\psi_{40}-1)+h_i^2(\psi_{04}-1)-2h_i(\psi_{22}-1)\right], i=2,3,4,5,6 \end{cases} \quad (3)$$

$$h_1=1, h_2=\frac{S_x^2}{S_x^2-C_x}, h_3=\frac{S_x^2}{S_x^2-\beta_2(x)}, h_4=\frac{S_x^2}{S_x^2\beta_2(x)-C_x}, h_5=\frac{S_x^2}{S_x^2C_x-\beta_2(x)}, h_6=\frac{S_x^2}{S_x^2+\beta_2(x)}$$

Where

$$s_y^2=\frac{1}{n-1}\sum_{i=1}^{n}(y_i-\bar{y})^2, s_x^2=\frac{1}{n-1}\sum_{i=1}^{n}(x_i=\bar{x})^2, S_y^2=\frac{1}{N-1}\sum_{i=1}^{N}(Y_i-\bar{Y})^2, S_x^2=\frac{1}{N-1}\sum_{i=1}^{N}(X_i-\bar{X})^2$$

$$\bar{X}=\frac{1}{N}\sum_{i=1}^{N}X_i,\quad \bar{Y}=\frac{1}{N}\sum_{i=1}^{N}Y_i,\quad \bar{x}=\frac{1}{n}\sum_{i=1}^{n}x_i,\quad \bar{y}=\frac{1}{n}\sum_{i=1}^{n}y_i,$$

$$\gamma=\frac{1}{n},\quad \psi_{rs}=\frac{\lambda_{rs}}{\lambda_{20}^{r/2}\lambda_{02}^{s/2}}\quad \text{and}\quad \lambda_{rs}=\frac{1}{N-1}\sum_{i=1}^{N}(Y_i-\bar{Y})^r(X_i-\bar{X})^s$$

The MSE of η to second order approximation is given below:

$$MSE(\eta)=\gamma S_y^4\left[(\psi_{40}-1)+\alpha^2\theta^2(\psi_{04}-1)-2\alpha\theta(\psi_{22}-1)\right] \quad (4)$$

The $MSE(\eta)$ expression is minimized for the optimum values of α given by equation (5). This is obtained by partial differentiation of equation (4) with respect to α.

$$MSE(\eta)_{\min}=\gamma S_y^4\left[(\psi_{40}-1)-\frac{(\psi_{22}-1)^2}{(\psi_{04}-1)}\right] \quad (5)$$

Audu et al. (2016) proposed improved class of ratio estimators for finite population variance by transforming both the sample variances of the study and auxiliary variables, after studying the work of Yadav and Kadilar (2013) and Adewara et al. (2012) estimators for population mean, and the estimators were given by:

$$\tau_1^*=k_1s_y^{2*}\left(\frac{S_x^2}{s_x^{2*}}\right), \tau_2^*=k_2s_y^{2*}\left(\frac{S_x^2-C_x}{s_x^{2*}-C_x}\right), \tau_3^*=k_3s_y^{2*}\left(\frac{S_x^2-\beta_2(x)}{s_x^{2*}-\beta_2(x)}\right), \tau_4^*=k_4s_y^{2*}\left(\frac{S_x^2\beta_2(x)-C_x}{s_x^{2*}\beta_2(x)-C_x}\right),$$

$$\tau_5^*=k_5s_y^{2*}\left(\frac{S_x^2C_x-\beta_2(x)}{s_x^{2*}C_x-\beta_2(x)}\right), \tau_6^*=k_6s_y^{2*}\left(\frac{S_x^2+\beta_2(x)}{s_x^{2*}+\beta_2(x)}\right) \quad .(6)$$

Where s_x^{2*} and s_y^{2*} are the respective sample finite variances of the auxiliary variable and study variables, having the relationship: (1) $S_x^2=s_x^2f+(1-f)s_x^{2*}$ (2) $S_y^2=s_y^2f+(1-f)s_y^{2*}$ with condition that $n<\frac{1}{2}N$.

