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FINANCIAL MODELLING AND DECISIONAL PROCESSES IN RISK MANAGEMENT

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FINANCIAL MODELLING AND DECISIONAL PROCESSES IN RISK MANAGEMENT(*)

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Abstract - In the field of the theoretical and practical developments about models of the integrate management of firm's pure risks,directed to the optimization of the economic and financial impact of the expected losses,we particularly remind the approaches through which,substantially,we can include pure risks in the firm's overall objectives.

In the configuration of the decisional processes concerning the strategies of risk management,one of the most significant model chosen ,basing on the diffusion of more remarkable results of studies about the capital market equilibrium and efficiency ,is the Capital Asset Pricing Model(C.A.P.M.).

Some of the simplifier hypothesis , originally introduced to give functionality to this approach,must be revisit and remeditate,because the consequent choices dynamically affect,for example, the size , the composition and the time distribution of firm's cash-flows , with all the phenomena which we can subsequently observe in the market.

So we think it's very interesting ,particularly ,to study at level of system of interrelationships among the whole policy of risk management,firm's market value,market price of a unit of standard deviation,financial risks,market's portfolio value(or of

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(*)Reelaboration of a paper presented in the fifth meeting of "Euro Working Group In Financial Modelling"-University of a market segment).

With these ideas we have delineated some first elaboration,also if we must still individuate all the interpretative keys,and above all to investigate the more suitable formulations ,configuring the whole or a part of the interrelationships,before pointed out, also taking into account the recent contributions about actuarial approach to the financial risks.

1- In the theoretical construction based on the principles of the Capital Budgeting and C.A.P.M. models,for which we refer to the original works of W.F.Sharpe,J.Lintner ,J.Mossin,we have already observed that the risk management problems are integrated in the context of the overall objectives of optimization of firm's market value.

For our purpose the fundamental equation in the two period model, as we known,is given by:

$$P_i = \frac{E(V_i) - \rho_{i,M} * \lambda_M * \sigma(V_i)}{1+r_f} \qquad (1)$$

where P_i represents the equilibrium market value of the i-th firm

,computed at risk free rate(r_f), at the beginning of period 1, which is expressed in function of:

- the expected value and standard deviation of the random variable $\tilde{V}_i$ -the i-th firm's market value at the beginning of period 2;

- the correlation coefficient ($\rho_{i,M}$) of the firm's return with market return;

- the market price(λ_M) for every unit of standard deviation.

$$\lambda_M = \frac{E(V_M)-P_M*(1+r_f)}{\sigma(V_M)}$$

is expressed with the new following components:

-the expected value and standard deviation of the random variable $\tilde{V}_M$ - the market value,at the beginning of period 2,of the market portfolio(or of a market segment);

-P_M -analogous market's portfolio value at the beginning of period 1;

In the synthetized model by the (1),we can observe the composite whole of the interrelationships between $\tilde{V}_i$,in some characteristic values of its distribution,and the market portfolio,particularly at level of correlation between market return and that of the generic i-th firm.

The model shows a clair decisional aim; so the value P_i can be maximized basing on different financial strategies and/or system's management of pure risks,with the purpose to individuate the strategy that shows,on the base of market influence in which the firm operates,the maximum value.

A decisional process of this kind shows the eventual dependence of the firm's market value by one or several components of the whole firm's risks(speculative risks,as business risks - financial risks ...etc...,pure or static risks, as product liability-commercial property,..etc..).

In the greater horizon outlined by the whole management cycle of the firm we can find situations of intersection among the relationships that characterize the structures,respectvely,of speculative risks and pure risks.

The division is not absolute,cannot be rigidly compartmentalized. It 's in this direction that we can elaborate a new reasoning in order to define the way through which this influence can be realized and,above all,in order to point out the effects on the base of the ratio of reciprocity among the different classes of risks.

Moreover, we can immediately observe that the happening of pure risks generates costs for the firm,so in the same way the option for a particular risk management strategy can affect the size, the composition and the time distribution of firm's cash-flows, with all the phenomena that we can also observe in the market. As the cost of pure risks system affects the resources of the firms finalized to the investments,so a different structure of the firm about value,solidity,solvency,can represent a determining influence on the objective to razionalize ex-ante the processes of financial and physical controls of pure risks.

So it ´s very interesting to study the system of interrelationships among whole risk management policy, firm´s market value, price of a unit of standard deviation, market´s portfolio value (or of a market segment).

In this paper, particularly, we shall write about the component $\rho_{i,M}$ showed in the equation (1).

The subsequent developments consent to extend the meaning of (1), pointing out, for example, the type of cost of the pure risks, or, also, searching for some possible logical connections between a decisional financial approach and a typical policy of risk management.

2 - From the strictly operative viewpoint we shall point out the $\tilde{V}_i$ in the following way:

$$\tilde{V}_i = \tilde{V}'_i - \tilde{C}_i \qquad (2)$$

where $\tilde{V}'_i$ represents the random variable - the net income of i-th firm inclusive of the costs linked to pure risks, here indicated with $\tilde{C}_i$ (specific costs of risk management).

