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9931
2022

September 2022

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CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
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CESifo *Working Paper No. 9931*

Screening Green Innovation through Carbon Pricing

Abstract

Green innovation is essential for climate change mitigation, but not all innovative projects deliver equal social value. We consider innovator heterogeneity in a model where the policy maker cannot observe innovation quality and directly subsidize the socially most valuable green innovations. We find that carbon pricing works as an innovation screening device; this creates a premium on the optimal carbon price, raising it above the Pigouvian level. We identify conditions for perfect screening and generalize results to screening policies under alternative intellectual property regimes and complementary policies.

JEL-Codes: O300, H230, Q550, Q580.

Keywords: carbon pricing, green innovation, optimal policy, R&D, screening.

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August 26, 2022
We thank Kris De Jaegher, Reyer Gerlagh, Matti Liski, Tuomas Takalo and participants at the 2022 EAERE Conference in Rimini, 2022 Oxford Economics of Sustainability Workshop, and VU Amsterdam Eureka seminar for valuable feedback. Ahlvik and van den Bijgaart are grateful for financial support by Formas, a Swedish Research Council for Sustainable Development [Dnr: 2020-00174].

1 Introduction

The development and adoption of green technologies is of key importance to achieve climate targets. As such, a comprehensive climate policy should not just include a carbon price to internalize the negative emission externalities, but also research and development (R&D) policies that reward innovations according to their social value. These instruments are implemented in markets with a substantial degree of heterogeneity both across and within technologies. Such heterogeneity is illustrated in Figure 1, which displays the number of forward citations of solar and wind power patents, a proxy for both innovation quality and spillovers. The figure shows that most patents receive zero or one citations, and more than 70% of all citations in solar and wind power technologies accrue to less than 10% of patents.[1]

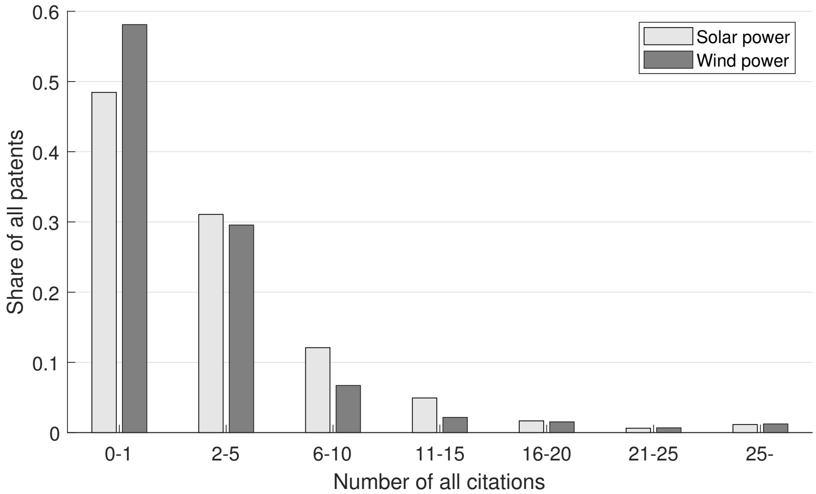

Figure 1: Citation distribution for solar and wind power technologies.

Notes: The figure uses EPO Patstat patent data for patents registered in 2016, and reports total number of forward citations over five years (2016-2021).

The success of climate policies depends not simply on whether they lead to more innovation, but particularly on whether they generate the *right* innovation. Whereas emission prices conveniently incentivize the adoption of lowest-cost abatement technologies, R&D subsidies do not, by themselves, incentivize the development of the socially most valuable technologies. Instead, such subsidies rely either on the policy maker's ability to 'pick winners' by targeting support to the most valuable technologies, or alternatively, amount to subsidies that are paid across-the-board and run the risk of being partly wasted on inferior projects.

This article seeks to explore a third alternative: a climate policy design that screens the most valuable innovations through a combination of subsidies, carbon pricing, and patenting rights. We develop a model where innovators privately observe their innovation cost and quality,

[1] Akin to Figure 1, Popp *et al.* (2013) and Dechezleprêtre *et al.* (2017) report significant asymmetries and skewness in the quality of green innovation. The finding that both the private returns to innovation and knowledge spillovers from innovation are strongly skewed applies more generally to other technology fields; see Trajtenberg (1990); Scherer and Harhoff (2000); Silverberg and Verspagen (2007).

and introduce green innovations to the market in order to license the technologies to polluting firms. In addition to private returns, innovation may generate spillovers, which are greater for high-quality technologies. Our framework features a trade-off between incentivizing the socially valuable, high-quality R&D and encouraging the take-up of the resulting emission-reducing innovations. A distinctive feature of *green* innovations, in contrast to general intellectual property, is that demand for clean technologies is not exogenous, but instead created by regulation.[2]

As the main result, we show that the policymaker can use carbon prices to screen the socially most valuable innovations. The intuition is as follows. Across-the-board R&D subsidies reward the technologies that are the cheapest to develop, socially valuable or not. In contrast, high carbon prices generate demand for CO_2 abatement technologies, and more so for the most valuable high-quality innovations that can spread more widely and thus have the largest market.[3] Moreover, a higher carbon price alleviates the distortion due to positive mark-ups on technology licenses created by the intellectual property rights system. These findings together imply that there exists a previously unidentified screening benefit to emission prices that gives rise to a 'carbon price premium', raising the optimal carbon price above the marginal emission damages.

Does the optimal policy resolve the problem of 'picking winners'? We show that in a special case the policy maker reaches the first-best using a combination of carbon prices and intellectual property rights that perfectly screen in the socially most valuable innovations. This is possible when there are no innovation spillovers and when firms' energy use is perfectly inelastic. If these restrictive conditions are not satisfied, the optimal policy will be second-best, and additionally rely on R&D subsidies that are based on prior knowledge about the distribution of innovation qualities. Still, we can show that, under more restrictive assumptions, the optimal policy always reduces the need for direct subsidies vis-a-vis a naive policy that does not exploit the screening benefit of carbon prices.

Our core result, that the policy maker can use high carbon prices as an instrument for screening the socially valuable innovations, is maintained under alternative assumptions regarding the intellectual property rights system and complementary policies; we reassess the optimal carbon price under exogenous patent systems, a patent buyout, and uptake subsidies for the abatement technologies.

Literature. A primary reason most economists favor carbon pricing over command-and-control policies is its informational simplicity: Carbon prices efficiently allocate abatement efforts in the presence of heterogeneity in abatement costs across sectors, firms and technolo-

[2]This idea has strong empirical support, for instance Calel and Dechezleprêtre (2016) and Calel (2020) establish a positive effect of carbon pricing on green patenting. See also Grubb *et al.* (2021) for a review.

[3]Empirical evidence for the positive relationship between innovation quality and private market value can be found in Harhoff *et al.* (1999), Hall *et al.* (2005) and Kogan *et al.* (2017). Particularly, Hall *et al.* (2005) find a positive relationship between firm valuation and patent citations, and Harhoff *et al.* (1999) and Kogan *et al.* (2017) establish a positive relationship between a patent's estimated economic value and the number of forward citations. In contrast to these articles, Abrams *et al.* (2013) find an inverted u-shaped relationship between economic value and forward citations. They explain this by widespread strategic patenting in high-value industries, aimed at discouraging follow-on innovation.

gies. In contrast, R&D policies are informationally demanding, as they require policy makers to know which innovations should be incentivized. Despite the large literature scrutinizing the twin environmental and innovation market failures,[4] there is strikingly little consideration of the policy implications of the substantial heterogeneity in green innovation.

Instead, this literature typically focuses on the case where innovators are homogeneous. In such a setting, a uniform R&D subsidy, if available, can accurately correct the positive externality from technology spillovers. Combining this R&D subsidy with an appropriate carbon price then allows the policymaker to adequately address both market failures (Gerlagh *et al.*, 2009, 2014; Acemoglu *et al.*, 2012, 2016; Greaker *et al.*, 2018). Research has then largely focused on second-best environments, where either (green) R&D subsidies (e.g., Hart, 2008; Gerlagh *et al.*, 2009; Greaker and Pade, 2009) or Pigouvian carbon prices (e.g., Fischer, 2008) are not available.[5] Our results do not rely on restricting the levels of carbon prices or R&D subsidies. Instead, it is the inability to specifically tailor R&D subsidies to heterogeneous innovators that creates a demand for using the carbon price as a screening instrument.[6] The screening effect we identify creates a carbon price premium that is new in the green innovation literature, but is related to the selection effect identified by Ahlvik and Liski (2022) in a setting where polluting firms can avoid carbon pricing by relocating production.

Innovator heterogeneity and the corresponding need to design an intellectual property right system that adequately encourages R&D under privately informed innovators are core features in the literature on general R&D policies (for instance, Scotchmer, 1999; Hopenhayn *et al.*, 2006; Weyl and Tirole, 2012). In this innovation policy literature, the demand curve is typically exogenous, and the optimal patenting system strikes a balance between incentivizing high-quality innovation and mitigating under-supply of technology. Green innovation as we consider in this article is fundamentally different from the types of R&D analyzed in this literature, as demand is not exogenous but instead created by the environmental policy. As such, the possibility to manipulate the policy-driven demand for innovation implies an additional policy tool which can be used to incentivize high-quality innovations and, under certain conditions, even implement the first-best allocation.

Our study also contributes to the recent literature on how government can more broadly design policies to screen the 'right' innovation and maximize welfare. This includes Acemoglu *et al.* (2018) and Akcigit *et al.* (2022), who consider optimal corporate taxes alongside R&D subsidies in contexts with firm dynamics and heterogeneity in research productivity and quality. In addition, Lach *et al.* (2021) study how government loan programs can be designed to screen

[4]See Popp *et al.* (2010a) and Popp (2019) for an overview of this literature.