The mean square errors of the estimators (MSEs) are given by:

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$$MSE\left(\tau_i^*\right)=\begin{cases} S_y^4\left\{\gamma v^2\begin{bmatrix} k_i^2\left(\psi_{40}-1\right)+\left(3k_i^2-2k_i\right)\left(\psi_{04}-1\right) \\ -2\left(2k_i^2-k_i\right)\left(\psi_{22}-1\right)\end{bmatrix}+\left(k_i-1\right)^2\right\}, i=1 \\ S_y^4\left\{\gamma v^2\begin{bmatrix} k_i^2\left(\psi_{40}-1\right)+\left(3k_i^2-2k_i\right)h_i^2\left(\psi_{04}-1\right) \\ -2\left(2k_i^2-k_i\right)h_i\left(\psi_{22}-1\right)\end{bmatrix}+\left(k_i-1\right)^2\right\}, i=2,3,4,5,6 \end{cases} \quad (7)$$

Where $k_i^*=\dfrac{v^2\gamma\left[\left(\psi_{04}-1\right)-\left(\psi_{22}-1\right)\right]+1}{v^2\gamma\left[3\left(\psi_{04}-1\right)-4\left(\psi_{22}-1\right)+\left(\psi_{40}-1\right)\right]+1}=\dfrac{A_1}{B_1}, i=1$ and

$$k_i^*=\frac{v^2\gamma\left[h_i^2\left(\psi_{04}-1\right)-h_i\left(\psi_{22}-1\right)\right]+1}{v^2\gamma\left[3h_i^2\left(\psi_{04}-1\right)-4h_i\left(\psi_{22}-1\right)+\left(\psi_{40}-1\right)\right]+1}=\frac{A_2}{B_2}, i=2,3,4,5,6. \quad v=\frac{n}{N-n}.$$

Replacing k_i by k_i^*, in equation (7), the minimum MSE of the estimators are:

$$MSE_{\min}\left(\tau_i^*\right)=\begin{cases} S_y^4\left(1-\dfrac{A_1^2}{B_1}\right), i=1 \\ S_y^4\left(1-\dfrac{A_2^2}{B_2}\right), i=2,3,4,5,6 \end{cases} \quad (8)$$

METHODOLOGY

Data

This section consist of explanation about the data used in this research. In order to investigate the merits of the proposed estimators, we have considered the following two natural populations to compare the Mean Square Error (MSEs) of the proposed and some existing estimators considered in the study. The datasets are given as:

Dataset 1: Subramani and Kumaranpandiyan (2012)

$$N=20, n=20, \bar{Y}=116.16, \bar{X}=98.6765, \rho=0.6904, S_y=98.8286$$

$$S_x=102.9709, C_y=0.8508, C_x=1.0435, \psi_{40}=4.9245, \psi_{04}=5.9878, \psi_{22}=4.6977$$

Dataset 2: Subramani and Kumaranpandiyan (2012)

$$N=20, n=20, \bar{Y}=116.16, \bar{X}=98.6765, \rho=0.6904, S_y=98.8286$$

$$S_x=102.9709, C_y=0.8508, C_x=1.0435, \psi_{40}=4.9245, \psi_{04}=5.9878, \psi_{22}=4.6977$$

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Model Development

Having study the estimator of Audu et al. (2016), we proposed some improved class of ratio estimators for finite population variance with known parameter as:

$$T_{AMi}^{*} = w_i s_y^{2*}\left(\frac{S_x^2+Q_i}{s_x^{2*}+Q_i}\right) \quad for \;\; i=1,2,3,4,5,6 \qquad (9)$$

Where $Q_1 = N,\; Q_2 = \left(N-C_x\right),\;\; Q_3 = \left(N-\beta_2(x)\right),\; Q_4 = \frac{\left(N-\beta_2(x)\right)}{C_x},\; Q_5 = \frac{\left(N-C_x\right)}{\beta_2(x)},$

$Q_6 = \left(N+\beta_2(x)\right)$. And the respective sample finite variances of the auxiliary and study variables $are\;\; s_x^{2*} \;\; and \;\; s_y^{2*}$, having the relationship: (1) $S_x^2 = s_x^2 f + \left(1-f\right)s_x^{2*}$ (2) $S_y^2 = s_y^2 f + \left(1-f\right)s_y^{2*}$ with condition that $n < \frac{1}{2}N$.