Now if we substitute to $\tilde{C}_i$ the costs of the different strategies of risk management, potentially utilizable, we can value the modalities of consequences that they could produce for the economic management of the firm,so that we can elaborate an articulate model of choice.

Generally,given a generic level α of retention,in a wider evaluation of insurance and risk retention programme , we observe that when we limit the effect of risk,$E(V_i)$ increases,($\frac{\partial E(V_i)}{\partial \alpha} > 0$) , but also generating the presuppositions for an increase of $\sigma(V_i)$, ($\frac{\partial \sigma(V_i)}{\partial \alpha} > 0$).

Of course ,for a maximum of P_i,we must necessarily verify the following two conditions:

$$\frac{\partial E(V_i)}{\partial \alpha} = \lambda_M * \frac{\partial \left[\rho_{i,M} * \sigma(V_i)\right]}{\partial \alpha} \qquad (3)$$

$$\frac{\partial^2 E(V_i)}{\partial \alpha^2} < \lambda_M * \frac{\partial^2 \left[\rho_{i,M} * \sigma(V_i)\right]}{\partial \alpha^2} \qquad (4)$$

with the hypothesys $\frac{\partial \lambda_M}{\partial \alpha} = 0$ that,if not supportable in a strictly analytical viewpoint, can be justified because the model is developed under the assumption of perfect competition, then the price of a unit of standard deviation represents a whole valuation of the market and hence also including the consequences for the i-th firm.

Moreover, the model is showed in all the cases in which the profitability of the single risk assets are strictly in relationship with the returns of market portfolio, and under the condition that they are always positively correlated to the variation in the returns of market portfolio.

The equation (3) can be ulteriorly developed in these terms:

$$\frac{\partial E(V_i)}{\partial \alpha} = \lambda_M * \left\{ \frac{\frac{\partial \operatorname{cov}(V_i, V_M)}{\partial \alpha}}{\sigma(V_M)} - \frac{\frac{\partial \sigma(V_i)}{\partial \alpha}}{\sigma(V_i) * \sigma(V_M)} * \operatorname{cov}(V_i, V_M) + \frac{\partial \sigma(V_i)}{\partial \alpha} * \rho_{i,M} \right\} \tag{5}$$

where $\frac{\partial \sigma(V_M)}{\partial \alpha}$ is equal to zero for the same reasons before showed for $\frac{\partial \lambda_M}{\partial \alpha}$.

By (5) now we derive that:

$$\frac{\partial E(V_i)}{\partial \operatorname{cov}(V_i, V_M)} = \frac{\lambda_M}{\sigma(V_M)} \qquad (6)$$

The marginal coefficient (6) will find its applications on the base of different risk management strategies , each one shows a particular structure of costs.

We shall apply the model for a classic strategy:the proportional retention,but in the same way we can proceed for the other strategies.

3 - Now,we assume that risk manager decides to transfer a part of risks through an insurance policy$(1-\alpha)$ and to retain the residual α (with $0<\alpha<1$).

Substantially the equation of the random variable $\tilde{C}_i$ is given by:

$$\tilde{C}_i = (1-\alpha)*G_i + \alpha * (\tilde{x}_i + I_i) \qquad (7)$$

where $\tilde{C}_i$ is expressed in function of:

- G_i - the insurance premium for the complete cover of risk ,$\tilde{x}_i$ - the amount of claims for the pure risks ; α -as before defined;I_i - the cost of administering the pure risks in the case of complete self-insurance,for which,however in a general formulation , we would point out the part depending on $\tilde{x}_i$.

I_i, for semplicity, can be considered as a known cost.

Substituting in (6), reminding formula (2), we derive:

$$\frac{\partial E(V_i)}{\partial \alpha} + G_i - E(x_i) - I_i = \lambda_M * \frac{\partial \rho_{i,M}}{\partial \alpha} * \sigma(V_i) + \lambda_M * \rho_{i,M} * \frac{\partial \sigma(V_i)}{\partial \alpha} \qquad (8)$$

where we have assumed $\frac{\partial \sigma(V_M)}{\partial \alpha}$ equal to zero, as before remarked.

Now, since $\frac{\partial \sigma(V_i)}{\partial \alpha} = \frac{\frac{\partial \sigma^2(V_i)}{\partial \alpha}}{2 * \sigma(V_i)}$ we can still obtain:

$$\frac{\partial E(V_i)}{\partial \alpha} + G_i - E(x_i) - I_i = \lambda_M * \sigma(V_i) * \frac{\partial \rho_{i,M}}{\partial \alpha} + \frac{\lambda_M * \rho_{i,M}}{2 * \sigma(V_i)} * \frac{\partial \sigma^2(V_i)}{\partial \alpha} + \frac{\alpha * \sigma^2(x_i) * \lambda_M * \rho_{i,M}}{\sigma(V_i)} - \frac{\lambda_M * \rho_{i,M} * \mathrm{cov}(V_i, x_i)}{\sigma(V_i)} \qquad (9)$$