[5]Some work explores both cases, see for instance Popp (2006), Fischer and Newell (2008) and Hart (2019). Further work also considers implications of unilateral policy making (Hémous, 2016; van den Bijgaart, 2017) and lack of policy commitment (Laffont and Tirole, 1994, 1996; Montero, 2011; Datta and Somanathan, 2016; Harstad, 2020).

[6]Both R&D subsidies and carbon pricing are widely used and even high carbon prices are observed in practice. In 2019, government support for business R&D amounted to 0.67 percent of GDP across the OECD on average (OECD, 2021). Globally, there are 57 carbon pricing initiatives either implemented or scheduled for implementation, with prices ranging from very low values (\$1/tCO_2 in Ukraine) to values that are above typical estimates for the Pigouvian level (\$ 130/tCO_2 in Sweden); see World Bank (2022).

the projects that generate positive expected social returns but would not be otherwise funded.

The remainder of the article is structured as follows. Section 2 presents the analytical model. Section 3 presents a first-best benchmark policy and solves for the second-best optimal combination of carbon pricing, innovation subsidies and technology prices. Section 4 considers alternative intellectual property rights regimes and complementary technology uptake subsidies, and Section 5 concludes.

2 Model

Final output and abatement Consider the model as follows. A competitive market produces the numeraire output according to the production function, $Y(E)$, where E denotes energy use and $Y'(E) > 0$ and $Y''(E) < 0$. Energy has a cost per unit of $\xi > 0$. Emissions are a byproduct from energy, and impose an external cost on society equal to $\Delta > 0$ per unit of emission. For convenience, we assume that each unit of energy generates one unit of emissions. Emissions can be reduced through abatement A, such that total emissions equal $E - A$, with

$$A = \frac{1}{\beta} \int_0^N \theta_i^{1-\beta} q_i^{\beta} di, \tag{1}$$

where $[0, N]$ is the mass of abatement technologies available on the market, q_i denotes the quantity used of abatement technology i, θ_i a measure of technology quality and we assume $\beta \in (0, 1)$. Firms purchase technology licenses at a price p per unit of technology. Operationalizing the technology has unit cost $\gamma > 0$. This captures, for instance, the cost of transporting equipment or the labor cost of putting the abatement technology in to operation. We assume $\Delta > \gamma$, that is, the social value of the reduced damages exceeds the cost of operationalizing an existing technology.

To incentivize emission mitigation, a policy maker can introduce a carbon price, τ.[7] Firms can then lower emission cost by either reducing energy use or adopting abatement technologies. Profit-maximizing firms will choose energy use such that the marginal benefit of energy equals its marginal cost: $Y'(E) = \xi + \tau$. Abatement is chosen similarly so that the marginal emission cost savings due to abatement equal the price of the technology, p_i plus the marginal cost of operationalizing the technology: $\tau\theta_i^{1-\beta} q_i^{\beta-1} = p_i + \gamma$. This expression can be rewritten to obtain the demand function for abatement technologies:

$$q = \theta \left(\frac{\tau}{p + \gamma} \right)^{\frac{1}{1-\beta}}, \tag{2}$$

where here and in the remainder we suppress the i subscript. The demand function has the following characteristics. First, demand for abatement technologies is downward sloping; a high license price p hinders the diffusion of the technology. Second, demand is created by regulation.

[7] In our model, a carbon price created by an emissions trading scheme or carbon tax are equivalent. We thereby abstract from the optimal choice amongst these policy instruments; see for example Montero (2002b), Requate (2005) and Popp *et al.* (2010b). We also abstract from imperfect competition in the output market, as studied by Montero (2002a).

Specifically, it increases in the emission price τ and is zero in the absence of environmental regulation: $q = 0$ if $\tau = 0$. Third, the quality of the abatement technology, θ, influences demand, with a higher demand for high-quality technologies.

Innovation Abatement technologies are developed and licensed by innovators. Once an innovation is made,[8] innovators can provide a licence at zero marginal cost. Then, innovator revenue from technology licensing is equal to

$$\pi = pq = p\theta \left(\frac{\tau}{p+\gamma}\right)^{\frac{1}{1-\beta}}. \tag{3}$$

Innovator revenue will be positive unless either the license price or the carbon price is zero, $p = 0$ or $\tau = 0$.

In addition to generating consumer surplus and profits, innovations may generate additional social value through innovation spillovers.[9] We allow for such spillovers and assume that they are linearly related to innovation quality: $\delta\theta$, with $\delta \geq 0$. This assumption implies a positive relationship between patents' private economic value and positive externalities through spillovers that is consistent with the empirical findings by Harhoff *et al.* (1999), Hall *et al.* (2005) and Kogan *et al.* (2017) (see also footnote 3).

From equation (3) we can solve the profit-maximizing price $p_M = \gamma(1-\beta)/\beta$, which is the same for all innovations regardless of their quality. If the innovator has full monopoly power in the intellectual property rights, it would choose this price. However, a policy maker may, either through the intellectual property rights system or direct regulations, restrict the price to below that level. For example, without any protection of intellectual property rights, competition will reduce the license price to zero: $p = 0$. Such free availability prevents the innovator from obtaining positive returns to its innovation, but maximizes (ex-post) spread. Intermediate prices $0 < p < p_M$ imply that the policy maker gives more protection against patent infringement. Throughout the article, we assume $p \leq p_M$ and, following Gilbert and Shapiro (1990), use p as a reduced-form for the strength of the intellectual property rights system.

The innovator will develop the technology whenever revenue exceeds the innovation cost, c, net of any innovation subsidies s:[10]

$$\pi \geq c - s. \tag{4}$$

Innovations are developed up to the point where the innovator breaks even. We define z as the maximum innovation cost c that innovators are willing to incur to develop the innovation.

[8] In the remainder, we interchangeably use the terms 'innovation' and 'technology'.

[9] For instance, knowledge generated by one innovation may aid subsequent innovation. Research suggests these spillovers are likely to be substantial. Myers and Lanahan (2022), for example, use R&D grants given out by the US Department of Energy and find that "*for every patent produced by grant recipients, three more are produced by others who benefit from spillovers.*" All in all, they estimate that only 25–50% of the value generated by a patent is captured by the patenting firm. Similarly, Zacchia (2020) finds that the marginal social returns to R&D are about 112% of the marginal private returns.

[10] We do not explicitly consider the uncertainty inherent in the innovation process. Assuming that innovators are risk neutral, one can interpret c as the (expected) cost incurred to obtain one successful innovation.

From (3) and (4), this gives

$$z(\tau, p, s, \theta) = s + p \left(\frac{\tau}{p + \gamma} \right)^{\frac{1}{1-\beta}} \theta. \tag{5}$$

For notational simplicity, most of the remaining exposition will suppress the arguments of $z(\tau, p, s, \theta)$.

The policy maker can incentivize innovation by offering greater direct subsidies s ('technology push'), or allowing innovators to choose a price p closer to the monopoly price p_M, and manipulating the demand for innovation by imposing a higher carbon price τ ('technology pull'). However, there is an important distinction between these strategies to increase innovation. A uniform subsidy rewards all innovators equally, but raising the license or carbon price is *more* valuable for those innovators with high θ. A uniform increase in s thus encourages more innovations of all qualities, whereas a similar increase in p or τ in particular induces innovation in high quality technologies. Our theoretical results below hinge on a testable prediction: that carbon pricing disproportionally increases the number of high-quality, highly cited patents.[11]

Distributions and information. Our key assumption is that innovations are heterogeneous in quality θ and cost c, and that these parameters are known only by innovators; policy makers know the distributions of θ and c, but cannot directly observe those parameters.[12] We assume that $\theta \in [\underline{\theta}, \overline{\theta}]$ is distributed based on density function $g(\theta)$ with cumulative distribution function $G(\theta)$, satisfying the standard monotone hazard rate assumption and $\underline{\theta} < \overline{\theta}$.[13] We assume that $c \in [0, \overline{c}]$ is distributed based on density function $f(c)$ and cumulative distribution $F(c)$, and innovation costs are assumed to be independent of θ, $g(\theta|c) = g(\theta)$. To avoid technical but uninteresting issues at the upper bound, we let $\overline{c} \to \infty$, implying that for any given finite subsidy level *some* innovations are left undeveloped.

Social welfare. A policy maker maximizes social welfare, which is given by

$$W = Y(E) - (\xi + \Delta)E + \int_{\underline{\theta}}^{\overline{\theta}} \left(\int_0^{z(\tau,p,s,\theta)} [v(\tau, p)\theta - c] f(c)dc \right) g(\theta)d\theta. \tag{6}$$

The first two terms capture the benefits of energy use $Y(E)$, net of its private (ξ) and social (Δ) costs. The integral gives the social value of innovations, $v(\tau, p)\theta$, net of development costs,

[11] Whether or not this prediction holds empirically is still an open question. Aghion *et al.* (2016) do report a larger elasticity of citation-weighted patents with respect to fuel prices compared to non-weighted patent counts, which is consistent with carbon pricing favoring high-citation patents.

[12] Although innovators may not know the exact quality or cost accurately ex-ante, it seems reasonable that it has better information than policy makers do. Innovation cost, c, should be understood broadly as the minimum reimbursement that the innovator would require to undertake the project. We follow the usual assumption in the literature (e.g., Scotchmer, 1999; Akcigit *et al.*, 2022) that the policy maker cannot observe innovation costs, at least not fully.