Properties (Bias and MSE) of the proposed estimators

In order to obtain the MSE, we defined $e_0 = \frac{s_y^2 - S_y^2}{S_y^2}$ and $e_1 = \frac{s_x^2 - S_x^2}{S_x^2}$ such that

$$\left.\begin{aligned} &E\left(e_0\right)=E\left(e_1\right)=0, E\left(e_0^2\right)=\gamma\left(\psi_{40}-1\right) \\ &E\left(e_1^2\right)=\gamma\left(\psi_{04}-1\right), E\left(e_0e_1\right)=\gamma\left(\psi_{22}-1\right) \end{aligned}\right\} \qquad (10)$$

Expressing $T_{AMi}^{*},\;\; i=1,2,3,4,5,6,$ in terms of $e_0 \;\; and \;\; e_1$, we have

$$T_{AMi}^{*} = w_i S_y^2\left(1-ue_0\right)\left(\frac{1}{1-\frac{uS_x^2e_1}{S_x^2+Q_i}}\right) \quad \text{for} \;\; \theta_i = \frac{S_x^2}{S_x^2+Q_i} \;\; , \;\; i=1,2,3,4,5,6 \qquad (11)$$

$$T_{AMi}^{*} = w_i S_y^2\left(1-ue_0\right)\left(1-u\theta_i e_1\right)^{-1} \;\; i=1,2,3,4,5,6 \qquad (12)$$

Now assume that $\left|u\theta_i e_1\right| < 1$ so that $\left(1-u\theta_i e_1\right)^{-1}$ is expandable. Expanding the right hand side of (12) up to second degree approximation, subtract S_y^2 from its both sides and taking expectation using the results in equation (10), we obtain the MSEs of the proposed estimators as:

$$MSE\left(T_{AMi}^{*}\right) = S_y^4\left\{1 + w_i^2\left[\begin{aligned} &1+\gamma\left\{3\theta_i^2\left(\psi_{04}-1\right)-4\theta_i u\left(\psi_{22}-1\right)+u^2\left(\psi_{40}-1\right)\right\} \\ &-2w_i\left[1+\gamma\left(\theta_i^2\left[\psi_{04}-1\right]-u\theta_i\left[\psi_{22}-1\right]\right)\right] \end{aligned}\right]\right\}, i=1,2,3,4,5,6 \quad (13)$$

The $MSE\left(T_{AMi}^{*}\right), i=1,2,3,4,5,6$ expressions are minimized for the optimum values of w_i given by

$$w_i = \frac{1+\gamma\left[\theta_i^2\left(\psi_{04}-1\right)-\theta_i u\left(\psi_{22}-1\right)\right]}{1+\gamma\left[3\theta_i^2\left(\psi_{04}-1\right)-4\theta_i u\left(\psi_{22}-1\right)+u^2\left(\psi_{40}-1\right)\right]} = \frac{C}{D}, i=1,2,3,4,5,6 \quad (14)$$

Where $u=\frac{n}{N-n}$ and $\theta_i=\frac{uS_x^2}{S_x^2+Q_i}, i=1,2,3,4,5,6$

Substituting $w_i, i=1,2,3,4,5,6$ into equation (13), we obtain the minimum MSE as

$$MSE_{\min}\left(T_{AMi}^*\right)=S_y^4\left(1-\frac{C^2}{D}\right), i=1,2,3,4,5,6 \quad (15)$$

Efficiency Comparisons

In this section efficiencies of the proposed estimators are compared with efficiencies of some estimators in the literature.

The $T_{AMi}^* - family$ of estimators of the population variance is more efficient than η_0 if,

$$MSE_{\min}\left(T_{AMi}^*\right)<Var\left(\eta_0\right) \quad i=1,2,3,4,5,6$$

$$\left(1-\frac{C^2}{D}\right)<\gamma\left[\left(\psi_{40}-1\right)\right] \quad i=1,2,3,4,5,6 \quad (16)$$

The $T_{AMi}^* - family$ of estimators of the population variance is more efficient than η_i if,

$$MSE_{\min}\left(T_{AMi}^*\right)<MSE\left(\eta_i\right) \quad i=1,2,3,4,5,6$$

$$\left(1-\frac{C^2}{D}\right)<\gamma\left[\left(\psi_{40}-1\right)+h_i^2\left(\psi_{04}-1\right)-2h_i\left(\psi_{22}-1\right)\right], i=1,2,3,4,5,6 \quad (17)$$

