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The Best Player Rule

Impact of Imitation and Memory in Dynamic Cournot Oligopolies

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In this thesis, we explore the impact of imitation with memory in the context of Cournot oligopolies. We model imitation with memory according to the best player rule by adjusting the model developed by Vega-Redondo (1997) and Alós-Ferrer (2004). We run computer simulations with this model to determine how the best player rule affects the market outcome in the long-run in a Cournot oligopoly and how this outcome is influenced by memory length. We show that this model has in fact only one stochastically stable state, namely the Walrasian equilibrium. Moreover, this result is obtained regardless of the number of firms in the market or the memory length and is not affected by changes in the model parameters, such as the slope and intercept of the inverse-demand function. It also seems that relative payoff considerations remain more important than absolute ones when memory is introduced to this model, explaining why all other quantities lack stability. We therefore conclude that the best player rule leads to the Walrasian equilibrium in the long-run in a Cournot oligopoly, regardless of the number of firms in the market or the memory length, and that this result is robust with respect to the model parameters.

1. Introduction

In the real, often complex world of oligopolies, firms have to operate without complete knowledge of the market and the price dynamics. Yet these firms still have to make difficult decisions without the capability of computing best replies, i.e., setting their production or price level whilst taking into account the level competing firms will settle at. One way firms cope with these uncertainties, is to use simple rules of thumb in their decision-making.

An example of such a rule of thumb is *imitation* with *trial and error*. Intuitively, imitation seems rather plausible. Since, in an oligopoly, firms are assumed to be aware of all decisions made in the market, including those of their competitors, the most profitable strategies observed are, as demonstrated by Vega-Redondo (1997), immediately imitated and no firm is going to deviate from this strategy. He argues that the underlying principle is that firms who manage to obtain higher payoffs thrive at the expense of their competitors. Imitation can therefore be determined by observed success, where "success" can be measured in terms of achieving the highest profit. From this point of view, an oligopoly can be seen as a game in which firms tend to achieve the highest payoffs by imitating actions from the past, the imitation game. Trial and error is used in this game to model the (small) probability of a firm trying something new, through an experiment, also called *mutation*. So together, imitation and trial and error can account for the behavior of firms in such situations.

While previous studies show that the use of imitation and trial and error has certain benefits, several challenges still remain. When we use this rule of thumb, it is for example not immediately clear how this will affect the long-run outcome in a Cournot oligopoly and whether the results will differ from those derived from the classical approach. Alós-Ferrer (2004), for instance, has made it apparent that the answers to the previous questions depend on the memory length, but makes no further remark about how often the different equilibria are reached. It is also still unclear how the outcome is affected when we consider alternative ways to incorporate memory in the model, or imitation rules. *The best player rule*, for example, is such an alternative rule, where firms choose the most recent action of the player that earned the highest profit *on average* in the past. We therefore incorporate this rule in this thesis to investigate whether such an alternative imitation rule will yield similar results, or that it will lead to a different outcome. But more importantly, we investigate why this, possibly different, outcome arises to gain a more detailed insight into the effects of memory in a Cournot oligopoly. To achieve this, this thesis aims to answer the following research questions:

How does the best player rule affect the market outcome in the long-run in a Cournot oligopoly, and how is this outcome influenced by memory length?

2. Modeling the imitation rule

2.1 Previous studies

The use of this rather simple rule of thumb has major implications for the study of the market outcome. Without the use of imitation and trial and error, we would have to follow the classical approach to study the Nash equilibrium. Vega-Redondo (1997) points out that this approach is based upon *absolute* payoff considerations, where no firm has any concern for its position relative to its competitors. In reality however, this concern for *relative* payoff considerations is very real and has a huge impact on price dynamics. We therefore favor a more dynamic approach that reckons with this concern, and the use of imitation and trial and error enables such an evolutionary approach. Vega-Redondo (1997), for example, shows that this evolutionary approach provides an alternative basis for Walrasian behavior. In an oligopoly, this concretely means that evolutionary forces will drive firms to behave in a Walrasian-like manner, resulting in behavior "as if" they confronted prices as price takers. Alós-Ferrer (2004) takes this approach even one step further by introducing finite memory. He shows that, in contrast to Vega-Redondo (1997), the Walrasian equilibrium is no longer the only long-run equilibrium, as long as memory takes at least one period into consideration. On the contrary, there appears to be a clear tension between the Walrasian and the Cournot equilibrium, resulting in the whole range of quantities between the two equilibria being stabilized. It is interesting to note that the same prediction arises in some oligopoly models with highly sophisticated firms, such as in Ferreira (2003) and in Delgado and Moreno (2004). Hence, it is apparent to see that memory has substantial consequences for the market outcome.