[13] In order to keep the demanded quantity in equation (2) positive, we rule out socially harmful innovations and guarantee that innovations always have a positive quality $\underline{\theta} > 0$. Note, though, that their ex-ante social value may still be negative because of the innovation costs (if $v\theta < c$).

c, integrated over the entire mass of innovations that are developed ($c \leq z$). The social value of innovation an innovation of quality θ is $v(\tau, p)\theta$, where

$$v(\tau, p) = \frac{\Delta}{\beta}\left(\frac{\tau}{p+\gamma}\right)^{\frac{\beta}{1-\beta}} - \gamma\left(\frac{\tau}{p+\gamma}\right)^{\frac{1}{1-\beta}} + \delta. \tag{7}$$

The first term in (7) captures the social value of abatement generated by innovation. This value is equal to marginal emission damages Δ, multiplied by the abatement from the use of technology, $\theta^{1-\beta}q^{\beta}/\beta$, see equation (1), with equilibrium q given by equation (2). The second term subtracts the cost of using the technology, which is equal to γq. The third term, δ, captures any positive spillovers from innovations that cannot be captured by the innovator.

The policy maker chooses carbon prices, license prices and innovation subsidies to maximize (6). In what follows, we make two different assumptions about the policy makers' constraints. In Section 3.1 we first consider a benchmark where the policy maker is able to *pick winners* by conditioning innovation subsidies on the true quality of innovation θ. Next, we assume that the policy maker cannot condition on θ (or c) and thereby the policies must be designed to *screen winners*. This inability to condition on θ is due to unobservability of θ on part of the policy maker; equivalently, it could be due to institutional constraints that inhibit the policy maker from differentiating its subsidies across innovators.[14] We assume throughout that carbon and license prices are common to all firms and technologies.[15]

3 Optimal climate policy with innovator heterogeneity

We begin with some general insights. The optimal policy in which the policy maker chooses carbon price τ and license price p to maximize social welfare, taking the innovation subsidy s, for now, as given. The carbon price that maximizes welfare (6) then satisfies:[16]

$$\frac{\partial W}{\partial \tau} = \underbrace{\frac{\Delta - \tau}{-Y''(E)}}_{\text{Pigovian effect}} + \underbrace{\int_{\underline{\theta}}^{\overline{\theta}} v'_{\tau}(\tau, p)\theta F(z)g(\theta)d\theta}_{\text{diffusion effect}} + \underbrace{\int_{\underline{\theta}}^{\overline{\theta}} \left[v(\tau, p)\theta - z\right] z'_{\tau} f(z)g(\theta)d\theta}_{\text{innovation effect } (\Omega_{\tau})} = 0, \tag{8}$$

where $v'_{\tau}(\tau, p)$ denotes the partial derivative of $v(\tau, p)$ with respect to τ, likewise z'_{τ}.

The carbon price serves three potential purposes, captured by the three effects in equation (8). Consider a small increase in τ. First, this changes energy use and affects direct climate damages; the first term is this Pigouvian effect which depends on the marginal damages from

[14] Our assumption that policy maker cannot observe innovation costs c implies our analysis abstracts from research subsidy schemes that are conditioned on c such as R&D tax credits. Although we acknowledge that a part of R&D costs may be observable and verifiable, unobservable R&D cost components, including R&D effort and managerial input, likely remain and typically not all expenses can be claimed for tax credits. Indeed, in the majority of OECD countries, less than half of business R&D expenditures qualify for tax credits (OECD, 2021). See also Scotchmer (1999), Lach *et al.* (2021) and Akcigit *et al.* (2022) for similar assumptions.

[15] In Section 4 we further assess the generalizability of our results under alternative intellectual property rights regimes.

[16] We use the implicit derivative of firms' inverse demand for energy, $Y'(E) = \xi + \tau$, to arrive at the first term.

emissions, Δ. Second, the social value of the technology, v, depends on how widely it is adopted. A higher carbon price can incentivize technology uptake and correct the distortion created by, for instance, a patenting system that allows innovators to set license prices above marginal cost. The second term gives this impact, aggregated over all technologies that enter the market. Third, a higher carbon price increases demand for the clean technology, and thereby makes innovation more profitable. The benefits of encouraging innovation are given by the gap between the social and private value of innovation $v(\tau,p)\theta - z$, multiplied by the marginal effect of carbon pricing on innovation incentives z'_τ, aggregated over potential innovations. Here, $f(z)g(\theta)$ captures the density of innovators with innovation quality θ and cost $c = z$.

We rewrite the innovation effect to decompose it into an average innovation effect $\bar{\Omega}_\tau$ and an innovation *screening* effect Ω^s_τ:[17]

$$\Omega_\tau = \underbrace{\mathbb{E}\left[(v(\tau,p)\theta - z)\,f(z)\right]\mathbb{E}\left[z'_\tau\right]}_{\text{average innovation effect }(\bar{\Omega}_\tau)} + \underbrace{Cov\left((v(\tau,p)\theta - z)f(z), z'_\tau\right)}_{\text{innovation screening effect }(\Omega^s_\tau)}, \tag{9}$$

with $\mathbb{E}\left[z'_\tau\right] \equiv \int_{\underline{\theta}}^{\overline{\theta}} z'_\tau g(\theta)d\theta$ the expected value of z'_τ (over θ) and similarly $\mathbb{E}\left[(v(\tau,p)\theta - z)f(z)\right] \equiv \int_{\underline{\theta}}^{\overline{\theta}} \left[v(\tau,p)\theta - z\right] f(z)g(\theta)d\theta$.

Equation (9) immediately highlights the implications of heterogeneity for the optimal carbon price: whenever heterogeneity implies a positive covariance between the effect of carbon pricing on innovation (z'_τ), and the gap between the social and private benefit of the additional innovation ($v(\tau,p)\theta - z$) with mass $f(z)$, the innovation screening effect Ω^s_τ will be positive, warranting a premium on the carbon price. In other words, carbon prices should be higher if they particularly incentivize the development of the socially most valuable technologies. We will show that this is true in our setting, as higher carbon prices increase demand for all technologies, creating the largest effect for technologies with the greatest uptake.

Similarly, the optimal p satisfies

$$\frac{\partial W}{\partial p} = \underbrace{\int_{\underline{\theta}}^{\overline{\theta}} v'_p(\tau,p)\theta F(z)g(\theta)d\theta}_{\text{diffusion effect}} + \underbrace{\int_{\underline{\theta}}^{\overline{\theta}} \left[v(\tau,p)\theta - z\right] z'_p f(z)g(\theta)d\theta}_{\text{innovation effect }(\Omega_p)} \geq 0, \tag{10}$$

holding with equality if $p < p_M$. As in (9), we can rewrite and decompose Ω_p into the average innovation effect $\bar{\Omega}_p$ and the innovation screening effect Ω^s_p:

$$\Omega_p = \underbrace{\mathbb{E}\left[(v(\tau,p)\theta - z)f(z)\right]\mathbb{E}\left[z'_p\right]}_{\text{average innovation effect }(\bar{\Omega}_p)} + \underbrace{Cov\left((v(\tau,p)\theta - z)f(z), z'_p\right)}_{\text{innovation screening effect }(\Omega^s_p)}, \tag{11}$$

where $\mathbb{E}\left[z'_p\right]$ is defined as $\mathbb{E}\left[z'_\tau\right]$.

The welfare-maximizing p optimally balances the cost and benefits of a marginal increase in p. First, such an increase has a negative impact on diffusion. Second, with positive carbon

[17] Here, Cov denotes covariance and this expression exploits that, by definition, for any two variables X and Y, $Cov(X,Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$.

pricing, a higher license price leads to higher revenue, especially for high θ innovations. This increases innovation incentives, and creates gains equal to $(v(\tau,p)\theta - z)f(z)$ for a technology of type θ. The term z'_p then captures the effect of a marginal increase in p on the mass of type-θ innovations that will be made.

Equations (8)-(11) make explicit how the consideration of heterogeneity influences the optimal policy prescription. We have, however, not yet considered innovation subsidies as part of the policy mix. Below, we show that whenever innovation subsidies are targeted, that is, conditioned on innovation quality τ, the innovation benefit of carbon prices is zero ($\Omega_\tau = 0$), and heterogeneity in innovation quality does not affect the optimal carbon price. This result however does not extend to the setting where only across-the-board innovation subsidies can be awarded. Even though such subsidies would ensure innovations are appropriately rewarded on average, the innovation screening effects in (9) and (11) remain.

3.1 Picking winners: Targeted R&D subsidies

We begin by considering a setting where the policy maker observes the type of each innovation, θ, and can condition R&D subsidies on this type, $s(\theta)$. The policy maker also chooses welfare-maximizing τ and p according to (8) and (10). From (6) we find that when the policy maker can type-target compensation, it will choose $s(\theta)$ such that the marginal social value of type-θ innovation is equal to its marginal private cost:

$$\frac{\partial W}{\partial s(\theta)} = v(\tau,p)\theta - z = 0. \tag{12}$$

Targeted innovation subsidies increase together with their quality: high-θ innovations receive higher subsidies, because their social value justifies a higher cost.

As optimal targeted subsidies equate the social and private value of innovation, it follows that the innovation effects of carbon pricing are zero. In fact, when the right innovations are in the market, there is no reason to distort the energy choice by deviating from the Pigouvian pricing ($\tau = \Delta$), and similarly, there is no reason to distort the diffusion of the new technology by setting a positive license price ($p = 0$). Innovations are then solely compensated through quality-dependent innovation subsidies, which equal the social value of the innovations. This result is summarized in the following proposition:

Proposition 1. ***(Picking winners)*** *If the innovation subsidy can be targeted based on innovation quality θ, then the optimal combination of policies is*

$$\begin{aligned}
&\tau = \Delta, && \textit{Pigouvian pricing}\\
&p = 0, && \textit{No patenting rights}\\
&s(\theta) = \Big[\Big(\frac{\Delta}{\gamma}\Big)^{\frac{\beta}{1-\beta}}\Big(\frac{\Delta}{\beta} - \Delta\Big) + \delta\Big]\theta, && \textit{Targeted R\&D subsidy}
\end{aligned}$$

with zero average innovation and innovation screening effects: $\bar{\Omega}_\tau = \bar{\Omega}_p = 0$ and $\Omega^s_\tau = \Omega^s_p = 0$.