The $T_{AMi}^* - family$ of estimators of the population variance is more efficient than η_i if,

$$MSE_{\min}\left(T_{AMi}^*\right)<MSE\left(\eta_i\right) \quad i=1,2,3,4,5,6$$

$$\left(1-\frac{C^2}{D}\right)<\gamma\left[\left(\psi_{40}-1\right)-\left(\psi_{04}-1\right)\right], i=1,2,3,4,5,6 \quad (18)$$

The $T_{AMi}^* - family$ of estimators of the population variance is more efficient than τ_i^* if,

$$MSE_{\min}\left(T_{AMi}^*\right)<MSE_{\min}\left(\tau_i^*\right) \quad i=1,2,3,4,5,6$$

$$\left.\begin{aligned}\left(1-\frac{C^2}{D}\right)&<\left(1-\frac{A_1^2}{B_1}\right), i=1\\ \left(1-\frac{C^2}{D}\right)&<\left(1-\frac{A_2^2}{B_2}\right), i=2,3,4,5,6\end{aligned}\right\} \quad (19)$$

When conditions (16), (17), (18) and (19) are satisfied, we can conclude that the family of proposed class of estimators is more efficient than the existing estimators.

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DATA ANALYSIS AND RESULTS

The numerical illustration to justify the appropriateness of the proposed estimators has been conducted using two data sets. Table 2 show the mean square errors of the proposed estimators and some existing estimators in literature and their percentage relative efficiencies of different estimators with respect to sample variance of the study variable respectively.

Table 2: Mean Square Errors (MSEs) and Percentage Relative Efficiencies (PREs) of proposed and some existing estimators considered in the study.

Estimators	MSE		PRE	
	Dataset 1	**Dataset 2**	**Dataset 1**	**Dataset 2**
η_0	18719098	7191.859	100	100
η_1	7235316	3924.948	258.7184	183.2345
η_2	7236528	4003.906	258.6751	179.6211
η_3	7242278	4249.508	258.4697	169.2398
η_4	7235519	3952.035	258.7112	181.9786
η_5	7241987	4372.127	258.4801	164.4934
η_6	7228378	3658.199	258.9668	196.5964
η_{opt}	5643700	2657.427	331.6813	270.6324
τ_1^*	3053941	429.9105	612.949	1672.873
τ_2^*	3054289	438.3024	612.8791	1640.844
τ_3^*	3055940	464.3576	612.5479	1548.776
τ_4^*	3053999	432.7903	612.9373	1661.742
τ_5^*	3055857	477.3385	612.5646	1506.658
τ_6^*	3051947	401.5034	613.3454	1791.233
T_{AM1}^*	**3037795**	**333.8646**	**616.2068**	**2154.1245**
T_{AM2}^*	**3038134**	**332.8637**	**616.1380**	**2160.6018**
T_{AM3}^*	**3039747**	**330.0603**	**615.8111**	**2178.9531**
T_{AM4}^*	**3051275**	**364.7056**	**613.4845**	**`1971.9629**
T_{AM5}^*	**3040333**	**299.4378**	**615.6924**	**2401.7873**
T_{AM6}^*	**3035848**	**337.7105**	**616.6020**	**2129.5930**

Table 2 Shows the Mean Square Errors (MSEs) and Percentage Relative Efficiencies (PREs) of the proposed and some existing estimators using the two natural datasets. The results revealed that the proposed estimator have the minimum MSE and highest PRE compare to the conventional and other existing estimators considered in the study. This implies that the proposed estimator is more proficient and can produce a better estimates of population variance than conventional and other existing estimators considered in the study.

CONCLUSION AND RECOMMENDATION

Conclusion

From section 3, the theoretical conditions obtained for the efficiencies of the proposed estimators supported by the numerical illustration in section 4 given in the Table 2. From the result, we conclude that the suggested variance estimators give a better estimate of finite population variance than the conventional and some existing estimators considered in the study in the sense of having higher percentage relative efficiency which implies lesser mean square error. This revealed that the proposed estimator have higher chances to produce estimates closer to the true value of mean for any population of interest.

Recommendation

The proposed estimator T_{AMi}^{*} is recommended for use in estimating finite population variance of any variable of interest.

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