In the articles of Vega-Redondo (1997) and Alós-Ferrer (2004), *imitate the best* is used as model, where simply the quantity that led to the highest profit in memory is imitated. In this thesis on the contrary, the best player rule is being used, for it provides a better formulation of imitation with memory with more applications for the real world. Secondly, we can note that although Alós-Ferrer (2004) provides information on the effect of memory on the market outcome in the long-run, he does not mention how often the different *stochastically stable* states are reached, or which memory length firms will choose when they have the opportunity to do so. In this thesis, we investigate the possible market outcome in a Cournot oligopoly, especially the stochastically stable states, under the best player rule and further analyze how this outcome is affected by memory.

2.2 The imitation game

2.2.1. The imitation model

We can now further explain the characteristics of the Cournot oligopoly model when we assume a well-behaved Cournot oligopoly. Time, for instance, is assumed to be discrete in this model. We also assume that all output levels that firms can announce belong to a common finite grid $\Gamma = \{\lambda\delta, (\lambda+1)\delta, ..., \nu\delta\}$ with $\delta > 0$ which is thought to be small, and $\lambda, \nu \in \mathbb{N}$, where $\lambda < \nu$, under the condition that q^W and q^C are in this grid. At each time period, firms play the Cournot game by simultaneously choosing their output levels from this grid Γ. They are assumed to have no knowledge of either the demand or the cost function. The only information firms have at their disposal when choosing an output level for period $t+1$ is which quantities were produced and the corresponding profits realized by all firms in the last $K+1$ periods with $K \geq 0$ including the current period. We define $q_i(t)$ as the output level of firm i

in period t, and $q_{-i}(t)$ as the vector of output levels of its competitors in that same period. The profit function of each firm i individually in period t is then given by

$$\pi_i(t) = \Pi\Big(q_i(t), q_{-i}(t)\Big) = P\Big(\sum_{j=1}^{N} q_j(t)\Big) q_i(t) - C\Big(q_i(t)\Big). \quad (1)$$

Note that the model without memory, that is with $K = 0$, corresponds to the model used by Vega-Redondo (1997). He uses the concept of stochastically stable states, which are those states that are in the support of the limit invariant distribution of the imitation process as the probability ε of experimentation approaches zero, to show that the Walrasian equilibrium is the only possible stochastically stable state. As explained in Schenk-Hoppé (2000) or Newton (2015), the interpretation of stochastically stable states is that, for small but positive ε , the process can be found in these states for most of the time.

2.2.2. The best player rule

We complete this model for the imitation game by specifying the imitation and experimentation process. The behavior of firms can be categorized into two types in this model, imitation or trial and error. Firms can, for example, simply imitate the quantity that led to the highest profit in the last $K+1$ periods, but, instead of imitating, a firm can also experiment by choosing a random quantity. This experimentation happens with a (small) probability of $\varepsilon \geq 0$ independent across time and firms, where all quantities in the output grid Γ have a positive probability of being selected. This implies that several firms might simultaneously decide to experiment in the same period, but that this occurrence is unlikely for small ε , since the probability of any two given firms experimenting in the same period is proportional to ε^2 .

We can also formalize imitation and trial and error mathematically, giving more insight in the implications of this rule of thumb. First, we fix a probability $\varepsilon \in [0,1)$ and define $q_i(t)$ and $q_{-i}(t)$ as described in the previous section. The behavior of each individual firm i can then be categorized as one of the following two types.

The first type of behavior to be considered is "imitate the best player". First, assume that imitation occurs with probability $1-\varepsilon$. Next, define B_t^K as the set of observed output levels produced by the player with the highest profit on average in memory. More formally,

$$B_t^K = \Big\{ q_j(t) | j \in \{1, ..., N\} \text{ and } \frac{\sum_{i=0}^{K} \pi_j(t-i)}{K+1} \geq \frac{\sum_{i=0}^{K} \pi_{j'}(t-i)}{K+1} \\ \forall j' \in \{1, ..., N\} \Big\}. \quad (2)$$

From this definition, it can be stated that $q_i(t+1) = q^*$, where $q^* \in B_t^K$, selected at random if B_t^K is not a singleton, according to an uniform probability distribution with full support on B_t^K .