Proof. See Appendix A.1. □

The policy mix in Proposition 1 implements the first-best allocation. As such, Proposition 1 can be considered a direct application of Tinbergen rule, where one policy tool is used for each policy target. Carbon pricing τ corrects the negative externality of emissions, targeted subsidies s ensure that innovators capture the social value of innovations and, thus, develop those technologies for which the social value exceeds innovation costs. This allows patents to be released for free, $p = 0$.

As the targeted subsidies ensure that the 'right' innovations will enter the market, there is no further need for the carbon price or the intellectual property rights system to act as a screening device; under the optimal policy combination, the innovation screening effects, Ω_τ^s and Ω_p^s are zero.

3.2 Screening winners: Across-the-board innovation subsidies

In actuality, policy makers often lack ready and reliable information about innovator and product characteristics, which inhibits their ability to accurately tailor policies to the most valuable innovations. Alternatively, the inability to differentiate R&D subsidies may stem from institutional constraints; differentiating subsidies across innovators within an industry may be prohibited by law, or it may be prohibitively expensive to implement. The inability to differentiate innovation subsidies across innovators implies that the policy maker is unable to implement the first-best allocation using the policy mix as described in Proposition 1. Instead, it must identify the constrained optimal combination of across-the-board subsidies and carbon pricing.

The optimal across-the-board subsidy, s, then satisfies the following first-order condition:

$$\frac{\partial W}{\partial s} = \int_{\underline{\theta}}^{\overline{\theta}} \left[v(\tau, p)\theta - z\right] f(z) g(\theta) d\theta = 0. \tag{13}$$

The subsidy balances two effects. First, a higher subsidy incentivizes innovation, which has value $v(\tau, p)\theta$; the first term. Second, the cost of this marginal innovation is $c = z$; the second term. The innovation subsidy then strikes a balance between the value and cost, averaged across all types θ, and taking into account the density of innovators at margin of innovating or not, $f(z)$.

In contrast to the type-targeted subsidy given by equation (12), the optimal across-the-board innovation subsidy is only correct 'on average'. Heterogeneity in innovation quality implies heterogeneity in the gap between the social and private returns to innovation; this creates a benefit to using carbon prices and intellectual property rights to incentivize the development of the best innovations. Mathematically, this is highlighted by the fact that whereas (13) ensures $\mathbb{E}\left[(v(\tau, p)\theta - z) f(z)\right] = 0$, and thus eliminates the average innovation effect $\bar{\Omega}_\tau$ in (9), the innovation screening effect Ω_τ^s remains positive: the optimal carbon price includes a premium because it rewards the most valuable, high quality innovations.

Likewise, we find that the policy maker finds it optimal to assign patenting rights to the innovator: the optimal p is strictly positive due to a positive innovation effect. Recall that in the setting with targeted R&D subsidies (Proposition 1), patents were not optimal, as they prevent the diffusion of new technologies. This result no longer holds under across-the-board subsidies.

Proposition 2. ***(Screening winners)*** *If the innovation subsidy cannot be targeted based on innovation quality θ, then the optimal combination of policies satisfies the following:*

$$\tau > \Delta, \qquad \textit{Higher-than Pigouvian pricing}$$

$$p > 0, \qquad \textit{Patenting rights}$$

$$s = \left[\left(\frac{\tau}{p+\gamma}\right)^{\frac{\beta}{1-\beta}}\left(\frac{\Delta}{\beta}-\tau\right)+\delta\right]\frac{\mathbb{E}[\theta f(z)]}{\mathbb{E}[f(z)]}, \qquad \textit{Across-the-board R\&D subsidy}$$

with positive innovation screening effects in carbon and license prices: Ω^s_τ, $\Omega^s_p > 0$, yet $\bar{\Omega}_\tau = \bar{\Omega}_p = 0$.

Proof. See Appendix A.2. □

Proposition 2 states our main result. When the policy maker cannot observe the true quality of innovation, the innovation effect is positive, and it is optimal to set a positive license price and a carbon price that is above the Pigouvian level. This is despite the fact that the policy maker subsidizes innovation; whereas the implementation of an optimal across-the-board subsidy eliminates the average innovation effects $\bar{\Omega}_\tau$ and $\bar{\Omega}_p$, the innovation screening effect remains: Ω^s_τ and Ω^s_p. This screening component then contributes to increasing the carbon price above marginal damages.

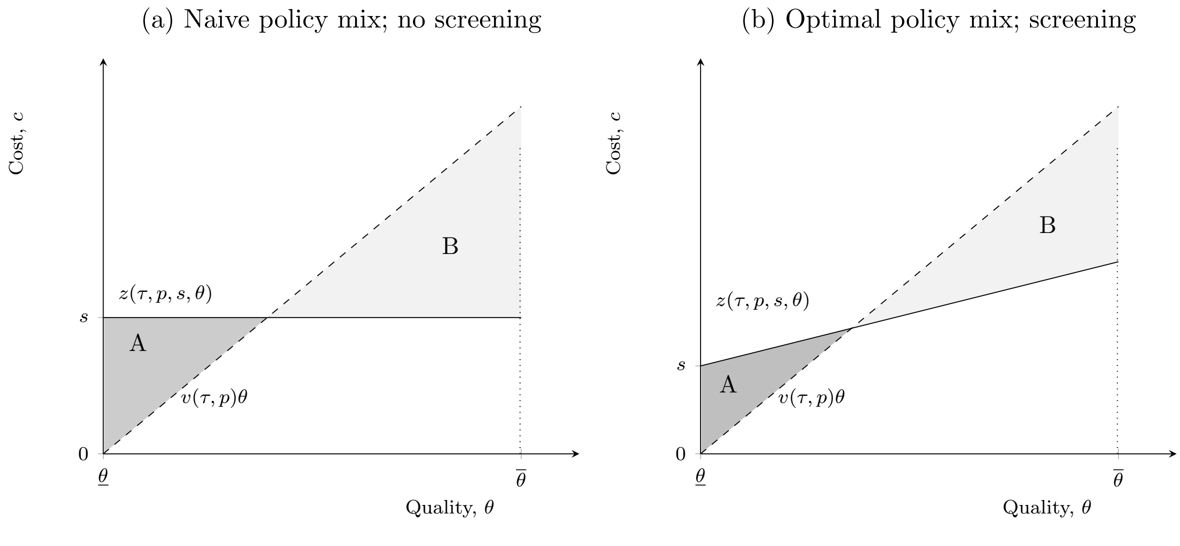

Figure 2: Graphical illustration of screening in the model.

Notes. Solid line $z(\tau, p, s, \theta)$ is the cut-off for developing innovations (eq. 5). Dashed line $v(p, \tau)\theta$ is the cut-off for socially beneficial innovations (eq. 7). Area A: Socially unbeneficial innovations that are developed. Area B: Socially beneficial innovations that are not developed.

Figure 1 illustrates our results. The policy mix partitions the type space into (i) innovations with cost below (or above) private returns, $c \leq z$ and (ii) innovations with cost below (or above) social returns, $c \leq v(\tau, p)\theta$. Figure 1a considers a naive policy, which is defined as a policy mix with Pigouvian pricing and no patenting ($\tau = \Delta, p = 0$), and R&D subsidies based on equation (13).[18] Under this naive policy mix, line z is horizontal and all innovations with cost below z are developed; see Figure 1a. This naive policy is costly, as it incentivizes the development of socially unbeneficial innovations (area A), yet fails to incentivize the development of some socially beneficial innovations (area B).

Figure 1b shows how the policy mix in Proposition 2 alleviates this problem. With higher-than-Pigouvian carbon pricing and patenting ($\tau > \Delta, p > 0$), the slope of z increases and the areas A and B become smaller. The optimal carbon price balances this screening benefit against deadweight loss associated with deviations from the Pigouvian price (eq. 8), and the optimal patenting balances the screening benefit against the monopoly distortion (eq. 10).

The inability to tailor R&D subsidies based on innovation quality implies that generally speaking, the policy mix described by Proposition 2 implements a second-best optimum. Under specific conditions, however, the policy can implement the first-best which includes incentivizing the development of the 'right' innovations (i.e., eliminating areas A and B in Figure 2). This is established in the proposition below:

Proposition 3. ***(Implementing first-best)*** *The policy mix described in Proposition 2 implements the first-best allocation if and only if $\delta = 0$ and $Y''(E) \to -\infty$. If those conditions hold, the optimal policy sets:*

$$\tau = \frac{\Delta}{\beta}, \qquad \textit{Higher-than-Pigouvian pricing}$$

$$p = \gamma \frac{1-\beta}{\beta}, \qquad \textit{Monopoly rights}$$

$$s = 0. \qquad \textit{No R\&D subsidy}$$

Proof. See Appendix A.3. □

The first-best is reached only if two conditions are met: $\delta = 0$ and $Y''(E) \to -\infty$. The first condition, $\delta = 0$, is that there are no innovation spillovers. If this is the case, innovators can capture the full social value of innovation through a combination of monopoly patent rights (that allow for $p = p_M$), and higher-than-Pigouvian carbon price equal to $\tau = \Delta/\beta$. These high carbon prices are needed because a high license price decreases the technology take-up. The policy maker can correct this under-provision problem by setting a higher-than-Pigouvian carbon price. Such above-Pigovian carbon prices however would normally create a distortion in energy demand. The second condition, $Y''(E) \to -\infty$, eliminates this distortion, as it implies the energy demand curve is vertical, and the market responds to regulation through abatement

[18] This policy mix is equivalent to the 'picking winners' policy described in Proposition 1, with the across-the-board R&D subsidy equal to a weighted average of the targeted subsidy of Proposition 1. One can show that if the policy maker naively ignores the innovation screening effects, she will consider this the optimal policy mix.

and innovation in abatement technologies, rather than by reducing energy use.[19] As the policy combination described in Proposition 3 ensures that the private and social value of innovation coincides, no further R&D subsidies are required.[20]