that, after simple elaborations, can be so written:

$$\alpha = \frac{\left\{ \frac{\partial E(V'_i)}{\partial \alpha} + \lambda_M * \rho_{i,M} * \left[\frac{cov(V'_i, x_i)}{\sigma(V_i)} - \frac{\sigma(V_i)}{\rho_{i,M}} * \frac{\partial \rho_{i,M}}{\partial \alpha} - \frac{\partial \sigma^2(V'_i)}{\partial \alpha} * \frac{1}{2*\sigma(V_i)} \right] \right\} * \sigma(V_i)}{\sigma^2(x_i) * \lambda_M * \rho_{i,M}} + \bar{\alpha} \qquad (10)$$

where $\bar{\alpha} = \dfrac{\sigma(V_i) * \left[G_i - E(x_i) - I_i \right]}{\lambda_M * \rho_{i,M} * \sigma^2(x_i)}$ represents the optimal level

of a retention programme under the assumption of no,direct or in- direct ,effect of risk management decisions on the correlation of the firm´s return with that one of the market.

Moreover ,by formula (10) we can immediately observe that α is inversely related to the variance of loss distribution of pure risks,besides the product between price of a unit of standard deviation and correlation coefficient between the company profitability and that of the market portfolio.

$\bar{\alpha}$ is also directly related to the standard deviation of the random variable firm´s market value multiplied by a quantity ,in the brackets,which ulteriorly specifies the interrelationships among parameters before indicated.

The size and the sign of this last quantity will be subsequently showed.

Clearly the ways through which we can point out this correlation and the modificative actions, represent the new level of analysis and elaboration of the problems before studied.

Therefore α is individuated by the sum of rate $\bar{\alpha}$, as before defined, and a quantity, subsequently indicated with z_i, which again shows the relationship, from the analytical viewpoint, among parameters of a risk management strategy, characteristic values of the firm and its interdependences with the return of the market portfolio.

z_i must respect the condition:

$$-\bar{\alpha} < z_i < 1-\bar{\alpha} \qquad (11)$$

that, taking into account the limit cases of complete self-insurance ($\alpha=1$) and complete insurance ($\alpha=0$), generally involves a range of variation between -1 and 1.

Moreover, in correspondence of the above-mentioned limit cases, we can observe a level of retention α, respectively, less or greater than $\bar{\alpha}$.

4 - Now we consider formula (10) where, substituting the following expression:

$$\frac{\partial \sigma^2(V'_i)}{\partial \alpha} = \frac{\partial \sigma^2(V_i)}{\partial \alpha} - 2*\alpha*\sigma^2(x_i) + 2*\mathrm{cov}(V'_i, x_i) \qquad (12)$$

and after a series of simple elaborations we can point out ,in equilibrium,on the base of parameters before showed and from the firm's viewpoint ,the insurance premium:

$$G_i = E(x_i) + I_i - \frac{\partial E(V'_i)}{\partial \alpha} + \frac{\partial \sigma(V_i)}{\partial \alpha} * \lambda_M * \rho_{i,M} + \sigma(V_i) * \lambda_M * \frac{\partial \rho_{i,M}}{\partial \alpha} \qquad (13)$$

This last formula can be usefully compared with the analogous equation of the insurance premium in absence of the effects of the risk management decisions on the system of classic risks of the firm(that is when we assume $\frac{\partial \rho_{i,M}}{\partial \alpha}$, $\frac{\partial E(V'_i)}{\partial \alpha}$, $\frac{\partial \sigma^2(V'_i)}{\partial \alpha}$ equal to zero),which subsequently is indicated with G^*_i.

We derive that,always from the firm's viewpoint:

$$\forall \alpha < \frac{\left[G_i - E(x_i) - I_i \right] * \sigma(V_i)}{\lambda_M * \rho_{i,M} * \sigma^2(x_i)} \Longrightarrow G_i(13) > G_i^*$$

and (14)

$$\forall \alpha > \frac{\left[G_i - E(x_i) - I_i \right] * \sigma(V_i)}{\lambda_M * \rho_{i,M} * \sigma^2(x_i)} \Longrightarrow G_i(13) < G_i^*$$

These first conclusions must be supported by a corrispondence of insurance supply,in order to the estimation of satisfactory tariff structures in the general conditions of equilibrium,and therefore,in decision making, interpreted on the base of the difference which exists between theoretical values and observed values,and last but not least by the attitude to risk (aversion,propension or neutrality).

A decision rule can obviously be given in many different ways.

In the meanwhile,these conclusions have showed,besides some indications about the approach,the whole description through which we can examine the problems on which we are studying.

The next developments will involve the search of the best formulations about the whole of the interrelationships before mentioned ,also on the base of suitable tools of the capital market,useful for the control of the management of financial risks,as well as on the base of recent contributions to financial modelling , financial risk analysis, through an actuarial approach,so-called of the third kind.

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