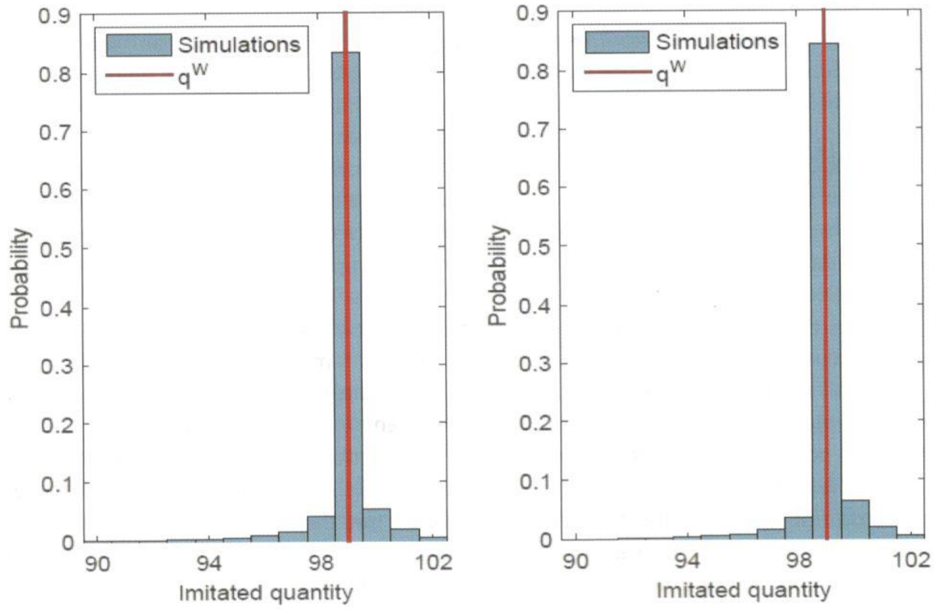
The second type of behavior to be considered is experimentation. First, assume that experimentation, or trial and error, occurs with probability ε . Here is $q_i(t+1)$ a randomly chosen quantity from the output grid Γ, according to an uniform probability distribution with full support on Γ.

3 Simulation Results

3.1 Simulation Analysis

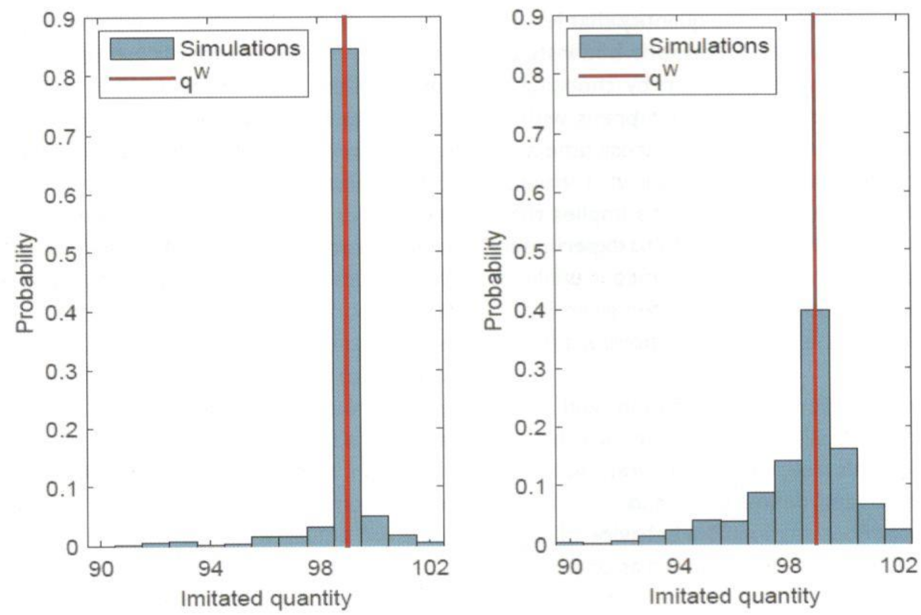
All production levels reported represent the imitated quantity in the last round for each initialization, where we have set the mutation parameter ε to 1%, and are expressed in units. The results of this research are divided into two separate cases, no memory versus short memory and short memory versus long memory.