If these conditions are not met, the policy maker cannot fully rely on screening, but must complement the policy by R&D subsidies. A relevant question is how the level of R&D subsidies in Proposition 2 compares to the average R&D subsidy level when subsidies can be targeted, or to the level implemented by a naive policy maker as described above. In other words, does using carbon pricing to screen winners also reduce spending on R&D subsidies?[21]

It is not straightforward to answer this question. On the one hand, by leveraging the market to encourage the most socially beneficial innovations, the policy mix in Proposition 2 leaves less need for direct subsidies. On the other hand, the different mix of policies implies that, for each quality level θ, the marginal innovator now operates at a different cost level c, with a potentially different density $f(c)$. To draw unambiguous conclusions from a comparison of subsidy levels, additional assumptions on the distribution $f(c)$ must be made. For the proposition below, we make such an assumption:

Proposition 4. ***(R&D subsidies)*** *Denote optimal targeted subsidies in Proposition 1 by* s^{target}*, optimal subsidies in Proposition 2 by* s^{opt}*, and naive subsidies by* s^{naive}*. If* $f(c)$ *is non-increasing in* c*, then*

$$\mathbb{E}[s^{opt}] < \mathbb{E}[s^{target}] = \mathbb{E}[s^{naive}].$$

Proof. See Appendix A.4. □

Proposition 4 states that, on average, the targeted subsidies described in Proposition 1 exceed the across-the-board subsidies from Proposition 2 and also the subsidies implemented by a naive policymaker. This result holds under specific distributions for innovation cost c, including the uniform and exponential distribution.

4 Extensions

In actuality, climate policy is generally determined separately from intellectual property rights policy. As such, the policy maker might not have the option to jointly optimize the carbon price τ and the patent regime, as proxied by the license price p. Additionally, the policy maker may have at its disposal alternative strategies for rewarding innovation. It may, for instance, offer the innovator to buy their patents (patent buyout), or subsidize the sales of technology licenses.

In the following, we explore such alternative contexts. We find that, apart from certain extreme cases, the innovation screening benefit remains positive: carbon prices continue to

[19] Our results show that the channel through which the market responds to climate regulation (as studied by e.g., Calel (2020) and Colmer *et al.* (2022)) have important implications for the optimal policy. In Appendix A.3 we show that the assumption $Y''(E) \to -\infty$ can be relaxed if the policy maker subsidizes energy use E.

[20] Indeed, in the appendix we formally prove that this policy can exactly replicate the outcome where subsidies can be type-targeted (shown in Proposition 1).

[21] We acknowledge that subsidy levels are not the only relevant metric in this context. A comparison of levels for instance does not account for the amount or quality of innovation incentivized though those subsidies.

contribute to screening the best innovators, leading to increased optimal carbon prices.

4.1 Exogenous license prices

Our main result in Proposition 2 assumes that the policy maker can jointly and simultaneously optimize three policy instruments, subsidies s, carbon pricing τ, and patenting p. The only constraint it faces in setting these instruments is that they are implemented across the board: they cannot be tailored to the specific innovator or technology quality. As highlighted above, the real-life environmental authority may be able to choose only the level of carbon pricing and the related innovation subsidies, yet having to take the intellectual property rights regime as given. Below we show that our main results regarding carbon pricing generalize to carry over to a setting where the license price is exogenously set:[22]

Proposition 5. ***(Exogenous license price.)*** *If the license price is exogenously set and the innovation subsidy cannot be targeted based on the innovation quality, the optimal policy can be summarized as follows*

- *If $p = 0$, then $\tau = \Delta$ with a zero innovation screening effect: $\Omega^s_\tau = 0$.*
- *If $p = (0, p_M]$, then $\tau > \Delta$ with a positive innovation screening effect: $\Omega^s_\tau > 0$.*

Proof. See Appendix A.5. □

The proposition states that whenever the intellectual property rights system is such that $p > 0$, the optimal carbon price increases due to the positive screening effect. Put differently, it is optimal for the policy maker to use the carbon price as an innovation screening device, irrespective of the intellectual property rights system in place. The exception is the case with $p = 0$. In this situation, no positive profits will be derived from innovation, and increasing the carbon price will not boost profits of the highest-quality technologies in particular. Therefore, the carbon price will not be able to contribute to the screening of the best innovations and should only be used to correct the environmental distortion (as $p = 0$ already ensures the optimal diffusion of the technology).

Analogous to Proposition 2, the optimal across-the-board innovation subsidy ensures that, on average, innovators are rewarded according to the social value of innovation. As such, the subsidy eliminates the average innovation effect $\overline{\Omega}_\tau$, but cannot be used to screen in the highest quality innovations.

4.2 Patent buyouts

A disadvantage of using intellectual property rights for rewarding innovation is that positive prices p reduce technology uptake. One proposal to enhance the diffusion of abatement technologies is for the government to buy patents and place them in the public domain (Kremer, 1998;

[22] As before, we consider only prices up to the monopoly price $p_M = \gamma(1-\beta)/\beta$; even if there were no restrictions on patenting rights, the innovator would never charge more than that price.

Galasso *et al.*, 2016). This would enhance the diffusion of green technologies by eliminating the monopoly distortion.

Below, we consider the implications of such a proposal for the optimal policy mix of carbon price τ, license prices p and across-the-board subsidies s. More specifically, we allow the policy maker to make a take-it-or-leave-it offer T to the innovators, who in turn decide whether to take the offer and sell their patent to the government, or retain it. As before, we assume that due to the lack of information or regulatory restrictions, the policy maker cannot (directly or indirectly) condition this offer on innovation quality θ. The innovator then accepts the offer if they expect to make less money by keeping the patent and licensing:

$$T \geq \pi = p\left(\frac{\tau}{p+\gamma}\right)^{\frac{1}{1-\beta}}\theta. \tag{14}$$

where π is profits as specified in (3). (14) shows that the government suffers from adverse selection: innovators are only willing to sell the low-quality innovations, and prefer to keep the revenue from licensing the high-quality innovations. We can derive the following:

Proposition 6. ***(Patent buyouts)*** *Define θ^* such that $T = \pi$ for $\theta = \theta^*$. It is then optimal to set T^* such that the policy maker:*

(i) Buys and releases patents ($p = 0$) in the interval $[\underline{\theta}, \theta^*]$.

(ii) Allows for patenting ($p > 0$) in the interval $[\theta^*, \overline{\theta}]$, where $\underline{\theta} \leq \theta^* < \overline{\theta}$.

In addition, $\tau > \Delta$ with a positive innovation screening effect $\Omega^s_\tau > 0$.

Proof. See Appendix A.6. □

In the proof of the proposition, we show formally that patent buyouts trade off two effects. On the one hand, buyouts are beneficial, because by placing patents in the public domain, they eliminate the monopoly distortion. On the other hand, buyouts incentivize the development of low-quality innovations with negative social value. The optimal buyout policy strikes a balance between these two effects. In spite of adverse selection it is always beneficial for the government to buy a subset of patents: Introducing a very small buyout price has first-order welfare effects due to wider diffusion of these patents, but only second-order effects to incentivizing low-quality patents. Nevertheless, the government never wants to buy all the patents, as this would lead to excessive development of technologies with negative social value.

4.3 Rewarding technology uptake

So far, the analysis has abstracted from policy tools that reward technology uptake. Examples of such tools are electric vehicle subsidies and tax credits for solar panels.[23]. It includes also policy options such as the advance market commitments, which have gained renewed interest and application in the COVID pandemic (Kremer, 2000; Kremer *et al.*, 2022), and are advocated as a potential tool for supporting negative emission technologies (e.g., Sarnoff, 2020).

[23]For instance, from April 1, 2022, the UK has reduced the VAT rate on solar panels and heat modules in residential application.

To assess the implications of such measures, we consider an exogenously set technology uptake subsidy σ, such that the licence price paid is $(1-\sigma)p$. We maintain the assumption that the policy maker can set any $p \leq p_M$, where with a technology uptake subsidy, we now have $p_M = \gamma(1-\beta)/(\beta(1-\sigma))$. We can then establish the following

Proposition 7. ***(Technology subsidies)*** *If the innovation subsidy cannot be targeted based on the innovation quality θ, and technology is subject to an exogenous uptake subsidy σ, then the optimal policy combination satisfies:*

- *If $\sigma \in [0,1)$, then $\tau > \Delta$ and $p > 0$ with a positive innovation screening effect: $\Omega^s_\tau > 0$.*
- *If $\sigma = 1$, then $\tau = \Delta$ and $p = \gamma(1-\beta)/\beta + \delta\left(\Delta/\gamma\right)^{-\frac{1}{1-\beta}}$ with a zero innovation screening effect: $\bar{\Omega}^s_\tau = 0$.*

Proof. See Appendix A.7. □

Although the innovation screening effect is zero under the optimal policy mix when $\sigma = 1$, it is important to recognize that in this scenario, market incentives are still heavily used to screen in the best innovations. With a fully subsidized abatement technology, the price paid for licenses is equal to the zero marginal license cost, implying diffusion is both maximal and optimal. In this setting, the license price p is used with the sole purpose of bringing in line the private and social value of innovation, thereby screening in the best innovations.

Setting $\sigma = 1$ may be unrealistic, as it requires that the government observes all occasions when a technology is used. A more realistic case may be one where the government is forced to set a lower-than-optimal level of the uptake subsidy $\sigma < 1$. In this case, we are back to the standard trade-off considered in Section 3: increasing the price level screens in the best technologies but at the cost of reduced uptake. For this reason, solely relying on intellectual property rights to screen innovation is insufficient, and it is optimal to additionally use carbon pricing for further screening.