In the first case, we compare no memory with short memory to determine if the best player rule selects a different imitating quantity if memory is introduced to the model. We show a visual representation of this comparison in Figure 1. From this figure we can conclude that introducing finite memory yields qualitatively the same outcome for the best player rule as before this introduction. It turns out that the imitated quantity is the Walrasian one in almost every run and that all other quantities lack stability. This implies that it is very much likely that, in the long-run, the only quantity observed will be the Walrasian one. In other words, the Walrasian equilibrium turns out to be the only stochastically stable state. It seems that, when using the best player rule, relative payoff considerations remain more important than absolute ones when memory is introduced to the model. The effects of spite causes us to favor the Walrasian equilibrium above the Cournot-Nash equilibrium rather quickly. This means that when a firm decides to deviate from the Walrasian equilibrium, it will consequently earn a higher profit, but the competitors who decided to remain in the Walrasian equilibrium will earn even higher profits. Spite therefore implies that, under the best player rule, deviating will result in a bad relative position. The remark can be made that Alós-Ferrer (2004) finds the opposite when using imitate the best as imitation rule. This explains why the introduction of memory has, in contrast to Alós-Ferrer (2004), no effect on the long-run outcome, since the same reasoning concerning the payoff considerations applies when no memory is present.



(a) $K = 0$ (b) $K = 1$

Figure 1. No memory versus short memory. The likelihood of imitating a quantity after 10.000 rounds with $N = 5$ firms is shown for $K = 0$ on the left and for $K = 1$ on the right. These probabilities are based on 1.000 initializations with the mutation parameter ε set to 1%. The red line illustrates the Walrasian output level.



(a) $K = 5$ (b) $K = 100$

Figure 2. Short memory versus long memory. The likelihood of imitating a quantity with $N = 5$ firms is shown for $K = 5$ after 10.000 rounds on the left and for $K = 100$ after 1.000.000 rounds on the right. These probabilities are based on 1.000 initializations with the mutation parameter ε set to 1%. The red line illustrates the Walrasian output level.

Moreover, because the two different imitation rules result in exactly the same selection procedure when there is no memory in the model, the same outcome arises as in the research of Vega-Redondo (1997), namely the Walrasian equilibrium.

We find similar results in the second case where we compare short memory with long memory to determine if the memory length affects the selection procedure of which quantity should be imitated under the best player rule. This comparison is visually represented in Figure 2 where we have set the memory length to a high value to represent long memory, namely $K = 100$. From this figure the conclusion can be drawn that the results remain qualitatively the same for the best player rule no matter what the memory length is. We should, however, point out that the Walrasian quantity is imitated a lot less in the long-run for $K = 100$. But we also notice that the probability of imitating the Walrasian quantity increases as we consider more and more rounds in each initialization. This means that we expect that the Walrasian quantity will be imitated at the end of almost every initialization regardless of the memory length if we consider enough rounds, and that any other quantity will lack stability. These results therefore indicate that the long-run market outcome in this Cournot oligopoly is independent of the memory length and, more specifically, that the Walrasian equilibrium is always the only stochastically stable state in this oligopoly. Moreover, it seems that relative payoff considerations are of far greater importance than absolute ones for any choice of memory length when we use the best player rule. Consequently, the memory length has no effect on the outcome under the best player rule and, again, the same outcome arises as in the research of Vega-Redondo (1997), namely the Walrasian equilibrium.

But the memory length could, nonetheless, still have an effect on the profit that firms earn on average in the long-run. To investigate this possible effect, we have considered the profit per firm earned on average over the last 1.000 rounds for each initialization. The results of this study are presented for different choices of memory length K in Figure 3 and Table 1. From this figure and this table it is apparent to see that the profit per firm earned on average over the last 1.000 rounds increases as the memory length increases and that this increase is actually statistically significant for larger values of K with respect to a Student's t-test with a 1% significance level. This profit increases since, if the memory length K is extended, we take more output levels into account when we select which quantity we should imitate. But, consequently, this will also mean that we will then take more mutations into account in this selection procedure, for if the memory length increases, more time is required before these mutations have disappeared from memory and the selection procedure. Moreover, the differences between the profits of firms earned on average in memory tend to grow bigger as the memory length K further increases, since the profits of non-mutating firms increase the most in the occurrence of a mutation.

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My name is Robert Verschuren. I am currently 21 years old, work as a junior consultant at Sprenkels & Verschuren and can be found at the athletics' track and field several times a week. After having obtained the Bachelor's degree in Econometrics by writing my thesis on the subject of Imitation and Memory in Dynamic Cournot Oligopolies, I am now pursuing the Master's degree in Financial Econometrics.

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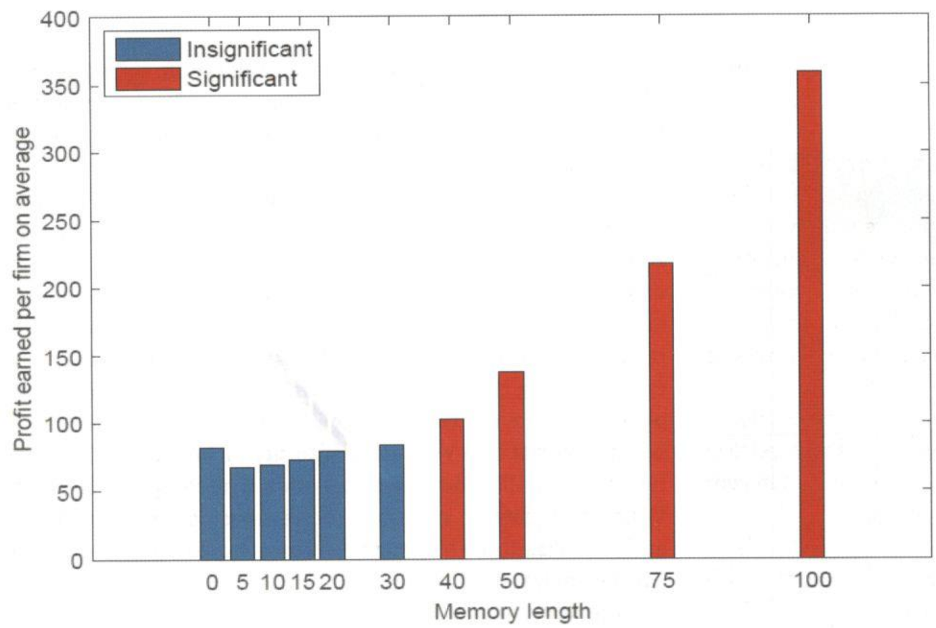

Figure 3. Profit per firm earned on average for each K. The profit per firm earned on average in the last 1.000 rounds $\bar{\pi}_k$ is shown for $N = 5$ firms for each value of $K = k$. These payoffs are based on 1.000 initializations with the mutation parameter ε set to 1%, and with 10.000 rounds if $K < 100$ and 1.000.000 rounds if $K = 100$. The blue bars indicate that the profit is not significantly larger than $\bar{\pi}_0$ at a significance level of 1%, whereas the red bars do represent significantly larger profits. Further information on the statistical significance can be found in Table 1.

Table 1
Statistical significance of the profit per firm for each K

The table presents the estimates, standard errors and the corresponding t-Statistics and p-values of the profits per firm earned on average shown in Figure 3. These values are used to test the null hypothesis of $\bar{\pi}_k = \bar{\pi}_0$ when $K = k$ against the alternative hypothesis of $\bar{\pi}_k > \bar{\pi}_0$ with a Student's t-test.

Parameter	Estimate	Standard Error	t-Statistic	Probability
$\bar{\pi}_0$	81.54	143.69	0.000	0.500
$\bar{\pi}_5$	67.28	132.53	−0.108	1.000
$\bar{\pi}_{10}$	69.58	141.33	−0.085	0.996
$\bar{\pi}_{15}$	73.06	143.36	−0.059	0.969
$\bar{\pi}_{20}$	79.31	146.28	−0.015	0.686
$\bar{\pi}_{30}$	83.19	164.51	0.010	0.376
$\bar{\pi}_{40}$	102.59	191.41	0.110	0.000
$\bar{\pi}_{50}$	137.57	235.31	0.238	0.000
$\bar{\pi}_{75}$	217.44	315.36	0.431	0.000
$\bar{\pi}_{100}$	359.13	403.61	0.688	0.000

If the mutating firm then by chance also happens to be the best player in that round, it could occur that this firm will remain the best player after its mutation, leading the market away from the Walrasian equilibrium and resulting in much higher payoffs. Such an occurrence will happen more frequently as the memory length K increases since the selection procedure will then take more mutations into account. Firms can therefore increase their profits earned on average in the long-run under the best player rule by selecting, as a group, a longer memory length.