5 Concluding comments

Since 2020, prices in the EU Emissions Trading System have rapidly increased, even coming close to \$100/tCO_2 in August 2022. This price is beyond most experts' mean estimates for the social cost of carbon (Drupp *et al.*, 2022). The high ETS prices are complemented by substantial support for green research and development at the national and European level: the EU Innovation Fund alone allocates roughly 40 billion Euro to innovation support for low-carbon technologies alone over 2020-2030 (EC, 2022). A natural question then arises, is this a sensible policy mix to steer the green transition?

The qualitative answer given in this article is 'yes'. Although the subsidies encourage the development of low-carbon technologies, the EU fund is unlikely able to perfectly target the most promising new innovations. The higher-than-Pigouvian carbon prices in the Emissions Trading System then complement the subsidies in the fund, by forcefully leveraging the market to reward the most valuable technologies, thereby 'screening winners'. Moreover, the presence of innovation spillovers, will imply that subsidies of the Innovation Fund remain a necessary complement to carbon pricing in the path towards net zero.

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A Proofs

We introduce some definitions and expressions that will be used in several proofs. For notational simplicity, the exposition in this section will suppress the arguments of $v(\tau, p)$ and $z(\tau, p, s, \theta)$.

The proofs will use $k \equiv p\big(\tau/(p+\gamma)\big)^{\frac{1}{1-\beta}}$ which is equal to the profit per unit of quality θ. This allows us to write the maximum innovation costs such that the innovator will innovate as $z = s + k\theta$. It is straightforward to verify that $k'_\tau = 0$ and thus $z'_\tau = 0$ if $\tau = 0$ or $p = 0$, and $k'_\tau > 0$ and $z'_\tau > 0$ whenever $\tau > 0$ and $p > 0$. Similarly, $k'_p = 0$ and thereby $z'_p = 0$ if $\tau = 0$ or $p = p_M$, and $k'_p > 0$ and $z'_p > 0$ if $\tau > 0$ and $p \in [0, p_M)$. As it is never optimal for the innovator to set $p > p_M$, we consider only solutions with $p \in [0, p_M]$.

In addition, we rewrite (8)-(11) as follows. First, define $\Gamma_\tau \equiv \int_{\underline{\theta}}^{\overline{\theta}} v'_\tau \theta F(z) g(\theta) d\theta$ as the diffusion effect of τ. This allows us to write the necessary condition (8) as

$$\frac{\partial W}{\partial \tau} = \frac{\Delta - \tau}{-Y''(E)} + \Gamma_\tau + \Omega_\tau = 0. \tag{A.1}$$

From here, we use (7), to write

$$\Gamma_\tau = \Big[\Delta - \tau\frac{\gamma}{p+\gamma}\Big]\frac{1}{(1-\beta)\tau}\Big(\frac{\tau}{p+\gamma}\Big)^{\frac{\beta}{1-\beta}}\int_{\underline{\theta}}^{\overline{\theta}} \theta F(z) g(\theta) d\theta, \tag{A.2}$$

and observe that the sign of Γ_τ is equal to the sign of $[\Delta - \tau\frac{\gamma}{p+\gamma}]$.

Similarly, define $\Gamma_p \equiv \int_{\underline{\theta}}^{\overline{\theta}} v'_p \theta F(z) g(\theta) d\theta$ as the diffusion effect of p. This allows us to write the necessary condition (10) as

$$\frac{\partial W}{\partial p} = \Gamma_p + \Omega_p \geq 0. \tag{A.3}$$

and holding with strict equality if $p < p_M$. Next, using (7), we can write Term Γ_p as:

$$\Gamma_p = -\Big[\Delta - \tau\frac{\gamma}{p+\gamma}\Big]\frac{1}{(1-\beta)(p+\gamma)}\Big(\frac{\tau}{p+\gamma}\Big)^{\frac{\beta}{1-\beta}}\int_{\underline{\theta}}^{\overline{\theta}} \theta F(z) g(\theta) d\theta, \tag{A.4}$$

or, by (A.2), $\Gamma_p = -\frac{\tau}{p+\gamma}\Gamma_\tau$. The sign of Γ_p is opposite to the sign of the term in brackets, and opposite to Γ_τ.

A.1 Proof of Proposition 1

The optimal p must satisfy (A.3). With $s(\theta)$ satisfying (12), the innovation effects are zero: by (11), $\Omega_p = \bar{\Omega}_p = \Omega^s_p = 0$. From (A.4), this implies $p = \gamma(\tau - \Delta)/\Delta$.

The optimal τ must satisfy (A.1). With $s(\theta)$ satisfying (12), the innovation effects are zero: by (9), $\Omega_\tau = \bar{\Omega}_\tau = \Omega^s_\tau = 0$. Similarly, with $p = \gamma(\tau - \Delta)/\Delta$, $\Gamma_\tau = 0$ (see (A.2)). By (A.1) this implies $\tau = \Delta$. From here it follows $p = 0$.

From (12) and the definitions of v from (7) and z from (5) we obtain the result for the optimal targeted subsidy:

$$s = \Big[\Big(\frac{\tau}{p+\gamma}\Big)^{\frac{\beta}{1-\beta}}\Big(\frac{\Delta}{\beta} - \tau\Big) + \delta\Big]\theta. \tag{A.5}$$

The optimal subsidy result is then obtained by plugging $\tau = \Delta$ and $p = 0$ into (A.5). □

A.2 Proof to Proposition 2

From (13), the optimal uniform subsidy implies $\mathbb{E}[(v\theta - z)f(z)] = 0$. From (9) and (11) it follows that the average innovation effects in Ω_τ and Ω_p are zero, and only the innovation screening effects remain: $\Omega_\tau = \Omega_\tau^s$ and $\Omega_p = \Omega_p^s$.

Next, define $\tilde{z} \equiv z(\tau, p, s, \tilde{\theta})$, with $\tilde{\theta}$ such that $v\tilde{\theta} - \tilde{z} = 0$: as both $v\theta$ and z are linear in θ, $\tilde{\theta}$ is uniquely defined. Then subtracting $\tilde{z}'_\tau \mathbb{E}[(v\theta - z)f(z)]$ from Ω_τ as defined in (8), gives

$$\Omega_\tau = \int_{\underline{\theta}}^{\overline{\theta}} (v\theta - z)(z'_\tau - \tilde{z}'_\tau) f(z) g(\theta) d\theta, \tag{A.6}$$

where we exploit the fact that, by (13), the optimal uniform subsidy is defined by $\mathbb{E}[(v\theta - z)f(z)] = 0$. In turn, we subtract $\mathbb{E}[(v\tilde{\theta} - \tilde{z})(z'_\tau - \tilde{z}'_\tau)f(z)]$ from (A.6). This gives

$$\Omega_\tau = k'_\tau [v - k] \int_{\underline{\theta}}^{\overline{\theta}} (\theta - \tilde{\theta})^2 f(z) g(\theta) d\theta, \tag{A.7}$$

where we exploit that $v\tilde{\theta} - \tilde{z} = 0$ by the definition of $\tilde{\theta}$. The integral contains a squared term and is thus necessarily positive. Therefore the sign of Ω_τ is equal to the sign of $k'_\tau[v - k]$.

Following steps similar to those used to derive (A.7), we can write Ω_p as

$$\Omega_p = k'_p [v - k] \int_{\underline{\theta}}^{\overline{\theta}} (\theta - \tilde{\theta})^2 f(z) g(\theta) d\theta, \tag{A.8}$$

or $\Omega_p = (k'_p / k'_\tau)\Omega_\tau$. As the integral term is positive, the sign of Ω_p is equal to the sign of $k'_p(v - k)$ as well as the sign of Ω_τ.

Proof: $p > 0$. Proof is by contradiction. Assume $p = 0$, then $z'_\tau = 0$ (and $k'_\tau = 0$) and by (A.7), $\Omega_\tau = 0$. We show that this leads to a contradiction with the first-order conditions (8),(10) and (13) which the optimal policy must necessarily satisfy. With $p = 0$, (A.2) becomes:

$$\Gamma_\tau = \frac{1}{(1-\beta)\tau} \Big[\Delta - \tau\Big] \Big(\frac{\tau}{\gamma}\Big)^{\frac{\beta}{1-\beta}} \int_{\underline{\theta}}^{\overline{\theta}} F(z)\theta g(\theta) d\theta.$$

Combining this with (A.1) and $\Omega_\tau = 0$ it follows that $\tau = \Delta$. With $\tau = \Delta$ and $p = 0$, (A.4) implies $\Gamma_p = 0$. Equation (A.3) then requires $\Omega_p = 0$ (with strict equality because $p = 0 < p_M$). Yet, at $p = 0$ and $\tau = \Delta$, $v - k = \frac{1-\beta}{\beta} \Delta^{\frac{1}{1-\beta}} \gamma^{-\frac{\beta}{1-\beta}} + \delta > 0$ which, using (A.8), gives $\Omega_p > 0$ and implies (A.3) cannot hold; a contradiction.

Proof: $\tau > \Delta$. Proof by contradiction. Assume $\tau \leq \Delta$. Then $\Gamma_\tau > 0$ by (A.2) and $\Gamma_p < 0$ by (A.4). Equations (A.1) and (A.3) then require $\Omega_\tau < 0$ and $\Omega_p > 0$, which cannot be simultaneously true.

Proof: $\Omega_\tau, \Omega_p > 0$. Proof by contradiction. By $\tau > \Delta$ and (A.1), $\Gamma_\tau + \Omega_\tau > 0$. Suppose $\Omega_\tau < 0$. Then $\Gamma_\tau > 0$. Yet this would imply $\Omega_p < 0$ and $\Gamma_p < 0$, which implies (A.3) is not satisfied. Similarly, if $\Omega_\tau = 0$, then (A.1) requires $\Gamma_\tau > 0$. In turn, this implies $\Omega_p = 0$ and $\Gamma_p < 0$ which is inconsistent with (A.3).