The question now arises whether a firm can also strengthen its position *relative* to its competitors by individually selecting a different memory length. If we for instance consider $N = 5$ firms active in the market and a single firm decides to select a different memory length, we would then have two groups who each select a different memory length, one with 4 firms and another with just one firm. It turns out that no group would in this case earn a higher profit than its competing group regardless of the memory length a group adopts, since relative payoffs are not only dominant between groups but also remain so between firms individually, and the Walrasian equilibrium would remain to be the only stochastically stable state in this oligopoly. Selecting a different memory length as an individual firm will therefore not gain any relative advantage. It actually turns out that this result remains qualitatively the same no matter the size of each group or the number of groups in total. Even if firms can decide on the memory length they adopt individually, they will still not gain any relative advantage. We can therefore conclude that selecting a different memory length, as an individual or as a subgroup, will not, under the best player rule, strengthen the position of a firm relative to its competitors and will still lead to the Walrasian equilibrium in the long-run.

3.2 Robustness

We have made clear in the previous section that the Walrasian equilibrium is the only stochastically stable state regardless of the memory length. But it is still unclear whether the same outcome arises for different values of N, that is the number of firms in the Cournot oligopoly.

In the first part of the analysis, we fixed the number of firms active in the market at a single value, namely $N = 5$. We have afterwards expanded this analysis by also evaluating the market with 2, 3, 4, 9, 10 and 11 firms. Each of these additional values of N yields qualitatively the same results as when we consider $N = 5$, meaning that the results are robust with respect to the number of firms N.

The number of firms active in the market does have an effect, however, on the profit that firms earn on average in the long-run. We can see from Figure 4 and Table 2 that the profit per firm earned on average over the last 1.000 rounds no longer increases significantly as the memory length increases, but that this profit is relatively high and actually remains more or less the same for each value of K.

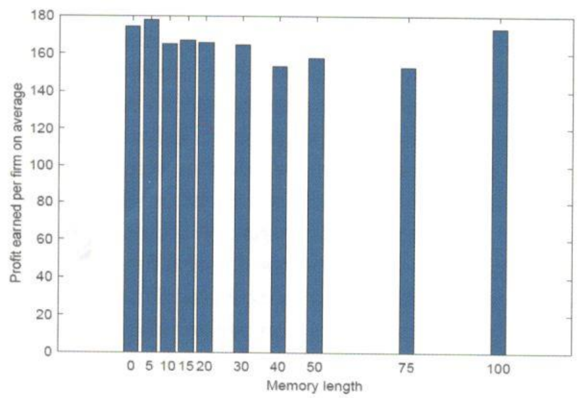

Figure 4. Profit per firm earned on average for each K with more firms. The profit per firm earned on average in the last 1.000 rounds $\bar{\pi}_k$ is shown for $N = 10$ firms for each value of $K = k$. These payoffs are based on 1.000 initializations with the mutation parameter ε set to 1%, and with 10.000 rounds if $K < 100$ and 1.000.000 rounds if $K = 100$. The blue bars indicate that the profit is not significantly larger than $\bar{\pi}_0$ at a significance level of 1%. Further information on the statistical significance can be found in Table 2.

Table 2
Statistical significance of the profit per firm with more firms

The table presents the estimates, standard errors and the corresponding t-Statistics and p-values of the profits per firm earned on average shown in Figure 4. These values are used to test the null hypothesis of $\bar{\pi}_k = \bar{\pi}_0$ when $K = k$ against the alternative hypothesis of $\bar{\pi}_k > \bar{\pi}_0$ with a Student's t-test.