Proof: $\overline{\Omega}_\tau = \overline{\Omega}_p = 0$ and $\Omega^s_\tau, \Omega^s_p > 0$. By (13), $\overline{\Omega}_\tau$ and $\overline{\Omega}_p$ as defined in (9) and (11) are zero. As $\Omega_\tau, \Omega_p > 0$, by (9) and (11) we must have Ω^s_τ and $\Omega^s_p > 0$.

Proof: s**.** The solution for s can in turn be obtained by substituting (5) and (7) in to (13) and rearranging the resulting expression. □

A.3 Proof of Proposition 3

A first necessary condition for first-best is that $v\theta = z$ holds for all θ; see eq. (12). From $z = s + k\theta$, this is equivalent to $s = 0$ and $v - k = 0$. Use the definition of v and k to write:

$$v - k - \delta = \frac{\Delta}{\beta}\Big(\frac{\tau}{p+\gamma}\Big)^{\frac{\beta}{1-\beta}} - (p+\gamma)\Big(\frac{\tau}{p+\gamma}\Big)^{\frac{1}{1-\beta}}$$

$$= \Big(\frac{\tau}{p+\gamma}\Big)^{\frac{\beta}{1-\beta}}\Big[\frac{\Delta}{\beta} - \tau\Big].$$

Observe that $v - k - \delta = 0$ holds when $\tau = 0$ and when $\tau = \Delta/\beta$. Differentiate $v - k - \delta$ with respect to τ:

$$\frac{\partial}{\partial\tau}(v - k - \delta) = \frac{1}{p+\gamma}\frac{1}{1-\beta}\Big(\frac{\tau}{p+\gamma}\Big)^{\frac{\beta}{1-\beta}-1}\Big[\Delta - \tau\Big].$$

The derivative is nonnegative if $\tau \in [0, \Delta]$ and negative if $\tau > \Delta$. In other words, $v - k - \delta$ takes an inverted U-shape with a peak at $\tau = \Delta$ and reaching zero at $\tau = \{0, \Delta/\beta\}$. Proposition 2 rules out $\tau < \Delta$. Thus, $v - k = 0$ requires

$$\tau \geq \Delta/\beta, \tag{A.9}$$

which holds with equality only if $\delta = 0$.

A second necessary (but not sufficient) condition for first-best, is that the first-order conditions for τ (eq. A.1) and p (eq. A.3) hold. In first-best, $v-k = 0$. By (A.7) and (A.8) this implies that $\Omega_\tau = \Omega_p = 0$. In turn, by (A.9), the first (Pigouvian) term in (A.1) is non-positive, meaning that first-best requires $\Gamma_\tau \geq 0$, which holds with equality only if $Y''(E) \to -\infty$. At the same time, $\Omega_p = 0$ and (A.3) imply that $\Gamma_p \geq 0$. By (A.2) and (A.4), $\Gamma_\tau \geq 0$ and $\Gamma_p \geq 0$ can only be simultaneously true if $\Gamma_\tau = \Gamma_p = 0$. This requires $Y''(E) \to -\infty$ and

$$\Delta - \tau\frac{\gamma}{p+\gamma} = 0.$$

As $p \leq p_M = \gamma(1-\beta)/\beta$, a necessary condition for the latter is $\tau \leq \Delta/\beta$. This is consistent with (A.9) only if $\tau = \Delta/\beta$, implying $\delta = 0$ and $p = p_M$.

We are left to show that the solution $\tau = \Delta/\beta$, $p = p_M = \gamma(1-\beta)/\beta$ and $s = 0$ indeed implements the same allocation as the first-best in Proposition 1 (with $\delta = 0$). In Proposition 1, $\tau = \Delta$ and $p = 0$. Hence, we require $v(\Delta, 0) = v(\Delta/\beta, p_M)$ and $s(\theta)$ from Proposition 1 equal to $z(\Delta/\beta, p_M, 0, \theta)$. One can straightforwardly confirm this is the case.

Last, note that an optimal tax on energy use E would set $Y'(E) = \xi + \Delta$ and eliminate the first term of (A.1). Therefore, if the policy maker can optimally set such an energy tax, we only require assumption $\delta = 0$; the other assumption $Y''(E) \Rightarrow -\infty$ can be dropped. □

A.4 Proof of Proposition 4

Proof is by contradiction. Suppose $s^{opt} \geq s^{naive}$. By Proposition 2, s^{opt} satisfies

$$s^{opt} = \Big[\Big(\frac{\tau}{p+\gamma}\Big)^{\frac{\beta}{1-\beta}}\Big(\frac{\Delta}{\beta} - \tau\Big) + \delta\Big]\frac{\mathbb{E}[\theta f(z)]}{\mathbb{E}[f(z)]}. \tag{A.10}$$

Next, s^{naive} satisfies (13), with $\tau = \Delta$ and $p = 0$, leading to $z = s$. It follows that $f(z)$ is constant, and we can write $\mathbb{E}[\theta f(z)]/\mathbb{E}[f(z)] = \mathbb{E}[\theta]$. This gives

$$s^{naive} = \Big[\Big(\frac{\Delta}{\gamma}\Big)^{\frac{\beta}{1-\beta}}\Big(\frac{\Delta}{\beta} - \Delta\Big) + \delta\Big]\mathbb{E}[\theta]. \tag{A.11}$$

Observe that this coincides with the expected subsidy, s^{target}, from Proposition 1: $s^{naive} = \mathbb{E}[s^{target}]$.

Consider z under the optimal subsidy. As $\tau > \Delta$ and $p > 0$ in this scenario (see Proposition 2), $z = s + k\theta$ is increasing in θ. Combined with the assumption that $f(c)$ is non-increasing in c, we have $Cov(\theta, f(z)) \leq 0$, implying:

$$\mathbb{E}[\theta f(z)] - \mathbb{E}[\theta]\mathbb{E}[f(z)] \leq 0 \;\; \Rightarrow \;\; \frac{\mathbb{E}[\theta f(z)]}{\mathbb{E}[f(z)]} \leq \mathbb{E}[\theta]$$

and with strict inequality if $f(z)$ is independent of z (uniform distribution $f(c)$). Therefore, from eqs. (A.10) and (A.11), for $s^{opt} \geq s^{naive}$ to hold, we must have

$$\Big(\frac{\tau}{p+\gamma}\Big)^{\frac{\beta}{1-\beta}}\Big(\frac{\Delta}{\beta} - \tau\Big) \geq \Big(\frac{\Delta}{\gamma}\Big)^{\frac{\beta}{1-\beta}}\Big(\frac{\Delta}{\beta} - \Delta\Big).$$

By $\tau > \Delta$ this implies

$$\frac{\tau}{p+\gamma} > \frac{\Delta}{\gamma}.$$

Yet by (A.4) and (A.8) this gives $\Gamma_p > 0$ and $\Omega_p > 0$, which implies the first-order condition (A.3) cannot hold; a contradiction. □

A.5 Proof of Proposition 5

Proof: s. The solution for s can be obtained by substituting (4) and (7) in to (13) and rearranging the resulting expression.

Proof: $\bar{\Omega}_\tau = 0$. From (13), the optimal uniform subsidy implies $\mathbb{E}[(v\theta - z)f(z)] = 0$. From here it follows that the average innovation effect in Ω_τ is zero, and only the innovation screening effect remains: $\Omega_\tau = \Omega_\tau^s$.

Proof: $\tau = \Delta$ and $\Omega_\tau^s = 0$ if $p = 0$. From (4), $z'_\tau = 0$ if $p = 0$. From (9) it follows that $\Omega_\tau^s = 0$ and thus $\Omega_\tau = 0$. The optimal carbon price $\tau = \Delta$ follows from (A.1) and (A.2).

Proof: $\tau > \Delta$ and $\Omega_\tau^s > 0$ if $p \in (0, p_m]$. The remainder of the proof is by contradiction, where we first prove that $\tau > \Delta$, and next $\Omega_\tau > 0$.

Suppose $\tau \leq \Delta$. Then by (A.1), we require $\Gamma_\tau + \Omega_\tau \leq 0$. Use (7), and following the same steps as in the Proof to Proposition 2 (see A.7), we can write Ω_τ as

$$\Omega_\tau = k'_\tau \left[\left(\frac{\tau}{p+\gamma}\right)^{\frac{\beta}{1-\beta}}\left(\frac{\Delta}{\beta} - \tau\right) + \delta\right]\int_{\underline{\theta}}^{\overline{\theta}} (\theta - \tilde{\theta})^2 f(z) g(\theta) d\theta, \tag{A.12}$$

where we know that by $p > 0$, $k'_\tau > 0$. Hence, $\tau \leq \Delta$ would imply $\Omega_\tau > 0$, and we require $\Gamma_\tau < 0$. Yet by (A.2), $p > 0$ and $\tau \leq \Delta$ imply $\Gamma_\tau > 0$; a contradiction.

Hence, we must have $\Delta > \tau$ and, by (A.1), $\Gamma_\tau + \Omega_\tau > 0$. Now suppose $\Omega_\tau \leq 0$. By (A.12), this requires $\Delta/\beta \leq \tau$. Next note that by (A.2) and $p \leq p_M$, we know

$$\Gamma_\tau \leq \left[\frac{\Delta}{\beta} - \tau\right] \frac{\beta}{(1-\beta)\tau} \left(\frac{\tau}{p+\gamma}\right)^{\frac{\beta}{1-\beta}} \int_{\underline{\theta}}^{\overline{\theta}} \theta F(z) g(\theta) d\theta, \tag{A.13}$$

which implies $\Gamma_\tau \leq 0$ for $\Delta/\beta \leq \tau$ and $\Gamma_\tau + \Omega_\tau \leq 0$: a contradiction. From here it follows that $\Omega_\tau > 0$ and thus $\Omega_\tau^s > 0$. □

A.6 Proof of Proposition 6

The policy maker makes a take-it-or-leave-it offer T, which innovators accept and sell if:

$$T \geq k\theta,$$

where we use the definition $k = p\big(\tau/(p+\gamma)\big)^{\frac{1}{1-\beta}}$. It follows that innovators with $\theta \leq \theta^*$ sell for T and the government gives the license out for free, $p = 0$. The cut-off type is

$$\theta^* = \frac{T}{k}, \tag{A.14}$$

and innovators with $\theta > \theta^*$ have $p > 0$. In the following analysis we treat θ^*, rather than T, as the choice variable and show the arguments of $v(\tau, p)$ to make the distinction between innovations bought ($p = 0$) and not bought ($p > 0$) clear.[24] We continue to use z as the maximum innovation cost an innovator is willing to incur, where we note that for those innovators that do not sell their innovation, z is still given by (4), whereas for innovators that sell their innovation $z = z^*$ given by

$$z^* = s + T. \tag{A.15}$$

Proof: $\theta^* < \overline{\theta}$. Observe that imposing $p = 0$ on innovators who do not accept the offer implies all innovators accept the offer and thus $\theta^* = \overline{\theta}$. Yet implementing $\theta^* = \overline{\theta}$ by choosing a sufficiently high T is equivalent to setting $p = 0$ with a sufficiently high s. From Proposition 2, this is not optimal: we must have $\theta^* < \overline{\theta}$, with $p > 0$ for innovators who do not sell their license.

Proof: p, τ, Ω_τ if $\theta^* = \underline{\theta}$. If in the optimum, $\theta^* = \underline{\theta}$, no innovator sells its patent, and the proof to Proposition 2 (Appendix A.2) applies.

Proof: p, τ, Ω_τ if $\theta^* \in \big(\underline{\theta}, \overline{\theta}\big)$. If in the optimum, $\theta^* \in \big(\underline{\theta}, \overline{\theta}\big)$ we can write the first-order conditions (8) and (10) as:

$$\begin{aligned}\frac{\partial W}{\partial \tau} =& \frac{\Delta - \tau}{-Y''(E)} + \underbrace{\int_{\underline{\theta}}^{\theta^*} v'_\tau(\tau, 0)\theta F(z^*) g(\theta) d\theta}_{\Gamma_{\tau,p=0}} + \underbrace{\int_{\theta^*}^{\overline{\theta}} v'_\tau(\tau, p)\theta F(z) g(\theta) d\theta}_{\Gamma_{\tau,p>0}} + \\ & \underbrace{\int_{\theta^*}^{\overline{\theta}} [v(\tau, p)\theta - z] z'_\tau) f(z) g(\theta) d\theta}_{\Omega_\tau} = 0,\end{aligned} \tag{A.16}$$

[24] We prove below that the policy maker indeed finds it optimal to set $p > 0$ for the innovators that did not accept the offer T.

and

$$\frac{\partial W}{\partial p} = \underbrace{\int_{\theta^*}^{\overline{\theta}} v_p'(\tau,p)\theta F(z))g(\theta)d(\theta)}_{\Gamma_{p,p>0}} + \underbrace{\int_{\theta^*}^{\overline{\theta}} [v(\tau,p)\theta - z]z_p' f(z)g(\theta)d\theta}_{\Omega_p} = 0. \tag{A.17}$$

The expression above defines the Λ separately for $\theta \leq \theta^*$ innovations (with $p = 0$), and $\theta > \theta^*$ innovations (with $p > 0$). As $p = 0$ implies the innovation decision is independent of θ for $\theta \leq \theta^*$ innovations, the innovation effect Ω includes only by $\theta > \theta^*$ innovations. The Γ_τ, Γ_p, Ω_τ and Ω_p can then be expressed akin to (A.2), (A.4), (A.7) and (A.8), respectively. Similarly, (13) now reads

$$\frac{\partial W}{\partial s} = \int_{\theta^*}^{\overline{\theta}} \left[v(\tau,p)\theta - z(\tau,p,s,\theta)\right] f(z)g(\theta)d\theta = 0. \tag{A.18}$$

The remainder of the proof closely follows the proof to Proposition 2:

Proof: $\tau > \Delta$. Proof by contradiction. Assume $\tau \leq \Delta$. Then $v_\tau'(\tau,0) \geq 0$ and $v_\tau'(\tau,p) > 0$ and $\Gamma_{\tau,p=0} \geq 0$ and $\Gamma_{\tau,p>0} > 0$. In addition, $v_p'(\tau,p) < 0$ and thus $\Gamma_{p,p>0} < 0$. Equations (A.16) and (A.17) then require that $\Omega_\tau < 0$ and $\Omega_p > 0$ which cannot be simultaneously true.

Proof: $\Omega_\tau, \Omega_p > 0$. Proof by contradiction. By $\tau > \Delta$ and (A.16), $\Gamma_{\tau,p=0} + \Gamma_{\tau,p>0} + \Omega_\tau > 0$. Suppose $\Omega_\tau < 0$. Then $\Gamma_{\tau,p=0} + \Gamma_{\tau,p>0} > 0$. Yet this would imply $\Omega_p < 0$ and $\Gamma_{p,p>0} < 0$, which implies (A.17) is not satisfied. Similarly, if $\Omega_\tau = 0$, then (A.16) requires $\Gamma_{\tau,p=0} + \Gamma_{\tau,p>0} > 0$. In turn, this implies $\Omega_p = 0$ and $\Gamma_{p,p>0} < 0$ which is inconsistent with (A.17).

Proof: $\overline{\Omega}_\tau = \overline{\Omega}_p = 0$ and $\Omega_\tau^s, \Omega_p^s > 0$. By (A.18), $\overline{\Omega}_\tau$ and $\overline{\Omega}_p$ are zero (see (9) and (11) for definitions, with the $p > 0$ subscript indicating these expressions are evaluated only for licenses that have not been sold). As $\Omega_\tau, \Omega_p > 0$, by (9) and (11) we must have Ω_τ^s and $\Omega_p^s > 0$.

A.7 Proof of Proposition 7

Under a technology uptake subsidy, demand (2) now reads

$$q = \theta\left(\frac{\tau}{(1-\sigma)p+\gamma}\right)^{\frac{1}{1-\beta}}. \tag{A.19}$$

Similarly, for the maximum innovation cost z and value of innovation $v\theta$ (see eqs. (5) and (7)) we now have

$$z(\tau,p,s,\theta) = s + k\theta, \tag{A.20}$$

with k now given by $k = p\left(\tau/((1-\sigma)p+\gamma)\right)^{\frac{1}{1-\beta}}$, and

$$v(\tau,p)\theta = \Big[\frac{\Delta}{\beta}\Big(\frac{\tau}{(1-\sigma)p+\gamma}\Big)^{\frac{\beta}{1-\beta}} - \gamma\left(\frac{\tau}{(1-\sigma)p+\gamma}\right)^{\frac{1}{1-\beta}} + \delta\Big]\theta. \tag{A.21}$$

As before, we have $k_\tau' > 0$ whenever $\tau > 0$, and $k_p' > 0$ whenever $\tau > 0$ and $p \in [0, p_M)$ with p_M now given by $p_M = \gamma(1-\beta)/(\beta(1-\sigma))$. As it is never optimal for the innovator to set $p > p_M$, we consider only solutions with $p \in [0, p_M]$. From here it again follows that $z_\tau' > 0$ if $\tau > 0$ and $p > 0$, $z_\tau' = 0$ if $\tau = 0$ or $p = 0$, $z_p' > 0$ if $\tau > 0$ and $p \in [0, p_M)$, and $z_p' = 0$ if $\tau = 0$ or $p = p_M$.

The optimal carbon price and license price must satisfy (A.1) and (A.3), with Γ_τ and Γ_p now given by

$$\Gamma_\tau = \left[\Delta - \tau\frac{\gamma}{(1-\sigma)p+\gamma}\right]\frac{1}{(1-\beta)\tau}\left(\frac{\tau}{(1-\sigma)p+\gamma}\right)^{\frac{\beta}{1-\beta}}\int_{\underline{\theta}}^{\overline{\theta}}\theta F(z)g(\theta)d\theta, \tag{A.22}$$

and

$$\Gamma_p = -\left[\Delta - \tau\frac{\gamma}{(1-\sigma)p+\gamma}\right]\frac{1-\sigma}{(1-\beta)(p+\gamma)}\left(\frac{\tau}{(1-\sigma)p+\gamma}\right)^{\frac{\beta}{1-\beta}}\int_{\underline{\theta}}^{\overline{\theta}}\theta F(z)g(\theta)d\theta. \tag{A.23}$$

The optimal subsidy similarly still satisfies (13), which gives $\mathbb{E}[(v\theta - z)f(z)] = 0$, and implies the average innovation effects are zero.

The remainder of the proof closely follows the Proof to Proposition 2, as presented in Appendix A.2. It is straightforward to verify that Ω_τ and Ω_p can again be expressed by (A.6) and (A.7), with k now specified as above.

Proof: $\sigma = 1$**.** If $\sigma = 1$, $\Gamma_p = 0$ and $p_M \to \infty$. (A.3) then requires $\Omega_p = 0$, which by (A.7) requires either $\tau = 0$ or $v = k$. Suppose that $v = k$. Then also $\Omega_\tau = 0$ and by (A.1) and (A.22) the optimal carbon price is given by $\tau = \Delta$. Given $\tau = \Delta$,

$$p = \gamma\frac{1-\beta}{\beta} + \delta\left(\frac{\Delta}{\gamma}\right)^{-\frac{1}{1-\beta}}$$

ensures that $[v - k] = 0$.

Proof: $\sigma < 1$**.** If $\sigma < 1$, $p > 0$, $\tau > \Delta$ and $\Omega^s_\tau, \Omega^s_p > 0$ with the proof following the same steps as the Proof to Proposition 2, as presented in Appendix A.2.

□