Parameter	Estimate	Standard Error	t-Statistic	Probability
$\bar{\pi}_0$	174.32	117.89	0.000	0.500
$\bar{\pi}_5$	177.91	116.78	0.031	0.166
$\bar{\pi}_{10}$	164.922	116.97	−0.080	0.994
$\bar{\pi}_{15}$	166.99	115.45	−0.064	0.978
$\bar{\pi}_{20}$	165.74	117.11	−0.073	0.990
$\bar{\pi}_{30}$	164.52	122.90	−0.080	0.994
$\bar{\pi}_{40}$	153.39	130.44	−0.161	1.000
$\bar{\pi}_{50}$	157.57	135.14	−0.124	1.000
$\bar{\pi}_{75}$	152.96	150.92	−0.142	1.000
$\bar{\pi}_{100}$	173.36	165.43	−0.006	0.573

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Intuitively, this difference is due to the fact that the number of firms active in the market has increased and that the profits of firms earned on average in memory vary less for a longer memory length. Naturally, an increase in the number of firms coincides with taking more output levels into account, and consequently also more mutations, when determining which quantity should be imitated. This explains why the overall profit is, regardless of the value of K , relatively high. The likelihood that a mutating firm also happens to be precisely that one firm that is the best player in that round now decreases since simply more firms could be mutating. Moreover, the memory length K will barely matter anymore for the probability that this occurrence takes place. A mutation will therefore lead the market away from the Walrasian equilibrium less often when the number of firms active in the market increases. The same reasoning can be followed in the opposite direction when the number of firms active in the market decreases, resulting in qualitatively the same outcome as for $N = 5$.

Even though we have made an extensive analysis of the best player rule, there are still some limitations in the research in this thesis. The main focus of this thesis was, for example, on a linear inverse-demand function and constant and equal marginal costs. It would therefore be interesting for future research to investigate whether the use of a non-linear inverse-demand function or non-constant and perhaps even unequal marginal costs will lead to qualitatively the same outcome. It would also be worth investigating if the same outcome will result in a Bertrand or a Stackelberg competition. Another possible subject of investigation could be if different imitation rules will lead to different behavior of firms.

4. Conclusion

The main objective of this thesis was to determine how the best player rule affects the market outcome in the long-run in a Cournot oligopoly and how this outcome is influenced by memory length. This is particularly interesting since in the real world of oligopolies, firms have to operate without complete knowledge of the market and the price dynamics. It therefore makes sense for firms to use simple rules of thumb in their decision-making to cope with these uncertainties, such as imitation with trial and error. Combining imitation with trial and error leads almost automatically to the best player rule. To analyze such an imitation rule, we have adjusted a pre-existing model for a Cournot oligopoly to incorporate this rule for simulation purposes.

After an extensive study, we have found that the Walrasian equilibrium is the only stochastically stable state in this oligopoly regardless of the memory length a firm adopts. In other words, we expect that the Walrasian quantity is the only quantity that will be observed in this market in the long-run, and that all other output levels lack stability and will therefore not be observed in the long-run. Intuitively, this is due to the fact that relative payoffs remain of greater importance than absolute ones when memory is introduced to the best player rule. This study was extended with some additional research, investigating the robustness of this outcome with respect to certain parameter changes such as the number of firms. It has turned out that changing the number of firms or the parameters of the inverse-demand and cost functions leads to qualitatively the same results, and that 10.000 rounds suffice for the imitation process to converge. We can therefore conclude that the best player rule yields the Walrasian outcome in the long-run in a Cournot oligopoly, regardless of the number of firms or the memory length, and that this result is robust with respect to the model parameters.

We were primarily focused on a linear inverse-demand function and constant and equal marginal costs in our research in this thesis. But, naturally, a non-linear inverse-demand function or non-constant marginal costs could also be of interest. This makes it particularly interesting for future research to investigate whether the results remain qualitatively the same when we impose less strict restrictions. Future research should also focus on the type of competition to investigate whether the same outcome will result in a Bertrand or a Stackelberg oligopoly. Another possible subject of interest could be to explore how the behavior of firms responds to other imitation rules.

The results presented in this thesis are not only theoretically relevant, but also empirically. Real firms in an oligopoly could base their strategic decisions on this simple rule of thumb without the need of computing best replies. But if these firms wish to maximize their profits, this would not be the right course of action for them, since they will most probably end up in the highly competitive and barely profitable Walrasian equilibrium in the long-run. Real firms in an oligopoly should therefore not incorporate the best player rule and are advised to follow a different imitation strategy.

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